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1. b) 6 - digits numbers : _ _ _ _ _ _

From 1,2,3,4,6,8 only 2,4,6,8 can be put at the last position => 4 ways

The first position : 5 ways

The second _ : 4 ways

third _ : 3 ways

fourth _ : 2 ways

fifth _ : 1 ways

=> Total number of even number = $4 \times 5 \times 4 \times 3 \times 2 \times 1 = 480$

c) Each time we flip the coin => 2 possibilities

Flip 15 times independently => 2^{15} possible outcomes

d) Minus 4 aces, we have 48 cards left

=> we have : 48 options for 1st draw

47 _____ 2nd _

46 _____ 3rd _

45 _____ 4th _

44 _____ 5th _____

=> Total : $48 . 47 . 46 . 45 . 44$

2. We consider 9 couples as 9 people, because all couples want to seat together:

For 9 people, we have $(9-1)!$ different ways

However, if a people change his/her seat to his/her partner, we will have a new way.

=> Total : $8! . 2^9$ ways

1 a) 7 yellow, 5 white, 8 blue balls

total $= 7+5+8 = 20$

$\Rightarrow$ There are $\dfrac{20!}{7! \times 5! \times 8!}$ ways we can pull them out of the box

4 b) $\left(x^5 + \dfrac{4}{x^8}\right)^{23}$

Fisrt, we will find n such that:

$$\left(x^5\right)^n \cdot \left(\frac{1}{x^8}\right)^{23-n} = x^{50}$$

$$\Leftrightarrow \quad \frac{x^{5n}}{x^{8(23-n)}} = x^{50}$$

$$\Leftrightarrow \quad 5n - 8(23-n) = 50$$

$$\Leftrightarrow \quad 5n - 8n - 184 = 50$$

$$\Leftrightarrow \quad 13n = 234$$

$$\Leftrightarrow \quad n = 18$$

$\Rightarrow$ Coefficient of x^{50} is:

$$\binom{23}{18} \cdot \left(x^5\right)^{18} \cdot \left(\frac{4}{x^8}\right)^5$$

$$= \binom{23}{18} \cdot 4^5 \cdot x^{50}$$

So, the coefficient of x^{50} is $\binom{23}{18} \cdot 4^5 = \ldots$

3) ② There are:

5 options for first digits (except 0)

5 _____ 2nd ___

4 _____ 3rd ___

3 _____ 4th ___

2 _____ 5th ___

1 _____ 6th ___

⇒ Total: 5.5.4.3.2.1 = 600 numbers

However, we have 2 numbers so each number will be duplicated

⇒ total: 600 : 2 = 300 numbers

* Divisible by 5:

- If the 6th number is 0

⇒ 5 options for 1st digit

4 _____ 2nd ___

⋮

⇒ Total: 5! numbers

- If the 6th number is 5

⇒ 4 options for 1st digit

4 options ___ 2nd ___

3 options ___ ...

⋮

⇒ total: 4.4.2.1.3

⇒ Total: (5! + 4.4.3.2.1) : 2

5 a) There are 2 vertices with 6 degree and 2 vertices degree 3. The best scenario is that those 4 vertices are connected.

⇒ There are in total 6 out going edges from those

But we only have 4 edges (2+1+1)

⇒ Contradiction.