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YZ49Y8
Discrete Math
Group 1

1 b) $x \in A \setminus (B \cup C)$

$\Leftrightarrow \begin{cases} x \in A \\ x \notin B \cup C \end{cases}$

$\Leftrightarrow \begin{cases} x \in A \\ x \notin B \\ x \notin C \end{cases}$

$\Leftrightarrow (x \in A \wedge x \notin B) \wedge (x \in A \wedge x \notin C)$

$\Leftrightarrow x \in A \setminus B \wedge x \in A \setminus C$

$\Leftrightarrow x \in (A \setminus B) \cap (A \setminus C)$

2 a) *) $\forall a \in X \Rightarrow (a,a) \in R \rightarrow R$ is reflective

*) we have $(1,2) \in R$ but $(2,1) \notin R$

$\Rightarrow$ R is not symmetric

*) R is not transitive if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$

Then R is not transitive since $(1,2) \in R \wedge (2,4) \in R$ but $(1,4) \notin R$

*) $\forall a,b \in X$, if $(a,b) \in R \wedge (b,a) \in R \Rightarrow a = b$

$\Leftrightarrow$ R is anti-symmetric

b) $R \subseteq \mathbb{Z} \times \mathbb{Z}$, $R = \{(x,y) \in \mathbb{Z} \times \mathbb{Z} \mid y^2 > 4x - 1\}$

1) a) I. $A = \{1,2,5\}$

$B = \{1,3,4\}$

$C = \{1,2,4,6\}$

$A \cup C = \{1,2,3,4,5,6\} \Rightarrow B \setminus (A \cup C) = \{3\}$

$A \cap C = \{1,2\} \Rightarrow (A \cap C) \setminus B = \{2\}$

$\circledast$ LHS $= (B \setminus (A \cup C)) \cup ((A \cap C) \setminus B)) = \{2,3\}$

$B \setminus C = \{3\} \Rightarrow \circledast$ RHS $= (A \cap C) \cup (B \setminus C) = \{1, 2, 3\}$

$\{2,3\} \neq \{1,2,3\}$

II. $A = \{1,2,5\}$

$B = \{3,4\}$

$C = \{1,2,4,6\}$

$A \cup C = \{1,2,4,5,6\} \Rightarrow B \setminus (A \cup C) = \{3\}$

$A \cap C = \{1,2\} \Rightarrow (A \cap C) \setminus B = \{1,2\}$

$\circledast$ LHS $= (B \setminus (A \cup C)) \cup ((A \cap C) \setminus B) = \{1,2,3\}$

$B \setminus C = \{3\}$

$\Rightarrow$ RHS $= (A \cap C) \cup (B \setminus C) = \{1,2,3)$

$\Rightarrow$ <u>LHS = RHS</u>

3 a) $R = \{(1,a), (1,b), (2,a), (2,b), (4,c), (4,a)\}$

$R^{-1}(\{a,b\}) = \{1,2,4\}$

$S = \{(a,2), (a,3), (b,2), (c,4), (d,3), (d,1)\}$

$\Rightarrow \quad S \circ R = \{(x,y) \mid \exists z: (x,z) \in R \wedge (z,y) \in S\}$

$= \{(1,2), (1,3), (2,2), (2,3), (4,2), (4,3), (4,4)\}$

5. $z = \dfrac{(\sqrt{2} - \sqrt{2}i)^9}{(1+\sqrt{3}i)^{17}}$

$\sqrt{2} - \sqrt{2}i = 2\left(\cos\frac{-\pi}{4} + i\sin\frac{-\pi}{4}\right) = 2e^{i\frac{-\pi}{4}}$

$1 + \sqrt{3}i = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2e^{i\frac{\pi}{3}}$

$\Rightarrow z = \dfrac{2^9 . e^{i\frac{-9\pi}{4}}}{2^{17} . e^{i.\frac{17\pi}{3}}} = \dfrac{1}{2^8} . e^{i\pi\left(\frac{-9}{4} - \frac{17}{3}\right)} = \dfrac{e^{i\pi . \frac{-95}{12}}}{2^8} = \dfrac{e^{i\pi . \frac{1}{12}}}{2^8}$

$z = \frac{1}{2^8} . e^{i\frac{\pi}{12}} = \frac{1}{2^8}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

$w^3 = z^{1/3} = \dfrac{1}{\sqrt[3]{2^8}}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)^{1/3} = \dfrac{1}{4\sqrt[3]{4}} . \cos\frac{\pi}{36} + \dfrac{i}{4\sqrt[3]{4}} . -\sin\frac{\pi}{36}$

3 b) R is called equivalence relation if it is $\begin{cases} \text{reflexive} \\ \text{symmetric} \\ \text{transitive} \end{cases}$

i) $R_1 \subseteq \mathbb{R}\times\mathbb{R}$, $R_1 = \{(x,y) \in \mathbb{R}\times\mathbb{R} \mid |x-y| \text{ is odd}\}$

$x \in \mathbb{R}$ we have xR_1x is not odd

$\rightarrow R_1$ is not reflexive $\rightarrow$ Not equivalence relation

ii) $R_2 \subset \mathbb{Z}\times\mathbb{Z}$, $R_2 = \{(x,y) \in \mathbb{Z}\times\mathbb{Z} \mid 2x - 2y \text{ is even}\}$

*) $\forall x \in \mathbb{Z}$, $(x,x) \in R_2$ since $2x - 2x = 0$ (even)

$\Rightarrow R_2$ is reflexive.

*) $\forall (x,y) \in R_2$ (that means $2x - 2y = 2(x-y)$ is even)

we have $(y,x) \in R_2$ since $yR_2x = 2y - 2x$

$= 2(y-x)$ is also even

$\Rightarrow R_2$ is symmetric

$\Rightarrow \forall (x,y) \in R_2 \wedge (y,z) \in R_2$: $2(\cancel{y}\, x - y)$ is even

and $2(y-z)$ is even

Consider $(x,z) \in R_2$ since $2x - 2z = 2(x-z)$ is even $\Rightarrow R_2$ is transitive 3

4)

$f \subseteq X \times Y$ is called a ~~fu~~ function if

$\forall x, y, y' : (\cancel{x,y}\ x, y) \in f \wedge (x, y') \in f \Rightarrow y = y'$

a) f_1 is a function

•) f_1 is injective since $\forall x, y \in X$ if $f_1(x) = f_1(y)$
$\Leftrightarrow x = y$

•) f_1 is not surjective since $rng(f_1) = \{a, b, c, d\} \neq X$

b) $f_2 \subset \mathbb{R} \times \mathbb{R}$, $f_2 = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid 2x^3 - 3 = y^2 + 2\}$

Consider $2x^3 - 3 = y^2 + 2$

$\Leftrightarrow y^2 = 2x^3 - 5$

$\Leftrightarrow \left[\begin{array}{l} y = \sqrt{2x^3 - 5} \\ y = -\sqrt{2x^3 - 5} \end{array}\right.$

Then for every $x > \sqrt[3]{5/2}$, there are 2 values of y

$\Rightarrow f_2$ is not a function.

6 a) $z = a + bi$, $Re z = Im z \Rightarrow a = b$.

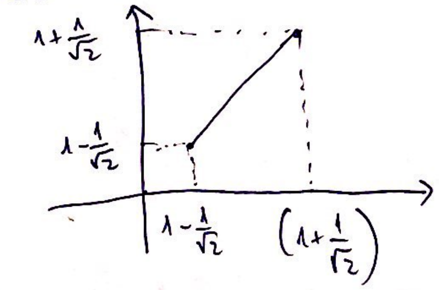
$z - 1 - i = (a-1) + (b-1)i$

$\rightarrow |z - 1 - i|^2 = (a-1)^2 + (b-1)^2 < 1$

$\Rightarrow 2(a-1)^2 < 1 \Leftrightarrow (a-1)^2 < \frac{1}{2} \rightarrow \frac{-1}{\sqrt{2}} < a - 1 < \frac{1}{\sqrt{2}}$

$\Rightarrow 1 - \frac{1}{\sqrt{2}} < a < 1 + \frac{1}{\sqrt{2}}$

$z = a + ai$



5.
$$z = \frac{(\sqrt{2} - \sqrt{2}i)^9}{(1+\sqrt{3}i)^{17}}$$

$$\sqrt{2} - \sqrt{2}i = 2\left(\cos\frac{-\pi}{4} + i\sin\frac{-\pi}{4}\right) = 2e^{i\frac{-\pi}{4}}$$

$$1+\sqrt{3}i = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2e^{i\frac{\pi}{3}}$$

$$\Rightarrow z = \frac{2^9 . e^{i\frac{-9\pi}{4}}}{2^{17} . e^{i.\frac{17\pi}{3}}} = \frac{1}{2^8} . e^{i\pi\left(\frac{-9}{4} - \frac{17}{3}\right)} = \frac{e^{i\pi\frac{-95}{12}}}{2^8} = \frac{e^{i\pi . \frac{1}{12}}}{2^8}$$

$$z = \frac{1}{2^8} . e^{\frac{i\pi}{12}} = \frac{1}{2^8}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

$$w^3 = z^{1/3} = \frac{1}{\sqrt[3]{2^8}}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)^{1/3} = \frac{1}{4\sqrt[3]{4}} . \cos\frac{\pi}{36} + \frac{i}{4\sqrt[3]{4}} . \sin\frac{\pi}{36}$$

$$z = \frac{1}{2^8}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

$$w^3 = z \Rightarrow w = z^{1/3} \Rightarrow w = \sqrt[3]{\frac{1}{2^8}} . \left[\cos\left(\frac{\pi}{36} + \frac{2k\pi}{3}\right) + i\sin\left(\frac{\pi}{36} + \frac{2k\pi}{3}\right)\right]$$

$(k = 0, 1, 2)$

$$\Rightarrow \quad w_1 = \frac{1}{\sqrt[3]{2^8}} . \left(\cos\frac{\pi}{36} + i\sin\frac{\pi}{36}\right)$$

$$w_2 = \frac{1}{\sqrt[3]{2^8}}\left(\cos\frac{25\pi}{36} + i\sin\frac{25\pi}{36}\right)$$

$$w_3 = \frac{1}{\sqrt[3]{2^8}} . \left(\cos\frac{49\pi}{36} + i\sin\frac{49\pi}{36}\right) = \frac{1}{\sqrt[3]{2^8}}\left[\cos\frac{-23\pi}{36} + i\sin\frac{-23\pi}{36}\right]$$

6b) $z = a+bi$

$z+2-i = (a+2)+(b-1)i \Rightarrow |z+2-i|^2 = (a+2)^2+(b-1)^2 \geq 16$

$z-2-i = (a-2)+(b-1)i \Rightarrow |z-2-i|^2 = (a-2)^2+(b-1)^2 \geq 16$

$\Rightarrow \begin{cases} (a+2)^2+(b-1)^2 \geq 16 \\ (a-2)^2+(b-1)^2 \geq 16 \end{cases} \Rightarrow (a+2)^2-(a-2)^2 \geq 0$

$\Leftrightarrow 4 \cdot 2a \geq 0 \Rightarrow a \geq 0.$

$\Rightarrow (a+2)^2+(b-1)^2 \geq 16 \ (a \geq 0)$

$\Leftrightarrow 4+(b-1)^2 \geq 16$

$\Leftrightarrow (b-1)^2 \geq 12$

$\Rightarrow \left[\begin{array}{l} b \geq \sqrt{12}+1 \\ b \leq -\sqrt{12}+1 \end{array}\right.$