

Homework 3 - Linear Programs, Sensitivity Analysis

MGCR 472 - Operations Management

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Prof. Rim Harris

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Question 1

Magnolium Bakery needs to decide how many and what types of cupcakes to make for this week. Currently, they offer two flavors of cupcakes: blueberry lemon and red velvet. Blueberry lemon cupcakes sell for a profit of \$4 per cupcake, while red velvet cupcakes sell for the same profit of \$4 per cupcake.

There are two main ingredients in making their cupcakes: flour and sugar. Each blueberry lemon cupcake requires 2 cups of flour and 4 cups of sugar. Each red velvet cupcake requires 4 cups of flour and 3 cups of sugar. Magnolium has a total of 100 cups of flour and 120 cups of sugar to use.

Also, they would like to make at least 5 blueberry lemon cupcakes.

Magnolium would like to determine how many of each type of cupcake to make.

- Formulate the problem as a linear program.
- Solve the linear program from part (a) above using Excel. Specifically, find the optimal number of each type of cupcake to make and the optimal profit of Magnolium. Please, submit a screenshot of your Excel spreadsheet including the solver parameters. Assume that fractions of cupcakes can be made.

Solution

- As a linear program

maximize

$$4x_1 + 4x_2$$

subject to

$$2x_1 + 4x_2 \leq 100$$

$$4x_1 + 3x_2 \leq 120$$

$$x_1 \geq 5$$

- Magnolium Bakery should bake **18 blueberry lemon** and **16 red velvet cupcakes** to obtain a max profit of **\$136**.

	A	B	C	D	E	F
1		x1	x2			
2	objective coefficient	4	4			
3	variables	18	16			
4						
5	maximize	136				
6	subject to	2	4	100	<=	100
7		4	3	120	<=	120
8		18			>=	5
9			16		>=	0
10						
11						
12						
13						
14						
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16						
17						
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27						
28						
29						

Solver Parameters

Set Objective: ↑

To: ☒ Max ☐ Min ☐ Value Of:

By Changing Variable Cells: ↑

Subject to the Constraints:

\$B\$8 >= \$F\$8
 \$C\$9 >= \$F\$9
 \$D\$6 <= \$F\$6
 \$D\$7 <= \$F\$7

Add
Change
Delete
Reset All
Load/Save

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method: Simplex LP Options

Solving Method
 Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Help
Solve
Close

Question 2

Magnolium Bakery would like to expand their operations by introducing a new cupcake flavor: caramel cake. Caramel cake cupcakes sell for a profit of \$5 per cupcake. In addition, the bakery now needs to keep track of a new ingredient for each of their cupcakes, butter. The number of cups of each ingredient needed to make each cupcake flavor are given in the chart below.

Ingredient	Blueberry Lemon	Red Velvet	Caramel Cake
Flour	2	4	2
Sugar	4	3	4
Butter	0.2	0.3	0.1

Magnolium Bakery has 100 cups of flour, 120 cups of sugar, and 5 cups of butter available to make their cupcakes. As before, they would like to make at least 5 blueberry lemon cupcakes. In order to maximize their profit, the bakery has entered the following LP into Excel.

$$\begin{aligned}
 &\text{maximize} && 4x_1 + 4x_2 + 5x_3 \\
 &\text{subject to} && \\
 &&& 2x_1 + 4x_2 + 2x_3 \leq 100 \\
 &&& 4x_1 + 3x_2 + 4x_3 \leq 120 \\
 &&& 0.2x_1 + 0.3x_2 + 0.1x_3 \leq 5 \\
 &&& x_1 \geq 5 \\
 &&& x_1, x_2, x_3 \geq 0
 \end{aligned}$$

Solve the above LP using Excel and answer the following questions.

- What is the optimal number of each cupcake flavor (fractions are allowed if necessary)? What is the maximum possible profit that Magnolium Bakery can earn? Please, print a copy of the Excel output.
- Suppose that the profit of each blueberry lemon cupcake increases to \$6. Will this change the optimal number of blueberry lemon cupcakes?
- By how much would the profit of a red velvet cupcake have to decrease before the bakery decides to reduce the number of red velvet cupcakes to make? (Assume that the profit of each blueberry lemon cupcake remains \$4.)
- Suppose now that Magnolium Bakery is given an offer to buy one additional cup of butter at a price of \$1. Should they accept this offer? Why or why not? If the answer is yes, then what will be the new optimal amounts of each cupcake flavor?

Solution

- Magnolium Bakery should bake **5 blueberry lemon, 20/3 red velvet** and **20 caramel cake cupcakes** to obtain a max profit of **\$146.67**.

	A	B	C	D	E	F	G
1		x1	x2	x3			
2	objective coefficient	4	4	5			
3	variables	5	6.666667	20			
4							
5	maximize	146.6667					
6	subject to	2	4	2	76.66667	<=	100
7		4	3	4	120	<=	120
8		0.2	0.3	0.1	5	<=	5
9		5				>=	5
10			6.666667			>=	0
11				20		>=	0
12							
13							
14							
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29							

Solver Parameters

Set Objective:

\$B\$5

↑

To:

☒ Max
 ☐ Min
 ☐ Value Of:

0

By Changing Variable Cells:

\$B\$3:\$D\$3

↑

Subject to the Constraints:

\$B\$9 >= \$G\$9

\$C\$10 >= \$G\$10

\$D\$11 >= \$G\$11

\$E\$6 <= \$G\$6

\$E\$7 <= \$G\$7

\$E\$8 <= \$G\$8

Add

Change

Delete

Reset All

Load/Save

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Simplex LP

Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Help

Solve

Close

- We observe from the sensitivity report:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$3	variables x1	5	0	4	1.111111111	1E+30
\$C\$3	variables x2	6.666666667	0	4	11	0.25
\$D\$3	variables x3	20	0	5	0.333333333	1.666666667

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$B\$9	x1	5	-1.111111111	5	15	5
\$C\$10	x2	6.666666667	0	0	6.666666667	1E+30
\$D\$11	x3	20	0	0	20	1E+30
\$E\$6	subject to	76.66666667	0	100	1E+30	23.33333333
\$E\$7		120	1.222222222	120	60	60
\$E\$8		5	1.111111111	5	2.1	1.5

The **allowable increase** of a blueberry lemon cupcake's profit/objective coefficient (\$4) is \$1.11.

Increasing its profit to \$6 implies adding \$2, which exceeds the allowable increase of \$1.11.

Therefore, the optimal number of blueberry lemon cupcakes would change.

c. The **allowable decrease** of a red velvet cupcake's profit/objective coefficient (\$4) is \$0.25.

Therefore, its profit would have to decrease more than \$0.25 to reduce the number of red velvet cupcakes to bake.

d. In the bottom row, from the **shadow price** (1.11) and **allowable increase** (2.1), adding one cup of butter yields a \$1.11 increase in profit.

Paying \$1 for this cup yields \$0.11 of net profit. Therefore, Magnolium Bakery should accept this offer.

The new optimal amounts for blueberry lemon, red velvet, and caramel cake cupcakes would then be 5, $11.111 = 100/9$, and $16.667 = 50/3$, respectively.