Recitation Session 3

Problem

1. *Calculus.*
   a. Calculate the derivative of $y = (\log(x))^{\log(x)}$
   b. Calculate $\int_0^\infty e^{\frac{-x^2}{2}} dx$
2. *Change of variables.*
   a. Let X be uniformly distributed over $[0, 1]$, and $Y = -\lambda^{-1} \log(X)$ where λ is positive. Find the distribution of Y
   b. Let $X \sim N(0, 1)$ and $Y = e^X$. Find the pdf of Y
3. *Inverse transform: discrete RV.* Figure 1 shows the cdf of a Binomial(2, 0.5) random variable.
   a. How would you obtain this cdf as a function $g(U)$ from the uniform $(0, 1)$ variable U?
   b. How would you obtain $Y \sim$ Binomial(10, 0.5) from (a)?
   c. How would you simulate it using fair coin flips?

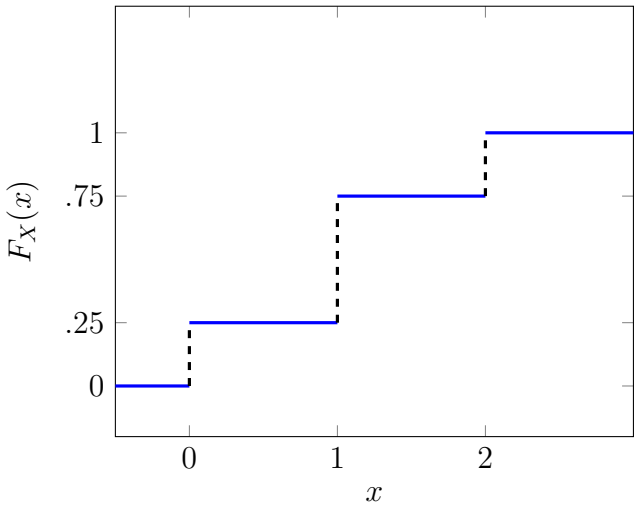

Figure 1: cdf of a binomial (2, 0.5) random variable.

4. *Laplace Distribution.* A continuous random variable X is said to have a Laplace distribution with parameter λ if its pdf is given by:

$$f(x) = A \exp(-\lambda|x|), \quad -\infty < x < \infty$$

for some constant A.

a. Can the parameter λ be negative or zero

b. Compute the constant A in terms of λ

c. Compute the cdf of X

d. For $s, t > 0$, compute $p(X > s + t | X > s)$

e. Compute the pdf of $Y = |X|$