Homework 2 - Waiting Lines

MGCR 472 - Operations Management

LE, Nhat Hung
McGill ID: 260793376
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Prof. Rim Harris
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Question 1

1. Customers arrive to the checkout area of New Navy such that the interarrival rate is exponential with rate equal to 100 customers per hour. At the checkout area, the service time of each customer is exponentially distributed, and on average each cashier can serve 10 customers per hour. The store has hired 12 cashiers to checkout customers and the checkout area consists of a single pool of cashiers with one common line in front of them.

a. What is the probability that a customer's service time is less than 19 minutes?

b. What is the average number of customers waiting in the queue to be checked out? This does not include the customers being checked out.

c. What is the average service time?

d. What is the average total number of customers in the checkout area? This includes the customers who are waiting to be checked out, and the customers being checked out.

e. [Extra Credit] What is the probability that there are 12 or more customers in the checkout area?

f. Suppose now that New Navy decides to replace the single common line for its customers with 12 separate lines, one line in front of each of the 12 cashiers. What is now the average number of customers waiting to be checked out? This only includes the customers who are waiting to be checked out (and not the customers being served).

Solution:

a. We have

$$\begin{gathered}\lambda = 100 \text{ customers/hr}\\ \mu = 10 \text{ customers/hr}\\ s = 12 \text{ cashiers}\\ 1 \text{ common line}\end{gathered}$$

Then, the probability that a customer's service time is less than 19 minutes is

$$Pr(\text{service time} < 19) = 1 - e^{-19\mu} = 1 - e^{-19(10/60)} = 0.96$$

b. From the M/M/s table, the average number of customers waiting in the queue to be checked out is

$$L_Q = 2.25 \text{ customers}$$

c. The average service time is

$$1/\mu = 1/10 = 0.1 \text{ hrs/customer} = 6 \text{ mins/customer}$$

d. The average number of customers in the checkout area is

$$L_S = \lambda W_S = \lambda(1/\mu + W_Q) = \lambda(1/\mu + L_Q/\lambda) = \lambda(1/\mu) + L_Q = 100(0.1) + 2.25 = 12.25 \text{ customers}$$

e. We know

$$\text{interarrival time } = 1/\lambda$$

$$\Rightarrow Pr(\text{interarrival time} > x) = Pr(1/\lambda > x) = e^{-\lambda x}$$

$$\text{avg system wait time } W_S = L_S/\lambda = 12.25/100 = 0.1225 \text{ hrs}$$

Therefore, the probability that there are ⩾12 customers in the checkout area is

$$Pr(L_s \geq 12) = Pr(\lambda W_S \geq 12) = Pr\left(\frac{1}{\lambda} \leq \frac{W_S}{12}\right) = 1 - Pr\left(\frac{1}{\lambda} \geq \frac{W_S}{12}\right) = 1 - e^{-\lambda \frac{W_S}{12}} = 1 - e^{-100\frac{0.1225}{12}} = 0.64$$

f. We now have 12 M/M/1 systems. The average number of customers waiting to be checked out in one queue is

$$L_Q = \frac{(\lambda/12)^2}{10(10-\lambda/12)} = \frac{(100/12)^2}{10(10-100/12)} \approx 4.167 \text{ customers}$$

With 12 queues, the total average number of customers waiting to be checked out is

$$12L_Q = 12\frac{(100/12)^2}{10(10-100/12)} = 50 \text{ customers}$$

Alternatively, in the M/M/s table, dividing lambda by 12 and looking at number of servers s = 1 yields the same L_Q for one queue, which amounts to 50 customers for all 12 queues.