COMP 546

Lecture 14

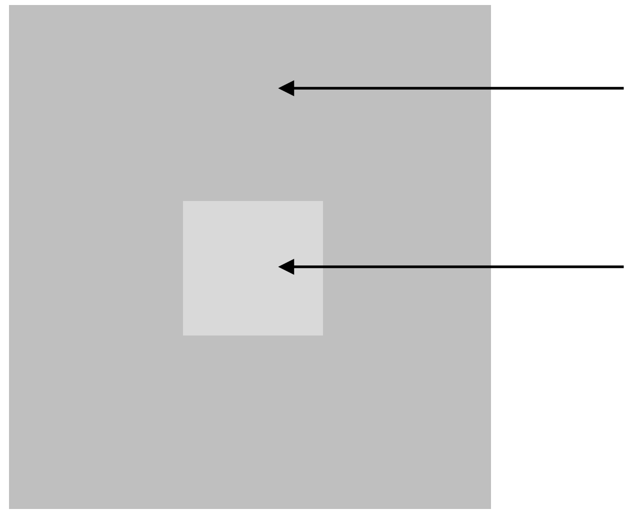
Likelihood

Tues. Feb. 26, 2019

Overview of today

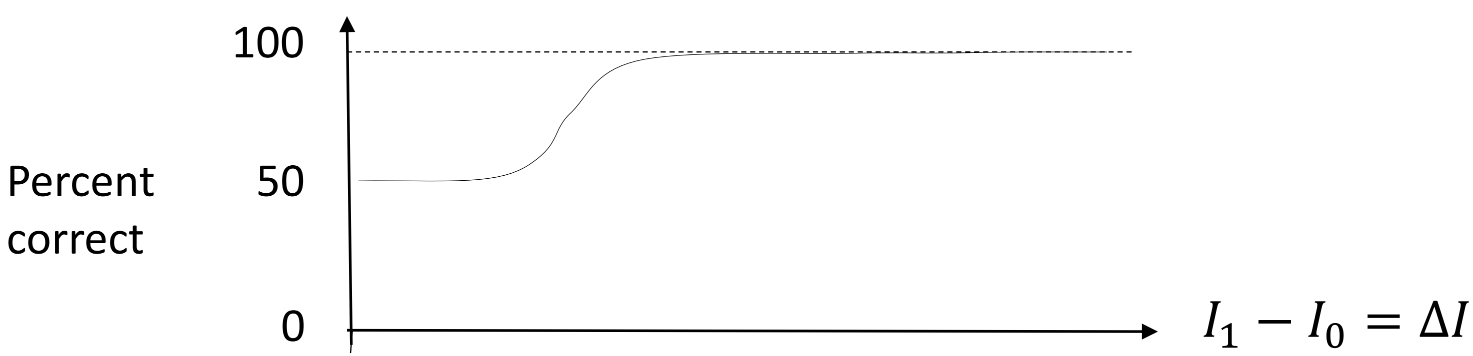
- Informal notion of likelihood
- Formal definition of likelihood as conditional probability
- Examples
  - Intensity increment
  - Orientation
  - Disparity
  - Slant and tilt

Task 1: detecting an intensity increment

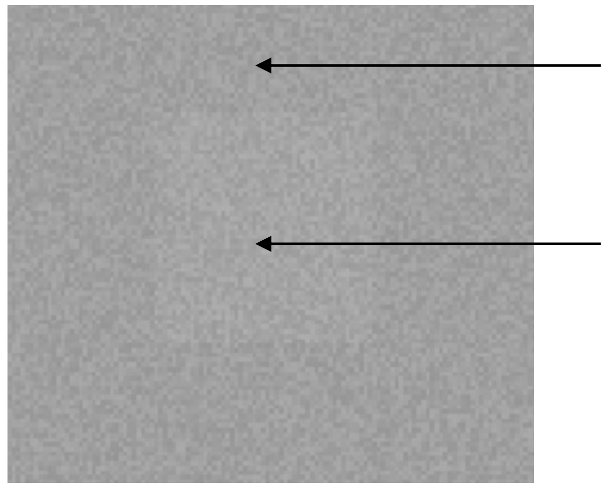


$$I_0$$

$$I_1 = I_0 + \Delta I$$

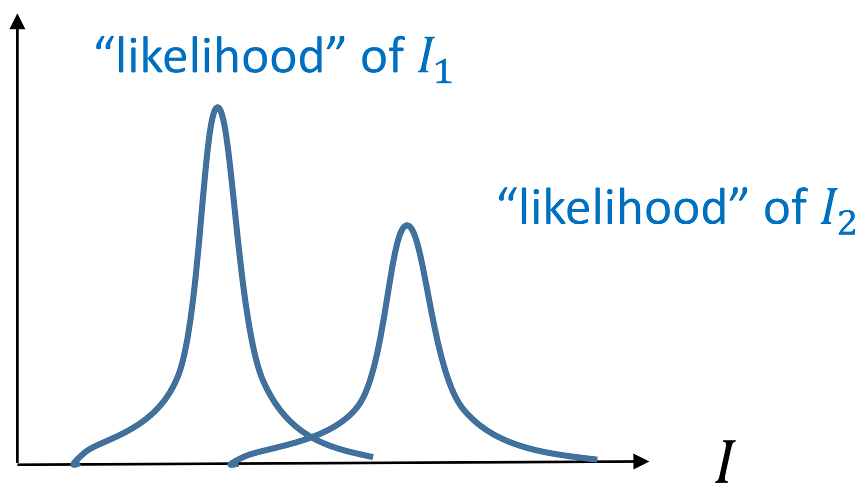


If ΔI is small and noise is big, then the task becomes more difficult.



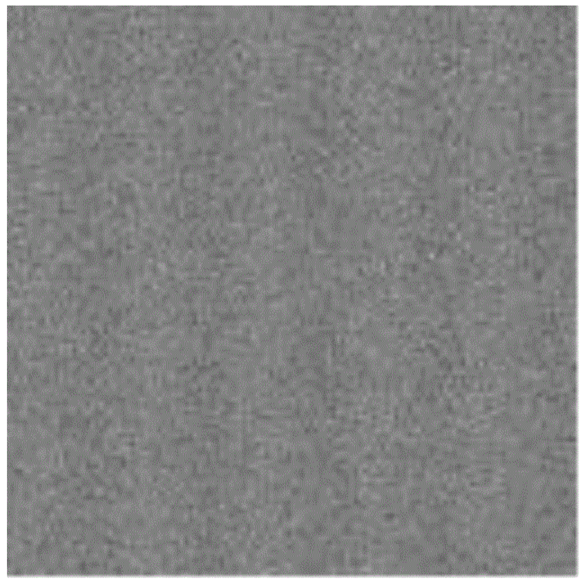
$$I_0 + noise(x, y)$$

$$I_1 + noise(x, y)$$



For now, think of these as observer's relatively certainty about values.

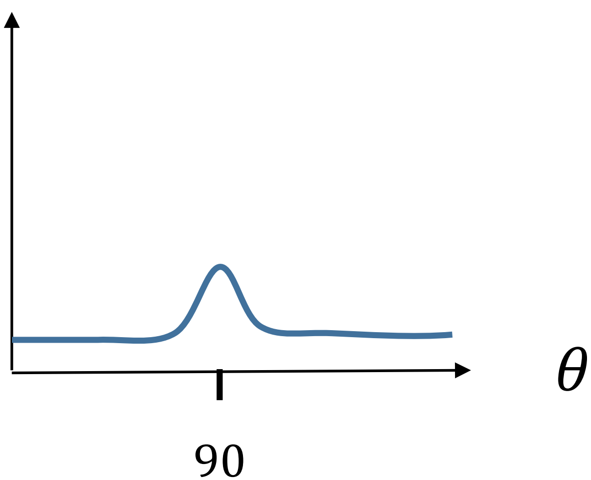
Task 2: estimate orientation of 2D sinusoid in noise



Last lecture: task was to judge horizontal or vertical.

But one could define many other tasks that require judging orientation.

likelihood of orientation

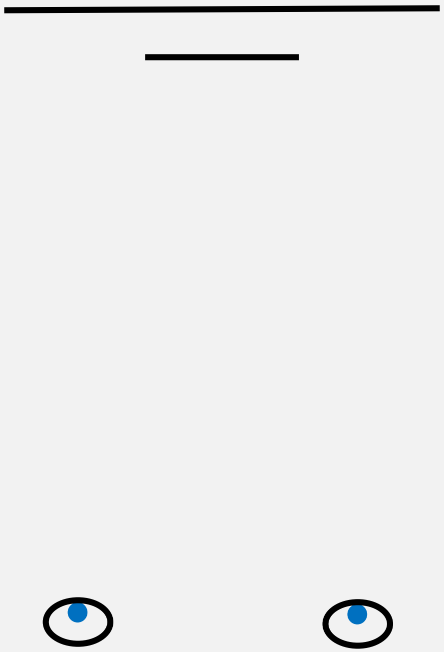


For now, think of these as observer's relatively certainty about values.

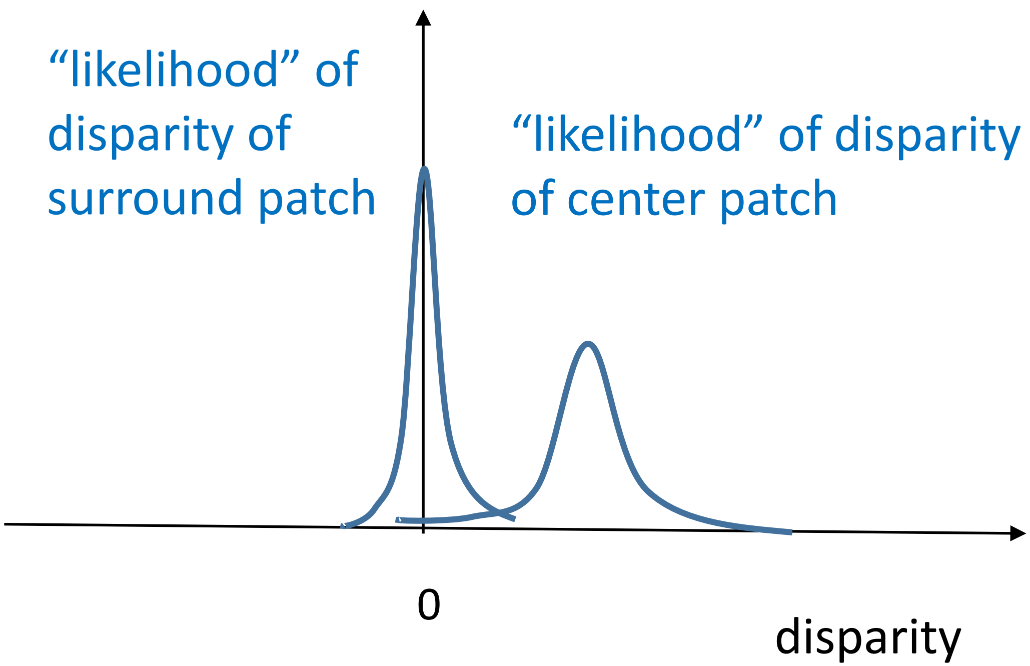
Task 3: estimate disparity of center and surround

left eye

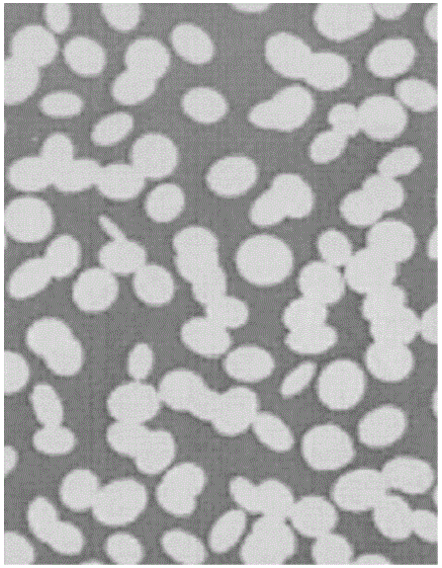
right eye

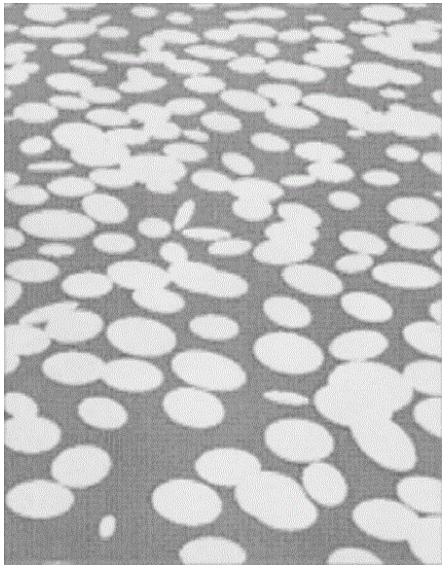


For now, think of these as observer’s relatively certainty about values.

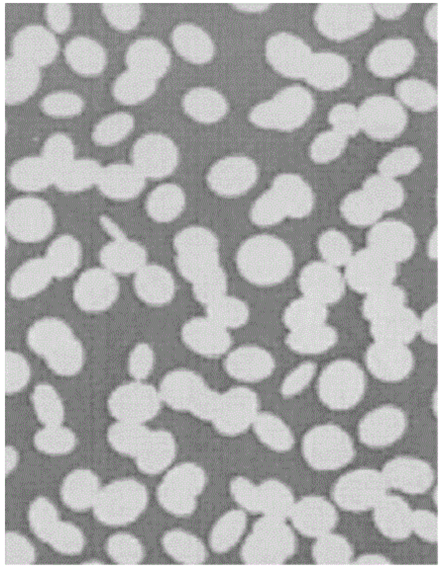


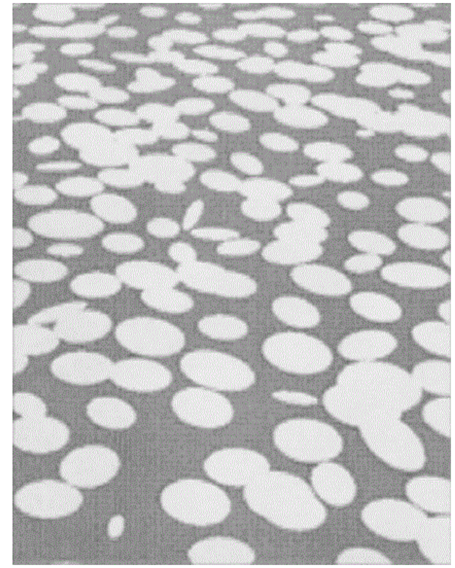
Task 4: estimate surface slant from texture



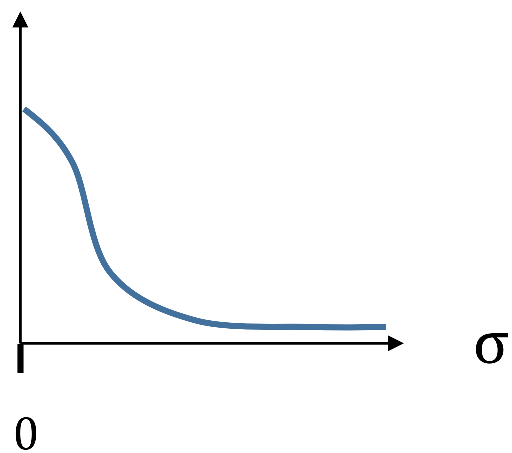


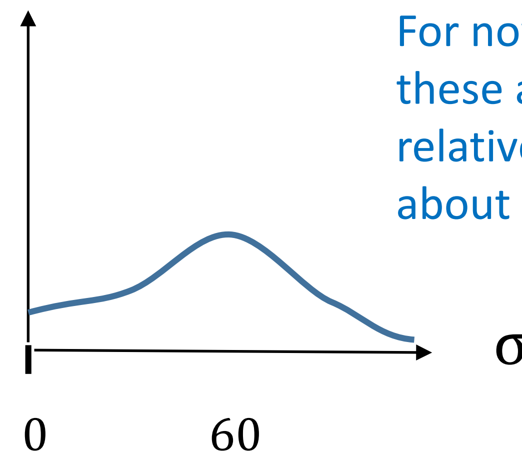
Random distribution of ellipse shapes and sizes
(not disks)





likelihood of slant σ





For now, think of these as observer's relatively certainty about values.

More *likely* to be frontoparallel

Most *likely* to be ~60 deg slant.

[Knill, 1998]

What is the formal definition of “likelihood” ?

(If you don’t remember your basic probability definitions, then you need to review them.)

We need to write down the variables of the problem:

Scene $\xrightarrow{\text{image formation}}$ Image $\xrightarrow{\text{vision}}$ Estimated Scene

S = s

$I = i$

S = $\hat{s}$

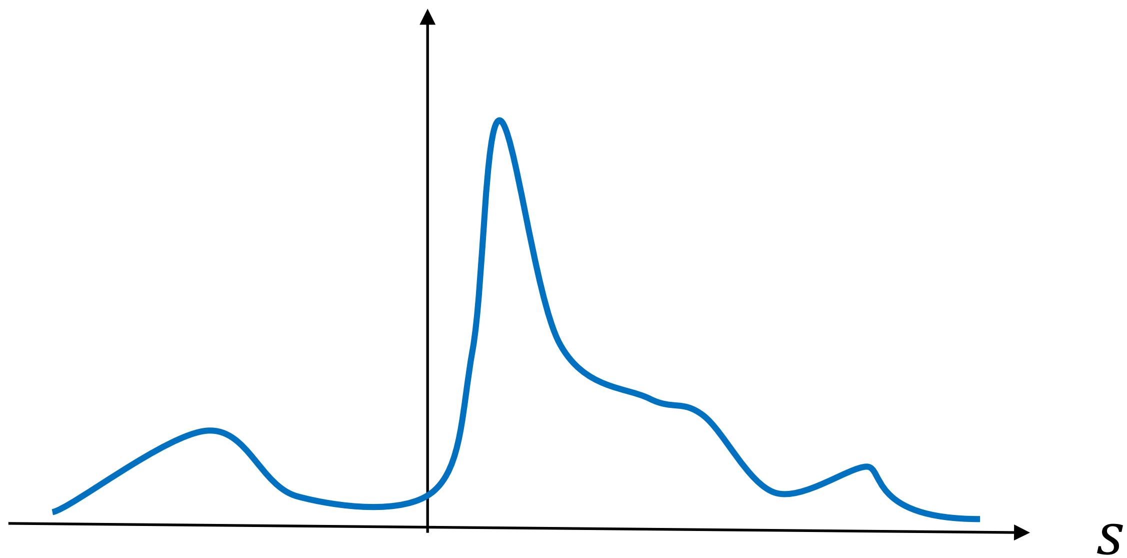
Scene	Image	Estimated Scene
luminance	image intensity	luminance
orientation	filter responses	orientation
disparity		disparity
2D velocity		2D velocity
surface slant, tilt		surface slant, tilt
…		…

Likelihood (from probability)

Let I and S be two random variables, representing some image and scene property, respectively. The conditional probability

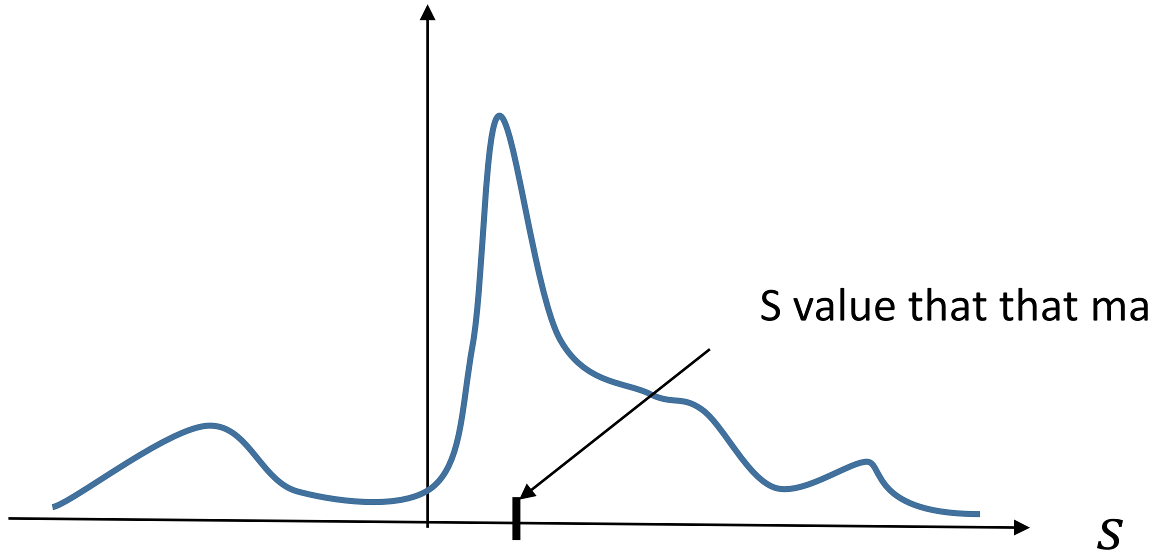
$$p(\, I = i \mid S = s \,)$$

is known as the "likelihood" of scene $S = s$, for that image $I = i$.



e.g. Maximum likelihood estimation:

Given an image $I = i$, choose the scene $S = s$ that maximizes $p(I = i \mid S = s)$.



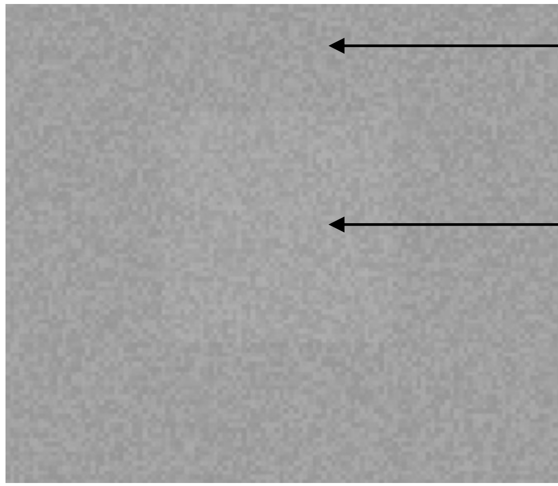
Overview of today

- Informal notion of likelihood

- Formal definition of likelihood as conditional probability

- Examples
  - Intensity increment (details)
  - Orientation (sketch only)
  - Disparity “
  - Slant and tilt “

Task 1 : detecting an intensity increment

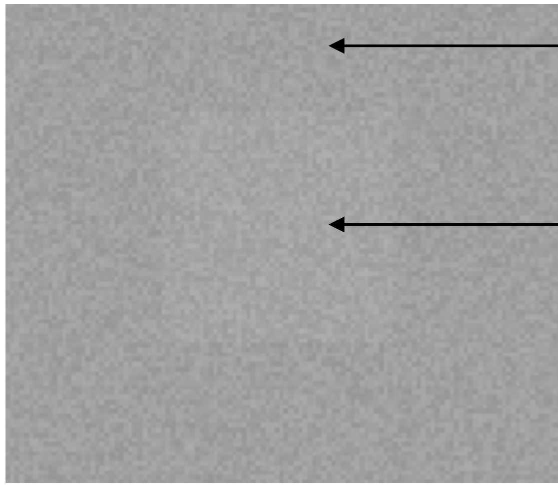


$I_0 + noise(x, y)$

$I_0 + \ \Delta I + noise(x, y)$

Here we could define the scene S = ΔI .

Task 1 : detecting an intensity increment



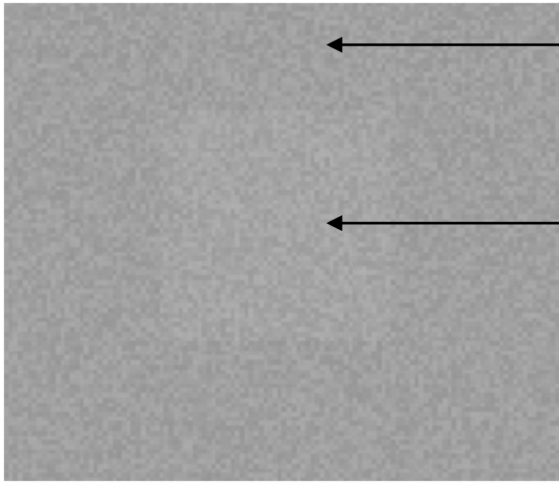
$I_0 + noise(x, y)$

$I_0 + \ \Delta I + noise(x, y)$

$$I_{surround}(x, y) = \ I_0 + noise(x, y)$$

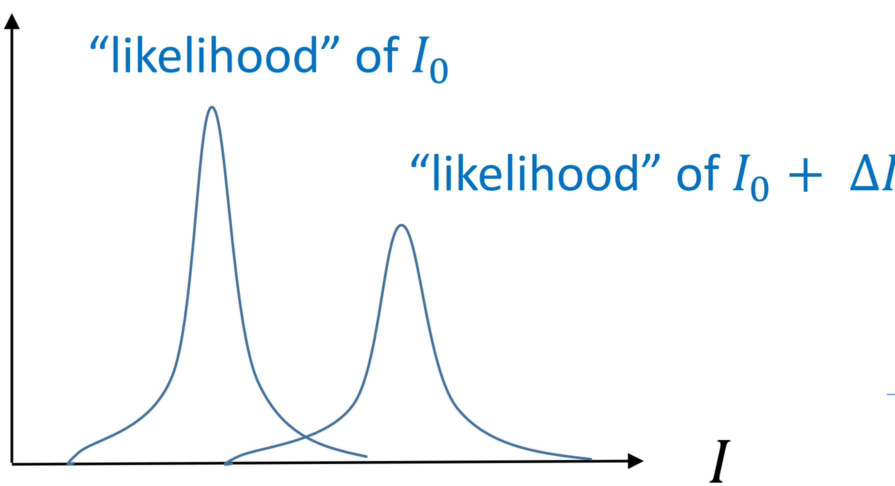
$$I_{center}(x, y) = \ I_0 + \Delta I + noise(x, y)$$

$noise(x, y)$ is Gaussian with mean 0 and variance ${\sigma_n}^2$.



$$I_0 + noise(x, y)$$

$$I_0 + \Delta I + noise(x, y)$$



Earlier, we thought of these as observer’s relatively certainty about values.

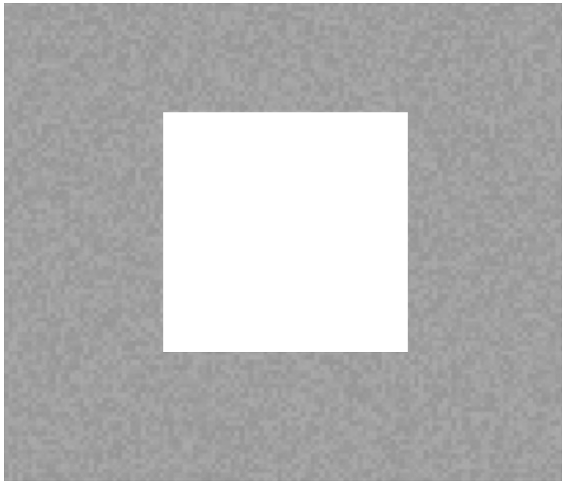
Now, let’s define these as a *likelihood function for a model observer.*

How ?

$$I_{surround}(x, y) = I_0 + n(x, y)$$

Here is the likelihood for $I_0 = i_0$, for each surround pixel (x, y):

$$p(\, I_{surround}(x, y) \mid I_0 = i_0 \,) = p\big(\, n(x, y)\big)$$



$$I_{surround}(x, y) = I_0 + n(x, y)$$

Here is the likelihood for $I_0 = i_0$, for *each* surround pixel (x, y):

$$p(\, I_{surround}(x, y) \mid I_0 = i_0 \,) = \underbrace{p\big(\, n(x, y)\big)}$$

Gaussian pixel noise

$$\frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{n(x,y)^2}{2\,\sigma_n{}^2}}$$

$$n(x, y) = I_{surround}(x, y) - i_0$$

Independent Random Variables

Two random variables X_1 and X_1 are independent if, for all values x_1 and x_2,

$$p(\, X_1 = x_1, X_2 = x_2) \;=\; p(\, X_1 = x_1)\;\, p(X_2 = x_2)$$

The same definition holds for many random variables.

The example here is pixel noise.

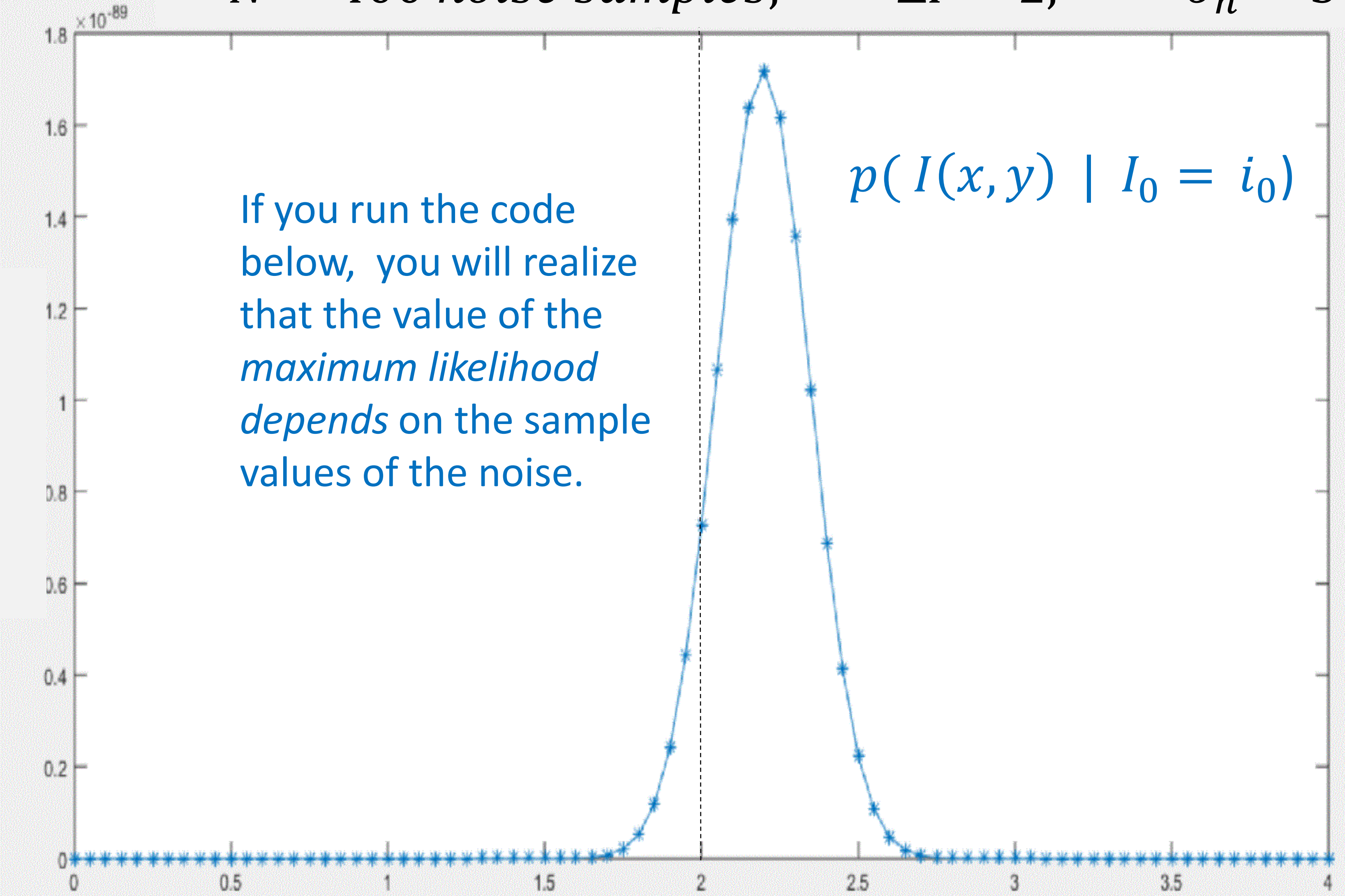
$$I_{surround}(x,y) = I_0 + n(x,y)$$

Here is the likelihood for $I_0 = i_0$ over all pixels in the surround:

$$p(\, I_{surround} \;\mid\; I_0 = i_0 \;) = \prod_{(x,y)} \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(I_{surround}(x,y) - i_0\,)^2}{2\,\sigma_n{}^2}}$$

For all pixels (x, y) in the surround.

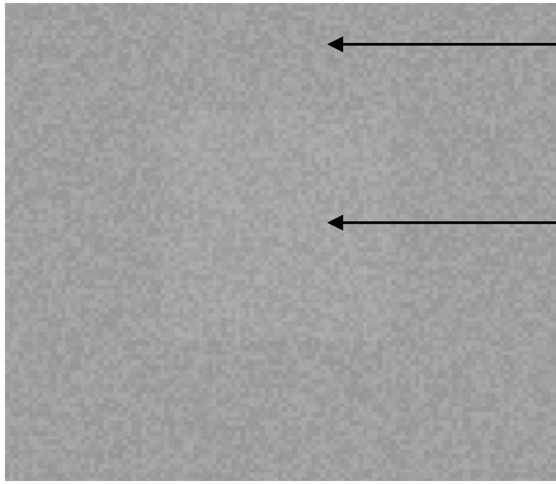
$N = 400 \; noise \; samples, \qquad \Delta I = 2, \qquad \sigma_n = 3$



If you run the code below, you will realize that the value of the *maximum likelihood depends* on the sample values of the noise.

i_0

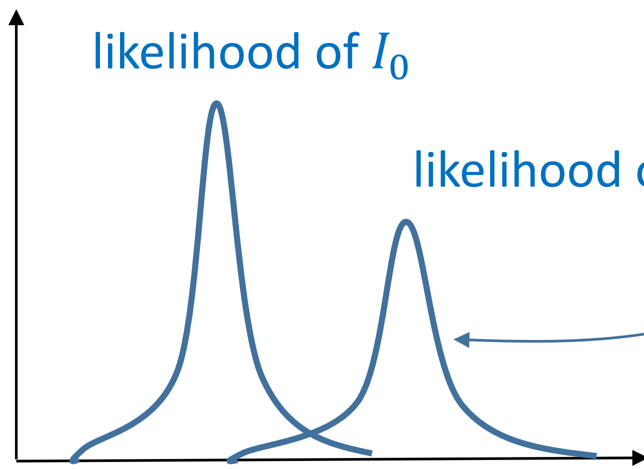
See exercises. http://www.cim.mcgill.ca/~langer/546/MATLAB/likelihood.m

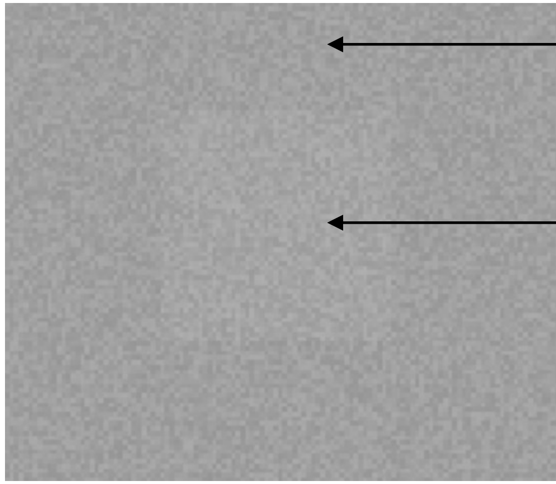


$I_0 + noise(x, y)$

$I_0 + \Delta I + noise(x, y)$

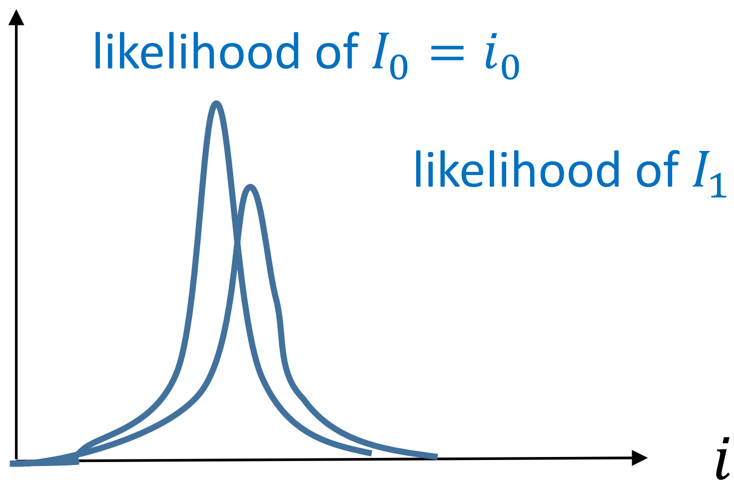
Similarly we can define a *model observer* likelihood function in the center.





$$I_0 + noise(x, y)$$

$$I_0 + \Delta I + noise(x, y)$$



If ΔI is small then the order of the maximum likelihoods will be less reliable indicator of the actual sign of ΔI, reducing performance level.

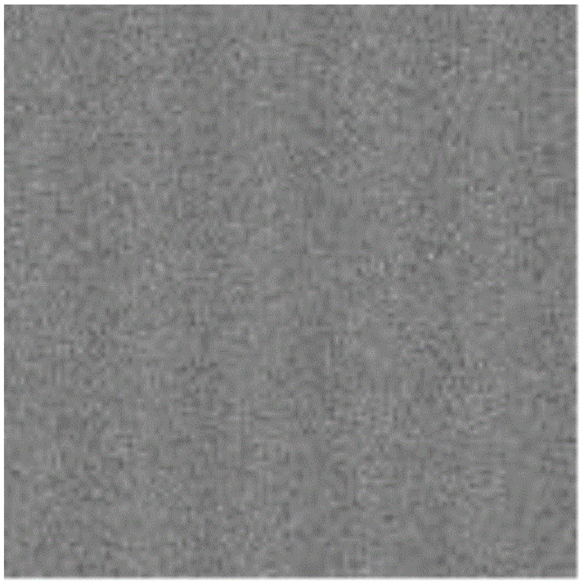
Why are we doing this?   The goal here is:

- to model the human observer's internal uncertainty,   by considering the inherent noise/randomness in some well defined vision task

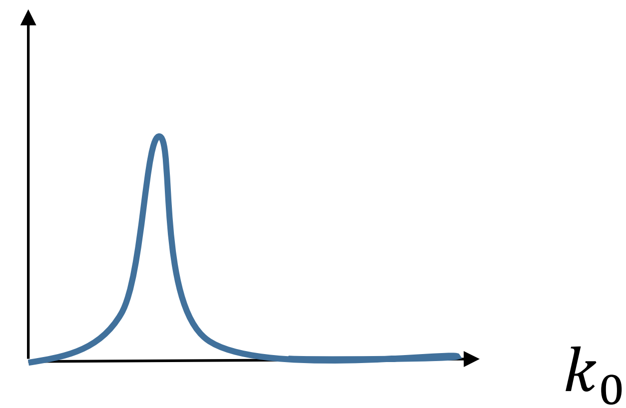
- to use this model to account for a human observer's performance in some task.   (later)

Task 2: estimate frequency of 2D sine in noise

(changed from orientation in order to simplify the math notation)



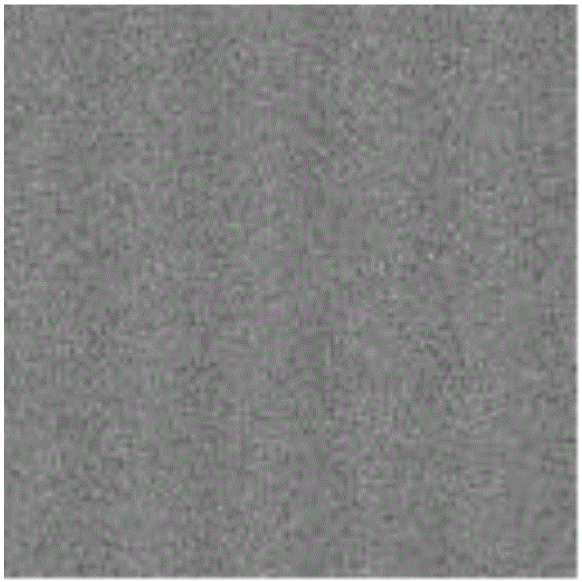
likelihood of spatial frequency



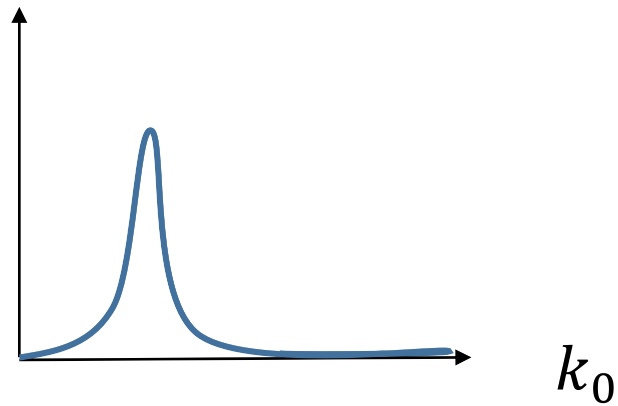
How to define such an orientation likelihood function?

Task 2: estimate frequency of 2D sine in noise

(changed from orientation in order to simplify the math notation)



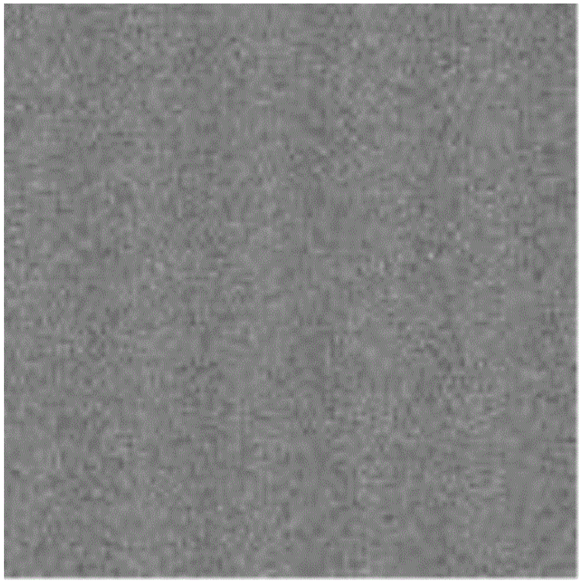
likelihood of spatial frequency (will depend on $\Delta I, I_0$ too)



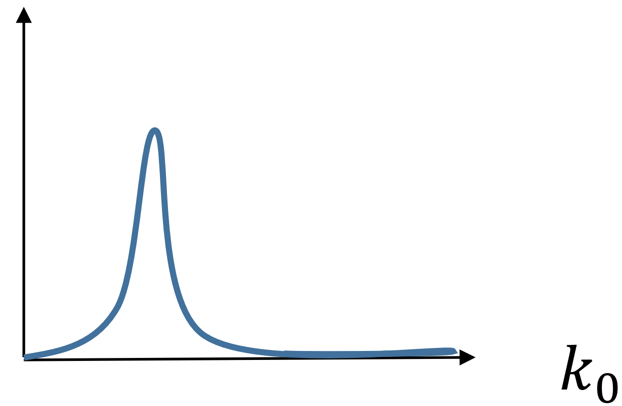
$$I(x,y) = I_0 + \Delta I \ sin\left(\frac{2\pi}{N} k_0 x\right) + noise(x,y)$$

Task 2: estimate frequency of 2D sine in noise

(changed from orientation in order to simplify the math notation)



likelihood of spatial frequency (will depend on $\Delta I, I_0$ too)



$$I(x,y) = I_0 + \Delta I \, sin\left(\frac{2\pi}{N} k_0 x\right) + noise(x,y)$$

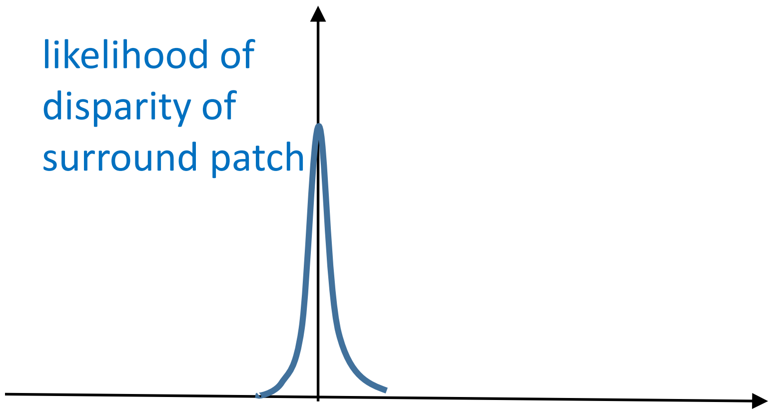
$$p(I \mid k = k_0) = \prod_{(x,y)} \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(noise(x,y))^2}{2\,\sigma_n{}^2}}$$

Task 3: estimate binocular disparity
(do center and surround separately)

left image

right image

(center patch and surround patch each have some disparity)



$$p(I_{left,} I_{right} \mid disparity = d\ ) = \prod_{(x,y)} \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(noise(x,y)\ )^2}{2\ \sigma_n{}^2}}$$

in surround

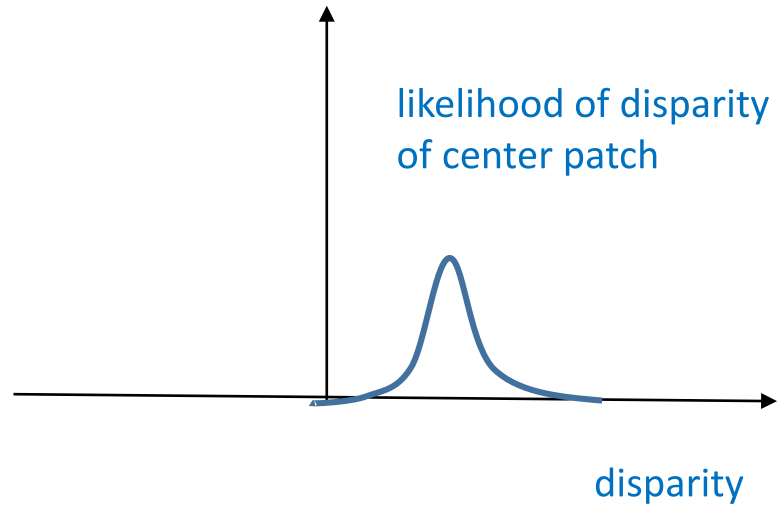
Task 3: estimate binocular disparity

(do center and surround separately)

left image

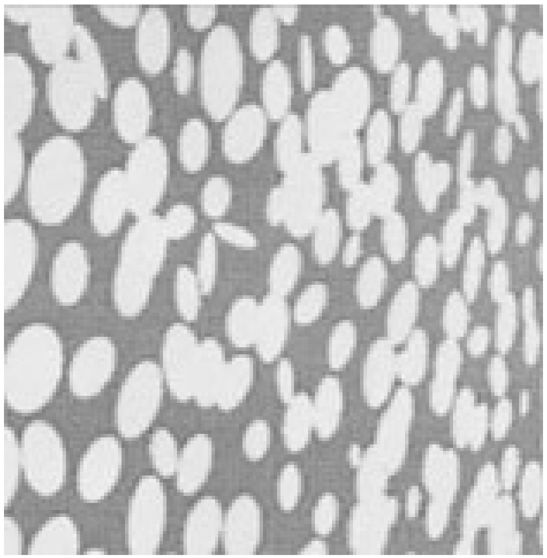
right image

(center patch and surround patch each have some disparity)



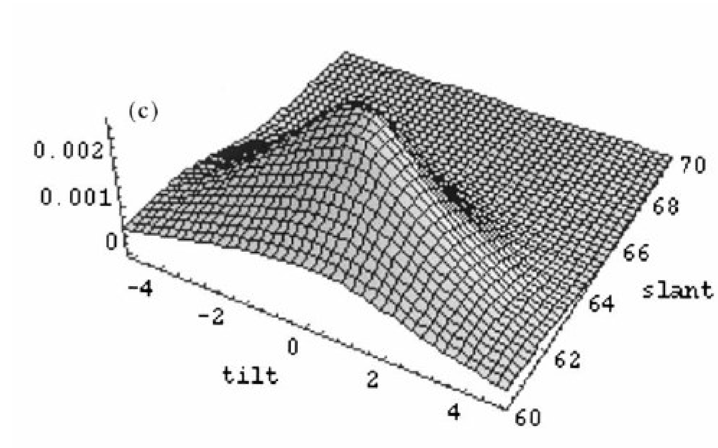
$$p(I_{left,} I_{right} \mid disparity = d\ ) = \prod_{\substack{(x,y) \\ \text{in center}}} \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{(noise(x,y)\ )^2}{2\ \sigma_n{}^2}}$$

Task 4: estimate surface orientation (slant and tilt) from texture



Set of ellipses
$\{\,(x, y, \ldots)\,\}$

position and
size and shape



Likelihoods
(slant and tilt)

Each image ellipse gives a likelihood function. Multiplying these likelihoods together for the different ellipses gives the overall likelihood function.

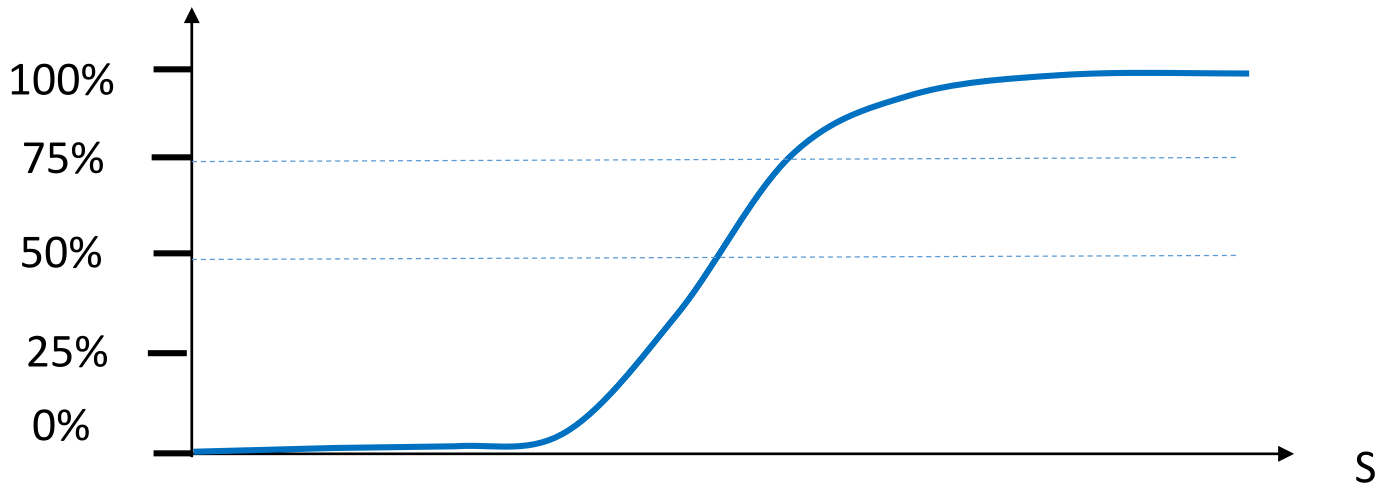
[Knill, 1998]

Why are we doing this? The goal here is:

- to model the human observer's internal uncertainty, by considering the inherent noise/randomness in some well defined vision task

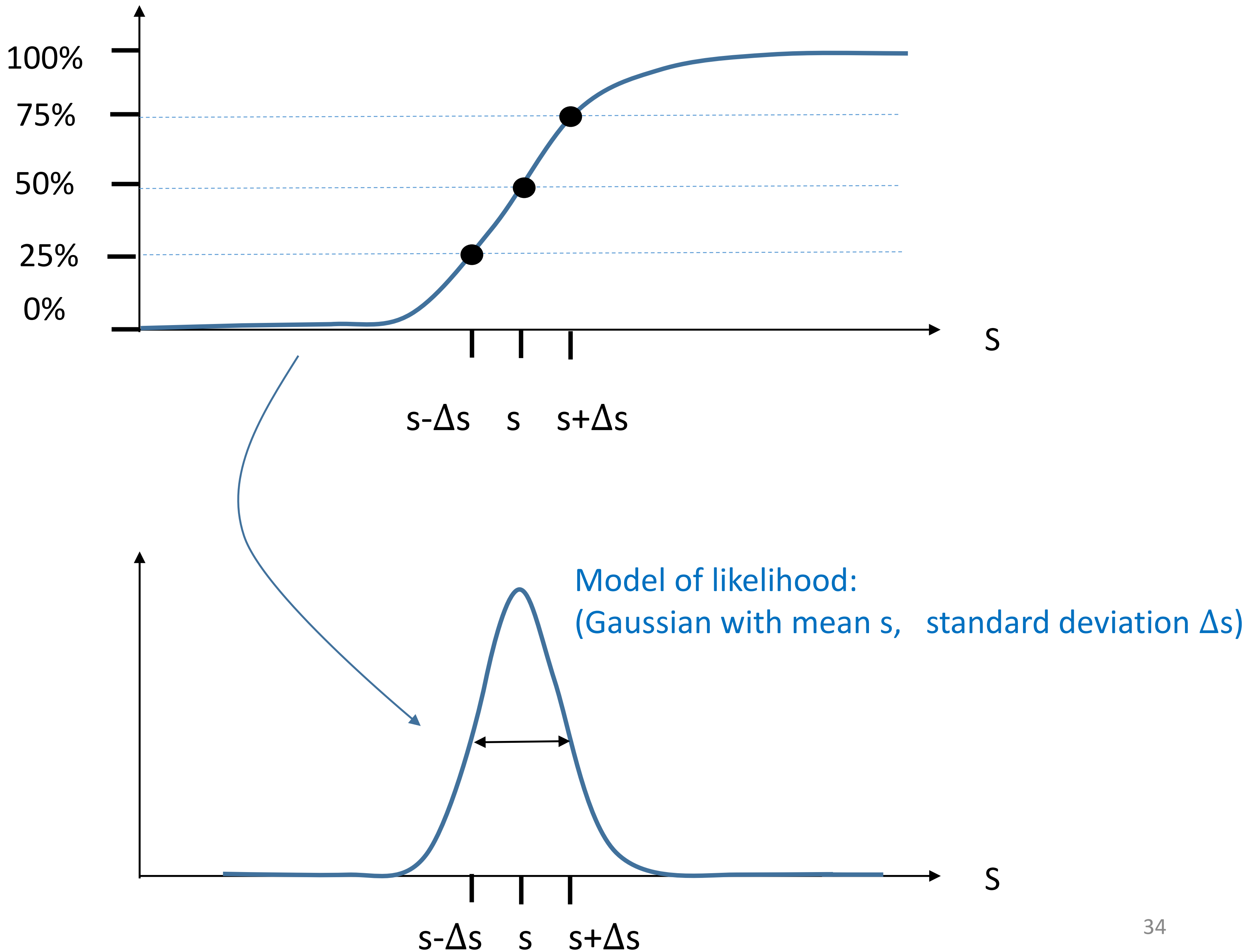
- to use this model to account for a human observer's performance in some task (coming next…)

Psychometric function (fit with cumulative Gaussian)



Given a human observer's psychometric function (measured in some experiment – see last lecture), use it to model the observer's "likelihood" function.

Psychometric function (fit with cumulative Gaussian)



Why are we doing this? The goal here is:

- to model the human observer's internal uncertainty, by considering the inherent noise/randomness in some well defined vision task

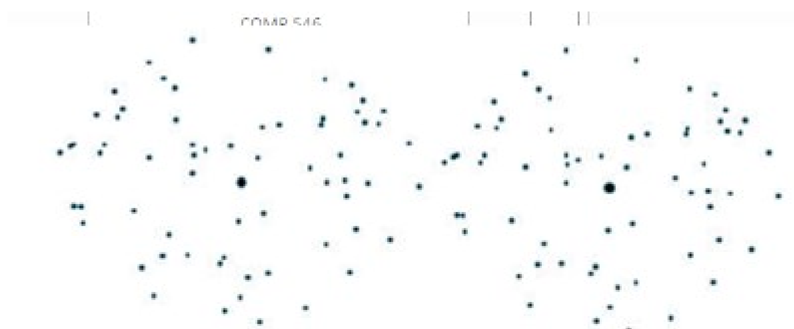
- to use this model to account for a human observer's performance in some task

  - Do the human observer's thresholds follow a similar pattern as the model observer's thresholds? *If not, then try to change the model observer so that you get the same pattern.* e.g. Use Gabor models + neural nets.

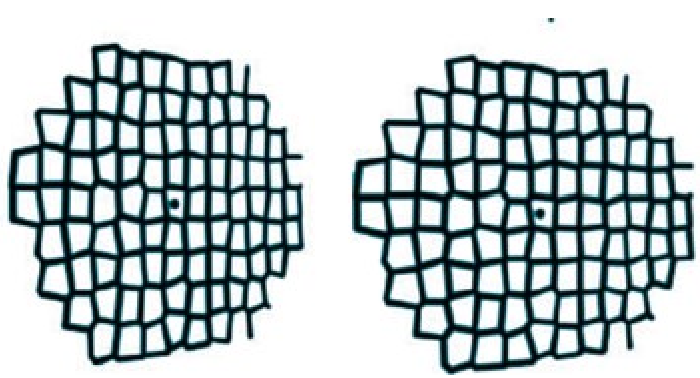
  - How do human observers combine cues? (next lecture)

Example (cue combination):

texture only
(monocular)



stereo only



texture and stereo

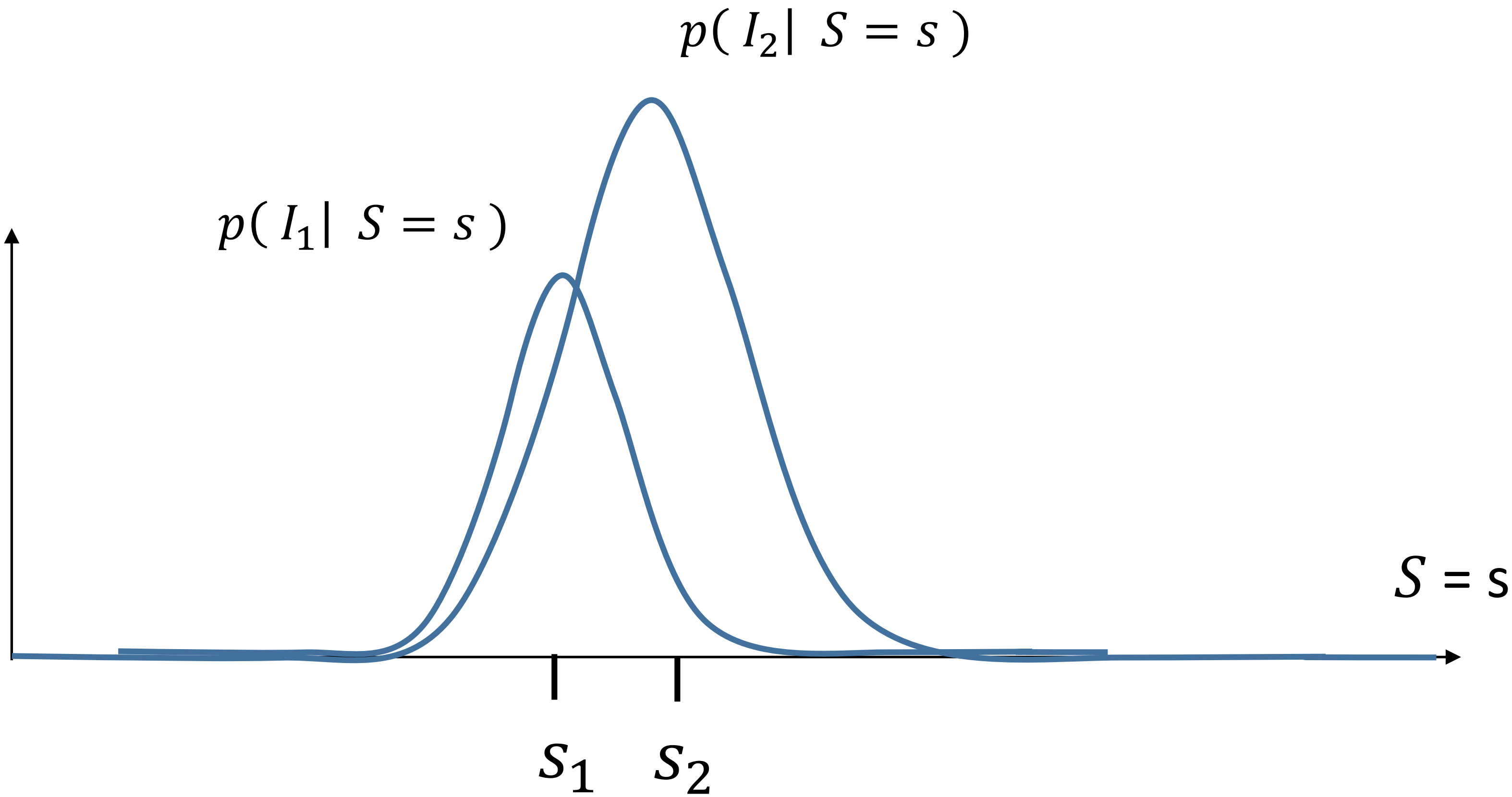
[Hillis 2004]

Assume likelihood function is "conditionally independent":

$$p(\, I_1, I_2 \mid S\,) = p(\, I_1 \mid S\,)\; p(\, I_2 \mid S\,)$$

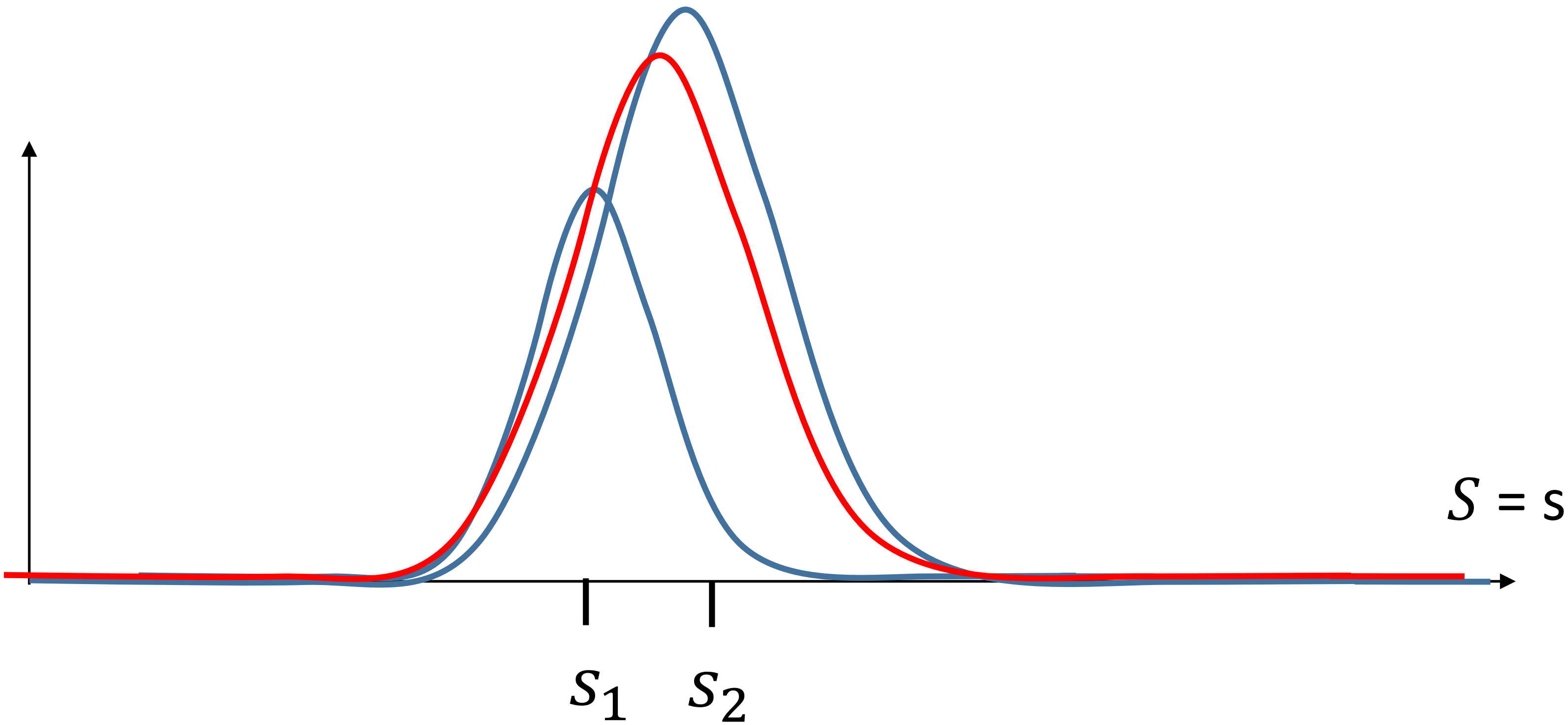
e.g. I_1 is texture cue.

I_2 is binocular disparity cue.



Their likelihood maxima might occur at different values of s.
This can happen if the likelihood *model* is incorrect (biased).

$$p(\, I_1, I_2 \mid S = s\,) \;=\; p(\, I_1 \mid S = s\,)\; p(\, I_2 \mid S = s\,)$$



Method of Cue Combination: show how/if human performance with both cues can be predicted from performance from each cue on its own. (Next lecture)