Exercises - April 1 2019

Exercise 1

- the time it takes a student to complete a TOLC-I, University orientation and evaluation test follows a density function of the form

$$f(X) = \begin{cases} c(t-1)(2-t) & 1 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

  where t is the time in hours.

a) using the `integrate()` R function, determine the contant c (and verify it analytically)

b) write the set of four R functions and plot the pdf and cdf, respectively

c) evaluate the probability that the student will finish the aptitude test in more than 75 minutes. And that it will take 90 and 120 minutes.

Exercise 2

- the lifetime of tires sold by an used tires shop is $10^4 \cdot X$ km, where x is a random variable following the distribution funcion

$$f(X) = \begin{cases} 2/x^2 & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

a) write the set of four R functions and plot the pdf and cdf, respectively

b) determine the probability that tires will last less than 15000 km

c) sample 3000 random variables from the distribution and determine the mean value and the variance, using the expression $Var(X) = E[X^2] - E[X]^2$

Exercise 3

- Marokov's inequality represents an upper boud to probability distributions:

$$P(X \geq k) \leq \frac{E[X]}{k} \text{ for } k > 0$$

- having defined a function

$$G(k) = 1 - F(k) \equiv P(X \geq k)$$

  plot $G(k)$ and the Markov's upper bound for

a) the exponential, $\text{Exp}(\lambda = 1)$, distribution function

b) the uniform, $\mathcal{U}(3, 5)$, distribution function

c) the binomial, $\text{Bin}(n = 1, p = 1/2)$, distribution function

d) a Poisson, $\text{Pois}(\lambda = 1/2)$, distribution function

Exercise 4

- Chebyshev's inequality tell us that

$$P(\left|X - \mu\right| \geq k\sigma) \leq \frac{1}{k^2}$$

- which can also be written as

$$P(\left|X - \mu\right| < k\sigma) \geq 1 - \frac{1}{k^2}$$

- use R to show, with a plot, that Chebyshev's inequality is is an upper bound to the following distributions:

a) a normal distribution, $\mathrm{N}(\mu = 3, \sigma = 5)$

a) an exponential distribution, $\mathrm{Exp}(\lambda = 1)$

b) a uniform distribution $\mathcal{U}(1 - \sqrt{2}, 1 + \sqrt{2})$

d) a Poisson, $\mathrm{Pois}(\lambda = 1/3)$, distribution function