COMP 350 Numerical Computing

Assignment #2: Floating point in C, overflow and underflow, numerical cancellation

Date Given: Tuesday, September 18. Date Due: Thursday, September 27, 2018, 11:59pm

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TA office hours: Thursday 4:00pm–5:30pm, Trottier 3110.

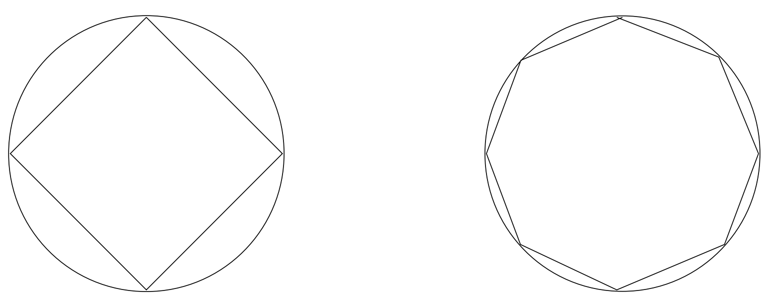
Submit your assignment including your code through myCourses.

1. (5 points) Write a C program to find the smallest positive integer x such that the floating point expression
$$1 \oslash (1 \oslash x)$$
is not equal to x, using single precision. Make sure that the variable x has type float, and assign the value of the expression $1 \oslash x$ to a float variable before doing the other division operation. Repeat with double precision.

2. (5 points) A calculus student was asked to determine $\lim_{n\to\infty} x_n$, where $x_n = (100^n)/n!$. He wrote a C program in single precision to evaluate x_n by using
$$x_1 = 100, \quad x_n = 100x_{n-1}/n, \quad n = 2, 3, \ldots, 70.$$
The numbers printed became ever larger and finally became ∞. So the student concluded that $\lim_{n\to\infty} x_n = \infty$. Please write a C program in single precision to verify the student's observation. The student's conclusion is actually wrong. What is the problem with his program?

   Bonus (2 points): Can you rewrite a C program to evaluate x_n so that you can make a right conclusion about $\lim_{n\to\infty} x_n$?

3. (10 points) In 250 B.C.E., the Greek mathematician Archimedes estimated the number π as follows. He looked as a circle with diameter 1, hence circumference π. Inside the circle he inscribed a square; see the following figure. The perimeter of the square is smaller than the



circumference of the circle, and so it is a lower bound for π. Archimedes then considered an inscribed octagon, 16-gon, etc., each time doubling the number of sides of the inscribed polygon, and producing ever better estimates for π. Using 96-sided inscribed and circumscribed polygons, he was able to show that $223/71 < \pi < 22/7$. There is a recursive formula for these estimates. Let p_n be the perimeter of the inscribed polygon with 2^n sides. Then p_2 is the perimeter of the inscribed square, $p_2 = 2\sqrt{2}$. In general
$$p_{n+1} = 2^n \sqrt{2(1 - \sqrt{1 - (p_n/2^n)^2})}.$$

(a) Write a program to compute p_n for $n = 3, 4, \ldots, 35$ in **double precision** by using the formula. Explain your results.

(b) Improve the formula to avoid the difficulty with it. Compute p_n for $n = 3, 4, \ldots, 35$ by your new formula in **double precision**. Comment on your results.

Note: Your program should not do unnecessary computation.