Introduction to Python - Lissajous figures and beats **Noah Huffman**
Ph 20

1 Lissajous Figures

In Python, I have defined the following functions,

$$X(t) = A_x cos(2\pi f_x t) \tag{1}$$

$$Y(t) = A_y sin(2\pi f_y t + \phi) \tag{2}$$

I numerically plotted these functions in Python by creating a sequence of values for X(t) and Y(t) for time steps given by Δt. Below, I have reproduced plots showing different Lissajous figures and discuss the effect of the parameters on the images.

$A_x = A_y = 1$ for all images for simplicity. It should be easy to see from equations 1 and 2 without a plot that A_x and A_y are amplitude scaling factors. If either parameter were different from unity, the resulting plot would be "stretched" in the respective dimension.

1.1 Effect of frequency ratio, $\frac{f_x}{f_y}$

Lissajous figures with rational $\frac{f_x}{f_y}$ ratios should be closed contours because after some number of rational cycles, Y and X will return to their initial states, and close the X, Y loop. For example, if Y's frequency is twice that of X, then after two periods of Y (one period of X), both X and Y will be in their original positions at the beginning of their periods again. This implies closed, periodic spacial motion of our object, which gives closed Lissajous figures.

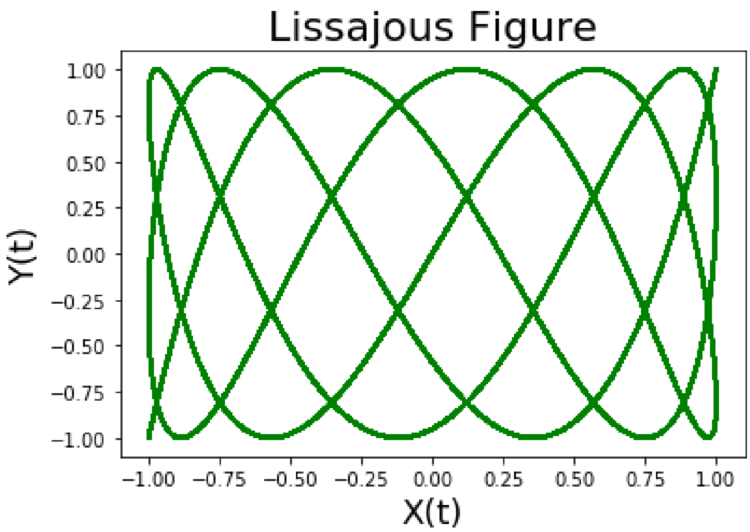
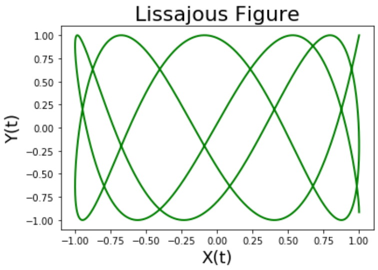
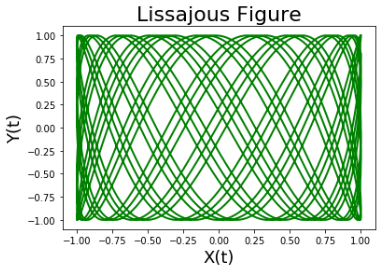


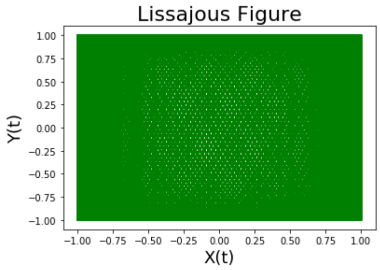
Figure 1: An example with $\phi = 0$, $f_x = 5$, and $f_y = 13$. Even after a long time ($t = \Delta t N = 100$), we see that the figure is closed.



(a) $t = 1$



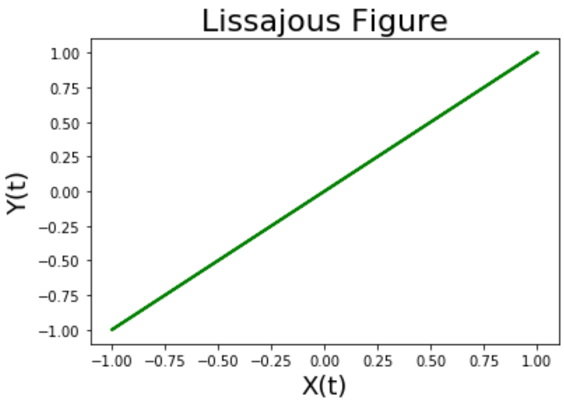
(b) $t = 2$



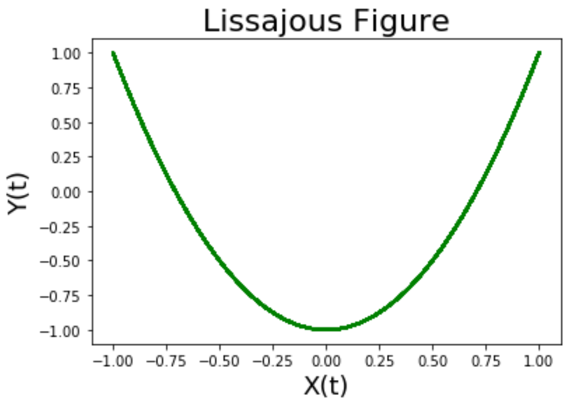
(c) $t = 100$

Figure 2: For the irrational ratio, $\frac{f_x}{f_y} = \frac{1}{e}$, the curve never closes and after a sufficient amount of time, fills all of the available space.

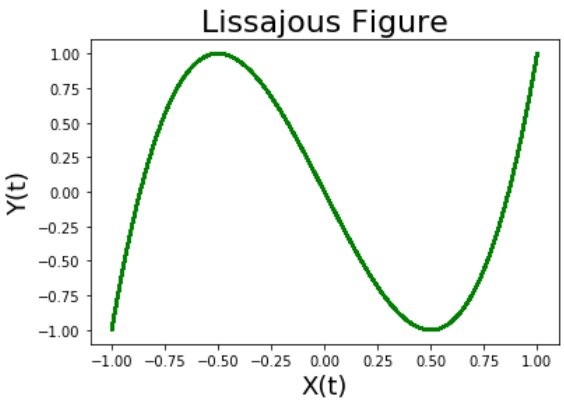
Holding $\phi = 0$, we can see the special case, $\frac{f_x}{f_y} = 1$, yields a line. Changing this ratio to $\frac{f_x}{f_y} = \frac{1}{n}$ yields a polynomial of order n.



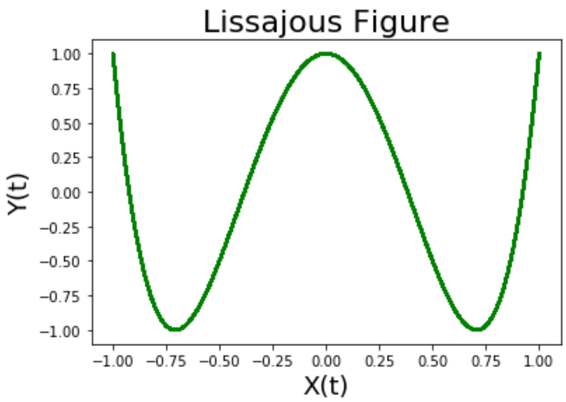
(a) $f_y = 1$



(b) $f_y = 2$



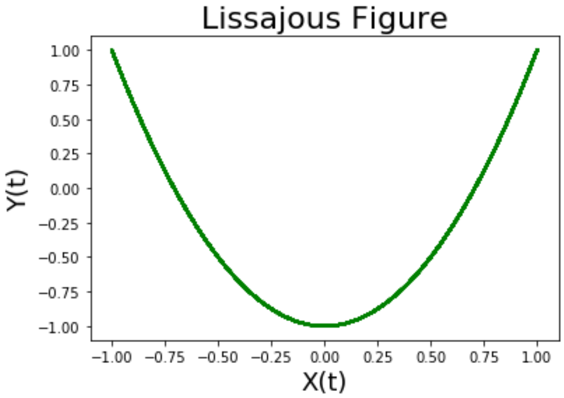
(c) $f_y = 3$



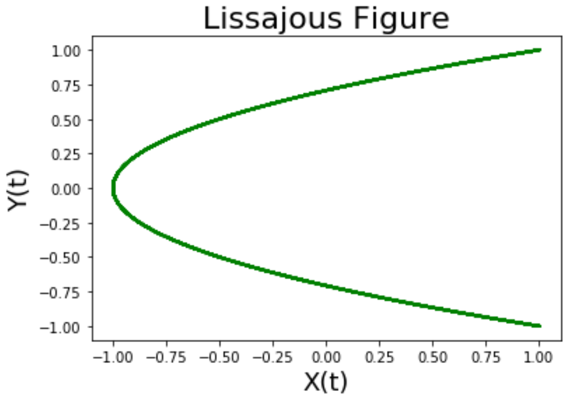
(d) $f_y = 4$

Figure 3: Effect of f_y on the Lissajous Figure for $f_x = 1$, $\phi = 0$.

Inverting the ratio $\frac{f_x}{f_y} = \frac{n}{1}$ yields a nth-order polynomial in Y(X).



(a) $\frac{f_x}{f_y} = \frac{1}{2}$

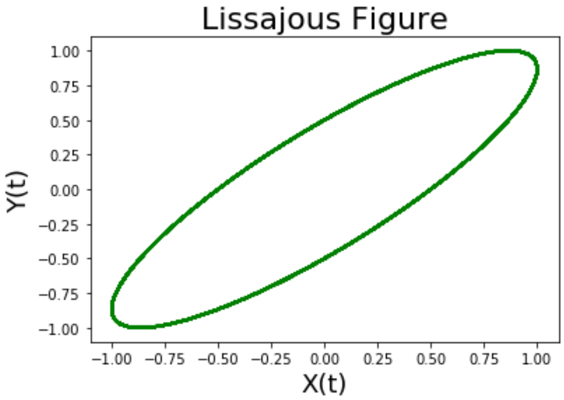


(b) $\frac{f_x}{f_y} = 2$

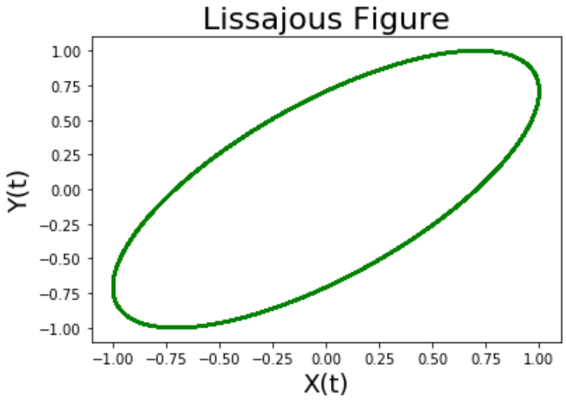
Figure 4: Effect of inverting the frequency ratio.

1.2 Effect of ϕ

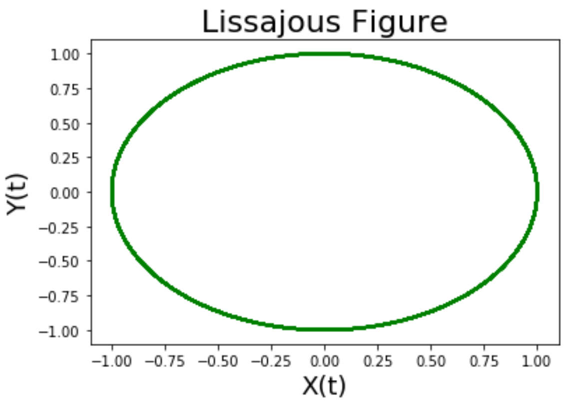
With $\frac{f_x}{f_y} = 1$, changing ϕ yields different ellipses, widening and rotating with increasing ϕ.



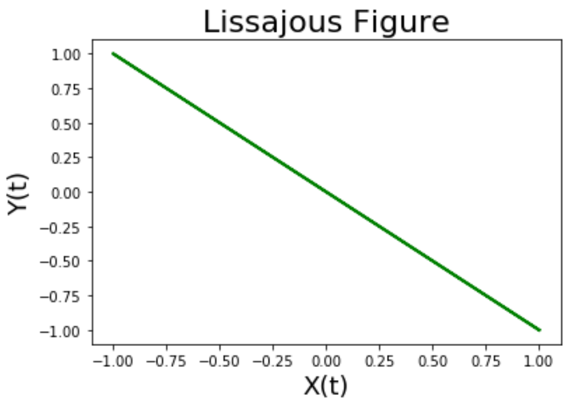
(a) $\phi = \frac{\pi}{6}$



(b) $\phi = \frac{\pi}{4}$



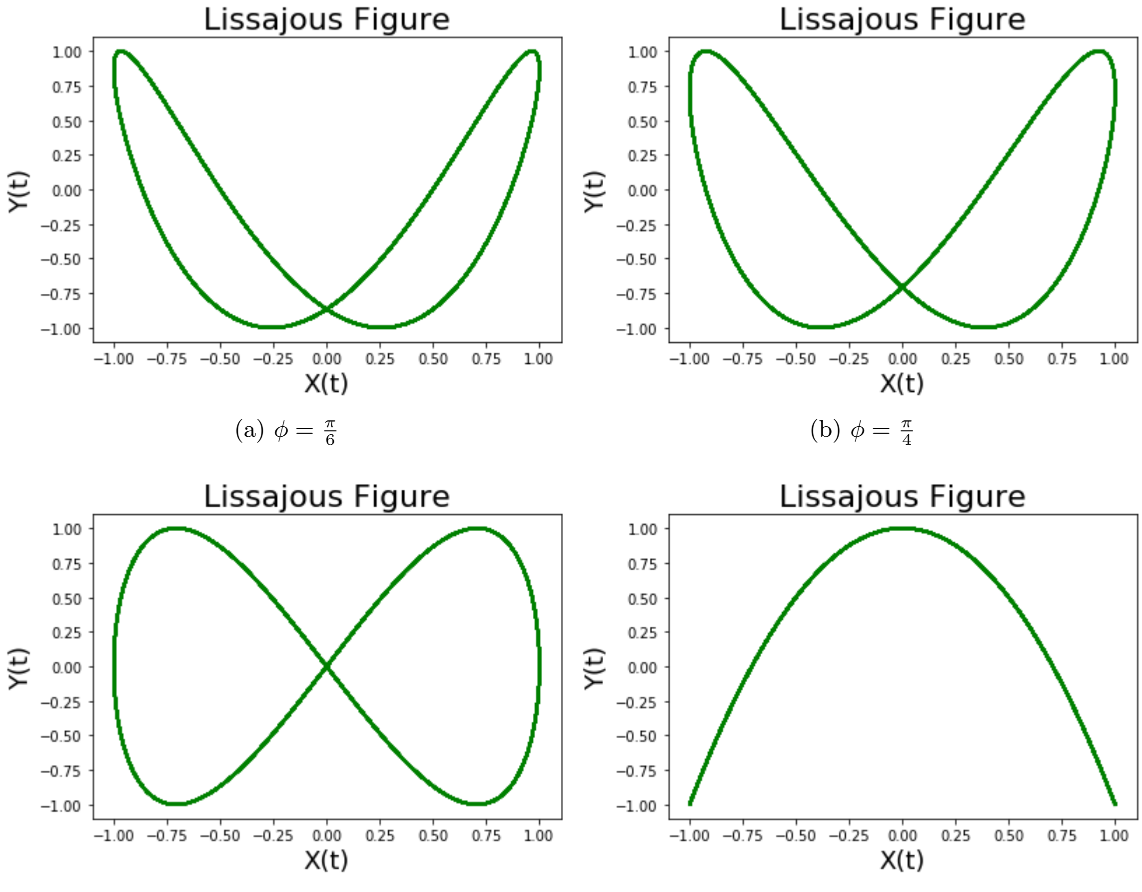
(c) $\phi = \frac{\pi}{2}$



(d) $\phi = \pi$

Figure 5: Effect of ϕ on the Lissajous Figure for $f_x = f_y = 1$.

The same ”widening” and inverting effect can be seen in higher orders. Below the effect of ϕ on the second order ratio is shown.



(a) $\phi = \frac{\pi}{6}$

(b) $\phi = \frac{\pi}{4}$

(c) $\phi = \frac{\pi}{2}$

(d) $\phi = \pi$

Figure 6: Effect of ϕ on the Lissajous Figure for $f_x = 1$, $f_y = 2$.

2 Beats

The phenomenon of beats occurs when two sinusoids are simultaneously active in a function as seen in the equation below.

$$cos(\omega_1 t) + cos(\omega_2 t) = 2cos(\frac{\omega_1 + \omega_2}{2}t)cos(\frac{\omega_1 - \omega_2}{2}t) \tag{3}$$

This can be achieved by adding the sinusoids we defined earlier.

$$Z(t) = X(t) + Y(t) \tag{4}$$

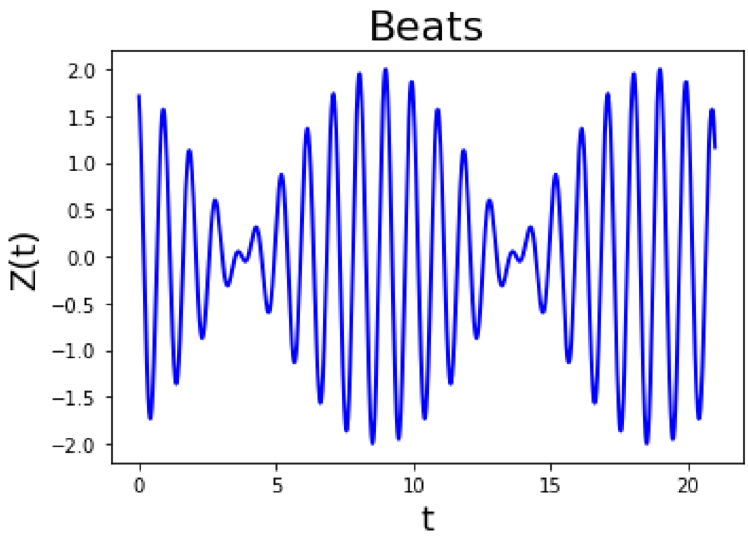


Figure 7: Example of the beats phenomenon. $f_x = 1$, $f_y = 1.1$, $\phi = \frac{\pi}{4}$

3 My Experience

This was my first time seeing Python **EVER**. I thought the assignment was difficult, mainly because I had no clue how to start, and spent a lot of time getting used to Python syntax. Friday office hours with Mike were very helpful. Talking to a person is infinitely better to learn a new language than looking things up on-line yourself.

I have previous coding experience in MATLAB, R, and Mathematica, and I found Python to be very different from what I know. From memory storage to data structures, it seems that everything is slightly different from what I know, and I think it will take some getting used to. Primarily, I think how functions work, and the prevalence of loops feels very strange to me. It is hard to put my finger on why I don't like how you write Python functions, but it feels weird. As for loops, it feels like they pop up everywhere in Python, where in MATLAB or Mathematica there are built-in functions to do things. For example, it felt weird to use a for loop to write the list data to a .txt. Numpy feels much more to what I am used to; just use a function to write an array to .txt.

I still think I need a lot of help to understand Python better. For example, I did this whole assignment in Jupyter notebook, and before the end of the class, I would like to learn more about how to run Python from the terminal.

I would comment on the Guido van Rossum article, but the link to the reading on the website returned an "Error 404: Not Found" message when I tried opening it.

4 Edits

You can use the Lissajous figure shapes to "tune" ϕ. The "wider" the shape is, the further one is from $\phi = n\pi$. The direction of the shape can tell you which direction high or low of $n\pi$ that ϕ is.

The apparent period seems to be $\frac{1}{\omega_1 - \omega_2}$, which is half of what it should be. this is because the carrier sinusoid has a period which spans two of the large "humps" shown in figure 7. Giving a period of $\frac{2}{\omega_1 - \omega_2}$, which matches the frequency we expect.