

Recitation Session 2

1. *Conditioning.* Throw 3 dice one by one, what is the probability that we obtain 3 numbers in strictly increasing order?
2. *Symmetry.* A and B toss a fair coin alternatively where A tosses first, and their outcome is recorded in one sequence of heads and tails. If there is a head followed by a tail (HT), the game ends and the person who tosses the tail win. What is the probability that A wins?
3. *Discrete uniform - binomial.* Suppose there are $N + 1$ boxes labeled by $b = 0, 1, 2, \dots, N$. Box b contains b black and $N - b$ white balls. A box is picked uniformly at random, and then n balls are drawn at random with replacement from whatever box is picked (the same box for each of the n draws). Let S_n denote the total number of black balls that appear among the n balls drawn. Find the distribution of S_n .
4. *Constant default probability.* The probability of observing at least one car on a highway during any 20 *min* interval is $\frac{609}{625}$. Assuming that the probability of seeing a car at any moment is uniform (constant) for the entire 20 *min*, then what is the probability of observing at least one car during any 5 *min* interval. (*Hint:* denote probability of observing a car in any 5-*min* interval to be p)
5. *Quantiles.* Define a Quantile function with respect to a random variable with a continuous and strictly monotonic distribution function:

Definition. Let X be a random variable with distribution function F , and let $p \in [0, 1]$, The quantile function $Q(p)$ returns x such that

$$F_X(x) := P(X \leq x) = p$$

Find the first and third quantile (i.e. $p = \frac{1}{4}$ and $\frac{3}{4}$) for exponential distribution with parameter β .