Test 1.

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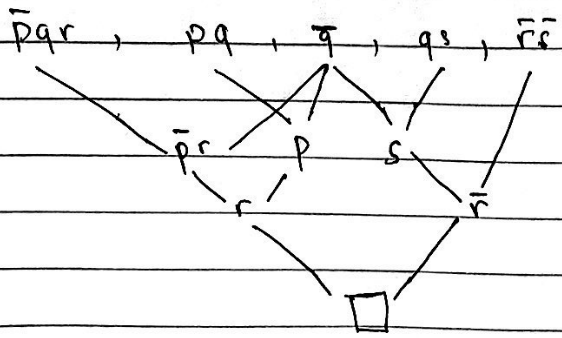
1. Truth table of $A = q \vee \neg p \rightarrow \neg q \wedge r$:

p	q	r	$q \vee \neg p \rightarrow \neg q \wedge r$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

2.
$$(\neg(p \wedge \neg r) \rightarrow \neg q) \rightarrow p$$
$$\equiv (\neg\neg(p \wedge \neg r) \vee \neg q) \rightarrow p.$$
$$\equiv \neg((p \wedge \neg r) \vee \neg q) \vee p.$$
$$\equiv (\neg(p \wedge \neg r) \wedge q) \vee p.$$
$$\equiv ((\neg p \vee r) \wedge q) \vee p.$$
$$\equiv (\neg p \vee r \vee p) \wedge (q \vee p)$$
$$\equiv r \wedge (q \vee p) = \text{CNF}.$$
$\Leftrightarrow \{\{r\}, \{q, p\}\}$ clausal form of CNF.

3. we have: $\{\neg p \vee q \vee r, \ p \vee q, \ \neg q, \ q \vee s, \ \neg r \vee \neg s\}$



→ unsatisfiable → PROVE!

4. $p = 2$;

$q = 1$; $a = \text{const}$

$f = 2$; $\sigma(x) = 1$; $\sigma(y) = 2$

$(\{0,1,2\}, \{p, q\}, \{f\}, \{1\})$

a) $D_{I,\sigma}(f(f(x, f(a,x)), y)) = 2$.

$f(\underbrace{1}_{x}, \underbrace{f(1,1)}_{2}) = 0$, $\; y = 2$

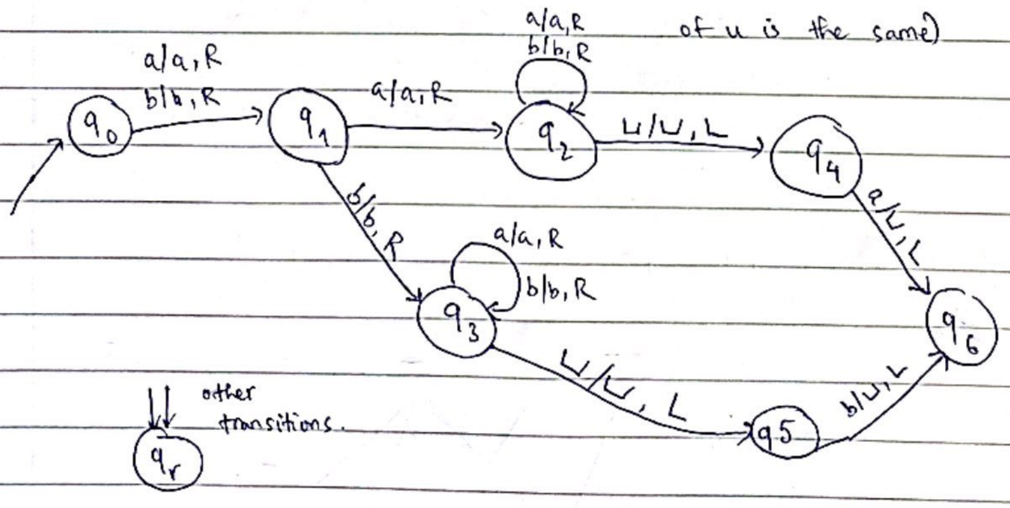
b) $v_{I,\sigma}(p(y,x) \vee q(x) \to \neg q(f(y,x))) = T$.

$p(2,1) = F$, $\; q(1) = T$, $\; f(2,1) = 0$, $\; q(0) = F$, $\; \neg q(0) = T$

c) $v_{I,\sigma}(\exists x p(x,y) \to \neg q(f(x,x))) = F$.

$f(1,1) = 2$, $\; q(2) = T$, $\; \neg q(2) = F$

6. $L = \{ u \in \{a,b\}^* \mid (|u| \geq 2) \wedge (\text{the } 2^{nd} \text{ and the last but 1 letter of } u \text{ is the same}) \}$



5. $f(n) = 2n\log_8 n$, $g(n) = (n + 3\log_2 n).\log_2 n$, $h(n) = n + \log_2(n^3)$

$= n + 3\log_2 n$

$\frac{f(n)}{g(n)} = \frac{2n\log_8 n}{(n+3\log_2 n).\log_2 n} = \frac{2/3\, n \cancel{\log_2 n}}{(n+3\log_2 n).\cancel{\log_2 n}} = \frac{2/3}{1 + 3.\underbrace{\frac{\log_2 n}{n}}_{0}} \to \frac{2}{3} = C$

$C > 0 \Rightarrow f(n) = \Theta(g(n))$

$\frac{g(n)}{f(n)} = \frac{(n+3\log_2 n).\log_2 n}{2n\log_8 n} = \frac{(n+3\log_2 n).\cancel{\log_2 n}}{2/3\, n \cancel{\log_2 n}} = \frac{1}{2/3} \to \frac{3}{2} = C$

$C > 0 \Rightarrow g(n) = \Theta(f(n))$

$\frac{g(n)}{h(n)} = \frac{(n + 3\log_2 n)\log_2 n}{n + 3\log_2 n} = \log_2 n \to +\infty$

$\Rightarrow g(n) \neq O(h(n))$ and $g(n) = \Omega(h(n))$

$\frac{h(n)}{g(n)} = \frac{n + 3\log_2 n}{(n + 3\log_2 n)\log_2 n} = \frac{1}{\log_2 n} \to 0$

$\Rightarrow h(n) = O(g(n))$ and $h(n) \neq \Omega(g(n))$