

Hàm ghép tron

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BÀI TOÁN

- Cho bộ điểm $(x_i, y_i)_{i=\overline{1,n}}$

- Tìm hàm ghép

$$S(x) = S_k(x), x \in [x_k, x_{k+1}]$$

- thoả mãn $S_k(x)$ là đa thức bậc p , $S(x)$ trơn đến cấp $p - 1$.

Spline tuyến tính

- Cho bộ điểm $(x_i, y_i)_{i=\overline{1,n}}$
- Tìm hàm ghép

$$S(x) = S_k(x), x \in [x_k, x_{k+1}]$$

- $\deg S_k(x) = 1$

$$S_k(x_k) = y_k = S_{k+1}(x_k)$$

Spline tuyến tính

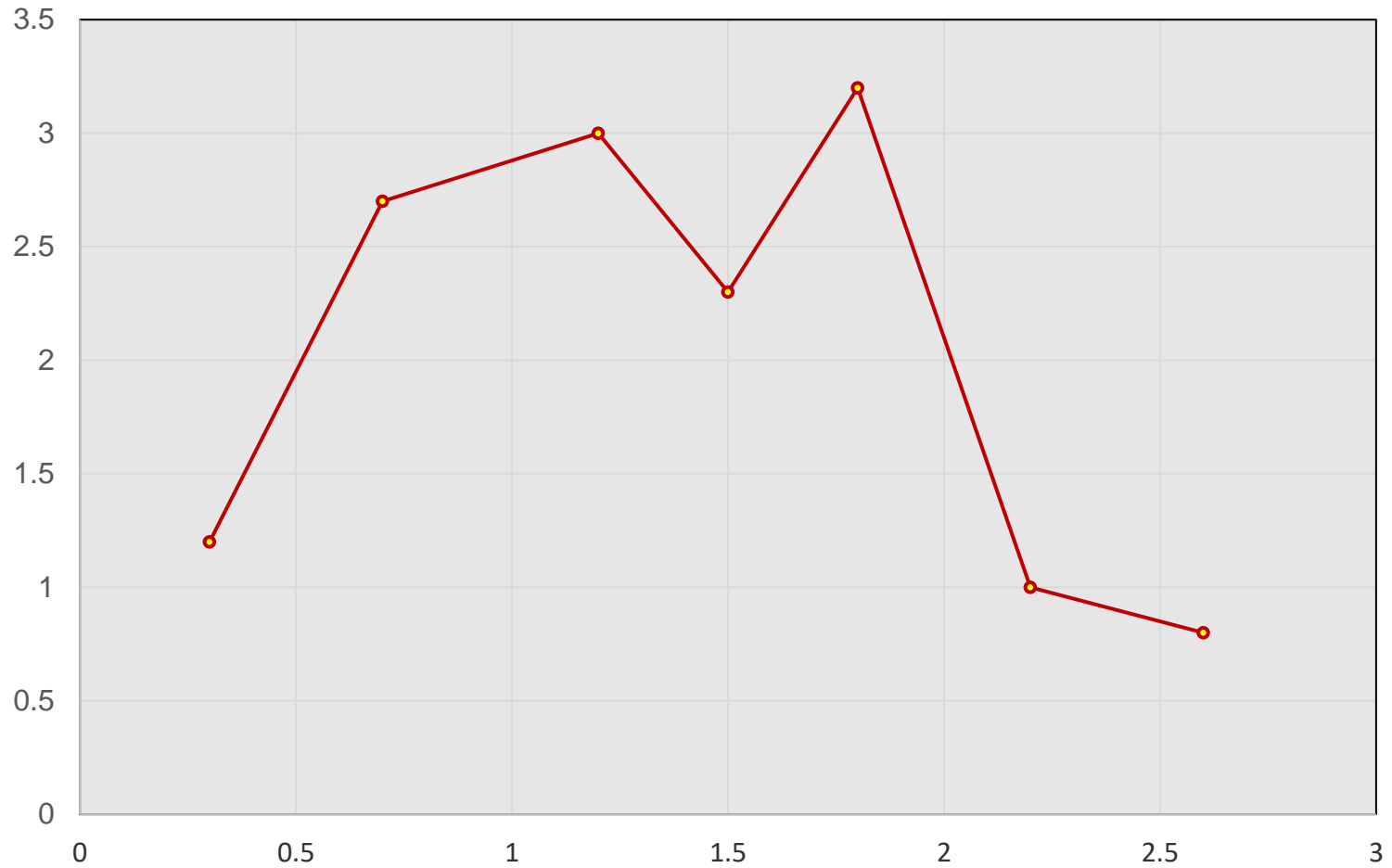
- $S_k(x), x \in [x_k, x_{k+1}]$ là đa thức nội suy bậc 1 với hai mốc nội suy:

$$(x_k, y_k), (x_{k+1}, y_{k+1})$$

$$S_k(x) = \frac{-y_k(x - x_{k+1}) + y_{k+1}(x - x_k)}{h_k} = a_k x + b_k$$

$$a_k = \frac{y_{k+1} - y_k}{h_k}; \quad b_k = \frac{y_k x_{k+1} - y_{k+1} x_k}{h_k}$$

Spline tuyến tính



Spline cấp 2

- Tìm hàm ghép $S(x) = S_k(x), x \in [x_k, x_{k+1}]$

$$\deg S_k(x) = 2$$

$$S_k(x_k) = y_k = S_{k+1}(x_k)$$

$$S'_k(x_k) = S'_{k+1}(x_k)$$

Spline cấp 2

- Đặt $m_k = S'_k(x_k) = S'_{k+1}(x_k)$

- Khi đó

$$S'_k(x) = \frac{-m_k(x - x_{k+1}) + m_{k+1}(x - x_k)}{h_k}$$

$$S_k(x) = \frac{-m_k(x - x_{k+1})^2 + m_{k+1}(x - x_k)^2}{2h_k} + \theta_k$$

$$S_k(x_k) = \frac{-m_k h_k}{2} + \theta_k = y_k \Leftrightarrow \theta_k = y_k + \frac{m_k h_k}{2}$$

$$S_k(x_{k+1}) = y_{k+1} \Leftrightarrow \boxed{m_{k+1} + m_k = 2 \frac{y_{k+1} - y_k}{h_k} = \gamma_1}$$

Spline cấp 2

- Ta thu được hệ $n-1$ phương trình, n ẩn

$$\begin{bmatrix} \color{red}{1} & \color{blue}{1} & 0 & & & & \\ 0 & \color{red}{1} & \color{blue}{1} & 0 & & & \\ & 0 & \color{red}{1} & \color{blue}{1} & 0 & & \\ & & 0 & \color{red}{1} & \color{blue}{1} & 0 & \\ & & & \ddots & \color{red}{\ddots} & \color{blue}{\ddots} & \ddots \\ & & & & 0 & \color{red}{1} & \color{blue}{1} & 0 \\ & & & & & 0 & \color{red}{1} & \color{blue}{1} \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \vdots \\ \gamma_{n-2} \\ \gamma_{n-1} \end{bmatrix}$$

Spline cấp 2

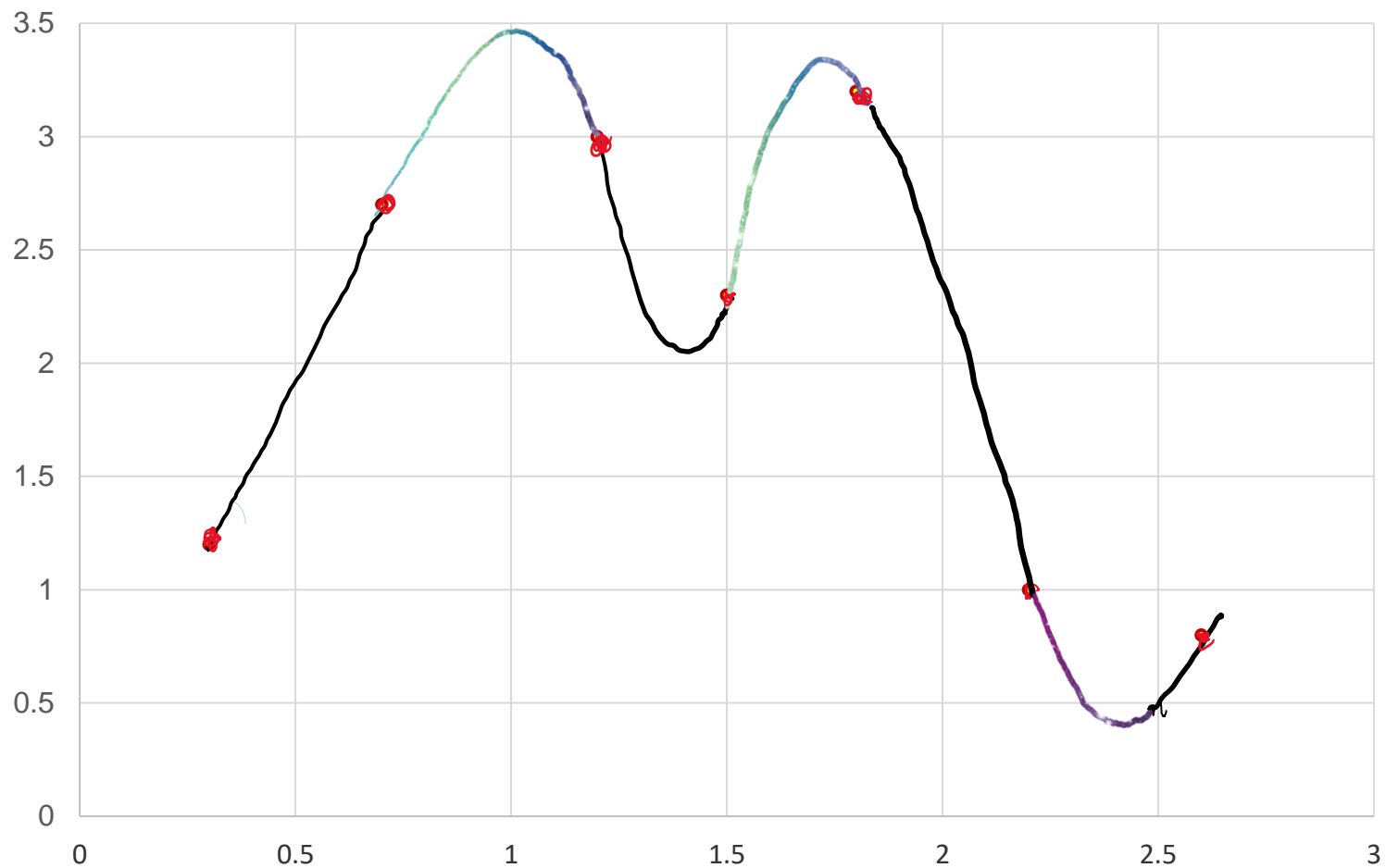
- $S_k(x) = a_k x^2 + b_k x + c_k$

$$a_k = \frac{m_{k+1} - m_k}{2h_k};$$

$$b_k = \frac{m_k x_{k+1} - m_{k+1} x_k}{h_k};$$

$$c_k = \frac{-m_k x_{k+1}^2 + m_{k+1} x_k^2}{2h_k} + y_k + \frac{m_k h_k}{2}$$

Spline tuyến tính



Spline cấp 3

- Tìm hàm ghép $S(x) = S_k(x), x \in [x_k, x_{k+1}]$

$$\deg S_k(x) = 3$$

$$S_k(x_k) = y_k = S_{k+1}(x_k)$$

$$S'_k(x_k) = S'_{k+1}(x_k)$$

$$S''_k(x_k) = S''_{k+1}(x_k)$$

Spline cấp 3

- Đặt
$$\alpha_k = S_k''(x_k) = S_{k+1}''(x_k)$$

- Khi đó

$$S_k''(x) = \frac{-\alpha_k(x - x_{k+1}) + \alpha_{k+1}(x - x_k)}{h_k}$$

$$S_k'(x) = \frac{-\alpha_k(x - x_{k+1})^2 + \alpha_{k+1}(x - x_k)^2}{2h_k} + \beta_k$$

$$S_k(x) = \frac{-\alpha_k(x - x_{k+1})^3 + \alpha_{k+1}(x - x_k)^3}{6h_k} + \beta_k x + \theta_k$$

Spline cấp 3

- $$S_k(x) = \frac{-\alpha_k(x - x_{k+1})^3 + \alpha_{k+1}(x - x_k)^3}{6h_k} + \bar{\beta}_k(x - x_k) - \bar{\theta}_k(x - x_{k+1})$$

$$S_k(x_k) = \frac{\alpha_k h_k^2}{6} + \bar{\theta}_k h_k = y_k \Leftrightarrow \bar{\theta}_k = \frac{y_k}{h_k} - \frac{\alpha_k h_k}{6}$$

$$S_k(x_{k+1}) = \frac{\alpha_{k+1} h_k^2}{6} + \bar{\beta}_k h_k = y_{k+1} \Leftrightarrow \bar{\beta}_k = \frac{y_{k+1}}{h_k} - \frac{\alpha_{k+1} h_k}{6}$$

Spline cấp 3

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$$S'_k(x) = \frac{-\alpha_k(x - x_{k+1})^2 + \alpha_{k+1}(x - x_k)^2}{2h_k} + \frac{y_{k+1}}{h_k} - \frac{\alpha_{k+1}h_k}{6} - \left(\frac{y_k}{h_k} - \frac{\alpha_k h_k}{6} \right)$$

$$S'_{k+1}(x_{k+1}) = \frac{-\alpha_{k+1}h_{k+1}}{2} + \frac{y_{k+2}}{h_{k+1}} - \frac{\alpha_{k+2}h_{k+1}}{6} - \left(\frac{y_{k+1}}{h_{k+1}} - \frac{\alpha_{k+1}h_{k+1}}{6} \right)$$

$$S_k(x_{k+1}) = \frac{\alpha_{k+1}h_k}{2} + \frac{y_{k+1}}{h_k} - \frac{\alpha_{k+1}h_k}{6} - \left(\frac{y_k}{h_k} - \frac{\alpha_k h_k}{6} \right)$$

Spline cấp 3

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$$S'_k(x_{k+1}) - S'_{k+1}(x_{k+1}) = 0$$

$$\Leftrightarrow \frac{h_{k+1}}{6} \alpha_{k+2} + \left(\frac{h_k + h_{k+1}}{3} \right) \alpha_{k+1} + \frac{h_k}{6} \alpha_k = \gamma_k$$

$$\gamma_k = \frac{y_{k+2} - y_{k+1}}{h_{k+1}} - \frac{y_{k+1} - y_k}{h_k}$$

Spline cấp 3

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$$S'_k(x_{k+1}) - S'_{k+1}(x_{k+1}) = 0$$

$$\Leftrightarrow \frac{h_{k+1}}{6} \alpha_{k+2} + \left(\frac{h_k + h_{k+1}}{3} \right) \alpha_{k+1} + \frac{h_k}{6} \alpha_k = \gamma_k$$

$$\gamma_k = \frac{y_{k+2} - y_{k+1}}{h_{k+1}} - \frac{y_{k+1} - y_k}{h_k}$$

Spline cấp 3

Hệ $n-2$ phương trình, n ẩn

$$\begin{bmatrix}
 \frac{h_1}{6} & \frac{h_1 + h_2}{3} & \frac{h_2}{6} & 0 & & & & & & \\
 0 & \frac{h_2}{6} & \frac{h_2 + h_3}{3} & \frac{h_3}{6} & 0 & & & & & \\
 & 0 & \frac{h_3}{6} & \frac{h_3 + h_4}{3} & \frac{h_4}{6} & 0 & & & & \\
 & & \ddots & \ddots & \ddots & \ddots & \ddots & & & \\
 & & & \ddots & \ddots & \ddots & \ddots & \ddots & & \\
 & & & & 0 & \frac{h_{n-3}}{6} & \frac{h_{n-3} + h_{n-2}}{3} & \frac{h_{n-2}}{6} & 0 & \\
 & & & & & 0 & \frac{h_{n-2}}{6} & \frac{h_{n-2} + h_{n-1}}{3} & \frac{h_{n-1}}{6} & \\
 & & & & & & & & &
 \end{bmatrix}
 \begin{bmatrix}
 \gamma_1 \\
 \gamma_2 \\
 \gamma_3 \\
 \vdots \\
 \vdots \\
 \gamma_{n-3} \\
 \gamma_{n-2}
 \end{bmatrix}$$

Spline cấp 3

$$S_k^\bullet(x) = a_k x^3 + b_k x^2 + c_k x + d_k$$

$$a_k = \frac{\alpha_{k+1} - \alpha_k}{6h_k};$$

$$b_k = \frac{3\alpha_k x_{k+1} - 3\alpha_{k+1} x_k}{6h_k};$$

$$c_k = \frac{-3\alpha_k x_{k+1}^2 + 3\alpha_{k+1} x_k^2}{6h_k} + \frac{y_{k+1} - y_k}{h_k} - \frac{(\alpha_{k+1} - \alpha_k)h_k}{6}$$

$$d_k = \frac{\alpha_k x_{k+1}^3 - \alpha_{k+1} x_k^3}{6h_k} + \frac{y_k x_{k+1} - y_{k+1} x_k}{h_k} + \frac{(\alpha_{k+1} x_k - \alpha_k x_{k+1})h_k}{6}$$

Spline tuyến tính

