**Recitation Session 5**

# Problem

1. *Minimum of Random Variables* Given a collection of independent exponentially distributed random variables $\{X_1; X_2; ...; X_n\}$ with parameter $\beta$. Let $Y = min\{X_1; X_2; ...; X_n\}$. Find the distribution of $Y$ .

2. *Independent Random Variables.* If the joint density function of $X$ and $Y$ is

$$f(x, y) = 6e^{-2x}e^{-3y}, \quad 0 < x < \infty,\ 0 < y < \infty$$

and is equal to 0 outside this region, are the random variables independent? What if the joint density function is

$$f(x, y) = 24xy, \quad 0 < x < 1,\ 0 < y < 1,\ 0 < x + y < 1$$

3. *Conditional Independence.* There is a disease which affects 2% of the population. A medical test is available which gives a false positive rate of 5% and a mis-detection rate of 3%. Compute the false discovery rate and the false omission rate of using this test.

4. *Generating Random Variables* Generate random numbers: suppose you are able to generate uniform random variables over $\{0, 1\}$. How to generate a sequence of independent uniform random variables over $\{0, 1, \cdots, 12\}$? Propose your algorithm and briefly explain your answer (You don't have to justify it rigorously).

# Extra Problem

1. *Transformations* Let $(X, Y)$ be uniformly distributed on the unit disk $\{(x, y) : x^2 + y^2 \leq 1\}$, i.e.,

$$f(x, y) = \begin{cases} \frac{1}{\pi}, & \text{if } x^2 + y^2 \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $R = \sqrt{X^2 + Y^2}$. Find the cdf and pdf of $R$.

2. *Generating Random Variables* We investigate how to generate discrete random variables given the pmf. For simplicity, we assume a random variable $X$ has its pmf as:

$$\Pr(X = k) = p_k > 0, \qquad k = 0, 1, 2, \cdots ,$$

and

$$\sum_{k=0}^{\infty} p_k = 1.$$

a. Write down the cdf of $X$ by using the partial sum $S_n = \sum_{k=0}^{n} p_k$.

b. Consider the generalized inverse of the cdf.

$$F_X^{-1}(x) := \inf\{t : F_X(t) \geq x\}, \quad 0 < x < 1,$$

which is the smallest value $t$ such that $F_X(t)$ is greater than or equal to $x$. Specify the following set:

$$A_n := \{x : F_X^{-1}(x) = n\}, \quad n \in \{0, 1, 2, \cdots\}.$$

Try to use the expression of the cdf obtained from (a). (You may use Geo(1/2) as a concrete example to help you think. Sketching the cdf is very helpful).

c. Suppose $U$ is a Unif[0,1] random variable. What is $\Pr(U \in A_n)$?

d. Can you provide a random number generator for Poisson($\lambda$)? You are encouraged to implement the algorithm but not required