Exploratory Data Analysis

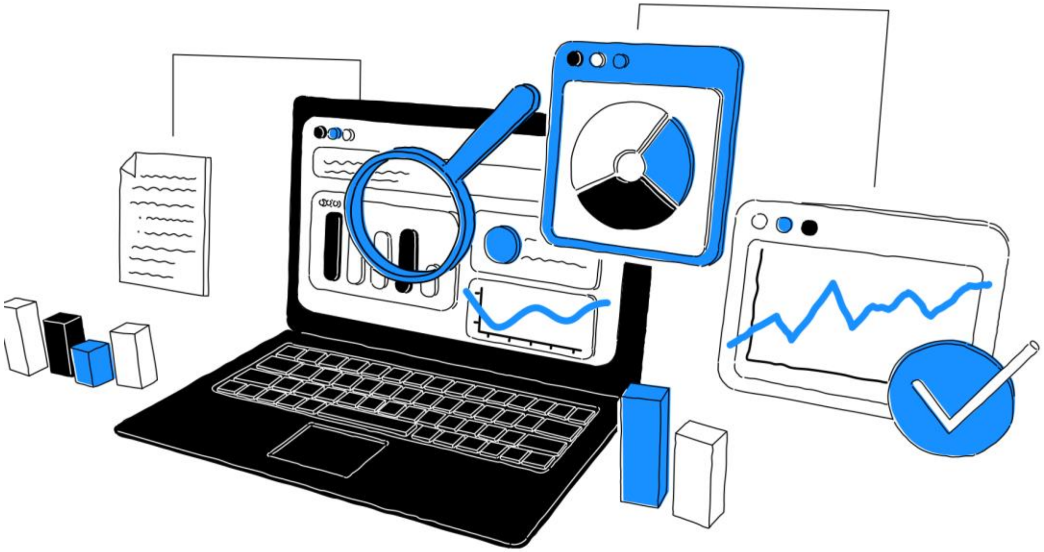
Nguyen Ngoc Thao
nnthao@fit.hcmus.edu.vn

Content outline

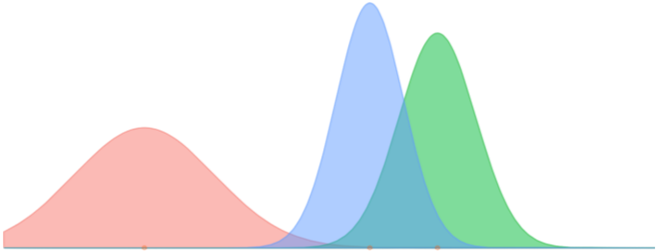
- Data objects and Attributes
- Basic statistical data descriptions
- Basic data visualization
- Data proximity measures

Exploratory data analysis (EDA)

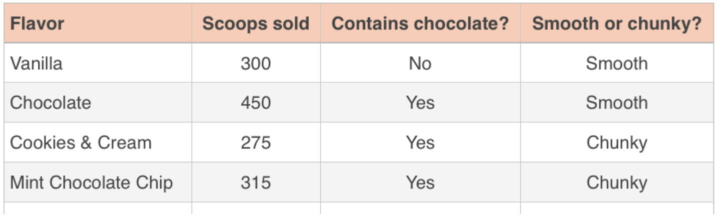
- EDA analyzes data to summarize their main characteristics, using statistical graphics and data visualization methods.
- It is used by data scientists to determine how best to handle data sources to get the answers they need,



Exploratory data analysis (EDA)

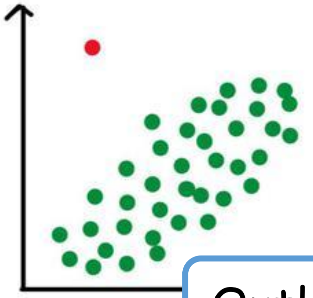


Data distribution

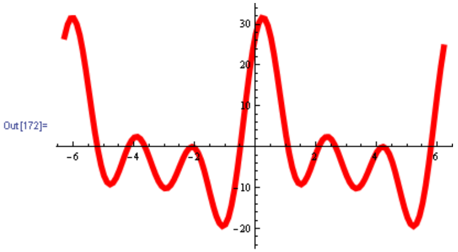


Flavor	Scoops sold	Contains chocolate?	Smooth or chunky?
Vanilla	300	No	Smooth
Chocolate	450	Yes	Smooth
Cookies & Cream	275	Yes	Chunky
Mint Chocolate Chip	315	Yes	Chunky

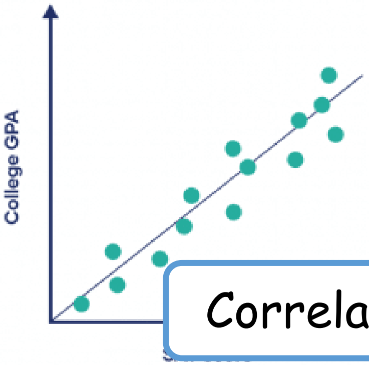
Data preprocessing



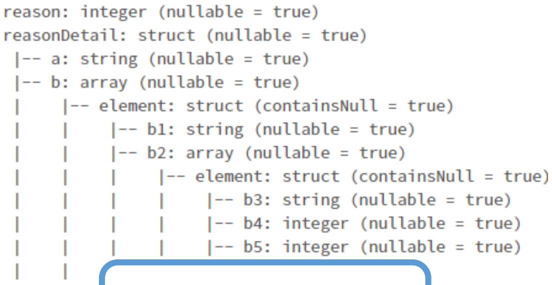
Outliers



Pattern discovery

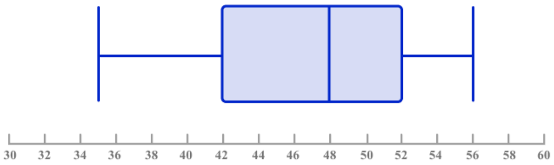


Correlation



```
reason: integer (nullable = true)
reasonDetail: struct (nullable = true)
 |-- a: string (nullable = true)
 |-- b: array (nullable = true)
 |    |-- element: struct (containsNull = true)
 |    |    |-- b1: string (nullable = true)
 |    |    |-- b2: array (nullable = true)
 |    |    |    |-- element: struct (containsNull = true)
 |    |    |    |    |-- b3: string (nullable = true)
 |    |    |    |    |-- b4: integer (nullable = true)
 |    |    |    |    |-- b5: integer (nullable = true)
 |    |
```

Data types



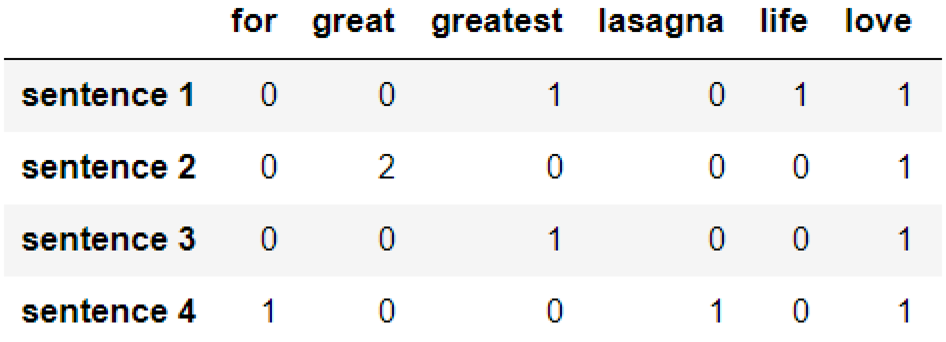
Data visualization

Data quality

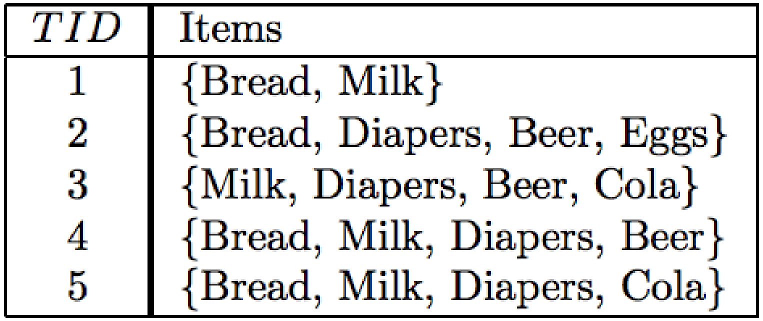
Data objects and Attributes

Data collection: Record datasets

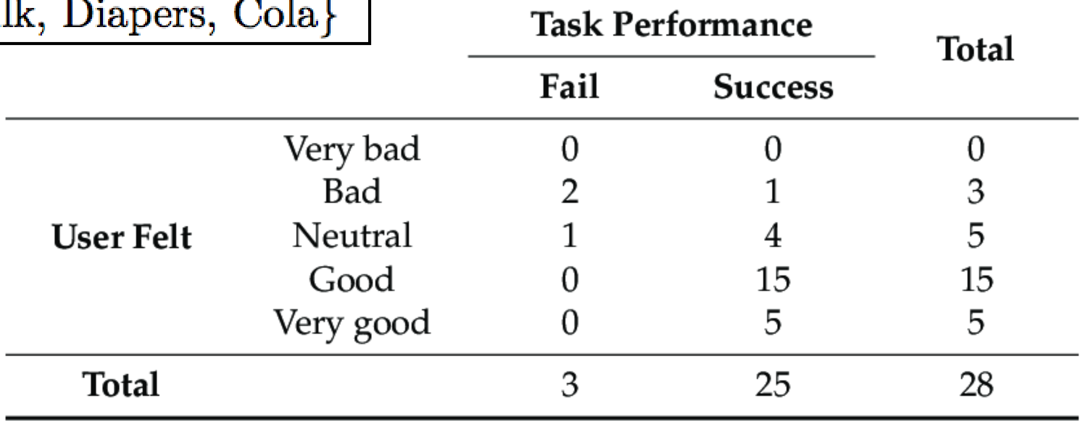
- Relational / transactional tuples
- Term-frequency vectors, numerical matrices, crosstabs



	for	great	greatest	lasagna	life	love
sentence 1	0	0	1	0	1	1
sentence 2	0	2	0	0	0	1
sentence 3	0	0	1	0	0	1
sentence 4	1	0	0	1	0	1



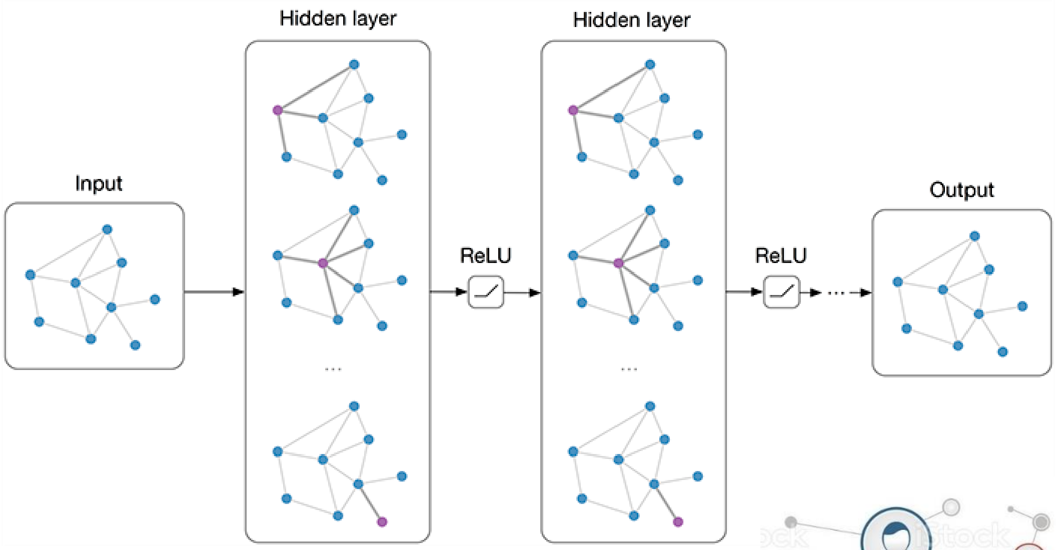
TID	Items
1	{Bread, Milk}
2	{Bread, Diapers, Beer, Eggs}
3	{Milk, Diapers, Beer, Cola}
4	{Bread, Milk, Diapers, Beer}
5	{Bread, Milk, Diapers, Cola}

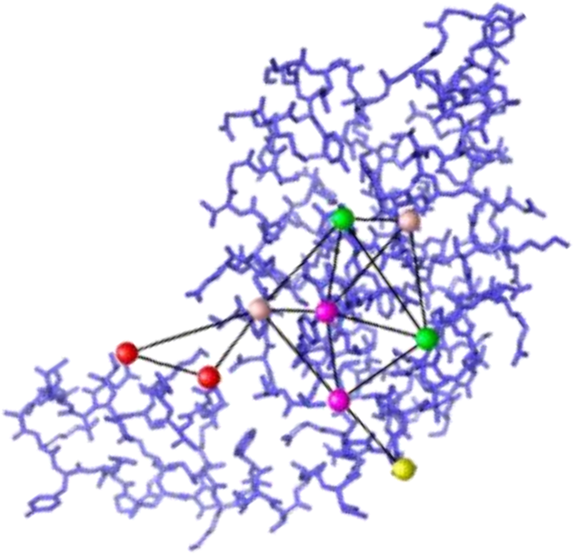


		Task Performance		**Total**
		Fail	**Success**	
User Felt	Very bad	0	0	0
	Bad	2	1	3
	Neutral	1	4	5
	Good	0	15	15
	Very good	0	5	5
Total		3	25	28

Data collection: Graph datasets

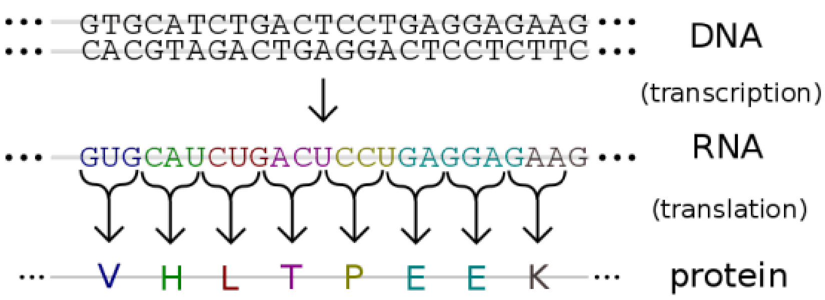
- The Internet, social networks, molecular structures



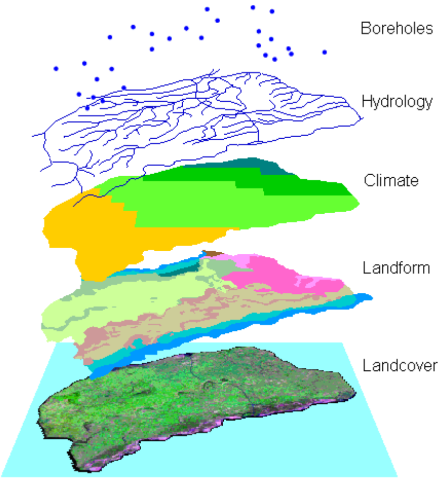


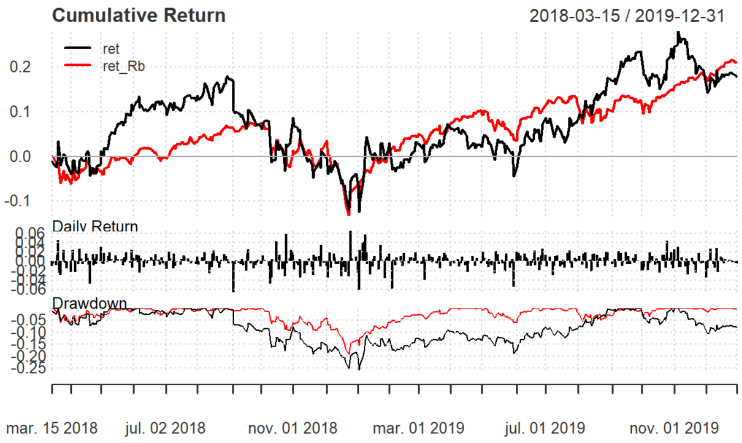
Data collection: Ordered datasets

- Sequential data: transaction sequences, genetic sequences
- Video data, temporal data, time-series data, etc.



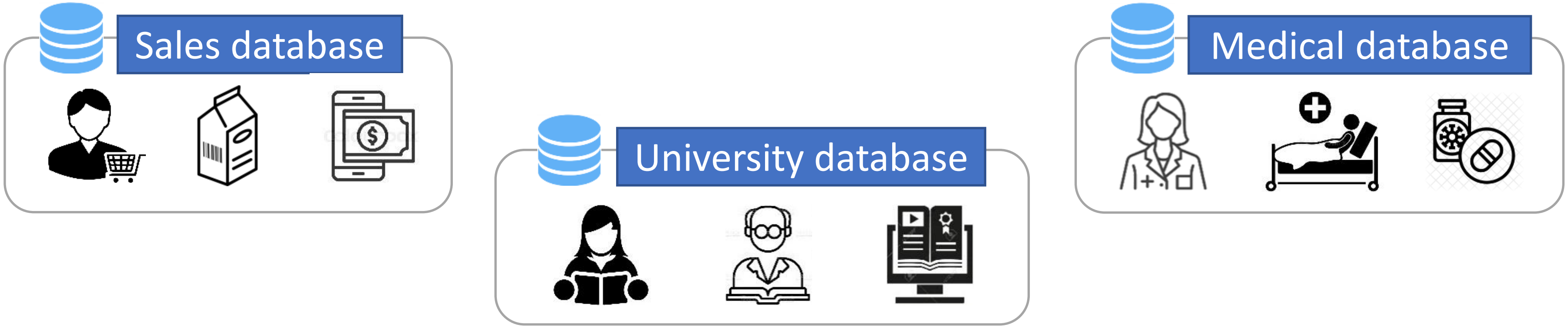
… GTGCATCTGACTCCTGAGGAGAAG …
… CACGTAGACTGAGGACTCCTCTTC …
DNA
(transcription)
… GUGCAUCUGACUCCUGAGGAGAAG …
RNA
(translation)
… V H L T P E E K …
protein





Data objects

- A data object depicts an entity, serving as the building block for a dataset.
  - Similar terms: sample, example, instance, data point, and tuple



Sales database

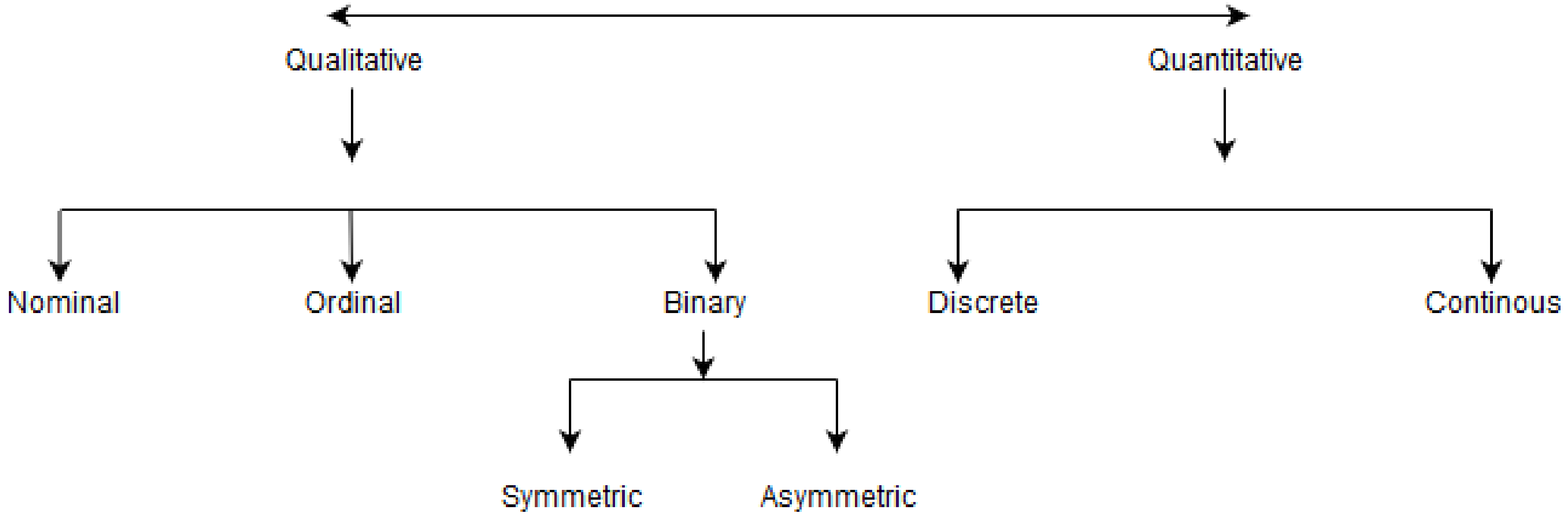
University database

Medical database

- Data objects are described by attributes.
  - In a database: rows → data objects, columns → attributes

Attributes

- An attribute shows some characteristic of a data object.
    - Similar terms: dimension, feature, and variable
    - E.g., a Customer object has 3 attributes {id, name, address}
- Observation: an observed value for a given attribute
- Feature vector: a set of attributes used to describe an object

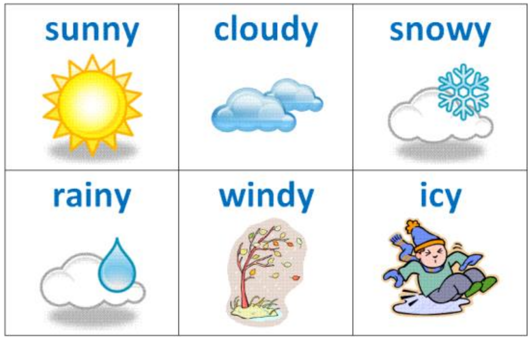


Attribute types: Nominal

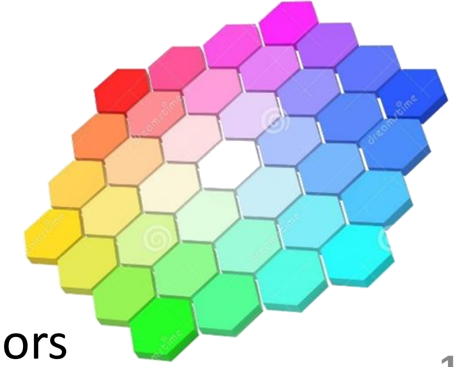
- Qualitative, values do not have any meaningful order
- Enumerations: categories, states, or “names of things”

Day and Night

Occupation



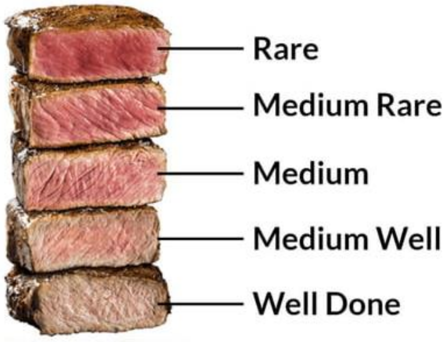
Weather

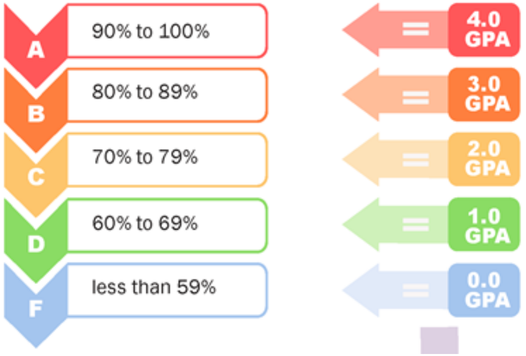


Colors

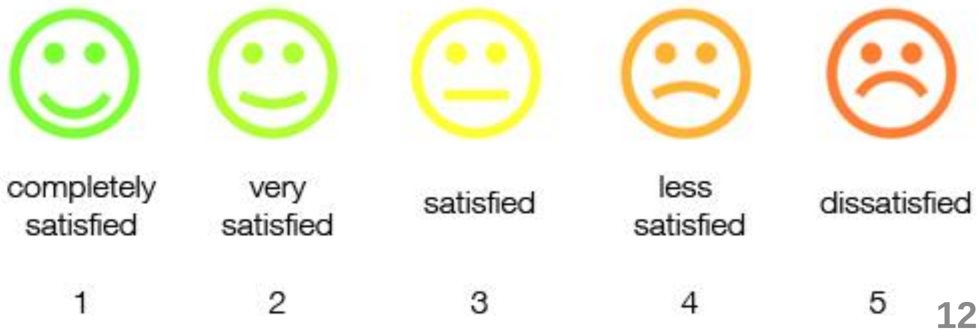
Attribute types: Ordinal

- Qualitative, values have a meaningful order (ranking) but magnitude between successive values is not known





- Useful for subjective assessments of qualities that cannot be measured objectively
  - E.g., customer satisfaction

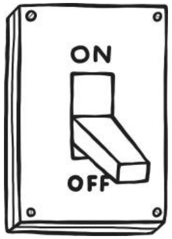


Attribute types: Binary

- Nominal attribute with only 2 states
- Symmetric binary: both outcomes equally important

Day and night

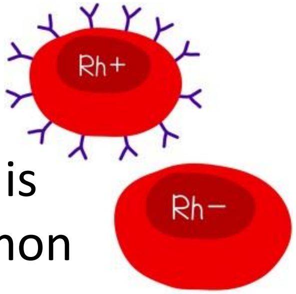
Male and Female



Switch light
On and Off

- Asymmetric binary: outcomes not equally important
  - Convention: assign 1 to the most important outcome (e.g., HIV test)

Rh positive is
more common



A positive result is more significant

Attribute types: Numeric

Interval numeric attribute

- Measured on a scale of **equal-sized units**
- Values have order (e.g., temperature in C˚ or F˚, calendar dates)
- No true **zero-point**: able to compute the difference – not able to talk of one value as being a multiple of another
  - E.g., 20˚C is five degrees higher than 15˚C (right), 10˚C is twice as warm as 5˚C (wrong)

Ratio numeric attribute

- Inherent **zero-point**
- Values can be considered as being an order of magnitude larger than the unit of measurement
  - E.g., temperature (10˚K is twice as high as 5˚K), monetary (you are 100 times richer with $100 than with $1), measurements (height, weight)

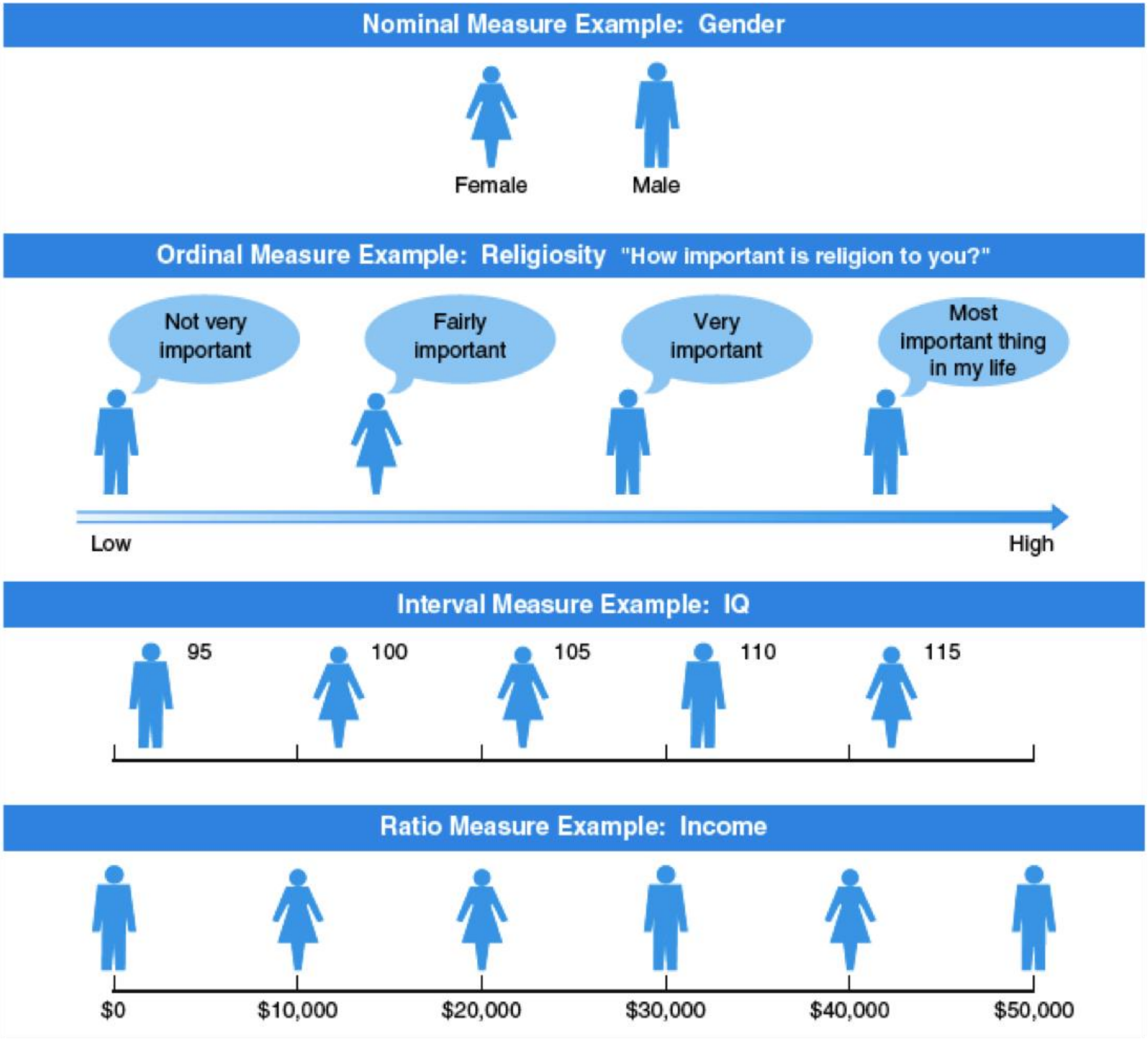

Nominal Measure Example: Gender
Female
Male
Ordinal Measure Example: Religiosity "How important is religion to you?"
Not very important
Fairly important
Very important
Most important thing in my life
Low
High
Interval Measure Example: IQ
95
100
105
110
115
Ratio Measure Example: Income
$0
$10,000
$20,000
$30,000
$40,000
$50,000

Image credit: Google Sites

Attributes: Discrete vs. Continuous

- There are many ways to organize attribute types, which are not mutually exclusive.
- Discrete attribute
  - Only a finite or countably infinite set of values
  - The values are sometimes represented as integers.
  - Binary attributes are a special case of discrete attributes.
- Continuous attribute
  - Real numbers of continuous domains
  - The values are usually represented using a finite number of digits

    → floating-point variables

Quiz 01: Data types

1. For each of the following data types, given an example.
   - Nominal data vs. Ordinal data.
   - Symmetric binary data vs. Asymmetric binary data
   - Interval numeric data vs. Ratio numeric data

2. How to check the data type of a variable in **Python**?
3. How to check the schema of a **pandas** Dataframe?

Basic statistical data descriptions

Central tendency: Arithmetic mean

- Let $x_1, x_2, \dots, x_N$ be a set of N values or observations for some numeric attribute X.
- The arithmetic mean is defined as $\boldsymbol{\mu = \frac{1}{N}\sum_{i=1}^{N} x_i}$
- The weighted arithmetic mean is written as $\mu^w = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$
  - where w_i is the weight value that associates with x_i.
- It is the most common and effective numeric measure

Central tendency: Arithmetic mean

- Consider the score records of John and Kelly.
- The (non-weighted) mean scores are

$$\mu_{John} = 82.6, \qquad \mu_{Kelly} = 84.6$$

John's record	
Homework	92
Quiz	74
Lab	83
Test	76
Final exam	88

Kelly's record	
Homework	100
Quiz	82
Lab	95
Test	70
Final exam	76

Homework	15 %
Quiz	10 %
Lab	20 %
Test	25 %
Final exam	30 %

- We now have the course grade distribution.
- The weighted mean scores are

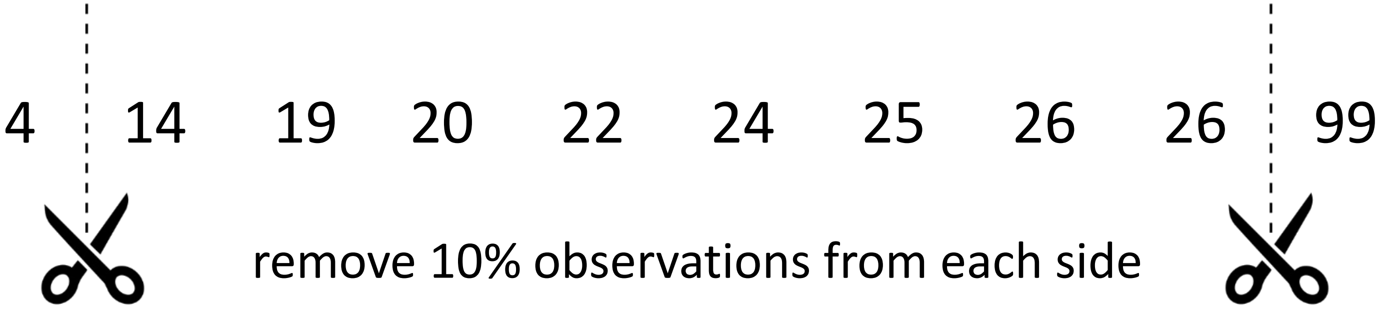
$$\mu^{w}_{John} = 83.2, \qquad \mu^{w}_{Kelly} = 82.5$$

$$\mu^{w}_{John} = \frac{0.15 \times 92 + 0.1 \times 74 + 0.2 \times 83 + 0.25 \times 76 + 0.3 \times 88}{0.15 + 0.1 + 0.2 + 0.25 + 0.3} = 83.2$$

Central tendency: Arithmetic mean

- Means are highly sensitive to extreme values (e.g., outlier).
- Trimmed mean: chop extreme values before calculating the regular mean

Typical mean: 27.9



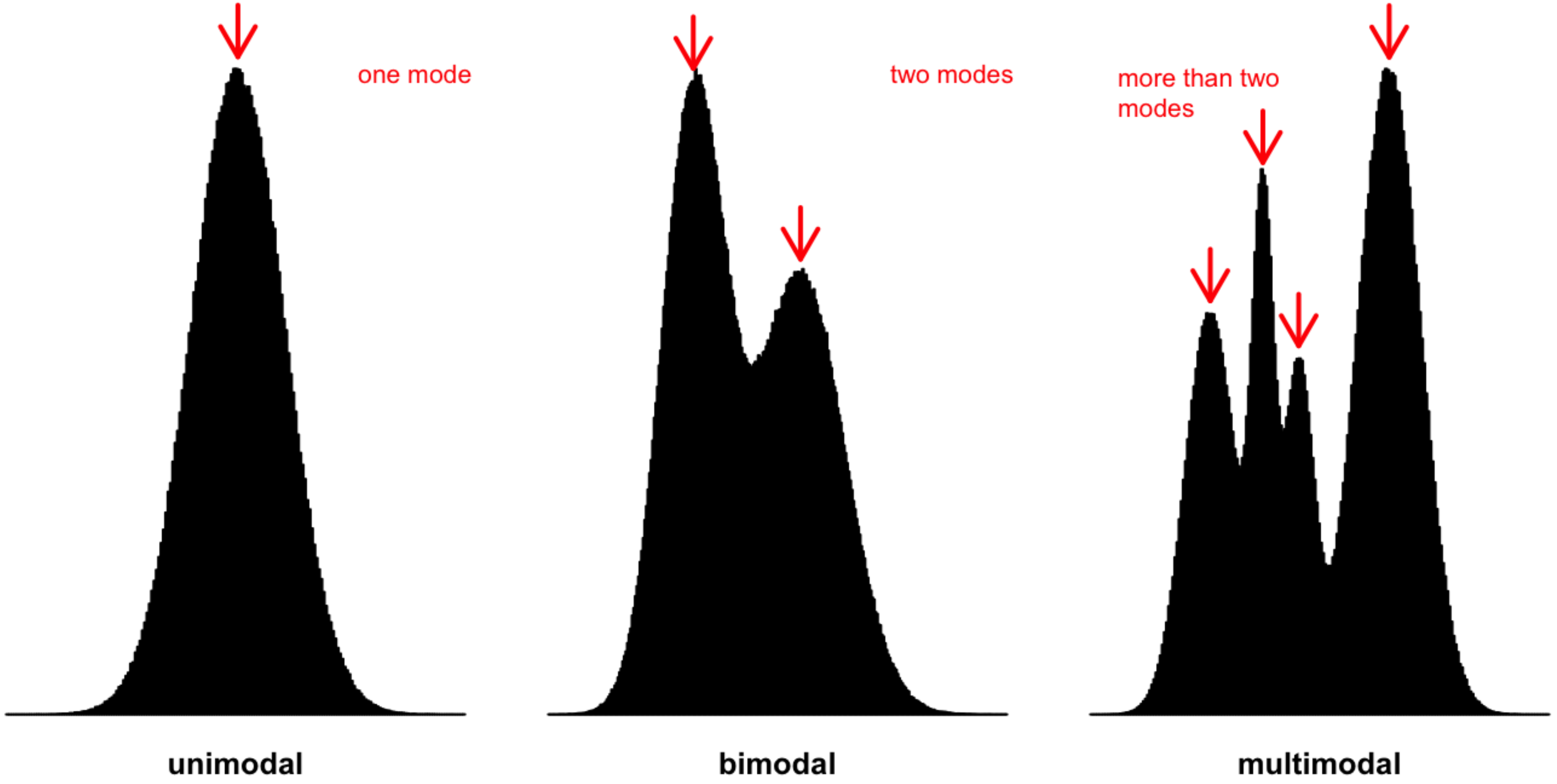
4 14 19 20 22 24 25 26 26 99

remove 10% observations from each side

Trimmed mean: 22

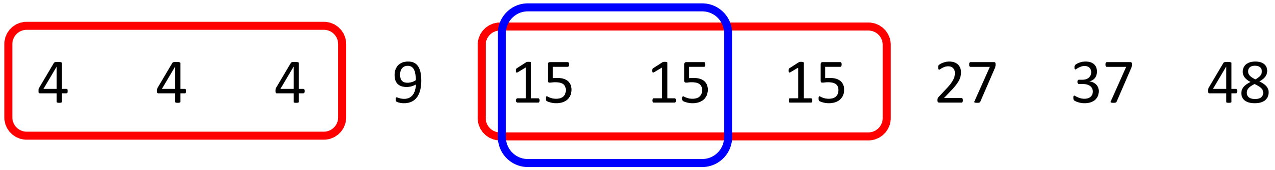
Central tendency: Mode

- Mode is the value that occurs most frequently in the data, defined for both qualitative and quantitative attributes.
  - If each data value occurs only once, then there is no mode



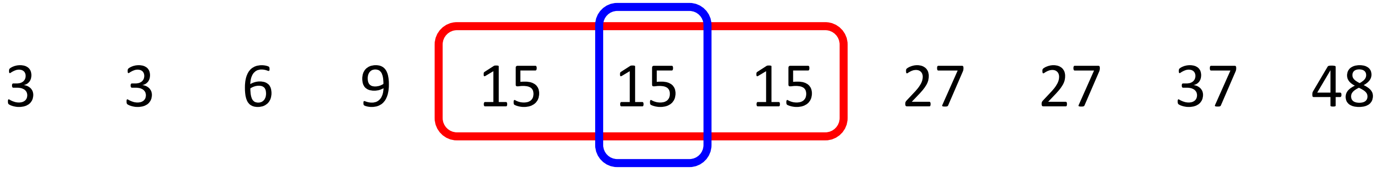
Central tendency: Median

- Suppose that the given set of N observations is sorted.
- Median is the middle value of the ordered set.
  - N is **odd**: pick the *exact middle value*; otherwise, take the *average of the two middlemost values*.
- Midrange is the average of the largest and smallest values in the set.



4 4 4 9 15 15 15 27 37 48

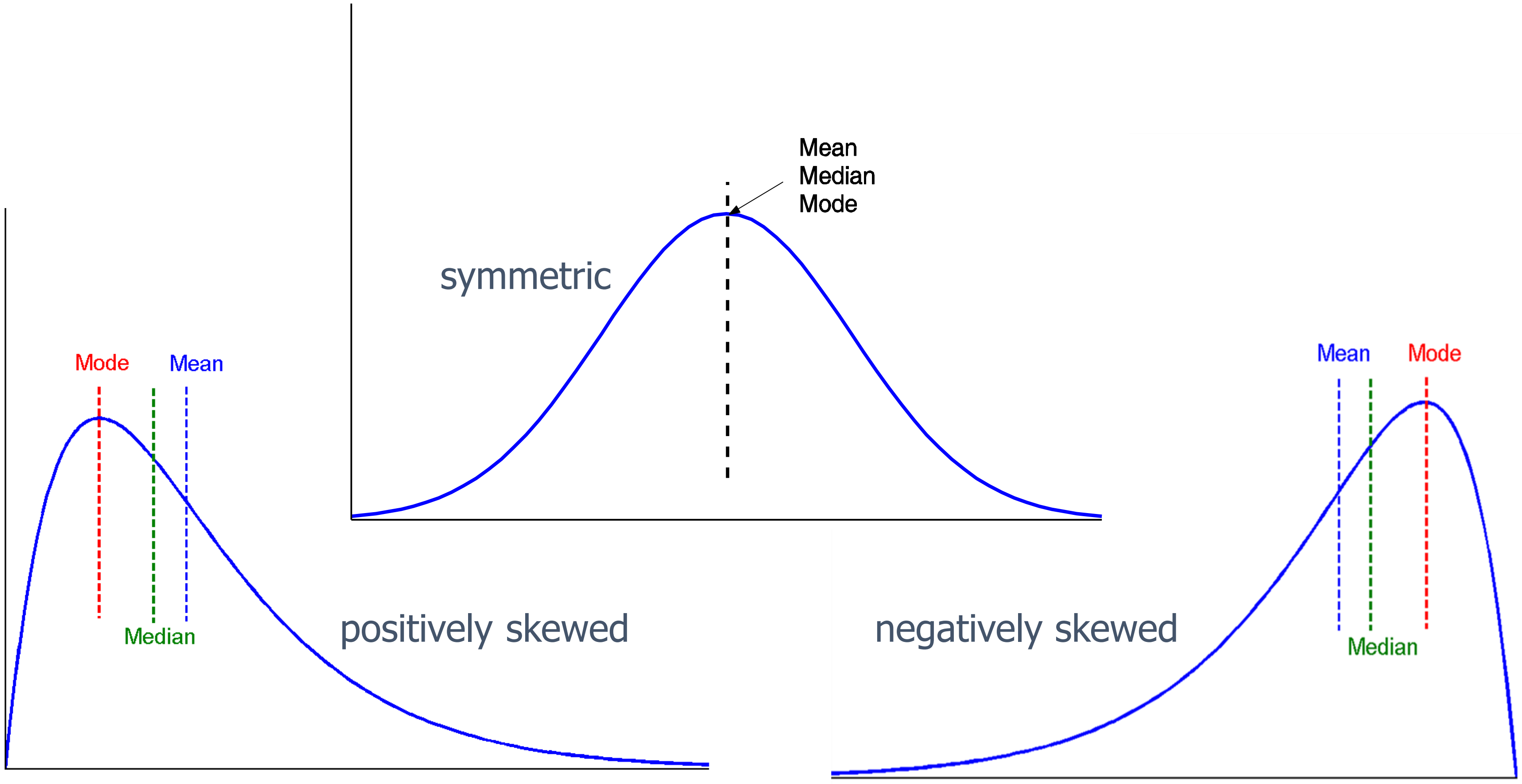
mean = 17.8 – mode: 4 and 15 – midrange = 26, median = (15+15)/2 = 15



3 3 6 9 15 15 15 27 27 37 48

mean = 18.636 – mode: 15 – midrange = 25.5, median = 15

Symmetric data vs. Skew data



- For moderately skewed unimodal numeric data, the empirical formula is

$$mean - mode \approx 3 \times (mean - median)$$

Quiz 02: Mean, mode, and Midrange

1. Consider the following 1D data series, which includes 13 data points.

   31, 40, 19, 45, 5, 18, 30, 5, 33, 33, 25, 5, 20

   Compute the following values: arithmetic mean, midrange, median, and mode.

2. For each of the following values, identify whether **pandas** supports and, if yes, by which function.

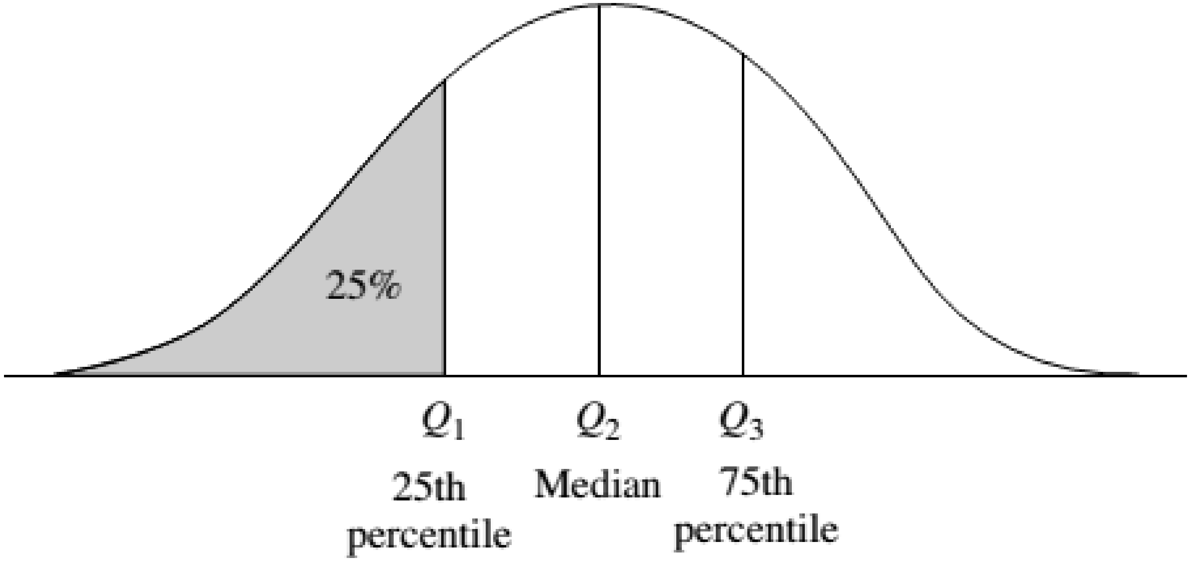
   - Arithmetic mean
   - Weighted arithmetic mean
   - Median, mode
   - Midrange

Data dispersion: Quantiles

- Let $x_1, x_2, \dots, x_N$ be a set of N observations sorted in increasing order for a numeric attribute X.
- Quantiles are points taken at regular intervals of a data distribution, dividing it into equal-sized consecutive sets.
- *k*th q-quantile $(0 < \mathrm{k} < q,\ k \in \mathbb{N}^*)$: a value x such that at most k/q data values $< x$ and at most $(q - k)/q$ of which $> x$.
    - There are $q - 1$ q-quantiles.

Data dispersion: Quantiles

- Quartiles (4-quantiles) split the data distribution into four equal parts.



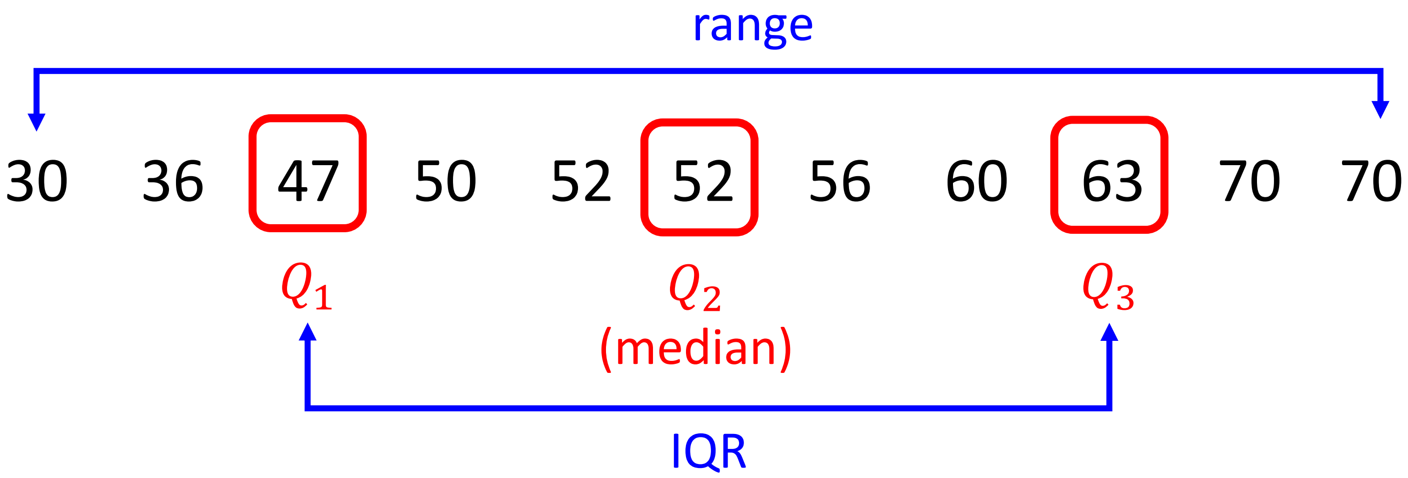
- Percentiles (100-quantiles): 100 equal-sized consecutive sets.
- 2-quantile is the median that splits the distribution into halves.

Data dispersion: Interquartile range

- Interquartile range (IQR) is the distance between the first and third quartiles.

$$IQR = Q_3 - Q_1$$

- Range is the difference between the largest and smallest values in the set.



range

30 36 47 50 52 52 56 60 63 70 70

Q_1

Q_2

(median)

Q_3

IQR

How to determine the quartile?

- Use the median to divide the ordered set into two halves.
  - If the original set has an even number of points, split it exactly in half
  - Otherwise, **do not include** the median in either half.
- Q_1 and Q_3 are the medians of the lower and upper halves, respectively.

$$6 \quad 7 \quad \underset{Q_1}{\boxed{15}} \quad 36 \quad 39 \quad \underset{Q_2}{\boxed{40}} \quad 41 \quad 42 \quad \underset{Q_3}{\boxed{43}} \quad 47 \quad 49$$

$$7 \quad \underset{Q_1}{\boxed{15}} \quad \underset{Q_2 = 37.5}{\boxed{36 \quad 39}} \quad \underset{Q_3}{\boxed{40}} \quad 41$$

Quiz 03: Quantiles

- You are given the following dataset representing the scores of 15 students in a math exam, already sorted.

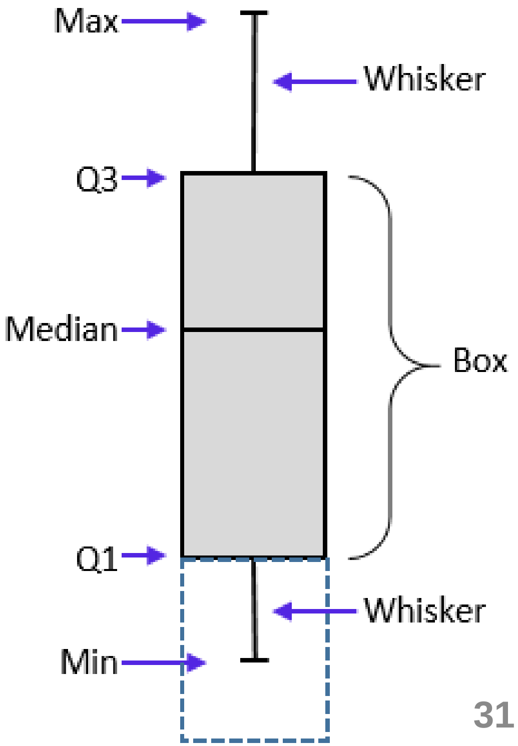
  45, 48, 52, 55, 62, 67, 70, 72, 75, 77, 80, 85, 87, 90, 95

- Compute the first quartile (Q1), second quartile (Q2), and third quartile (Q3), and IQR.

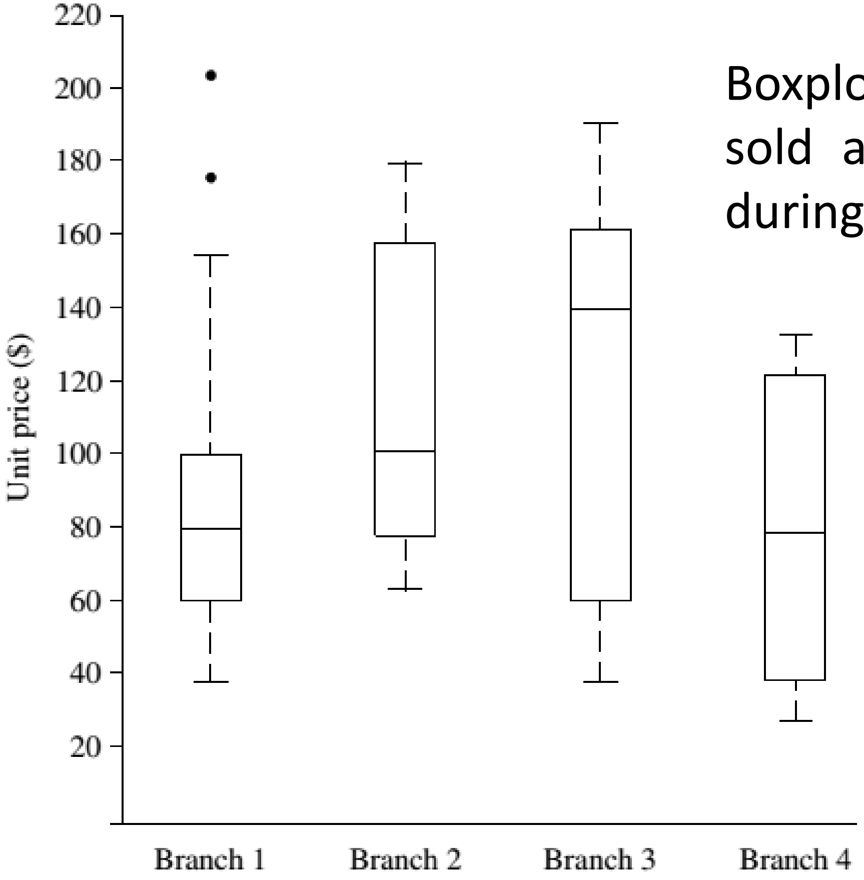
Data dispersion: Boxplot

- A five-number summary of a distribution includes
  - The median ($\boldsymbol{Q_2}$), the quartiles $\boldsymbol{Q_1}$ and $\boldsymbol{Q_3}$,
  - The smallest ($\boldsymbol{Min}$) and largest ($\boldsymbol{Max}$) individual values.

- The summary is presented by a boxplot.
  - Outliers: points that are out the range $[-1.5 \times IQR, 1.5 \times IQR]$, plotted individually



Data dispersion: Boxplot



Boxplot for the unit price data for items sold at four branches of AllElectronics during a given time period

- For Branch 1, the median price of items sold is $80, Q_1 is $60, and Q_3 is $100. Notice that two outlying observations, 175 and 202, were plotted individually as they are more than 1.5 × IQR.

Quiz 04: Draw a box plot

1. Consider the following 1D data series, which includes 15 data points sorted in ascending order.

   21, 25, 27, 29, 32, 36, 36, 48, 67, 70, 74, 75, 79, 150, 197

   - Define the five-number summary for the above data.
   - Draw the boxplot representing the above five-number summary. Note the vertical axis and all the values.

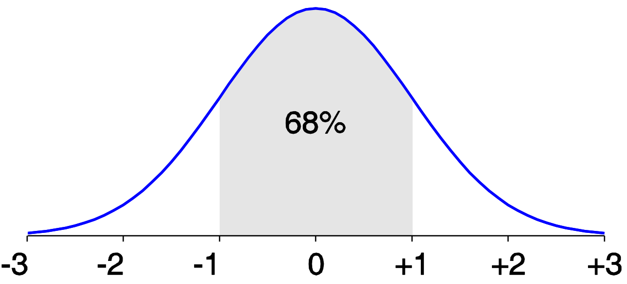
2. How to draw a boxplot in **Python**?
3. Can **scikit-learn** be used to draw a **boxplot**? If yes, how?

Data dispersion: Variance

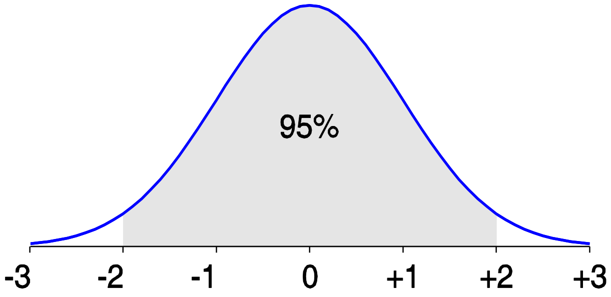
- The (population) variance is defined as

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2 = \left(\frac{1}{N}\sum_{i=1}^{N}{x_i}^2\right) - \bar{x}^2$$

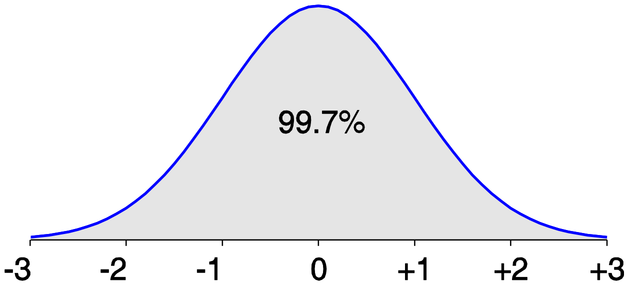
- The standard deviation is the square root of the variance.
  - *Low* $\sigma \rightarrow$ the data tends to be very *close to the mean.*
  - *High* $\sigma \rightarrow$ the data *spreads out over a large range* of values.



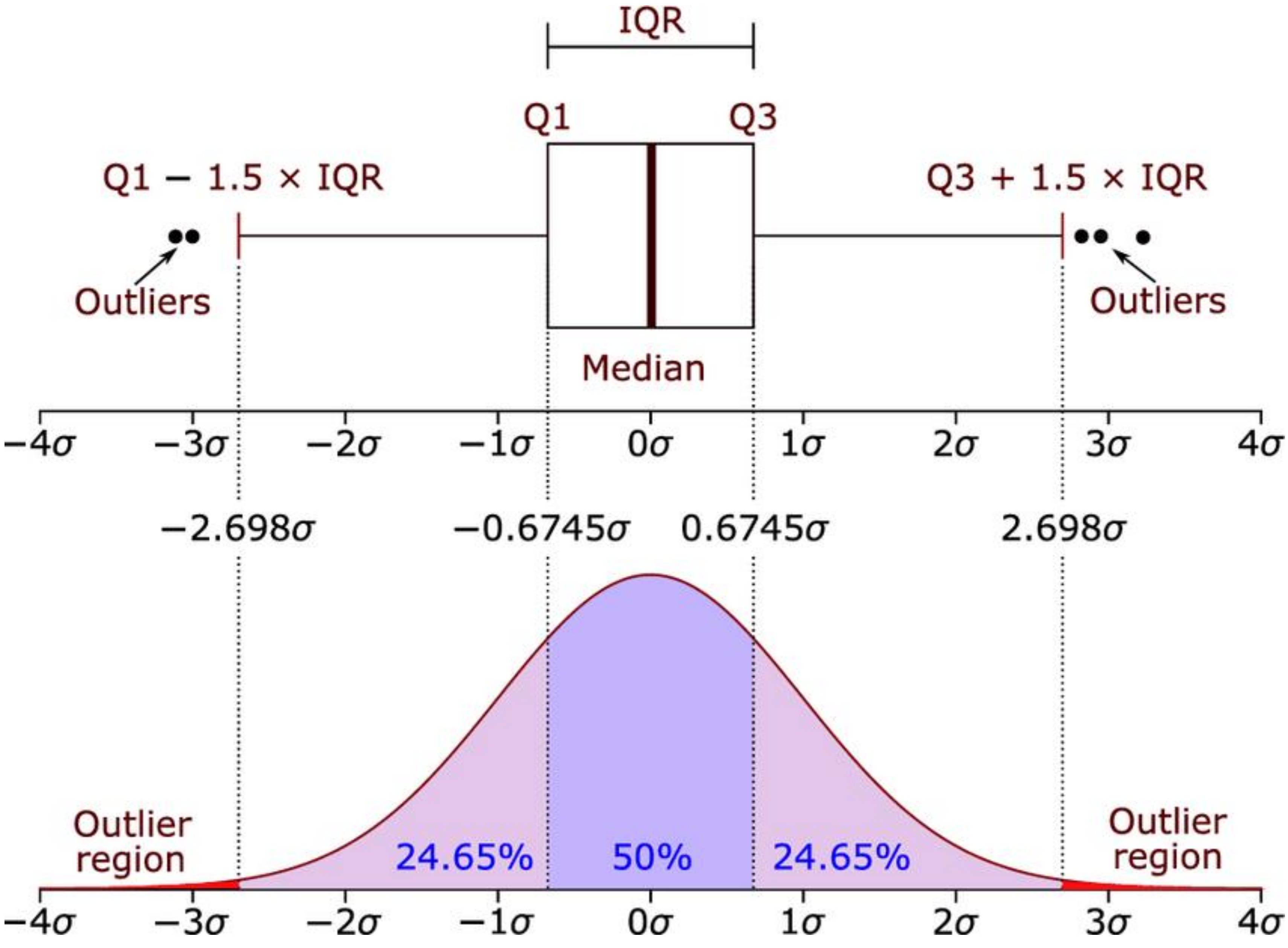
$[\mu - \sigma,\ \mu + \sigma]$



$[\mu - 2\sigma,\ \mu + 2\sigma]$



$[\mu - 3\sigma,\ \mu + 3\sigma]$



Box plot and probability density function of a normal distribution.

Image credit: ResearchGate

Quiz 05: Variance and standard deviation

1. Consider the following 1D data series, which includes 15 data points sorted in ascending order.

   21, 25, 27, 29, 32, 36, 36, 48, 67, 80, 84, 85, 89, 92, 97

2. Does **pandas** provide the variance and standard deviation for a dataframe? If yes, how?

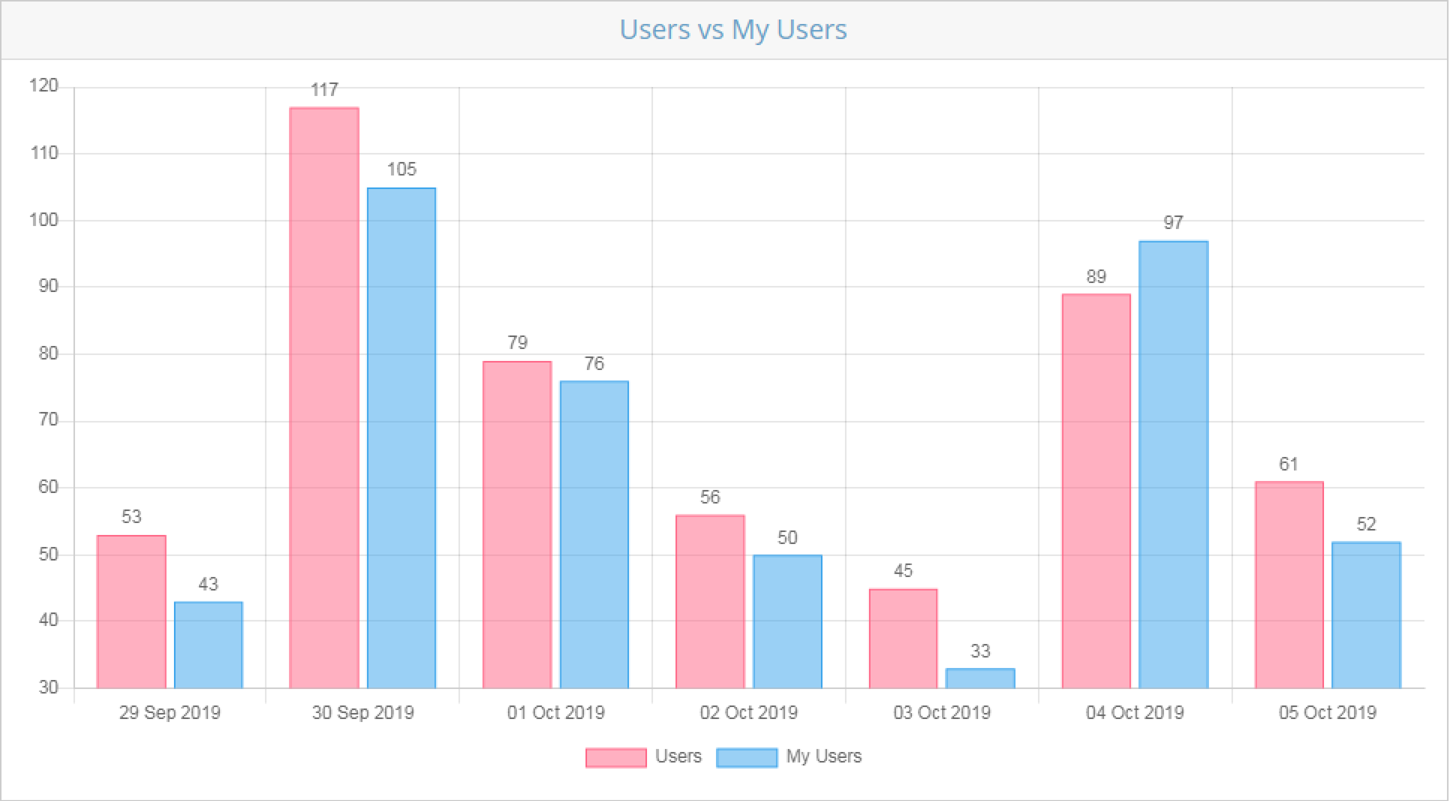
Basic data visualization

Why data visualization?

- Gain insight into an information space by mapping data onto graphical primitives
- Provide qualitative overview of large datasets
- Search for patterns, trends, irregularities, relationships
- Help find interesting regions and suitable parameters for further quantitative analysis
- Provide a visual proof of computer representations derived

Bar chart

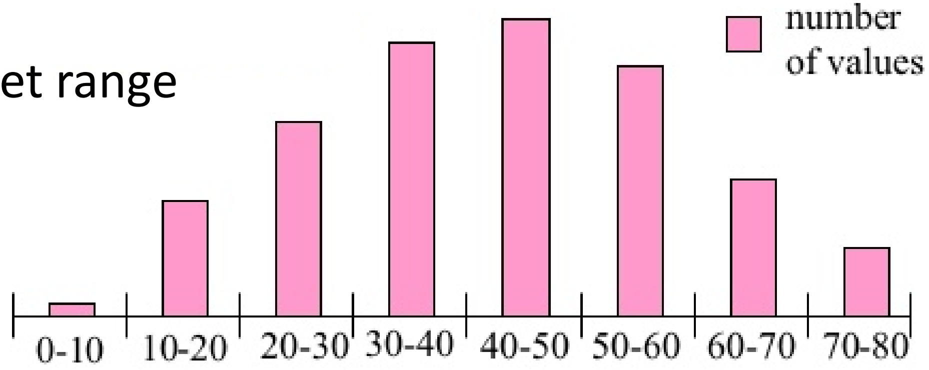
- A bar chart presents **nominal data** by using rectangular bars with heights proportional to the values represented.

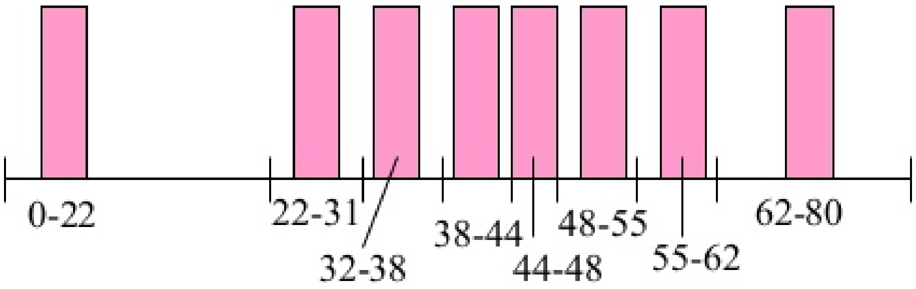


Histogram

- The range of values for a numeric attribute X is partitioned into disjoint consecutive subranges, called **buckets** or **bins**.
- Each bar is for a subrange such that its height represents the total items within the subrange.

Equal-width: equal bucket range



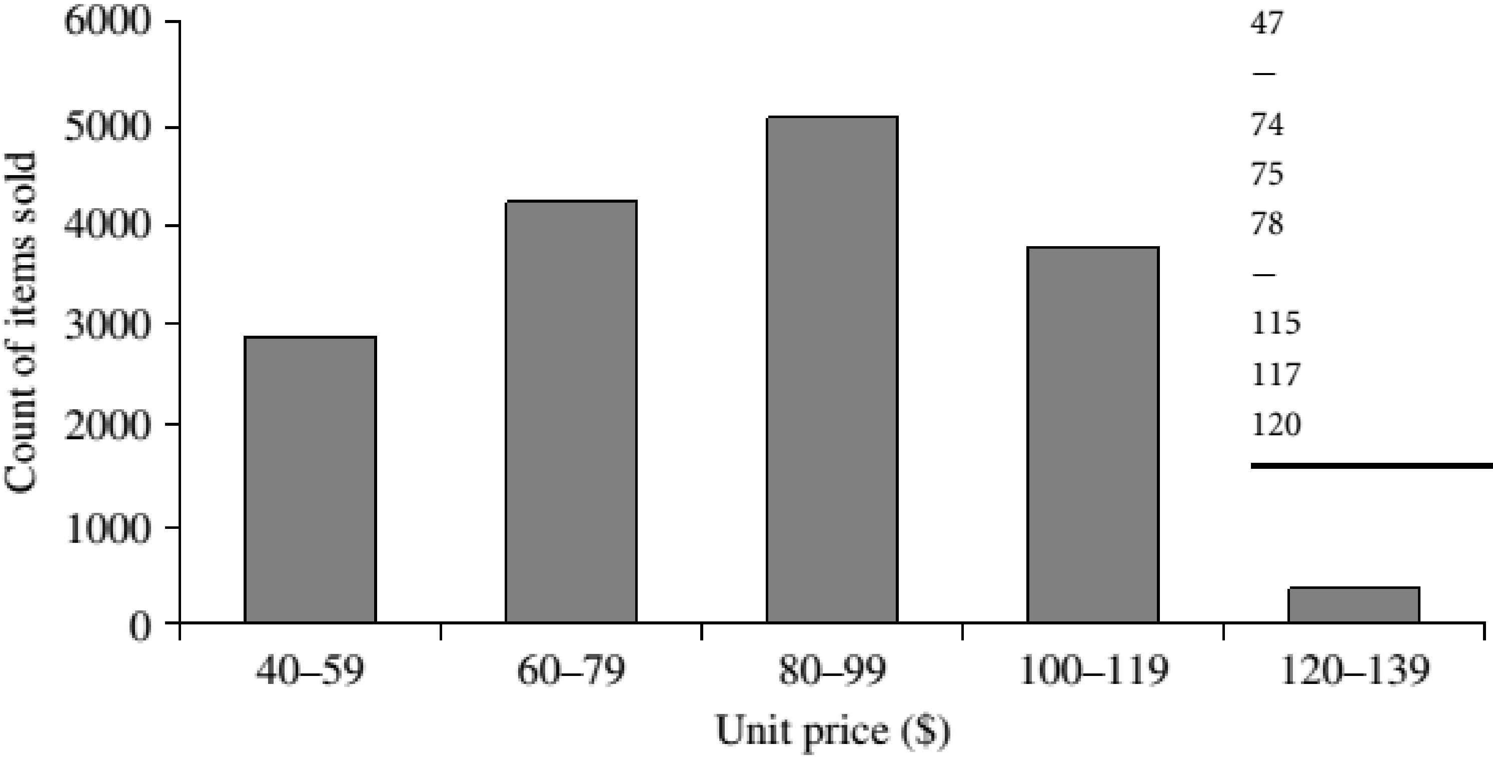


Equal-frequency: equal bucket depth

Histogram: An example

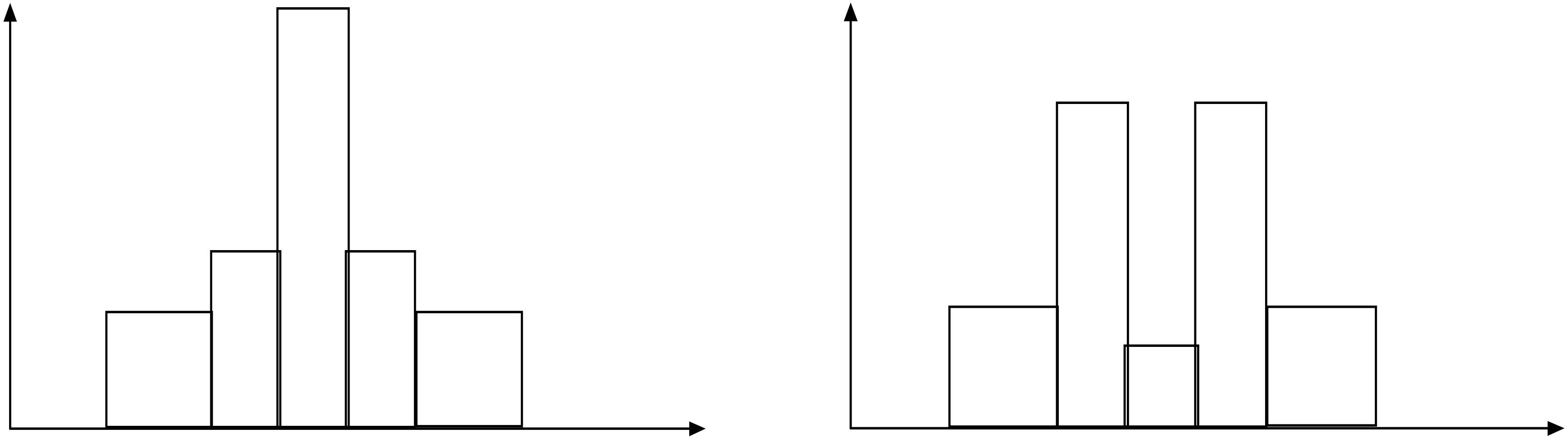
A Set of Unit Price Data for Items Sold at a Branch of *AllElectronics*

Unit price ($)	*Count of items sold*
40	275
43	300
47	250
—	—
74	360
75	515
78	540
—	—
115	320
117	270
120	350



Histogram over boxplot

- The two following histograms may have the same boxplot.
- However, they represent rather different data distributions.

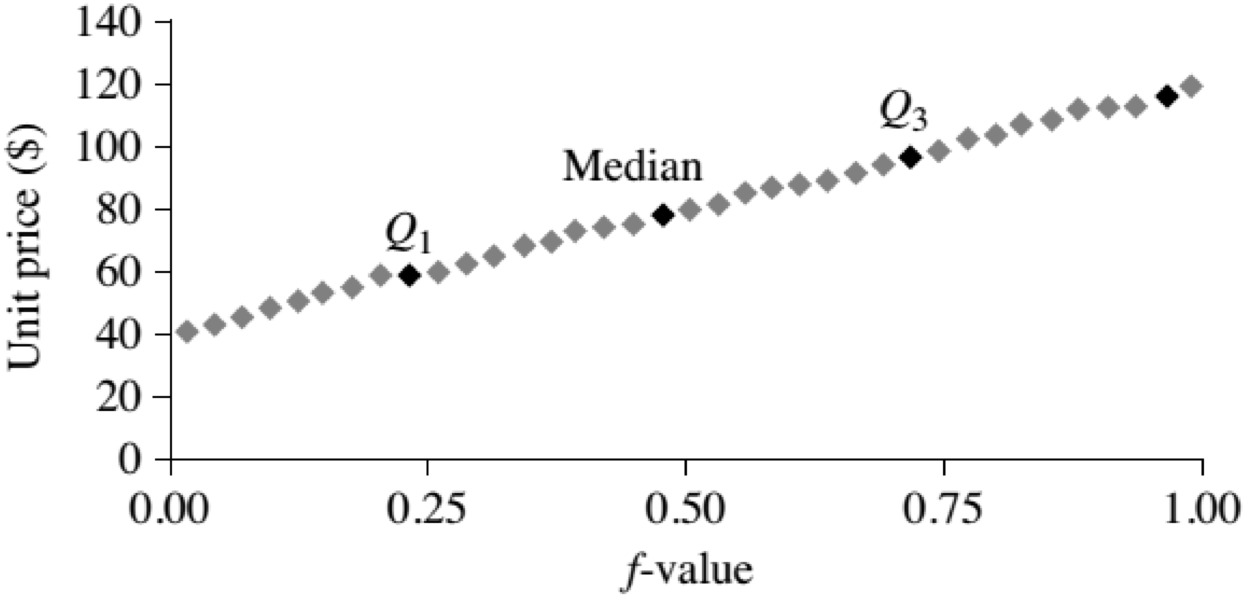


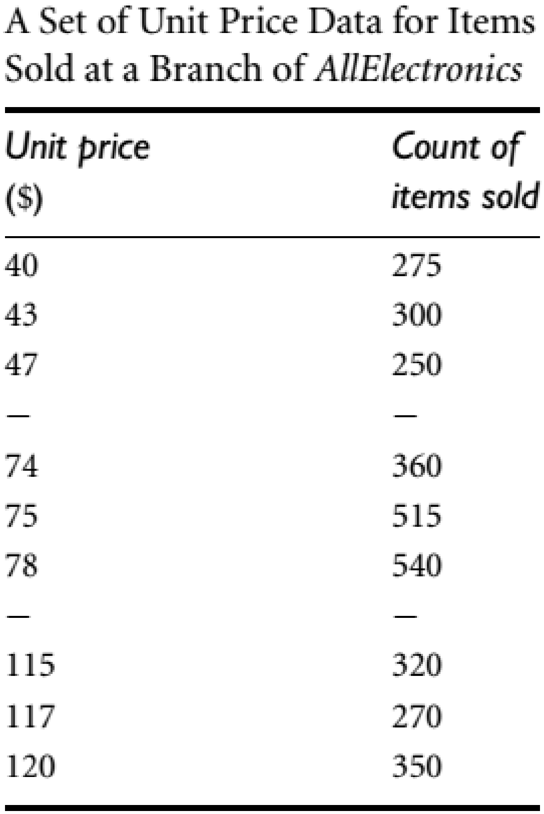
Quantile plot

- A quantile plot presents the plot quantile information for a univariate data distribution.
  - It allows access to both overall behavior and unusual occurrences.
- Let $x_1, x_2, \ldots, x_N$ be the data observations sorted in increasing order for some ordinal or numeric attribute X.
- Each value x_i is paired with $f_i = \frac{i-0.5}{N}$, indicating that approximately $f_i \times 100\%$ of data are $\leq x_i$.

Quantile plot: An example

Quantile plot for the unit price data





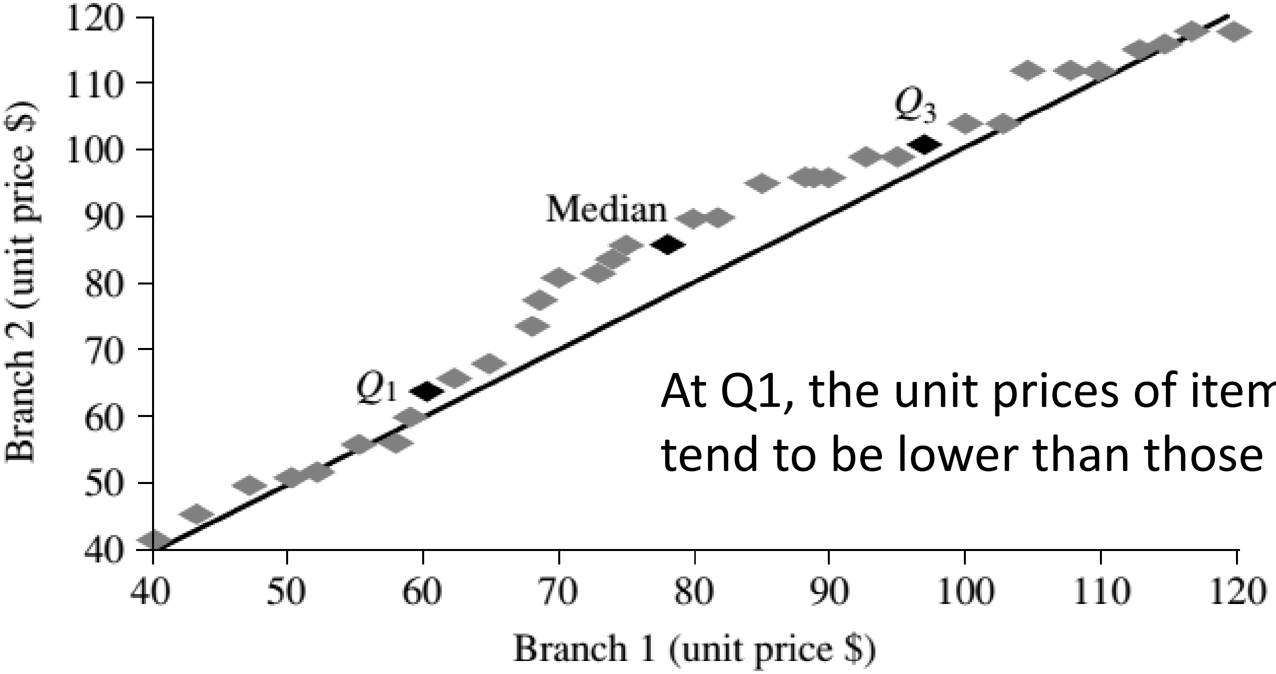
A Set of Unit Price Data for Items Sold at a Branch of *AllElectronics*

Unit price ($)	*Count of items sold*
40	275
43	300
47	250
—	—
74	360
75	515
78	540
—	—
115	320
117	270
120	350

Quantile-Quantile plot

- A quantile-quantile plot draws the quantiles of one univariate distribution against the corresponding quantiles of another.

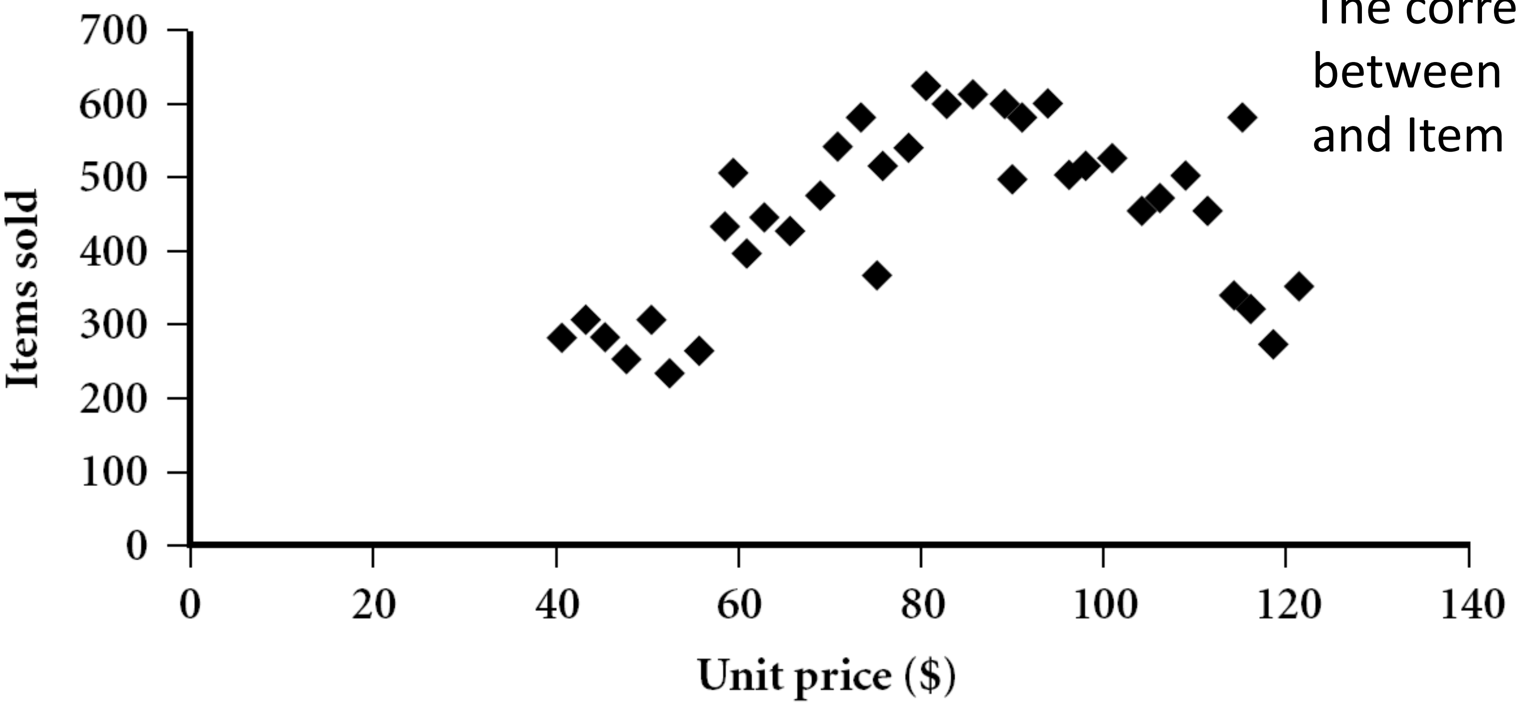
Is there a shift in going from one distribution to another?



At Q1, the unit prices of items sold at Branch 1 tend to be lower than those at Branch 2

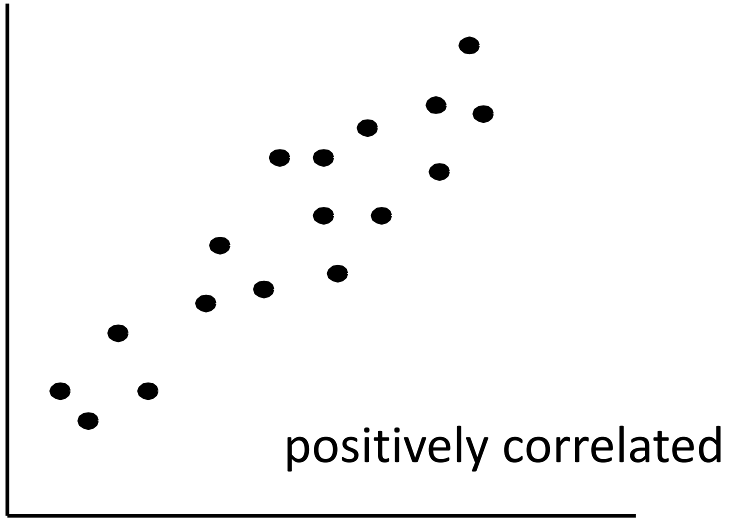
Scatter plot

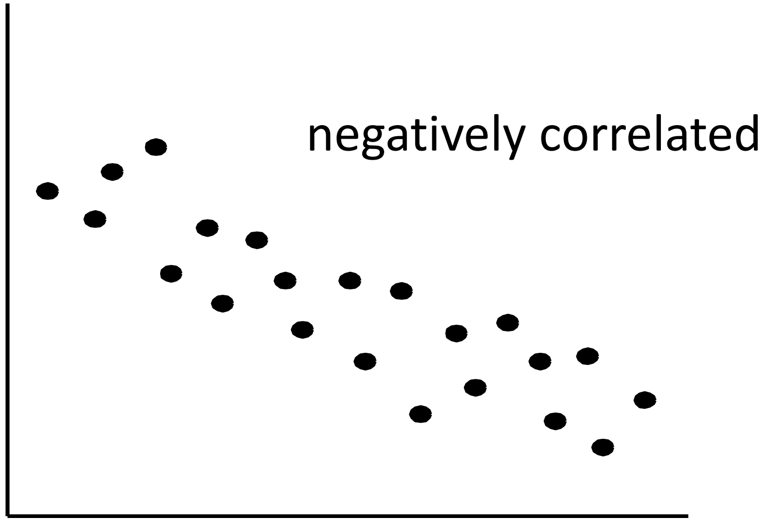
- A scatter plot looks at the bivariate data to see clusters of points or outliers
  - Each pair of values is treated as a pair of coordinates and plotted as points in the plane.

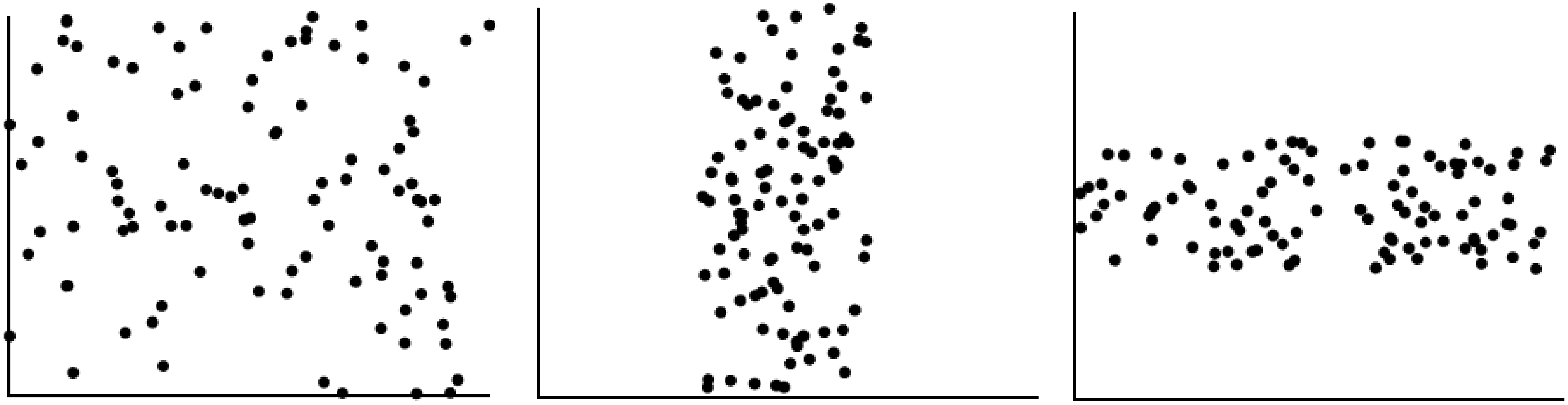


The correlation between Unit prices and Item sold

Scatter plot: Data correlation


positively correlated


negatively correlated


uncorrelated data

Quiz 06: Scatter plot

1. Consider the following data table, in which there are five tuples of two attributes, A and B.

No.	Attributes	
	A	B
1	19	16
2	25	10
3	13	26
4	12	29
5	16	20

   Draw the scatter plot, whose the horizontal axis denotes attribute A, and the vertical axis represents attribute B.

2. How to draw a scatter plot using **Python library**?

Data proximity measures

Similarity and Dissimilarity

Similarity

- A numerical measure of how alike two data objects, i and j, are
- Values often falls in the range [0,1]: 0 – unalike → 1 – identical

Dissimilarity (distance)

- A numerical measure of how different two data objects are
- It works in an opposite direction to some similarity measure
- The lower bound is often 0, while the upper limit varies

Proximity

- This refers to either similarity or dissimilarity

Feature matrix vs. Dissimilarity matrix

- Feature matrices are essential to most machine learning task.

Feature matrix

$$\begin{bmatrix} x_{11} & \cdots & x_{1f} & \cdots & x_{1p} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{i1} & \cdots & x_{if} & \cdots & x_{ip} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n1} & \cdots & x_{nf} & \cdots & x_{np} \end{bmatrix}$$

- n data points with p dimensions
- Object-by-attribute structure

Dissimilarity matrix

$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ : & : & : & & \\ d(n,1) & d(n,2) & \cdots & \cdots & 0 \end{bmatrix}$$

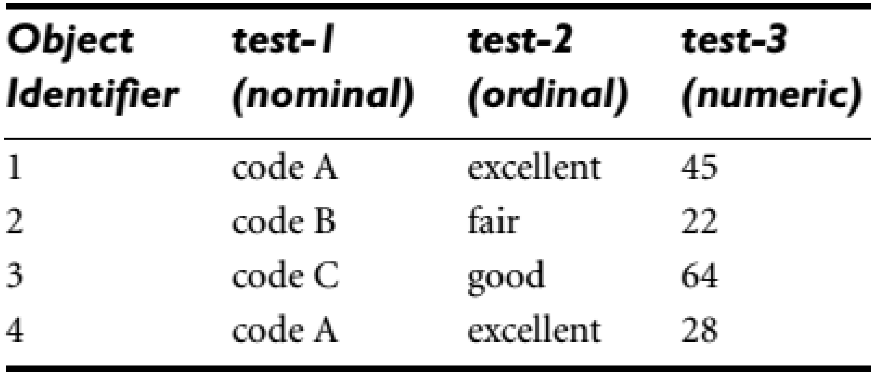
- A collection of distances for all pairs of n objects
- Object-by-object structure

- Many nearest-neighbor algorithms use dissimilarity matrices.

Measures for nominal attributes

- Let the number of states of a nominal attribute be M
- Method 1: Simple matching $\boldsymbol{d(i,j) = \frac{p-m}{p}}$
  - m: the number of attributes for which i and j are in the same state,
  - p: the total number of attributes describing the objects

$$\begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ 1 & 1 & 0 & \\ 0 & 1 & 1 & 0 \end{bmatrix}$$



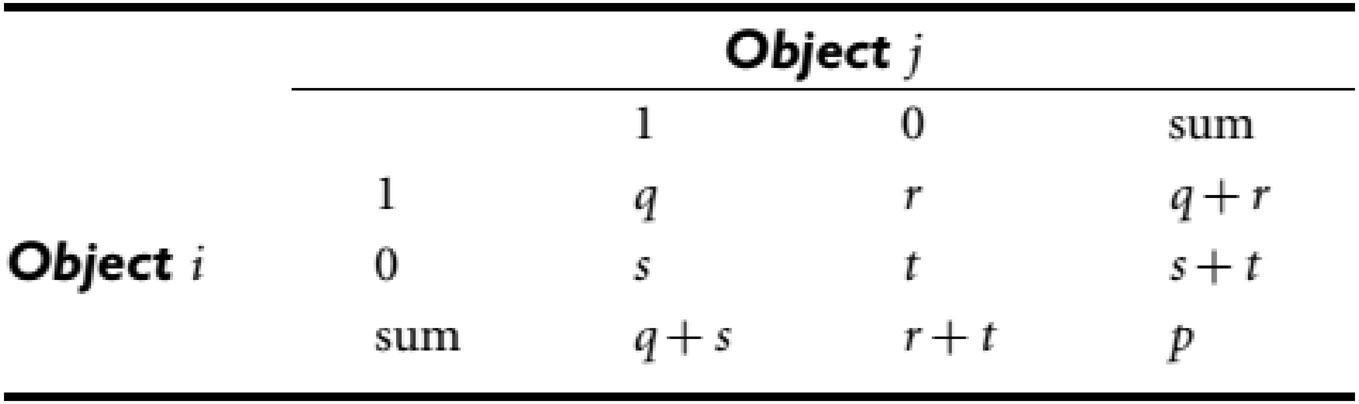
Object Identifier	test-1 (nominal)	test-2 (ordinal)	test-3 (numeric)
1	code A	excellent	45
2	code B	fair	22
3	code C	good	64
4	code A	excellent	28

$$\begin{bmatrix} 0 & & & \\ d(2,1) & 0 & & \\ d(3,1) & d(3,2) & 0 & \\ d(4,1) & d(4,2) & d(4,3) & 0 \end{bmatrix}$$

- Method 2: Create a binary attribute for each of the M states
- Measures of similarity $\boldsymbol{sim(i,j) = 1 - d(i,j) = \frac{m}{p}}$

Measures for binary attributes

- Contingency table



		Object j		
		1	0	sum
Object i	1	q	r	$q+r$
	0	s	t	$s+t$
	sum	$q+s$	$r+t$	p

- Symmetric binary variable

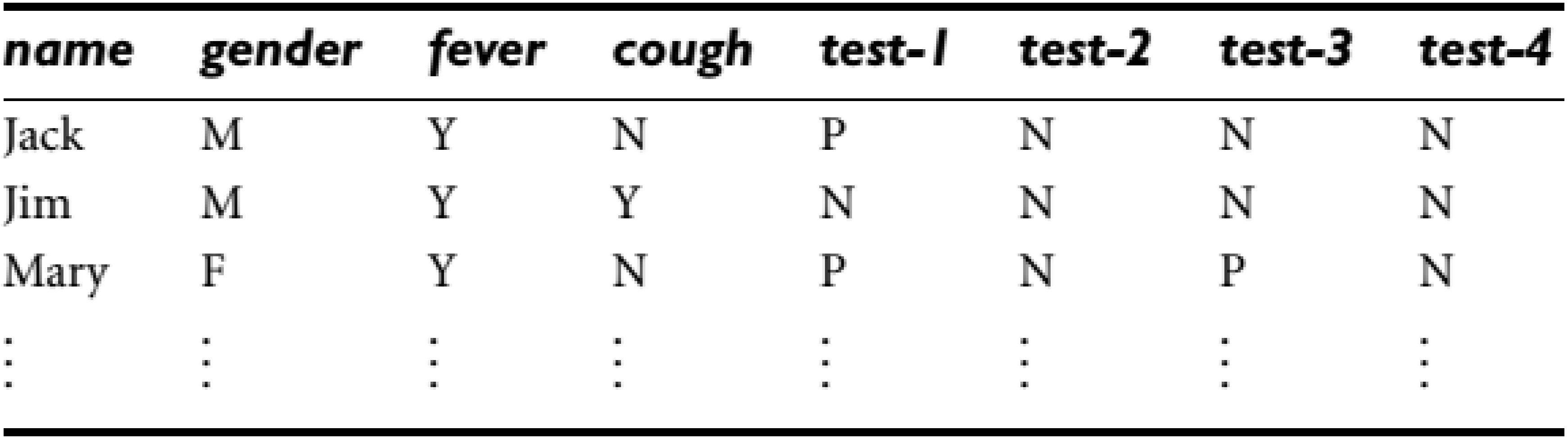
$$d(i,j) = \frac{r+s}{q+r+s+t}$$

- Asymmetric binary variable

$$d(i,j) = \frac{r+s}{q+r+s}$$

- Jaccard coefficient: $sim(i,j) = 1 - d(i,j) = \frac{q}{q+r+s}$

Measures for binary attributes



name	*gender*	*fever*	*cough*	*test-1*	*test-2*	*test-3*	*test-4*
Jack	M	Y	N	P	N	N	N
Jim	M	Y	Y	N	N	N	N
Mary	F	Y	N	P	N	P	N
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

- Gender is symmetric binary, the remaining attributes are asymmetric
- Let the values Y and P be 1 and the value N be 0.
- Suppose that the distance between objects (patients) is computed based only on the asymmetric attributes
- $d(Jack, Jim) = \frac{1+1}{1+1+1} = 0.67,$ $d(Jack, Mary) = \frac{0+1}{2+0+1} = 0.33$

  $d(Jim, Mary) = \frac{1+2}{1+1+2} = 0.75$

Measures for numeric attributes

- Consider two data points of p-dimensional

$$i = \left(x_{i1}, x_{i2}, \dots, x_{ip}\right) \text{ and } j = \left(x_{j1}, x_{j2}, \dots, x_{ij}\right)$$

- Minkowski distance (L_h norm)

$$d(i,j) = \sqrt[h]{\left|x_{i1} - x_{j1}\right|^h + \left|x_{i2} - x_{j2}\right|^h + \cdots + \left|x_{ip} - x_{jp}\right|^h}$$

  - where h is the order

Measures for numeric attributes

- $h = 1$: Manhattan (city block, L_1 norm) distance

$$d(i,j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| + \cdots + |x_{ip} - x_{jp}|$$

- $h = 2$: Euclidean (L_2 norm) distance

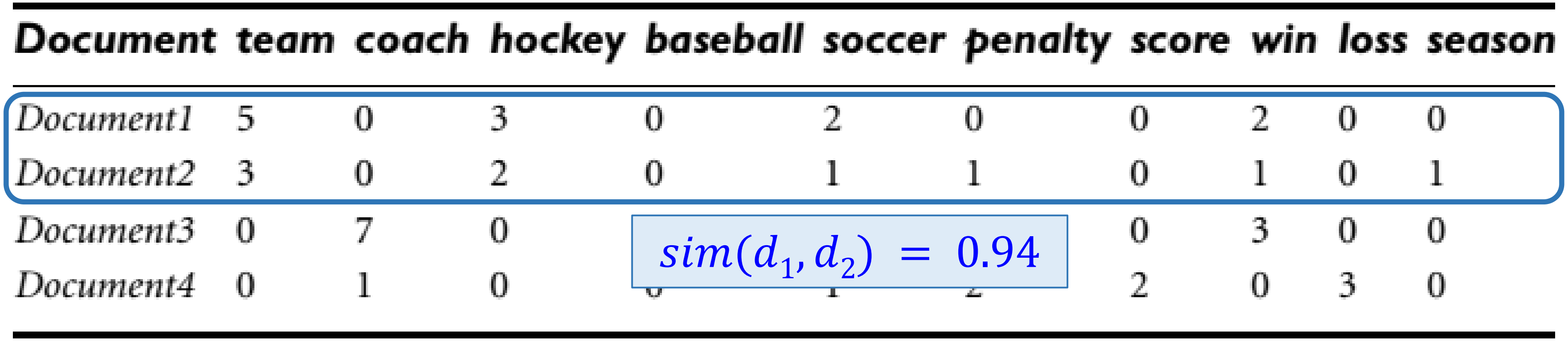
$$d(i,j) = \sqrt{|x_{i1} - x_{j1}|^2 + |x_{i2} - x_{j2}|^2 + \cdots + |x_{ip} - x_{jp}|^2}$$

- $h \to \infty$: "supremum" (L_{max} / L_∞ norm, Chebyshev) distance

$$d(i,j) = \lim_{h\to\infty}\left(\sum_{f=1}^{p} |x_{if} - x_{jf}|^h\right)^{1/h} = \max_f^p |x_{if} - x_{jf}|$$

Cosine similarity

- A document can be represented by thousands of keywords in the document.

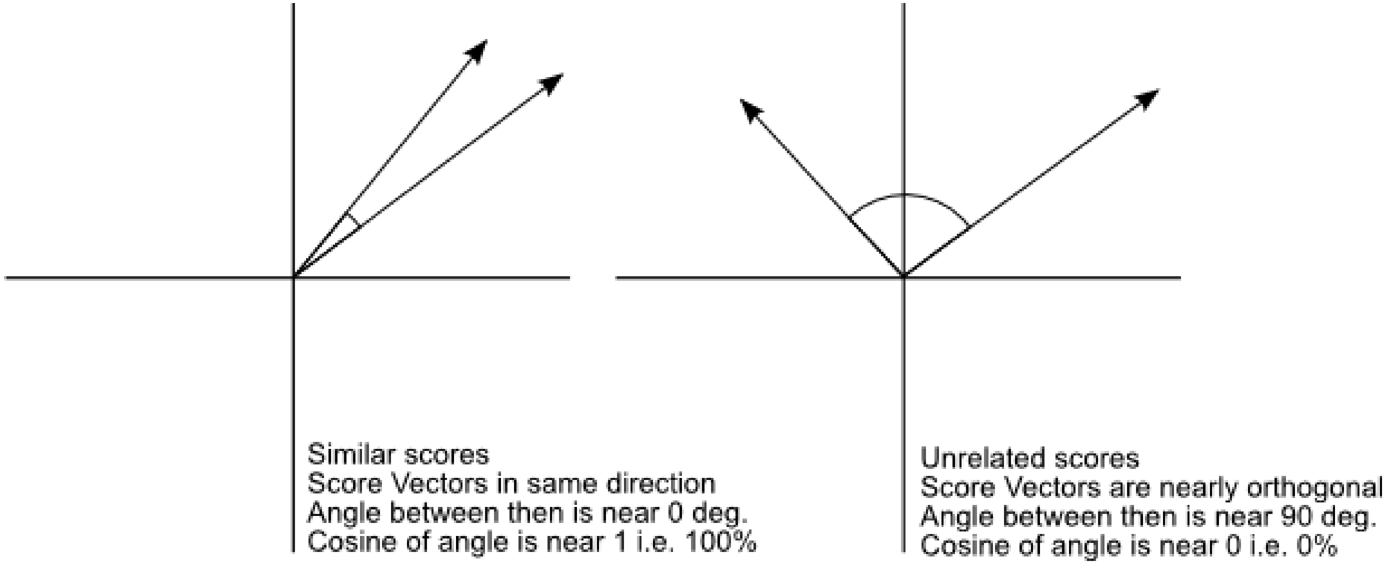


Document	team	coach	hockey	baseball	soccer	penalty	score	win	loss	season
Document1	5	0	3	0	2	0	0	2	0	0
Document2	3	0	2	0	1	1	0	1	0	1
Document3	0	7	0				0	3	0	0
Document4	0	1	0	0	1	2	2	0	3	0

$sim(d_1, d_2) \ = \ 0.94$

Cosine similarity

- Let d_1 and d_2 are two vectors (e.g., term-frequency vectors).
- Cosine similarity is non-metric: $sim(d_1, d_2) = \dfrac{d_1 \cdot d_2}{\|d_1\|\|d_3\|}$
  - where $\cdot$ is vector dot product, $\|d\|$ is the length of vector d
  - sim = 0 means no match, while sim = 1 means a complete match.



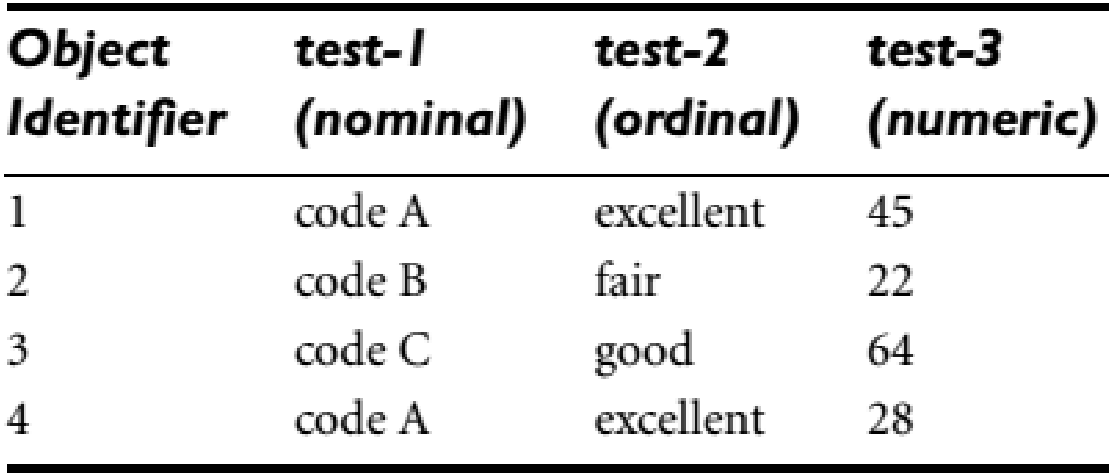
Similar scores
Score Vectors in same direction
Angle between then is near 0 deg.
Cosine of angle is near 1 i.e. 100%

Unrelated scores
Score Vectors are nearly orthogonal
Angle between then is near 90 deg.
Cosine of angle is near 0 i.e. 0%

Measures for ordinal attributes

- The range of a numeric attribute can be mapped to an ordinal attribute f having M_f states.
  - E.g., temperate: cold (-30^{o}C – 10^{o}C), moderate (-10^{o}C – 10^{o}C), and warm (10^{o}C – 30^{o}C)
- Let M represent the number of possible ordered states, which define the ranking $1, \ldots, M_f$
- Replace each x_{if} by its corresponding rank, $r_{if} \in \{1, \ldots, M_f\}$
- Replace rank r_{if} of i^{th} object by $z_{if} = \dfrac{r_{if} - 1}{M_f - 1}$
- Continue with any measure for numeric attributes

Measures for ordinal attributes



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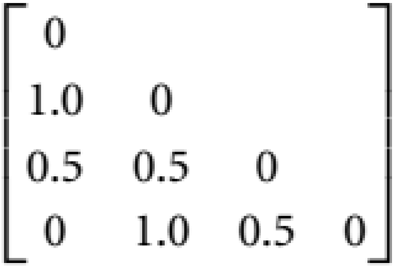
1	code A	excellent	45
2	code B	fair	22
3	code C	good	64
4	code A	excellent	28

- test-2 = {fair, good, excellent}, i.e., $M_f = 3$
- The ranks of four objects are 3, 1, 2, and 3, respectively
- Map the rank 1 to 0.0, rank 2 to 0.5, and rank 3 to 1.0
- Dissimilarity matrix using Euclidean distance



$$\begin{bmatrix} 0 & & & \\ 1.0 & 0 & & \\ 0.5 & 0.5 & 0 & \\ 0 & 1.0 & 0.5 & 0 \end{bmatrix}$$

Measures for attributes of mixed types

- Suppose that the dataset has p attributes of mixed type.
- The distance between objects i and j is $\boldsymbol{d(i,j)=\frac{\sum_{f=1}^{p}\delta_{ij}^{(f)}d_{ij}^{(f)}}{\sum_{f=1}^{p}\delta_{ij}^{(f)}}}$
  - $\delta_{ij}^{(f)}=0$ if (1) x_{if} or x_{jf} is missing, or (2) $x_{if}=x_{jf}=0$ and attribute f is asymmetric binary. Otherwise, $\delta_{ij}^{(f)}=1$
  - If f is numeric: $d_{ij}^{(f)}=\frac{|x_{if}-x_{jf}|}{\max_h x_{hf}-\min_h x_{hf}}$, where h runs over all nonmissing objects for attribute f
  - If f is nominal or binary: $d_{ij}^{(f)}=0$ if $x_{if}=x_{jf}$; otherwise, $d_{ij}^{(f)}=1$
  - If f is ordinal: compute r_{if} and treat $z_{if}=\frac{r_{if}-1}{M_f-1}$ as numeric

Measures for attributes of mixed types

Dissimilarity matrix of test-1

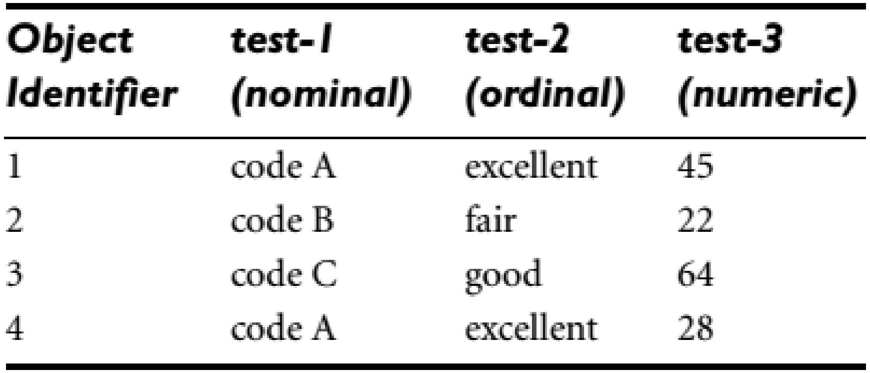
$$\begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ 1 & 1 & 0 & \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Dissimilarity matrix of test-2

$$\begin{bmatrix} 0 & & & \\ 1.0 & 0 & & \\ 0.5 & 0.5 & 0 & \\ 0 & 1.0 & 0.5 & 0 \end{bmatrix}$$

Dissimilarity matrix of test-3

$$\begin{bmatrix} 0 & & & \\ 0.55 & 0 & & \\ 0.45 & 1.00 & 0 & \\ 0.40 & 0.14 & 0.86 & 0 \end{bmatrix}$$



Object Identifier	*test-1 (nominal)*	*test-2 (ordinal)*	*test-3 (numeric)*
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4	code A	excellent	28

- $\delta_{ij}^{(f)} = 1$ for each attribute f
- $d(3,1) = \frac{1(1)+1(0.50)+1(0.45)}{3} = 0.65$
- The resulting dissimilarity matrix

$$\begin{bmatrix} 0 & & & \\ 0.85 & 0 & & \\ 0.65 & 0.83 & 0 & \\ 0.13 & 0.71 & 0.79 & 0 \end{bmatrix}$$

Quiz 06: Jaccard coefficient

1. Calculate the similarity between these two observations, in which all the attributes are binary asymmetric.

IDs	fever	cough	breathing difficulty	fatigue	headache	loss of taste	sore throat
1	1	1	1	0	0	1	1
2	0	1	0	0	1	1	1

2. Explore the distance and similarity metrics are supported in **scikit-learn**.

References

- Jiawei Han, Micheline Kamber, and Jian Pei, 2011. Data Mining: Concepts and Techniques (3rd ed.). Morgan Kaufmann Publishers Inc. Chapter 2.

...the end.