MATH 140 - BOOKLET SOLUTIONS

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antoine.prep101@gmail.com

CHAPTER ONE:

#1 (a) $\lim_{x\to 1^+} \frac{(x-1)}{(x-1)} = 1$ (Note: $x\to 1^+ \Rightarrow x>1 \Rightarrow x-1>0$)

(b) $\lim_{x\to -2^-} \frac{-(x+2)}{x(x+2)} = \lim_{x\to -2^-} \frac{-1}{x} = \frac{1}{2}$ (Note: $x\to -2^- \Rightarrow x+2<0$)

(c) $x\to 7^-$ or $x\to 7^+$ $x>0$

$\lim_{x\to 7^-} \frac{7-x}{7-x} = 1$

(d) $x\to -\infty \Rightarrow x<0 \Rightarrow |x| = -x$

$\lim_{x\to -\infty} \frac{-x}{x} = -1.$

#2 (a) $\lim_{x\to 0} \frac{\sin(x)}{x} \cdot \frac{1}{4} = 1\left(\frac{1}{4}\right) = \frac{1}{4}.$

(b) $\lim_{x\to 1} \frac{\sin((1-x)(1+x))}{(1-x)(1+x)} \cdot (1+x) = 1\,(1+1) = 2$

Note: $\lim_{x\to 1} \frac{\sin(1-x^2)}{(1-x^2)} = 1$

(c) $\lim_{x\to 0} \frac{\sin(5x)}{\sin(3x)} = \lim_{x\to 0} \frac{\sin(5x)}{5x} \cdot 5x \cdot \frac{3x}{\sin(3x)} \cdot \frac{1}{3x}$

$= \lim_{x\to 0} \frac{\sin(5x)}{5x} \cdot \frac{1}{\frac{\sin(3x)}{3x}} \cdot \frac{5x}{3x} = (1)(1)\left(\frac{5}{3}\right) = \frac{5}{3}$

(d) $\lim_{x\to 0} \frac{\sin^2 x}{x^2(x-3)} = \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 \cdot \frac{1}{x-3} = 1^2 \cdot \left(\frac{1}{-3}\right) = -\frac{1}{3}$

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(e) $\lim_{x\to 0}\left(\frac{\tan(3x)}{x}\right)^2 = \lim_{x\to 0}\left(\frac{\sin(3x)}{x}\cdot\frac{1}{\cos(3x)}\right)^2$

$= \lim_{x\to 0}\left(3\cdot\frac{\sin(3x)}{3x}\cdot\frac{1}{\cos(3x)}\right)^2 = \left(3(1)\left(\frac{1}{1}\right)\right)^2 = 9.$

(f) $\lim_{x\to\infty}\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = 1$ $\left(\frac{1}{x}\to 0 \text{ as } x\to\infty\right)$

#3 (a) $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}\cdot\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \lim_{x\to 3}\frac{(x-3)}{(x-3)(\sqrt{x+1}+2)}$

$= \lim_{x\to 3}\frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}.$

(b) $\lim_{x\to -1}\frac{2x+\sqrt{x^2-3x}}{x+1}\cdot\frac{2x-\sqrt{x^2-3x}}{2x-\sqrt{x^2-3x}} = \lim_{x\to -1}\frac{4x^2-x^2+3x}{(2x-\sqrt{x^2-3x})(x+1)}$

$= \lim_{x\to -1}\frac{3x(x+1)}{(x+1)(2x-\sqrt{x^2-3x})} = \lim_{x\to -1}\frac{3x}{2x-\sqrt{x^2-3x}} = \frac{-3}{-2-2} = \frac{3}{4}.$

(c) $\lim_{x\to 0}\frac{\cos x-\sqrt{1+\sin^2 x}}{x^2(2+x^2)}\cdot\frac{\cos x+\sqrt{1+\sin^2 x}}{\cos x+\sqrt{1+\sin^2 x}}$

$= \lim_{x\to 0}\frac{\cos^2 x-1-\sin^2 x}{x^2(2+x^2)(\cos x+\sqrt{1+\sin^2 x})} = \lim_{x\to 0}\frac{-2\sin^2 x}{x^2(2+x^2)(\cos x+\sqrt{1+\sin^2 x})}$

$= \lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2\cdot\frac{-2}{(2+x^2)(\cos x+\sqrt{1+\sin^2 x})} = 1\cdot\frac{-2}{2\,(2)} = -\frac{1}{2}$

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#4 (a) $\lim_{x\to 1} \dfrac{\frac{1-x}{x}}{x-1} = \lim_{x\to 1} \dfrac{(1-x)}{x(x-1)}$

$= \lim_{x\to 1} -\dfrac{1}{x} = -1$

#4(b) $\lim_{x\to 2} \dfrac{\frac{8-2(6-x)}{6-x}}{(x-2)(x+2)} = \lim_{x\to 2} \dfrac{-4+2x}{(6-x)(x-2)(x+2)} = \lim_{x\to 2} \dfrac{2(x-2)}{(6-x)(x-2)(x+2)}$

$= \lim_{x\to 2} \dfrac{2}{(6-x)(x+2)} = \dfrac{2}{4(4)} = \dfrac{1}{8}$.

#5 (a) $\lim_{x\to -3} \dfrac{x(x-3)(x+3)}{(x+3)(x-1)} = \dfrac{-3(-3-3)}{-3-1} = \dfrac{18}{-4} = -\dfrac{9}{2}$

Note: may also use L' HR $\lim_{x\to -3} \dfrac{3x^2-9}{2x+2} = \dfrac{18}{-4} = -\dfrac{9}{2}$.

(b) $\lim_{x\to 2} \dfrac{3x^2-8x+4}{x^2+9x-22} \overset{HR}{=} \lim_{x\to 2} \dfrac{6x-8}{2x+9} = \dfrac{4}{13}$

(c) $\lim_{x\to 0} \dfrac{x^3+6x^2+12x+8-8}{x} = \lim_{x\to 0} x^2+6x+12 = 12$

OR $\lim_{x\to 0} \dfrac{3(x+2)^2}{1} = 12$.

(d) $\lim_{x\to -2} \dfrac{-4x-4}{2x+1} = \dfrac{-4}{3}$

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#6(a) $f(x) = \frac{x(x^2-9)}{(x+3)(x-1)} = \frac{x(x-3)(x+3)}{(x+3)(x-1)}$

$f(x) = \frac{x(x-3)}{(x-1)}$ V.A. $x = 1$

$\lim_{x\to1^-} f(x) = \frac{1(-2)}{0^-} = +\infty$

$\lim_{x\to1^+} f(x) = \frac{1(-2)}{0^+} = -\infty$

(b) $f(x) = \frac{x}{(x-2)(x+2)}$ V.A. $x=2$, $x=-2$

$\lim_{x\to-2^-} \frac{-2}{-4(0^-)} = -\infty$ $\lim_{x\to-2^+} \frac{-2}{-4(0^+)} = +\infty$

$\lim_{x\to2^-} \frac{2}{0^-(4)} = -\infty$ $\lim_{x\to2^+} \frac{2}{0^+(4)} = +\infty$.

(c) $f(x) = \frac{\sqrt{x4+9}}{(x-2)(x+1)}$ V.A. $x = 2$, $x = -1$

$\lim_{x\to2^-} f(x) = \frac{\sqrt{25}}{0^-(3)} = -\infty$ $\lim_{x\to2^+} f(x) = \frac{\sqrt{25}}{0^+(3)} = +\infty$.

$\lim_{x\to-1^-} f(x) = \frac{\sqrt{10}}{-3(0^-)} = +\infty$ $\lim_{x\to-1^+} f(x) = \frac{\sqrt{10}}{-3(0^+)} = -\infty$.

(d) $f(x) = \ln\left(\frac{|x-1|}{|(x-4)(x-3)|}\right)$ V.A. $x=1$, $x=4$, $x=3$

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$$\lim_{x\to 1} f(x) = \ln\left(\frac{0^+}{6}\right) = -\infty.$$

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$$\lim_{x\to 4^-} f(x) = \ln\left(\left|\frac{3}{0^-(1)}\right|\right) = \ln(\infty) = \infty.$$

$$\lim_{x\to 4^+} f(x) = \ln\left(\left|\frac{3}{0^+(1)}\right|\right) = \ln(\infty) = \infty.$$

$$\lim_{x\to 3^-} f(x) = \ln\left(\left|\frac{2}{-1(0^-)}\right|\right) = \ln(\infty) = \infty$$

$$\lim_{x\to 3^+} f(x) = \ln\left(\left|\frac{2}{-1(0^+)}\right|\right) = \ln(\infty) = \infty.$$

#7 (a) H.A. $y = -\frac{3}{7}$.

$$\lim_{x\to\infty} \frac{x^3\left(3 - \frac{4}{x^2} + \frac{2}{x^3}\right)}{x^3\left(\frac{4}{x^3} - 7\right)} = \frac{3}{-7} = -\frac{3}{7}$$

(same limit for $x\to-\infty$)

Note: you may also use L'HR.

(b) deg(num) > deg(den) $\Rightarrow$ No H.A.

$$\lim_{x\to\infty} \frac{x^2\left(9 + \frac{5}{x} - \frac{2}{x^2}\right)}{x\left(4 + \frac{1}{x}\right)} = \lim_{x\to\infty} \frac{9x}{4} = \infty.$$

$$\lim_{x\to-\infty} \frac{x^2\left(9 + \frac{5}{x} - \frac{2}{x^2}\right)}{x\left(4 + \frac{1}{x}\right)} = \lim_{x\to-\infty} \frac{9x}{4} = -\infty$$

(c) degree (num) < deg. (denominator) $\Rightarrow$ $y = 0$ H.A.

$$\lim_{x\to\infty} \frac{7x^2 - 2x + 1}{2x^5 - x^4 + 3} = \lim_{x\to\infty} \frac{x^2\left(7 - \frac{2}{x} + \frac{1}{x^2}\right)}{x^5\left(2 - \frac{1}{x} + \frac{3}{x^5}\right)} = \lim_{x\to\infty} \frac{7}{2x^3} = 0$$

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Same Limit for $x \to -\infty$.

(d) $$\lim_{x\to\infty} \frac{x^3\left(15+\frac{4}{x^2}\right)}{\sqrt{x^6\left(9+\frac{1}{x^6}\right)}} = \lim_{x\to\infty} \frac{x^3\left(15+\frac{4}{x^2}\right)}{|x^3|\sqrt{9+\frac{1}{x^6}}}$$

$$= \lim_{x\to\infty} \frac{x^3\left(15+\frac{4}{x^2}\right)}{x^3\left(\sqrt{9+\frac{1}{x^6}}\right)} = \frac{15}{\sqrt{9}} = 5 \qquad y=5 \text{ H.A. @ } \infty.$$

$$\lim_{x\to-\infty} \frac{x^3\left(15+\frac{4}{x^2}\right)}{\sqrt{x^6\left(9+\frac{1}{x^6}\right)}} = \lim_{x\to-\infty} \frac{x^3\left(15+\frac{4}{x^2}\right)}{|x^3|\sqrt{9+\frac{1}{x^6}}}$$

$$= \lim_{x\to-\infty} \frac{x^3\left(15+\frac{4}{x^2}\right)}{-x^3\left(\sqrt{9+\frac{1}{x^6}}\right)} = \frac{15}{-\sqrt{9}} = -5 \qquad y=-5 \text{ HA @ } -\infty$$

(e) $$\lim_{x\to\infty} \frac{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{x\left(2+\frac{3}{x}\right)} = \lim_{x\to\infty} \frac{|x|\sqrt{1+\frac{1}{x^2}}}{x\left(2+\frac{3}{x}\right)} = \lim_{x\to\infty} \frac{x\sqrt{1+\frac{1}{x^2}}}{x\left(2+\frac{3}{x}\right)} = \frac{1}{2}$$

$y = \frac{1}{2}$ H.A. @ ∞

$$\lim_{x\to-\infty} \frac{|x|\sqrt{1+\frac{1}{x^2}}}{x\left(2+\frac{3}{x}\right)} = \lim_{x\to-\infty} \frac{-x\sqrt{1+\frac{1}{x^2}}}{x\left(2+\frac{3}{x}\right)} = -\frac{1}{2}$$

$y = -\frac{1}{2}$ H.A. @ $-\infty$

(f) $$\lim_{x\to\infty} \frac{x^2\left(15+\frac{4}{x}\right)}{\sqrt{x^4\left(9+\frac{1}{x^4}\right)}} = \lim_{x\to\infty} \frac{x^2\left(15+\frac{4}{x}\right)}{x^2\sqrt{9+\frac{1}{x^4}}} = \frac{15}{\sqrt{9}} = 5$$

(Same limit for $x \to -\infty$
$\sqrt{x^4} = |x^2| = x^2$ for all x)

$y = 5$ H.A.

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antoine.prep101@gmail.com

#8(a) $$\lim_{x\to\infty} \frac{\left(\sqrt{9x^2+5}-\sqrt{x^2+x}\right)\left(\sqrt{9x^2+5}+\sqrt{x^2+x}\right)}{\sqrt{9x^2+5}+\sqrt{x^2+x}}$$

$$= \lim_{x\to\infty} \frac{9x^2+5-x^2-x}{\sqrt{9x^2+5}+\sqrt{x^2+x}}$$

$$= \lim_{x\to\infty} \frac{8x^2-x+5}{x\sqrt{9+\frac{5}{x^2}}+x\sqrt{1+\frac{1}{x}}} = \lim_{x\to\infty} \frac{x^2\left(8-\frac{1}{x}+\frac{5}{x^2}\right)}{x\left(\sqrt{9+\frac{5}{x^2}}+\sqrt{1+\frac{1}{x}}\right)}$$

$$= \lim_{x\to\infty} \frac{8x}{\sqrt{9}+1} = \infty.$$

(b) $$\lim_{x\to-\infty} \overbrace{x+\sqrt{x^2-x}}^{-\infty+\infty} = \lim_{x\to-\infty} x+\sqrt{x^2-x} \cdot \frac{x-\sqrt{x^2-x}}{x-\sqrt{x^2-x}}$$

$$= \lim_{x\to-\infty} \frac{x^2-x^2+x}{x-\sqrt{x^2-x}} = \lim_{x\to-\infty} \frac{x}{x-|x|\sqrt{1-\frac{1}{x}}}$$

$$= \lim_{x\to-\infty} \frac{x}{x+x\sqrt{1-\frac{1}{x}}} = \lim_{x\to-\infty} \frac{x}{x\left(1+\sqrt{1-\frac{1}{x}}\right)} = \frac{1}{1+\sqrt{1}} = \frac{1}{2}$$

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#9 $$\lim_{x\to 3} \frac{\left(\sqrt{7+\sqrt{x+1}}-3\right)\left(\sqrt{7+\sqrt{x+1}}+3\right)}{(x-3)(x+2)\left(\sqrt{7+\sqrt{x+1}}+3\right)}$$

$$= \lim_{x\to 3} \frac{7+\sqrt{x+1}-9}{(x-3)(x+2)\left(\sqrt{7+\sqrt{x+1}}+3\right)}$$

$$= \lim_{x\to 3} \frac{\left(\sqrt{x+1}-2\right)\left(\sqrt{x+1}+2\right)}{(x-3)(x+2)\left(\sqrt{7+\sqrt{x+1}}+3\right)\left(\sqrt{x+1}+2\right)}$$

$$= \lim_{x\to 3} \frac{(x+1-4)}{(x-3)(x+2)\left(\sqrt{7+\sqrt{x+1}}+3\right)\left(\sqrt{x+1}+2\right)}$$

$$= \lim_{x\to 3} \frac{1}{(x+2)\left(\sqrt{7+\sqrt{x+1}}+3\right)\left(\sqrt{x+1}+2\right)}$$

$$= \frac{1}{5(6)(4)} = \frac{1}{120}.$$

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#10 $$\lim_{x\to\infty} \frac{\left(\sqrt{ax^2+bx}-\sqrt{ax^2-bx}\right)\left(\sqrt{ax^2+bx}+\sqrt{ax^2-bx}\right)}{\sqrt{ax^2+bx}+\sqrt{ax^2-bx}}$$

$$= \lim_{x\to\infty} \frac{ax^2+bx-ax^2+bx}{|x|\sqrt{a+\frac{b}{x}}+|x|\sqrt{a-\frac{b}{x}}}$$

$$= \lim_{x\to\infty} \frac{2bx}{x\sqrt{a+\frac{b}{x}}+x\sqrt{a-\frac{b}{x}}} = \lim_{x\to\infty} \frac{2bx}{x\left(\sqrt{a+\frac{b}{x}}+\sqrt{a-\frac{b}{x}}\right)}$$

$$= \frac{2b}{2\sqrt{a}} = \frac{b}{\sqrt{a}}.$$

#11 $$\lim_{x\to 1} \frac{(x^{\frac{1}{3}}-1)(x^{\frac{1}{3}}+1)}{(x^{\frac{1}{3}}-1)(x^{\frac{2}{3}}+x^{\frac{1}{3}}+1)}$$

$$= \lim_{x\to 1} \frac{x^{\frac{1}{3}}+1}{x^{2/3}+x^{1/3}+1} = \frac{2}{3}$$

OR use L'HR.

$$\lim_{x\to 1} \frac{\frac{2}{3}x^{-1/3}}{1} = \frac{2}{3}.$$

we can also use the definition of the derivative for f(x) = x^(2/3) at x=0

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CHAPTER TWO:

antoine.prep101@gmail.com

#1(a) $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{2(x+h)^2 - 5(x+h) + 3 - (2x^2 - 5x + 3)}{h}$$

$$= \lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 - 5x - 5h + 3 - 2x^2 + 5x - 3}{h}$$

$$= \lim_{h\to 0} \frac{4xh + 2h^2 - 5h}{h}$$

$$= \lim_{h\to 0} \frac{h(4x + 2h - 5)}{h} = 4x - 5.$$

(b) $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{2(x+h)-1} - \sqrt{2x-1}\right)\left(\sqrt{2x+2h-1} + \sqrt{2x-1}\right)}{h\left(\sqrt{2x+2h-1} + \sqrt{2x-1}\right)}$$

$$= \lim_{h\to 0} \frac{2x + 2h - 1 - 2x + 1}{h\left(\sqrt{2x+2h-1} + \sqrt{2x-1}\right)}$$

$$= \lim_{h\to 0} \frac{2h}{h\left(\sqrt{2x+2h-1} + \sqrt{2x-1}\right)} = \frac{2}{2\sqrt{2x-1}}$$

$$= \frac{1}{\sqrt{2x-1}}.$$

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(c) $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{5}{3x+3h+1} - \frac{5}{3x+1}}{h}$$

$$= \lim_{h\to 0} \frac{5(3x+1) - 5(3x+3h+1)}{h(3x+3h+1)(3x+1)}$$

$$= \lim_{h\to 0} \frac{-15h}{h(3x+3h+1)(3x+1)}$$

$$= \frac{-15}{(3x+1)^2}.$$

#2(a) $$y' = 10x e^x + (5x^2+1)e^x = e^x(5x^2+10x+1)$$

(b) $$y' = \frac{1}{5x^3+4x-2}(15x^2+4) = \frac{15x^2+4}{5x^3+4x-2}$$

(c) $$y' = (10x+3)\, e^{5x^2+3x-1}$$

(d) $$y' = 4(5x - \ln x)^3 \cdot \left(5 - \frac{1}{x}\right)$$

(f) $$y' = \frac{2x(x^5-x) - (5x^4-1)(x^2+1)}{(x^5-x)^2} = \frac{2x^6 - 2x^2 - 5x^6 - 5x^4 + x^2 + 1}{(x^5-x)^2}$$

$$= \frac{-3x^6 - 5x^4 - x^2 + 1}{(x^5-x)^2}$$

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(g) $y' = \dfrac{5 - \frac{1}{x} + 2e^{2x}}{2\sqrt{5x - \ln x + e^{2x}}}$

(h) $y = 5x^2 + 3x - 1$; $y' = 10x + 3$

(i) $y = 5 \ln(5x+4) \Rightarrow y' = 5 \cdot \dfrac{5}{5x+4} = \dfrac{25}{5x+4}$

(j) $y' = 5\left[\ln(5x+4)\right]^4 \cdot \dfrac{5}{5x+4}$.

#3(a) $y' = \dfrac{\cos x(xe^x) - \sin x\,(e^x + xe^x)}{(xe^x)^2}$

$= \dfrac{e^x\left(x\cos x - \sin x\,(1+x)\right)}{x^2 e^{2x}} = \dfrac{x\cos x - \sin x\,(1+x)}{x^2 e^x}$.

(b) $y' = \left(4e^x + (4x-1)e^x\right)\ln(3x+1) + (4x-1)e^x \dfrac{3}{3x+1}$.

$= e^x(4x+3)\ln(3x+1) + \dfrac{3e^x(4x-1)}{3x+1}$

(c) $y' = \dfrac{\frac{1}{1+x^2}\cdot(1+x^2)^2 - \arctan x \cdot 2(1+x^2)\cdot 2x}{(1+x^2)^4}$

$= \dfrac{(1+x^2) - 4x(1+x^2)\arctan x}{(1+x^2)^4}$

$= \dfrac{1 - 4x\arctan x}{(1+x^2)^3}$.

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(d) $y' = \dfrac{1}{\ln(x\ln x)} \cdot \left(\ln(x\ln x)\right)'$

$= \dfrac{1}{\ln(x\ln x)} \cdot \dfrac{1}{x\ln x} \cdot (x\ln x)'$

$= \dfrac{\ln x + x\left(\frac{1}{x}\right)}{\ln(x\ln x) \cdot x\ln x} = \dfrac{\ln x + 1}{x\ln x \ \ln(x\ln x)}$.

(e) $y' = 3\sec^2 x \sec x \tan x \cdot \sin(3x) + \sec^3(x) \cdot 3\cos(3x)$

$= 3\sec^3 x \tan x \sin(3x) + 3\sec^3(x)\cos(3x)$

(f) $y' = \sec^2\left(\sqrt{x^3+1}\right) \cdot \dfrac{1}{2\sqrt{x^3+1}} \cdot 3x^2$.

(g) $y' = \sinh\left(e^{\tan(4x)}\right) \cdot e^{\tan(4x)} \cdot \sec^2(4x) \cdot 4$.

(i) $y' = 5^{4x^2+3x} \cdot \ln 5 \cdot (8x+3)$

(j) $y' = 8^{7^x} \cdot \ln 8 \cdot 7^x \ln 7$.

(k) $y = \ln(2x-1) - \ln(3x+2)$

$y' = \dfrac{2}{2x-1} - \dfrac{3}{3x+2}$.

(h) $y' = \{1 - \tanh^2(x^2+1)\}(2x)$.

OR

$= \operatorname{sech}^2(x^2-1) \cdot 2x$

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(l) For this function it is easier to use Log. diff

$$\ln y = 2\ln(x^2-3) + 4\ln(5x+2) + \frac{1}{2}\ln(7x-1) + 5x^2 - 3x + 2$$

$$\therefore \frac{1}{y}\cdot y' = 2\cdot\frac{2x}{x^2-3} + 4\cdot\frac{5}{5x-2} + \frac{1}{2}\frac{7}{7x-1} + 10x - 3$$

$$\therefore y' = y\left(\frac{4x}{x^2-3} + \frac{20}{5x-2} + \frac{7}{2(7x-1)} + 10x - 3\right)$$

$$= (x^2-3)^2(5x+2)^4\sqrt{7x-1}\; e^{5x^2-3x+2}\left(\frac{4x}{x^2-3} + \frac{20}{5x-2} + \frac{7}{2(7x-1)} + 10x-3\right)$$

#4 (a) $5y^4\frac{dy}{dx} + 2xy^3 + x^2\cdot 3y^2\frac{dy}{dx} - 2 = 0$

$$\therefore (5y^4 + 3x^2y^2)\frac{dy}{dx} = 2 - 2xy^3$$

$$\frac{dy}{dx} = \frac{2-2xy^3}{5y^4+3x^2y^2}$$

(b) $\frac{1}{x+y}\left(1+\frac{dy}{dx}\right) - 2xy^2 - x^2\cdot 2y\frac{dy}{dx} = 0$

$$\frac{1}{x+y} + \frac{1}{x+y}\frac{dy}{dx} - 2xy^2 - 2x^2y\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = \frac{2xy^2 - \frac{1}{x+y}}{\frac{1}{x+y} - 2x^2y}.$$

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(c) $2\frac{dy}{dx}\cos(2y) + \sec^2(xy)\left(y + x\frac{dy}{dx}\right) = 0$

$2\cos(2y)\cdot\frac{dy}{dx} + x\sec^2(xy)\frac{dy}{dx} = -y\sec^2(xy)$

$\therefore \frac{dy}{dx} = \frac{-y\sec^2(xy)}{2\cos(2y) + x\sec^2(xy)}$.

#6 (a) $2\ln(xy)\cdot\frac{1}{xy}\left(y + x\frac{dy}{dx}\right) = 0$

$\therefore \frac{dy}{dx} = -\frac{y}{x}$ @ $(1,2)$ $\frac{dy}{dx} = -2$.

Slope of tan

$m = -2 \quad x = 1,\ y = 2$

$y = mx + b \Rightarrow 2 = -2(1) + b \Rightarrow b = 4$

tan. line $y = -2x + 4$

(b) $y = x^{3/2} \Rightarrow y' = \frac{3}{2}x^{1/2}$

tangent // $y = 15x + 1 \Rightarrow$ slope of tan $= 15 = m$

$\frac{3}{2}x^{1/2} = 15 \Rightarrow x^{1/2} = 10 \Rightarrow x = 100$

so $y = 1000$

$1000 = 15(100) + b$

$-500 = b$ $\qquad y = 15x - 500$.

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#5

$$6x\frac{dx}{dy} - \frac{1}{1+(x^4y^4)}\left(2xy^2\frac{dx}{dt} + 2x^2y\right) = 7$$

$$6x\frac{dx}{dy} - \frac{2xy^2}{1+x^4y^4}\frac{dx}{dy} - \frac{2x^2y}{1+x^4y^4} = 7$$

$$\frac{dx}{dy} = \frac{7+\frac{2x^2y}{1+x^4y^4}}{6x-\frac{2xy^2}{1+x^4y^4}}.$$

#7 (a) $h'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

$$h'(4) = \frac{f'(4)g(4) - f(4)g'(4)}{(g(4))^2}$$

$$= \frac{3\cdot(-7) - (-6)(2)}{(-7)^2} = \frac{-21+12}{49} = -\frac{9}{49}.$$

(b) $h'(x) = \dfrac{2x\, f(x)g(x) - x^2\left(f'(x)g(x) + f(x)g'(x)\right)}{(f(x)\,g(x))^2}$

$$h'(3) = \frac{2(3)(4)(5) - (3)^2\left(-3(5) + (4)(2)\right)}{((4)(5))^2}$$

$$= \frac{120 - 9(-7)}{400} = \frac{183}{400}.$$

#8 (a) $h'(x) = \dfrac{\left(f'(x)g(x) + f(x)g'(x)\right)\left(f(x)+g(x)\right) - \left(f(x)\,g(x)\right)\left(f'(x) + g'(x)\right)}{\left(f(x)+g(x)\right)^2}$

$$h'(5) = \frac{[3(1) + (-2)(-1)][-2+1] - (-2)(1)(3+(-1))}{(-2+1)^2}$$

$$= \frac{-5+4}{1} = -1.$$

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from this point onward in this chpater, the topics are not on the midterm

#9(a) $\ln y = \ln(x^2+3x)^{x^3-x} = (x^3-x)\ln(x^2+3x)$

$$\frac{1}{y}y' = (3x^2-1)\ln(x^2+3x) + (x^3-x)\cdot\frac{2x+3}{x^2+3x}$$

$$y' = y\left((3x^2-1)\ln(x^2+3x) + \frac{(x^2-1)(2x+3)}{x+3}\right)$$

$$= (x^2+3x)^{x^3-x}\left[(3x^2+1)\ln(x^2+3x) + \frac{(x^2-1)(2x+3)}{x+3}\right]$$

(b) $\ln y = \ln\left(4x(\sin x)^{\ln x}\right) = \ln 4 + \ln x + \ln(\sin x)^{\ln x}$

$$= \ln 4 + \ln x + \ln x\,\ln(\sin x)$$

$$\frac{1}{y}y' = \frac{1}{x} + \frac{1}{x}\ln(\sin x) + \ln x\cdot\frac{\cos x}{\sin x}$$

$$y' = 4x(\sin x)^{\ln x}\left[\frac{1}{x} + \frac{1}{x}\ln(\sin x) + \ln x\cot x\right].$$

(c) $\ln y = \ln\left(8x^x(\ln x)^{\ln x}\right) = \ln 8 + x\ln x + \ln x\ln(\ln x)$

$$\frac{1}{y}y' = \ln x + x\left(\frac{1}{x}\right) + \frac{1}{x}\ln(\ln x) + \ln x\cdot\frac{1}{\ln x}\cdot\frac{1}{x}$$

$$y' = y\left(\ln x + 1 + \frac{1}{x}\ln(\ln x) + \frac{1}{x}\right)$$

$$= 8x^x(\ln x)^{\ln x}\left(\ln x + 1 + \frac{1}{x}\ln(\ln x) + \frac{1}{x}\right).$$

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#10 (a) $f(x) = \sqrt{x}$ $a = 25$

$f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow f'(a) = \frac{1}{2\sqrt{25}} = \frac{1}{10}$

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antoine.prep101@gmail.com

$f(a) = \sqrt{25} = 5$

$L(x) = f(a) + f'(a)(x-a) = 5 + \frac{1}{10}(x-25)$

$\sqrt{25.1} \approx L(25.1) = 5 + \frac{1}{10}(25.1 - 25) = 5 + \frac{1}{10}(0.1)$

$= 5.01$.

Differentials: $\sqrt{25.1} = \sqrt{25+0.1} \approx \sqrt{25} + dy$

$dy = \frac{1}{2\sqrt{x}}dx$ @ 25 $dy = \frac{1}{2\sqrt{25}}(0.1) = 0.01$

$\therefore \sqrt{25.1} \approx 5 + 0.01 = 5.01$.

(b) $f(x) = \sqrt[4]{x}$ $a = 16$ $f(16) = \sqrt[4]{16} = 2$

$f'(x) = \frac{1}{4}x^{-3/4}$ $f'(16) = \frac{1}{4}(16)^{-3/4} = \frac{1}{32}$

$L(x) = 2 + \frac{1}{32}(x - 16)$

$\sqrt[4]{16.08} \approx L(16.08) = 2 + \frac{1}{32}(16.08 - 16)$

$= 2 + \frac{0.08}{32} = 2 + \frac{0.01}{4} = 2.0025$

(c) $\ln(0.8) = \ln(1-0.2) \approx \ln 1 + dy$

$y = \ln x$ $dy = \frac{1}{x}dx$

$\Rightarrow \ln(0.8) \approx \ln 1 + \frac{1}{1}(-0.2) = -0.2$

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(d) $\sin\left(\frac{62.\pi}{180}\text{rd}\right) \approx ?$

$y = f(x) = \sin x \qquad a = \frac{\pi}{3} \qquad f(a) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$f'(x) = \cos x \qquad f'\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$L(x) = \frac{\sqrt{3}}{2} + \frac{1}{2}\left(x - \frac{\pi}{3}\right)$

$\sin(62°) = \sin\left(\frac{62\pi}{180}\right) \approx \frac{\sqrt{3}}{2} + \frac{1}{2}\left(\frac{62\pi}{180} - \frac{\pi}{3}\right)$

$= \frac{\sqrt{3}}{2} + \frac{1}{2}\left(\frac{2\pi}{180}\right) = \frac{\sqrt{3}}{2} + \frac{\pi}{180}.$

#11 (a) $2 + y' = 6(x-y)^2(1-y')$

$\therefore \quad 2 + y' = 6(x-y)^2 - 6y'(x-y)^2$

$y' = \dfrac{6(x-y)^2 - 2}{1 + 6(x-y)^2}$ @ $(1,0)$ $\quad y' = \dfrac{6(1)^2 - 2}{1 + 6(1)^2}$

$y' = \frac{4}{7}.$

$$y'' = \frac{12(x-y)(1-y')\left(1 + 6(x-y)^2\right) - 12(x-y)(1-y')\left(6(x-y)^2 - 2\right)}{\left(1 + 6(x-y)^2\right)^2}$$

$y'' = \dfrac{12(x-y)(1-y')(3)}{\left(1 + 6(x-y)^2\right)^2}$ @ $(1,0)$ $\quad y'' = \dfrac{36(1)\left(\frac{3}{7}\right)}{7}$

$= 108/49.$

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$f(x) = y'$ @ $a=1$ $f(a) = f(1) = y'(1) = \frac{4}{7}$

$$f'(a) = y''(a) = \frac{108}{49}$$

$$L(x) = f(a) + f'(a)(x-a)$$
$$= \frac{4}{7} + \frac{108}{49}(x-1)$$

Slope of tan. line @ $x = 0.95 = y'(0.95)$

$$\approx L(0.95) = \frac{4}{7} + \frac{108}{49}(-0.05).$$

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CHAPTER THREE :

#1 $\lim_{x\to -1^-} f(x) = (-1)^2 - a^2(-1) = 1+a^2$

$\lim_{x\to -1^+} f(x) = -1+6 = 5$

$f(-1) = (-1)^2 - a^2(-1) = 1+a^2$

$\therefore$ f cont @ $x=-1$ and thus everywhere if $1+a^2 = 5$

$a^2 = 4 \Rightarrow a = \pm 2$

#2 $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin(ax)}{ax} \cdot a = a$

$\lim_{x\to 0^+} f(x) = -2$

$f(0) = -2$

$\therefore$ f cont @ $x=0$ and thus f cont everywhere if

$a = -2$

#3 Continuity @ $x=1$

$\lim_{x\to 1^-} f(x) = a(1)^2 - 1 + b = a+b-1$

$\lim_{x\to 1^+} f(x) = 4(1) - 2 = 2$

$f(1) = a+b-1$

$a+b-1 = 2$

$\Rightarrow a+b = 3$

$\therefore$ f cont @ $x=1$ if $a+b=3$

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f is continuous @ $x = 1$.

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antoine.prep101@gmail.com

$x < 1$, $f'(x) = 2ax - 1$, @ $x = 1$: $2a - 1$

$x > 1$, $f'(x) = 4$, @ $x = 1$: 4

$\therefore$ f differentiable @ $x = 1$ if $2a - 1 = 4$

$\Rightarrow a = \frac{5}{2}$

$a + b = 3 \Rightarrow b = 3 - \frac{5}{2} = \frac{1}{2}$.

#4 $\lim_{x \to 1^-} f(x) = 8(1) - 1 = 7$

$\lim_{x \to 1^+} f(x) = 8(1)^3 - 4b = 8 - 4b$

$f(1) = a$

$\therefore$ f cont @ $x = 1$ if

a = 7 = 8-4b

therefore a=7 and b=1/4

#5 Continuity @ $x = 2$

$\lim_{x \to 2^-} f(x) = 2^2 = 4$

$\lim_{x \to 2^+} f(x) = m(2) + b = 2m + b$

$\therefore$ f cont @ $x = 2$ if

$2m + b = 4$.

$f(2) = 4$

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f continuous @ $x=2$.

$x<2$ $\quad f'(x) = 2x$ $\quad$ @ $x=2$: $\quad 4$

$x>2$ $\quad f'(x) = m$ $\quad$ @ $x=2$: $\quad m$

$\therefore$ f diff at $x=2$ iff $m=4$

$2m+b=4 \Rightarrow 8+b=4 \Rightarrow b=-4.$

#6 @ $x=0$

$\lim_{x\to 0^-} f(x) = 4(0)-1 = -1$; $\lim_{x\to 0^+} f(x) = 0^2+0-1 = -1$; $f(0)=2$

f NOT cont @ $x=0$. This is a removable discontinuity

@ $x=2$

$\lim_{x\to 2^-} f(x) = (2)^2+2-1 = 5$; $\lim_{x\to 2^+} f(x) = \frac{1}{2-4} = \frac{-1}{2}$; $f(2) = -\frac{1}{2}$

f NOT continuous @ $x=2$. This is a jump disc (right continuous)

@ $x=4$

f NOT cont. @ $x=4$ because $f(4)$ NOT defined ($x=4$ V.A.)

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