6.

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TEST 1

$$A = \begin{bmatrix} 2 & -1 & 5 \\ -1 & 2 & 0 \\ -1 & 2 & -2 \end{bmatrix} = QR, \quad Q = ? \quad R = ?$$

$$q_1' = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} = a_1$$

$$r_{11} = \|q_1'\| = \sqrt{2^2 + (-1)^2 + (-1)^2} = \sqrt{6}$$

$$q_1 = \frac{1}{\|q_1'\|} \cdot q_1' = \frac{1}{\sqrt{6}} \cdot \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2/\sqrt{6} \\ -1/\sqrt{6} \\ -1/\sqrt{6} \end{bmatrix}$$

$$r_{12} = q_1^T \cdot a_2 = \begin{bmatrix} 2/\sqrt{6} & -1/\sqrt{6} & -1/\sqrt{6} \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} = -\sqrt{6}$$

$$q_2' = a_2 - r_{12} \cdot q_1 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} + \sqrt{6} \cdot \begin{bmatrix} 2/\sqrt{6} \\ -1/\sqrt{6} \\ -1/\sqrt{6} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$r_{22} = \|q_2'\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$q_2 = \frac{1}{\|q_2'\|} \cdot q_2' = \frac{1}{\sqrt{3}} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

$$r_{13} = q_1^T \cdot a_3 = \begin{bmatrix} 2/\sqrt{6} & -1/\sqrt{6} & -1/\sqrt{6} \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix} = 2\sqrt{6}$$

$$r_{23} = q_2^T \cdot a_3 = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix} = \sqrt{3}$$

$$q_3' = a_3 - r_{13} \cdot q_1 - r_{23} \cdot q_2 = \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix} - 2\sqrt{6} \cdot \begin{bmatrix} 2/\sqrt{6} \\ -1/\sqrt{6} \\ -1/\sqrt{6} \end{bmatrix} - \sqrt{3} \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$r_{33} = \|q_3'\| = \sqrt{0^2 + 1^2 + (-1)^2} = \sqrt{2}$$

$$q_3 = \frac{1}{\|q_3'\|} \cdot q_3' = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

⑥ $\Rightarrow \tilde{Q} = \begin{bmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{bmatrix}, \quad Q = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$

$\tilde{R} = \begin{bmatrix} \sqrt{6} & -\sqrt{6} & 2\sqrt{6} \\ 0 & \sqrt{3} & \sqrt{3} \\ 0 & 0 & \sqrt{2} \end{bmatrix}, \quad R = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

③

$$A = \begin{bmatrix} 2 & 2 & -2 \\ -2 & 2 & 0 \end{bmatrix}$$

$\|A\|_\infty = \max\{2+2+2,\ 2+2+0\} = \max\{6,4\} = 6.$

$\|A\|_1 = \max\{2+2,\ 2+2,\ 2+0\} = \max\{4,4,2\} = 4$

$\|A\|_2 = 12$

$$A^TA = \begin{bmatrix} 2 & -2 \\ 2 & 2 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2 & -2 \\ -2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 0 & -4 \\ 0 & 8 & -4 \\ -4 & -4 & 4 \end{bmatrix}$$

$\exists v \neq 0: Av \to \lambda v$

$\Leftrightarrow \det(A - \lambda I) = 0 \Leftrightarrow \begin{vmatrix} 8-\lambda & 0 & -4 \\ 0 & 8-\lambda & -4 \\ -4 & -4 & 4-\lambda \end{vmatrix} = 0$

$\Leftrightarrow (8-\lambda)[(8-\lambda)(4-\lambda) - 16] - 6.[0(4-\lambda) - 16] + (-4).[0.(-4) + 4(8-\lambda)] = 0$

$\Leftrightarrow (8-\lambda)(16 - 12\lambda + \lambda^2) - 4(32 - 4\lambda) = 0.$

$\Leftrightarrow 128 - 96\lambda + 8\lambda^2 - 16\lambda + 12\lambda^2 - \lambda^3 - 128 + 16\lambda = 0$

$\Leftrightarrow -\lambda(\lambda^2 - 20\lambda + 96) = 0$

$\Leftrightarrow \left[\begin{array}{l} \lambda = 0 \\ \lambda^2 - 20\lambda + 96 = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} 0 \\ 12 \\ 8 \end{array}\right. \Rightarrow \rho(A^TA) = \max\{0,8,12\} = 12$

(4) $A = \begin{bmatrix} 2 & 3 & -1 & 2 \\ -4 & -8 & 3 & -1 \\ 6 & 11 & -1 & 2 \\ 2 & 7 & 6 & -3 \end{bmatrix} \quad b = \begin{bmatrix} 5 \\ -1 \\ 9 \\ 8 \end{bmatrix}$

$A = LU$

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 0 & -2 & 1 & 3 \\ 6 & 11 & -1 & 2 \\ 2 & 7 & 6 & -3 \end{bmatrix} +2R_1 + R_2$$

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 0 & -2 & 1 & 3 \\ 0 & 2 & 2 & -4 \\ 2 & 7 & 6 & -3 \end{bmatrix} -3R_1 + R_3$$

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 0 & -2 & 1 & 3 \\ 0 & 2 & 2 & -4 \\ 0 & 4 & 7 & -5 \end{bmatrix} -R_1 + R_2$$

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 0 & -2 & 1 & 3 \\ 0 & 0 & 3 & -1 \\ 0 & 4 & 7 & -5 \end{bmatrix} R_2 + R_3$$

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 0 & -2 & 1 & 3 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 9 & 1 \end{bmatrix} 2R_2 + R_4$$

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 0 & -2 & 1 & 3 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix} -3R_3 + R_4 \quad = U$$

$$\Rightarrow L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & -1 & 1 & 0 \\ 1 & -2 & 3 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 3 & -1 & 2 \\ 0 & -2 & 1 & 3 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$\Rightarrow Ax = b$

$LUx = b$

$\Leftrightarrow Ly = b$

$Ly = b \ (1)$

$Ux = y \ (2)$

(1) $Ly = b$

$$\Leftrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & -1 & 1 & 0 \\ 1 & -2 & 3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ 9 \\ 8 \end{bmatrix}$$

$$\Rightarrow \begin{cases} y_1 = 5 \\ -2y_1 + y_2 = -1 \Rightarrow y_2 = 9 \\ 3y_1 - y_2 + y_3 = 9 \Rightarrow y_3 = 3 \\ y_1 - 2y_2 + 3y_3 + y_4 = 8 \Rightarrow y_4 = 12 \end{cases}$$

(2) $Ux = y$

$$\Leftrightarrow \begin{bmatrix} 2 & 3 & -1 & 2 \\ 0 & -2 & 1 & 3 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \\ 3 \\ 12 \end{bmatrix}$$

(4)

$$\Rightarrow \begin{cases} 4x_4 = 12 \Rightarrow x_4 = 3 \\ 3x_3 - x_4 = 3 \Rightarrow x_3 = 2 \\ -2x_2 + x_3 + 3x_4 = 9 \Rightarrow x_2 = 1 \\ 2x_1 + 3x_2 - x_3 + 2x_4 = 5 \Rightarrow x_1 = -1 \end{cases} \qquad \Rightarrow x = \begin{bmatrix} -1 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

(5)

$$\begin{bmatrix} 2 & -4 & 6 & 2 \\ -4 & 1 & -15 & -10 \\ 6 & -15 & 23 & 18 \\ 2 & -10 & 18 & 31 \end{bmatrix} = LDL^T$$

(I did this with $(2,2) = 1$. I haven't read your message telling to change $(2,2) = 11$ while solving this.)

$$\rightarrow \begin{bmatrix} 2 & -4 & 6 & 2 \\ -2 & -7 & -3 & -6 \\ 3 & -3 & 5 & 12 \\ 1 & -6 & 12 & 29 \end{bmatrix} \rightarrow \begin{bmatrix} \cancel{-7} & \cancel{-3} & \cancel{-6} \\ \cancel{-3} & & \\ \cancel{-6} & & \end{bmatrix} \begin{bmatrix} -7 & -3 & -6 \\ 3/7 & 44/7 & 102/7 \\ 6/7 & 102/7 & 239/7 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 44/7 & 102/7 \\ 51/22 & 4/11 \end{bmatrix} \rightarrow \begin{bmatrix} 4/11 \end{bmatrix}$$

$$\Rightarrow L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & 3/7 & 1 & 0 \\ 1 & 6/7 & 51/22 & 1 \end{bmatrix} ; \quad D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -7 & 0 & 0 \\ 0 & 0 & 44/7 & 0 \\ 0 & 0 & 0 & 4/11 \end{bmatrix}$$

All the pivots are not positive, so it is not positive definite matrix.

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$$A = \begin{bmatrix} 5 & 1 & -2 & 0 \\ 2 & 7 & -2 & 1 \\ 2 & 0 & 4 & -1 \\ -3 & -2 & 4 & 9 \end{bmatrix}.$$

$$\|B_J\|_\infty = \max\left\{ \frac{1+2+0}{5} ; \frac{2+2+1}{7} ; \frac{2+0+1}{4} , \frac{3+2+4}{9} \right\}$$

$$= \max\left\{ 3/5 , 5/7 , 3/4 , 1 \right\} = 1.$$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ -3 & -2 & 4 & 1 \end{bmatrix} \qquad A_1 = \begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

$$\|B_G\|_\infty \leq \max\left\{ \frac{1+2}{(5-0)} ; \frac{2+1}{(7-2)} ; \frac{1}{(4-2)} \right\}$$

$$\max\left\{ \frac{3}{5} ; \frac{3}{5} ; \frac{1}{2} \right\} = \frac{3}{5}.$$

This is diagonally dominant matrix.

1) $M = M(t; k^-; k^+) = M(7, -6, 6)$

a) $\varepsilon_0 = \frac{1}{2} \cdot 2^{k^-} = \frac{1}{2} \cdot 2^{-6} = 2^{-7} = \frac{1}{128} = 0{,}007812$

$M_\infty = (1-2^{-t})\, 2^{k^+} = (1-2^{-7})\, 2^{6} = \frac{127}{2} = 63{,}5$

$\varepsilon_M = 2^{-t} = 2^{-7} = \frac{1}{128} = 0{,}007812$

b) 0.24 $\quad fl(0.24) = 0.1110100 \cdot 2^{-2}$

$\varepsilon_0 < 0.24 < M_\infty$

	0.24
0	48
0	96
1	82
1	64
1	28
0	56
1	12
0	24
0	48
0	96

23.56

23	1
11	1
5	1
2	0
1	1

	0.56
1	12
0	24
0	48

$fl(23.56) = 0.1011110 \cdot 2^{-5}$

$\varepsilon_0 < 23.56 < M_\infty$

$10111.10|0 \rightarrow 0.1011110 \cdot 2^{-5}$

2

a) $\Delta x = 10^{-4}$

$\Delta f \leq M_1 \Delta x$

$$M_1 = \max_{x \in (x-\Delta x, x+\Delta x)} |f'(x)| = \max_{x \in (-10^{-4}, \frac{\pi}{2}+10^{-4})} |\cos x| = 1$$

$\Rightarrow \quad \Delta f \leq 1.10^{-4}$

b) $|\delta f| \simeq C(f, x)|x|$

$$C = \max \frac{x f'(x)}{f(x)} = \max\left(\frac{x \cos x}{\sin x}\right) \simeq 1$$

$\Rightarrow |\delta f| \leq 1.6.10^{-5}$