Fundamentals of theory of computation 2

1st (midterm) test

1. $A = q \to \neg p \vee \neg q \wedge r$

   Make the truth table of A.

2. Give a formula in conjunctive normal form (CNF) equivalent with $(\neg(\neg r \wedge p) \to q) \to \neg r$.

3. Prove by resolution procedure, that the following set of formulas is unsatisfiable.

   $$\{p \vee \neg q \vee r, \quad \neg p \vee \neg q, \quad q, \quad \neg q \vee \neg s, \quad \neg r \vee s\}.$$

4. In first order logic suppose, that p is a predicate symbol of arity 2, q is a predicate symbol of arity 1, f is a function symbol of arity 2, and a is a constant symbol.

   Consider the interpretation $\langle \{0,1,2\}, \{P, Q\}, \{F\}, \{1\} \rangle$ (i.e., predicate and function symbols denoted by small letters are interpreted as relations and functions denoted by the corresponding capital letters, and a is interpreted as 1), where

$P^{\mathcal{I}}$	0	1	2
0	h	i	i
1	h	i	i
2	h	h	h

$Q^{\mathcal{I}}$	
0	h
1	i
2	i

$f^{\mathcal{I}}$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

   (Rows correspond to 1st argument, columns correspond to 2nd argument.) For the assignment σ let $\sigma(x) = 2, \sigma(y) = 1$. Determine the following values (give your calculations as well, not just your final answer).

   (a) $\mathcal{D}_{I,\sigma}(f(f(y, f(a, x)), y))$

   (b) $v_{I,\sigma}(p(x, y) \vee q(a) \to \neg q(f(x, y)))$

   (c) $v_{I,\sigma}(\forall y p(y, x) \to \neg q(f(y, y)))$

5. Consider the following functions

   $$f(n) = 3^n + 2n^3, \qquad g(n) = n^2 3^n + n^3 2^n, \qquad h(n) = (n^2 + n)3^n.$$

   Which of the following statements hold?

   $$f(n) = \Omega(g(n)), \qquad g(n) = \Omega(f(n)), \qquad g(n) = O(h(n)), \qquad h(n) = O(g(n)).$$

   Support your answers with an argument.

6. Give a Turing machine deciding the following language L.

   $$L = \{u \in \{a, b, c\}^* \mid \text{ the last letter of } u \text{ occurs no more in } u \ \}.$$