Recitation Session 8

Problem

1. *Sinusoidal Signal with Random Phase.* Define $X(t) = \alpha\cos(\omega t + \Theta)$ where $t \geq 0$, $\Theta \sim Unif[0, 2\pi]$ and α, ω are constant.
   a. Find CDF and PDF of $X(t)$
   b. Find mean of $X(t)$
2. *Dice Rolling.* You can roll a die 3 times. You win X where X is the last roll you get. After each roll, you decide whether you should continue rolling or stop.
   a. What is the expected value of each roll?
   b. How should you decide if you want to re-roll the die to maximize your winnings?
   c. What is the expected value of your winnings?
3. *Waiting Time.* Suppose $H_i \sim Unif[a, b]$ i.i.d. with $a < b$ and $i = 0, 1, 2...$ is the height of person you observed, sequentially. Let H_0 be your initial observation, and denote T the number of observations (in addition to H_0) it takes to find someone taller. What is $E(T)$? (Hint: (a) The distribution of H_i does not matter (b) Ordered Statistics)
4. *Spaghetti in a Bowl.* Suppose you have a plate of spaghetti, no sauce, where you randomly choose two ends and tie them together until there's no end. Find the expected number of loops given n noddles.