Project 4

How many correct digits are you guaranteed when solving systems of equations?

As we learned in Chapter 0, small rounding errors (of relative size $\epsilon_{\text{mach}} \approx 2^{-52} \approx 10^{-16}$) are made on each floating point operation, leading (in Chapter 1) to loss of correct significant digits in root-finding. In this project, you will determine whether it is possible for these errors to accumulate enough to significantly reduce the accuracy of solutions of systems of linear equations. Worst case: Is it possible to **lose all correct digits**?

1. Implement the Matlab code for Gaussian Elimination of Section 2.1 of the text (keep it simple: no row exchanges should be done). First check that the code gives the correct solution for a small system of equations, of your choice. Check that the backward error of your answer is near machine epsilon, by multiplying `A` times the output solution `x` and comparing with your chosen `b`. Note that you need to save copies of A and b to do this check, if your Gaussian elimination code changes their entries.

2. Let `A` be the $n \times n$ matrix whose `(i,j)` entry is $A_{ij} = \tan(3i + 4j)$. Define the vector `c` to have all components equal to 1, or `c=ones(n,1)` in Matlab. Define the vector `b=A*c`. You have constructed a system $Ax = b$ with a known exact solution c. Now solve $Ax = b$ for x using your Gaussian elimination code, and determine the relative forward error (RFE) and relative backward error (RBE) of your computed solution x. (Caution: Use the `clear` command before constructing A and b.)

   For the solution to be correct to double precision, RFE should be close to machine precision, or 10^{-16}. To check this, make a table of RFE, RBE, EMF, and the condition number `cond(A,inf)`, where

   $$\text{EMF = Error Magnification Factor} = \frac{\text{RFE}}{\text{RBE}}$$

   for $n = 8, 12$ and 16. Also answer the question: How large would n have to be to lose half of the correct digits, i.e. for RFE to exceed 0.5×10^{-8}?

3. Repeat Step 2 for the second matrix $A_{ij} = \cos(\sin(3i + 4j))$.

4. Compare the results you obtained from the matrices A in Step 2 and Step 3. Which of the 6 systems in Steps 2 and 3 could be solved with at least some correct digits, and which were total failures?

Begin your report by answering the four questions above. Please upload one file (PDF) only to Blackboard. Include the Matlab code used and your Matlab session. You may edit your session to limit the length of your report – please do not print out all the large matrices!

Due: Thurs., Oct. 10