Assignment 3 | **LE, Nhat Hung**
MATH 324 - Statistics | McGill ID: 260793376
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10.8 p. 495

A two-stage clinical trial is planned for testing $H_0 : p = .10$ versus $H_a : p > .10$, where p is the proportion of responders among patients who were treated by the protocol treatment. At the first stage, 15 patients are accrued and treated. If 4 or more responders are observed among the (first) 15 patients, H_0 is rejected, the study is terminated, and no more patients are accrued. Otherwise, another 15 patients will be accrued and treated in the second stage. If a total of 6 or more responders are observed among the 30 patients accrued in the two stages (15 in the first stage and 15 more in the second stage), then H_0 is rejected. For example, if 5 responders are found among the first-stage patients, H_0 is rejected and the study is over. However, if 2 responders are found among the first-stage patients, 15 second-stage patients are accrued, and an additional 4 or more responders (for a total of 6 or more among the 30) are identified, H_0 is rejected and the study is over.

a. Use the binomial table to find the numerical value of α for this testing procedure.

Let Y_1, Y_2 such that

$$Y_1, Y_2 \sim \text{Binomial}(n = 15, p = .10)$$

Then

$$\begin{aligned}
\alpha &= P(\text{reject } H_0 | H_0) \\
&= P(\text{reject } H_0 \text{ in stage } 1 | H_0) + P(\text{reject } H_0 \text{ in stage } 2 | H_0) \\
&= P(Y_1 \geq 4) + P(Y_1 + Y_2 \geq 6, Y_1 \leq 3) \\
&= 1 - P(Y_1 \leq 3) + \sum_{k=0}^{3} P(Y_1 + Y_2 \geq 6, Y_1 = k) \\
&= 1 - .944 + \sum_{k=0}^{3} P(Y_2 \geq 6 - k) P(Y_1 = k) \\
&= .056 + \sum_{k=0}^{3} (1 - P(Y_2 \leq 6 - k - 1)) p_{Y_1}(k) \\
&= .099
\end{aligned}$$

$P(Y_2 \leq 6 - k - 1)$ from Table 1, Appendix 3, and $p_{Y_1}(k)$ from calculator.

b. Use the binomial table to find the probability of rejecting the null hypothesis when using this rejection region if $p = .30$.

$$Y_1, Y_2 \sim \text{Binomial}(n = 15, p = .30)$$

Similar to part a,

$$\alpha = 1 - P(Y_1 \leq 3) + \sum_{k=0}^{3} (1 - P(Y_2 \leq 6 - k - 1)) p_{Y_1}(k) = .932$$

c. For the rejection region defined above, find β if $p = .30$.

$$\beta = P(Y_1 + Y_2 \leq 5, Y_1 \leq 3) = \sum_{k=0}^{3} P(Y_2 \leq 5 - k) p_{Y_1}(k) = .068$$

Or, more simply

$$\beta = 1 - \alpha = 1 - .932 = .068$$

10.20 p. 504

The Rockwell hardness index for steel is determined by pressing a diamond point into the steel and measuring the depth of penetration. For 50 specimens of an alloy of steel, the Rockwell hardness index averaged 62 with standard deviation 8. The manufacturer claims that this alloy has an average hardness index of at least 64. Is there sufficient evidence to refute the manufacturer's claim at the 1% significance level?

$$H_0 : \mu = 64$$
$$H_a : \mu < 64$$
$$Z = \frac{\hat{\mu} - \mu_0}{s/\sqrt{n}} = \frac{62 - 64}{8/\sqrt{50}} \approx -1.77$$
$$\alpha = .01 \Rightarrow -z_{.01} \approx -2.33 \Rightarrow \text{Rejection region} = \{z < 2.33\}$$

$$Z \not< -2.33 \Rightarrow Z \notin \text{RR}$$

Therefore, no sufficient evidence refutes the manufacturer's claim at the 1% significance level.

10.21 p. 504

Shear strength measurements derived from unconfined compression tests for two types of soils gave the results shown in the following table (measurements in tons per square foot). Do the soils appear to differ with respect to average shear strength, at the 1% significance level?

Soil Type I	Soil Type II
$n_1 = 30$ $\overline{y}_1 = 1.65$ $s_1 = 0.26$	$n_2 = 35$ $\overline{y}_2 = 1.43$ $s_2 = 0.22$

$$Z = \frac{(\overline{y}_1 - \overline{y}_2) - 0}{\sigma_{\mu_1 - \mu_2}} = \frac{\overline{y}_1 - \overline{y}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1.65 - 1.43}{\sqrt{\frac{.26^2}{30} + \frac{.22^2}{35}}} = 3.65$$
$$z_{\alpha/2} = z_{.01/2} = z_{.005} \approx 2.58$$

$$|Z| > z_{\alpha/2}$$

Therefore, the soils do appear to differ with respect to average shear strength, at the 1% significance level.

10.23 p. 505

Studies of the habits of white-tailed deer indicate that these deer live and feed within very limited ranges, approximately 150 to 205 acres. To determine whether the ranges of deer located in two different geographical areas differ, researchers caught, tagged, and fitted 40 deer with small radio transmitters. Several months later, the deer were tracked and identified, and the distance y from the release point was recorded. The mean and standard deviation of the distances from the release point were as given in the accompanying table.

	Location	
	1	2
Sample size	40	40
Sample mean (ft)	2980	3205
Sample standard deviation (ft)	1140	963
Population mean	μ_1	μ_2

a. If you have no preconceived reason for believing that one population mean is larger than the other, what would you choose for your alternative hypothesis? Your null hypothesis?

$$H_a : \mu_1 - \mu_2 \neq 0$$
$$H_0 : \mu_1 - \mu_2 = 0$$

b. Would your alternative hypothesis in part (a) imply a one-tailed or a two-tailed test? Explain.

The alternative hypothesis $\mu_1 - \mu_2 \neq 0$ implies a two-tailed test, because we want to detect a change in $\mu_1 - \mu_2$ both above and below 0.

c. Do the data provide sufficient evidence to indicate that the mean distances differ for the two geographical locations? Test using $\alpha = .10$.

$$Z = \frac{2980 - 3205}{\sqrt{\frac{1140^2}{40} + \frac{963^2}{40}}} \approx -.95$$
$$z_{\alpha/2} = z_{.05} = 1.65$$

$$|Z| \not> z_{\alpha/2}$$

Therefore, the data lacks sufficient evidence to indicate the mean distances differ for the two geographical locations with $\alpha = 0.10$.

10.49 p. 512

Refer to Exercise 10.19. Construct a 95% upper confidence bound for the average voltage reading.

$$\text{CI} = \hat{\mu} \pm z_{.05}(s/\sqrt{n}) = 128.6 \pm 1.96(2.1/\sqrt{40}) = [127.95, 129.25]$$

a. How does the value $\mu = 130$ compare to this upper bound?

$$\mu = 130 > 129.25$$

b. Based on the upper bound in part (a), should the alternative hypothesis of Exercise 10.19 be accepted?

$$H_a : \mu < 130$$

From the upper bound a, H_a can be accepted

c. Is there any conflict between the answer in part (b) and your answer to Exercise 10.19?

In 10.19,

$$Z = \frac{128.6 - 130}{2.1/\sqrt{40}} = -4.22$$

$$z_\alpha = z_{.05} \approx 1.96$$

$$Z < -z_\alpha$$

Therefore, we reject the null hypothesis and accept the alternative hypothesis, similar to part b.

10.50 p. 516

High airline occupancy rates on scheduled flights are essential for profitability. Suppose that a scheduled flight must average at least 60% occupancy to be profitable and that an examination of the occupancy rates for 120 10:00 A.M. flights from Atlanta to Dallas showed mean occupancy rate per flight of 58% and standard deviation 11%. Test to see if sufficient evidence exists to support a claim that the flight is unprofitable. Find the *p*-value associated with the test. What would you conclude if you wished to implement the test at the $\alpha = .10$ level?

$$H_0 : \mu = .6$$

$$H_a : \mu < .6$$

$$z = \frac{.58 - .6}{.11/\sqrt{120}} \approx -1.99$$

$$\text{p-value} = P(Z \leq z) = P(Z \leq -1.99) = .0233$$

$$\alpha = .10 > \text{p-value}$$

Therefore, with $\alpha = .10$, H_0 would be rejected.

10.53 p. 516

How would you like to live to be 200 years old? For centuries, humankind has sought the key to the mystery of aging. What causes aging? How can aging be slowed? Studies have focused on *biomarkers*, physical or biological changes that occur at a predictable time in a person's life. The theory is that, if ways can be found to delay the occurrence of these biomarkers, human life can be extended. A key biomarker, according to scientists, is forced vital capacity (FVC), the volume of air that a person can expel after taking a deep breath. A study of 5209 men and women aged 30 to 62 showed that FVC declined, on the average, 3.8 deciliters (dl) per decade for men and 3.1 deciliters per decade for women. Suppose that you wished to determine whether a physical fitness program for men and women aged 50 to 60 would delay aging; to do so, you measured the FVC for 30 men and 30 women participating in the fitness program at the beginning and end of the 50- to 60-year age interval and recorded the drop in FVC for each person. A summary of the data appears in the accompanying table.

	Men	Women
Sample size	30	30
Sample average drop in FVC (dl)	3.6	2.7
Sample standard deviation (dl)	1.1	1.2
Population mean drop in FVC	μ_1	μ_2

a. Do the data provide sufficient evidence to indicate that the decrease in the mean FVC over the decade for the men on the physical fitness program is less than 3.8 dl? Find the attained significance level for the test.

$$H_0 : \mu_1 = 3.8$$
$$H_a : \mu_1 < 3.8$$
$$z = \frac{3.6 - 3.8}{1.1/\sqrt{30}} \approx -1$$

$$\text{p-value} = P(Z \leq -1) = .1587$$

b. Refer to part (a). If you choose $\alpha = .05$, do the data support the contention that the mean decrease in FVC is less than 3.8 dl?

$$\alpha = .05 < \text{p-value}$$

Therefore, with $\alpha = .05$, H_0 can't be rejected.

c. Test to determine whether the FVC drop for women on the physical fitness program was less than 3.1 dl for the decade. Find the attained significance level for the test.

$$\text{p-value} = P(Z \leq \frac{2.7 - 3.1}{1.2/\sqrt{30}}) = P(Z \leq -1.83) = .0336$$

d. Refer to part (c). If you choose $\alpha = .05$, do the data support the contention that the mean decrease in FVC is less than 3.1 dl?

$$\alpha = .05 > \text{p-value}$$

Therefore, we can reject the null hypothesis and conclude that the data support the contention that the mean decrease in FVC is less than 3.1 dl.

10.83 p. 539

The manager of a dairy is in the market for a new bottle-filling machine and is considering machines manufactured by companies A and B. If ruggedness, cost, and convenience are comparable in the two machines, the deciding factor will be the variability of fills (the machine producing fills with the smaller variance being preferable). Let σ_1^2 and σ_2^2 be the fill variances for machines produced by companies A and B, respectively. Now consider various tests of the null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$. Obtaining samples of fills from the two machines and using the test statistic S_1^2 / S_2^2, we could set up as the rejection region an upper-tail area, a lower-tail area, or a two-tailed area of the F distribution, depending on the interests to be served. Identify the type of rejection region that would be most favored by the following persons, and explain why.

a. The manager of the dairy

The manager of the dairy would prefer

$$H_a : \sigma_1^2 \neq \sigma_2^2$$

Therefore, a two-tailed area of the F distribution should be used as the rejection region.

b. A salesperson for company A

The manager prefers lower variance. Therefore, company A's salespersons would prefer

$$H_a : \sigma_1^2 < \sigma_2^2$$

Therefore, a lower-tailed area is of the F distribution should be used as the rejection region.

c. A salesperson for company B

Conversely, in company B's best interests, an upper-tail area of the F distribution should be used as the rejection region.