Fundamentals of theory of computation I

Test 1

Send your solutions as a single NEPTUNCODE.zip file to ktichler@inf.elte.hu or tichlerk@gmail.com

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Exercise 1: Let $L_1 = \{b^n a^{2n} \mid n \in \mathbb{N}\}$, $L_2 = \{(ba)^n b^n \mid n \in \mathbb{N}\}$. Determine the following languages.

$$L_1 L_2 = ? \qquad L_1 \cap L_2 = ? \qquad \mathrm{Pre}(L_2) = ?$$

($\mathrm{Pre}(L_2)$ is the prefix language of L_2, the set of prefixes of the words of L_2.)

Exercise 2: Let $G = \langle \{S, A, B\}, \{a, b\}, P, S \rangle$ be a grammar, where the set of production rules P is as follows.

$$S \to aAa \mid bAb \mid \lambda$$
$$A \to aBa \mid bBb \mid \lambda$$
$$B \to aSa \mid bSb$$

Exercise 3: Let $|u|_t$ denote the number of t's in a word u. Make a deterministic finite automaton accepting the following language L.

$$L = \big\{u \in \{0,1\}^* \,\big|\, (\text{the last letter of } u \text{ is } 1) \wedge (|u|_0 \text{ is even})\big\}.$$

Exercise 4: Make a deterministic finite automaton recognizing the same language as the following nondeterministic one. $A = \langle \{q_0, q_1, q_2\}, \{a, b\}, M, \{q_0\}, \{q_0, q_1\} \rangle$, where the transition function M is given as follows.

		a	b
$\rightleftarrows$	q_0	$\{q_1, q_2\}$	$\emptyset$
$\leftarrow$	q_1	$\{q_0, q_1\}$	$\{q_2\}$
	q_2	$\emptyset$	$\{q_0, q_1\}$

Exercise 5: Make a minimal automaton, according to the algorithm we learnt (leaving unreachable states then contracing equivalent ones), recognizing the same language as the following automaton A. $A = \langle \{1,2,3,4,5,6,7,8,9\}, \{a,b\}, M, 1, \{4,5,6,7,8\} \rangle$, where the transition function M is given as follows.

		a	b
$\rightarrow$	1	9	3
	2	4	1
	3	9	5
$\leftarrow$	4	1	8
$\leftarrow$	5	3	4
$\leftarrow$	6	6	2
$\leftarrow$	7	9	8
$\leftarrow$	8	3	7
	9	1	3

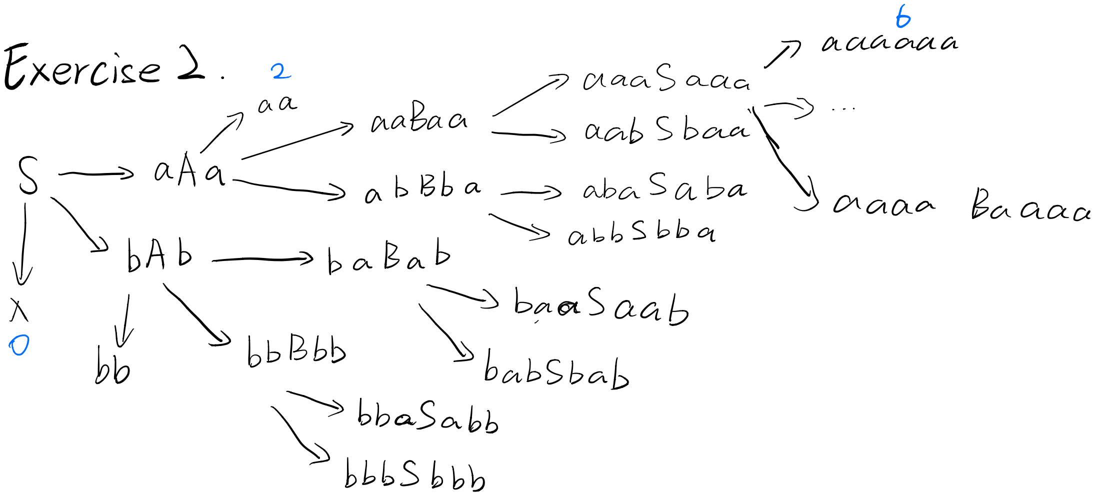
Exercise 1.

$L_1L_2 = \{ b^n a^{2n} (ba)^m b^m \mid m, n \in \mathbb{N} \}$

$L_1 \cap L_2 = \{ \varepsilon \}$

$Pre(L_2) = \{ (ba)^n b^n \mid n \in \mathbb{N} \} \cup \{ (ba)^n \mid n \in \mathbb{N} \} \cup \{ b \}$

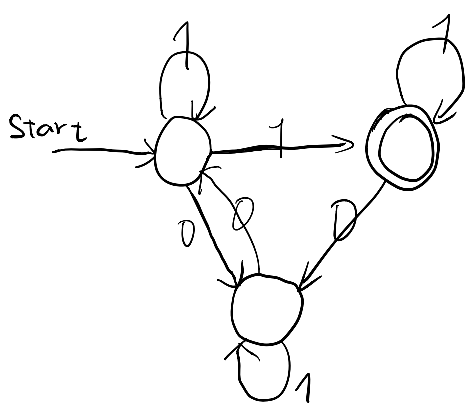
Exercise 2.



$L(G) = \{ w \in \{a,b\}^* \mid w = w^{-1} \wedge |w| \in (0, 2, 6, 8, \ldots) \}$

This grammar generates palindromes, as basic, if $w \in \{a,b\}^*$, $w = w^{-1}$. From step by step, we can see the length of w is following 0, 2, 6, 8, 12, 14 ....

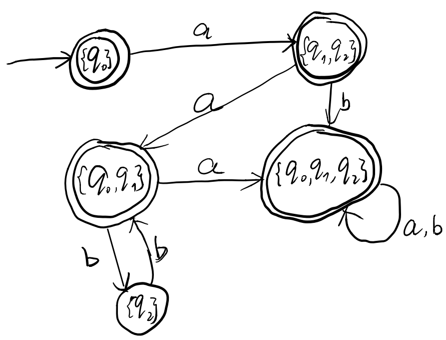
Exercise 3.



	0	1
$\rightarrow q_0$	q_1	q_2
q_1	q_0	q_1
$\leftarrow q_2$	q_1	q_2

Exercise 4.

	a	b
$\rightleftarrows \{q_0\}$	$\{q_1, q_2\}$	$\emptyset$
$\leftarrow \{q_1, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$
$\leftarrow \{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_2\}$
$\leftarrow \{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_2\}$	$\emptyset$	$\{q_0, q_1\}$



Exercise 5.

$\underset{\sim}{0}$: $\{1,2,3,9\}$, $\{4,5,6,7,8\}$

	a	b
$\leftarrow 4$	$\{1,2,3,9\}$	$\{4,5,6,7,8\}$
$\leftarrow 5$	$\{1,2,3,9\}$	$\{4,5,6,7,8\}$
$\leftarrow 6$	$\{4,5,6,7,8\}$	$\{1,2,3,9\}$
$\leftarrow 7$	$\{1,2,3,9\}$	$\{4,5,6,7,8\}$
$\leftarrow 8$	$\{1,2,3,9\}$	$\{4,5,6,7,8\}$

	a	b
1	$\{1,2,3,9\}$	$\{1,2,3,9\}$
2	$\{4,5,6,7,8\}$	$\{1,2,3,9\}$
3	$\{1,2,3,9\}$	$\{4,5,6,7,8\}$
9	$\{1,2,3,9\}$	$\{1,2,3,9\}$

$\underset{\sim}{1}$: $\{1,9\}$, $\{2\}$, $\{3\}$, $\{6\}$, $\{4,5,7,8\}$

	a	b
$\leftarrow 4$	$\{1,9\}$	$\{4,5,7,8\}$
$\leftarrow 5$	$\{3\}$	$\{4,5,7,8\}$
$\leftarrow 7$	$\{1,9\}$	$\{4,5,7,8\}$
$\leftarrow 8$	$\{3\}$	$\{4,5,7,8\}$

	a	b
1	$\{1,9\}$	$\{3\}$
9	$\{1,9\}$	$\{3\}$

$\overset{2}{\sim}$: $\{1,9\}, \{2\}, \{3\}, \{6\}, \{4,7\}, \{5,8\}$

	a	b
← 4	$\{1,9\}$	$\{5,8\}$
← 7	$\{1,9\}$	$\{5,8\}$

	a	b
← 5	$\{3\}$	$\{4,7\}$
← 8	$\{3\}$	$\{4,7\}$

$\overset{3}{\sim} = \overset{2}{\sim} = \sim$.

The minimal automaton

	a	b
→ $\{1,9\}$	$\{1,9\}$	$\{3\}$
$\{2\}$	$\{4,7\}$	$\{1,9\}$
$\{3\}$	$\{1,9\}$	$\{5,8\}$
← $\{6\}$	$\{6\}$	$\{2\}$
← $\{4,7\}$	$\{1,9\}$	$\{5,8\}$
← $\{5,8\}$	$\{3\}$	$\{4,7\}$