validate_model

November 12, 2021

[1]:
```python
import os

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.dates as mdates
import jax
import jax.numpy as jnp
import numpyro

from jax import random
from numpyro.infer import MCMC, NUTS, Predictive, init_to_median
from collections import defaultdict
from scipy.stats import percentileofscore
from sklearn.metrics import mean_absolute_error as mae

from forecasting.time_series_model import gp
from forecasting.forecast import UnivariateScaler
from forecasting.utils import load_timeseries, load_holidays
from forecasting.plotting import ribbon_plot
```

[2]:
```python
%config InlineBackend.figure_format = "retina"

NUM_CPUS = int(os.environ.get("NUM_CPUS", os.cpu_count()))
numpyro.set_host_device_count(NUM_CPUS)

# Set a random seed
rng_key = random.PRNGKey(42)
```

1 Model validation

In order to check the performance of the demand predictor, we need to validate the predictions against historic admissions data. We can do this by training the model on historic patient admissions data, for example on the 120 days between 01/01/2021 and 30/04/2021. We then generate a forecast for the next 7 days, in this example between 01/05/2021 and 07/05/2021, and compare it to historic data from that time period. We can look at where the admitted number of patients for each hour lies compared to different confidence intervals and check that the correct amount of historic data lies within each confidence interval, for example 50% of the actual patients that arrived should fall within the 50% confidence interval predicted by the model.

[3]:
```python
# Load holiday data
HOLIDAYS = load_holidays()
```

[4]:
```python
# Validation class
class Validation:
    COVID_START_DATE = pd.to_datetime("2020-Mar-20")

    def __init__(self, mcmc, training_hours=2880, forecast_hours=168):
        self.training_hours = training_hours
        self.forecast_hours = forecast_hours
        self.timeseries = load_timeseries()
        self.mcmc = mcmc
        # These are instantiated at training time
        self.x_scaler = None
        self.L = None
        self.training_start_date = None

    def run(self, rng_key, dates):
        subkeys = jax.random.split(rng_key, num=len(dates))
        results = {}
        for key, date in zip(subkeys, dates):
            y_train, y_test = self.split(date)
            training_data = self.prepare_data_dictionary(y_train,␣
↪is_training=True)
            forecast_data = self.prepare_data_dictionary(y_test,␣
↪is_training=False)
            self.mcmc.run(key, **training_data)
            prediction = self.predict(key, **forecast_data)
            results[date] = (y_test, prediction, mcmc.get_samples())
        return results

    def predict(self, rng_key, *args, **kwargs):
        predictive = Predictive(self.mcmc.sampler.model, posterior_samples=self.
↪mcmc.get_samples())
        prediction = predictive(rng_key, *args, **kwargs)
        return prediction["y"]

    def split(self, date, validate=True):
        self._validate_split_date(date)
        past_datetimes = self.timeseries.loc[:date].index
        future_datetimes = self.timeseries.index.difference(past_datetimes)
        training_datetimes = past_datetimes[-self.training_hours:]
        forecast_datetimes = future_datetimes[:self.forecast_hours]
```

```
        y_train = self.timeseries.loc[training_datetimes]
        y_test = self.timeseries.loc[forecast_datetimes]
        return y_train, y_test

    def prepare_data_dictionary(self, y, is_training=True):
        if is_training:
            # Reset the scaler in each training, re-use if validating
            self.x_scaler = UnivariateScaler()
            self.training_start_date = y.index.min()

        x = (y.index - self.training_start_date) / pd.Timedelta("1H")
        xsd = self.x_scaler.fit_transform(x)

        if is_training:
            self.L = 1.5 * max(xsd)

        day_of_week = y.index.day_of_week
        hour_of_day = y.index.hour
        is_holiday = [d.date() in HOLIDAYS.date for d in y.index]

        return {
            "y": jnp.array(y.values) if is_training else None,
            "x": jnp.array(xsd),
            "day_of_week": jnp.array(day_of_week),
            "hour_of_day": jnp.array(hour_of_day),
            "is_holiday": jnp.array(is_holiday),
            "L": self.L,
            "M": 10,
        }

    def _validate_split_date(self, date):
        """
        Do not split if Covid is very recently
        in the past or in the very near future.
        """
        if date >= self.COVID_START_DATE:
            dt_past = (date - self.COVID_START_DATE) / pd.Timedelta("1H")
            dt_future = (self.timeseries.index.max() - date) / pd.
↪Timedelta("1H")

            msg1 = (
                f"At least {self.training_hours} training hours are required"
                f" but it's only been {dt_past} hours since Covid started"
            )
            msg2 = (
                f"At least {self.forecast_hours} validation hours are required"
                f" but there's only {dt_future} hours left in the future."
```

```
            )
            assert (dt_past > self.training_hours), msg1
            assert (dt_future > self.forecast_hours), msg2
        else:
            dt_future = (self.COVID_START_DATE - date) / pd.Timedelta("1H")
            dt_past = (date - self.timeseries.index.min()) / pd.Timedelta("1H")
            msg1 = (
                f"At least {self.forecast_hours} validation hours are required"
                f" but there's only {dt_future} hours until Covid starts."
            )
            msg2 = (
                f"At least {self.training_hours} training hours are required"
                f" but it's only been {dt_past} hours since data collection␣
↪started."
            )
            assert (dt_future > self.forecast_hours), msg1
            assert (dt_past > self.training_hours), msg2
```

[5]:
```
# Initialise model
mcmc = MCMC(NUTS(gp, init_strategy=init_to_median), num_warmup=2000,␣
↪num_samples=2000, num_chains=4)
```

[6]:
```
# Initalise class instance
cv = Validation(mcmc)
```

2 Select validation dates

We chose 20 random dates during the period that we have data for (01/07/2016 - 01/07/2021) to perform validation on. For each of these 20 dates, we checked that there was sufficient data before that date to train the model (120 days) and sufficient data after that date to validate the model (7 days). We also checked for each of the dates the training and validation periods did not overlap with the start of the COVID-19 pandemic (defined for this purpose as 20/03/2020). The reason that we did not extensively test this model's performance over the pandemic period is that this model was not designed to predict the sudden drop in planned admissions and rise of COVID-19 admissions caused by the pandemic. For modelling admissions during COVID-19 a different functional form is recommended, such as the epidemiologically inspired approach adopted in the Early Warning System.

[7]:
```
# Select 20 random dates to perform validation
selected_dates = pd.to_datetime(sorted(np.random.choice(cv.timeseries.index,␣
↪size=20, replace=False)))
selected_dates = list(selected_dates)
```

[8]:
```
selected_dates
```

[8]: [Timestamp('2016-12-10 14:00:00'),
 Timestamp('2017-03-29 19:00:00'),
 Timestamp('2017-05-13 02:00:00'),
 Timestamp('2017-06-18 11:00:00'),
 Timestamp('2017-09-27 10:00:00'),
 Timestamp('2017-12-27 18:00:00'),
 Timestamp('2017-12-31 22:00:00'),
 Timestamp('2018-06-02 22:00:00'),
 Timestamp('2018-06-25 04:00:00'),
 Timestamp('2018-11-19 03:00:00'),
 Timestamp('2019-03-16 21:00:00'),
 Timestamp('2020-01-06 08:00:00'),
 Timestamp('2020-10-08 15:00:00'),
 Timestamp('2020-11-06 09:00:00'),
 Timestamp('2021-02-16 05:00:00'),
 Timestamp('2021-03-17 11:00:00'),
 Timestamp('2021-04-15 11:00:00'),
 Timestamp('2021-05-04 19:00:00'),
 Timestamp('2021-05-05 15:00:00'),
 Timestamp('2021-05-06 02:00:00')]

[9]:
```
# Test if any of these dates are too close to start of COVID
validation_dates = []
for date in selected_dates:
    try:
        cv.split(date)
    except AssertionError:
        print(f"skipping {date}")
        continue
    else:
        validation_dates.append(date)
```

[10]:
```
validation_dates
```

[10]: [Timestamp('2016-12-10 14:00:00'),
 Timestamp('2017-03-29 19:00:00'),
 Timestamp('2017-05-13 02:00:00'),
 Timestamp('2017-06-18 11:00:00'),
 Timestamp('2017-09-27 10:00:00'),
 Timestamp('2017-12-27 18:00:00'),
 Timestamp('2017-12-31 22:00:00'),
 Timestamp('2018-06-02 22:00:00'),
 Timestamp('2018-06-25 04:00:00'),
 Timestamp('2018-11-19 03:00:00'),
 Timestamp('2019-03-16 21:00:00'),
 Timestamp('2020-01-06 08:00:00'),
 Timestamp('2020-10-08 15:00:00'),

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 Timestamp('2020-11-06 09:00:00'),
 Timestamp('2021-02-16 05:00:00'),
 Timestamp('2021-03-17 11:00:00'),
 Timestamp('2021-04-15 11:00:00'),
 Timestamp('2021-05-04 19:00:00'),
 Timestamp('2021-05-05 15:00:00'),
 Timestamp('2021-05-06 02:00:00')]
```

3 Run validation

[11]:
```python
# Set random key generator
cv_rng_key = jax.random.PRNGKey(1)
```

[12]:
```python
# Run validation
result = cv.run(cv_rng_key, validation_dates)
```

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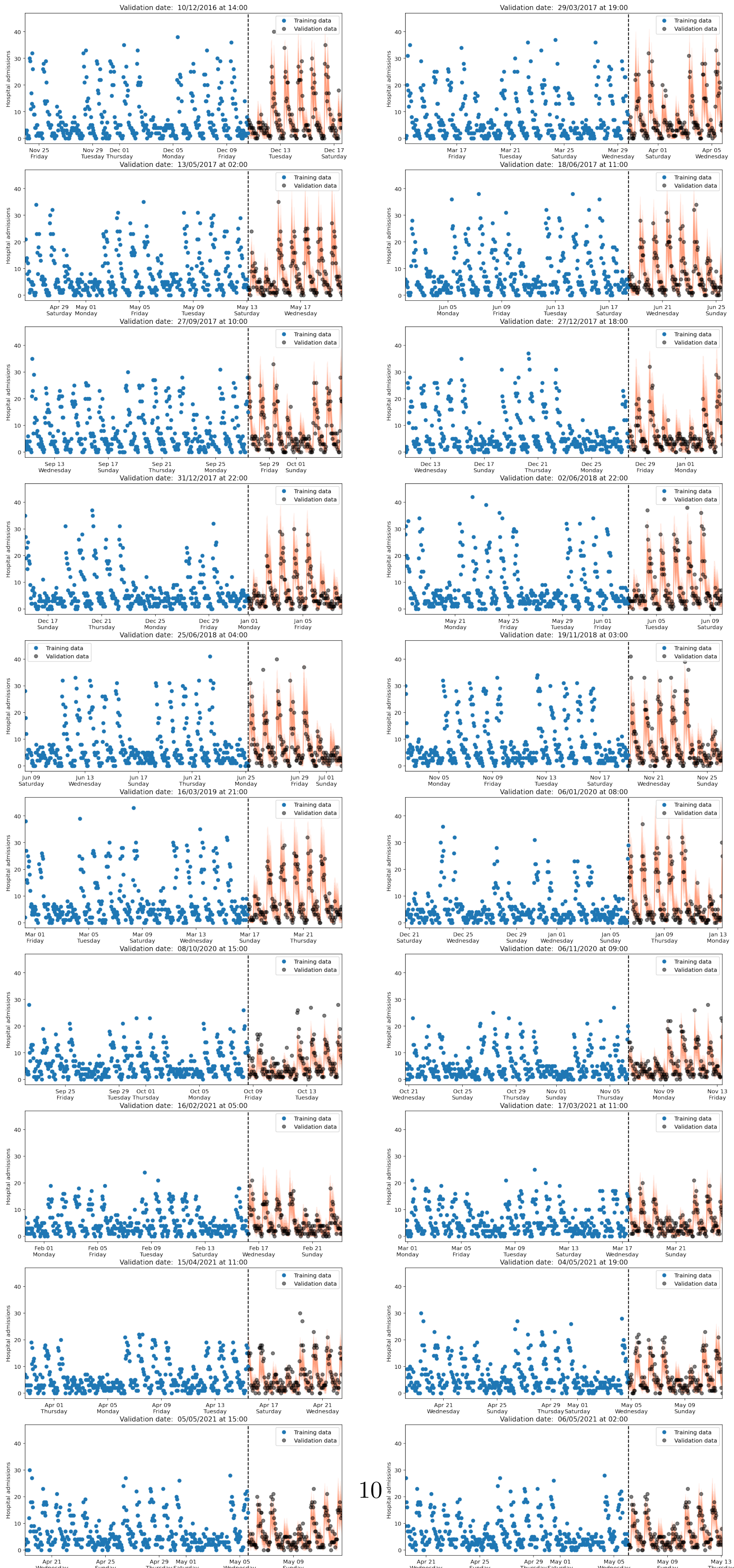
```

4 Plot forecast for validation dates

The figure below shows the validation plots on different dates, with the date for each plot given in the title and shown in the plot as a black dashed line. In each of these plots, the blue points show a subset of the 120 days worth of training data before each date, and the grey points are 7 days worth of validation data after each date, with the training and validation data both having come from the data provided to us by KGH. The coral bands then show the 95% confidence intervals for the model. The day of the week and hour of the day effects are evident within these plots, as you can see the drop in admissions on weekends and during the night. The effect of the long term trend is also evident, as the forecast reflects the drop in admissions after COVID compared to before.

[13]:
```python
# Plot forecast with validation data for each date
ax_date_format = mdates.DateFormatter('%b %d\n%A')
f, axes = plt.subplots(len(validation_dates) // 2, 2, figsize=(22,␣
↪len(validation_dates) * 5 // 2))
for ax, date in zip(axes.flatten(), validation_dates):
    y_train, _  = cv.split(date, validate=False)
    y, forecast, _ = result[date]
    ax.plot(y_train.iloc[-400:], marker='o', lw=0, label='Training data')
    ax.plot(y, marker='o', lw=0, color='k', alpha=0.5, label='Validation data')
    ax.vlines(date, -2, 47, color='black', linestyles='dashed')
    ribbon_plot(y.index, forecast, plot_median=False, ax=ax,␣
↪ribbon_color='coral')
    ax.xaxis.set_major_formatter(ax_date_format)
    ax.set_title(f"Validation date: {date: %d/%m/%Y at %H:%M}")
    ax.legend()
    ax.set_ylim([-2, 47])
    ax.set_xlim([min(y_train.iloc[-400:].index), max(y.index)])
    ax.set_ylabel("Hospital admissions")
```

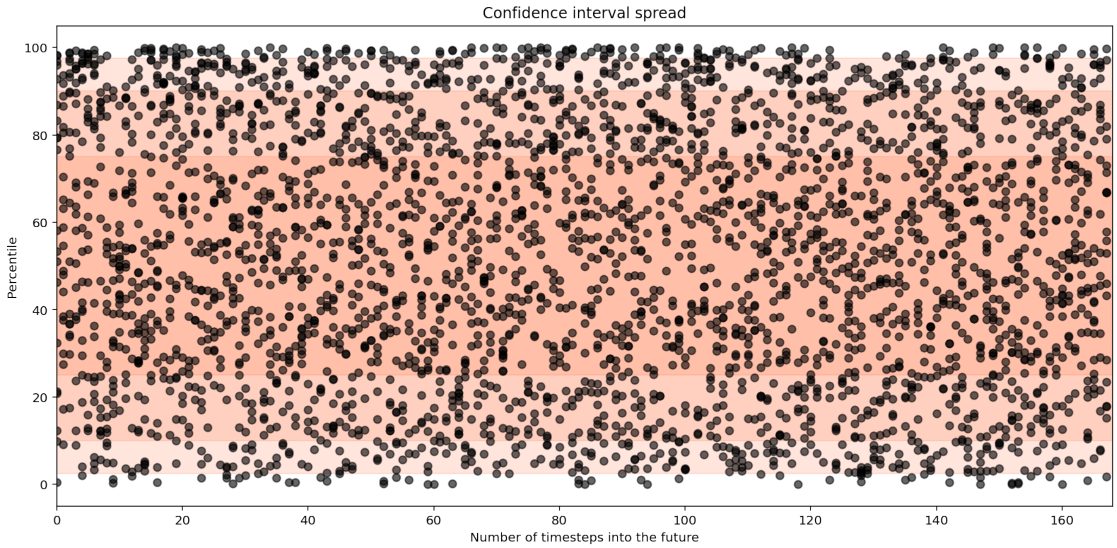

Validation date: 10/12/2016 at 14:00
Validation date: 29/03/2017 at 19:00
Validation date: 13/05/2017 at 02:00
Validation date: 18/06/2017 at 11:00
Validation date: 27/09/2017 at 10:00
Validation date: 27/12/2017 at 18:00
Validation date: 31/12/2017 at 22:00
Validation date: 02/06/2018 at 22:00
Validation date: 25/06/2018 at 04:00
Validation date: 19/11/2018 at 03:00
Validation date: 16/03/2019 at 21:00
Validation date: 06/01/2020 at 08:00
Validation date: 08/10/2020 at 15:00
Validation date: 06/11/2020 at 09:00
Validation date: 16/02/2021 at 05:00
Validation date: 17/03/2021 at 11:00
Validation date: 15/04/2021 at 11:00
Validation date: 04/05/2021 at 19:00
Validation date: 05/05/2021 at 15:00
Validation date: 06/05/2021 at 02:00
Training data
Validation data
Hospital admissions

5 Model calibration

To get a measure of how well our model is performing, we can generate plots to show where the validation data points land relative to the confidence intervals from the forecast, such as the one in the figure below. For example, if a validation data point was right in the centre of the forecast band it would be placed on percentile 50 in this plot. The x axis then shows the time in hours between the validation data point and the date the validation was performed on. For a perfect model, 50% of the points would lie within the darkest coral band, 80% within the next band and then 95% within the lightest band. The figure at the bottom shows the results of this test - 48%, 78% and 94% of the validation points fall within the 50%, 80% and 95% confidence intervals respectively. In summary, the results of these tests show that our demand predictor is incredibly accurate and provides the users with a reliable indication of incoming patient demand.

[14]:
```
# Find where each validation data point lies compared to percentiles from␣
 ↪forecast
percentiles = defaultdict(list)
for y, forecast, _ in result.values():
    for n, val in enumerate(y.values):
        percentiles[n].append(percentileofscore(forecast[:, n], val))
```

[15]:
```
# Plots validation data points together compared to which percentile they on
plt.figure(figsize=(15, 7))
for n, ps in percentiles.items():
    plt.plot([n] * len(ps), ps, marker='o', lw=0, color='k', alpha=0.6)
plt.axhspan(2.5, 97.5, alpha=0.2, color='coral')
plt.axhspan(10, 90, alpha=0.2, color='coral')
plt.axhspan(25, 75, alpha=0.2, color='coral')
plt.title("Confidence interval spread")
plt.xlabel("Number of timesteps into the future")
plt.ylabel("Percentile")
plt.xlim([0, 168])
plt.show()
```

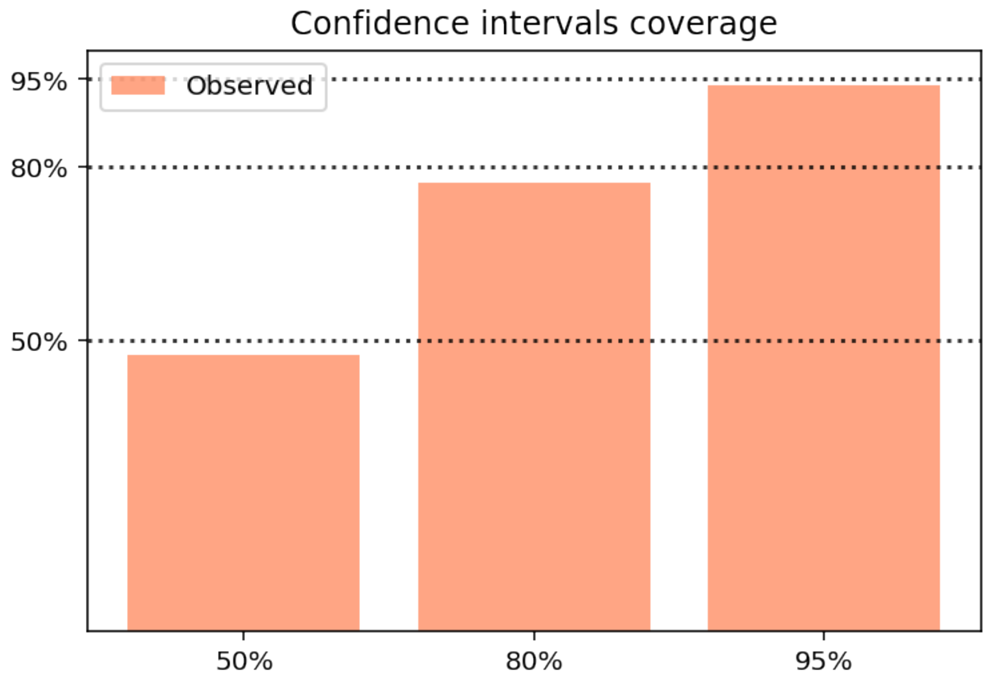


[16]:
```
# For each calibration data points, checks whether it is within the 50%, 80%
↪and 95% confidence intervals
calibration = {50: [], 80: [], 95: []}
for n, ps in percentiles.items():
    calibration[50].extend([25 <= val <= 75 for val in ps])
    calibration[80].extend([10 <= val <= 90 for val in ps])
    calibration[95].extend([2.5 <= val <= 97.5 for val in ps])
```

[17]:
```
# Percentage of validation data points within 50%, 80% and 95% confidence
↪intervals respectively
sum(calibration[50]) / len(calibration[50]), sum(calibration[80]) /
↪len(calibration[80]), sum(calibration[95]) / len(calibration[95])
```

[17]: (0.475297619047619, 0.7708333333333334, 0.9389880952380952)

[18]:
```
# Plots amount of validation data points within 50%, 80% and 95% confidence
↪intervals
plt.title("Confidence intervals coverage")
plt.bar([1, 2, 3], [np.mean(c) for c in calibration.values()],
↪label='Observed', color='coral', alpha=0.7)
plt.xticks([1, 2, 3], ["50%", "80%", "95%"])
plt.yticks([0.50, 0.80, 0.95], ["50%", "80%", "95%"])
for p in [50, 80, 95]:
    plt.axhline(y=p/100, linestyle=':', color='k', alpha=0.8)
plt.ylim([0, 1])
plt.legend()
plt.show()
```


Confidence intervals coverage
Observed
95%
80%
50%
50%
80%
95%