DISCRETE MATHEMATICS 1, TEST 1 GROUP 1

Please write your name and Neptun code on the paper. You have 90+15 minutes for the test. If you are ready, please upload the photos of your solutions (with explanation!) to Canvas. Grade boundaries: 42, 34, 25 and 17 points for grades 5, 4, 3 and 2, respectively.

1. (a) (4 marks) Give three example sets A, B and C, for which $(B\backslash(A \cup C)) \cup ((A \cap C)\backslash B) = (A \cap C) \cup (B\backslash C)$ is **NOT true**. Then give three, for which it is **true**.
   (b) (4 marks) Prove by definition, that for arbitrary sets A, B and C the following statement holds: $A\backslash(B \cup C) = (A\backslash B) \cap (A\backslash C)$.
2. (a) (4 marks) Let R be a homogeneous binary relation on set X. Decide, whether R is reflexive, symmetric, transitive and anti-symmetric, if the relation is the following:
   $$R = \{(1,1),(1,2),(1,3),(2,2),(2,4),(3,3),(4,3),(4,1),(4,4)\} \quad X = \{1,2,3,4\}.$$
   (b) (4 marks) A homogeneous binary relation $R \subseteq \mathbb{Z} \times \mathbb{Z}$, $R = \{(x,y) \in \mathbb{Z} \times \mathbb{Z} | \quad y^2 = 4x - 1\}$ is given. Find $dmn(R)$, $rng(R)$ and $R(\{0,7\})$.
3. (a) (4 marks) Let $R = \{(1,a),(1,b),(2,a),(2,b),(4,c),(4,a)\}$ and $S = \{(a,2),(a,3),(b,2),(c,4),(d,3),(d,1)\}$ be binary relations.
      i. Find the inverse image $R^{-1}(\{a,b\})$
      ii. Find the relation $S \circ R$ and write it down as a set of ordered pairs.
   (b) (4 marks) For each of the following examples, decide if the relation is an equivalence relation, justifying your answer.
      i. $R_1 \subseteq \mathbb{R} \times \mathbb{R}$, $R_1 = \{(x,y) \in \mathbb{R} \times \mathbb{R} | \quad |x-y| \text{ is odd}\}$
      ii. $R_2 \subseteq \mathbb{Z} \times \mathbb{Z}$, $R_2 = \{(x,y) \in \mathbb{Z} \times \mathbb{Z} | \quad 2x - 2y \text{ is even}\}$
4. (8 marks) Decide about each of the relations below if it is a function, justifying your answer. For each function, decide if it is injective, surjective and/or bijective.
   (a) $f_1 \subset X \times X$, $f_1 = \{(a,c),(b,b),(d,d),(e,a)\}$, where $X = \{a,b,c,d,e,f\}$.
   (b) $f_2 \subset \mathbb{R} \times \mathbb{R}$, $f_2 = \{(x,y) \in \mathbb{R} \times \mathbb{R} | \quad 2x^3 - 3 = y^2 + 2\}$.
5. (10 marks) Give the polar form of the following complex number:
   $$z = \frac{(\sqrt{2} - \sqrt{2}i)^9}{(1 + \sqrt{3}i)^{17}}.$$
   Find all complex numbers w such that $w^3 = z$.
6. (8 marks) Represent the following sets in the Gaussian plane:
   (a) $\{z \in \mathbb{C} | \quad Rez = Imz \wedge |z - 1 - i| < 1\}$.
   (b) $\{z \in \mathbb{C} | \quad |z + 2 - i| \geq 4 \wedge |z - 2 - i| \geq 4\}$.