2nd part :

1. Statement about the inverse of the composition.

$$(R \circ S)^{-1} = S^{-1} \circ R^{-1}$$

$$(z,x) \in (R \circ S)^{-1} \Leftrightarrow (x,z) \in R \circ S \Leftrightarrow \exists y : (x,y) \in S \wedge (y,z) \in R$$

$$\Leftrightarrow \exists y : (y,x) \in S^{-1} \wedge (\cancel{x,y}\ z,y) \in R^{-1} = (z,x) \in S^{-1} \circ R^{-1}$$

LE THANH DUC
YZ49Y8

2. Theorem about the number of permutations with repetition

+ A permutation with repetition is a sequence of m different kinds of elements containing k_1 number of elements of first kind, k_2 number of elements of the second kind, ..., k_m number of elements of the m^{th} kind, and the number of these are:

$$\frac{n!}{k_1! \cdot k_2! \ldots k_m!} \quad \text{where } n = k_1 + k_2 + \ldots + k_m$$

+ Proof: if we distinguish between all elements, then there are $n! = (k_1 + k_2 + \ldots + k_m)!$ possible sequences of n elements.

However, we donot want to distinguish between elements of the same kind, but for each i we are only interested in the set of operations occupied ~~in~~ by the elements of i^{th} kind. If we fix the k_i positions for the elements of the i^{th} kind, we can permute these elements in three positions in $k_i!$ ways. Hence, in $n!$, each sequence has been counted $k_1! \cdot k_2! \cdot k_3! \ldots k_m!$ times. Therefore, the number of permutations with repetition is:

$$\frac{n!}{k_1! \cdot k_2! \ldots k_m!}$$

Scanned with CamScanner

3. Euler - formular :

⊕ Theorem : Let $G = (\varphi, E, V)$ be a connected ~~graph~~ planar graph. Then for any planar embedding of G:

$$|E| + 2 = |V| + f.$$

where f. denotes the number of faces in the planar embedding.

⊕ Proof :

Suppose there is a cycle in G. By deleting an edge of cycle, two faces are merge, so both f and $|E|$ are reduced by 1. In the end, we obtain a tree for which the equation holds.

part 1:

1. ~~Wa~~ 3 properties of operation of set union.

For any sets A and B:

1. $A \cup \emptyset = A$.
2. $A \cup (B \cup C) = (A \cup B) \cup C$: associativity.
3. $A \cup B = B \cup A$: commutativity.
4. $A \cup A = A$ (idempotence)
5. $A \subseteq B \Leftrightarrow A \cup B = B$.

$R \subseteq A \times B$

2. Let A and B be sets. If $R \subseteq A \times B$ then we call R a relation ~~from~~ from A to B. If $A = B$, then we say that R is a relation on X and in this case we say that R is a homogeneous binary relation.

3. $R \subseteq A \times A$

Scanned with CamScanner

4. A function $f: X \to Y$ is called injective if :

$$\forall x_1, x_2 \in X : f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2 ;$$

5. A binary relation on a set X is called partial order if it is reflexive, transitive and anti-symmetric.

6. Real part and imaginary part of complex number $3i$.

we have formular : $z = a + bi$ $(a, b \in \mathbb{R})$ is a complex number.

Then: the real part of z is $Re(z) = a \in \mathbb{R}$

the imaginary of z is $Im(z) = b \in \mathbb{R}$.

$\Rightarrow$ Apply it to $3i$ $\Leftrightarrow$ $Re(z) = 0$.

$Im(z) = 3$.

7. De Moivre's formular

- Let $z, w \in \mathbb{C}$ be nonzero complex numbers : $z = |z|(\cos\varphi + i\sin\varphi)$, $w = |w|(\cos\omega + i\sin\omega)$, and let $n \in \mathbb{N}^+$.

$\Rightarrow$ 1. $zw = |z||w|(\cos(\varphi + \omega) + i\sin(\varphi + \omega))$;

2. $\dfrac{z}{w} = \dfrac{|z|}{|w|} \cdot (\cos(\varphi - \omega) + i\sin(\varphi - \omega))$

3. $z^n = |z|^n(\cos n\varphi + i\sin n\varphi)$

8. Let $z = |z|(\cos\varphi + i\sin\varphi)$, $n \in \mathbb{N}^+$.

The n^{th} roots of z are :

$$w_k = \sqrt[n]{|z|}\left(\cos\left(\frac{\varphi}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\varphi}{n} + \frac{2k\pi}{n}\right)\right)$$

$(k = 0, 1, \ldots n-1)$

9. Binomial theorem

For any $x, y \in \mathbb{R}$ and $n \in \mathbb{N}$ we have

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

Scanned with CamScanner

11. A graph $G' = (\varphi', E', V')$ is called a subgraph of a graph $G = (\varphi, E, V)$ if $E' \subseteq E$, $V' \subseteq V$ and $\varphi' \subseteq \varphi$. We also say in this case G is a ~~sup~~ supergraph of G'.

12. Equivalenet characterisations of trees using the number of edges.

- For simple graph G on n vertices ($n \in \mathbb{N}^+$) the following conditions are equivalent:
  + G is a tree.
  + G contains no ~~g~~ cycles and it has $n-1$ edges.
  + G is connected and it has $n-1$ edges.

Scanned with CamScanner