# MATH 323 - MID TERM
SAMPLE QUESTIONS

1. (a) Four letters are to be selected from the first eight letters of the alphabet, *abcdefgh*. How many distinct letter patterns can be formed if the selection is made **without replacement**? How many patterns are possible if the selection is made **with replacement**? 4 Marks

   (b) How many **distinct** letter patterns can be created by rearranging the letters *abcccdde*? 3 Marks

   (c) In each month of a given year, a company selects one from six contractors to fulfil a contract in that month, with all selections being equally likely within a month, and with selections made in successive months being independent.

   Compute the probability that a given contractor is selected **fewer than three times** during the year. 3 Marks

2. A total of 123 people were interviewed as part of an urban mass transportation study.

   - Some people live more that 10km from city centre (event A),
   - some people regularly drive to work (event B),
   - some people would use a public mass transportation if it was available (event C).

   The Venn diagram below represents the counts of people that fall into each partition element:
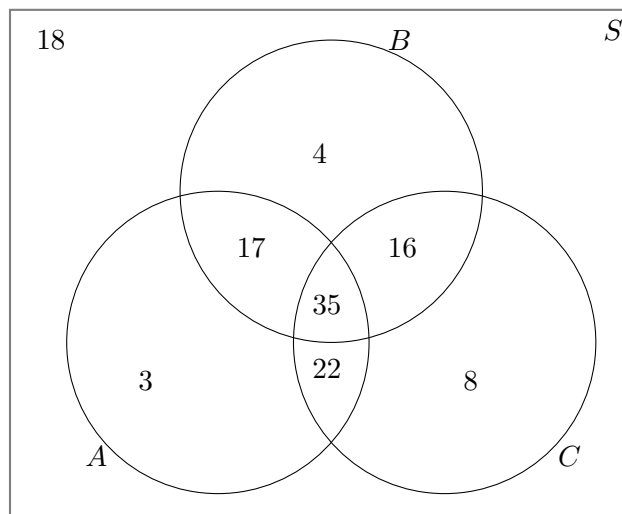
   A further questionnaire is going to be given to one person who is selected from the 123 people in the study population. Compute the probabilities of the following events:

   (a) the selected person drives to work; 2 Marks

   (b) the selected person drives to work and would use a public mass transportation system if it were available; 2 Marks

   (c) the selected person lives more than 10km from the city centre; 2 Marks

   (d) the selected person lives more than 10km from the city centre, drives to work, but would not use a mass transportation system.
   2 Marks

   Write down a symbolic expression of the event "exactly of two of A, B and C occur", and compute the probability of this event. 2 Marks

3. In a certain population, a genetic marker at a given genomic locus takes one of three forms (or *genotypes*), AA, Aa, or aa, and the genotype for an individual is thought to determine their risk of a specific disease in later life.

   - 60% of genotype AA individuals develop the disease;
   - 35% of genotype Aa individuals develop the disease;
   - 5% of genotype aa individuals develop the disease.

   The proportions of type AA and aa individuals are 0.04 and 0.64 respectively.

   Assuming that the disease mechanism described above is correct:

   (a) What proportion of the population develop the disease ?   3 Marks

   (b) Given that a person develops the disease, what is the probability that their genotype is AA ?
                                                                                              3 Marks

   Suppose that new research determines that in fact, the disease can also develop due to non genetic reasons: each person may develop the disease with probability 0.1 **irrespective** of their genotype, and only 70% of cases are determined by genetics.

   In light of the new research, what proportion of population develop the disease ?   4 Marks

4. A University committee is to comprise five members drawn from Faculties of Arts, Engineering and Science. Each Faculty nominates four professors for consideration for the committee, and the committee is selected with all selections of five members from the nominees being equally likely.

   (a) Let discrete random variable Y denote the number of Science professors selected for the committee. Write down the set of possible values, denoted $\mathcal{Y}$, that Y can take and for which the probability is positive.   2 Marks

   (b) Write down the form of the probability mass function (pmf), $p(y)$, for Y.   3 Marks

   (c) Suppose that discrete random variable Z records the number of different Faculties represented on the committee. Identify the set of possible values, denoted $\mathcal{Z}$, that Z can take and for which the probability is positive, and find $p(z)$ for $z \in \mathcal{Z}$.   5 Marks