MATH 323 - ASSIGNMENT 4

Please submit your assignment by 11.59 pm on Friday 30th November by uploading a pdf to myCourses.

NOTE THAT THIS ASSIGNMENT CARRIES 30 MARKS, AND WILL COUNT 1.5 TIMES AS MUCH AS ASSIGNMENTS 1 AND 2.

1. Suppose Y is a continuous random variable with the following cdf:

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ y/2 & 0 \le y \le 2 \\ 1 & y > 2. \end{cases}$$

Let $X = Y^2$. Find

   (a) $P(1 \le Y \le 2)$.   1 MARK

   (b) $P(X \le Y)$;   2 MARKS

   (c) $P(Y \le 2X)$;   2 MARKS

   (d) $P(X + Y \le 3/4)$;   2 MARKS

   (e) the *covariance* between Y and X, $\text{Cov}[Y, X]$, defined by   3 MARKS

$$\text{Cov}[Y, X] = \mathbb{E}[YX] - \mathbb{E}[Y]\mathbb{E}[X].$$

   Note that here, as $X = Y^2$, we have that

$$\mathbb{E}[YX] \equiv \mathbb{E}[Y^3].$$

2. Suppose that Y_1 and Y_2 are continuous random variables with joint pdf given by

$$f_{Y_1,Y_2}(y_1, y_2) = c(3y_1y_2 + y_1^2 + y_2^2) \qquad 0 < y_1 < 1, 0 < y_2 < 1$$

   and zero otherwise, for some constant $c > 0$.

   (a) Find the value of c.   2 MARKS

   (b) Find the joint cdf, $F_{Y_1,Y_2}(y_1, y_2)$, for all values $(y_1, y_2) \in \mathbb{R}^2$.   5 MARKS

   (c) Find the marginal pdf of Y_1, $f_{Y_1}(y_1)$ (taking care to note the support of this pdf).   2 MARKS

   (d) Are Y_1 and Y_2 independent ? Justify your answer.   1 MARK

Question 3 is on page 2.

3. Suppose that Y_1 and Y_2 are continuous random variables with joint pdf given by

$$f_{Y_1,Y_2}(y_1, y_2) = cy_1^2 \exp\{-4(y_1 + y_2)\} \qquad y_1 > 0, y_2 > 0$$

and zero otherwise, for some constant $c > 0$.

(a) Find the value of c. 3 MARKS

(b) Are Y_1 and Y_2 independent ? Justify your answer. 1 MARK

(c) Let $Y = Y_1 + Y_2$. Compute the probability $P(Y \leq 3)$. 3 MARKS

(d) Let U and V be independent continuous random variables having the same (marginal) distribution as Y_2. Identify the distribution of random variable W defined by

$$W = U + V.$$

3 MARKS