Assignment 2 — **LE, Nhat Hung**

MATH 240 — February 2, 2018

1. (a) i. $\exists x(D(x) \wedge R(x))$
   ii. $\forall x(\neg D(x) \vee \neg R(x))$
   iii. Everything isn't in the state of Denmark or isn't rotten.

   (b) i. $\forall \epsilon \in \mathbb{R}_{>0}[\exists N \in \mathbb{R}_{>0}(B(x,N) \Rightarrow B(\epsilon, |f(x)-L|)]$
   ii. $\exists \epsilon \in \mathbb{R}_{>0}[\forall N \in \mathbb{R}_{>0}(B(x,N) \wedge \neg B(\epsilon, |f(x)-L|)]$
   iii. There exists a positive real ϵ such that for all positive real N, $x > N$ and $|f(x)-L| \geq \epsilon$.

2. (a) The table below shows for all truth value combinations of P, Q, R, $(P \Rightarrow Q) \wedge (Q \Rightarrow R) \Rightarrow (P \Rightarrow R)$ is true, i.e. the latter is a tautology.

P	Q	R	$P \Rightarrow Q$	$Q \Rightarrow R$	$P \Rightarrow R$	$(P \Rightarrow Q) \wedge (Q \Rightarrow R) \Rightarrow (P \Rightarrow R)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

   (b)

$$
\begin{aligned}
(P \Rightarrow Q) \wedge \neg Q \Rightarrow \neg P &\equiv (\neg P \vee Q) \wedge \neg Q \Rightarrow \neg P && \textit{conditional} \\
&\equiv (\neg P \wedge \neg Q) \vee (Q \wedge \neg Q) \Rightarrow \neg P && \textit{distribution} \\
&\equiv (\neg P \wedge \neg Q) \vee \mathbb{F} \Rightarrow \neg P && \textit{complement} \\
&\equiv (\neg P \wedge \neg Q) \Rightarrow \neg P && \textit{identity} \\
&\equiv P \vee Q \vee \neg P && conditional, DeMorgan's \\
&\equiv Q \vee \mathbb{T} && \textit{complement} \\
&\equiv \mathbb{T} && \textit{domination}
\end{aligned}
$$

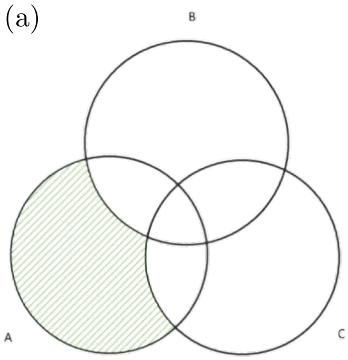
   (c) S: study, P: pass, M: watch a movie.

$$
\begin{aligned}
(S \Rightarrow P) \wedge (\neg M \Rightarrow S) \wedge \neg P \Rightarrow M &\equiv (S \wedge \neg P) \vee (\neg M \wedge \neg S) \vee P \vee M && \textit{conditional,} \\
& && DeMorgan's \\
&\equiv [(S \vee P) \wedge (\neg P \vee P)] \vee && \textit{association,} \\
&\quad [(\neg M \vee M) \wedge (\neg S \vee M)] && \textit{distribution} \\
&\equiv [(S \vee P) \wedge \mathbb{T}] \vee [\mathbb{T} \wedge (\neg S \vee M)] && \textit{complement} \\
&\equiv \mathbb{T} \wedge [(S \vee P) \vee (\neg S \vee M)] && \textit{distribution} \\
&\equiv (S \vee P) \vee (\neg S \vee M) && \textit{identity} \\
&\equiv \mathbb{T} \vee P \vee M && \textit{association,} \\
& && \textit{complement} \\
&\equiv \mathbb{T} && \textit{domination}
\end{aligned}
$$

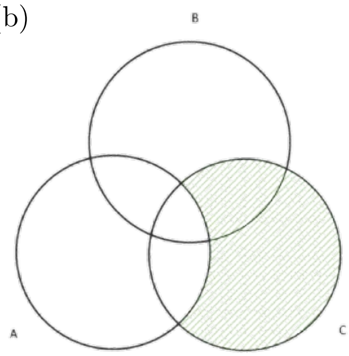
   The argument's thus valid.

3. (a) $A \oplus C = \{-1, 0, 2, 3, 4, 5, 6\}$
   (b) $A \cap D = \{1, 2\}$
   (c) $C \cup D = \{-1, 0, 1, 2, \{1, 2\}\}$
   (d) $\{1, \{2\}\} \cup D = \{1, 2, \{2\}, \{1, 2\}\}$
   (e) $B \backslash (A \oplus C) = \{q \in \mathbb{Q} | q \in (1, 2) \cup (2, 3)\}$
   (f) $\{\emptyset\} \backslash \wp(A) = \emptyset$
   (g) $(A \backslash B) \backslash (C \backslash \overline{D}) = \{1, 3, 4, 5, 6\} \backslash \{1\} = \{3, 4, 5, 6\}$
   (h) $\wp(D) \cap D = \{\{1, 2\}\}$

4. (a)



(b)



(c)