CSE 6740 A/ISyE 6740, Fall 2023, Georgia Tech

Homework 0: Background review

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- This is an assessment of your linear algebra and probability background.
- The maximum time you are allowed to spend on this is **30 minutes**.
- This assessment will help us determine whether this class will be appropriate for you this semester or if you should take consider taking it later.
- To keep the assessment meaningful, please attempt to solve the questions yourself, without taking help from others and online resources.
- Please turn in your work at the end of class on a sheet of paper or on Gradescope.
- If you decide to take this class and get both the questions right, you will obtain a 1% bonus to your final grade.

1. (Trefethen and Bau 5.4) If $A \in \mathbb{C}^{m\times m}$, and the SVD of A is given by $A = U\Sigma V^*$, give an eigen decomposition of the $2m \times 2m$ matrix,

$$B = \begin{bmatrix} 0 & A^* \\ A & 0 \end{bmatrix}. \tag{1}$$

2. A student is given a surprise quiz consisting of n questions. The student is 50% sure of the first answer. For the ith question, the student knows the answer with probability $p_i = (i-1)/n, \;\; n \geq i > 1$. Define $s := 0.25 + \sum_{i=1}^{n-1} i(n-i)/n^2$. Estimate an upper bound for the probability that the student's score falls outside the range $(n/2 - 2\sqrt{s}, n/2 + 2\sqrt{s})$. Hint: use Chebyshev's inequality.