Recitation Session 6

Problem

1. *Markov's inequality* Suppose we know that the number of items produced in a factory during a week is a random variable with mean 500.
   a. What can be said about the probability that this week's production will be at least 1000? (Hint: Markov's inequality)
   b. If the variance of a week's product is known to equal 100, what is the probability that this week's production will be between 400 and 600?
2. *Mean and Variance.* Let $X \sim \text{Exp}(\beta)$. Use the definition to compute the following quantities.
   a. Compute $\text{E}(X)$. (Hint: use u-substitution).
   b. Compute $\text{Var}(X)$. (Hint: use Gamma function in homework 2 or integration by parts).
3. *Expected value and variance of sample average* Suppose $\{X_1, \cdots, X_n\}$ are n independent identically distributed (they are of the same distribution) random variables with finite expectation μ, variance σ^2, and cdf $F_X(x)$. Denote the sample average as

$$\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i.$$

   a. Compute the expectation of $\overline{X}_n$.
   b. Compute the variance of $\overline{X}_n$.
4. *Generating Random Variables* Every package of some intrinsically dull commodity includes a small and exciting plastic object. There are n different types of object, and each package is equally likely to contain any given type. You buy one package each day.
   a. Let X_k be the number of days which elapse between the acquisitions of the $(k-1)$-th new type of object and the k-th new type. What is the distribution of X_k, $1 \leq k \leq n$? (Hint: In particular, we have $X_1 = 1$. The distribution of X_k should depend on k).
   b. What is the mean of X_k? (Hint: use the result in (a).)
   c. Let X be the number of days which elapse before you have a full set of objects. What is the expectation of X?