Math 240: Discrete Structures I (W18) − Assignment 1

Solutions must typed or very neatly written and uploaded to MyCourses no later than **6 pm** on **Friday, February 2, 2018**. Up to 4 bonus marks will be awarded for solutions typeset in LaTeX; both the .tex file and .pdf file must be uploaded.

[12]   1. **Negation of predicates.** For each of the sentences below,

      i. write it using symbolic logic notation, using the indicated predicates;

      ii. find the negation of your sentence from (i) and simplify as much as possible; and

      iii. re-write your answer from (ii) in English (with appropriate mathematical notation where applicable). Your answer should match the one from (ii) exactly; answers which differ will not receive full marks, even if logically equivalent.

    (a) Something is rotten in the state of Denmark. [$D(x)$: x is in Denmark; $R(x)$: x is rotten]

    (b) For every real number $\varepsilon > 0$ there is a positive real number N such that $x > N$ implies that $|f(x) - L| < \varepsilon$. [$B(x, y)$: $x > y$]

[10]   2. **Rules of inference.**

    (a) Verify the transitivity inference rule by using a truth table (do not just give the table, but also clearly state why your table shows the argument is valid).

    (b) Verify the modus tollens inference rule by showing an appropriate statement is a tautology using only logical identities.

    (c) Use the rules of inference given on the handout to determine if the following argument is valid. Clearly state which rules you are using (you may symbolize if it is helpful).

        If I study, then I will pass.
        If I do not go to a movie, then I will study.
        I did not pass.
        ―――――――――――――――――――――
        Therefore, I went to a movie.

[12]   3. **Set operations.** Let $A = \{n \in \mathbb{N} \mid n < 7\}$, $B = \{q \in \mathbb{Q} \mid |q - 2| < 1\}$, $C = \{r \in \mathbb{R} \mid r^3 - r = 0\}$ and $D = \{1, 2, \{1, 2\}\}$. Find each of the following (recall that $\mathcal{P}(X)$ denotes the power set of X). Do not use "..." in your answer; give clear rules for set membership if you need them.

    (a) $A \oplus C$ (c) $C \cup D$ (e) $B \setminus (A \oplus C)$ (g) $(A \setminus B) \setminus (C \setminus \overline{D})$

    (b) $A \cap D$ (d) $\{1, \{2\}\} \cup D$ (f) $\{\emptyset\} \setminus \mathcal{P}(A)$ (h) $\mathcal{P}(D) \cap D$

[6]   4. **Venn diagrams.** Draw a Venn diagram for each of the following. If using LaTeX, you may include graphics for your diagrams (if hand-drawn, make sure they are very neat).

    (a) $A \cap \overline{(B \cup C)}$ (b) $\overline{A} \setminus (B \cup \overline{C})$ (c) $(A \cup B) \oplus (C \setminus B)$