

# Assignment 2 - Bond Returns and the Expectations Hypothesis, Bond Duration, Portfolio Theory, Stock Valuation, Comparables, Capital Budgeting

MGCR 341 - Introduction to Financial Accounting

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## Bond Returns and the Expectations Hypothesis

### 1)

You and your colleagues, Elmo and Ronald, are working as summer interns at the treasury department of Pepsi Co. The company wants to raise \$1bn via a bond issuance at  $t = 0$  to finance a project that will generate a large cash flow at  $t = 2$ . The CFO has asked you and the other interns for your thoughts on the following 3 options:

- i) issue a 2-year 0-cp bond at  $t = 0$
- ii) issue a 1-year 0-cp bond at  $t = 0$ , with the intention of paying off that bond's face value at  $t = 1$  with the proceeds from another 1-year 0-cp bond issuance at  $t = 1$
- iii) issue a 1-year 0-cp bond at  $t = 0$ , and also lock in the forward interest rate  $f_{1,1}$ , for borrowing \$1.05bn at  $t = 1$  for 1 year.

Given:  $r_{0,1} = 5\%$ ,  $r_{0,2} = 6\%$ .  $f_{1,1}$  is set such that there is no arbitrage in the market. Pepsi only cares about minimizing the expected total repayment at  $t = 2$ .

Elmo responded with: "Based on current labour market conditions, and its impact on inflation, and based on what the Federal Reserve has said, I believe the 1-year rate at  $t = 1$  will be 6.5%. As such, it would be wise to issue the 2-year bond, which has a 6% yield."

Ronald responded with: "We should go with option iii) as this would save us a lot of money compared to option i)."

- a) Explain the flaw in Elmo's logic.
- b) Is Ronald right?
- c) Suppose your own personal view is that  $r_{1,1}$  will equal 6.8%. What option would you recommend to the CFO. Justify by showing how much money you would expect to save compared to the other options in terms of total repayment at  $t = 2$ . (Assume that the CFO trusts your expected forecast of  $r_{1,1}$ .)

### Solution:

- a) According to Elmo,  $r_{1,1} = 6.5\%$

Then, the total repayment at  $t = 2$  of option i is

$$1\text{bn}(r_{0,2})^2 = 1\text{bn}(1.06^2) = 1.1236\text{bn}$$

However, the repayment of option ii is

$$1\text{bn}(r_{0,1})(r_{1,1}) = 1\text{bn}(1.05)(1.065) = 1.11825\text{bn}$$

which is lower.

- b)

$$f_{1,1} = \frac{(1 + r_{0,2})^2}{1 + r_{0,1}} - 1 = \frac{1.06^2}{1.05} - 1 = 7.01\%$$

Repayment of option iii is

$$1\text{bn}(r_{0,1})(f_{1,1}) = 1\text{bn}(1.05)(1.0701) = 1.1236\text{bn}$$

which is equal to the repayment of option i. Therefore, Ronald is wrong.

c)

With  $r_{1,1} = 6.8\%$ , the repayment of option ii is

$$1\text{bn}(1.05)(1.068) = 1.1214\text{bn}$$

which is still lower than options i and iii.

Therefore, option ii is optimal, saving  $1.1236\text{bn} - 1.1214\text{bn} = \$2.2\text{M}$  compared to the other options.

## Bond Duration

### 2)

(This is the same question as from Assignment 2 Summer1 2015. So as long as you attempt it you will receive full credit.)

At  $t = 0$  the yield curve is flat at 5%. Imagine the following very simplified pension fund:

At  $t = 0$  the pension fund takes in a \$10,000 contribution from an employee. The pension fund is a defined benefit plan and guarantees that it will pay the employee  $\$5,000(1.05)^3$  at  $t = 3$  and  $\$5,000(1.05)^9$  at  $t = 9$ .

If the pension fund invests \$5,000 in a 3-year zero coupon bond and \$5,000 in a 9-year zero coupon bond the pension fund is not taking on any risk and will be able to pay the employee the promised cash flows.

Suppose, however, that the fund manager running the pension plan decides to invest \$8,000 in a 14-year zero coupon bond and \$2,000 in a 16-year zero coupon bond instead (perhaps because the fund manager thinks that interest rates will go down).

Now, suppose that at  $t = 0$  the yield curve shifts up to 5.3% (parallel shift of the entire yield curve).

a) Calculate the exact amount by which the pension plan is underfunded at  $t = 0$  after the yield curve has shifted up to 5.3%. At any point in time, the amount of underfunding is equal to the present value of the liabilities minus the present value of the assets. (Hint: the assets are the 14-year and 16-year bonds, and you may treat the liabilities as a 3 and 9 year zero coupon bond with face values  $\$5,000(1.05)^3$  and  $\$5,000(1.05)^9$ , respectively.) Answer this question by **directly computing** the new values of the assets and liabilities, and not by using the Modified Duration approach.

b) Answer part a again, but this time using the **Modified Duration methodology**. (Hint: You can view the pension plan as a portfolio consisting of a long position in 14-year and 16-year bonds, and a short position in the 3-year and 9-year bonds. The amount of underfunding is thus given by the negative of the change in the value of the portfolio resulting from the shift in the yield curve. (Note that your answers to parts a and b will differ slightly because the Modified Duration methodology involves linear approximations to reality.)

The answer to part a is the amount of money the employer would have to contribute into the fund at  $t = 0$ , after the yield curve has shifted, in order for the fund to no longer be underfunded.

Suppose that after the yield curve shift the pension fund decides it no longer wants to be exposed to parallel shifts in the yield curve. One option would be to liquidate the position in the 14-year and 16-year bonds, and invest those proceeds, plus your answer to part a, in 3-year and 9-year zero coupon bonds with face values of  $\$5,000(1.05)^3$  and  $\$5,000(1.05)^9$ , respectively. Another option would be to liquidate the position in the 2 long bonds, and invest those proceeds, plus your answer to part a in a T-year zero coupon bond.

c) What would T have to be in order for the fund to no longer be exposed to parallel shifts in the yield curve? (Comment: Note that now the yield curve is already at 5.3%, and thus 5.3% is the new "initial" YTM of the bonds for the purposes of calculating MD.)

d) Based on your answer to part c: Although you are now protected against parallel shifts in the yield curve, you are not protected against changes in the slope of the yield curve, or changes in the curvature of the yield curve. Would it make sense

to take the long position in the T-year bond if the fund manager believed that right after buying the T-year bond, there would be a change in the curvature of the yield curve such that:  $\Delta y_9 = \Delta y_3$  and  $\Delta y_T = \Delta y_3 - 1\%$ ? Specifically, what would the  $t = 0$  profit or loss be based on the change in curvature described in the previous sentence? (Hint: consider  $\Delta P_{total}$  of the portfolio of long the T-year bond, and “short” 3 and 9 year bonds. Substitute in for the change in yields as described above. You should get some cancellations and be able to compute a numerical profit or loss.)

(As a side note, defined benefit pension plans may also invest in stocks. A decrease in the value of those stocks is another common source of pension underfunding or shortfalls. Indeed, many pension plans had significant shortfalls following the 2007-2009 stock market decline, requiring employers to make significant contributions to their pension funds.

For further optional information, this might be a good time to look up “pension” on Wikipedia, noting the difference between a “defined benefit” and “defined contribution” pension plan. You may also be interested in some of the information at [http://en.wikipedia.org/wiki/Pension\\_Benefit\\_Guaranty\\_Corporation](http://en.wikipedia.org/wiki/Pension_Benefit_Guaranty_Corporation), particularly the point about Enron.)

### Solution:

a)

$$\begin{aligned}\text{underfunding} &= PV(\text{liabilities}) - PV(\text{assets}) \\ &= \frac{5000(1.05^3)}{1.053^3} + \frac{5000(1.05^9)}{1.053^9} - \frac{8000(1.05^{14})}{1.053^{14}} - \frac{2000(1.05^{16})}{1.053^{16}} = 233.12\end{aligned}$$

b)

$$\begin{aligned}\Delta P_{total} &= \Delta P_{14} + \Delta P_{16} - \Delta P_3 - \Delta P_9 \\ &= P_{0,14}(-MD_{14})\Delta YTM + P_{0,16}(-MD_{16})\Delta YTM - P_{0,3}(-MD_3)\Delta YTM - P_{0,9}(-MD_9)\Delta YTM \\ &= P_{0,14}(-D_{14}/(1 + YTM_{initial}))\Delta YTM + P_{0,16}(-D_{16}/(1 + YTM_{initial}))\Delta YTM \\ &\quad - P_{0,3}(-D_3/(1 + YTM_{initial}))\Delta YTM - P_{0,9}(-D_9/(1 + YTM_{initial}))\Delta YTM \\ &= 8000(-14/1.05)(0.003) + 2000(-16/1.05)(0.003) - 5000(-3/1.05)(0.003) - 5000(-9/1.05)(0.003) \\ &= -240\end{aligned}$$

Therefore, the approximate underfunding is 240.

c)

$$\begin{aligned}P_{0,T} &= \frac{8000(1.05^{14})}{1.053^{14}} + \frac{2000(1.05^{16})}{1.053^{16}} + 233.12 = 9830.63 \\ \Delta P_{total} &= \Delta P_T - \Delta P_3 - \Delta P_9 \\ &= 9830.63(-T/1.053)\Delta y_T - \frac{5000(1.05^3)}{1.053^3}(-3/1.053)\Delta y_3 - \frac{5000(1.05^9)}{1.053^9}(-9/1.053)\Delta y_9\end{aligned}$$

Parallel shift of yield curve  $\Rightarrow$  all delta y's are identical. We now simply denote this value as delta y.

$$\begin{aligned}\Delta P_{total} &= 9830.63(-T/1.053)\Delta y - \frac{5000(1.05^3)}{1.053^3}(-3/1.053)\Delta y - \frac{5000(1.05^9)}{1.053^9}(-9/1.053)\Delta y \\ \Delta P_{total} = 0 &\Rightarrow 9830.63(-T/1.053)\Delta y - \frac{5000(1.05^3)}{1.053^3}(-3/1.053)\Delta y - \frac{5000(1.05^9)}{1.053^9}(-9/1.053)\Delta y = 0 \\ &\Rightarrow 9830.63(-T/1.053) - \frac{5000(1.05^3)}{1.053^3}(-3/1.053) - \frac{5000(1.05^9)}{1.053^9}(-9/1.053) = 0 \\ &\Rightarrow T = 5.97\end{aligned}$$

d)

$$\begin{aligned}
\Delta P_{total} &= 9830.63(-5.97/1.053)(\Delta y_3 - 0.01) - \frac{5000(1.05^3)}{1.053^3}(-3/1.053)\Delta y_3 - \frac{5000(1.05^9)}{1.053^9}(-9/1.053)\Delta y_3 \\
&= \left[ 9830.63(-5.97/1.053)\Delta y_3 - \frac{5000(1.05^3)}{1.053^3}(-3/1.053)\Delta y_3 - \frac{5000(1.05^9)}{1.053^9}(-9/1.053)\Delta y_3 \right] \\
&\quad + 9830.63(-5.97/1.053)(-0.01) \\
&= 0 + 9830.63(-5.97/1.053)(-0.01) = 557.75
\end{aligned}$$

### 3)

(This is the same question as from Summer 2016. So as long as you attempt the question you will receive full credit.)

At  $t = 0$ ,  $r_{0,2} = 4\%$  and  $r_{0,10} = 8\%$ . You want to put on a yield curve flattening trade, such that for every 1% of flattening you will make \$1,000 profit. You can trade 2-year and 10-year 0-coupon bonds at  $t = 0$ . For each bond, specify how much you are trading in PV terms and whether you are long or short. (Note: a 1% flattening implies that  $\Delta y_{10} = \Delta y_2 - 1\%$ .)

#### Solution:

Betting on flattening  $\Rightarrow$  short the short term bond and long the long term bond

$$\Delta P_{total} = -\Delta P_2 + \Delta P_{10}$$

Ensure position will break even if there's no change in the slope i.e. parallel shift of yield curve:

$$\Delta P_{total} = 0 \Rightarrow -P_{0,2}(-MD_2)\Delta y + P_{0,10}(-MD_{10})\Delta y = 0 \Rightarrow P_{0,2} = \frac{MD_{10}}{MD_2}P_{0,10}$$

Set profit = 1000:

$$\Delta P_{total} = 1000 \Rightarrow -P_{0,2}(-MD_2)\Delta y_2 + P_{0,10}(-MD_{10})(\Delta y_2 - 0.01) = 1000$$

$$\Rightarrow P_{0,10}(-MD_{10})(-0.01) = P_{0,10}\left(-\frac{10}{1.08}\right)(-0.01) = 1000 \Rightarrow P_{0,10} = 10800$$

$$\Rightarrow P_{0,2} = \frac{MD_{10}}{MD_2}P_{0,10} = \frac{10/1.08}{2/1.04}10800 = 52000$$

## Portfolio Theory

### 4)

Answer True or False. No explanations are required.

- In the single index model, if two firms have uncorrelated firm specific risk, then a 50%-50% portfolio of the two stocks would have all firm specific risk diversified away.
- In the single index model, firm specific risk is assumed to have a negative correlation with the market return.
- For a 30%-70% portfolio of two stocks, the portfolio standard deviation will be less than a 30%-70% weighted average of the two stocks' standard deviations (assume the correlation between the stocks is less than 1).
- In the single-index model, portfolio standard deviation converges to zero as the number of stocks in your portfolio goes to infinity.
- If you have mean-variance preferences, but the CAPM does not hold, then an optimal portfolio for you would still involve mixing the tangency portfolio with the risk free asset, even though the Tangency portfolio may not be the market portfolio.
- In the CAPM, if Stock A has a higher expected return than Stock B, it must also have a higher Beta.
- In the CAPM, if Stock A has a higher expected return than Stock B, it must also have a higher standard deviation.

h) A mean-variance investor will necessarily prefer investing 100% in portfolio A rather than investing 100% in portfolio B as long as A has a higher Sharpe ratio than B.

**Solution:**

- a) **False.** In order for the firm specific risk to get diversified away you need a portfolio of many stocks, not just 2.
- b) **False.** The correlation is 0.
- c) **True.**
- d) **False.** It converges to the beta of the portfolio times the standard deviation of the market. Only firm specific risk is diversified away.
- e) **True.** Mean-variance investors want to be on the steepest Capital Allocation Line.
- f) **True.**
- g) **False.**
- h) **False.** For example, A might have a higher standard deviation than B, and a particularly risk averse investors might thus very well prefer B.

**5)**

You are given the following information:

- a. Stocks A, B, and C have standard deviations of 50%, 60%, and 70%, respectively
- b. Stocks A and B have expected returns of 10% and 15%, respectively
- c. Stock C has a beta of 1.5
- d. Stocks A, B, and C are all uncorrelated (correlation = 0)
- e. The risk free rate is 4%
- f. The expected return on the market is 14%
- g. The standard deviation of the market is 20%
- h. Portfolio Z consists of 60% in A, 30% in B, 50% in C, and -40% in the risk free asset.
- i. The CAPM holds

- a) What is the expected return of stock C?
- b) What is the Beta of stock A?
- c) What is the expected return on Portfolio Z?
- d) What is the beta of Portfolio Z?
- e) What is the standard deviation of Portfolio Z?
- f) Construct an optimal portfolio that has the same standard deviation as Portfolio Z.
- g) What is the expected return of the portfolio you constructed in part f?
- h) Construct an optimal portfolio that has the same expected return as Portfolio Z.
- i) What is the Sharpe ratio of the portfolio you constructed in part h?

**Solution:**

a)

$$E[r_C] = r_f + \beta_C(E[r_m] - r_f) = 0.04 + 1.5(0.14 - 0.04) = 19\%$$

b)

$$0.1 = 0.04 + \beta_A(0.14 - 0.04) \Rightarrow \beta_A = 0.6$$

c)

$$E[r_Z] = 0.6(0.1) + 0.3(0.15) + 0.5(0.19) - 0.4(0.04) = 18.4\%$$

d)

$$0.184 = 0.04 + \beta_Z(0.14 - 0.04) \Rightarrow \beta_Z = 1.44$$

e)

$$\sigma_Z^2 = \sum_i w_i^2 \sigma_i^2 + 2 \sum_{i < j} w_i w_j \text{Cov}[r_i, r_j] = 0.6^2(0.5^2) + 0.3^2(0.6^2) + 0.5^2(0.7^2) \\ \Rightarrow \sigma_Z = 49.49\%$$

f) Want tangency portfolio. CAPM holds  $\Rightarrow$  tangency portfolio is the market portfolio.

$$0.4949 = w_m 0.2 \Rightarrow w_m = 2.474, w_{rf} = -1.474$$

g)

$$2.474(0.14) - 1.474(0.04) = 28.75\%$$

h)

$$0.184 = w_m(0.14) + (1 - w_m)(0.04) \Rightarrow w_m = 1.44, w_{rf} = -0.44$$

i) The optimal portfolio in h is on the same CAL line as the market portfolio and therefore has the same Sharpe's ratio as the market's.

$$S = \frac{E[r_m] - r_f}{\sigma_m} = \frac{0.14 - 0.04}{0.2} = 0.5$$

## 6)

Given:

- i) Stock X: expected return = 25%, standard deviation = 40%
- ii) Stock Y: standard deviation = 60%
- iii) Risk free rate = 5%

Hodor has mean-variance preferences and was faced with the following 2 investment options:

- A) Mix the risk free asset with stock X (any weights you want)
- B) Mix the risk free asset with stock Y (any weights you want)

Given that Hodor selected option B, what can you say about the expected return of stock Y?

### Solution:

When mixing the risk free asset with stocks X or Y, want to mix with the one that has the higher Sharpe ratio to be on the steeper Capital Allocation Line.

Therefore:

$$S_Y \geq S_X \Rightarrow (E[r_Y] - 0.05)/0.6 \geq (0.25 - 0.05)/0.4 \Rightarrow E[r_Y] \geq 35\%$$

## Stock Valuation

### 7)

(This is the same question as from Assignment 3 Winter 2013. So as long as you attempt it you will receive full credit.)

Assume the risk free rate is 5% and that investors are risk neutral. Today is  $t = 0$  and Yahoo is trading at \$20 a share. Take it as given that everyone knows Yahoo will not pay any dividends before  $t = 5$ . Your colleague tells you that based on an analysis of the PV of all future dividends, the fair price of Yahoo is \$15. He thus recommends shorting the stock.

You agree with your colleague's analysis in terms of the expectations of all future dividends. You're concerned, however, about a possible takeover bid that Microsoft will make for Yahoo at  $t = 1$ . Specifically, you believe that at  $t = 1$  Microsoft will announce that they are willing to purchase Yahoo at a price of \$30 a share at  $t = 2$ . Your colleague tells you that even if Microsoft announces a takeover bid at  $t = 1$ , the deal will ultimately fail because the board of directors at Yahoo will not approve the deal.

You agree that ultimately the deal will not go through. But, you believe that at  $t = 1$ , the market will have the following beliefs about the takeover: there is a 90% chance that the deal will go through at  $t = 2$  and therefore a 90% chance the stock price at  $t = 2$  will equal \$30; there is a 10% chance that the deal will not go through and that the stock price will come back to \$20 at  $t = 2$ .

Based on the above information, and assuming a 1-year investment horizon, what would the minimum price for Yahoo have to be at  $t = 0$  for you to want to short it at  $t = 0$ ?

### Solution:

Expected stock price at  $t = 1$  to cover short position:

$$E[P_1] = \frac{0.9(30) + 0.1(20)}{1.05} = 27.62$$

Lowest price where you'd be willing to short the stock at  $t = 0$  is:

$$E[P_1]/1.05 = 27.62/1.05 = 26.3$$

### 8)

(This is the same question as from Summer 2014 Final Exam. So as long as you attempt it you will receive full credit.)

Today is  $t = 0$ . The market believes Morgan Stanley will pay a \$2 dividend at  $t = 1$ . But the market also believes that Morgan Stanley's  $t = 2$  dividend will be cut to \$1 due to real estate related losses. The market believes that thereafter dividends are expected to grow at a 3% rate until  $t = 5$ , and then grow at a 6% rate until  $t = 11$ , and then grow at a rate of  $g$  in perpetuity. The market also believes that at  $t = 9$  the stock price will be trading based on a dividend yield of  $\text{Div}_{10}/P_9 = 9\%$ . The discount rate is 10%.

a) Write down an equation where the only unknown is  $P_0$ , the price of Morgan Stanley at  $t = 0$ ? You do not need to solve for  $P_0$ .

b) Write down an equation where the only unknown is  $g$ ? You do not need to isolate or solve for  $g$ .

c) What is the expected return (including the dividend) on the stock from  $t = 0$  to  $t = 1$ ?

d) Suppose you believe the following: Morgan Stanley will pay a \$3 dividend at  $t = 1$ , a \$2 dividend at  $t = 2$ , a \$4 dividend at  $t = 3$  and that dividends will remain constant thereafter. You also believe that at  $t = 2$  the market will revise its views on Morgan Stanley. Specifically, you believe that at  $t = 2$  the market will believe that  $\text{Div}_3$  will equal \$5,  $\text{Div}_4$  will equal \$6,  $\text{Div}_5$  will equal \$7,  $\text{Div}_6$  will equal \$9 and that the  $P_3/\text{Div}_4$  ratio will equal 20. Assuming you have a 2 year investment horizon, write down an equation where the only unknown is  $H_0$ , the most you would be willing to pay for Morgan Stanley at  $t = 0$ ?

### Solution:

a)

$$P_0 = \frac{2}{1.1} + \frac{1}{0.1 - 0.03} \left( 1 - \left( \frac{1.03}{1.1} \right)^{5-2+1} \right) \frac{1}{1.1} + \frac{1.03^3(1.06)}{0.1 - 0.06} \left( 1 - \left( \frac{1.06}{1.1} \right)^{9-6+1} \right) \frac{1}{1.1^5} + \frac{P_9}{1.1^9}$$

$$\frac{P_9}{1.1^9} = \frac{\text{Div}_{10}}{0.09(1.1^9)} = \frac{1.03^3(1.06^5)}{0.09(1.1^9)}$$

Therefore,

$$P_0 = \frac{2}{1.1} + \frac{1}{0.1 - 0.03} \left( 1 - \left( \frac{1.03}{1.1} \right)^{5-2+1} \right) \frac{1}{1.1} + \frac{1.03^3(1.06)}{0.1 - 0.06} \left( 1 - \left( \frac{1.06}{1.1} \right)^{9-6+1} \right) \frac{1}{1.1^5} + \frac{1.03^3(1.06^5)}{0.09(1.1^9)}$$

b)

$$P_0 = \frac{2}{1.1} + \frac{1}{0.1 - 0.03} \left( 1 - \left( \frac{1.03}{1.1} \right)^{5-2+1} \right) \frac{1}{1.1} + \frac{1.03^3(1.06)}{0.1 - 0.06} \left( 1 - \left( \frac{1.06}{1.1} \right)^{11-6+1} \right) \frac{1}{1.1^5} + \frac{1.03^3(1.06^6)(1+g)}{0.1 - g} \frac{1}{1.1^{11}}$$

c) Since everything is fairly priced the expected return equals the discount rate of 10%.

d)

$$H_0 = \frac{Div_1}{1.1} + \frac{Div_2}{1.1^2} + \frac{Div_3 + P_3}{1.1^3} = \frac{3}{1.1} + \frac{2}{1.1^2} + \frac{5 + 20(Div_4)}{1.1^3} = \frac{3}{1.1} + \frac{2}{1.1^2} + \frac{5 + 20(6)}{1.1^3}$$

## Comparables

9)

The companies listed below are all expected to pay dividends in the form of a growing perpetuity. All earnings are paid out as dividends, so earnings = dividends. For parts a to f, select the best answer. Justify briefly.

a) Company A has a higher growth rate and a higher required rate of return than company B. Which company should have a higher dividend yield?

i) A

ii) B

iii) Can't tell

b) Company C has the same dividend growth rate and the same required rate of return as company D. But company C has a lower profit margin than company D. (Profit margin is equal to earnings as a fraction of sales. Assume that the profit margin will remain constant.) Which company should have a higher PE ratio?

i) C

ii) D

iii) Stock C and D should have the same PE ratio

c) Companies E and F have the same growth rate and the same required rate of return, but company E has a lower profit margin than company F. (Profit margin is equal to earnings as a fraction of sales. Assume that the profit margin will remain constant.) Which company should be trading at a higher multiple of sales? (The company with the higher ratio of Price/Sales is said to be trading at a higher multiple of sales.)

i) E

ii) F

iii) Can't tell

d) Website company G is expected to grow users more rapidly than company H and both companies have the same required rate of return, and the same profit margin. Company H is able to generate more sales revenue per user than company G. Which company should be trading at a higher multiple of users? (The company with the higher ratio of Price/Users is said to be trading at a higher multiple of users.)

i) G

ii) H

iii) Can't tell

(To be clear, when thinking of P here, think of the value of the entire company, not per share, as we don't usually think in terms of "users per share", like we might with earnings per share.)



e) Website companies I and J have the same growth rate and the same required rate of return. I has a higher profit margin than J (assume their profit margins remain constant). Company J is able to generate more sales revenue per user than company I. For each of the below variables, indicate which company should trade at a higher multiple of the variable, or if you can't tell. Explain briefly.

i) Earnings

ii) Sales

iii) Users

f) Assume now that companies K and L have the same required rate of return, and the same growth rate until  $t = 5$ , but that thereafter K will have a higher growth rate than L in perpetuity. Which company should have a higher dividend yield at  $t = 0$ ?

i) K

ii) L

iii) Can't tell

### Solution:

a) **iii) Can't tell**

$$\text{Dividend yield} = Div_1 / P_0$$

In case of growing perpetuity, recall:

$$P_0 = Div_1 / (r_E - g) \Rightarrow Div_1 / P_0 = r_E - g$$

Both the required rate of return and growth rate of company A are higher, but we don't know by how much.

b) **iii) Stock C and D should have the same PE ratio**

$$\text{PE ratio} = \frac{P_0}{E_1} = \frac{\rho}{r_E - g}$$

Disregarding  $\rho$ , the required rate of return and growth rate is the same for both companies.

c) **ii) F**

Higher profit margin  $\Rightarrow$  willing to pay more per dollar of current sales.

d) **iii) Can't tell**

Lower user growth rate, but higher sales revenue per user  $\Rightarrow$  not sure which one dominates.

e)

i) **Same multiple of earnings** because growth rates and required rates of return are the same

ii) **Company I should trade at higher multiple of sales** due to higher profit margin

iii) **Can't tell for multiple of users** because J generates for sales per user but has lower profit margin

f) **ii) L**

Higher growth rate  $\Rightarrow$  lower dividend yield

## Capital Budgeting

### 10)

(This is the same question as from Winter 2013 Assignment 3. So as long as you attempt it you will receive full credit.)

Today is  $t = 0$ . Your steel company has an existing blast furnace and you are evaluating a proposal to replace it with a new furnace.

Ten years ago, at  $t = -10$ , you paid \$20M for the old furnace, and for depreciation purposes you assumed a 20-year asset life, and a \$0 salvage value.

The new furnace costs \$42M at  $t = 0$ , and for depreciation purposes you will assume a 10-year asset life, and a \$2M salvage value.

Regardless of whether you replace the furnace or not at  $t = 0$ , at  $t = 2$  you will sell whichever furnace you have left. At  $t = 2$ , you believe you could sell the old furnace for \$12M (before taxes), if you still have it, or the new furnace for \$36M (before taxes). Note, however, that if you replace the furnace at  $t = 0$ , then you will sell the old furnace at  $t = 0$  for \$16M (before taxes).

Replacing the furnace will result in \$3.2M in additional new sales per year and will save \$2.8M per year in operating expenses, from  $t = 1$  to  $t = 2$ . The current sales and costs are \$90M and \$25M per year, respectively.

Assume for sales that customers pay 40% in cash at the time of purchase and the remaining 60% is paid in cash 1 year later. Assume for operating expenses that the company pays 20% in cash in the year the expense is incurred, and the remaining 80% is paid in cash the following year. (Capital purchases and sales, however, are entirely in cash at the time of purchase or sale.)

The company's tax rate is 40%. Assume that if the company does not replace the furnace, it will continue operations with the existing furnace until  $t = 2$ , and then sell the old furnace at  $t = 2$ . Fill in the table below indicating the incremental values associated with replacing the furnace as appropriate. Show your calculations separately.

### **Solution:**

**$t = 0$ :**

Purchase of new furnace:

Outflow = 42M

Sale of old furnace:

Inflow = 16M - taxes = 16M - 40%(gain on disposal) = 16M - 40%(proceeds - book value) = 16M - 40%(16M - (20M - 10(1M))) = 16M - 40%(6M) = 13.6M

**$t = 2$ :**

Sale of new furnace:

Inflow = 36M - taxes = 36M - 40%(36M - book value) = 36M - 40%(36M - (42M - 2(4M))) = 35.2M

Opportunity cost of not being able to sell old furnace (since you would have sold it at  $t = 0$ ):

= 12M - taxes = 12M - 40%(12M - book value) = 12M - 40%(12M - (20M - 12(1M))) = 10.4M

### **Accounts Receivable (AR):**

**$t = 1$ :**

60% of 3.2M of AR are created. This increase in AR needs to be subtracted.

**$t = 2$ :**

60% of 3.2M of AR are created, but the 60% of 3.2M of AR that were created at  $t = 1$  are being paid off. So at  $t = 2$  there is no net change in AR.

**$t = 3$ :**

No new AR are created, but the 60% of 3.2M of AR that were created at  $t = 2$  will be paid off. This decrease in AR needs to be added back to net income.

### **Accounts Payable (AP):**

In this example there are fewer expenses if going with the new furnace. But these benefits are not all cash benefits since 80% of expenses are paid on credit.

Thus at  $t = 1$ , have to make a downward adjustment of 80%(2.8M).

As with the AR, there is no net change in AP at  $t = 2$ .

One way to think about  $t = 3$  is that 80% of the 2.8M lower  $t = 2$  expense is realized as a cash benefit at  $t = 3$ .

	Incremental Value	t = 0	t = 1	t = 2	t = 3
	Revenue		3.2M	3.2M	
-	Operating Expenses		2.8M	2.8M	
-	Depreciation		(3M)	(3M)	
=	Operating Profit		3M	3M	
-	Taxes		(1.2M)	(1.2M)	
	Net Income (excluding profit from				
=	equipment sale)		1.8M	1.8M	
	Depreciation		3M	3M	
	Purchase/Sale t = 0	-42M + 16M			
	Purchase/Opportunity cost t = 2			35.2M - 10.4M	
	A/R		-60% * 3.2M		0 60% * 3.2M
	A/P		-80% * 2.8M		0 80% * 2.8M
=	Net Cash Flow	(28.4M)	0.64M	29.6M	4.16M