Lecture 17: Midterm 1 and problem-solving

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- Today: Midterm 1 and problem-solving
- After this: CNNs, VAEs, feature extraction/Dimension reduction

Asking questions

Understanding Deep Learning (Still) Requires Rethinking Generalization

By **Chiyuan Zhang**, **Samy Bengio**, **Moritz Hardt**, **Benjamin Recht**, and **Oriol Vinyals**

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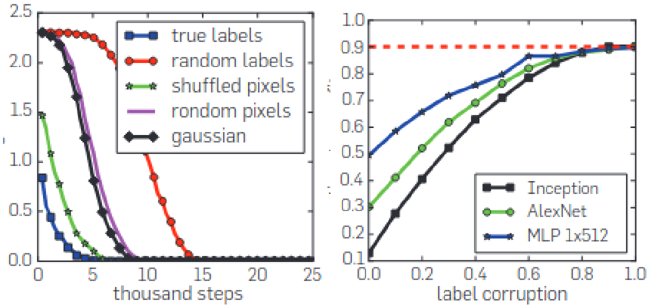
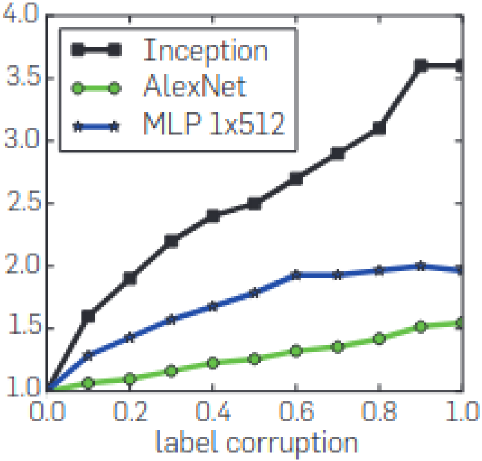
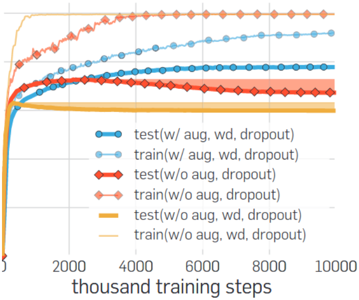


Does our understanding of generalization hold?

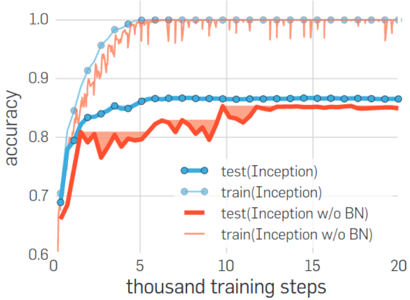


- Hand-wavy: test error is higher when we expect complexity to be higher.

How to improve generalization?



(a) Inception on ImageNet



(b) Inception on CIFAR10

- Early stopping (implicit regularization)

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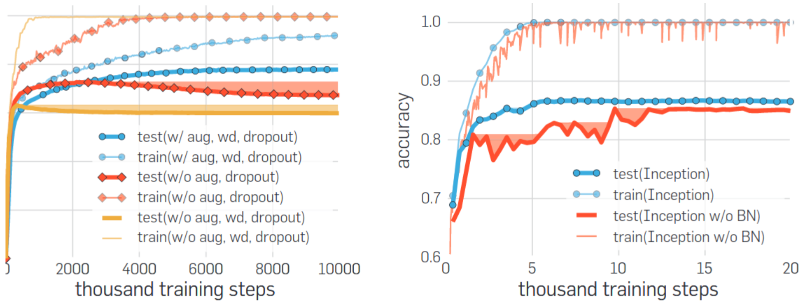

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- Batch normalization: normalize the inputs to each layer for each mini-batch, i.e., make the inputs have zero mean and unit variance.

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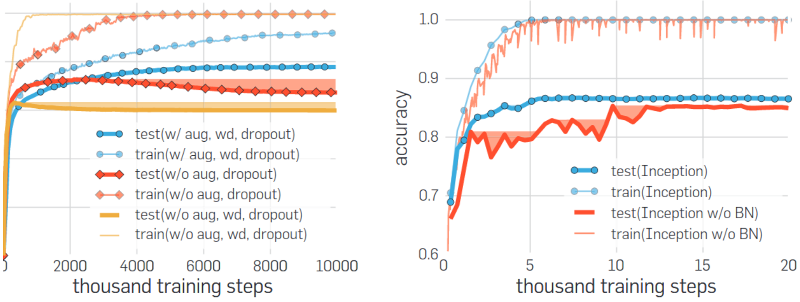

(a) Inception on ImageNet

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- Early stopping (implicit regularization)
- Batch normalization: normalize the inputs to each layer for each mini-batch, i.e., make the inputs have zero mean and unit variance.
- Dropout: randomly set some activations to zero.

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- The role of the data distribution...
- Bubeck Sellke 2021:

2. *The distribution μ of the covariates x_i satisfies isoperimetry (or is a mixture theoreof).*
3. *The expected conditional variance of the output (i.e., the "noise level") is strictly positive, denoted $\sigma^2 := \mathbb{E}^{\mu}[Var[y|x]] > 0$.*

Then, with high probability over the sampling of the data, one has simultaneously for all $f \in \mathcal{F}$:

$$\frac{1}{n}\sum_{i=1}^{n}(f(x_i) - y_i)^2 \leq \sigma^2 - \epsilon \;\Rightarrow\; \mathrm{Lip}(f) \geq \widetilde{\Omega}\left(\epsilon\sqrt{\frac{nd}{p}}\right).$$

What are the assumptions?

- Isoperimetry: if for an l-Lipschitz function $h : \mathbb{R}^d \to \mathbb{R}$, $\mathbb{P}[|h(X) - Eh| \geqslant t] \leqslant 2e^{(-dt^2)/(2cl^2)}$, then, the distribution of X is c-isoperimetric.
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- For imagenet, Bubeck and Sellke estimate needing $\mathcal{O}(10^{10} - 10^{11})$ parameters.

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- ℓ^2 regularization leads to better generalization. Why?

Regularization and generalization

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- Rademacher complexity of linear class (on features) lower.

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- Repetition helps!

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- Flourishing? “Mathematics for human flourishing” - Francis Su