Assignment 2 - Divide & Conqur
COMP 251 - Algorithms and Data Structures
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Part 1: Programming exercise

1. (50 points)

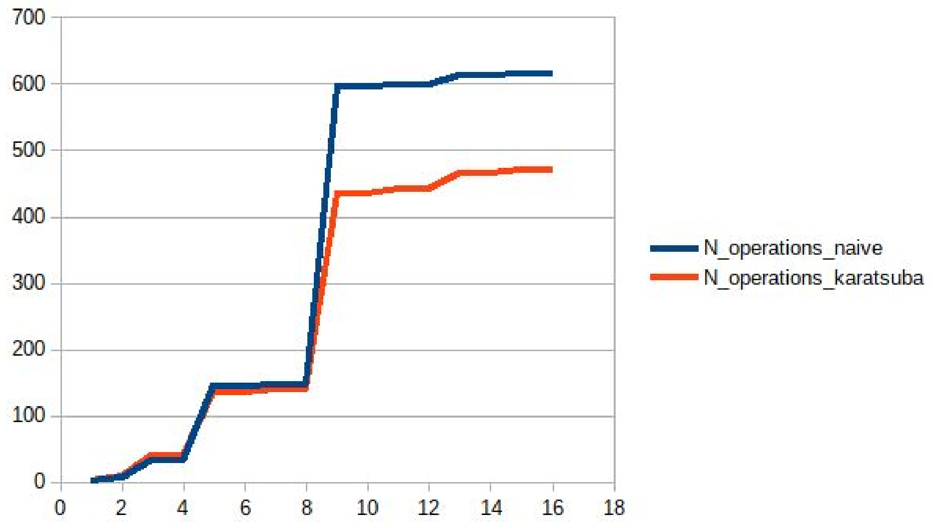

Figure 1: Comparison of costs of naive and Karatsuba multiplication, size 1 to 16

We observe no stark contrast between the pair of methods' costs when the integer size is lower than 8. Karatsuba outperforms the naive method in some intervals, but costs more in others.

However, with larger sized integers, clearer differences reveal themselves.

Starting from size = 8, a clear and maintained gap between the cost of Karatsuba and that of naive multiplication shows itself. The naive method rises above the haters, but at what cost?

A higher one than Karatsuba's, leaving the latter the overall superior performer.

Part 2: Master theorem

2. (25 points) Apply the master theorem.

Master theorem:

If

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$
$$f(n) = n^d \log^p n$$
$$a \geq 1,\ b > 1,\ f(n) > 0$$

then

$$T(n) = \begin{cases} \Theta(n^d log^p n) & \text{if} \quad d > \log_b a,\ p \geq 0 \\ \Theta(n^d) & \text{if} \quad d > \log_b a,\ p < 0 \\ \Theta(n^d \log^{p+1} n) & \text{if} \quad d = \log_b a,\ p > -1 \\ \Theta(n^d \log\log n) & \text{if} \quad d = \log_b a,\ p = -1 \\ \Theta(n^d) & \text{if} \quad d = \log_b a,\ p < -1 \\ \Theta(n^{\log_b a}) & \text{if} \quad d < \log_b a \end{cases}$$

(a) (5 points) T(n) = 25 · T(n/5) + n

$$\log_5 25 = 2$$
$$d = 1,\ d < 2$$
$$\text{Therefore, } T(n) = \Theta(n^2)$$

(b) (5 points) T(n) = 2 · T(n/3) + n · log(n)

$$d = 1,\ > log_3 2$$
$$p = 1,\ \geq 0$$
$$\text{Therefore, } T(n) = \Theta(n \log n)$$

(c) (5 points) T(n) = T(3n/4) + 1

$$d = 0,\ = log_{\frac{4}{3}} 1$$
$$p = 0,\ > -1$$
$$\text{Therefore, } T(n) = \Theta(\log n)$$

(d) (5 points) T(n) = 7 · T(n/3) + n^3

$$d = 3,\ > \log_3 7$$
$$p = 0,\ \geq 0$$
$$\text{Therefore, } T(n) = \Theta(n^3)$$

(e) (5 points) T(n) = T(n/2) + n(2 − cos n)

Non-applicable. The regularity condition is violated.

3. (25 points)

$$T_A(n) = 7T_A\left(\frac{n}{2}\right) + n^2$$
$$T_B(n) = \alpha T_B\left(\frac{n}{4}\right) + n^2$$

Find largest integer α such that algorithm B is asymptotically faster than A.

From master theorem,

$$T_A(n) = \Theta(n^{\log_2 7}) = \Theta(n^{2.807\ldots}).$$

Want

$$T_B(n) < \Theta(n^{\log_2 7})$$

Which boils down to

$$\log_2 7 > \log_4 \alpha$$

$$\log_2 7 > \log_4 \alpha \Rightarrow 4^{log_2 7} > \alpha \Rightarrow 49 > \alpha \Rightarrow \alpha = 48$$

In conclusion, the largest integer α such that algorithm B is asymptotically faster than A is 48.