# Math 240: Discrete Structures I (W18) – Assignment 8

Solutions must typed or very neatly written and uploaded to MyCourses no later than **6 pm** on **Saturday, March 31, 2018**. Up to 4 bonus marks will be awarded for solutions typeset in LaTeX; both the .tex file and .pdf file must be uploaded.

*You may use theorems proven or stated in class, but you must state the theorem you are using. All work must be shown for full marks.*

[14] 1. **Principle of Inclusion-Exculsion .**

(a) Determine the number of integers from 1 to $1,000,000$ which are relatively prime to $990,099$. [Note: there is a formula that one can use to calculate this. You may, by all means, read up about it but you may not use it to answer the problem.]

(b) Out of a group of 30 people, 20 people play flute, 8 play piano, and 25 play violin. If 20 people play at least two of the three instruments and 6 play all three, how many of the 30 people play none of the three instruments?

[14] 2. **Pigeon Hole Principle.**

(a) Let $X$ be a subset of the integers from 1 to 1997 such that $|X| \geq 34$. Show that there exist distinct $a, b, c \in X$ and distinct $x, y, z \in X$ such that $a + b + c = x + y + z$ and $\{a, b, c\} \neq \{x, y, z\}$.

(b) Let $n$ be any positive integer, and let $d$ be a positive integer such that $1 \leq d \leq 9$. Show that some multiple of $n$ can be expressed in base 10 using only the digits 0 and $d$.

[12] 3. **Graphs.** For each of the following degree sequences, either show that no graph exists having that degree sequence or construct an example showing that a graph does exist with that degree sequence. If you give an example, give a set representation of its vertices and edges as well as a drawing of the graph.

(a) $0, 2, 3, 3, 4, 5, 6, 6, 7$

(b) $0, 2, 3, 3, 4, 5, 6, 7, 8$

(c) $1, 2, 3, 3, 4, 5, 6, 7, 8$

(d) $1, 2, 3, 3, 4, 5, 6, 8, 8$