Assignment 1 — **LE, Nhat Hung**
MATH 323 - Probability — McGill ID: 260793376
Prof. David Stephens — Date: September 26, 2018
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1. Consider two events A and B in sample space S, and suppose $P(A) = p_A$, $P(B) = p_B$.

(a) Find the largest number l_1 and the smallest number u_1 such that

$$l_1 \leq P(A \cap B) \leq u_1 .$$

$l_1 = 0$
$u_1 = min(p_A, p_B)$

(b) Find the largest number l_2 and the smallest number u_2 such that

$$l_2 \leq P(A \cup B) \leq u_2 .$$

$l_2 = max(p_A, p_B)$
$u_2 = p_A + p_B$

2. Three newspapers, denoted A, B and C are published in a city. A comprehensive survey indicates that amongst the adult population of the city, 20% read A, 16% read B and 14% read C, but the survey also reveals that 8% read both A and B, 5% read both A and C, and 4% read both B and C, and that 2% read all three newspapers. A person is selected at random from the adult population of the city (that is, all adults are equally likely to be selected).

Use formal notation to express the following events in terms of A, B, C, and find their probabilities.

(a) the selected person reads none of the newspapers:

$$\begin{aligned} P(A' \cap B' \cap C') = \; & 1 - P(A \cup B \cup C) \\ = \; & 1 - \\ & [\, P(A) + P(B) + P(C) \\ & - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ & + P(A \cap B \cap C) \,] \\ \\ = \; & 1 - 0.2 - 0.16 - 0.14 + 0.08 + 0.05 + 0.04 - 0.02 \\ = \; & 1 - 0.35 \\ = \; & 0.65 \end{aligned}$$

(b) the selected person reads precisely one of the newspapers:

$$a : (A \cap B' \cap C') \cup (A' \cap B \cap C') \cup (A' \cap B' \cap C)$$

$$\alpha : A \cap B' \cap C',$$

$$\beta : A' \cap B \cap C',$$
$$\gamma : A' \cap B' \cap C.$$

$$\begin{aligned}
P(a) = \; & P(\alpha) + P(\beta) + P(\gamma) - P(\alpha \cap \beta) - P(\alpha \cap \gamma) \\
& - P(\beta \cap \gamma) + P(\alpha \cap \beta \cap \gamma) \\
= \; & P(\alpha) + P(\beta) + P(\gamma) - 0 - 0 - 0 + 0 \\
= \; & P(A) + P(A) + P(B) \\
& - 2P(A \cap B) - 2P(A \cap C) - 2P(B \cap C) \\
& + 3P(A \cap B \cap C) \\
= \; & 0.2 + 0.16 + 0.14 - 0.16 - 0.1 - 0.08 + 0.06 \\
= \; & 0.22
\end{aligned}$$

(c) the selected person reads at least A and B, given that they read at least one newspaper.

At least A and B is
$$a : A \cap B,$$

read at least one newspaper is
$$b : A \cup B \cup C.$$

$$\begin{aligned}
P(a \mid b) = \; & P(a \cap b) \div P(b) \\
= \; & P(A \cap B) \div P(A \cup B \cup C) \\
= \; & 0.08 \div 0.35 \\
\simeq \; & 0.2286
\end{aligned}$$

3. (a) Eighteen students enrol in a course in which the Professor utilizes the following novel method of evaluation. Eighteen balls are placed in a bag: one is black, the other seventeen are red. Students are each to select one ball from the bag, without replacement. The student who selects the black ball will receive an A, all other students will receive a B.

If a student prefers a higher grade to a lower grade, should they prefer to select first, fifth or eighteenth in the sequence of selections ?

A: first ball selected is black
B: fifth ball selected is black
C: eighteenth ball selected is black

$$P(A) = \frac{1}{18}$$

$$P(B) = \left(\frac{17}{18}\right)\left(\frac{16}{17}\right)\left(\frac{15}{16}\right)\left(\frac{14}{15}\right)\left(\frac{1}{4}\right) = \frac{1}{18}$$

$$P(C) = \frac{17!}{18!} = \frac{1}{18}$$

The chance of getting an A is always $\frac{1}{18}$ .The grading scheme is “fair”.

(b) After complaints, the Professor agrees to modify the method of evaluation in (a), and does so by replacing one red ball by another black ball, with the concession of awarding As to each of the two students who select a black ball, with the other students receiving Bs.

Under the new method of evaluation, should a student prefer to select first, fifth or eighteenth ?

$$P(A) = \frac{1}{9}$$

We will now calculate $P(B)$:

We can represent all 18 selections as a sequence: RRRBRBRR… for example. R for a red ball and B for a black ball.

Fix a black ball at the fifth position. Then, the number of outcomes which has B at the fifth position is the number of different sequences we can create with the remaining 17 balls. With only 1 black ball among these 17 balls, the number of different sequences we can make is simply 17.

Thus, we have 17 outcomes where the fifth selected ball is black.

The total number of different sequences possible with 2 black balls and 16 red balls is C_2^{18} .

We then have:

$$P(B) = \frac{17}{C_2^{18}} = \frac{17}{153} = \frac{1}{9}$$

The calculation for $P(C)$ is the same.

$$P(C) = \frac{17}{C_2^{18}} = \frac{1}{9}$$

Again, it does not matter when the student select a ball.