Math 240: Discrete Structures I (W18) – Assignment 7

Solutions must typed or very neatly written and uploaded to MyCourses no later than **6 pm** on **Saturday, March 24, 2018**. Up to 4 bonus marks will be awarded for solutions typeset in LaTeX; both the .tex file and .pdf file must be uploaded.

You may use theorems proven or stated in class, but you must state the theorem you are using. All work must be shown for full marks.

[16] 1. **Combinatorics.** Three families have decided to go on vacation together. Family A has 1 adult and 2 children, Family B has 3 adults and 6 children, and Family C has 2 adults and 4 children.

(a) It's time to board the plane! In how many ways can the whole group line up together to present their tickets if every family must stay together?

(b) Five rooms have been reserved for the families at their hotel. In how many ways can the rooms be assigned if no more than 4 people are allowed in a room? What if we require at least one adult in each room?

(c) For each of the five days the families are on vacation, they will be given a stack of 18 towels. Each family will get their towels in a matching colour and no two families have the same colour on the same day. How many different ways could the towels be presented to them over the five days if the hotel carries 7 different colours of towel? What if Family A insists that it does not receive the same colour on consecutive days?

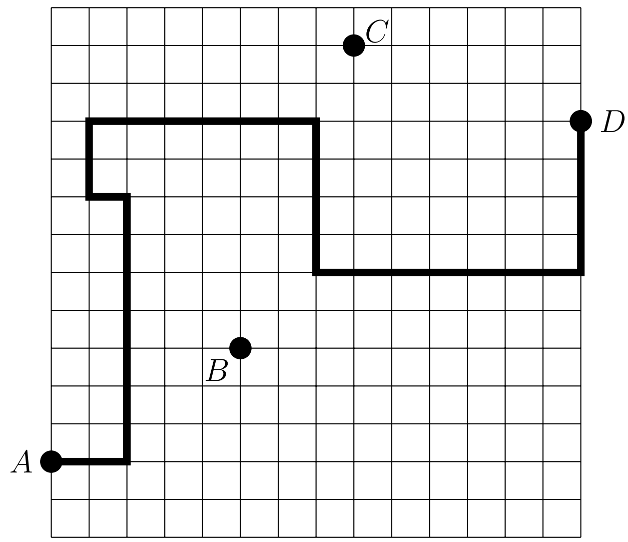
(d) This afternoon, the families have a chance to go on an excursion. In how many ways could a group be formed to go if it must have at least 2 adults and 2 children?

(e) It's dinner time. A large circular table is reserved to seat all 18 vacationers. How many different seating arrangements are there if rotations are ignored? In other words, if everyone seated gets up and moves k seats to the left, it's still considered the same seating arrangement. What if the children from Family B insist on sitting consecutively?

(f) It's time for the stage show! A hypnotist invites 5 of the vacationers onstage to occupy 5 chairs. How many possible seating arrangements are there? What if we insist that at least one member from each family must go on stage?

(g) Time for the vacation photo! Unfortunately, Family C has had a falling out over who was to blame for puncturing the inflatable unicorn in the pool. How many ways can the vacationers line up for the photo if no two members of Family C are next to one another?

[12] 2. **More combinatorics.** Consider the unit grid below (i.e. each square has unit length). A path between two points is a sequence of line segments which follow the grid (an example is shown).



(a) How many paths are there from A to D if you can only move right or up?

(b) How many of the paths from (a) pass through the point B?

(c) How many paths from (a) have the property that every vertical segment has length exactly 1?

(d) How many paths are there from A to D if you never move left and your path never revisits itself?

(e) How many paths from (d) are there which pass through C?

(f) How many paths from (d) have the property that you use exactly 5 horizontal line segments?

(g) How many paths from (f) have the property that no horizontal segment has length 1?

[12] 3. **Proving identities.** Let m be a fixed positive integer, and let n be an arbitrary integer such that $n \geq m$. Prove

$$P(n,m)2^{n-m} = \sum_{k=m}^{n} \binom{n}{k} P(k,m)$$

(a) by a combinatorial argument;

(b) by using the Binomial Theorem.

Two notes on part (b):

- You will need some calculus.
- If you make a claim about something being true for all positive integers, you must prove it.