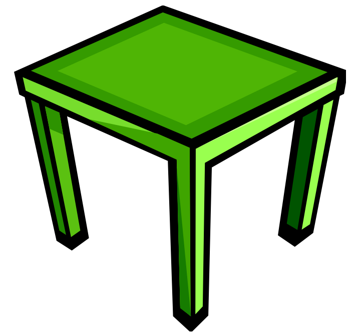
Operations Management





Session 6:
Formulating a Linear Program

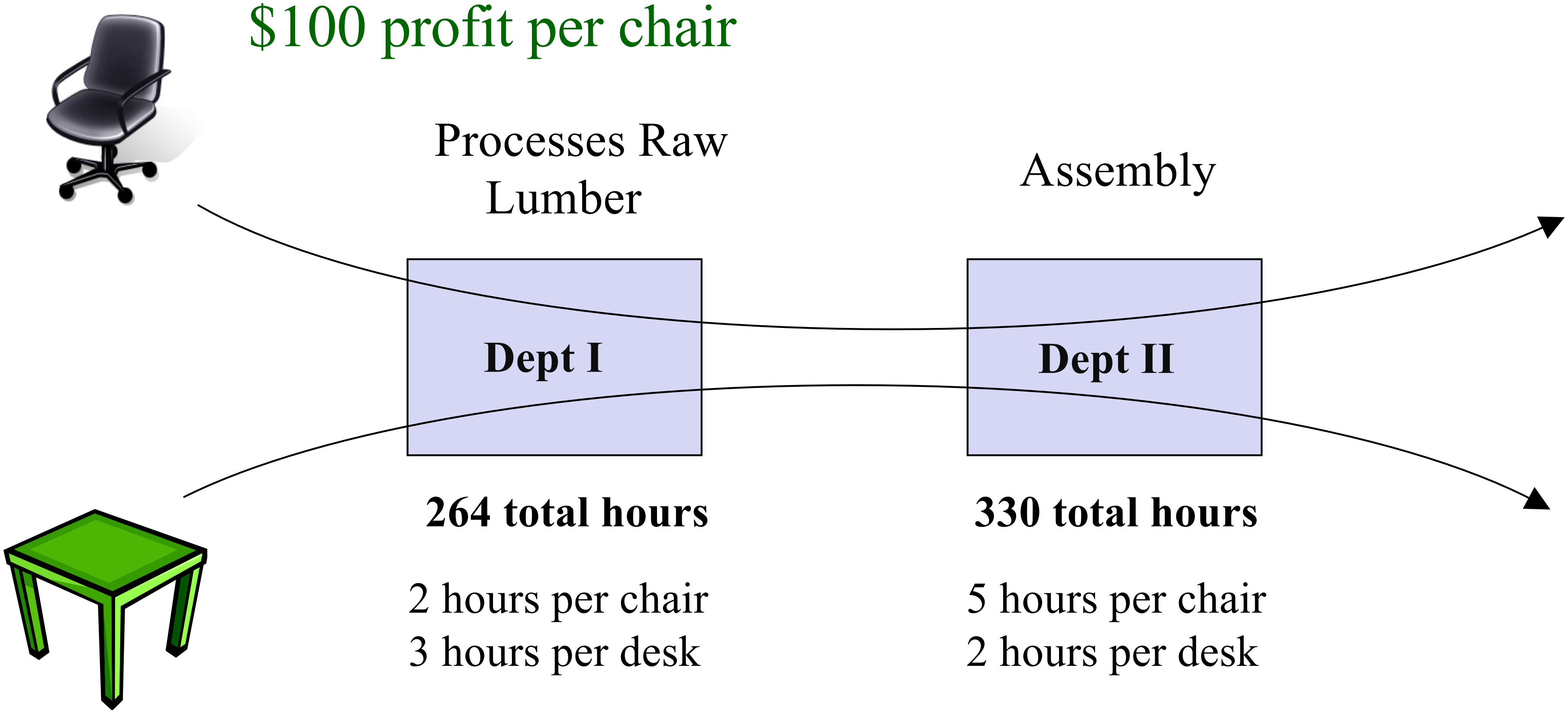
Where Are We?

<u>Module 3:</u> Optimal Resource Allocation

- Session 6: Formulating a LP – Today
- Session 7: Solving a LP using Excel – Jan. 28
- Session 8: Applying LP to real-estate – Jan. 30
- Session 9: Guest Lecture – Feb. 4
- Session 10: Midterm Review – Feb. 6
- Midterm – **Friday, February 14**

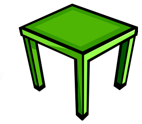
Problem 1: Office Supplies Inc.

$100 profit per chair

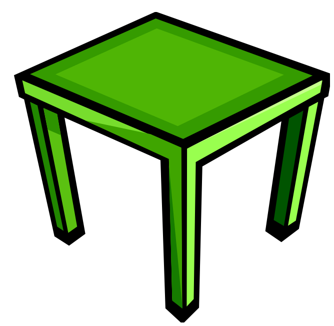


$125 profit per desk

Problem 1: Office Supplies Inc.



How many chairs and how many desks should Office Supplies Inc. make in order to maximize their profit?



How to Formulate a LP

1. Identify decision variables.

2. Write out objective function.

3. Write out constraints.

4. Write the LP.

Step 1: Identify Decision Variables

- Decision variables represent what has to be decided.
- Example:
  - How many products to produce/buy/consume?
- Each decision variable is written as an unknown, usually $x_1, x_2, \ldots, x_n$.
- Find the decision variables by looking at the problem.

Step 2: Write Out Objective Function

- What should be optimized in the linear program?
- Do we want to **maximize or minimize**?
- Examples:
  - Minimize costs, Maximize profit, Maximize revenue, etc.
- Data:
  - Objective function coefficient: contribution of each variable to the objective function.
- The objective function is of the type:

$$\boxed{\text{Min } z = c_1 x_1 + \ldots + c_n x_n}$$

or

$$\boxed{\text{Max } z = c_1 x_1 + \ldots + c_n x_n}$$

where $c_1, \ldots, c_n$ are real numbers.

Step 3: Write Out Constraints

- What is restricting the objective?
- Examples:
  - A resource can only be used for a certain number of hours.
- Constraints are of the type:

$$\boxed{a_1 x_1 + \ldots + a_n x_n \geq b}$$

or

$$\boxed{a_1 x_1 + \ldots + a_n x_n \leq b}$$

where $a_1,\ldots,a_n$ and b are real numbers.

Step 4: Write the LP

Minimization Problem			Maximization Problem	
Min	$z = c_1 x_1 + \ldots + c_n x_n$	Objective Function	Max	$z = c_1 x_1 + \ldots + c_n x_n$
s.t.	$a_{11} x_1 + \ldots + a_{1n} x_n \leq b_1$	Constraints	s.t.	$a_{11} x_1 + \ldots + a_{1n} x_n \leq b_1$
	$a_{21} x_1 + \ldots + a_{2n} x_n \leq b_2$			$a_{21} x_1 + \ldots + a_{2n} x_n \leq b_2$
	$\ldots$			$\ldots$
	$x_1 \geq 0$	Non-negativity Constraints (almost always)		$x_1 \geq 0$
	$\ldots$			$\ldots$
	$x_n \geq 0$			$x_n \geq 0$

Terminology

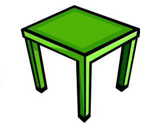
- A set of values for $x_1,\ldots, x_n$ is called:
  - A **feasible solution** if it satisfies all the constraints (including non-negativity).
  - An **optimal solution** if it satisfies all the constraints (including non-negativity) AND gives the best value of the objective function.
- The best value of the objective function is called the **optimal objective function value**.
- There could be more than one optimal solution and sometimes there is no optimal solution.

Problem 1: Decision Variables

- The decision variables for Office Supplies Inc. are how many chairs and how many desks to produce in this production period.

- Let
  - x_1 be the number of chairs produced in this production period.
  - x_2 be the number of desks produced in this production period.

Problem 1: Objective Function

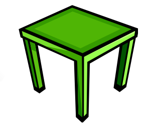
- The objective of Office Supplies Inc. is to maximize profit.

- Contribution of each decision variable to the profit:
  - \$100 per chair.
  - \$125 per desk.

- Objective function:

$$\boxed{\text{Max } z = 100\, x_1 + 125\, x_2}$$

Problem 1: Constraints

- Limited resources:
  - 264 hours in Dept I.
  - 330 hours in Dept II.

- Utilization of resources for each variable:
  - Dept I: 2 hours per chair, 3 hours per desk.
  - Dept II: 5 hours per chair, 2 hours per desk.

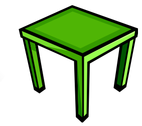
- Constraints:

$$\boxed{\begin{array}{c} 2\,x_1 + 3\,x_2 \leq 264 \\ 5\,x_1 + 2\,x_2 \leq 330 \end{array}} \quad \begin{array}{l} \text{Dept I} \\ \text{Dept II} \end{array}$$

Problem 1: Write the LP

$$
\begin{array}{ll}
\text{Max} & z = 100\,x_1 + 125\,x_2 \\
\text{s.t.} & 2\,x_1 + 3\,x_2 \leq 264 \\
 & 5\,x_1 + 2\,x_2 \leq 330 \\
 & x_1 \geq 0 \\
 & x_2 \geq 0
\end{array}
$$

Problem 1: Office Supplies Inc.

- Could Office Supplies Inc. produce 55 chairs and 33 desks? In other words, is $x_1 = 55$ and $x_2 = 33$ a **feasible solution** to the LP?

No, because it does not satisfy the Dept. II constraint:

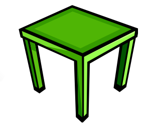
$$5 \times 55 + 2 \times 33 = 341 > 330.$$

- Is $x_1 = 33$ and $x_2 = 66$ a **feasible solution** to the LP?

Yes, because it satisfies all the constraints:

$$2 \times 33 + 3 \times 66 = 264 \leq 264$$
$$5 \times 33 + 2 \times 66 = 297 \leq 330$$
$$33 \geq 0$$
$$66 \geq 0$$

Problem 1: Office Supplies Inc.

- What is the profit if Office Supplies Inc. produces 33 chairs and 66 desks?

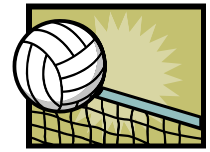
$$z = \$100 \times 33 + \$125 \times 66 = \$11{,}550.$$

- What can you say about the optimal objective function value?

The optimal objective function value is greater than or equal to \$11,550 as we found a feasible solution that attains this value.

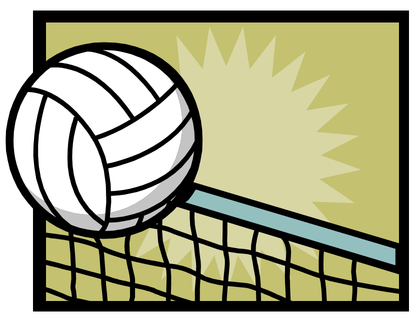
We say that \$11,550 is a **lower bound** on the optimal objective function value of the LP.

Problem 2: Protein Milk

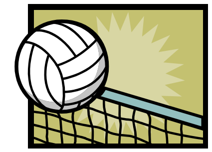


A 16-ounce bottle of Protein Milk must contain protein, carbohydrates, and fats in at least the following amounts:

Protein	Carbs	Fat
3 oz.	5 oz.	4 oz.



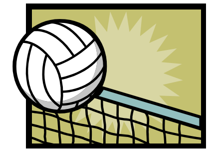
Problem 2: Protein Milk



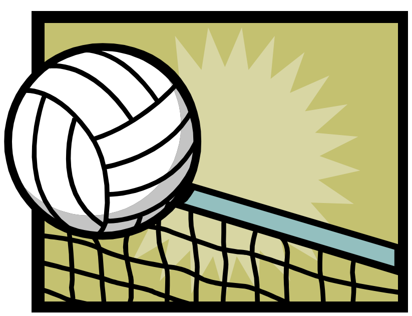
Four mixes may be blended together in various proportions to produce a bottle. The contents and prices of each mix are as follows.

	Contents and price per ounce of mix			
Mix	**Protein Content (oz)**	**Carbohydrate Content (oz)**	**Fat Content (oz)**	**Price ($)**
1	**3/16**	**7/16**	**5/16**	**4/16**
2	**5/16**	**4/16**	**6/16**	**6/16**
3	**2/16**	**2/16**	**6/16**	**3/16**
4	**3/16**	**8/16**	**2/16**	**2/16**

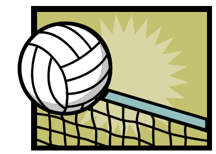
Problem 2: Protein Milk



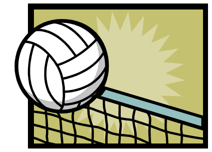
How much of each mix should be added to a 16 oz. bottle of Protein Milk in order to minimize the cost?



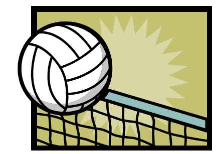
Problem 2: Decision Variables



Problem 2: Write the LP



Problem 2: Protein Milk



- Find a feasible solution:

- The objective function value associated with this solution is:

Summary

- Formulating a **Linear Program** (LP):
  - Decision variables.
  - Objective function.
  - Constraints.

- Feasible solutions versus optimal solutions.

- Very useful tool to guide decision making:
  - Large scale problems.
  - Very large number of applications.

Next Class

- Bring your laptops to class.
- Install Solver in Excel (see instructions on myCourses).
- Download the Excel file from myCourses before the class (LP_Spreadsheet.xls).