Project 1

Can you get ten correct digits from the Bisection Method?

The function

$$f(x) \quad = \quad 98e^{5x^2} - (1225x^4 + 280x^2 + 66)e^{3/7}$$

has exactly one root. Your assignment is to use the Bisection Method to calculate this root accurately to 10 decimal places, if possible. We know that in theory, the Bisection Method finds a root to as much precision as we ask for. MATLAB does all calculations in double precision floating point, equivalent to about 16 decimal digits.

1. Use the Intermediate Value Theorem to show that a root is guaranteed in the interval $[0, 1]$. Apply the Bisection Method on the starting interval $[0, 1]$ to find the root to 10 correct places. If using the textbook's code, choose a tolerance `TOL` that will guarantee at least 10 correct digits. Report your approximate root r, **rounded to 10 digits** after the decimal point. Repeat the Bisection Method for at least two other starting intervals $[0, b]$ where $1 < b < 1.5$. Do your results for different starting intervals agree with one another, up to 10 decimal places?

2. The backward error, or checking error, is $|f(r)|$. Calculate it for the approximate root(s) you found in Step 1. Are you confident you have the correct first 10 digits of the root?

3. Now repeat Steps 1 and 2 after replacing the 66 in the function with 65. Just as for $f(x)$, *the revised function* $f_{\text{new}}(x)$ *has exactly one root in the interval* $0 \leq x \leq 1$.

4. Summarize any differences between the two cases, f and $f_{\text{new}}(x)$. Do you believe that you have found the root to 10 correct places in both cases? If not 10, how many? Explain your reasoning.

Begin your report by stating your conclusions about the four questions above. Save the Matlab code used and your Matlab session, and include these with your report. Save your report as a .pdf file and upload it to Blackboard.

Due: Thurs., Sept. 5