DISCRETE MATHEMATICS 1, TEST 2 GROUP 1

Please write your name and Neptun code on the papers. You have 90+15 minutes for the test. If you are ready, please upload the photos of your solutions (with explanation!) to Canvas. **The allowed file format is JPG. If you do not submit your solutions until deadline, then it is a fail.** Grade boundaries: 42, 34, 25 and 17 points for grades 5, 4, 3 and 2, respectively.

1. (a) (3 marks) A box contains 7 yellow, 5 white and 8 blue balls. In how many different ways can we pull out them from the box?
   (b) (3 marks) How many different 6-digits even numbers can be formed from the numbers 1, 2, 3, 4, 6, 8, if we use every digits exactly once?
   (c) (3 marks) We flip a coin 15 times. How many possible outcomes exist?
   (d) (3 marks) How many different ways can we draw 5 cards from a standard deck of 52 French cards, if we would **not** like to have an ace among the cards? (The order does not matter.)
2. (8 marks) In how many different ways can 9 couples be seated around a round table, if everyone wants to sit next to his/her partner.
3. (8 marks) How many different 6 digit number can be formed from the digits 0, 1, 3, 3, 5, 6, if we can use every digits exactly once? How many of these are divisible by 5?
4. (a) (6 marks) A cupboard contains 8 yellow, 8 green and 8 red shoes. How many socks must be taken out to be sure that we choose a matched pair.
   (b) (6 marks) Find the coefficient of the term x^{50} in the expansion of $\left(x^5 + \frac{4}{x^8}\right)^{23}$.
5. (a) (5 marks) Does there exist a simple graph with the following degree sequence: 1, 1, 2, 3, 3, 6, 6?
   (b) (5 marks) Prove, that in a tree the product of the number of vertices and the number of edges is even.

.