Fundamentals of theory of computation 2

1st (midterm) test

1. $A = q \vee \neg p \to \neg q \wedge r$

   Make the truth table of A.

2. Give a formula in conjunctive normal form (CNF) equivalent with $(\neg(p \wedge \neg r) \to \neg q) \to p$.

3. Prove by resolution procedure, that the following set of formulas is unsatisfiable.

   $$\{\neg p \vee q \vee r, \quad p \vee q, \quad \neg q, \quad q \vee s, \quad \neg r \vee \neg s\}.$$

4. In first order logic suppose, that p is a predicate symbol of arity 2, q is a predicate symbol of arity 1, f is a function symbol of arity 2, and a is a constant symbol.

   Consider the interpretation $\langle \{0,1,2\}, \{P,Q\}, \{F\}, \{1\} \rangle$ (i.e., predicate and function symbols denoted by small letters are interpreted as relations and functions denoted by the corresponding capital letters, and a is interpreted as 1), where

P	0	1	2
0	F	T	T
1	F	T	T
2	F	F	F

Q	
0	F
1	T
2	T

F	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

   (Rows correspond to 1st argument, columns correspond to 2nd argument.) For the assignment σ let $\sigma(x) = 1, \sigma(y) = 2$. Determine the following values (give your calculations as well, not just your final answer).

   (a) $\mathcal{D}_{I,\sigma}(f(f(x, f(a, x)), y))$

   (b) $v_{I,\sigma}(p(y, x) \vee q(a) \to \neg q(f(y, x)))$

   (c) $v_{I,\sigma}(\exists x p(x, y) \to \neg q(f(x, x)))$

5. Consider the following functions

   $$f(n) = 2n \log_8 n, \qquad g(n) = (n + 3\log_2 n) \cdot \log_2 n, \qquad h(n) = n + \log_2(n^3).$$

   Which of the following statements hold?

   $$f(n) = \Omega(g(n)), \qquad g(n) = \Omega(f(n)), \qquad g(n) = O(h(n)), \qquad h(n) = O(g(n)).$$

   Support your answers with an argument.

6. Give a Turing machine deciding the following language L.

   $$L = \{u \in \{a,b\}^* \mid (|u| \geq 2) \wedge (\text{the 2nd and the last but one letter of } u \text{ is the same})\}.$$