Exercises - April 29 2019

Exercise 1

- Using Monte Carlo Methods evaluate the following integral

$$\iint_{\Omega} \sin \sqrt{\ln(x+y+1)}\ dx\ dy$$

- where

$$\Omega = \left\{ (x,y) : \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 \le \frac{1}{4} \right\}$$

Exercise 2

- using Monte Carlo methods, evaluate the volume of the region whose points satisfy the following inequalities:

$$\begin{cases} 0 \le x \le 1 \qquad 0 \le y \le \quad 1 \qquad 0 \le z \le 1 \\ x^2 + \sin y \le z \\ x - z + \mathrm{e}^y \le 1 \end{cases}$$

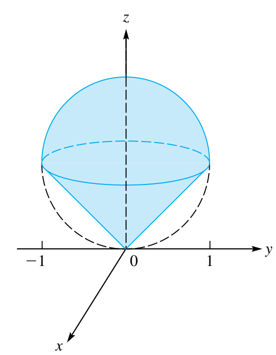
Exercise 3

- Consider the volume above the cone $z^2 = x^2 + y^2$ and inside the sphere

$$x^2 + y^2 + (z-1)^2 = 1$$

- the volume is contained in the box bounded by the inequalities

$$-1 \le x \le 1 \qquad -1 \le y \le 1 \qquad 0 \le z \le 2$$



Exercise 4

- Using the importance sampling method, Evaluate the integral

$$\int_b^\infty x^{\alpha-1} \mathrm{e}^{-x}\ dx$$

- with $\alpha > 1$ and $b > 0$

1) one possibility is to use as sampling function $g(x) = \mathrm{e}^{-x}$ in the domain $[b, \infty]$
2) a more efficient method, especially for large b, is to use $g(x) = \lambda \mathrm{e}^{-\lambda(x-b)}$

Exercise 5

- Using the importance sampling method, Evaluate the integral

$$\int_b^\infty x^{\alpha-1} \mathrm{e}^{-x}\ dx$$

- with $\alpha \le 1$ and $b > 0$

Hint: use the sampling function $g(x) = \mathrm{e}^{-(x-b)}$ in the domain $[b, \infty]$