

## MATH 240 – Assignment 1

1. a. Contingency.

$$(P \vee Q) \Rightarrow \neg P$$

$P$	$Q$	$P \vee Q$	$\neg P$	$(P \vee Q) \Rightarrow \neg P$
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	T

b. Tautology.

$$(P \Leftrightarrow Q) \wedge (Q \Leftrightarrow R) \Rightarrow (P \Leftrightarrow R)$$

$P$	$Q$	$R$	$P \Leftrightarrow Q$	$Q \Leftrightarrow R$	$(P \Leftrightarrow Q) \wedge (Q \Leftrightarrow R)$	$P \Leftrightarrow R$	$(P \Leftrightarrow Q) \wedge (Q \Leftrightarrow R) \Rightarrow (P \Leftrightarrow R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	F	T	T
T	F	F	F	T	F	F	T
F	T	T	F	T	F	F	T
F	T	F	F	F	F	T	T
F	F	T	T	F	F	F	T
F	F	F	T	T	T	T	T

c. Contradiction.

$$[(P \oplus Q) \oplus \neg Q] \Leftrightarrow P$$

$P$	$Q$	$P \oplus Q$	$[(P \oplus Q) \oplus \neg Q]$	$[(P \oplus Q) \oplus \neg Q] \Leftrightarrow P$
T	T	F	F	F
T	F	T	F	F
F	T	T	T	F
F	F	F	T	F

2. a.

$$\begin{aligned}
 [(P \Rightarrow Q) \wedge P] \Rightarrow Q &\equiv [(\neg P \vee Q) \wedge P] \Rightarrow Q && \text{implies} \\
 &\equiv [(P \wedge \neg P) \vee (P \wedge Q)] \Rightarrow Q && \text{distributive} \\
 &\equiv [\mathbb{F} \vee (P \wedge Q)] \Rightarrow Q && \text{complement} \\
 &\equiv (P \wedge Q) \Rightarrow Q && \text{identity} \\
 &\equiv \neg P \vee \neg Q \vee Q && \text{implies} \\
 &\equiv \neg P \vee \mathbb{T} && \text{complement} \\
 &\equiv \mathbb{T} \blacksquare && \text{identity}
 \end{aligned}$$

b.

$$\begin{aligned}
 \neg(P \wedge Q) \wedge (Q \Rightarrow P) &\equiv (\neg P \vee \neg Q) \wedge (\neg Q \vee P) && \text{DeMorgan's, implies} \\
 &\equiv \neg Q \vee (\neg P \wedge P) && \text{distributive} \\
 &\equiv \neg Q \vee \mathbb{F} && \text{complement} \\
 &\equiv \neg Q \blacksquare && \text{identity}
 \end{aligned}$$

c.

$$\begin{aligned}
 \neg[(P \vee Q) \vee [(Q \vee \neg R) \wedge (P \vee R)]] &\equiv \neg[(P \vee Q \vee Q \vee \neg R) \wedge (P \vee Q \vee P \vee R)] && \text{distributive,} \\
 &\equiv \neg[(P \vee Q \vee \neg R) \wedge (P \vee Q \vee R)] && \text{associative} \\
 &&& \text{idempotent}
 \end{aligned}$$

$$\begin{aligned}
&\equiv \neg[(P \vee Q) \vee (\neg R \wedge R)] \\
&\equiv \neg[(P \vee Q) \vee \mathbb{F}] \\
&\equiv \neg[P \vee Q] \\
&\equiv \neg P \wedge \neg Q \blacksquare
\end{aligned}$$

distributive  
complement  
identity  
DeMorgan's

3. a. True.

$$\begin{aligned}
&\pi \text{ is an integer if and only if } \sqrt{e+3} \text{ is a vowel} \\
&\equiv \pi \text{ is an integer} \Leftrightarrow \sqrt{e+3} \text{ is a vowel} \\
&\equiv \mathbb{F} \Leftrightarrow \mathbb{F} \\
&\equiv \mathbb{T} \blacksquare
\end{aligned}$$

b. False.

$$\begin{aligned}
&0 > 1 \text{ whenever } 2 + 2 = 4 \\
&\equiv 2 + 2 = 4 \Rightarrow 0 > 1 \\
&\equiv \mathbb{T} \Rightarrow \mathbb{F} \\
&\equiv \mathbb{F} \blacksquare
\end{aligned}$$

c. True.

$$\begin{aligned}
&\text{If (a) implies (b), then pigs cannot fly} \\
&\equiv ((a) \Rightarrow (b)) \Rightarrow \text{pigs cannot fly} \\
&\equiv (\mathbb{T} \Rightarrow \mathbb{F}) \Rightarrow \mathbb{T} \\
&\equiv \mathbb{F} \Rightarrow \mathbb{T} \\
&\equiv \mathbb{T} \blacksquare
\end{aligned}$$

4. a.  $T \Leftrightarrow R$

Players only love you when they're playing.

$$\text{b. } \forall x \left( \exists y \left( P(y) \Rightarrow (B(y, x) \wedge B(2x + 1, y)) \right) \right)$$

$$\text{c. } \forall x (E(x) \wedge T(x) \Rightarrow P(x)) \Rightarrow G$$