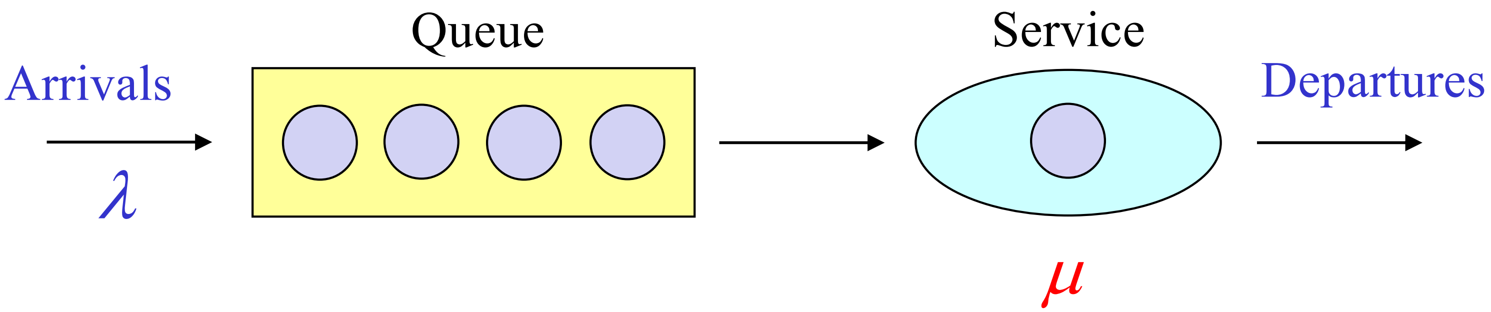
Operations Management

Session 5: First City National Bank

Announcements

- HW1 is due on Thursday January 23rd
- HW2 will be posted on *my*Courses today-due on Thursday January 30th
- Last lecture on Queuing/ Waiting lines
- Next module (3 lectures) on Linear Programming (LP)

Single Server Queue



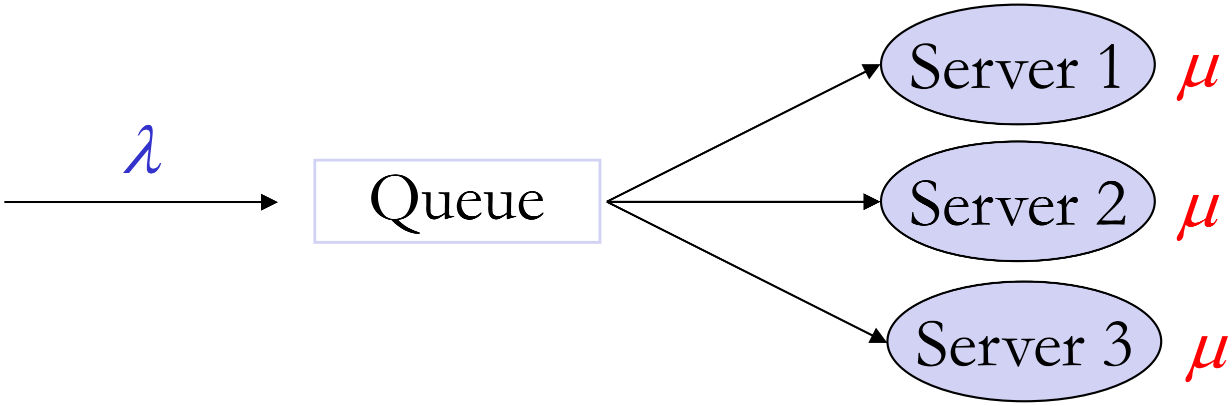
Single Server Queue Model Equations

M/M/1

$\lambda<\mu$

Average number of units in System	$L_S = \dfrac{\lambda}{\mu - \lambda}$
Average Time in System	$W_S = \dfrac{1}{\mu - \lambda}$
Average Number of Units in Queue	$L_Q = \dfrac{\lambda^2}{\mu(\mu - \lambda)}$
Average Time in Queue	$W_Q = \dfrac{\lambda}{\mu(\mu - \lambda)}$
System Utilization	$\rho = \dfrac{\lambda}{\mu}$

Multi-Server Queueing System



Single Queue & Three Identical Server System (Flexible Servers)

Model Equations

M/M/s

$\lambda < s\mu$

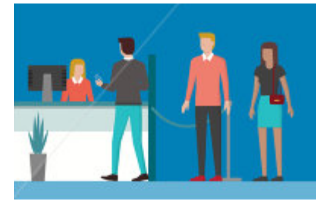
Average number of units in Queue	L_Q: ***from table***
Average Time in Queue	$W_Q = \frac{L_Q}{\lambda}$
Average Number of Units in System	$L_S = \lambda W_S$
Average Time in System	$W_S = \frac{L_S}{\lambda}$
Average Service Time	$\frac{1}{\mu} = W_S - W_Q$
System Utilization	$\rho = \frac{\lambda}{s\mu}$

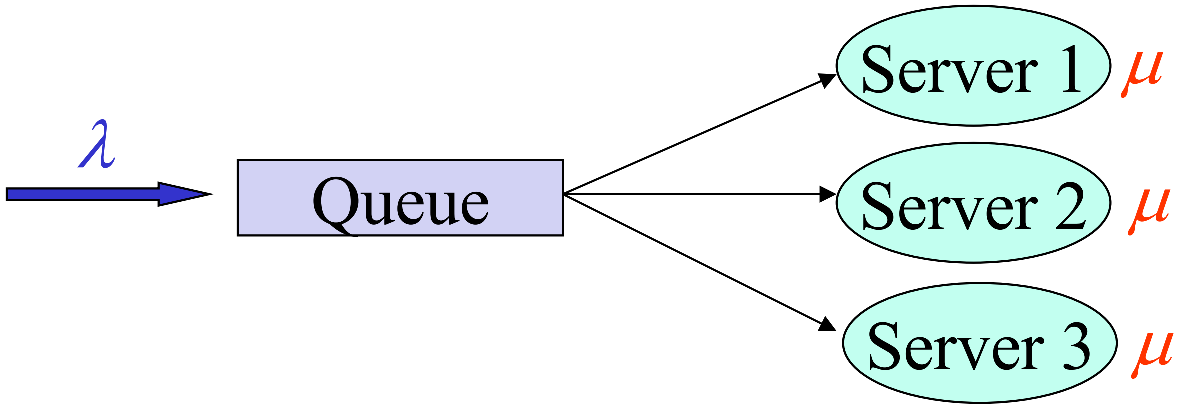
L_Q Table

<u>Example:</u>
Suppose
$\lambda = 1.2/\text{min}$
$\mu = 1/\text{min}$
$s = 2$
$Ws = ?$

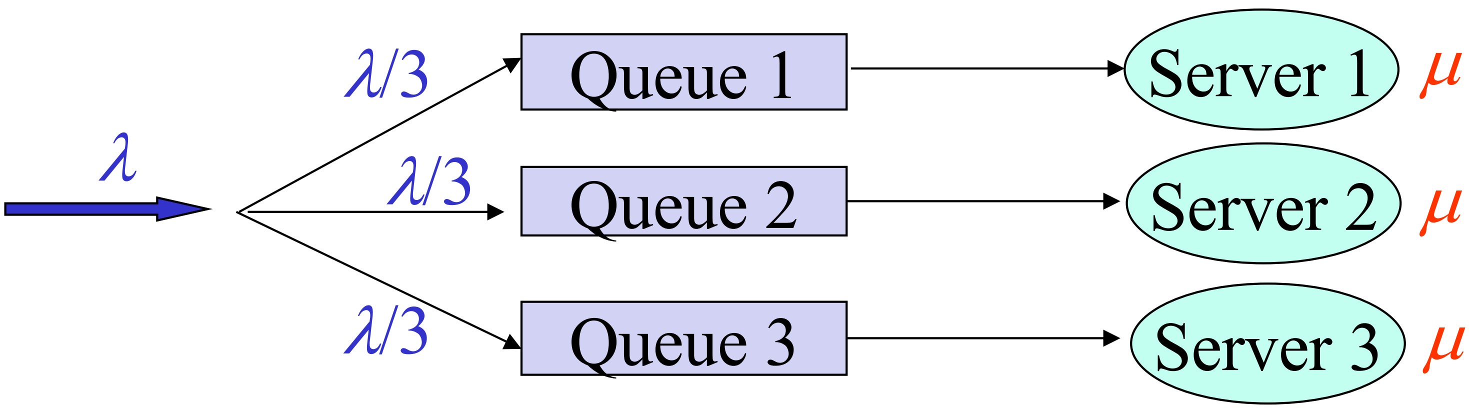
λ/μ	1	2	3	4	5
0.9	8.1000	0.2285	0.0300	0.0041	
0.95	18.0500	0.2767	0.0371	0.0053	
1.0		0.3333	0.0454	0.0067	
1.2		0.6748	0.0904	0.0158	
1.6		2.8444	0.3128	0.0604	0.0121
2.0			0.8888	0.1730	0.0390
2.4			2.1261	0.4205	0.1047
2.8			12.2724	1.0000	0.2411
3.2				2.3855	0.5128
3.6				7.0893	1.0550
4.0					2.2164
4.4					5.2675
4.8					21.6384

Multi-Server Queueing System





System 1: Single Queue & Three Identical Server System (Flexible Servers)

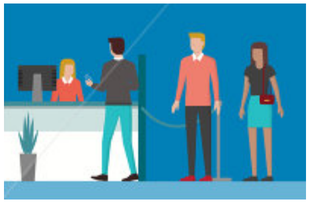


System 2: Three Single-Queue-Single-Server System (Specialized Servers)

Multiple Lines

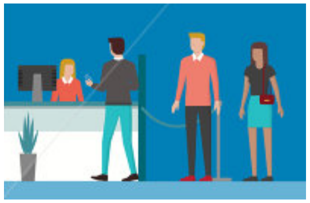
Single Line

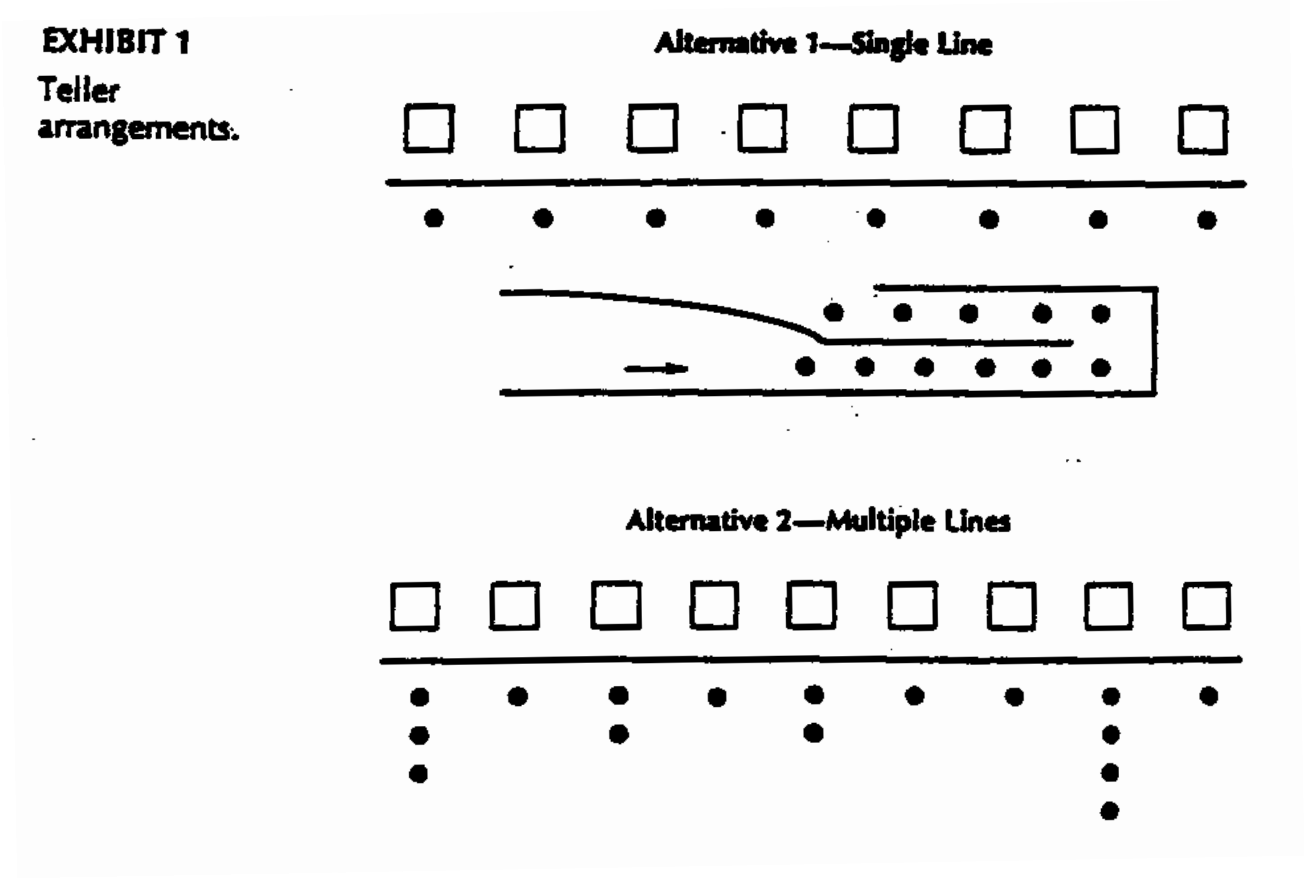
First City National Bank



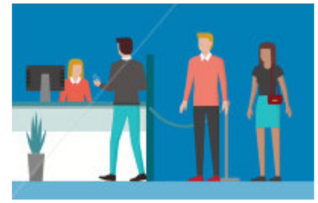
- Verification of model assumptions (using data).
- Practical issues in determining model parameters.
- Finding the number of servers.
- Design issues
  - **Single line or multiple lines?**
- Staffing.

Queueing Options





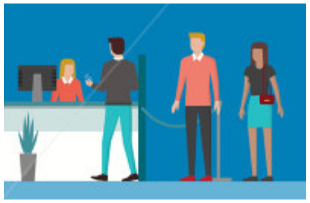
Arrival Rate

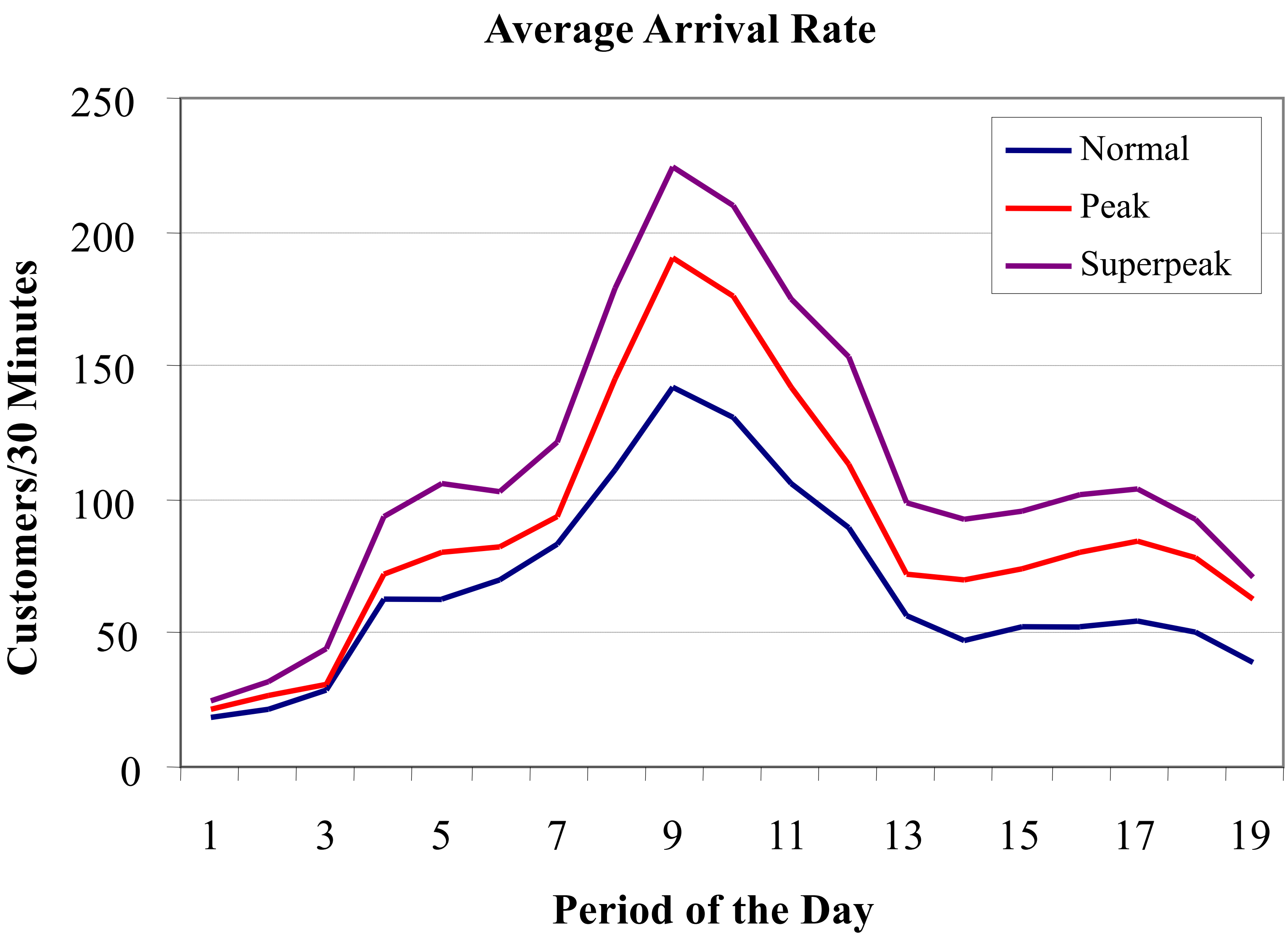


Time of Day	Normal Days: Total Number of Arrivals	Normal Days: Average Arrival Rate*	Peak Days: Total Number of Arrivals	Peak Days: Average Arrival Rate*	Superpeak Days: Total Number of Arrivals	Superpeak Days: Average Arrival Rate*
8-8:30	803	19	625	22	331	25
8:30-9	919	22	758	27	418	32
9-9:30	1207	29	863	31	571	44
9:30-10	2580	63	2033	72	1228	94
10-10:30	2599	63	2237	80	1382	106
10:30-11	2870	70	2283	82	1337	103
11-11:30	3384	83	2625	94	1577	121
11:30-12	4548	111	4060	145	2325	179
12-12:30	5804	142	5329	190	2908	224
12:30-1	5351	131	4923	176	2724	210
1-1:30	4355	106	3983	142	2271	175
1:30-2	3632	89	3150	113	1991	153
2-2:30	2321	57	2012	72	1282	99
2:30-3	1935	47	1960	70	1206	93
3-3:30	2151	52	2064	74	1250	96
3:30-4	2115	52	2238	80	1328	102
4-4:30	2291	55	2340	84	1346	104
4:30-5	2054	50	2191	78	1216	93
5-5:30	1598	39	1763	63	924	71

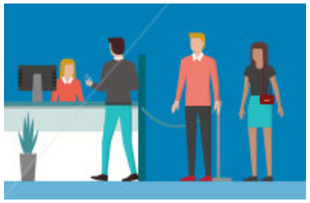
Total normal days = 41, total peak days = 28, total superpeak days = 13.
*The total number of arrivals is divided by the number of days to arrive at the average arrival rate.

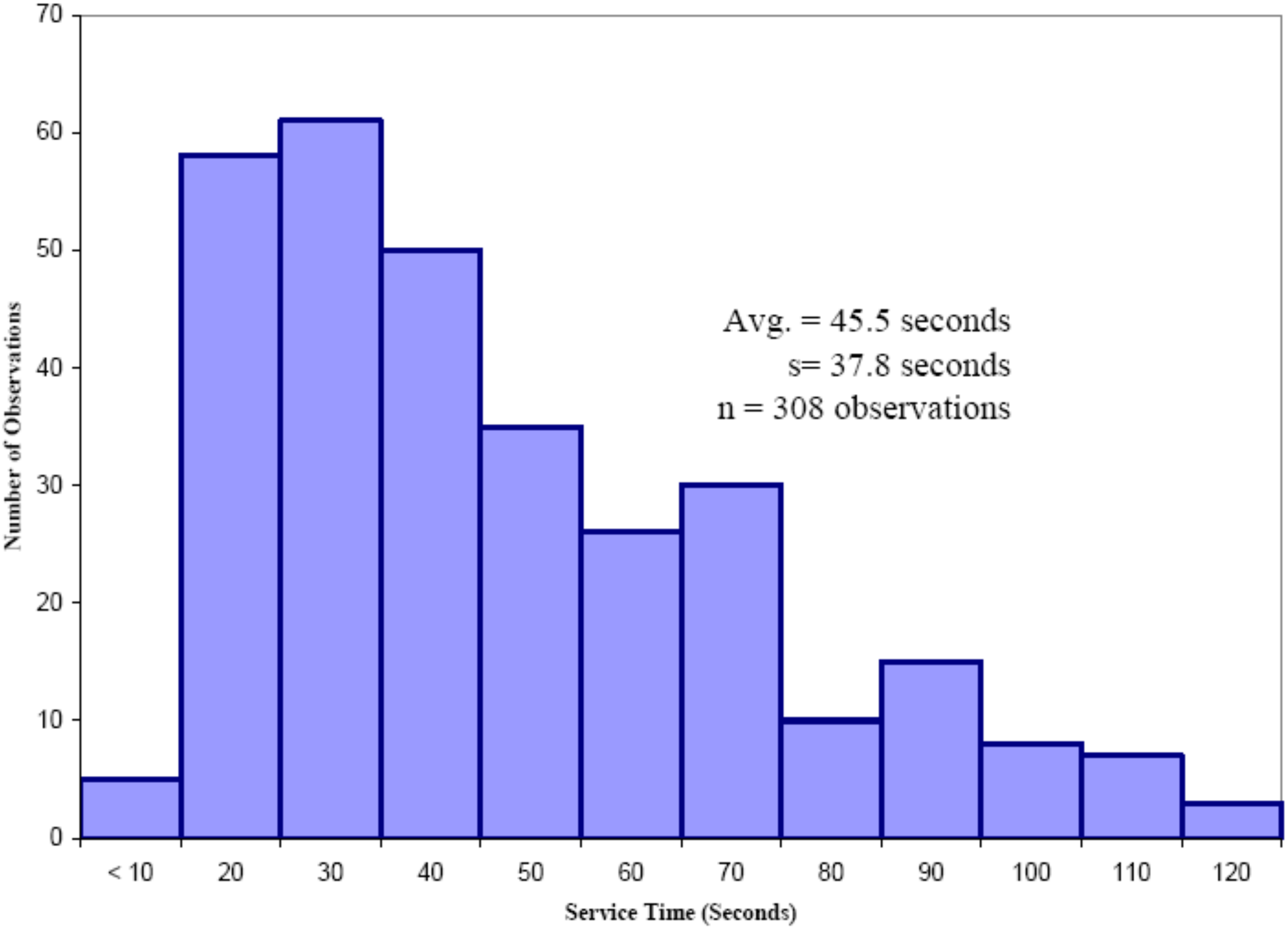
Arrival Rate Patterns



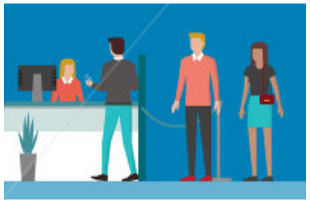


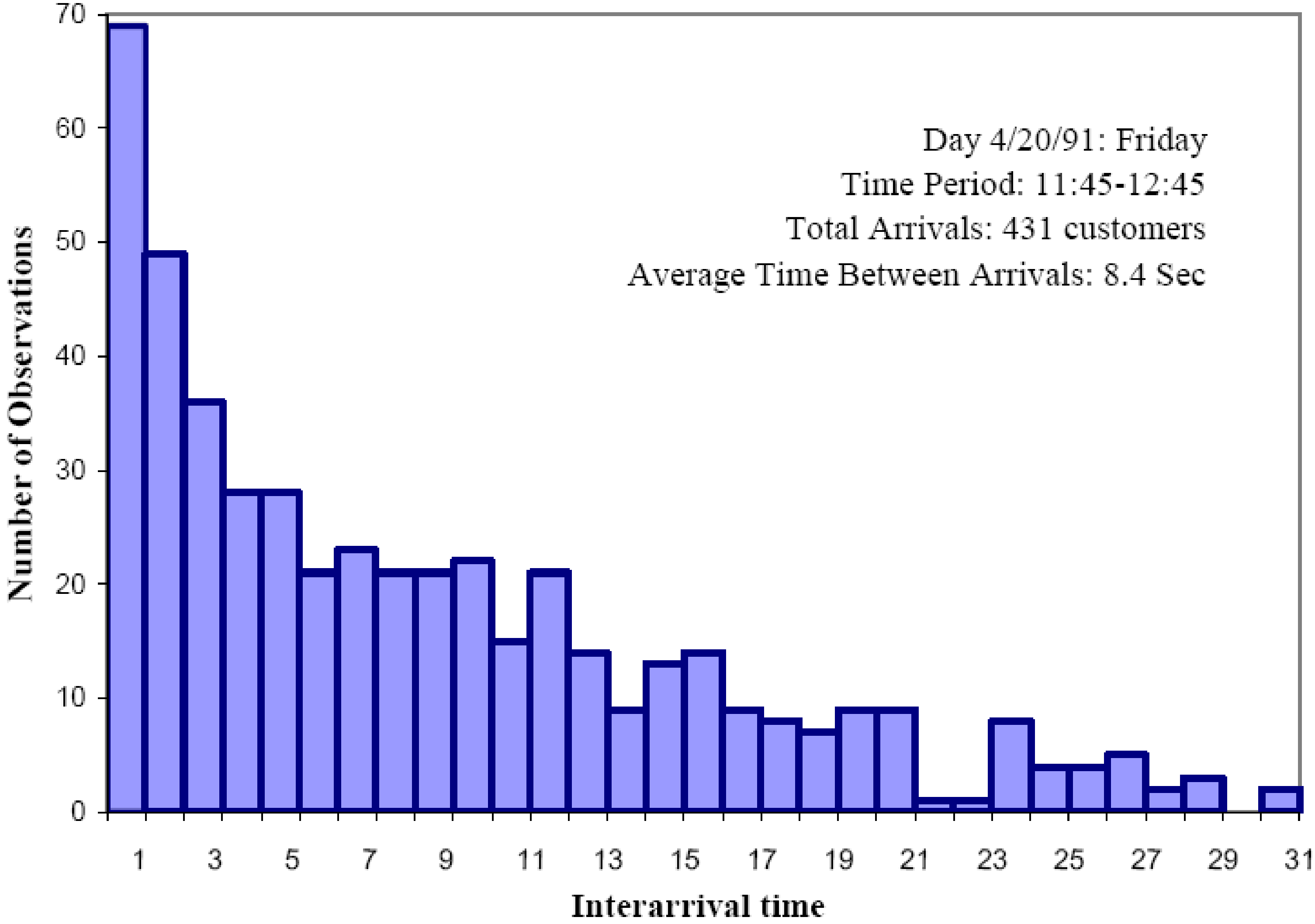
Service Time





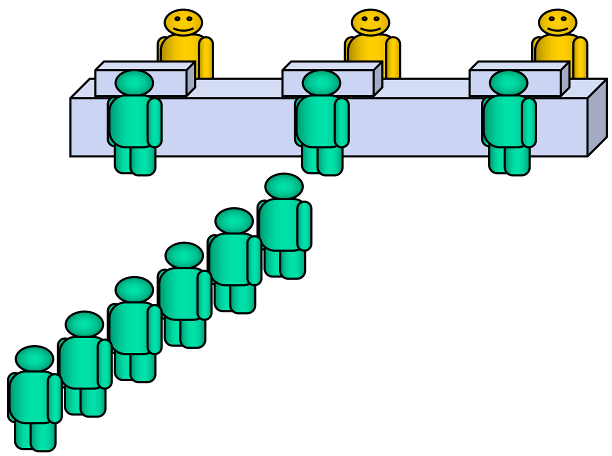
Interarrival Time



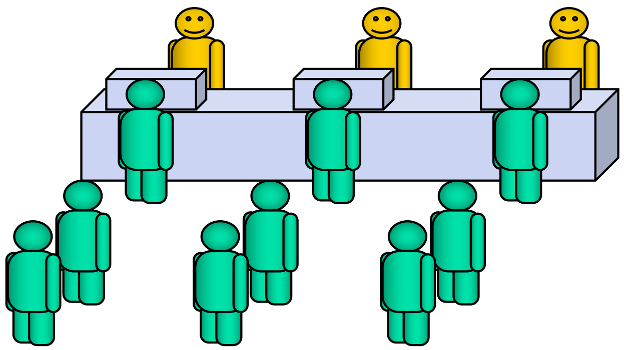


Alternative 1 or 2?

Alternative 1: Common Queue



Alternative 2: Separate Queues

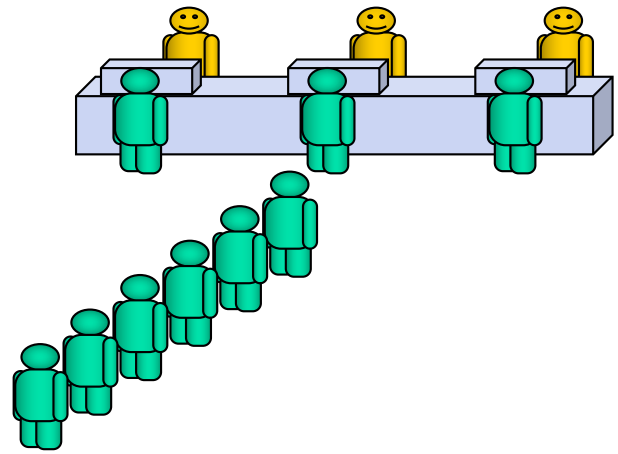


How to Solve the Problem?

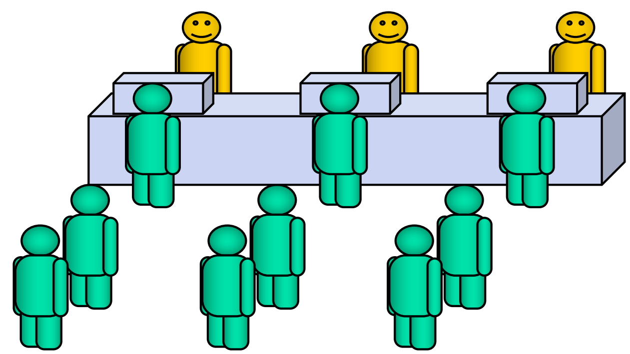
Identify arrival rate and service rate for a given time interval (e.g., normal day 12-12:30).

Use M/M/s Spreadsheet to analyze Alternatives 1 and 2.

Alternative 1: Common Queue



Alternative 2: Separate Queues

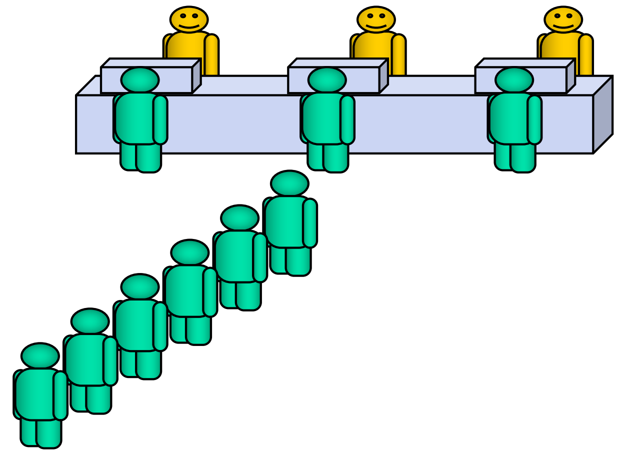


Use M/M/s to Analyze Alternatives 1 and 2

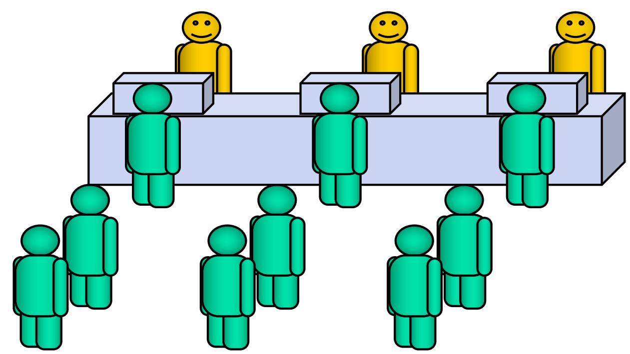
λ : Arrival rate to the system.
μ : Service rate of each server.
s : Number of servers.

$$\lambda_1 = \lambda$$
$$\mu_1 = \mu$$
$$s_1 = s$$

Alternative 1: Common Queue



Alternative 2: Separate Queues



$$\lambda_2 = \boldsymbol{\lambda/s}$$
$$\mu_2 = \mu$$
$$s_2 = 1$$

Example: A Normal Day 12-12:30

Arrival rate =

Service rate of each server =

How many servers?

s =

Alternative 1: Common Queue	**Alternative 2: 4 Separate Queues**
$\lambda =$	$\lambda =$
$\mu =$	$\mu =$
$s =$	$s =$
$\rho =$	$\rho =$
$W_q =$	$W_q =$

Example

Alternative 1: Common Queue

$\lambda = 4.73$ cust/min
$\mu = 1.32$ cust/min
$s = 4$

ρ =
W_Q =

Inputs:

lambda	4.73
mu	1.32

Definitions of terms:

lambda = arrival rate
mu = service rate
s = number of servers
Lq = average number in the queue
Ls = average number in the system
Wq = average wait in the queue
Ws = average wait in the system
P(0) = probability of zero customers in the system
P(delay) = probability that an arriving customer has to wait

Outputs:

s	Lq	Ls	Wq	Ws	P(0)	P(delay)	Utilization
0							
1	infinity	infinity	infinity	infinity	0.0000	1.0000	1.0000
2	infinity	infinity	infinity	infinity	0.0000	1.0000	1.0000
3	infinity	infinity	infinity	infinity	0.0000	1.0000	1.0000
4	6.7024	10.2857	1.4170	2.1746	0.0118	0.7793	0.8958
5	1.0241	4.6074	0.2165	0.9741	0.0233	0.4049	0.7167
6	0.2866	3.8700	0.0606	0.8182	0.0265	0.1933	0.5972
7	0.0886	3.6720	0.0187	0.7763	0.0274	0.0845	0.5119
8	0.0274	3.6108	0.0058	0.7634	0.0277	0.0338	0.4479
9	0.0082	3.5915	0.0017	0.7593	0.0278	0.0124	0.3981
10	0.0023	3.5857	0.0005	0.7581	0.0278	0.0042	0.3583
11	0.0006	3.5840	0.0001	0.7577	0.0278	0.0013	0.3258
12	0.0002	3.5835	0.0000	0.7576	0.0278	0.0004	0.2986

Example

Alternative 2: 4 Separate Queues

$\lambda = 4.73/4 = 1.1825$ cust/min

$\mu = 1.32$ cust/min

$s = 1$

$\rho =$

$W_Q =$

Inputs:

lambda	1.1825
mu	1.32

Definitions of terms:

lambda = arrival rate
mu = service rate
s = number of servers
Lq = average number in the queue
Ls = average number in the system
Wq = average wait in the queue
Ws = average wait in the system
P(0) = probability of zero customers in the system
P(delay) = probability that an arriving customer has to wait

Outputs:

s	Lq	Ls	Wq	Ws	P(0)	P(delay)	Utilization
0							
1	7.7042	8.6000	6.5152	7.2727	0.1042	0.8958	0.8958
2	0.2248	1.1207	0.1901	0.9477	0.3813	0.2771	0.4479
3	0.0295	0.9253	0.0249	0.7825	0.4052	0.0692	0.2986
4	0.0041	0.8999	0.0034	0.7610	0.4079	0.0141	0.2240
5	0.0005	0.8964	0.0004	0.7580	0.4082	0.0024	0.1792
6	0.0001	0.8959	0.0001	0.7576	0.4083	0.0003	0.1493
7	0.0000	0.8958	0.0000	0.7576	0.4083	0.0000	0.1280
8	0.0000	0.8958	0.0000	0.7576	0.4083	0.0000	0.1120
9	0.0000	0.8958	0.0000	0.7576	0.4083	0.0000	0.0995
10	0.0000	0.8958	0.0000	0.7576	0.4083	0.0000	0.0896
11	0.0000	0.8958	0.0000	0.7576	0.4083	0.0000	0.0814
12	0.0000	0.8958	0.0000	0.7576	0.4083	0.0000	0.0747

Example - Recap

Arrival rate = **4.73** cust/min.

Service rate of each server = **1.32** cust/min.

Alternative 1:
Common Queue

$\lambda = 4.73$ cust/min
$\mu = 1.32$ cust/min
$s = 4$

$\rho = 0.896$
$W_Q = 1.417$ min

Alternative 2:
4 Separate Queues

$\lambda = 4.73/4 = 1.1825$ cust/min
$\mu = 1.32$ cust/min
$s = 1$

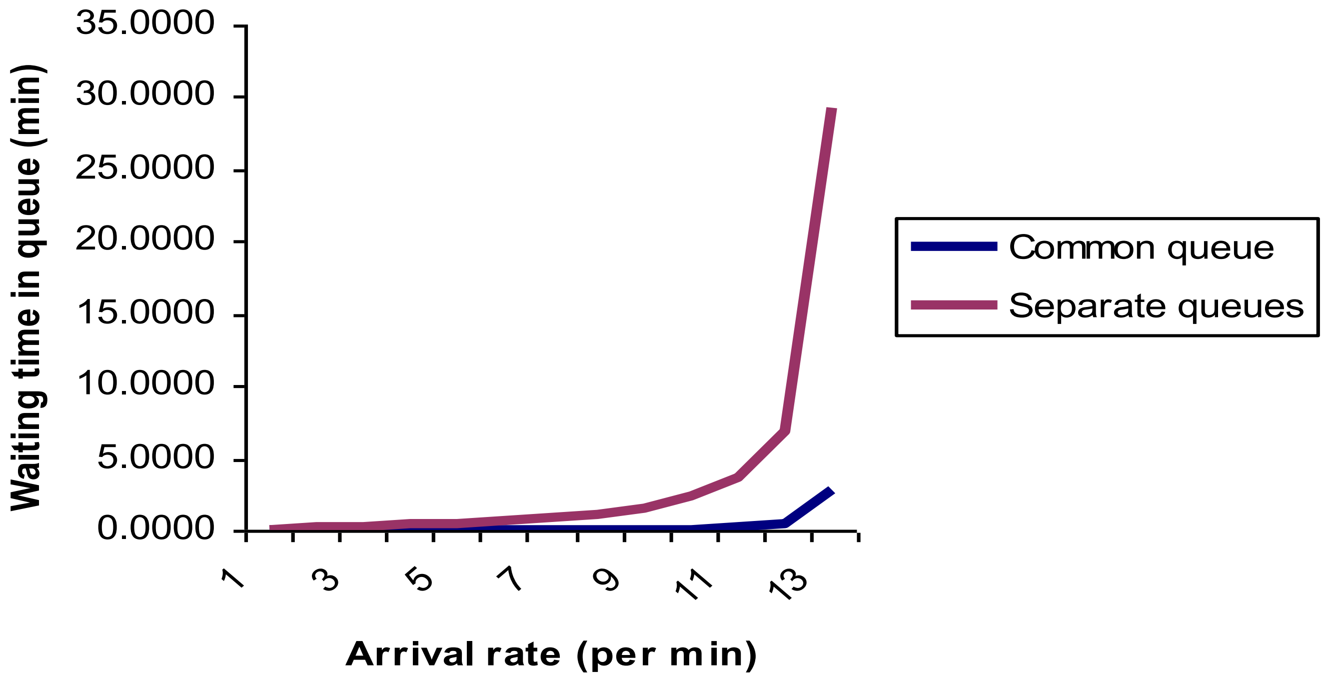
$\rho = 0.896$
$W_Q = 6.515$ min

Observations:

1. Same utilization.
2. **Lower waiting time in the common queue.**

Is it Always True?

- **Input data:** **μ=1.32** and **s=10.**
- Common queue vs. separate queues: waiting time.



Lower waiting time in the common queue!

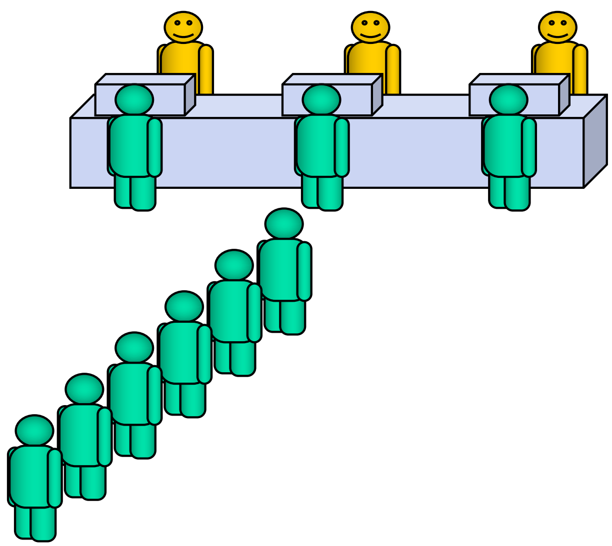
Intuition

Some people are waiting in line, while some servers are idle?

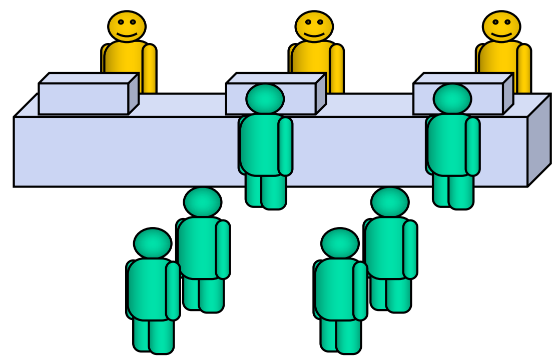
No, impossible

Yes, possible

Alternative 1: Common Queue



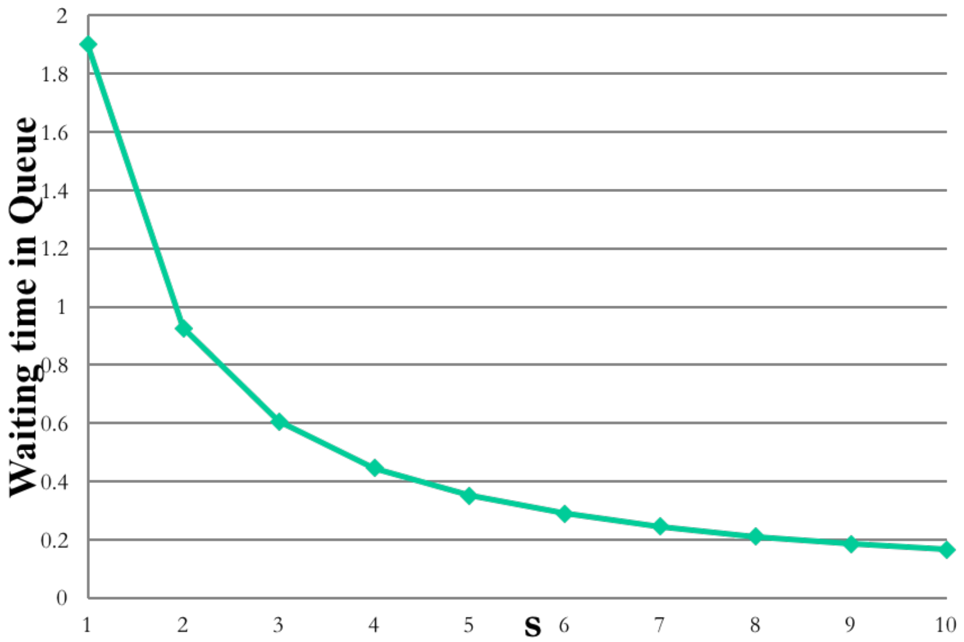
Alternative 2: Separate Queues



<u>Take-away message:</u> **Pooling resources is more efficient.**

Deciding the Number of Servers

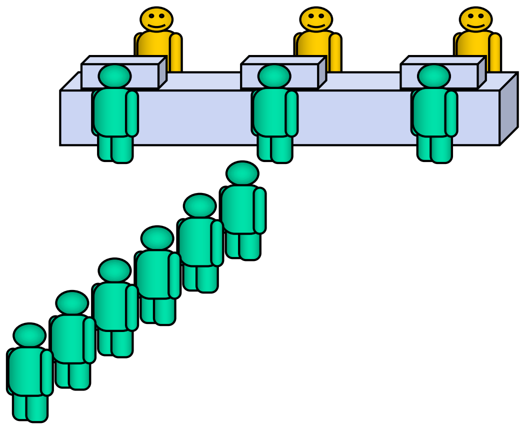
$\rho = 0.95, \mu = 10$



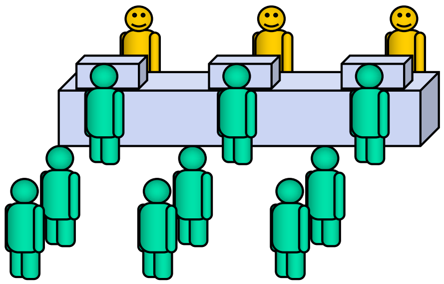
How shall we decide the value of s?

Pro and Con of Common Queue

Alternative 1: Common Queue



Alternative 2: Separate Queues



Pro: Lower waiting time
Con: A longer queue

Any suggestion to this problem?

Whole Foods

- How are the lines arranged?

- How to improve?

Summary

- Waiting lines form due to **variability**.
- Basic tradeoff between cost and quality.
- Decisions:
  - Service capacity: service rate, number of servers.
  - System configuration.
- Performance measures:
  - Under Exponential service times and interarrival times.
  - Use of **M/M/s Spreadsheet** to examine system performance.
- Common queue is better than separate queues in terms of reducing waiting time.