

Math 240: Discrete Structures I (W18) – Assignment 3

Solutions must typed or very neatly written and uploaded to MyCourses no later than **6 pm** on **Saturday, February 10, 2018**. Up to 4 bonus marks will be awarded for solutions typeset in L^AT_EX; both the .tex file and .pdf file must be uploaded.

You may use theorems proven or stated in class, but you must state the theorem you are using.

- [15] 1. **Proofs with sets.** Let A, B, C be arbitrary sets. For each of the following statements, either prove it is true (without a Venn diagram) or give a counterexample to show that it is false.
- (a) $(A \setminus B) \setminus C = A \setminus (B \cup C)$
 - (b) $(A \oplus B = A \oplus C \text{ and } A \cap B = A \cap C) \Rightarrow B = C$
 - (c) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$
2. **Relations.**
- [11] (a) Determine whether or not each relation is reflexive, symmetric, antisymmetric, and/or transitive. For each property, if the relation has that property, prove it. If it doesn't have that property, give a counterexample. State if the relation is a total order, partial order but not a total order, or neither; justify your answer.
- i. $\mathcal{R} = \{(X, Y) \in (\mathcal{P}(A))^2 \mid X \cap Y \neq \emptyset\}$ where A is some arbitrary set
 - ii. $\mathcal{R} = \{(a, b) \in \mathbb{N}^2 \mid a \text{ divides } b\}$ (a divides b means that there is some integer k such that $b = ka$)
- [6] (b) For $a, b \in \mathbb{R} \setminus \{0\}$, define $a \sim b$ iff $\frac{a}{b} \in \mathbb{Q}$. Prove that \sim defines an equivalence relation on $\mathbb{R} \setminus \{0\}$. Show that $\left\lceil \frac{9-\sqrt{5}}{1-\sqrt{5}} \right\rceil = \left\lceil \frac{2}{3-6\sqrt{5}} \right\rceil$.
- [8] 3. **Proof techniques.** Prove the following statements using the method of your choice (direct proof, proof of the contrapositive, proof by contradiction).
- (a) Let $a, b \in \mathbb{R}$. If $a \in \mathbb{Q}$ and $b \notin \mathbb{Q}$, then $a \pm b \notin \mathbb{Q}$.
 - (b) If the average of 4 distinct integers is 10, then at least one of the integers is greater than 11.