COMP 546

Lecture 8

image motion

Tues. Feb. 5, 2019

What is motion ?

Time $t + \Delta t$

Time t

(X_2, Y_2, Z_2)

(X_1, Y_1, Z_1)

"Motion" is a change in position over time.

What is image motion ?

Time $t + \Delta t$

Time t

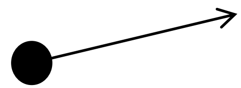
(x_2, y_2)

(x_1, y_1)

"Image motion" is a change in image position over time.

What is image velocity ?

Time t

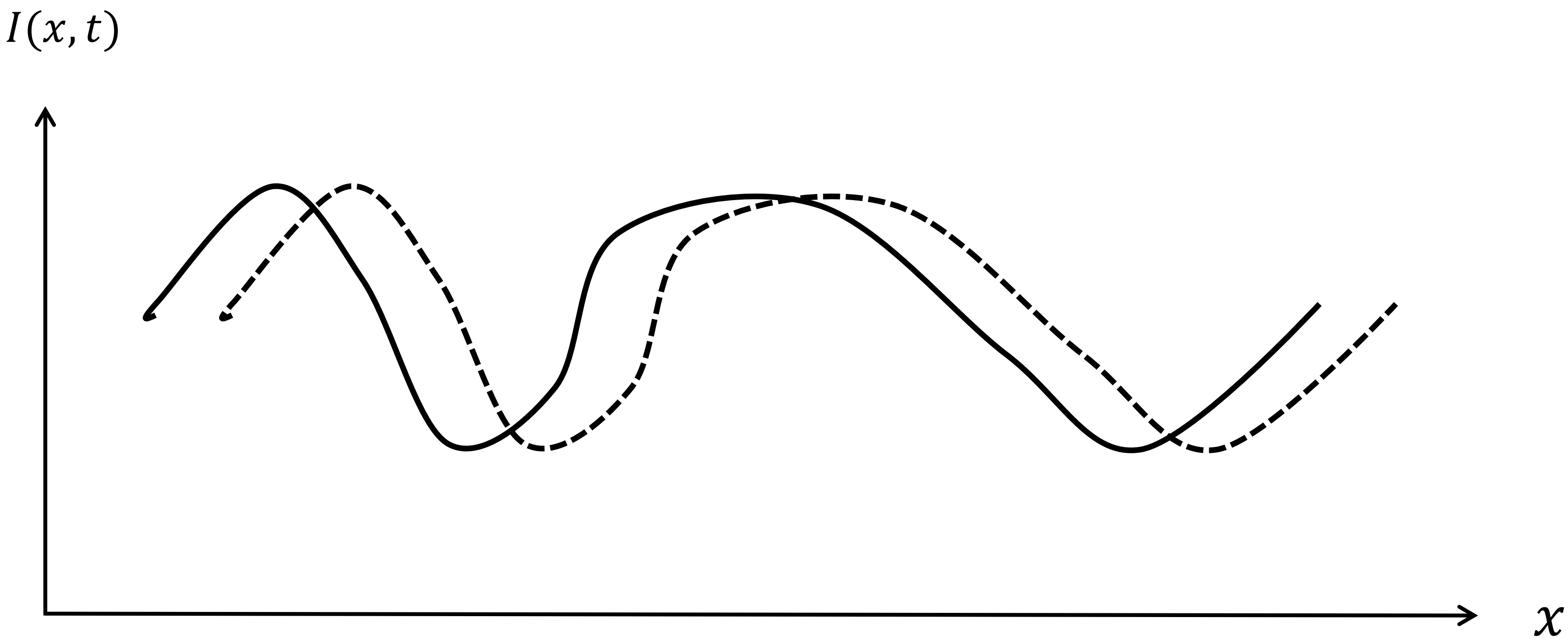


$$(v_x, v_y) = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)$$

"Image velocity" is a rate of change of image position over time.

But a change in position of *what*?

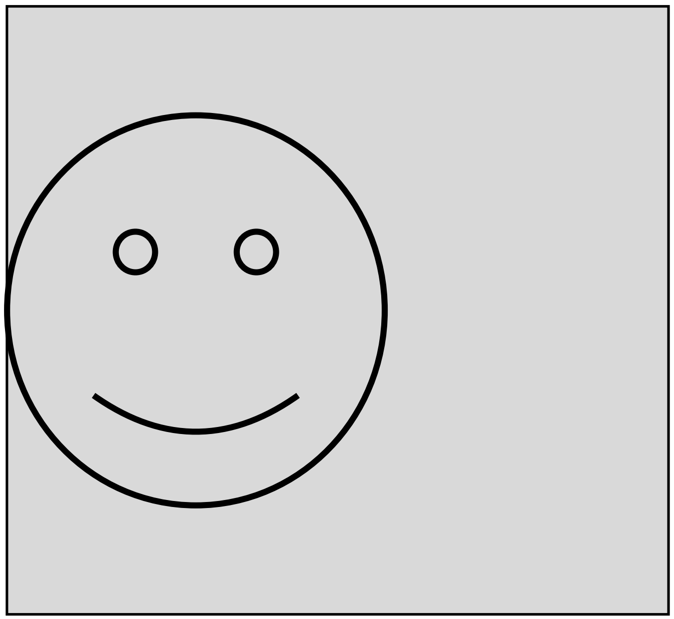
e.g. Motion for 1D image $I(x, t)$



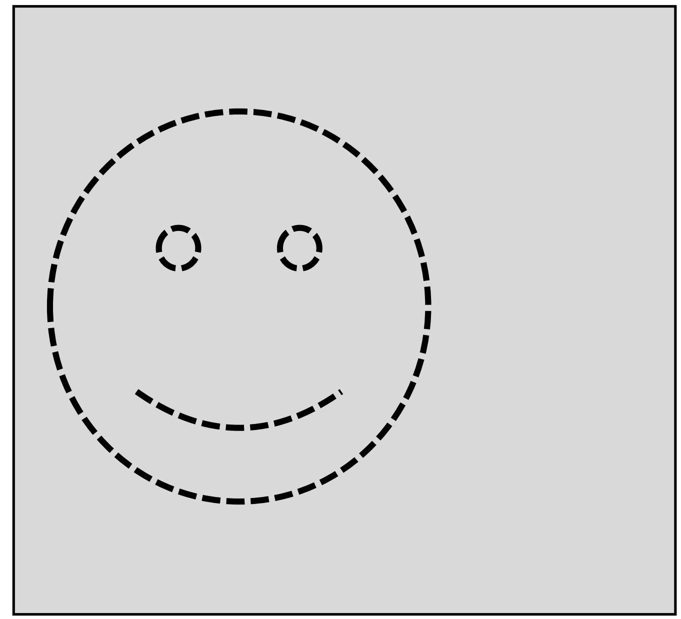
Suppose the *intensities* shift position from time t_0 to time $t_0 + \Delta t$.

Motion for 2D image $I(x, y, t)$

$I(x, y, t_0)$



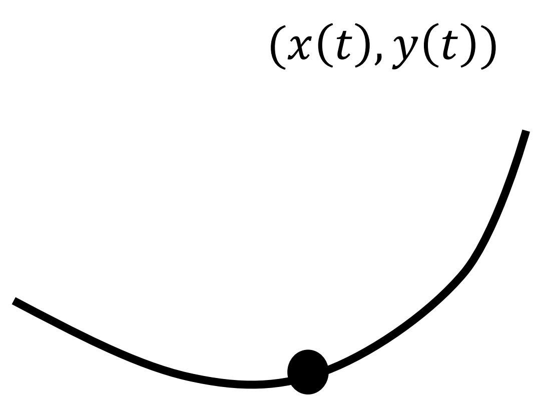
$I(x, y, t_0 + \Delta t)$



Suppose the *intensities* shift position from time t_0 to time $t_0 + \Delta t$.

The shift property is commonly expressed in 2D images as follows.

Let $(x(t), y(t))$ be the *path* of a particular moving point.



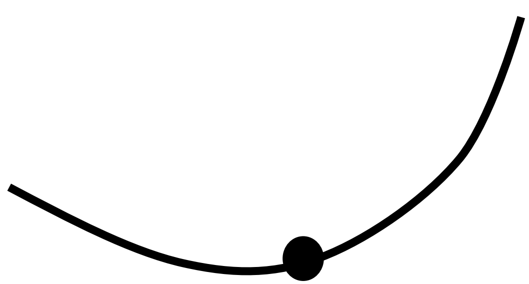
Intensity conservation assumption

The shift property is commonly expressed in 2D images as follows.

Let $(x(t), y(t))$ be the *path* of a particular moving point.

Assume the moving point's intensity doesn't change over time:

$$\frac{d}{dt} I(x(t), y(t), t) = 0$$



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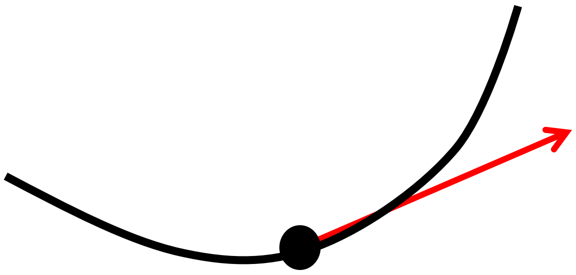

$$(v_x, v_y) = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)$$

Image velocity

We can write the changing intensities along any path as follows:

$$\frac{d\,I(x(t),y(t),t)}{dt} = \frac{\partial I}{\partial x}\frac{dx(t)}{dt} + \frac{\partial I}{\partial y}\frac{dy(t)}{dt} + \frac{\partial I}{\partial t}$$

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The intensity conservation assumption says that:

$$\frac{d\, I(x(t), y(t), t)}{dt} = \frac{\partial I}{\partial x}\underbrace{\frac{dx(t)}{dt}}_{v_x} + \frac{\partial I}{\partial y}\underbrace{\frac{dy(t)}{dt}}_{v_y} + \frac{\partial I}{\partial t} = 0$$

Motion Constraint Equation

$$\frac{\partial I}{\partial x} \; {\color{red} v_x} + \frac{\partial I}{\partial y} \; {\color{red} v_y} + \frac{\partial I}{\partial t} = 0$$

i.e. Partial derivatives in intensity give a linear constraint on the image velocity.

Note: you can measure these partial derivatives.

Motion Constraint Equation

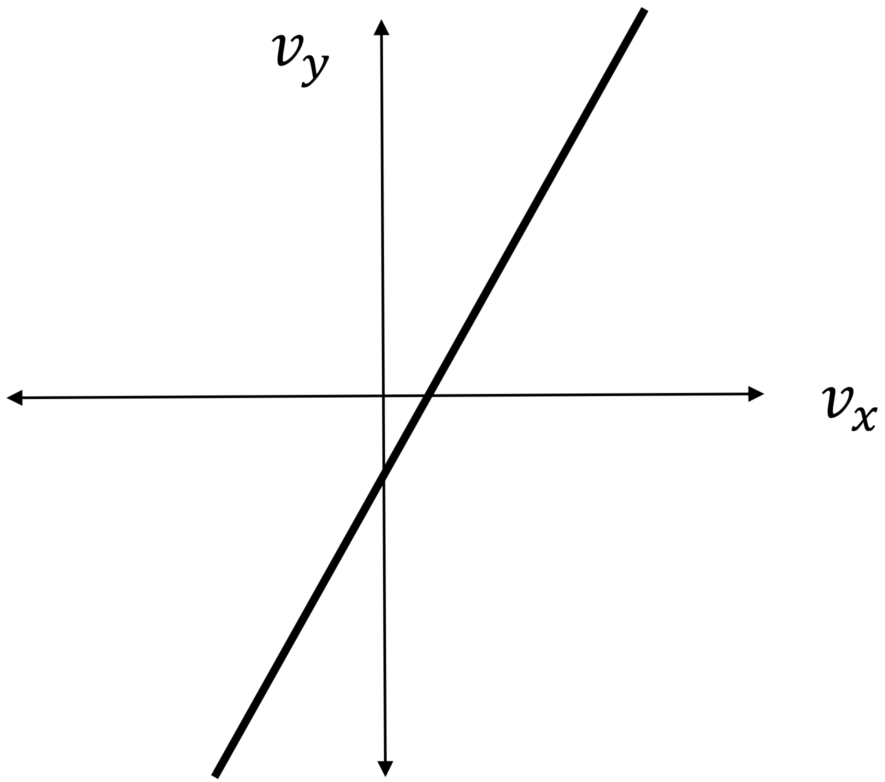
$$\frac{\partial I}{\partial x} \; v_x + \frac{\partial I}{\partial y} \; v_y + \frac{\partial I}{\partial t} = 0$$

$$\left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right) \cdot \left(v_x, v_y, \right) + \frac{\partial I}{dt} = 0$$

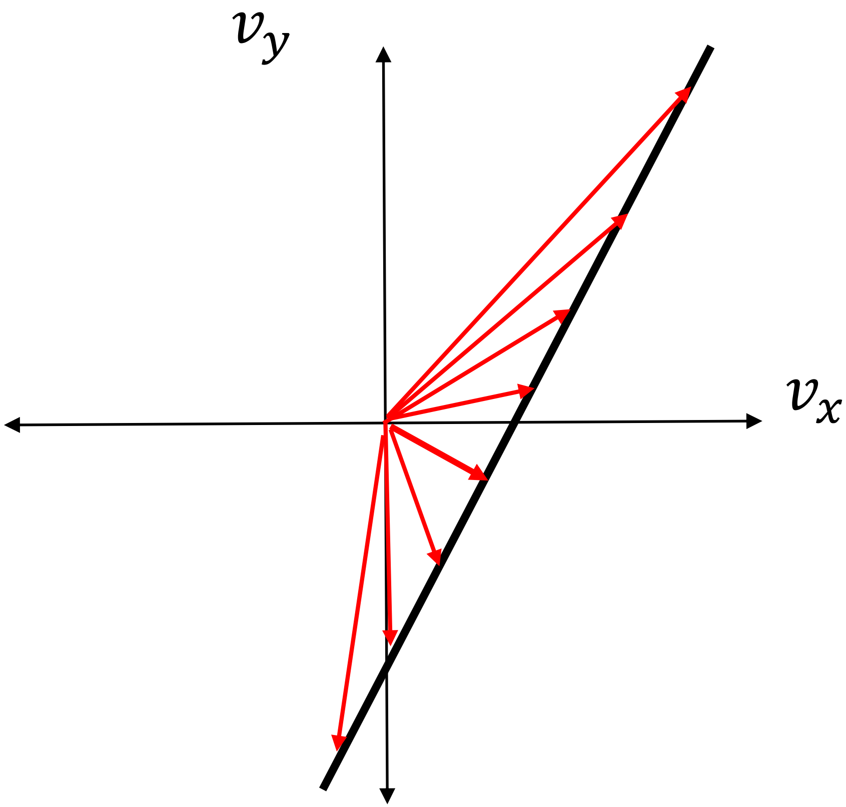
There is *no constraint* on the component of velocity that is perpendicular to spatial derivative.

Motion constraint equation is a line in *velocity space*

$$\frac{\partial I}{\partial x} v_x + \frac{\partial I}{\partial y} v_y + \frac{\partial I}{\partial t} = 0$$



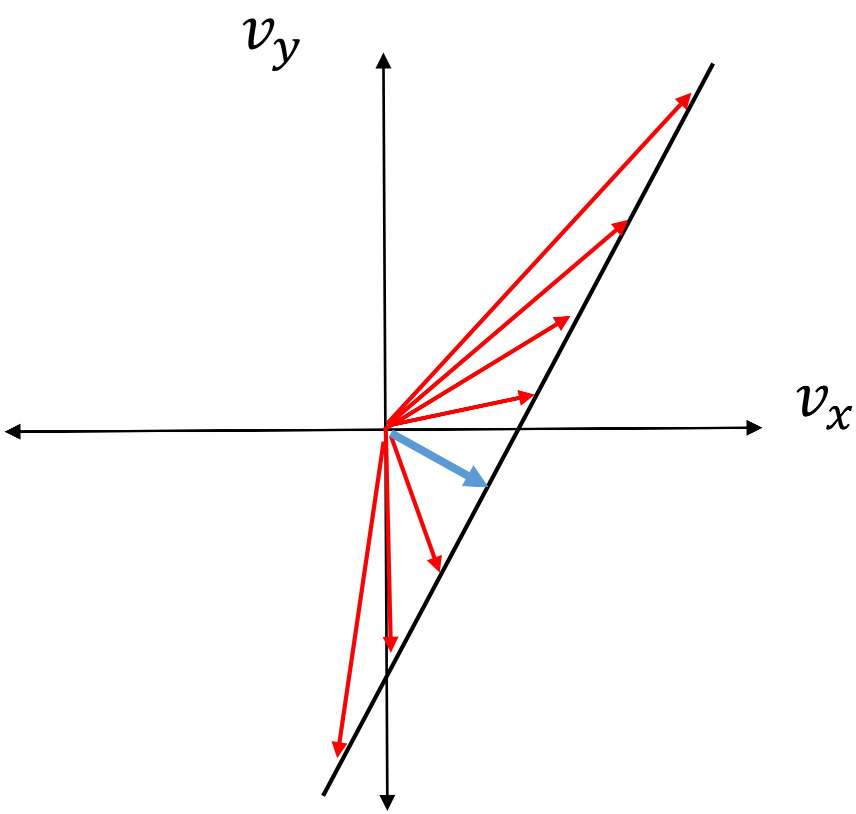
$$\frac{\partial I}{\partial x} v_x + \frac{\partial I}{\partial y} v_y + \frac{\partial I}{\partial t} = 0$$



Many velocities (v_x, v_y) satisfy any given motion constraint equation.

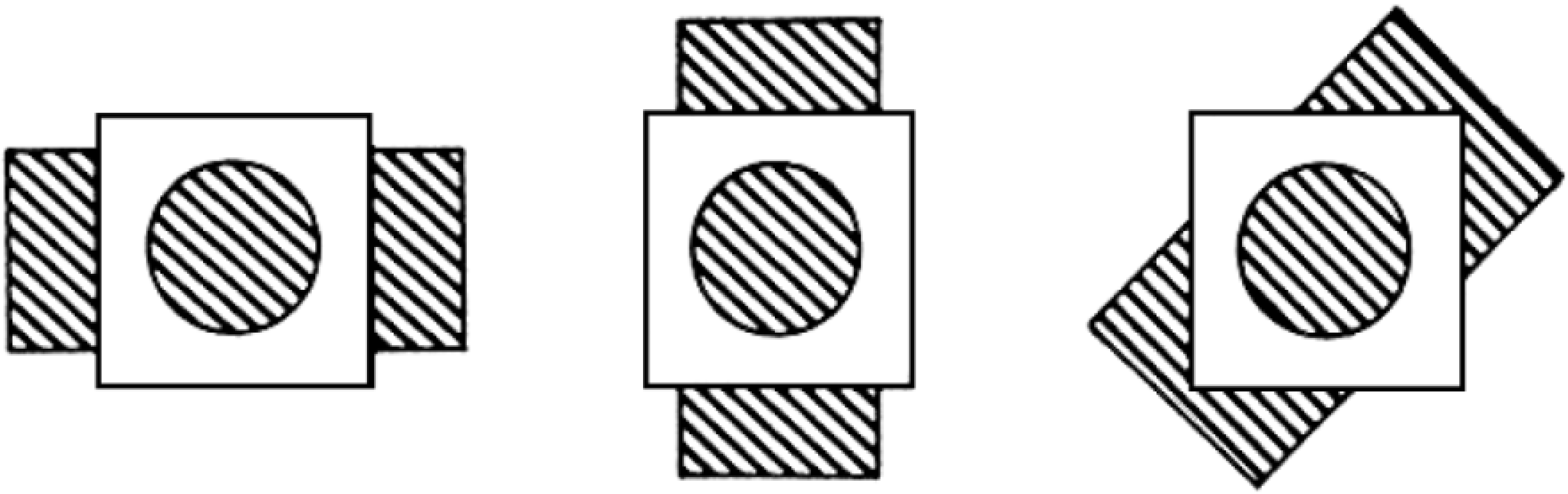
Normal velocity

$$\frac{\partial I}{\partial x} \; v_x + \frac{\partial I}{\partial y} \; v_y + \frac{\partial I}{\partial t} = 0$$



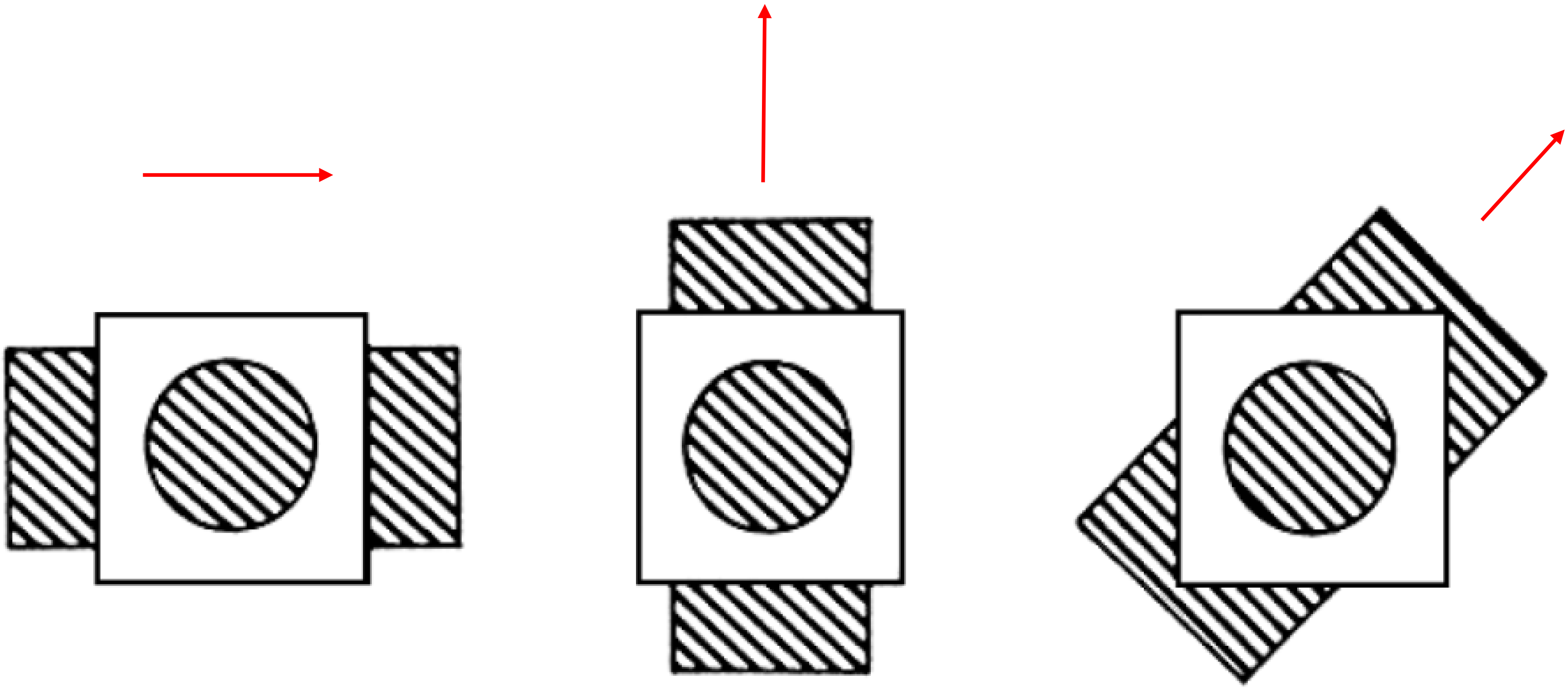
"Normal velocity" is the (v_x, v_y) that is in the direction of the XY gradient.

“Aperture Problem”



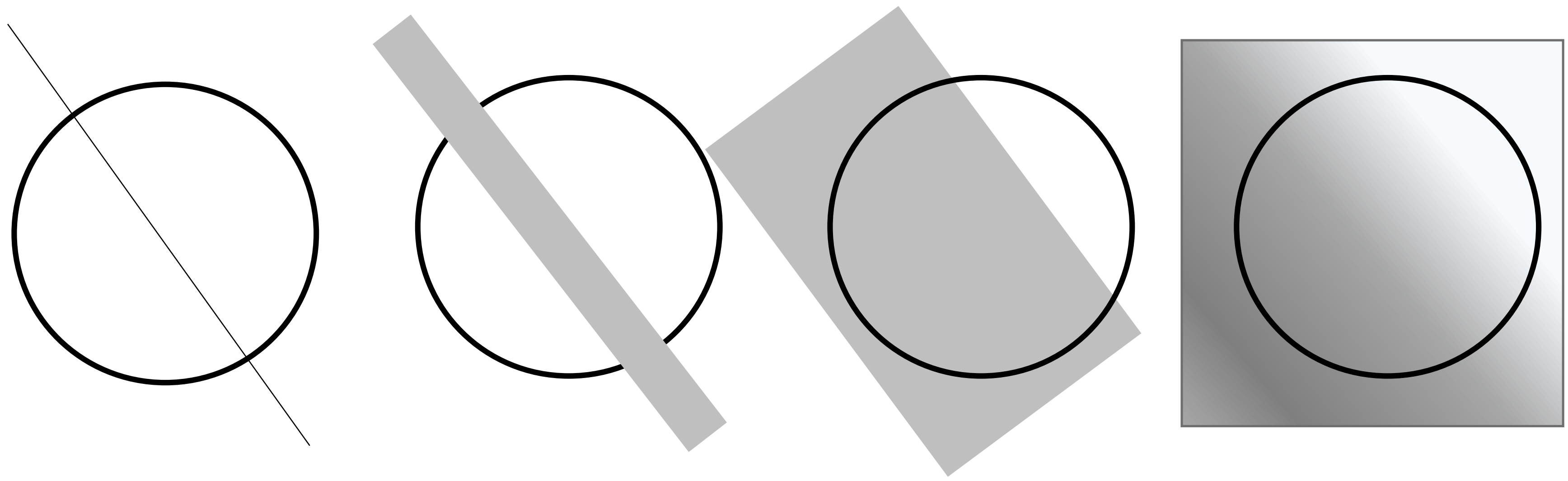
It is possible to have different image velocities (v_x, v_y) in three cases, which produce the same motion in the three apertures. How ?

“Aperture Problem”



All three image velocities (v_x, v_y) would produce the same image motion within the aperture.

"Aperture Problem"



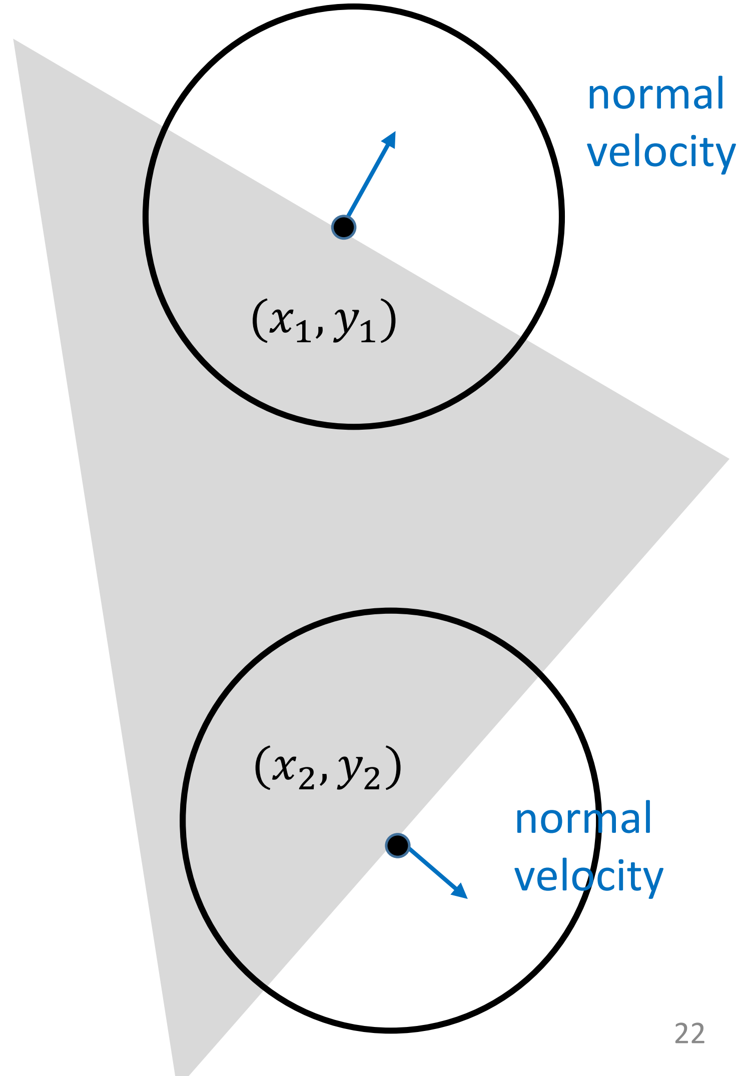
For any translating pattern that has image gradients in one direction only, the visual system can only measure the normal velocity, i.e. the velocity component in the direction of the spatial gradient.

To solve the aperture problem, you need more than one image gradient direction.

Let's look at this first as a computational problem. Then we'll sketch out how the visual system solves this problem.

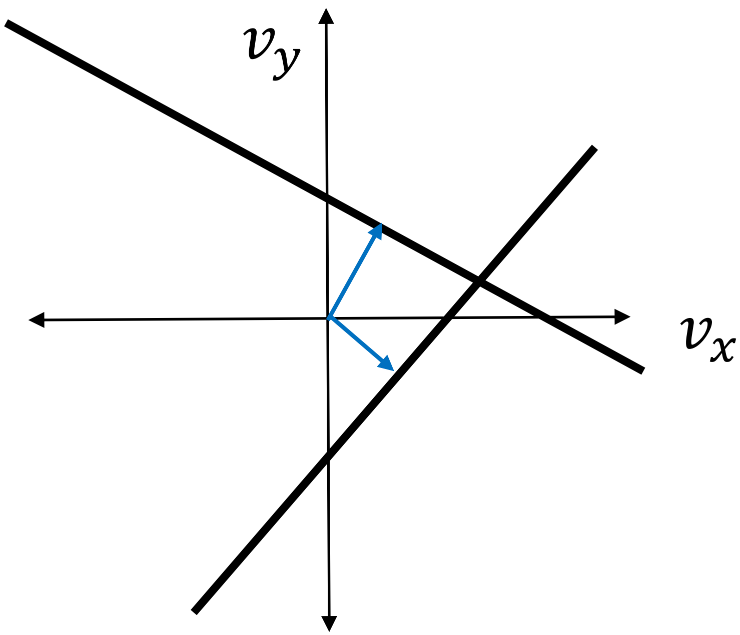
“Intersection of Constraints” (IOC)

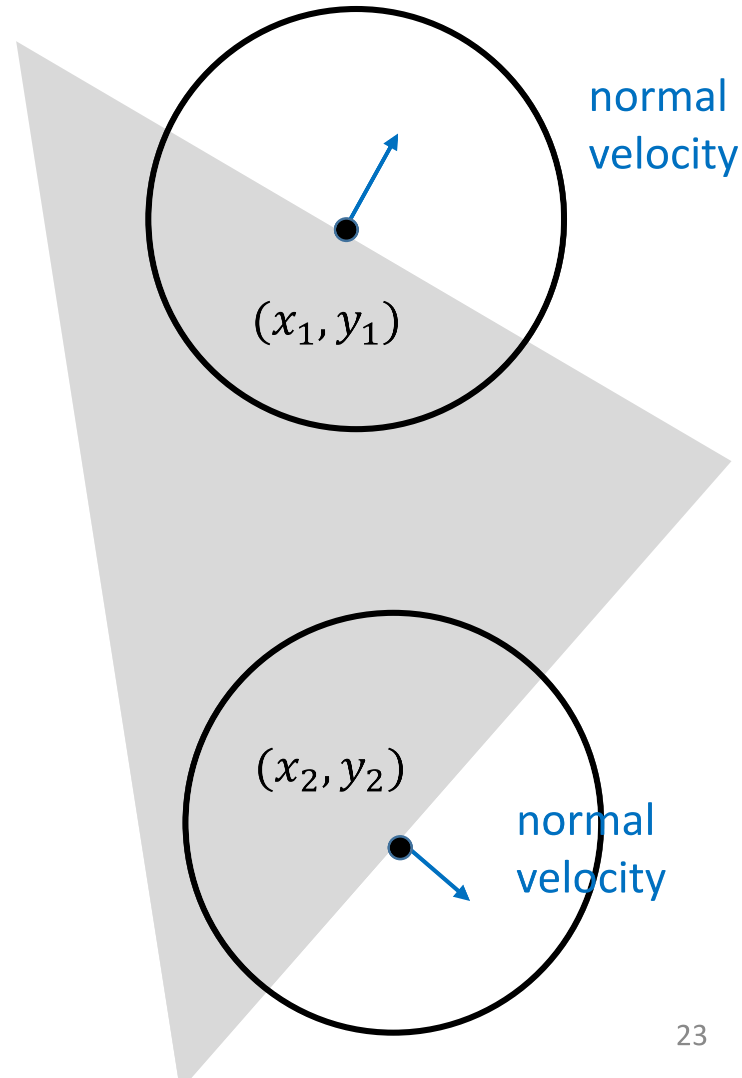
Suppose two nearby image points have two *different spatiotemporal gradients which define two different normal velocities.*



"Intersection of Constraints" (IOC)

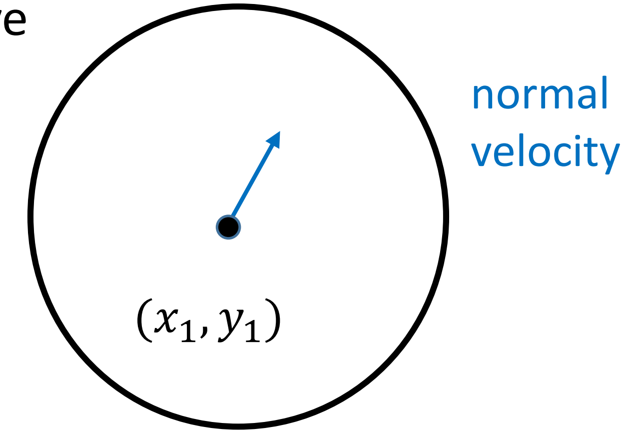
Suppose two nearby image points have two *different spatiotemporal gradients which define two different normal velocities. This defines two motion constraint equations.*

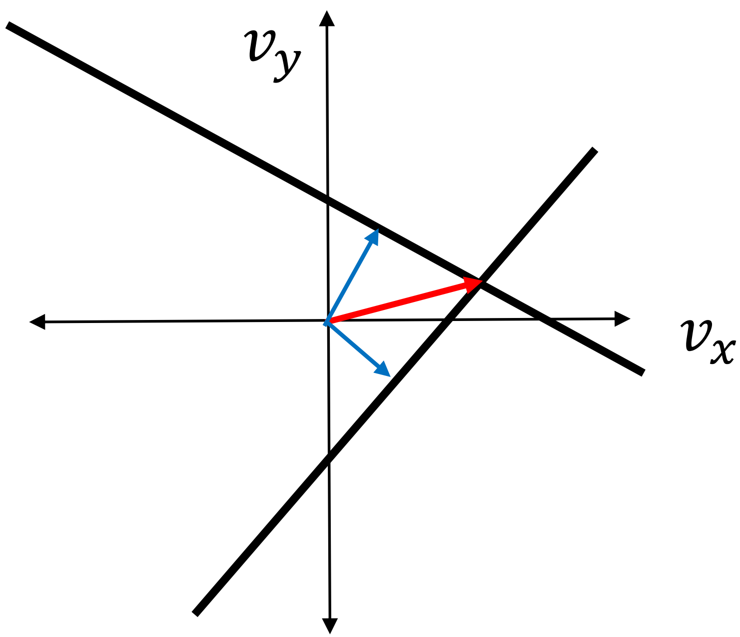


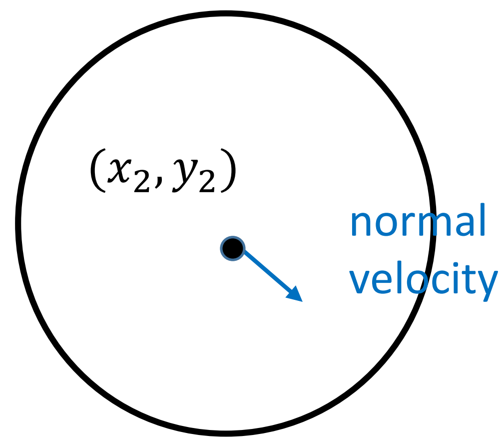


Intersection of Constraints (IOC)

If we assume these two points (regions) have the same image velocity, then we can in principle solve for this velocity using linear algebra:
find the intersection of the two motion constraint lines. IOC gives a unique solution.

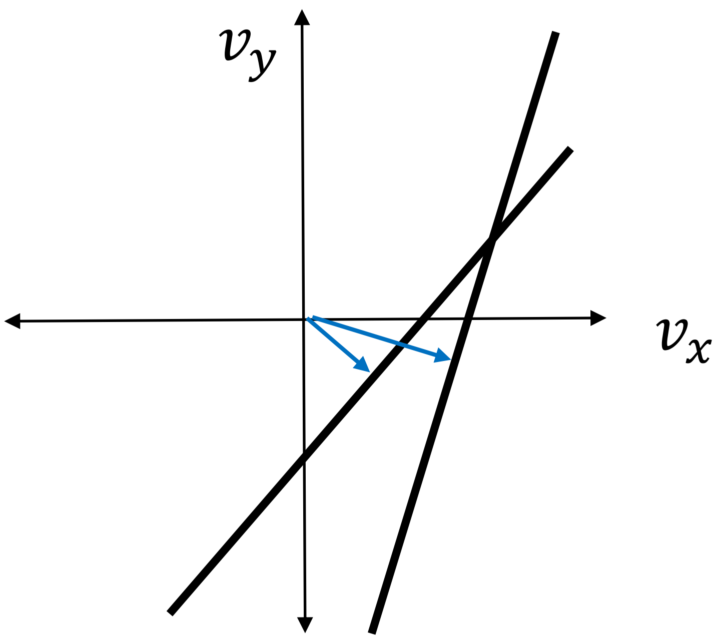


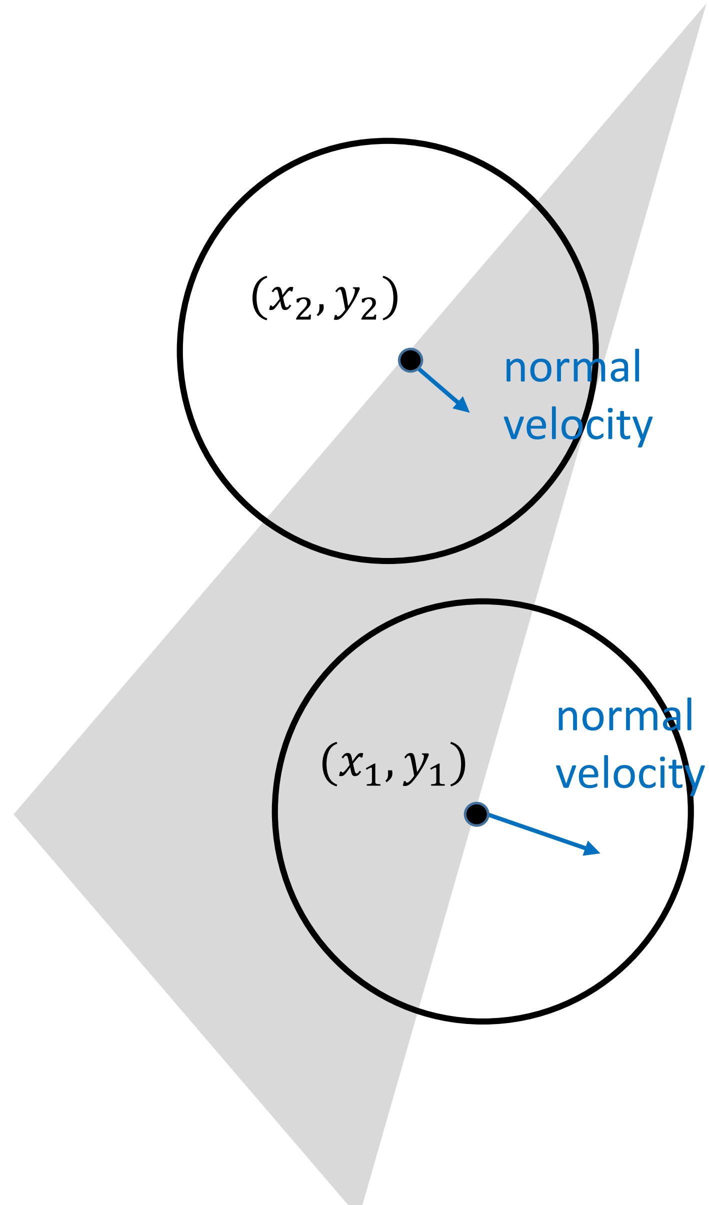




Another Example (counterintuitive)

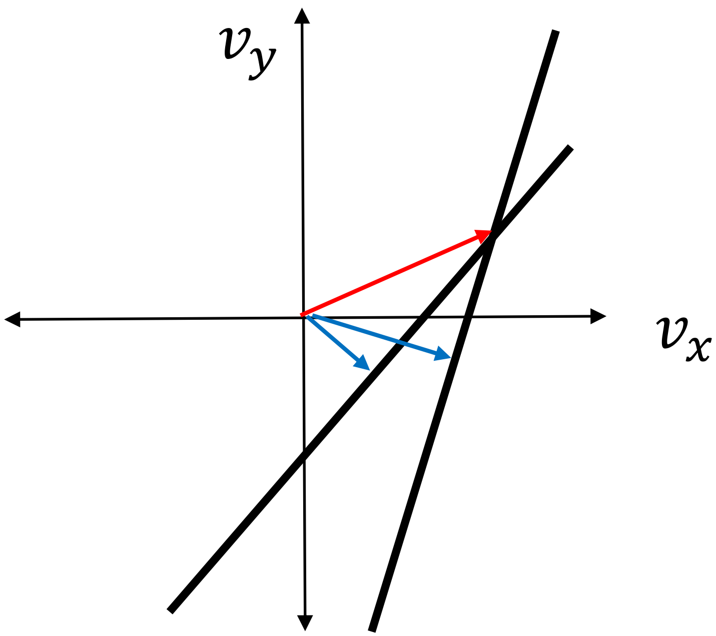
Both normal velocities are downward to the right.

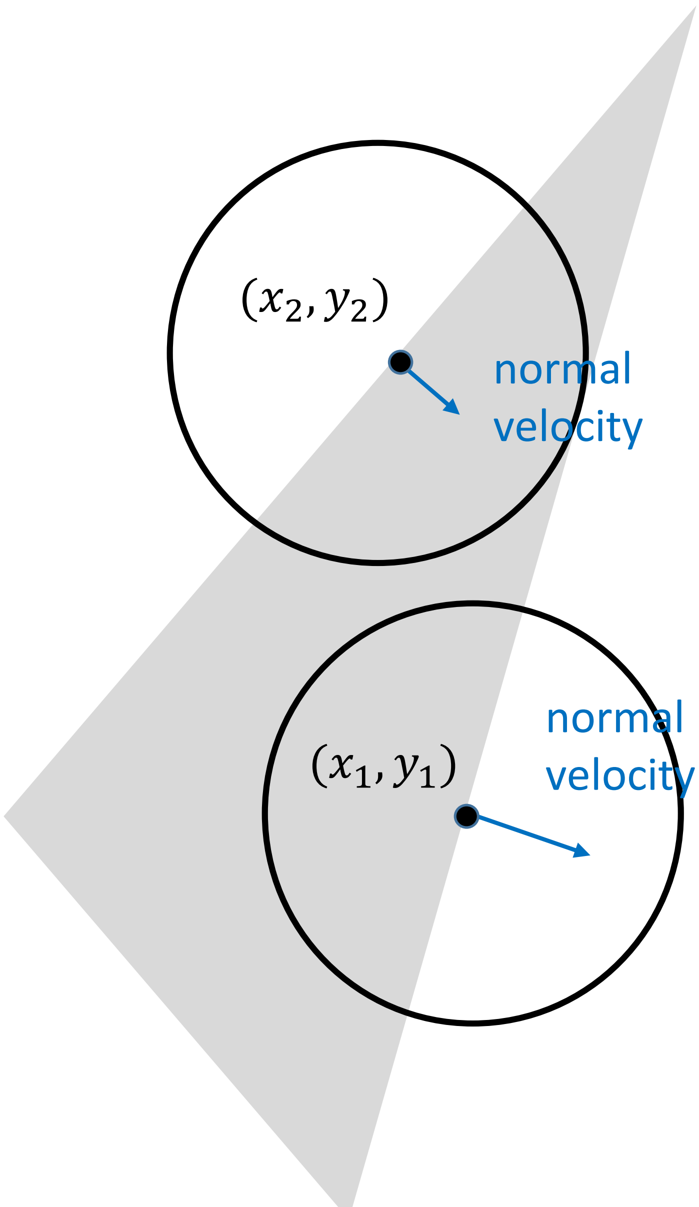




Another Example (counterintuitive)

But the intersection of constraint solution is upward to the right.



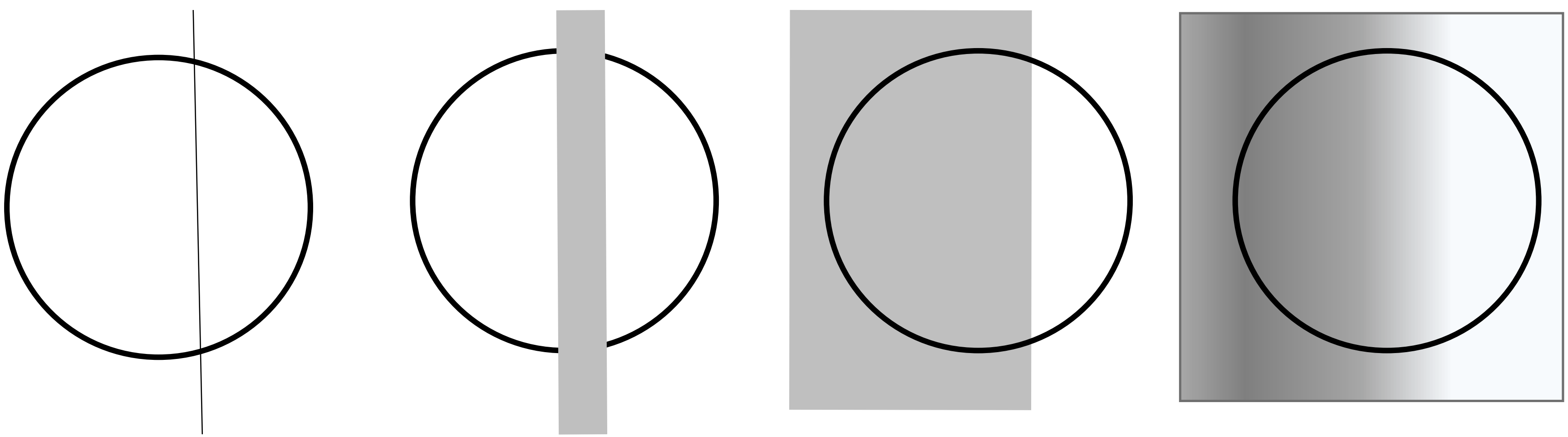


To briefly summarize, we have considered the motion information that is available in a small XYT neighborhood, namely from partial derivatives of image intensity at a point.

Let's now try to relate this to what is measured by cells in V1 of the brain.

Consider the case that image gradient(s) are in *x* direction.

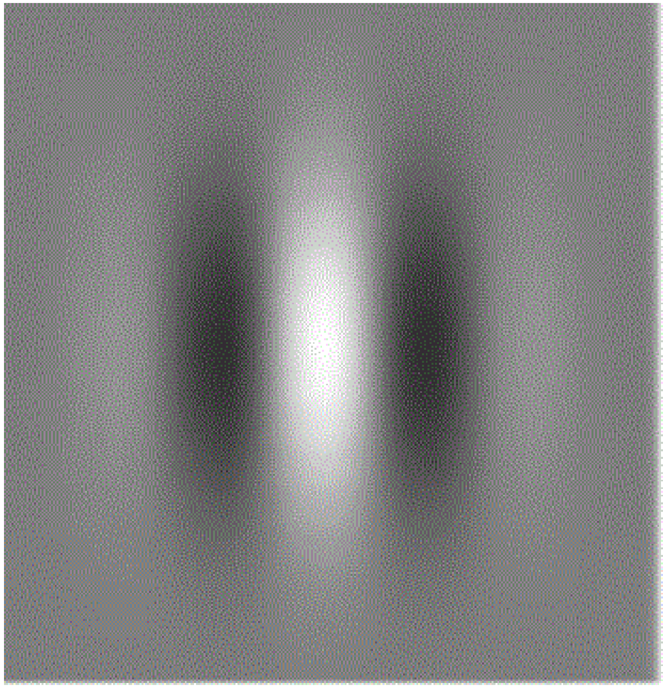
For example:



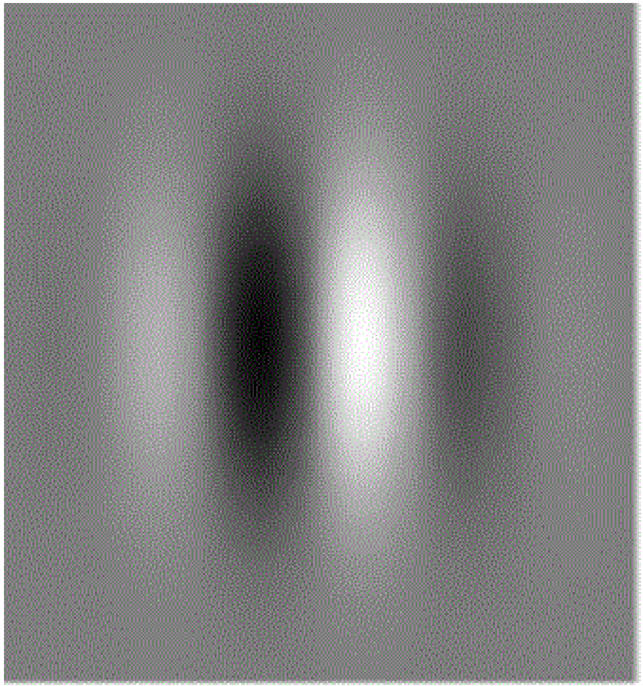
line bar edge ramp

Recall: V1 cells detect oriented structure, and some V1 cells are also motion sensitive (next slide).

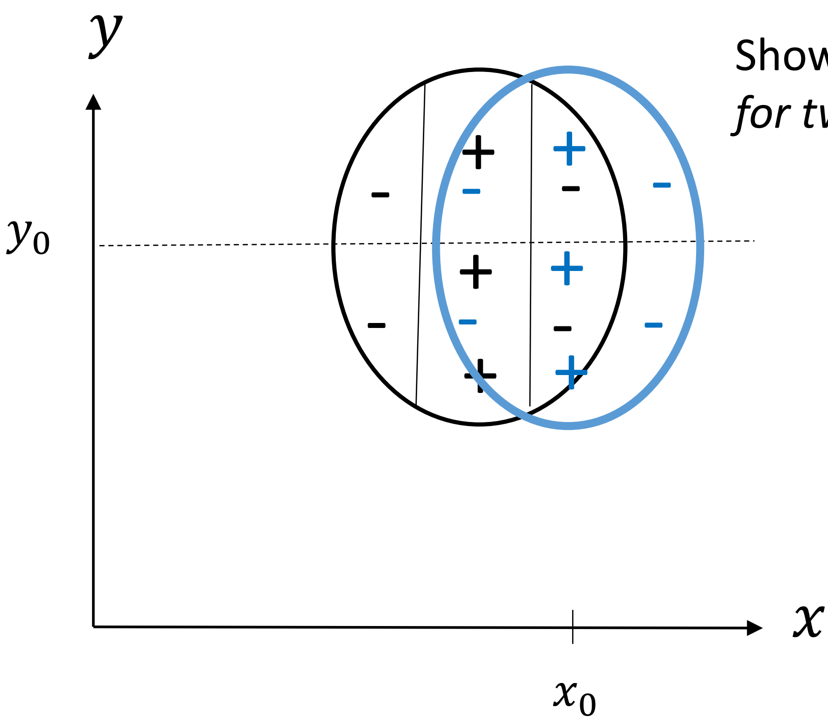
cosGabor(x,y)



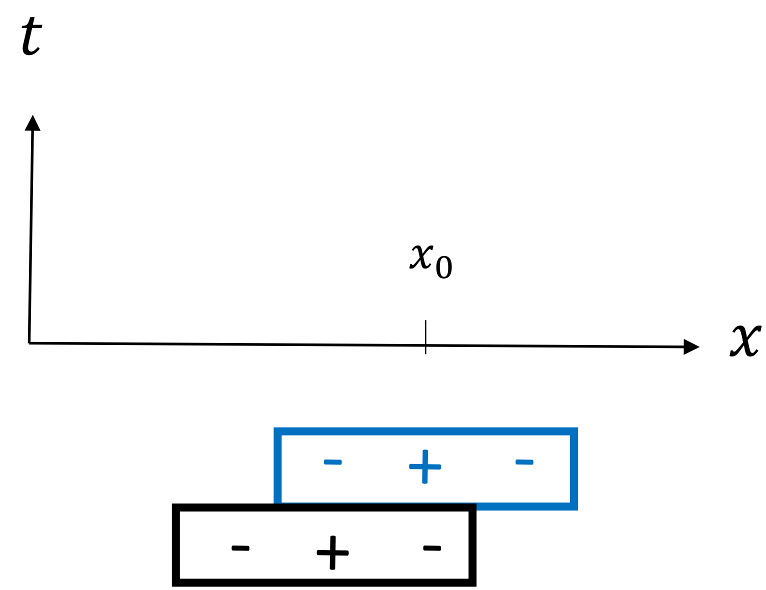
sinGabor(x,y)



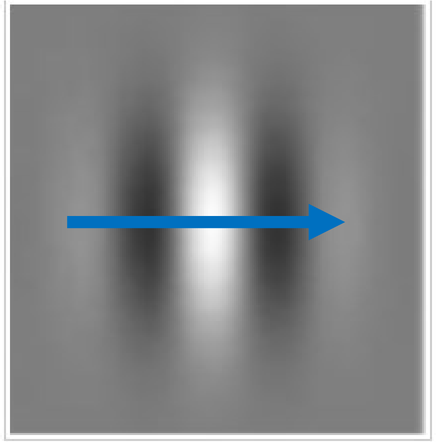
Orientation *and* direction tuned cells in V1

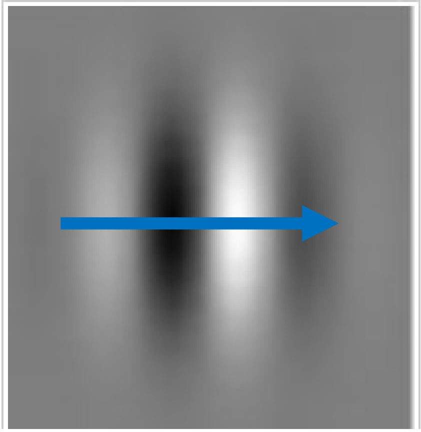


Shows XY receptive fields
for two time slices



Such a cell is selective for vertical line that moves to the right.
This is analogous to binocular disparity cells (last lecture).



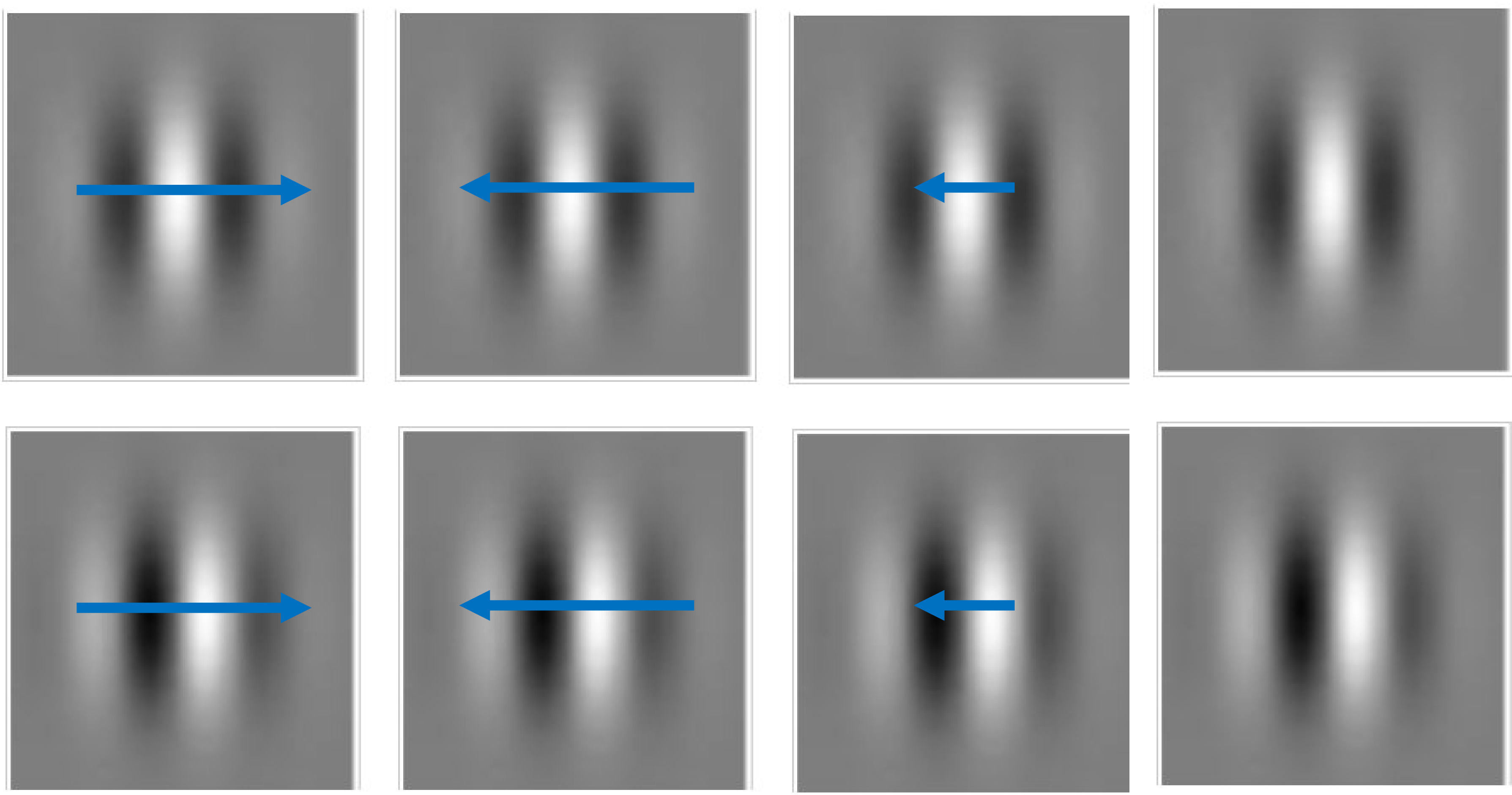


Fact: V1 cells only respond to the motion component that is normal (perpendicular) to their preferred orientation.

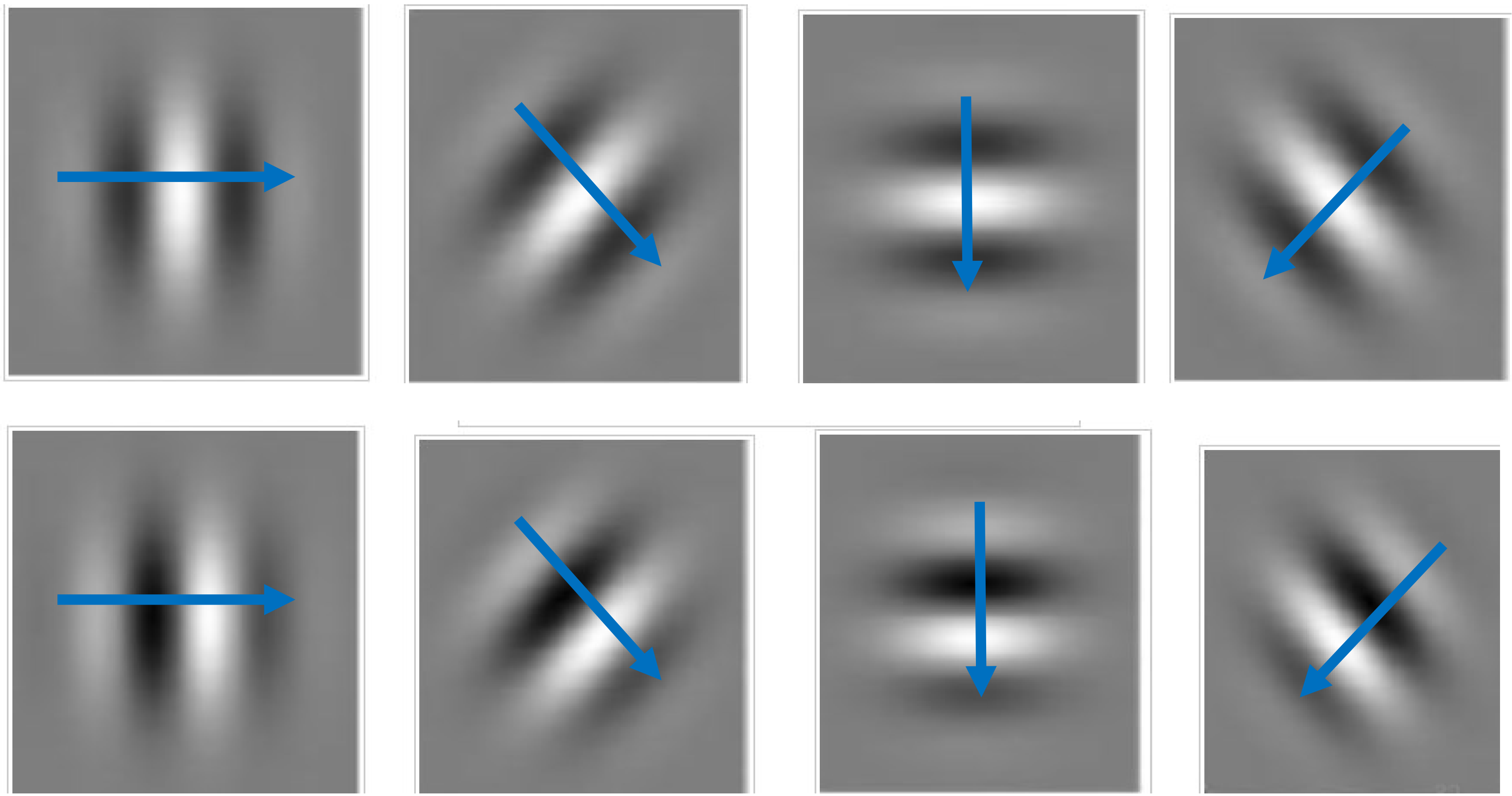
Here we again have the idea of a “normal velocity”.

These cells prefer….

Fast motion to the right | Fast motion to the left | Slow motion to the left | No motion (static)

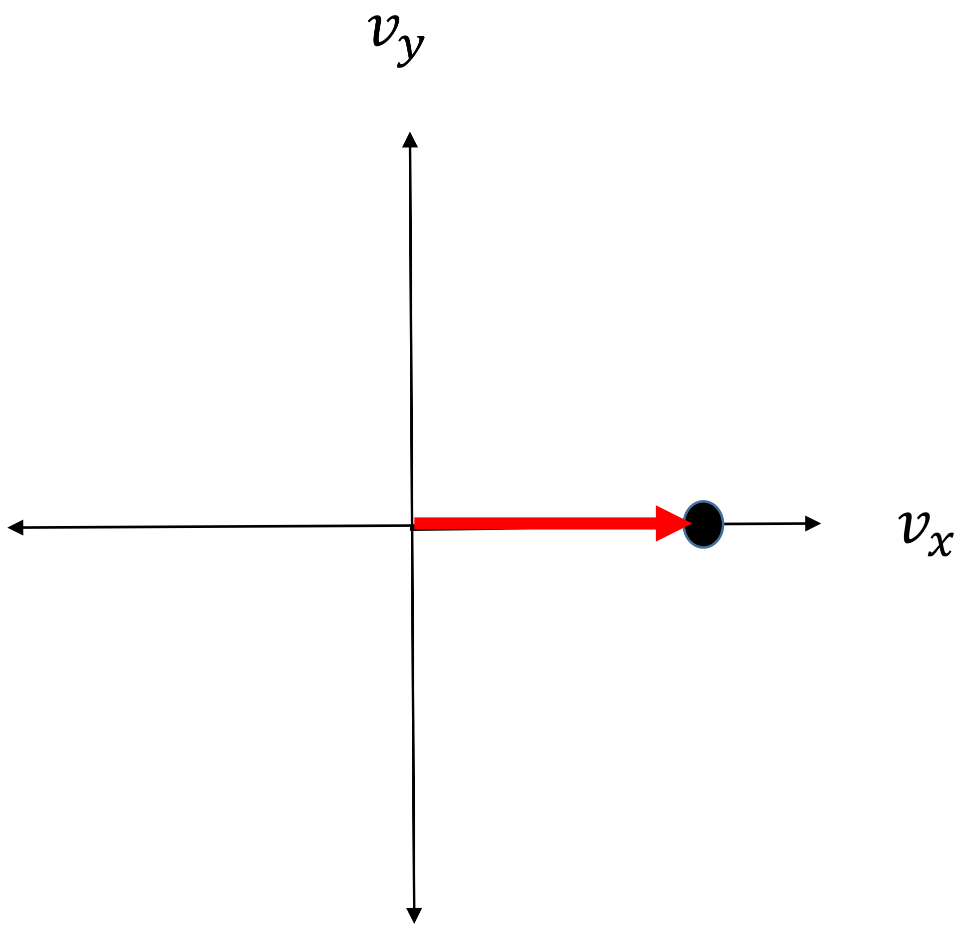


These cells prefer fast motion in different directions, but always normal to the preferred orientation.

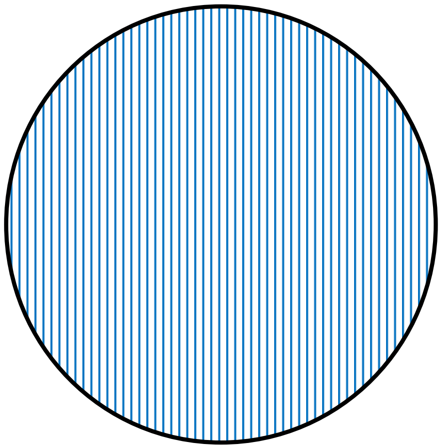


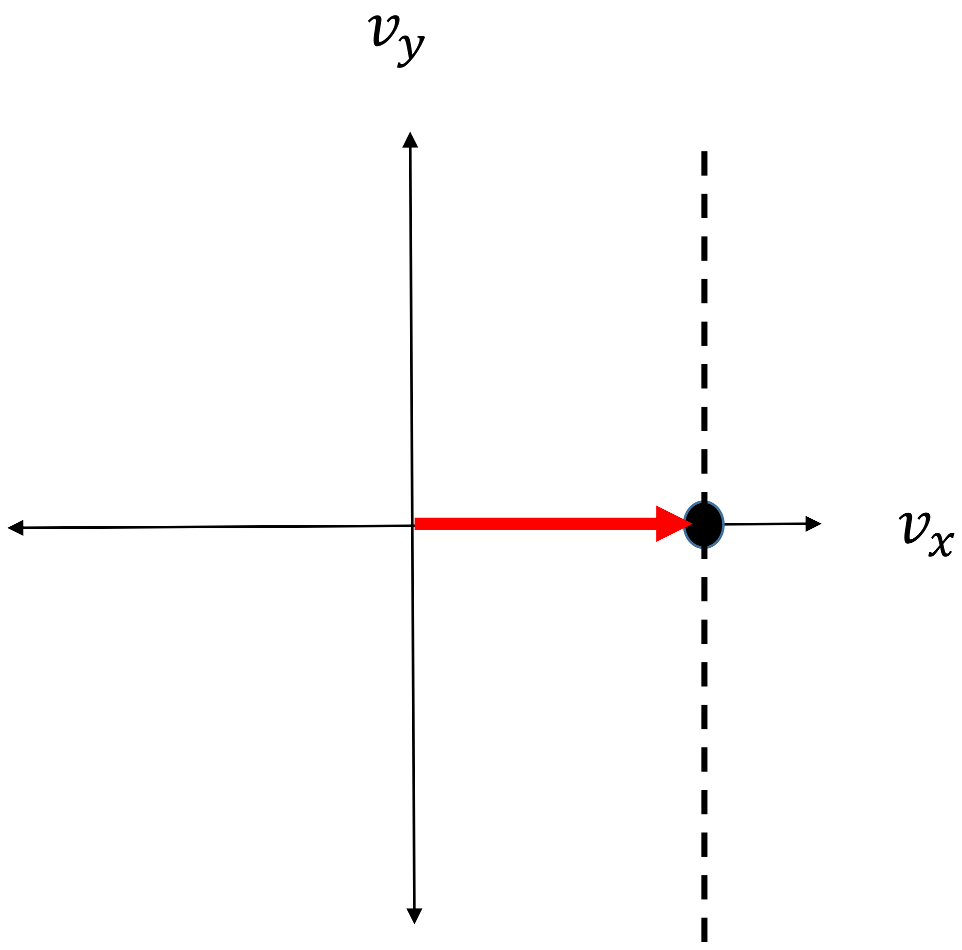
Suppose the true image velocity (v_x, v_y) is toward the right.

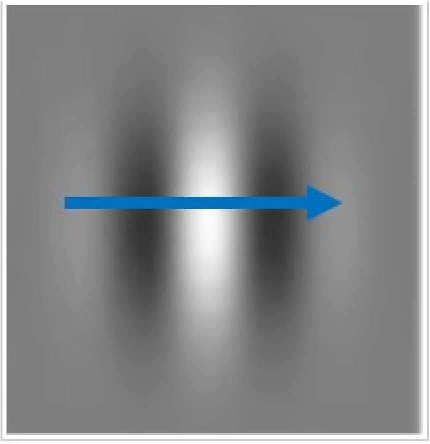
Which V1 motion cells could have a large response in this situation?



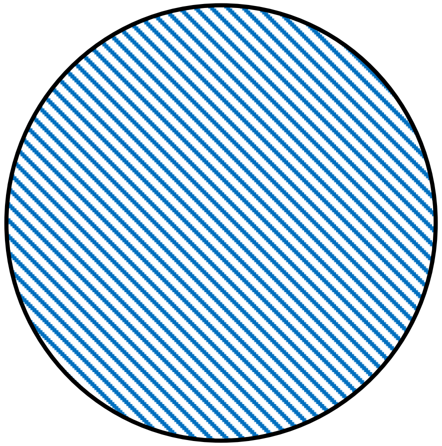
A vertically oriented cell that is sensitive to this horizontal speed could respond well to the horizontal gradient components.

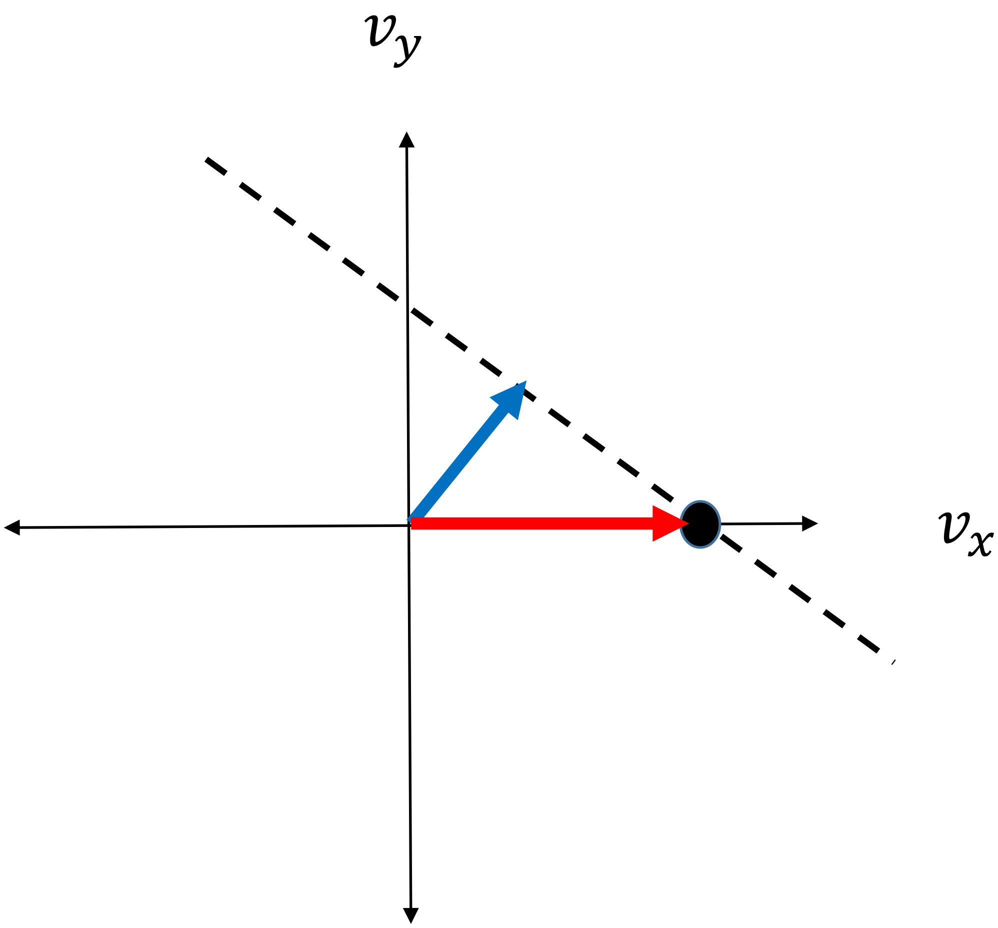


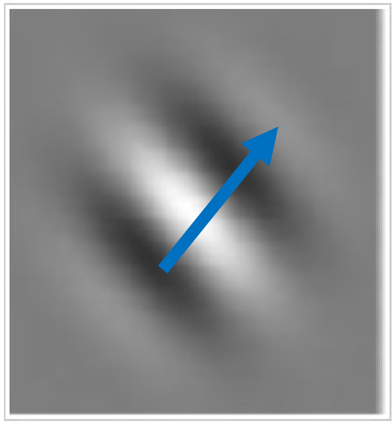




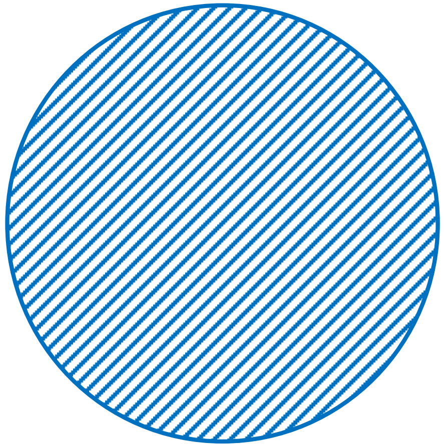
A diagonally oriented cell that is sensitive to a lower image speed (shorter vector) could respond well to the image gradient components in the upper right direction.

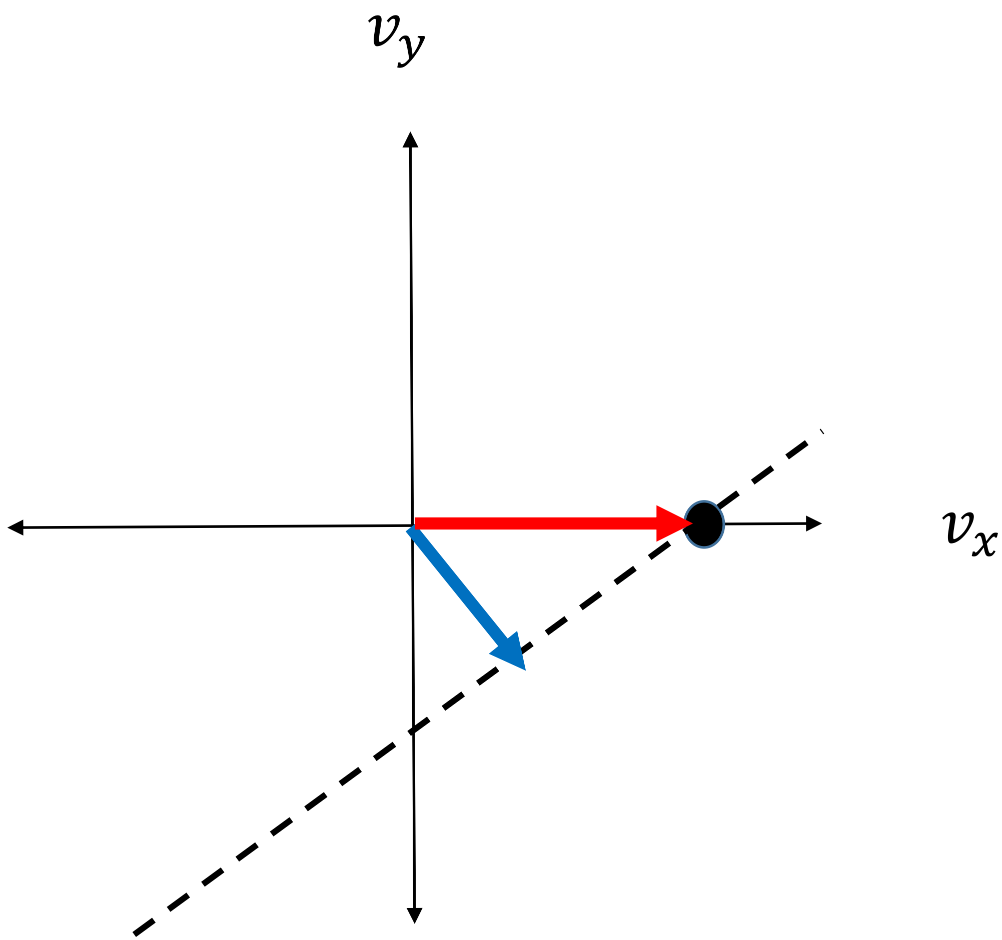


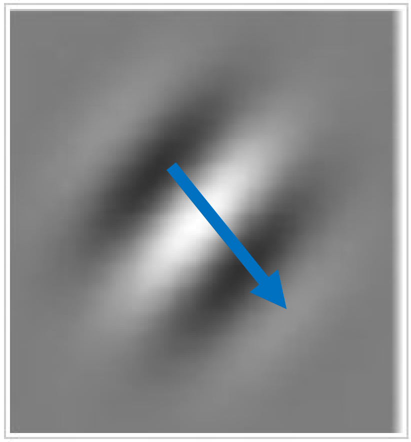




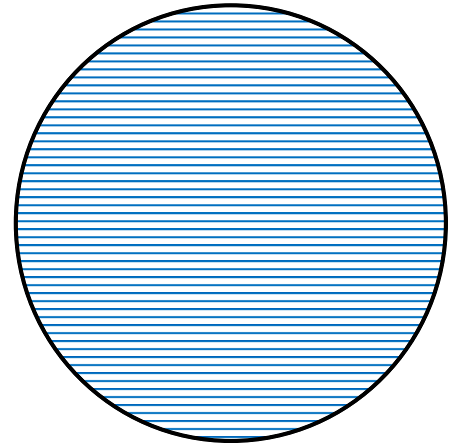
A diagonally oriented cell that is sensitive to a down-right-ward normal velocity could respond well, namely to gradients in downward right diagonal direction.







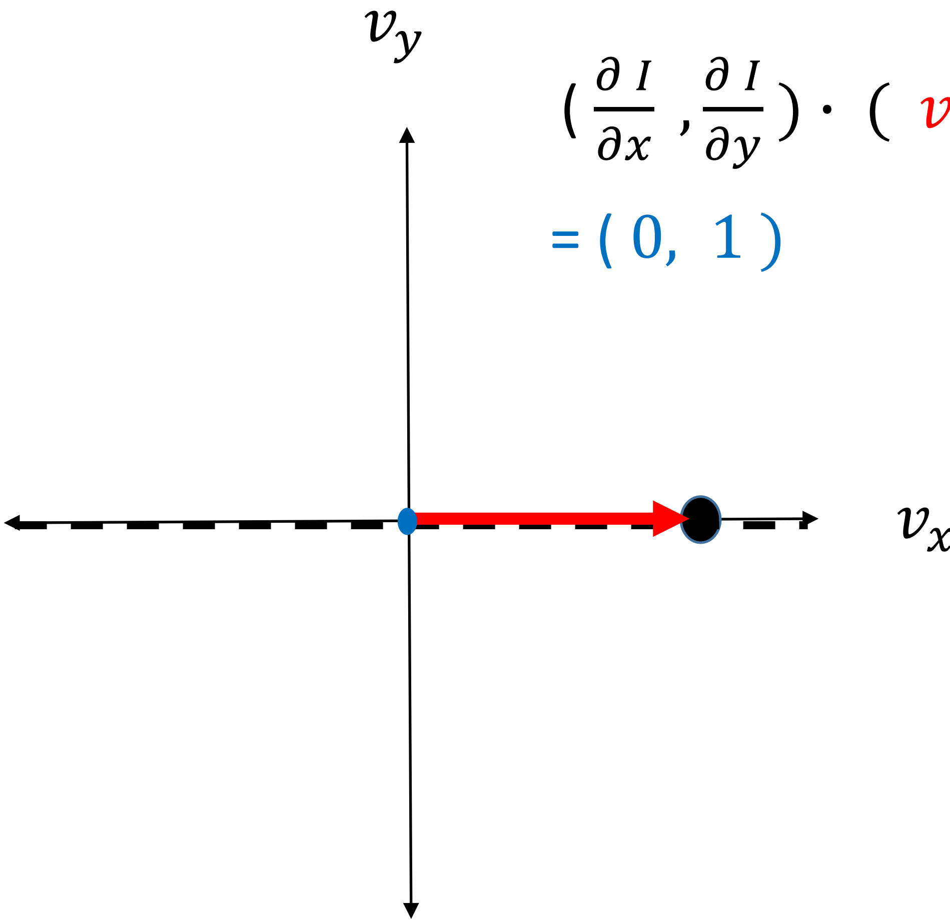
A horizontally oriented cell that prefers static horizontal structure could give a good response, namely to vertical gradient components.

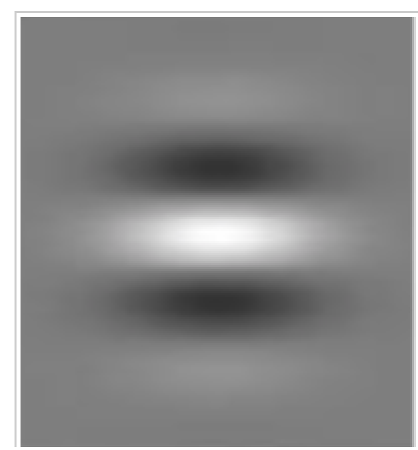


Note the motion constraint line passes through the origin.

$$\left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right) \cdot (v_x, v_y) + \frac{\partial I}{dt} = 0$$

$= (0,\ 1)$





Summary

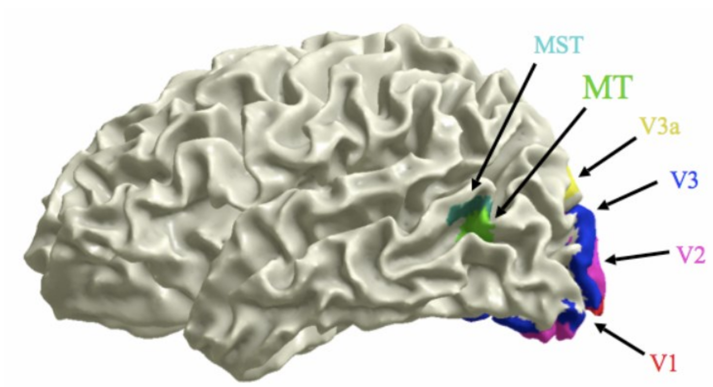
- Many cells in V1 are sensitive to orientation and (normal) velocity.

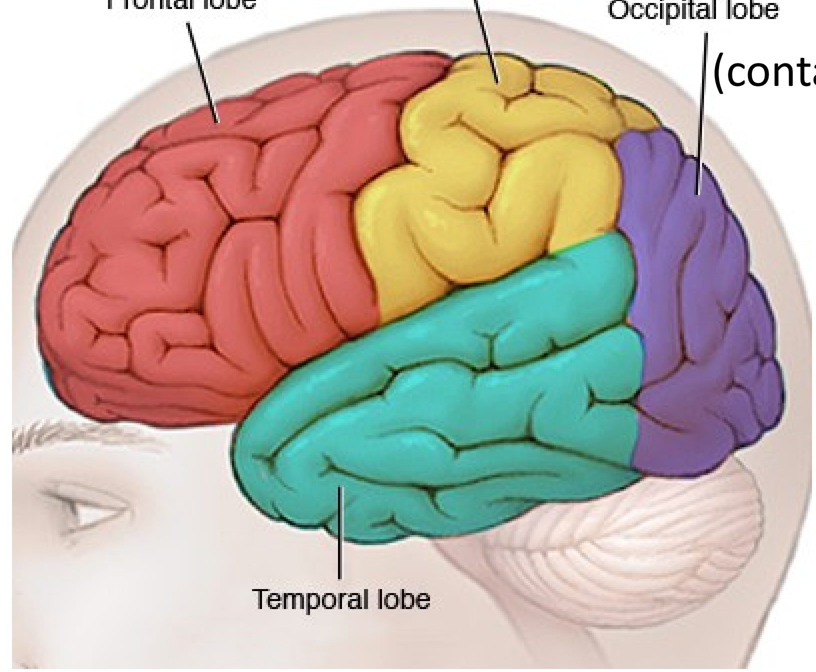
- For the visual system to estimate the image velocity (v_x, v_y) at a location (x, y), it needs to combine the responses of cells tuned to different orientations and speeds.

Computationally, this corresponds to the intersection of constraint (IOC) solution.

You will take this further in Assignment 2.

ASIDE: Motion pathway in the brain





MST ⟵ MT ⟵V1

(temporal lobe contains motion area MT and MST : "middle temporal" & "medial superior temporal")

V1 → MT

MT cells receive inputs from orientation+motion tuned V1 cells.

Many MT cells are *velocity tuned*. They respond well to image patterns with multiple gradient directions, moving with some preferred velocity (v_x, v_y).