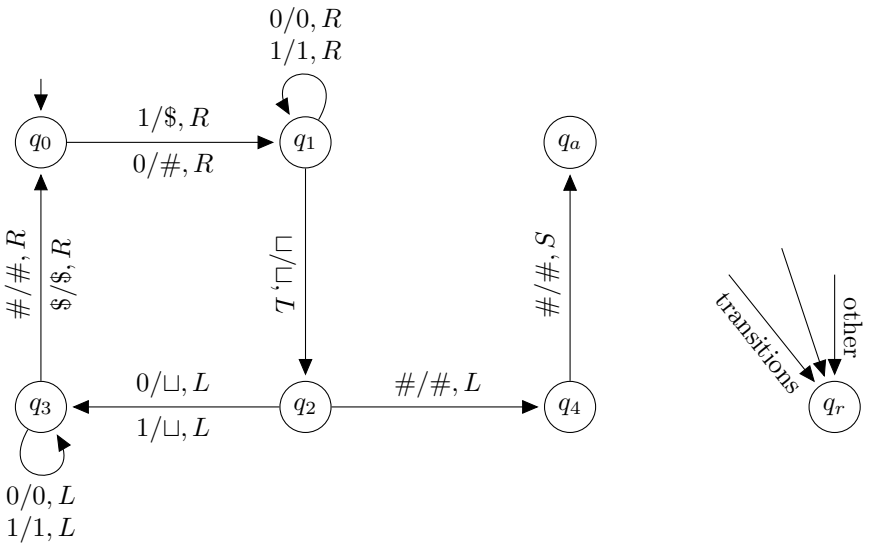
Fundamentals of theory of computation 2 – 2nd test

1. Transitions of a deterministic Turing machine $M = \langle \{q_0, q_1, q_2, q_3, q_4, q_a, q_r\}, \{0,1\}, \{0,1,\sqcup,\#,\$\}, \delta, q_0, q_i, q_n\rangle$ is given by the following transition diagram.



   (a) Give the sequence of configurations for input 100 till a halting configuration.

   (b) Determine $L(M)$, the language recognized by M. Support your answer by an argument.

   (c) Give an asymptotically sharp asymptotic upper bound for the time complexity of M. (8 points)

2. (a) Give a Turing machine computing the function $f(b^n) = \begin{cases} b^{n-1}ab^{n-1} & \text{if } n > 0 \\ c & \text{if } n = 0 \end{cases}$.

   (b) Give an asymptotically sharp asymptotic upper bound for the time complexity of your machine.

   (7 points)

3. (a) Give a Turing machine recognizing $L = \left\{ a^k b^n \;\middle|\; 1 \le n \le 2k-1 \right\}$.

   (b) Give an asymptotically sharp asymptotic upper bound for the time complexity of your machine.

   (8 points)

4. Is there an instance D of Post Correspondence Problem satisfying all of the following properies?

   (i) $|D| \ge 3$,

   (i) the alphabet of the dominos is $\{a, b\}$,

   (ii) D has a solution,

   (iv) there is a domino d of D, such that if we reverse the bottom word of d and leave its top word and other dominos unchanged, then the new set of dominos D' has no solution.

   Support your answer by an argument.

   (Remember: Reverse (or mirror image) of $u = abbab$ is $u^{-1} = babba$.) (6 points)

Choose one from the following 2 exercises. You do not have to solve the other one. Points will be given only for the chosen one.

5A. Prove, that $\{\langle M, M'\rangle \,|\, L(M) \cap L(M') \neq \emptyset\} \notin R$, where M, M' are TM's over input alphabet $\{0,1\}$ and $\langle M, M'\rangle$ is a code of a 2-tuple (M, M').

OR

5B. HALFHC $= \{\langle G\rangle \,|\,$ directed graph G has a directed cycle containing at least half of the vertices$\}$

HC $= \{\langle G\rangle \,|\,$ directed graph G has a directed Hamiltonian cycle$\}$

Prove, that HC $\le_p$ HALFHC. (7 points)

Turing machines in exercies 2 and 3 can have multiple tapes, but they should be deterministic.