

Homework 5 - Statistical Process Control (SPC), \bar{x} , R, and p-Charts

MGCR 472 - Operations Management

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Question 1

Suppose that a window manufacturer is monitoring the process used to make its windows. Every 15 minutes, the manufacturer measures the width of 4 windows. Over the course of two and a half hours, the following 10 samples (each of size 4) are taken. All measurements are in meters.

Sample	x_1	x_2	x_3	x_4	\bar{x}	R
1	1.11	1.03	1.33	1.30	1.20	0.30
2	1.21	1.28	1.05	1.30	1.21	0.25
3	1.31	1.03	1.19	1.07	1.15	0.28
4	1.21	1.12	1.11	0.97	1.10	0.24
5	1.27	1.20	1.33	1.01	1.20	0.32
6	0.96	1.00	1.09	1.03	1.02	0.13
7	1.03	1.15	1.20	1.15	1.13	0.17
8	1.12	0.99	1.02	1.27	1.10	0.28
9	1.30	1.11	1.27	1.05	1.18	0.25
10	1.04	1.26	1.04	1.09	1.11	0.22

a. Calculate the upper and lower control limits for the \bar{x} -chart and the R -chart associated with this data. See below for the table of the D_2 , D_3 , and D_4 values.

Sample Size	D_2	D_3	D_4
2	1.128	0	3.268
3	1.693	0	2.574
4	2.059	0	2.282
5	2.326	0	2.115
6	2.534	0	2.004
7	2.704	.0761	1.924

b. Plot the \bar{x} -chart and the R -chart. Determine whether or not the process is in control. If it is not in control, state what looks wrong on the control chart and what could possibly be causing this.

Solution

a, Recall the control limits formulae

$$CL_{\bar{x}} = \bar{\bar{x}} \pm \frac{3\bar{R}}{D_2\sqrt{n}}$$

$$CL_R = (D_3\bar{R}, D_4\bar{R})$$

From the tables above

$$\bar{\bar{x}} = 1.14, \bar{R} = 0.244$$

With sample size $n = 4$

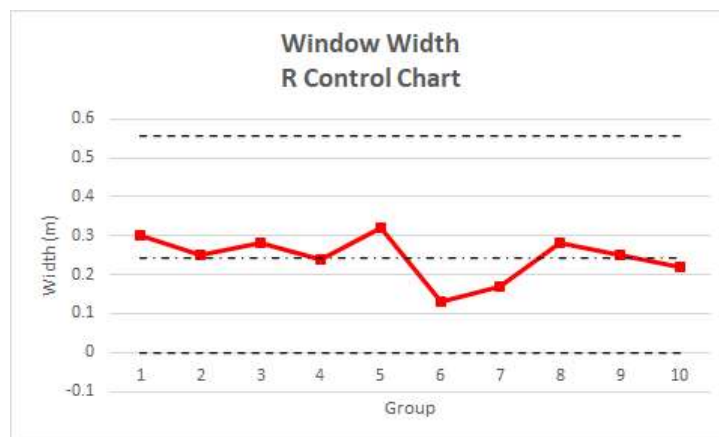
$$D_2, D_3, D_4 = 2.059, 0, 2.282$$

Therefore, the control limits are

$$CL_{\bar{x}} = 1.14 \pm \frac{3(0.244)}{2.059\sqrt{4}} = (0.962, 1.318)$$

$$CL_R = (D_3\bar{R}, D_4\bar{R}) = (0, 0.557)$$

b. The \bar{x} and R-charts are as follows



For both charts, there are

- No points close to reaching outside the control limits
- No more than 3 points in a row (<7) are on one side of the center line, or follow increasing or decreasing trends
- No non-random looking patterns

Therefore, the process seems to be in control.

Question 2

Detroit Central Hospital is trying to improve its image by providing a positive experience to its patients and their relatives. Part of the program involves providing meals that are both tasty and healthy. A questionnaire accompanies each meal served, asking the patient whether s/he is satisfied or unsatisfied with the meal. A 250-patient sample of the survey results over the past 10 days yielded the following data (see next page).

Day	Sample Size	No. of Unsatisfied Patients
1	250	228
2	250	106
3	250	155
4	250	153
5	250	129
6	250	213
7	250	237
8	250	199
9	250	59
10	250	65

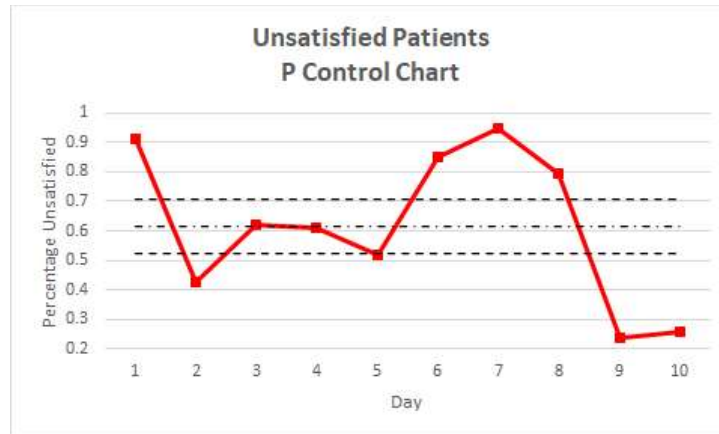
Construct a p -chart that plots the percentage of patients unsatisfied with their meals. Set the control limits to include 99.73% of the random variation in meal satisfaction. Is this process in control or out of control? Justify carefully your answer.

Solution

The p -chart control limits are

$$\begin{aligned}
 CL_p &= \bar{p} \pm z\sigma_p \\
 LCL_p &\geq 0 \\
 \text{standard deviation } \sigma_p &= \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \\
 n &= 250 \\
 \text{include 99.73\% of random variation} &\Rightarrow z = 3
 \end{aligned}$$

The resulting p -chart is



We observe

- 8 out of 10 days, the proportion of unsatisfied patients is outside of control limits
- 6 points in a row out of control limits (day 5-10)
- Proportions exceed control limits on a large margin, and wildly fluctuate
- Irregular, short but strong bursts of up and down-trends

Therefore, the process is out of control.