Math 240: Discrete Structures I (W18) – Assignment 1

Solutions must typed or very neatly written and uploaded to MyCourses no later than **6 pm** on **Friday, January 26, 2018**.

1. Use a truth table to determine if each statement is a tautology, contradiction, or contingency.

[3] (a) $(P \vee Q) \Rightarrow \neg P$

[4] (b) $(P \Leftrightarrow Q) \wedge (Q \Leftrightarrow R) \Rightarrow (P \Leftrightarrow R)$

[4] (c) $\Big[(P \oplus Q) \oplus \neg Q\Big] \Leftrightarrow P$, where $P \oplus Q$ is defined by the following truth table:

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

Note: $\oplus$ is known as "exclusive-or"; $P \oplus Q$ means "P or Q are true but not both".

2. Verify the following statements using only identities (see the list posted on MyCourses). Show all of your work and name the identity or identities used in each step.

[4] (a) $\Big[(P \Rightarrow Q) \wedge P\Big] \Rightarrow Q$ is a tautology

[4] (b) $\neg(P \wedge Q) \wedge (Q \Rightarrow P) \equiv \neg Q$

[4] (c) $\neg\Big[(P \vee Q) \vee \big[(Q \vee \neg R) \wedge (P \vee R)\big]\Big] \equiv \neg P \wedge \neg Q$

3. Of the following conditional and biconditional statements, which are true and which are false? Briefly justify your answers.

[2] (a) π is an integer if and only if $\sqrt{e+3}$ is a vowel.

[2] (b) $0 > 1$ whenever $2 + 2 = 4$.

[3] (c) If (a) implies (b), then pigs cannot fly.

4. Symbolize the following English sentences in logic, using the abbreviation scheme provided.

[2] (a) "Thunder only happens when it's raining."
T : thunder happens; R : it's raining

[4] (b) "For every positive integer n there is a prime number that is bigger than n but at most $2n$."
$I(x) : x$ is a positive integer; $P(x) : x$ is a prime number; $B(x, y) : x$ is bigger than y.

[4] (c) "Goldbach's Conjecture is true if every even integer greater than 2 can be written as the sum of two primes."
G : Goldbach's Conjecture is true; $E(x) : x$ is an even integer; $T(x) : x$ is greater than 2; $P(x) : x$ is the sum of two primes.