Project 5
Approximating functions by polynomial interpolation

In this project you will build interpolating polynomials, with the goal of approximating the function $f(x) = \sin 7x + \cos 4x$ on the domain $[0, 4]$ as closely as possible. Newton's divided differences will be used to construct the polynomial.

1. The Matlab command
   `x0=4*(0:(n-1))/(n-1);`
   defines n equally-spaced points in the interval $[0, 4]$. Take `y0` values from the function $f(x)$ at these `x0` coordinates, using "exact" values from Matlab's library function for `sin`. The Matlab m-files `nest.m` and `newtdd.m` can be used to find the degree $n-1$ interpolating polynomial $P_{n-1}(x)$ that passes through the n points. Plot the actual $f(x)$ versus $P_{n-1}(x)$ on $[0, 4]$ for $n = 9$ points. (Use a plotting grid of width 0.01 or less to get a good plot.) Include the interpolating points, plotted as circles. In a separate figure, plot the interpolation error of $P_8(x)$ on $[0, 4]$, using Matlab's `semilogy` command, and calculate the **maximum** interpolation error on the domain $[0, 4]$.

2. Find the smallest n that makes the **maximum** interpolation error on the domain $[0, 4]$ less than 0.5×10^{-6}, if possible. For this n, show the same two plots as in Step 1, the interpolation polynomial plot and the semilog interpolation error plot. Where along the interval $[0, 4]$ is the error typically the largest?

3. Change the base points from equally-spaced to the "Chebyshev points" on $[0, 4]$, and redo Step 2. Find the smallest n that makes the error less than 0.5×10^{-6} and show the same two plots. The Chebyshev points are generated by replacing the command above by the Matlab command
   `x0=2+2*cos((1:2:2*n-1)*pi/(2*n));`

4. What can you say about the comparison between interpolation with equally-spaced interpolation points and Chebyshev points? If one was better, can you give an intuitive explanation of why?

Begin your report by stating your conclusions about the questions above and show the plots. Save the Matlab code used and your Matlab session, and include these with your report. Save your report as a .pdf file and upload it to Blackboard. (You are allowed to edit your Matlab session as long as you include the important results.)

BONUS
Two extra points will be awarded to the paper with the smallest maximum interpolation error, where you are allowed to use any set of points `x0` that you want. Please state BONUS somewhere on your paper so I know you are attempting this.

Due: Tues., Nov. 5