Homework 6 - Economic Order Quantity (EOQ), Newsvendor Model

MGCR 472 - Operations Management

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Question 1

B&H needs to decide how to manage its inventory of cameras. The demand for cameras at B&H is 200 cameras per week. Each time that B&H places an order for a new shipment of cameras, it must pay $80 in fixed processing fees. A camera costs B&H $60 to purchase. The cost for B&H to hold a camera in its store for one week is $4. Assume that the lead time for the delivery of a camera is 0 weeks.

a. Suppose that B&H places orders for cameras in quantities of 50 cameras at a time and places a new order for cameras each time that it runs out. Draw a graph showing the number of cameras that B&H has on-hand in inventory at each point in time up until the time when it places its fourth order. Label the points in time at which B&H places a new order. Assume that B&H places its first order for 50 cameras on day 0.

b. Suppose again that B&H places orders for 50 cameras at a time. What will be B&H's average holding costs per week? What will be B&H's average fixed ordering costs per week?

c. What is the optimal number of cameras for B&H to order each time that it places an order? Assuming that B&H orders according to its optimal ordering quantity (EOQ), what will be B&H's average fixed ordering costs over the course of a week? What will be B&H's average holding costs over the course of a week?

d. Suppose now that the lead time for the delivery of a shipment of cameras increases to 0.2 weeks. Assuming that B&H orders according to its optimal order quantity from part c above, what should be B&H's reorder point for a shipment of cameras?

e. Suppose that the lead time for the delivery of cameras is still 0.2 weeks. However, rather than placing an order when B&H's inventory of cameras reaches its reorder point calculated in part d above, B&H decides to place a new order for cameras whenever it has 60 cameras in inventory. How much additional holding costs (over the EOQ holding costs) will B&H incur as a result of this ordering policy? (Suppose that B&H orders according to the EOQ level each time that it places an order.)

f. [Extra Credit] Suppose now that the lead time for the delivery of a shipment of cameras increases to 0.5 weeks. Assuming that B&H orders according to its optimal order quantity from part
c above, what should be B&H's reorder point for a shipment of cameras? Explain carefully your answer.

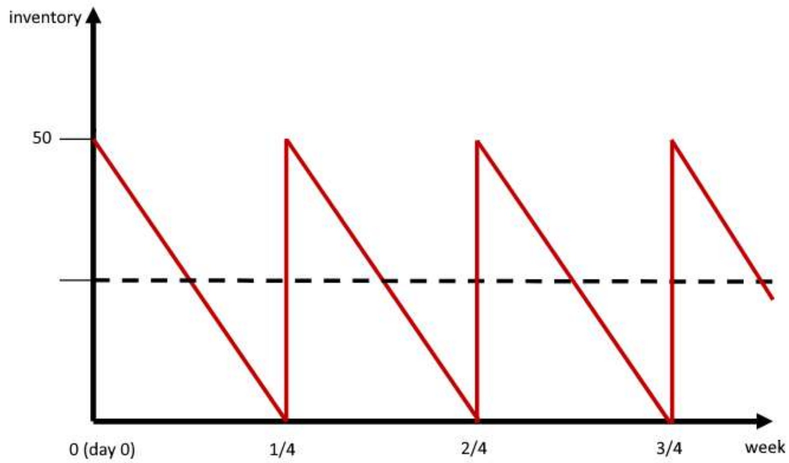
Solution

a. Demand D = $200/week

Ordering cost S = $80/order

Purchasing cost P = $60/unit

Holding cost H = $4/unit/week



b. Order quantity Q = 50 units

Average inventory = Q/2 = 25 units

$$\text{Average holding cost } \overline{H} = HQ/2 = 4(50/2) = \$100$$

Number of cycles N = D/Q = 4

$$\text{Average ordering cost } \overline{S} = SN = 80(4) = \$320$$

c. The optimal order quantity (EOQ solution) is

$$Q^* = \sqrt{\frac{2SD}{H}} = \sqrt{\frac{2(80)(200)}{4}} = 40\sqrt{5} = 89.44 \text{ units}$$

Therefore, the optimal ordering and holding costs are

$$\overline{S}^* = SN^* = SD/Q^* = 80(200/89.44) = \$178.89$$

$$\overline{H}^* = HQ^*/2 = 4(89.44/2) = \$178.88$$

d. Lead time L = 0.2 weeks

The reorder point (ROP) is

$$\text{ROP} = L \times \text{weekly demand} = LD = 0.2(200) = 40 \text{ units}$$

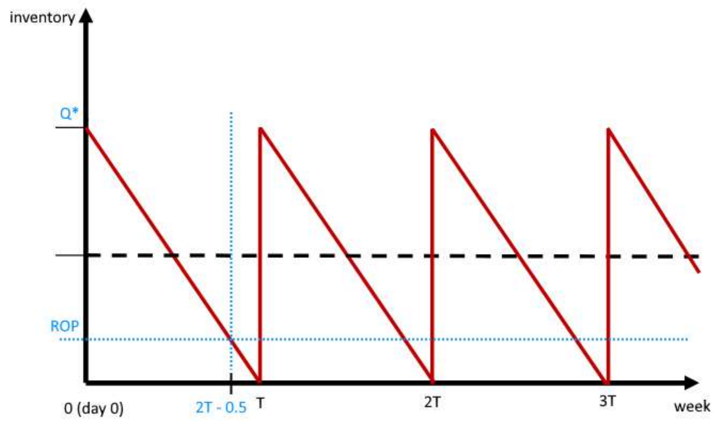
e. With ROP = 60 units, the inventory will always be refilled when it still has 20 units left. The extra holding cost to cover these 20 units is

$$20H = 20(4) = \$80$$

f. The cycle length T is

$$T = Q^*/D = 40\sqrt{5}/200 = \sqrt{5}/5 = 0.45 \text{ weeks}$$

Lead time L = 0.5 weeks, and L > T



From above, we can interpret the lead time L' to be

$$L' = T - (2T - 0.5) = 0.5 - T = 0.5 - \sqrt{5}/5 = 0.053 \text{ weeks}$$

From above, the ROP is

$$\text{ROP} = L'D = (0.5 - \sqrt{5}/5)(200) = 10.56 \text{ units}$$

Question 2

Every evening, Empire News needs to determine how many copies of the newspaper to purchase for the following morning. A newspaper costs $0.50 to purchase and sells for $2.25. Any unsold newspapers can be recycled at the end of the day at a value of $0.25 per newspaper. The following table provides Empire News' daily demand distribution for newspapers.

Number of newspapers	160	170	175	185	200	205	210	220
Probability	0.2	0.1	0.05	0.2	0.15	0.05	0.20	0.05

a. Suppose that Empire News is considering the purchase of 200 newspapers. What is the marginal value from a 201st newspaper? Is it profitable for Empire News to purchase its 201st newspaper? Justify your answer.

b. What is the optimal number of newspapers for Empire News to purchase each evening?

Solution

a. Unit cost $0.5

Unit price $2.25

Salvage value $0.25

Probability that demand D ≤ 200 = 0.2+0.1+0.05+0.2+0.15=0.7

Therefore, the marginal value of from a 201st newspaper is

$$0.7(0.25 - 0.5) + 0.3(2.25 - 0.5) = \$0.35$$

which is positive.

Therefore, purchasing a 201st newspaper is profitable.

b. The costs of overstocking and understocking are

$$c_o = 0.5 - 0.25 = \$0.25$$
$$c_u = 2.25 - 0.5 = \$1.75$$

From the **Newsvendor formula,** the **critical formula/service level** is

$$c_u/(c_u + c_o) = 1.75/2 = 0.875$$

Because demand here is **discrete**, we want to find the smallest X, the optimal number of newspapers, such that

$$\text{P}\{D \leq X\} = \text{CDF}(X) \geq 0.875$$

From the CDF of each demand

Demand	160	170	175	185	200	205	210	220
Pdf	0.2	0.1	0.05	0.2	0.15	0.05	0.2	0.05
Cdf	0.2	0.3	0.35	0.55	0.7	0.75	0.95	1

The optimal number of newspapers is 210.

Question 3

A pastry shop is considering how much hot chocolate to prepare each morning. Hot chocolate costs \$0.10 per oz to make and sells for \$0.40 per oz. Customers can buy hot chocolate in any number of ounces that they wish. Any hot chocolate not sold by the end of the day is discarded. The daily demand for hot chocolate is normally distributed with a mean of 1,200 oz and a standard deviation of 150 oz. How much hot chocolate should the pastry shop make each morning? You may use a z-table for this question.

Solution

The understocking and overstocking costs are

$$c_u = 0.4 - 0.1 = 0.3$$
$$c_o = 0.1$$

Demand is normally distributed with mean and standard deviation

$$\mu = 1200, \sigma = 150$$

Want X such that

$$\text{CDF}(X) = c_u/(c_u + c_o) = 0.3/0.4 = 0.75$$
$$X = \mu + \sigma \mathbf{z} = 1200 + 150\mathbf{z}$$

From the z-table, CDF = 0.75 ⇒ **z** = 0.68

Therefore

$$X = 1200 + 150(0.68) = 1302$$

The pastry shop should make 1302 oz of hot chocolate each morning.