COMP 350 Numerical Computing
Assignment #6, Numerical Integration

Date given: Tuesday, Nov 13. Date due: 11:55pm, Thursday, Nov 29, 2018

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1. (a) (4 points) Using the recursive trapezoid rule to compute $\int_0^{2\pi}(\cos(2x)/e^x)dx$. Stop the iteration until the difference between two consecutive computed integrals is smaller than or equal to 10^{-4}.

   (b) (6 points) Using the adaptive Simpson's method to compute $\int_0^{2\pi}(\cos(2x)/e^x)dx$ by taking $\epsilon = 10^{-4}$ and `level_max=20`. Try to avoid redundant function evaluation.

   For both methods, report the number of function evaluations and print the final results and the MATLAB codes as well.

   Note: The exact integral is $(1 - e^{-2\pi})/5$. You can use this to check if your answer is reasonable.

2. (a) (6 points) Construct a rule of the form

$$\int_{-1}^{1} f(x)x^2 dx \approx af(-\alpha) + bf(0) + cf(\alpha)$$

   such that it is exact for all polynomials of as high a degree as possible.

   Hint: Use one of the approaches we used in class to drive the Gaussian quadrature rule for $n = 2$.

   (b) (4 points) Suppose we want to compute $\int_a^b f(x)x^2 dx$. We divide the interval $[a, b]$ into n equal subintervals $[x_i, x_{i+1}]$, $i = 0, 1 \ldots, n-1$. For each subinterval we apply the above quadrature rule (you need to do interval exchange transformations), leading to the composite quadrature rule. Derive this composite rule.