

MATH 323: PROBABILITY

READING THE NORMAL DISTRIBUTION TABLES

The standard Normal cdf (for $\mu = 0$ and $\sigma = 1$) can be written in integral form

$$F(y) = \int_{-\infty}^y \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}t^2\right\} dt \quad y \in \mathbb{R}$$

but the integration cannot be performed analytically. The Table on page 2 displays the numerical value of $F(y)$ for $0 \leq y \leq 3.09$. For example we have

- $F(0.52) = P(Y \leq 0.52) = 0.6985$;
- $F(1.86) = P(Y \leq 1.86) = 0.9686$;
- $F(2.41) = P(Y \leq 2.41) = 0.9920$.

We also have that $P(Y > y) = 1 - P(Y \leq y)$, so

- $P(Y > 0.21) = 1 - F(0.21) = 1 - 0.5832 = 0.4168$;
- $P(Y > 1.60) = 1 - F(1.60) = 1 - 0.9452 = 0.0548$;
- $P(Y > 2.27) = 1 - F(2.27) = 1 - 0.9884 = 0.0116$.

Finally, for $y < 0$ we have by the symmetry of the Normal pdf that

$$P(Y \leq y) = P(Y > -y) = 1 - P(Y \leq -y) = 1 - F(-y).$$

Therefore

- $P(Y \leq -0.21) = 1 - F(0.21) = 1 - 0.5832 = 0.4168$;
- $P(Y \leq -1.50) = 1 - F(1.50) = 1 - 0.9332 = 0.0668$;
- $P(Y \leq -2.04) = 1 - F(2.04) = 1 - 0.9793 = 0.0207$.

For a general Normal pdf with parameters μ and σ , we have that if $Y \sim \text{Normal}(\mu, \sigma^2)$ and

$$Z = \frac{Y - \mu}{\sigma}$$

then Z has a standard Normal distribution. Hence we have that

$$P(Y \leq y) = F_Y(y) = F_Z\left(\frac{y - \mu}{\sigma}\right) = P(Z \leq (y - \mu)/\sigma)$$

with $F_Z(\cdot)$ tabulated in the standard Normal table. Therefore

- $Y \sim \text{Normal}(1, 1^2)$: $P(Y \leq 2) = P(Z \leq (2 - 1)/1) = P(Z \leq 1) = 0.8413$;
- $Y \sim \text{Normal}(0, 2^2)$: $P(Y \leq 1) = P(Z \leq (1 - 0)/2) = P(Z \leq 0.5) = 0.6915$;
- $Y \sim \text{Normal}(3, 5^2)$:

$$P(Y > 6) = 1 - P(Y \leq 6) = 1 - P(Z \leq (6 - 3)/5) = 1 - P(Z \leq 0.6) = 1 - 0.7257 = 0.2743;$$

Table of the standard Normal distribution

Entries in table are $P(Y \leq y)$

y	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990