

Recitation Session 12

Problem

1. *Parametric Model Estimation* Let $X_1, \dots, X_n \sim \text{Poisson}(\lambda)$ and $\{X_i\}$ are i.i.d.
 - a. (6 pts) Find the method of moment estimator $\hat{\lambda}_{MM}$ of λ .
 - b. (7 pts) Find the maximum likelihood estimator $\hat{\lambda}_{ML}$ of λ .
2. *More Parametric Model Estimation* Let $X_1, \dots, X_n \sim \text{Unif}(0, \theta)$ where $\theta > 0$ is unknown and $\{X_i\}$ are i.i.d.
 - a. Find the method of moments estimator $\hat{\theta}_{MM}$ of θ .
 - b. Find the MSE, bias, and variance of $\hat{\theta}_{MM}$.
 - c. The maximum likelihood estimator is given by $\hat{\theta}_{ML} = \max_{1 \leq i \leq n} X_i$. Find the MSE, bias, and variance of $\hat{\theta}_{ML}$. You may use the fact that the pdf of $\hat{\theta}_{ML}$ is
$$f_{\hat{\theta}_{ML}}(x) = \frac{n}{\theta^n} x^{n-1}, \quad 0 < x < \theta.$$
 - d. Are $\hat{\theta}_{ML}$ and $\hat{\theta}_{MM}$ consistent estimators of θ ? Which estimator has smaller MSE?
3. *Bayesian Statistics* Suppose $X_1, \dots, X_n \sim \text{Geo}(\theta)$ and we assume the prior distribution of θ is $\text{Beta}(a, b)$.
 - a. Write down the posterior distribution $\pi(\theta|)$. (You don't need to specify the normalization constant).
 - b. Find the Minimum mean-square-error estimation MMSE, $\hat{\theta}_{MMSE, n}$. What is the limit of $\hat{\theta}_{MMSE, n}$ when $n \rightarrow \infty$?
 - c. Find the MAP, $\hat{\theta}_{MAP, n}$.