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1. $(-1,2),(0,3),(1,1)$

$$l_i(x) = \prod_{j=0,\, j\neq i}^{2} \frac{x-x_j}{x_i-x_j} \quad ; \quad L(x) = \sum_{i=0}^{2} f(x_i)\cdot l_i(x)$$

$$l_0(x) = \frac{x-0}{-1-0}\cdot\frac{x-1}{-1-1} = \frac{x(x-1)}{2} \quad ; \quad l_1(x) = \frac{x+1}{0+1}\cdot\frac{x-1}{0-1} = \frac{(x+1)(x-1)}{-1}$$

$$l_2(x) = \frac{x+1}{1+1}\cdot\frac{x-0}{1-0} = \frac{x(x+1)}{2}$$

$$L(x) = 2\cdot\frac{x(x-1)}{2} + \frac{3(x+1)(x-1)}{-1} + \frac{x(x+1)}{2} = x(x-1) - 3(x+1)(x-1) + \frac{1}{2}x(x+1)$$

$$f(x_0,x_1) = \frac{f(x_1)-f(x_0)}{x_1-x_0} = 1$$

$$f(x_1,x_2) = \frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{1-3}{1-0} = -2$$

$$f(x_0,x_1,x_2) = \frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0} = \frac{-2-1}{1+1} = \frac{-3}{2}$$

	$k=0$	1	2
$-1 = x_0$	2		
$0 = x_1$	3	1	
$1 = x_2$	1	-2	$-\frac{3}{2}$

$$P(x) = f(x_0) + f(x_0,x_1)(x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$$

$$= 2 + 1(x+1) - \frac{3}{2}(x+1)x$$

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2. $f(x) = (x+3)^{-1}$ $\quad x \in [-2,1]$

$$M_n = \max_{x\in[-2,1]} |f^{(n)}(x)| \quad ; \quad f^{(n)}(x) = \pm n!\,(x+3)^{-n-1}$$

$$M_n = \max_{x\in[-2,1]} |\pm n!\,(x+3)^{-n-1}| = n!\cdot 1^{-n-1} \quad \text{for } x=-2$$

$$M_n = n!$$

Error at some x

$$|f(x) - L_n(x)| \leqslant \frac{M_{n+1}}{(n+1)!} |W_n(x)|$$

$$\| f(x) - L_n(x) \|_\infty \leqslant \frac{M_{n+1}}{(n+1)!} \|W_n\|_\infty$$

maximal error

maximal $W_n(x)$

$$W_n(x) = \prod_{i=0}^{n} (x - x_i)$$

3. $(x_0; f_0, f_0', f_0'') = (1, 2, 1, -2)$

$(x_1; f_1; f_1') = (2; -3, 1)$

	k= 0	1	2	3	4
$x_0=1$	2				
$x_0=1$	2	1			
$x_0=1$	2	1	-1		
$x_1=2$	-3	-5	-6	-5	
$x_1=2$	-3	1	6	12	17

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$$f[1,2] = \frac{-3-2}{2-1} = -5$$

$$\frac{12+5}{2-1} = \frac{17}{1} = 17.$$

$$\frac{f'(x_1) - f[1,2]}{2-1} = \frac{1+5}{1} = 6$$

$$f[1,2] - f'(x_0) = -6$$

$$\frac{\left(f[1,2] - f(x_0)'\right) - \frac{f(x_0)''}{2}}{2-1} = \frac{-6+1}{1} = -5$$

$$\frac{\left(f(x_1') - f[1,2]\right) - \left(\frac{f[1,2] - f'(x_0)}{2-1}\right)}{2-1} = \frac{6+6}{2-1} = 12$$

4. $\cos x - 2x + 1 = 0$

$$2x = \cos x + 1$$

$$x = \frac{\cos x + 1}{2} \Rightarrow F(x) = \frac{\cos x + 1}{2}$$

$$|F'(x)| = \left(\frac{\cos x + 1}{2}\right)' = \left|-\frac{\sin x}{2}\right| \leq \frac{1}{2} < 1 \Rightarrow$$ F is contraction for all real x

Banach's fix point theorem: $\exists x^* \in \mathbb{R}$ (unique). $f(x^*) = 0$.

$x_{k+1} = F(x_k)$ and $\| x^{(k)} - x^* \| \leq \frac{e^k}{1-2} \| x - x^{(0)}$

$f(x^*) = 0$: root.

$$x_0 = 0 \Rightarrow x_1 = F(0) = \frac{1+1}{2} = 1$$

$$\| 1 - x^* \| \leq \frac{\frac{1}{2}}{1-\frac{1}{2}} \| x_1 - x_0 \|$$

$$\| 1 - x^* \| \leq \frac{\frac{1}{2}}{\frac{1}{2}} \| 1 - 0 \| = 1$$

$\| 1 - x^* \| \leq 1 \Rightarrow$ Distance of x_1 to the solution is less or equal to 1

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$$|x_0 - x^*| < \frac{2 \cdot \min_{x \in \mathbb{R}} |f'(x)|}{\max_{x \in \mathbb{R}} |f''(x)|} = \frac{2 \cdot \min_{x \in \mathbb{R}} |-\sin x - 2|}{\max_{x \in \mathbb{R}} |f \cos x|} = \frac{2 \cdot 1}{1} = 2$$

$f'(x) = -\sin x - 2$

$f''(x) = -\cos x$

we have : $x^* \approx 0.83$

$|0 - 0.83| < 2 \Rightarrow$ starting fom x_0 the Newton method coverges to x^*

5. $$\int_0^2 (x^2 - 3x + 1)\, dx = (2-0)(1^2 - 3(1) + 1) + \frac{(2-0)^3}{24} (\eta^2 - 3\eta + 1)''$$

$$= 2(1 - 3 + 1) + \frac{8}{24}(2\eta - 3)'$$

$$= -2 + \frac{1}{3} \cdot 2$$

$$= \frac{-4}{3} \Rightarrow \text{yes, we can !}$$

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