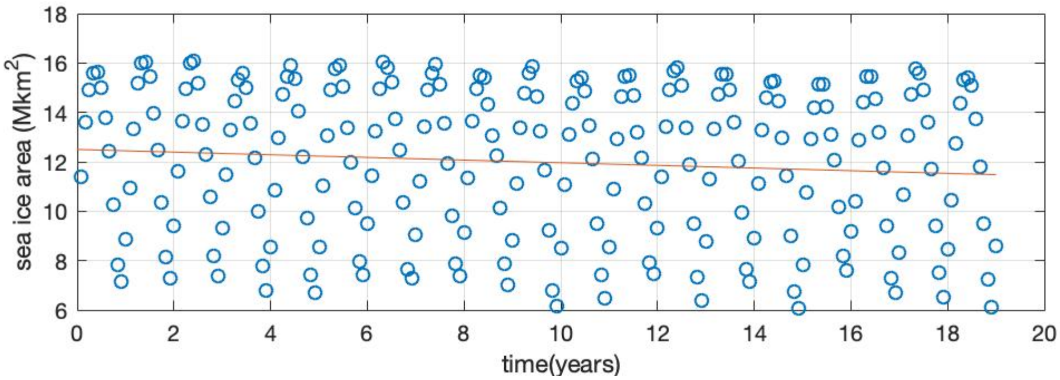
Ngoc Huynh
MATH 446
Project 6
Are We Melting?

Question 1: $y = c_1 + c_2t$

From Nov. 1980 to Oct. 1999
After loading the file seaice1.txt and denoting the sea ice area equals to the date in the file seaice1.txt. I input the following commands to obtain the linear model:

```
>> t=(19*(1:228)/228)';
>> n=228;
>> a = [ones(n,1) t];
>> b = y;
>> ata = a'*a

ata =

   1.0e+04 *

    0.0228    0.2175
    0.2175    2.7617

>> atb = a'*b

atb =

   1.0e+04 *

    0.2733
    2.5709
>> c = ata\atb

c =

   12.4996
   -0.0537

>> t1=0:.01:19;
>> y1=c(1)+c(2)*t1;
>> plot(t,y,'o',t1,y1)
xlabel('time(years)')
ylabel('sea ice area (Mkm^2)')
>> grid
```




```
>> c(2)

ans =

   -0.0537
```

Compute the Root Mean Squared Error (RMSE):

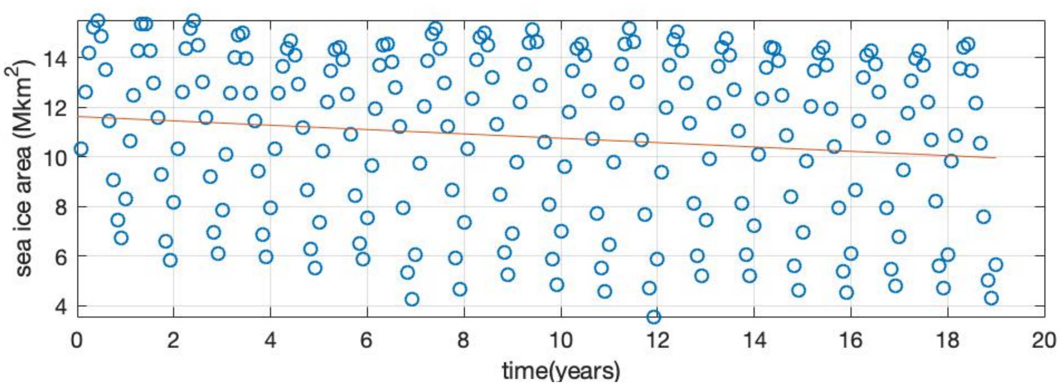
```
>> RMSE=norm(c(1)+c(2)*t-y)/sqrt(n)

RMSE =

    2.9808
```

From Nov. 2000 to Oct. 2019

After loading the file seaice2.txt and denoting the sea ice area equals to the date in the file seaice2.txt. I input the following commands to obtain the linear model:

```
>> t=(19*(1:228)/228)';
>> n=228;
>> b = y;
>> a = [ones(n,1) t];
>> ata = a'*a

ata =

   1.0e+04 *

    0.0228    0.2175
    0.2175    2.7617

>> atb = a'*b

atb =

   1.0e+04 *

    0.2460
    2.2876

>> c = ata\atb

c =

   11.6270
   -0.0876
```

```
>> t1=0:.01:19;
>> y1=c(1)+c(2)*t1;
plot(t,y,'o',t1,y1)
xlabel('time(years)')
ylabel('sea ice area (Mkm^2)')
```



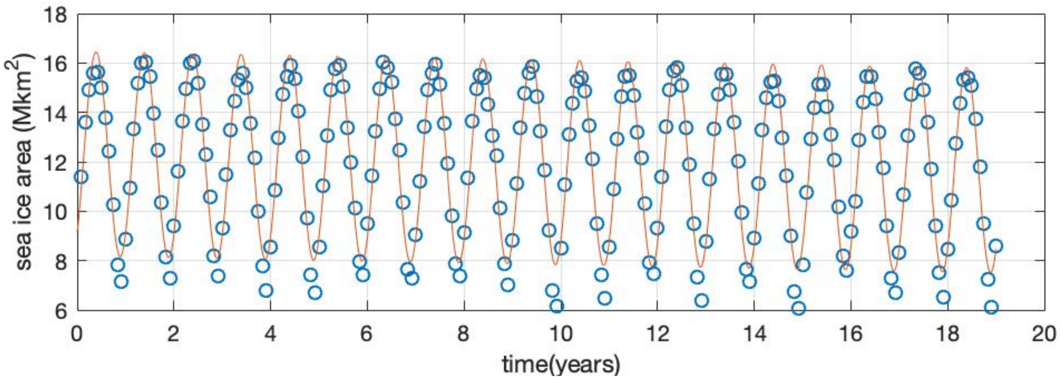
```
>> c(2)

ans =

   -0.0876

>> RMSE=norm(c(1)+c(2)*t-y)/sqrt(n)

RMSE =

     3.3342
```

<u>Question 2:</u> $y= c_1+ c_2t+c_3\cos2\pi t+c_4\sin2\pi t$

From Nov. 1980 to Oct. 1999

After loading the file seaice1.txt and denoting the sea ice area equals to the date in the file seaice1.txt. I input the following commands to obtain the linear model:

```
>> a = [ones(n,1) t cos(2*pi*t) sin(2*pi*t)];
>> b = y;
>> ata = a'*a

ata =

   1.0e+04 *

    0.0228    0.2175    0.0000   -0.0000
    0.2175    2.7617    0.0010   -0.0035
    0.0000    0.0010    0.0114   -0.0000
   -0.0000   -0.0035   -0.0000    0.0114

>> atb = a'*b

atb =

   1.0e+04 *

    0.2733
    2.5709
   -0.0363
    0.0303

>> c = ata\atb

c =

   12.3271
   -0.0356
   -3.1832
    2.6447

>> t1=0:.01:19;
y1=c(1)+c(2)*t1+c(3)*cos(2*pi*t1)+c(4)*sin(2*pi*t1);
plot(t,y,'o',t1,y1)
xlabel('time(years)')
ylabel('sea ice area (Mkm^2)')
>> grid
```



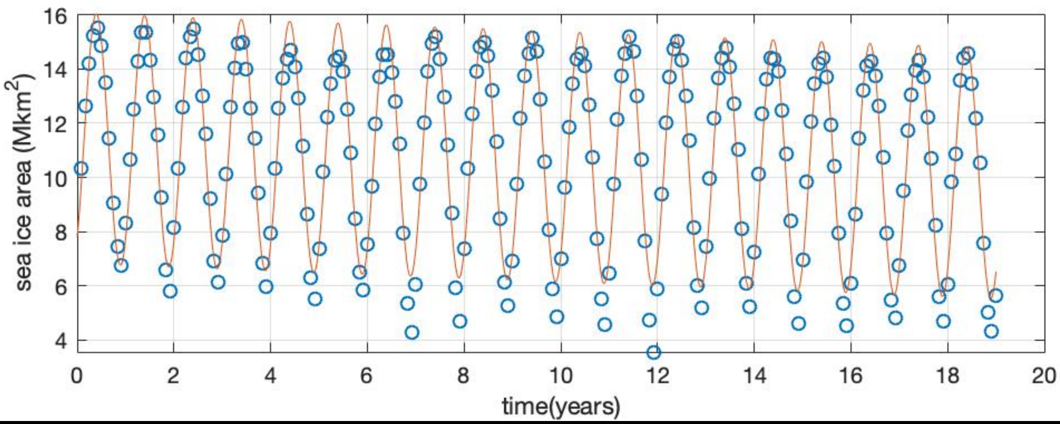

```
>> c(2)

ans =

   -0.0356
>> RMSE = norm(c(1)+c(2)*t+c(3)*cos(2*pi*t)+c(4)*sin(2*pi*t)-y)/sqrt(n)

RMSE =

    0.5757
```

From Nov. 2000 to Oct. 2019

After loading the file seaice2.txt and denoting the sea ice area equals to the date in the file seaice2.txt. I input the following commands to obtain the linear model:

```
>> a = [ones(n,1) t cos(2*pi*t) sin(2*pi*t)];
>> b = y;
>> ata = a'*a

ata =

   1.0e+04 *

    0.0228    0.2175    0.0000   -0.0000
    0.2175    2.7617    0.0010   -0.0035
    0.0000    0.0010    0.0114   -0.0000
   -0.0000   -0.0035   -0.0000    0.0114

>> atb = a'*b

atb =

   1.0e+04 *

    0.2460
    2.2876
   -0.0414
    0.0327

>> c = ata\atb

c =

   11.4385
   -0.0678
   -3.6279
    2.8496
```

```
>> t1=0:.01:19;
y1=c(1)+c(2)*t1+c(3)*cos(2*pi*t1)+c(4)*sin(2*pi*t1);
plot(t,y,'o',t1,y1)
xlabel('time(years)')
ylabel('sea ice area (Mkm^2)')
grid
```




```
>> c(2)

ans =

   -0.0678
```

```
>> RMSE = norm(c(1)+c(2)*t+c(3)*cos(2*pi*t)+c(4)*sin(2*pi*t)-y)/sqrt(n)

RMSE =

    0.6983
```

Comparing Table

	$y = c_1 + c_2t$	$y= c_1+ c_2t+c_3\cos2\pi t+c_4\sin2\pi t$
c_2 (***Nov. 1980 to Oct. 1999***)	-0.0537	-0.0356
c_2 (***Nov. 2000 to Oct. 2019***)	-0.0876	-0.0678
RMSE (***Nov. 1980 to Oct. 1999***)	2.9808	0.5757
RMSE (***Nov. 2000 to Oct. 2019***)	3.3342	0.6983

→ Looking at the table above, the values of c_2 become larger (or also known as getting closer to 0) in the more complex model, and the values of RMSE become smaller in the more complex model. I consider the c_2 of $y= c_1+ c_2t+c_3\cos2\pi t+c_4\sin2\pi t$ to be more accurate than the one in $y = c_1 + c_2t$ because the RMSE is smaller.

Question 3: $y=c_1+c_2t+c_3\cos2\pi t+c_4\sin2\pi t+c_5\cos4\pi t+c_6\sin4\pi t+c_7\cos6\pi t+c_8\sin6\pi t$

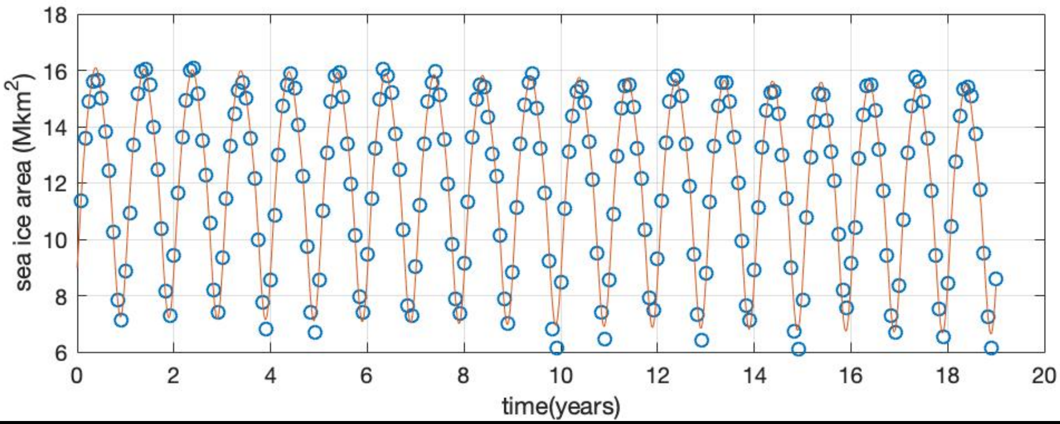
From Nov. 1980 to Oct. 1999

After loading the file seaice1.txt and denoting the sea ice area equals to the date in the file seaice1.txt. I input the following commands to obtain the linear model:

```
>> a=[ones(n,1) t cos(2*pi*t) sin(2*pi*t) cos(4*pi*t) sin(4*pi*t) cos(6*pi*t) sin(6*pi*t)];
>> b = y;
ata = a'*a

ata =

   1.0e+04 *

    0.0228    0.2175    0.0000   -0.0000   -0.0000   -0.0000   -0.0000   -0.0000
    0.2175    2.7617    0.0010   -0.0035    0.0009   -0.0016    0.0009   -0.0010
    0.0000    0.0010    0.0114   -0.0000    0.0000   -0.0000    0.0000    0.0000
   -0.0000   -0.0035   -0.0000    0.0114   -0.0000    0.0000   -0.0000    0.0000
   -0.0000    0.0009    0.0000   -0.0000    0.0114   -0.0000   -0.0000    0.0000
   -0.0000   -0.0016   -0.0000    0.0000   -0.0000    0.0114   -0.0000   -0.0000
   -0.0000    0.0009    0.0000   -0.0000   -0.0000   -0.0000    0.0114   -0.0000
   -0.0000   -0.0010    0.0000    0.0000    0.0000   -0.0000   -0.0000    0.0114

>> atb = a'*b

atb =

   1.0e+04 *

    0.2733
    2.5709
   -0.0363
    0.0303
   -0.0029
    0.0067
    0.0013
    0.0028
```

```
>> c=ata\atb

c =

   12.3086
   -0.0337
   -3.1834
    2.6453
   -0.2510
    0.5858
    0.1166
    0.2453
```

```
>> y1=c(1)+c(2)*t1+c(3)*cos(2*pi*t1)+c(4)*sin(2*pi*t1)+c(5)*cos(4*pi*t1)+ c(6)*sin(4*pi*t1)+ c(7)*cos(6*pi*t1)+ c(8)*sin(6*pi*t1);
>> t1=0:.01:19;
y1=c(1)+c(2)*t1+c(3)*cos(2*pi*t1)+c(4)*sin(2*pi*t1)+c(5)*cos(4*pi*t1)+ c(6)*sin(4*pi*t1)+ c(7)*cos(6*pi*t1)+ c(8)*sin(6*pi*t1);
plot(t,y,'o',t1,y1)
xlabel('time(years)')
ylabel('sea ice area (Mkm^2)')
>> grid
```




```
>> c(2)

ans =

   -0.0337
```

```
>> RMSE = norm(c(1)+c(2)*t+c(3)*cos(2*pi*t)+c(4)*sin(2*pi*t)+c(5)*cos(4*pi*t)+ c(6)*sin(4*pi*t)+ c(7)*cos(6*pi*t)+ c(8)*sin(6*pi*t)-y)/sqrt(n)

RMSE =

    0.3026
```

So, RMSE = 0.3026

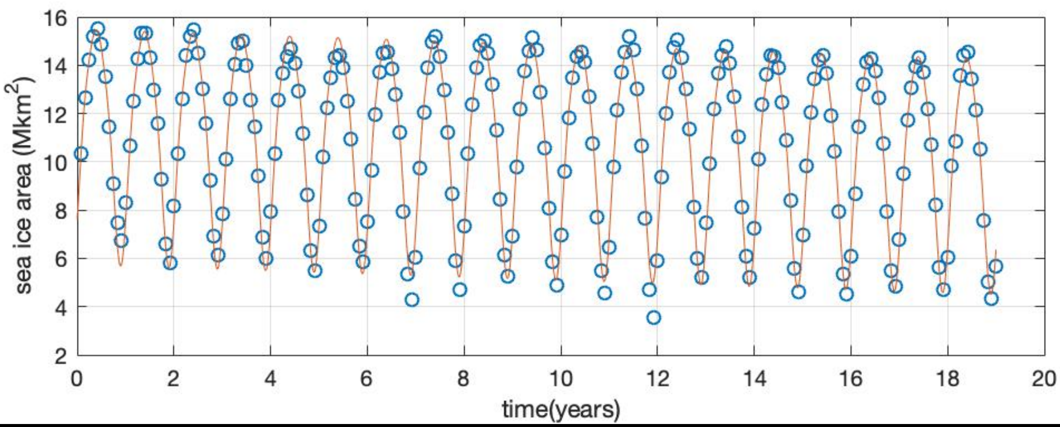
From Nov. 2000 to Oct. 2019

After loading the file seaice2.txt and denoting the sea ice area equals to the date in the file seaice2.txt. I input the following commands to obtain the linear model:

```
>> a=[ones(n,1) t cos(2*pi*t) sin(2*pi*t) cos(4*pi*t) sin(4*pi*t) cos(6*pi*t) sin(6*pi*t)];
b = y;
ata = a'*a

ata =

   1.0e+04 *

    0.0228    0.2175    0.0000   -0.0000   -0.0000   -0.0000   -0.0000   -0.0000
    0.2175    2.7617    0.0010   -0.0035    0.0009   -0.0016    0.0009   -0.0010
    0.0000    0.0010    0.0114   -0.0000    0.0000   -0.0000    0.0000    0.0000
   -0.0000   -0.0035   -0.0000    0.0114   -0.0000    0.0000   -0.0000    0.0000
   -0.0000    0.0009    0.0000   -0.0000    0.0114   -0.0000   -0.0000    0.0000
   -0.0000   -0.0016   -0.0000    0.0000   -0.0000    0.0114   -0.0000   -0.0000
   -0.0000    0.0009    0.0000   -0.0000   -0.0000   -0.0000    0.0114   -0.0000
   -0.0000   -0.0010    0.0000    0.0000    0.0000   -0.0000   -0.0000    0.0114

>> atb = a'*b

atb =

   1.0e+04 *

    0.2460
    2.2876
   -0.0414
    0.0327
   -0.0029
    0.0088
    0.0008
    0.0029
```

```
>> c=ata\atb

c =

   11.4154
   -0.0654
   -3.6282
    2.8503
   -0.2488
    0.7586
    0.0732
    0.2531
```

```
>> t1=0:.01:19;
y1=c(1)+c(2)*t1+c(3)*cos(2*pi*t1)+c(4)*sin(2*pi*t1)+c(5)*cos(4*pi*t1)+ c(6)*sin(4*pi*t1)+ c(7)*cos(6*pi*t1)+ c(8)*sin(6*pi*t1);
plot(t,y,'o',t1,y1)
xlabel('time(years)')
ylabel('sea ice area (Mkm^2)')
grid
```




```
>> c(2)

ans =

   -0.0654
```

```
>> RMSE = norm(c(1)+c(2)*t+c(3)*cos(2*pi*t)+c(4)*sin(2*pi*t)+c(5)*cos(4*pi*t)+ c(6)*sin(4*pi*t)+ c(7)*cos(6*pi*t)+ c(8)*sin(6*pi*t)-y)/sqrt(n)

RMSE =

    0.3667
```

So RMSE = 0.3667

Comparing Table

	$y = c_1 + c_2t$	$y=c_1+c_2t+c_3\cos 2\pi t+c_4\sin 2\pi t+c_5\cos 4\pi t+c_6\sin 4\pi t+c_7\cos 6\pi t+c_8\sin 6\pi t$
c_2 (***Nov. 1980 to Oct. 1999***)	-0.0537	-0.0337
c_2 (***Nov. 2000 to Oct. 2019***)	-0.0876	-0.0654
RMSE (***Nov. 1980 to Oct. 1999***)	2.9808	0.3026
RMSE (***Nov. 2000 to Oct. 2019***)	3.3342	0.3667

→ Looking at the table above, the values of c_2 become larger (or also known as getting closer to 0) in the more complex model, and the values of RMSE become smaller in the more complex model. I consider the c_2 of $y = c_1 + c_2t + c_3\cos 2\pi t + c_4\sin 2\pi t + c_5\cos 4\pi t + c_6\sin 4\pi t + c_7\cos 6\pi t + c_8\sin 6\pi t$ to be more accurate than the one in $y = c_1 + c_2t$ because the RMSE is smaller.

<u>Question 4:</u>

	$y = c_1 + c_2t$	$y= c_1+ c_2t + c_3\cos 2\pi t + c_4\sin 2\pi t$	$y=c_1+c_2t+c_3\cos 2\pi t+c_4\sin 2\pi t+c_5\cos 4\pi t+c_6\sin 4\pi t+c_7\cos 6\pi t+c_8\sin 6\pi t$
c_2 (***Nov. 1980 to Oct. 1999***)	-0.0537	-0.0356	-0.0337

c_2 (***Nov. 2000 to Oct. 2019***)	-0.0876	-0.0678	-0.0654
RMSE (***Nov. 1980 to Oct. 1999***)	2.9808	0.5757	0.3026
RMSE (***Nov. 2000 to Oct. 2019***)	3.3342	0.6983	0.3667

As I increase the complex model, RMSE is getting smaller.
The sea ice is decreasing because all c2 are lesser than 0.
The rate of change c2 is different between the two 19-year periods. For all three models, c2 of the second period are all smaller than the first period, so the ice is melting more in the second period.