MATH 223 - LINEAR ALGEBRA **FALL 2017**

MIDTERM EXAM

Date: October 25th
Duration: 1 hour and 30 minutes
Note: This sheet has to be returned with the exam booklet.

Last Name & ID : ______________________________

Question 1. True or false? Circle the answer, no justification is needed.
Throughout, V is a finite-dimensional vector space.

T / F : If W is a subspace of V, then each basis of W is contained in some basis of V.
T / F : If W is a subspace of V, then each basis of V contains a basis of W.
T / F : If V is the sum of two subspaces, $V = W_1 + W_2$, then $\dim V = \dim W_1 + \dim W_2$.
T / F : V has a finite number of bases.

Question 2. Define the notion of trace for a square matrix. Show that $tr(AB) = tr(BA)$ whenever A and B are square matrices of the same size.

Question 3. The following three complex matrices are known as the *Pauli matrices*:

$$P_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad P_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad P_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Check that the Pauli matrices are hermitian. Show that every hermitian matrix in $M_{2\times 2}(\mathbb{C})$ is a linear combination of I, P_1, P_2, P_3 with real coefficients.

Question 4. Define the notion of invertible matrix. Using the definition, show that a diagonal matrix is invertible if and only if each one of its diagonal entries is non-zero.

Question 5. Define the kernel and the range of a linear mapping $F : V \to W$, and show that they are subspaces.