Project 3
Convergence of Newton's Method and Multiplicity of Roots

Let's return to the equation

$$f(x) = 98e^{5x^2} - (1225x^4 + 280x^2 + 66)e^{3/7} = 0$$

from Project 1. It has one real solution. You are asked to find the solution using Newton's method, and study the convergence to the solution. Then we'll repeat for the second version of the equation.

1. Use Newton's method to calculate the root to as many correct decimal places as you can. For each root, underline or highlight the digits you are confident are correct.

2. If Newton's Method converges to the root quadratically, calculate the ratio

$$\lim_{i\to\infty} \frac{e_{i+1}}{e_i^2}$$

   from the Newton iteration and compare with the theoretical value (from calculus). The ratio and the theoretical value should agree to at least a couple of decimal places.

   On the other hand, if Newton's Method converges to the root linearly, calculate the ratio

$$\lim_{i\to\infty} \frac{e_{i+1}}{e_i}$$

   from the Newton iteration, and deduce the multiplicity of the root from the ratios.

3. Repeat Steps 1 and 2 for the equation $f_{\text{new}}(x) = 0$ obtained by replacing 66 with 65. To compute the errors accurately, you may use the fact that the root of $f_{\text{new}}(x) = 0$ is known to us to be exactly $\sqrt{3/35}$.

Begin your report by answering the questions 1 – 3 above. Please upload one file (PDF) only to Blackboard. Include the Matlab code used and your Matlab session.

Due: Thurs., Sept. 19