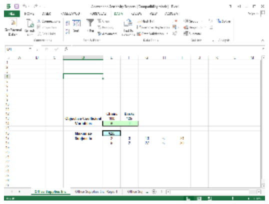
Operations Management

Session 8:
Otto Development Corporation

Recap on Solving LPs Using Excel

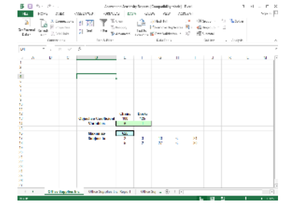


- **Simple process:** formulate LP in spreadsheet, setup the solver, and press solve.

- Find optimal solution as well as **additional useful information** (two new Excel worksheets are created).

- Allowing to perform **sensitivity analyses**: saving time and drawing useful managerial insights.

Sensitivity Analysis



How sensitive is the optimal objective function value and the optimal solution to **parameter changes**?

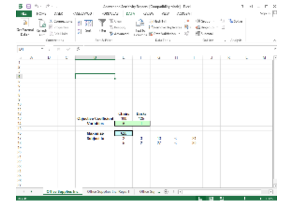
Two types of sensitivity analyses:

– If the objective function coefficients change.

– If the righthand side of a constraint changes.

Always making <u>one change at a time</u>.

Sensitivity Analysis - Summary



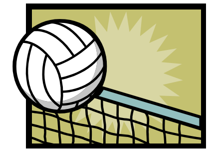
Changing <u>**one objective function coefficients:**</u>

- Check if the change is within the allowable decrease or increase.
- If yes, the <u>optimal solution remains the same</u>.
- Compute the new optimal objective function (using the optimal solution and the new objective function coefficients).

Changing <u>**the righthand side of one constraint:**</u>

- Check if the change is within the allowable decrease or increase.
- If yes, <u>constant shadow price and same set of binding constraints</u>.
- Compute the change in the optimal objective as the shadow price multiplied by the change in the RHS.
- Compute the new optimal solution by solving the system of equations obtained from the binding constraints.

Review of Protein Milk Example



Going over the Protein Milk example one more time (slides from previous lecture).

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Have a maximum capacity of 800 condominiums to build, 1,500 total sq. ft and 450 sq. ft. of BKE space per condo.

Type	Luxury	High-end	Average
Demand	300	200	500
Selling Price	$700 sq. ft.	$500 sq. ft.	$400 sq. ft.
Rating	8.0	6.0	4.0

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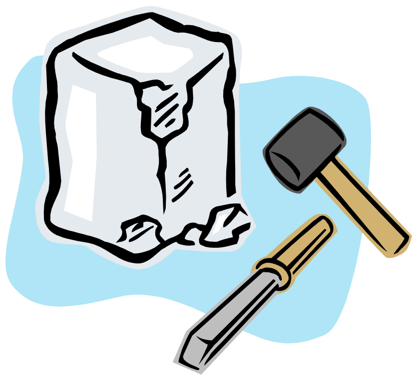
Materials available are:



Type	Marble	Ceramic	Wood
Availability	50,000 sq. ft.	100,000 sq. ft.	unlimited sq. ft.
Rating	9.0	6.0	4.0

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$$\text{Rating of Apt.} = \frac{9 \text{ x BKE sq. ft. marble} + 6 \text{ x BKE sq. ft. ceramic} + 4 \text{ x BKE sq. ft. wood}}{450 \text{ sq. ft.}}$$

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How many of each type of condo should Otto build in order to maximize their revenue?

Formulating a Linear Program

1. Identify decision variables.

2. Write out objective function.

3. Write out constraints.

4. Write the LP.

Decision Variables

Number of each type of condo that Otto will make:

- x_1: number of luxury condos.

- x_2: number of high-end condos.

- x_3: number of average-market condos.

Decision Variables

Amount of each type of material to use in their condos:

- M_1: quantity of marble used in all the luxury condos.
- M_2: quantity of marble used in all the high-end condos.
- M_3: quantity of marble used in all the average-market condos.
- C_1: quantity of ceramic used in all the luxury condos.
- C_2: quantity of ceramic used in all the high-end condos.
- C_3: quantity of ceramic used in all the average-market condos.
- W_1: quantity of wood used in all the luxury condos.
- W_2: quantity of wood used in all the high-end condos.
- W_3: quantity of wood used in all the average-market condos.

Every quantity is in number of square-feet.

Objective Function

Maximize revenue:

- $700 per sq. ft. for luxury condo.
- $500 per sq. ft. for high-end condo.
- $400 per sq. ft. for average-market condo.
- 1,500 sq. ft. per condo.

$$\text{Max } z = 700 \times 1{,}500\, x_1 + 500 \times 1{,}500\, x_2 + 400 \times 1{,}500\, x_3$$

Equivalently:

$$\text{Max } z = 1{,}050{,}000\, x_1 + 750{,}000\, x_2 + 600{,}000\, x_3$$

Constraints

- Number of condos (1).
- Demand (3).
- Ratings (3).
- Surface area of BKE (3).
- Availability of materials (2).

<u>Total:</u> 12 constraints + 12 non-negativity constraints.

Number of Condos Constraint

At most 800 condos:

Demand Constraints

- Max demand for luxury: 300.
- Max demand for high-end: 200.
- Max demand for average-market: 500.

Ratings Constraints

Minimum rating:

- 8 for luxury condo.
- 6 for high-end condo.
- 4 for average-market condo.

Ratings Constraints

Surface Area of BKE Constraints

- Need a total of 450 sq. ft. of BKE space per condo.
- But M_i, C_i, W_i in sq. ft. for all condos.

Materials Constraints

- Maximum 50,000 sq. ft. of marble.
- Maximum 100,000 sq. ft. of ceramic.
- No limit on wood.

Linear Program

$$
\begin{aligned}
\max \quad & 1{,}050{,}000x_1 + 750{,}000x_2 + 600{,}000x_3 \\
s.t. \quad & x_1 + x_2 + x_3 \le 800 \\
& M_1 + C_1 + W_1 - 450x_1 = 0 \\
& M_2 + C_2 + W_2 - 450x_2 = 0 \\
& M_3 + C_3 + W_3 - 450x_3 = 0 \\
& M_1 + M_2 + M_3 \le 50{,}000 \\
& C_1 + C_2 + C_3 \le 100{,}000 \\
& x_1 \le 300 \\
& x_2 \le 200 \\
& x_3 \le 500 \\
& 9M_1 + 6C_1 + 4W_1 - 3600x_1 \ge 0 \\
& 9M_2 + 6C_2 + 4W_2 - 2700x_2 \ge 0 \\
& 9M_3 + 6C_3 + 4W_3 - 1800x_3 \ge 0
\end{aligned}
$$

+ 12 non-negativity constraints

Time to solve the problem using Excel

Optimal Solution

Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C3	Variable x1	166.6666667	0	1050000	1E+30	150000
D3	Variable x2	166.6666667	0	750000	75000	150000
E3	Variable x3	466.6666667	0	600000	150000	150000
F3	Variable M1	50000	0	0	1E+30	500
G3	Variable M2	0	0	0	500	1E+30
H3	Variable M3	0	-500	0	500	1E+30
I3	Variable C1	25000	0	0	1E+30	166.6666667
J3	Variable C2	75000	0	0	166.6666667	333.3333333
K3	Variable C3	-1.51582E-13	0	0	333.3333333	200
L3	Variable W1	0	-333.3333333	0	333.3333333	1E+30
M3	Variable W2	0	-333.3333333	0	333.3333333	1E+30
N3	Variable W3	210000	0	0	333.3333333	333.3333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
O10	marble avail.	50000	1333.333333	50000	40000	15000
O11	ceramic avail.	100000	333.3333333	100000	15000	15000
O12	demand L	166.6666667	0	300	1E+30	133.3333333
O13	demand H	166.6666667	0	200	1E+30	33.33333333
O14	demand A	466.6666667	0	500	1E+30	33.33333333
O15	rating L	0	-333.3333333	0	30000	120000
O16	rating H	1.16415E-10	-333.3333333	0	30000	0
O17	rating A	0	-166.6666667	0	30000	0
O6	#condos	800	600000	800	33.33333333	466.6666667
O7	KBE area L	0	1666.666667	0	0	3750
O8	KBE area H	1.45519E-11	1666.666667	0	0	3750
O9	KBE area A	0	666.6666667	0	0	7500

Sensitivity Analysis

What is the optimal solution and optimal revenue if the revenue per sq. ft. of a luxury condo drops to \$650 (was initially \$700)?

Sensitivity Analysis

What is the optimal solution and optimal revenue if the revenue per sq. ft. of luxury condo drops to $550?

Sensitivity Analysis

What is the optimal solution and optimal revenue if the demand for luxury condos increases by 100?

Sensitivity Analysis

What is the optimal solution and optimal revenue if 1,000 additional sq. ft. of marble are available?

Sensitivity Analysis

What is the optimal solution and optimal revenue if the number of condos to build

– Has to be less or equal than 820 (was initially 800)?

Sensitivity Analysis

What is the optimal solution and optimal revenue if the number of condos to build

- Can be anything we want?

Summary

- "Realistic" real-estate example of solving an LP.
- Sensitivity analysis may be very useful.
- Excel Spreadsheet is available online.
- Next class: Midterm review.