Project 6
Are we melting?

The amount of sea ice floating in the Arctic ocean is a critical factor in the stability of the earth's climate. In particular, a decrease in sea ice could cause (1) increased ocean levels, (2) a drastic change in the North Atlantic Meridional Overturning Circulation (MOC) due to melted freshwater ice being dumped into the ocean, and (3) reduced albedo of the planet leading to the reduction in the amount of sun's energy being reflected back into space. In this project we try to determine from data how the total area of Arctic sea ice has changed over the last 40 years, and whether the rate of change is accelerating.

You will use least squares to fit some models to monthly sea ice measurements from two 19-year periods, courtesy of satellites that monitor the coverage of the ice pack daily. The data is stored at the National Snow and Ice Data Center at the University of Colorado at Boulder. The files `seaice1.txt` and `seaice2.txt`, available at our Blackboard page, are each a file of length 228 consisting of the total Arctic sea ice area in million of square kilometers, measured monthly over the two periods Nov. 1980 to Oct. 1999, and Nov. 2000 to Oct. 2019, respectively.

The fact that the data has an annual cycle (melting in the summer, freezing in the winter) complicates our analysis. We could take a straight average of the annual sea ice coverage area and fit a best line to them. But due to the obvious annual oscillation, a straight average is a poor fit to the data. Is it more appropriate to make a better model of the data, and look for at linear part of the model? We will investigate simple models and compare results with more complicated models, that try to take account of the annual oscillation.

1. Fit both of the monthly time series with the model $y = c_1 + c_2 t$, where y denotes the sea ice area (in 10^6 km^2) and t represents time in years, $0 \leq t \leq 19$. The Matlab command `t = (19*(1:228)/228)';` will define the time variable with units of years. The command `load filename` will load the data into your Matlab window. What are the two approximate slopes c_2, representing the rate of increase (in million km per yr.) over the two time periods? Plot both time series along with the corresponding linear model, and compute the root mean squared error (RMSE) of each of the fits. Use the `grid` command after the plot to see grid markings.

2. Your plot in Step 1 should show a clear annual periodicity in the data. To fit the annual cycle, repeat Step 1 with the trigonometric model $y = c_1 + c_2 t + c_3 \cos 2\pi t + c_4 \sin 2\pi t$. Show the plots, and analyze the new c_2 and RMSE – how did they change with the more complex model? Do you consider the new c_2 to be more or less accurate than the one in Step 1?

3. Repeat Step 2, but with the improved trig model
$y = c_1 + c_2 t + c_3 \cos 2\pi t + c_4 \sin 2\pi t + c_5 \cos 4\pi t + c_6 \sin 4\pi t + c_7 \cos 6\pi t + c_8 \sin 6\pi t$.

4. Summarize your conclusions. What happened to the RMSE as you increased the complexity of the model? Is the sea ice increasing or decreasing? Is the rate of change c_2 approximately stable, or is it different between the two 19-year periods?

Due: Tues., Nov. 26.