Computing to understand complex systems

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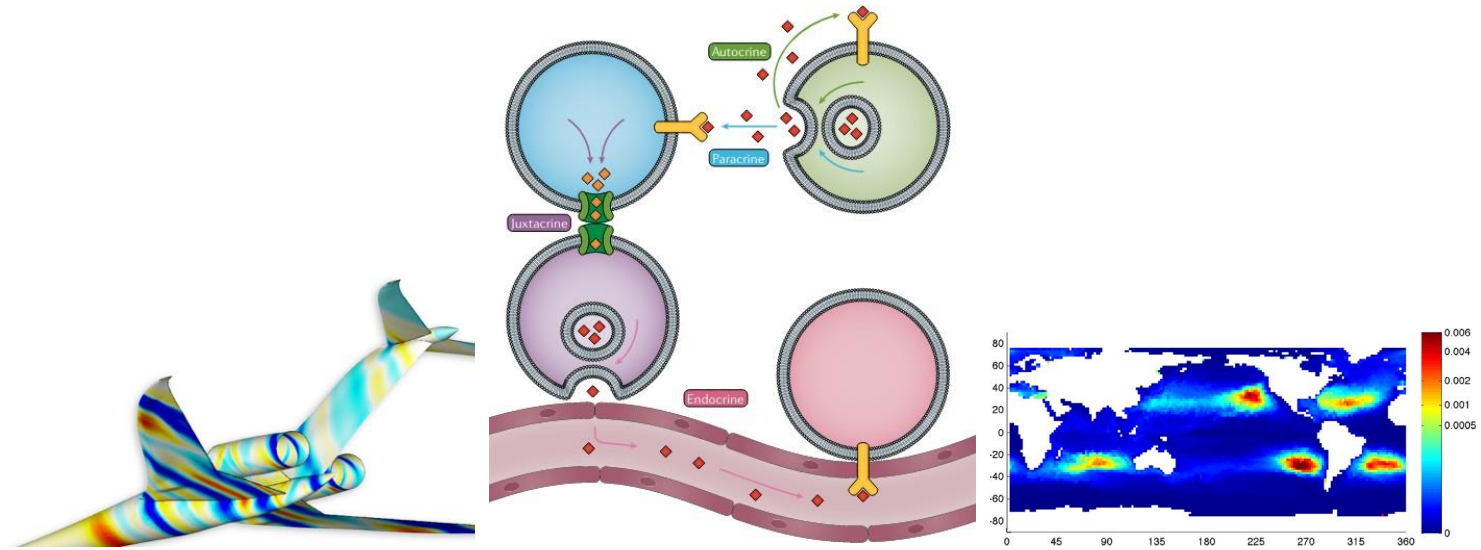
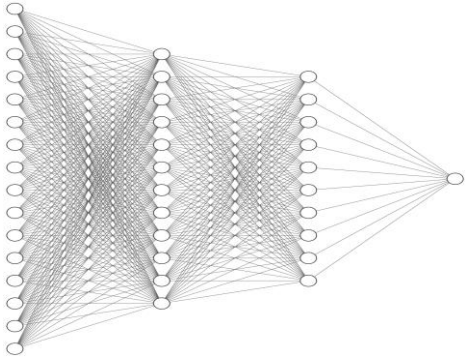
Computational dynamics + statistics

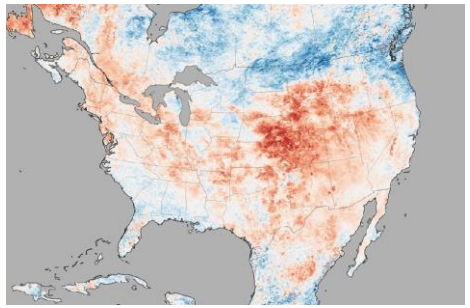
- **Goal**: To answer fundamental scientific questions around complex dynamics



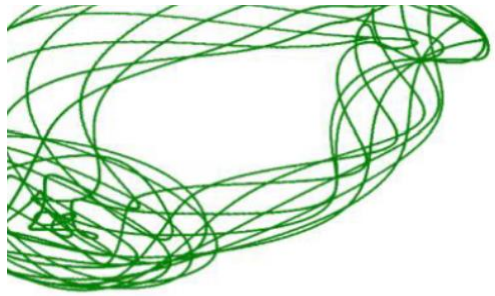
Left: Aftosmis et al 2015; Center: Armingol et al 2021; Right: Froyland et al 2014

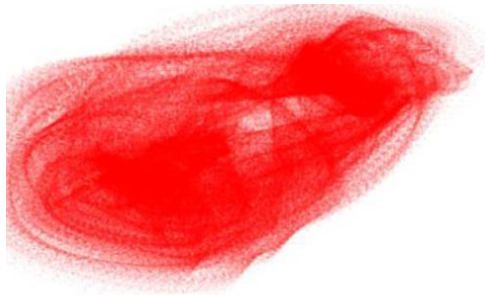
Application areas



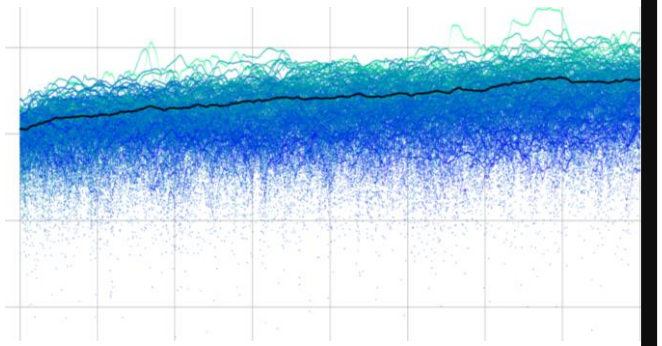


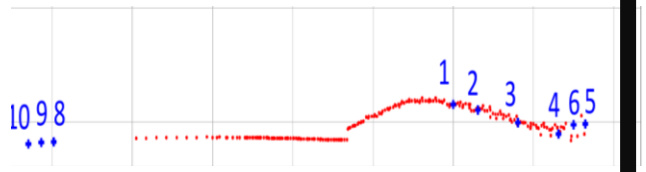
Information from numerical models





Computational methods development

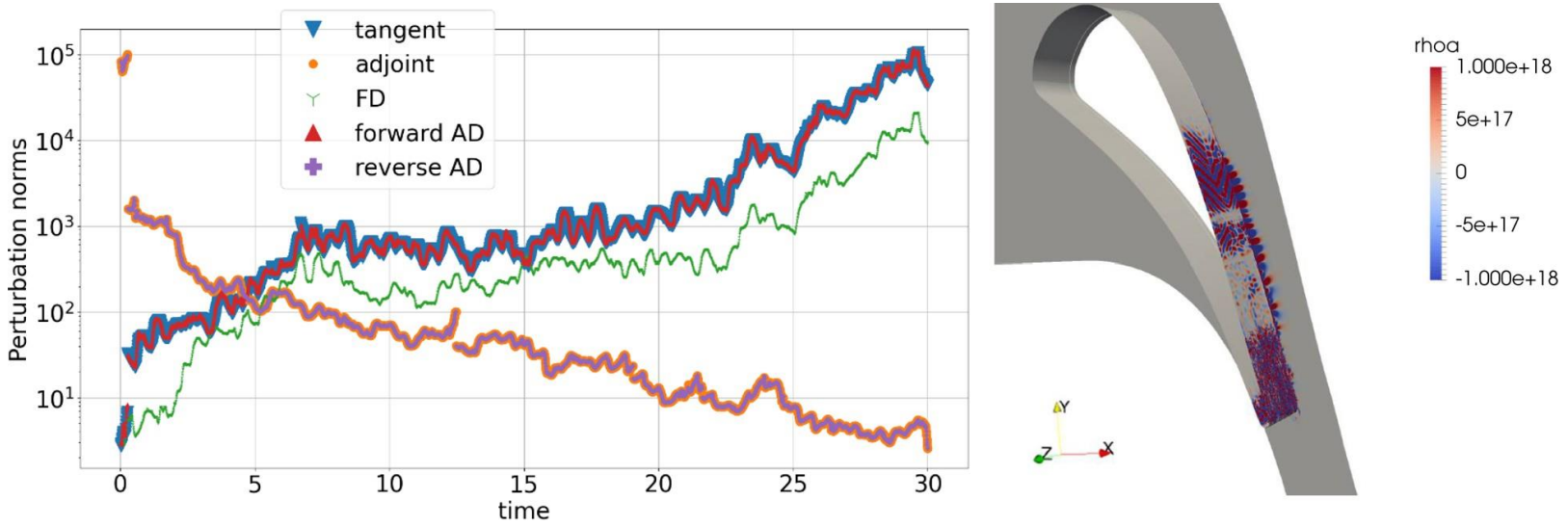




- Optimization and design
- Sensitivity analysis and control
- Order reduction
- Data assimilation

Insights obtained from rigorous algorithms and analyses

Computing with **chaotic** systems

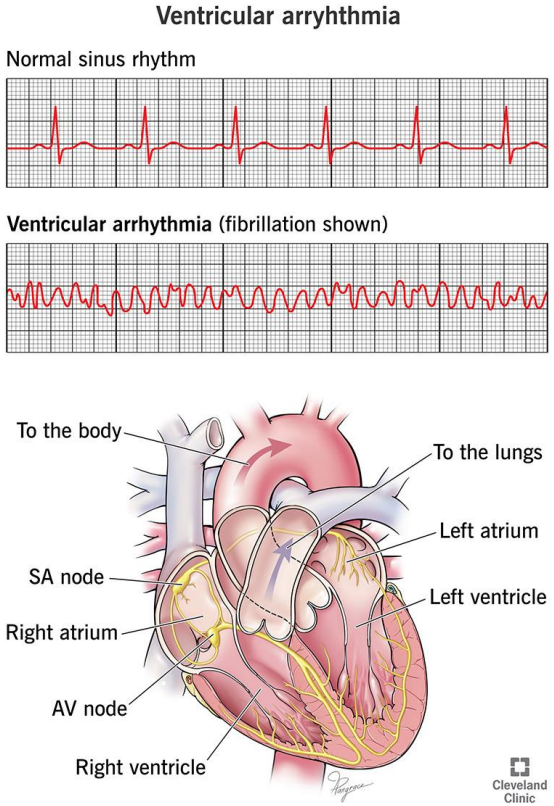


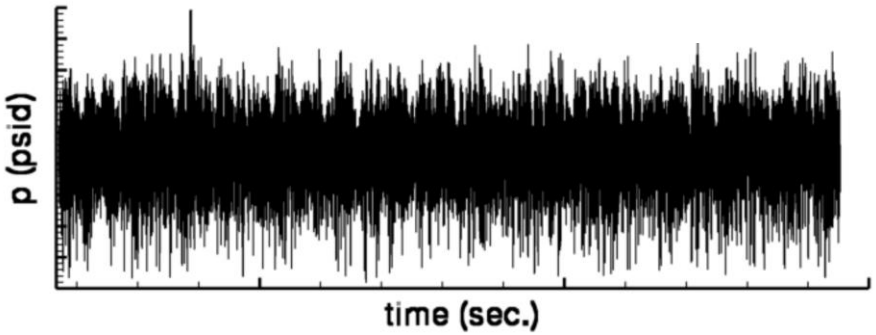
Goal: Tractable computation of **linear response**

$$\frac{d}{ds} \lim_{N \to \infty} \langle J \rangle_N , \tag{1}$$

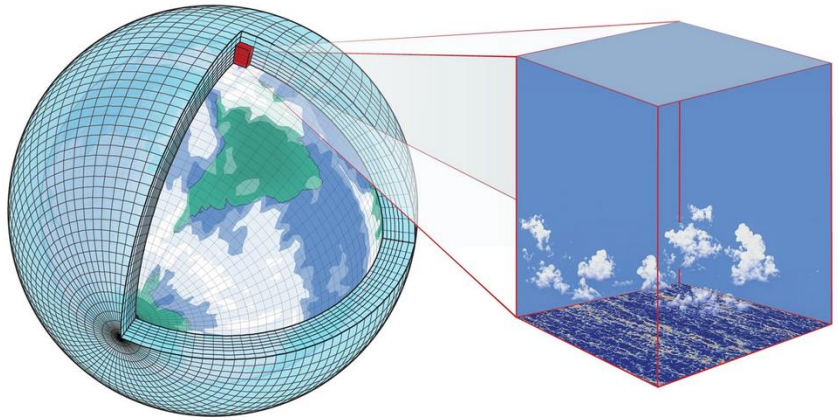
s: Design input, $\langle J \rangle_N$:N-time average of an observable J.
Downstream applications: optimization, data assimilation, model selection, mechanistic understanding etc.

Computing **sensitivities** will help with UQ, optimization





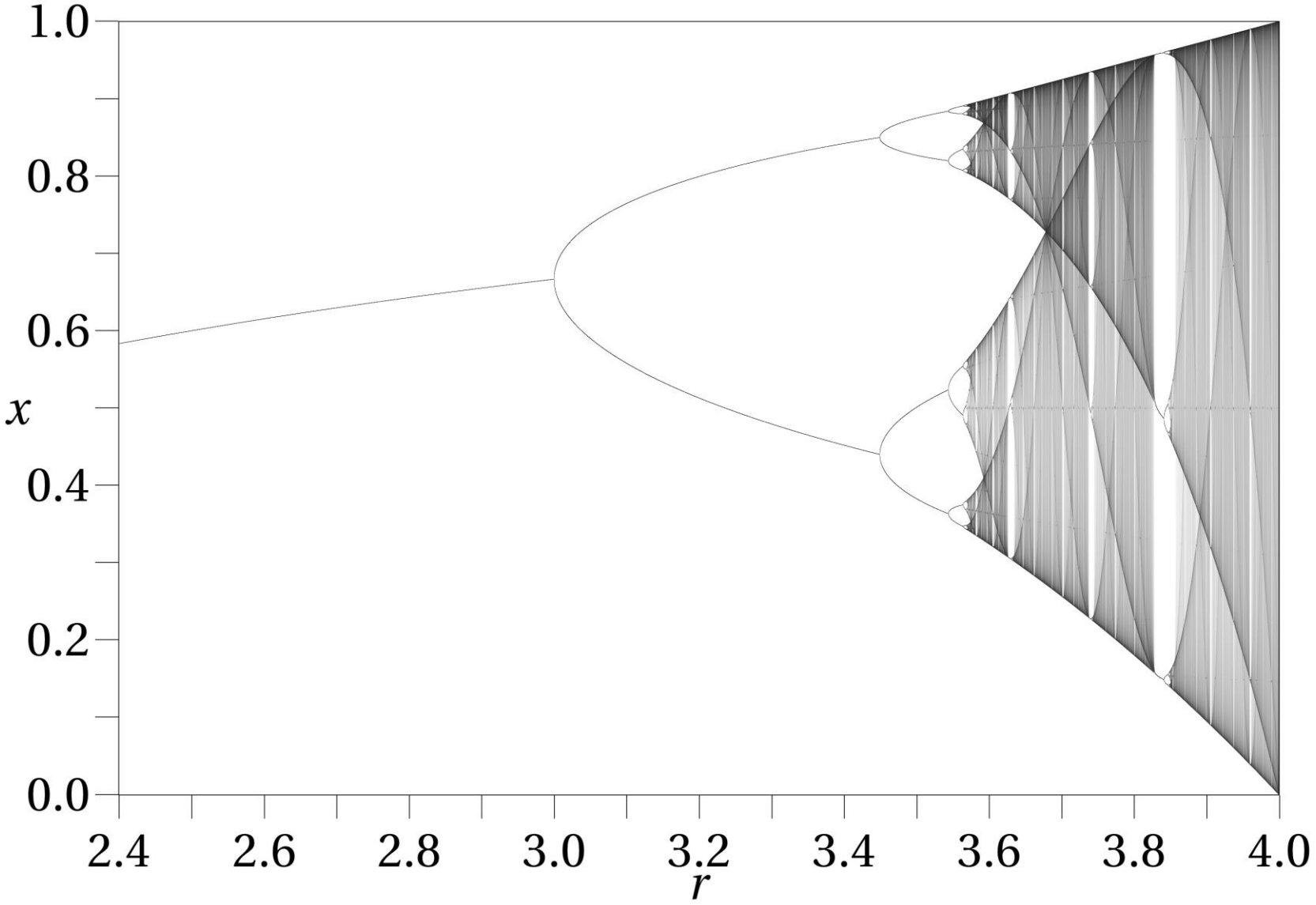
Alter et al 2015



Schneider, T., Teixeira, J., Bretherton, C. et al. 2017

Today: the logistic map

$$\varphi(x) = r\,x\,(1-x), \qquad x \in (0,1)$$



Today: the logistic map

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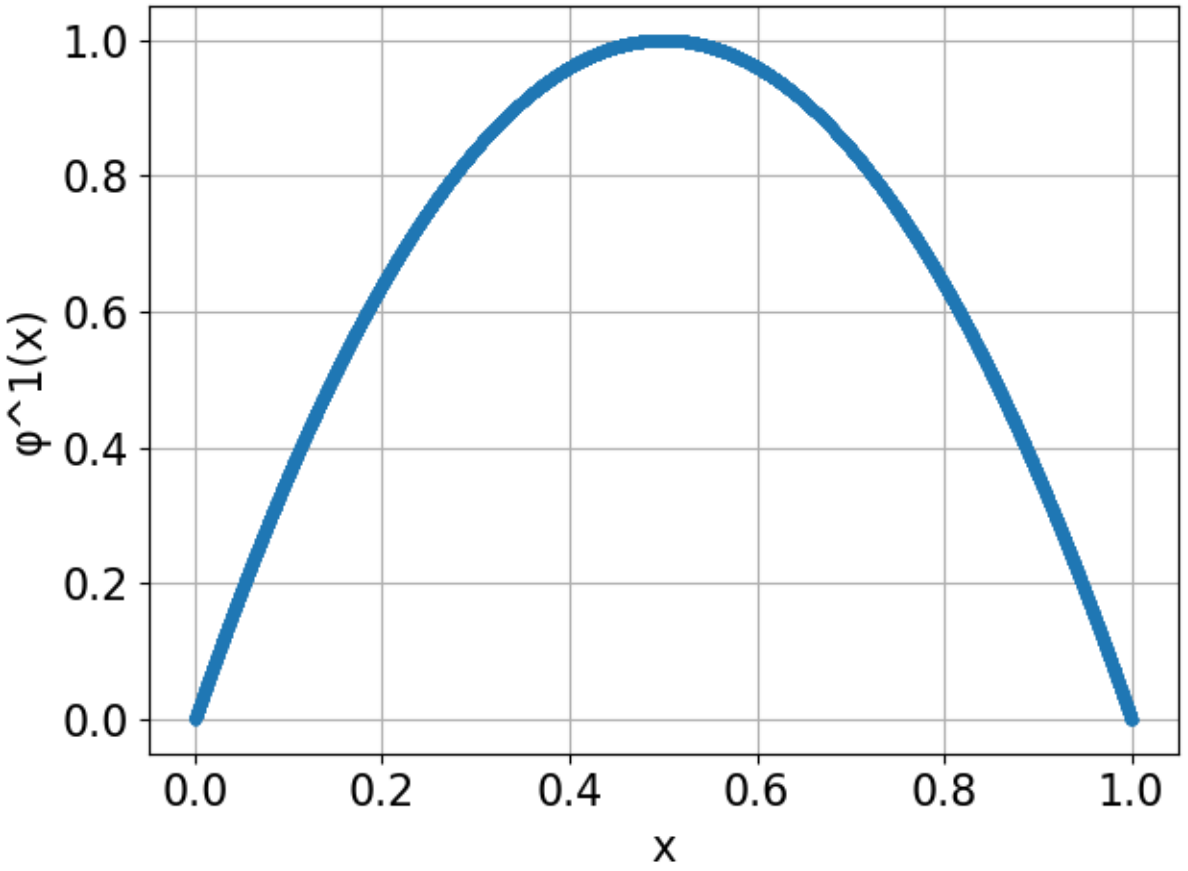
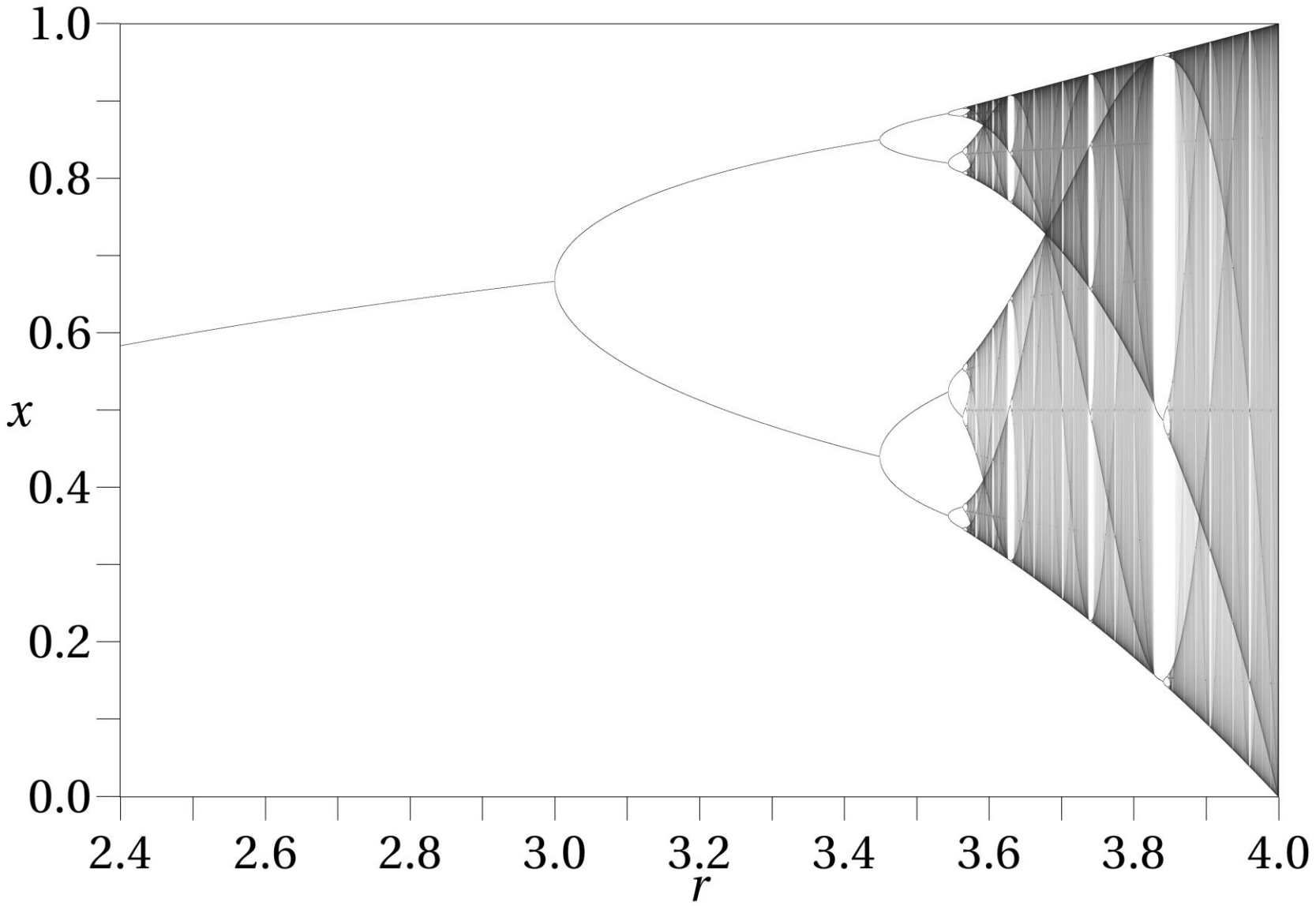
Orbit: $x, \varphi(x), \varphi^2(x), \varphi^3(x), \varphi^4(x), \ldots, \varphi^{10}(x), \ldots, \varphi^{1000}(x), \ldots$

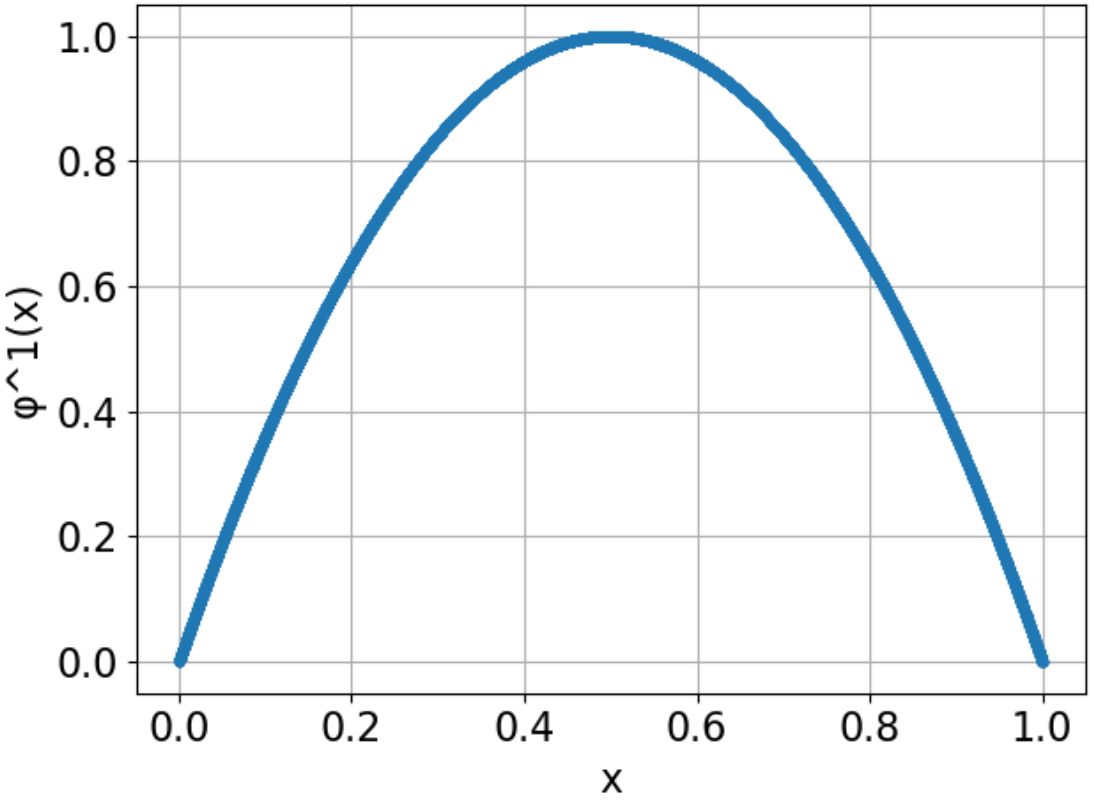


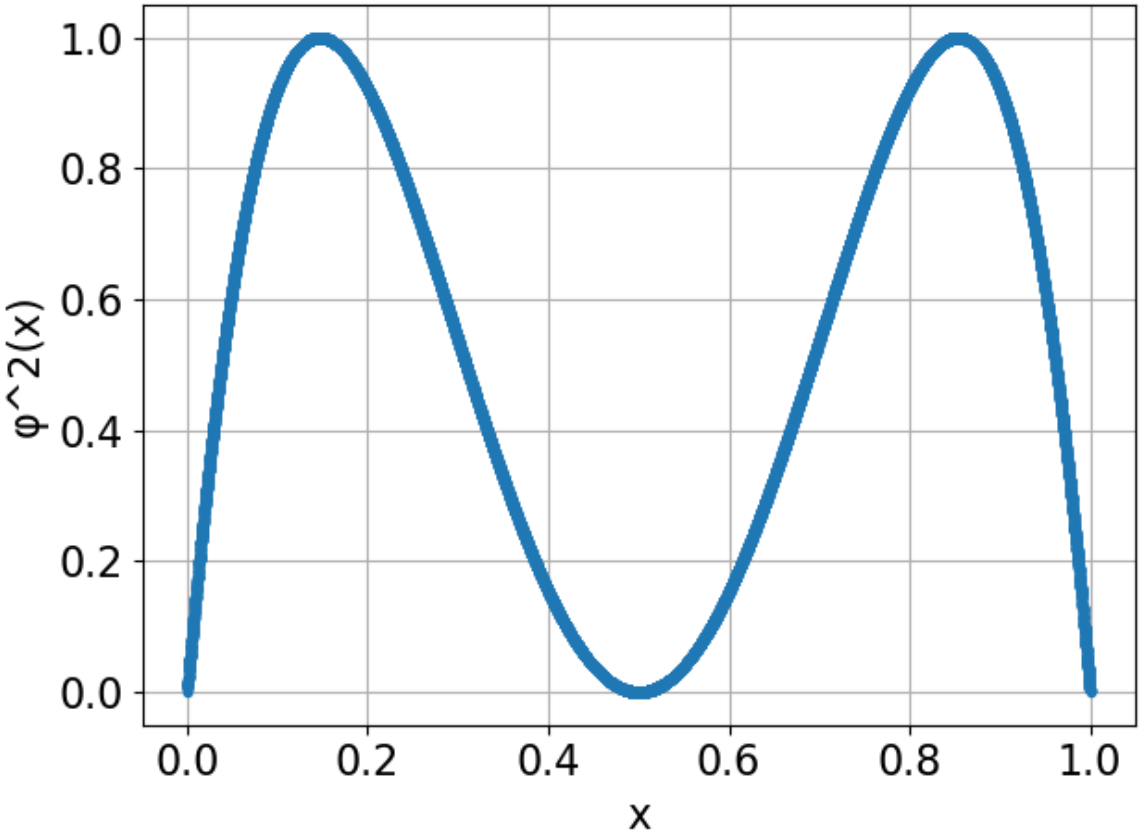
Bifurcations: qualitatively different behavior of orbits as parameters are varied



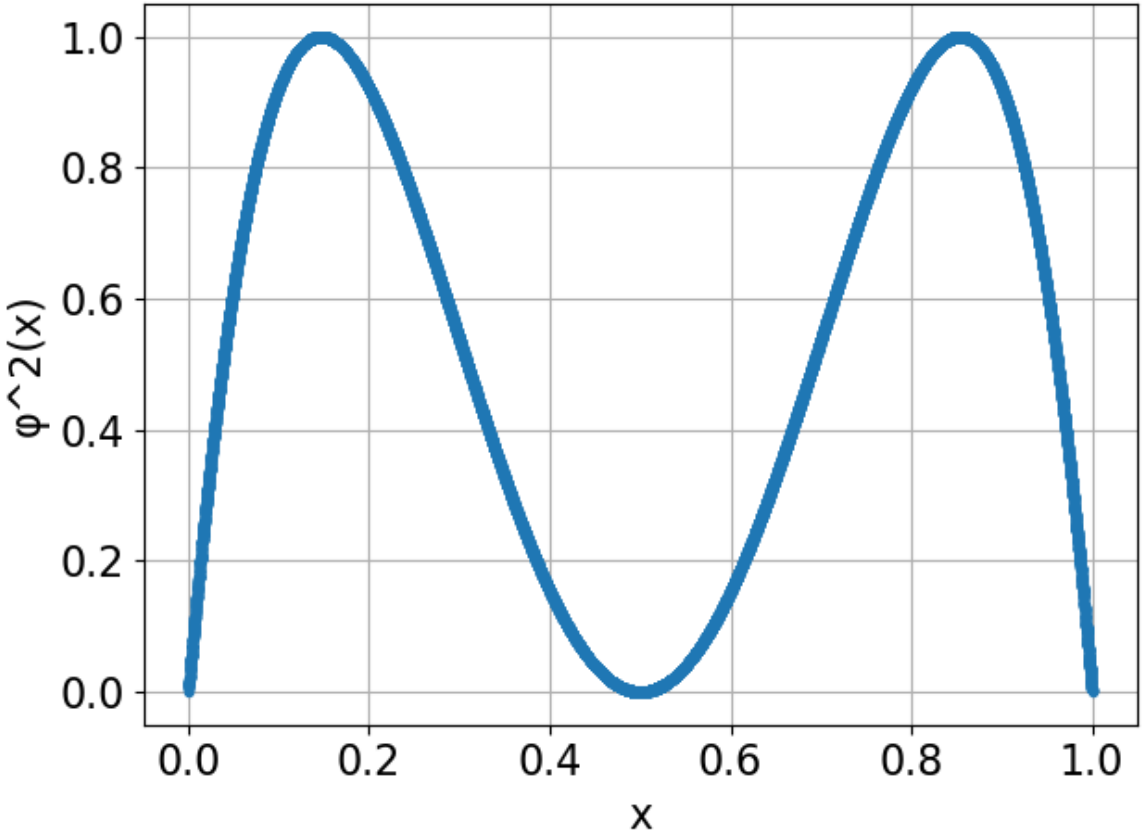
- Introduced chaos to the world of ecology
- Model of competition within a population
- Number of competing interactions quadratic
- X refers to percent of maximum population
- Fix rate r = 4
- Question: at a given time, is the population as likely to be < 50 percent as > 50 percent?

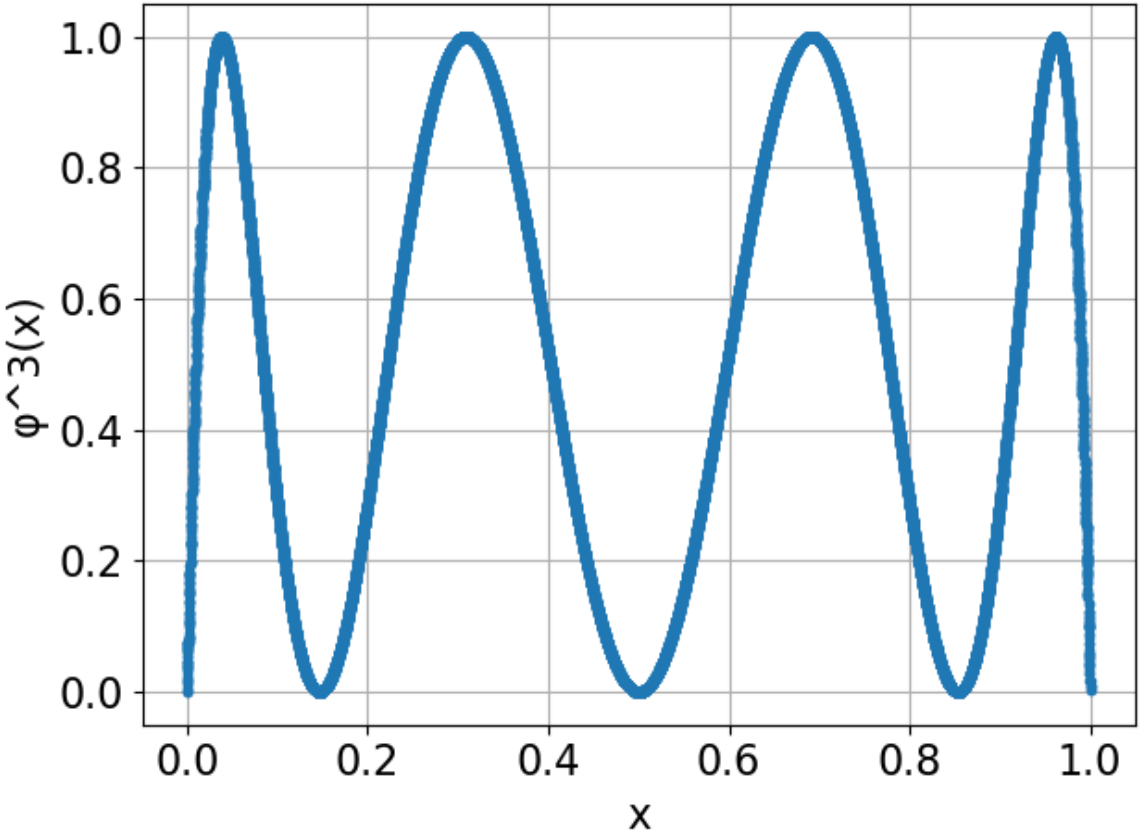
Evolution of ensembles under the dynamics



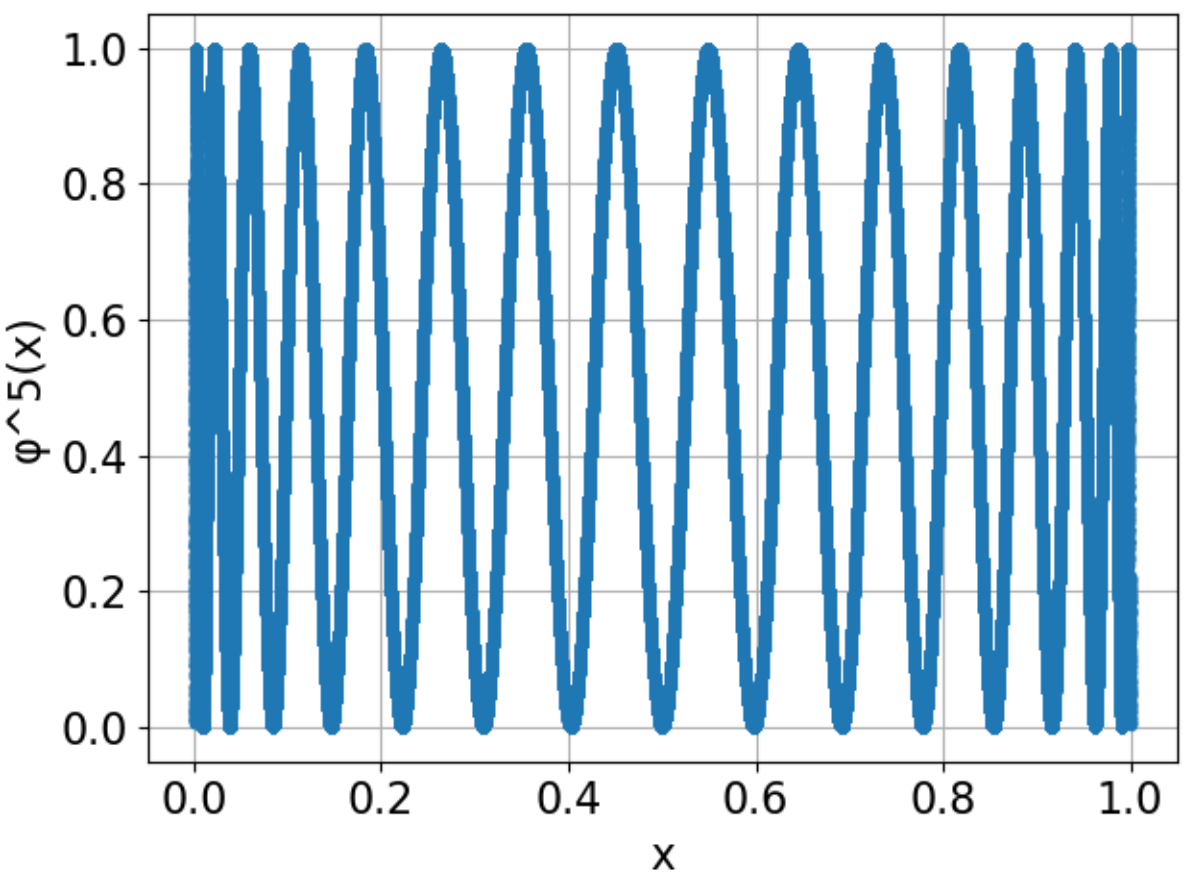


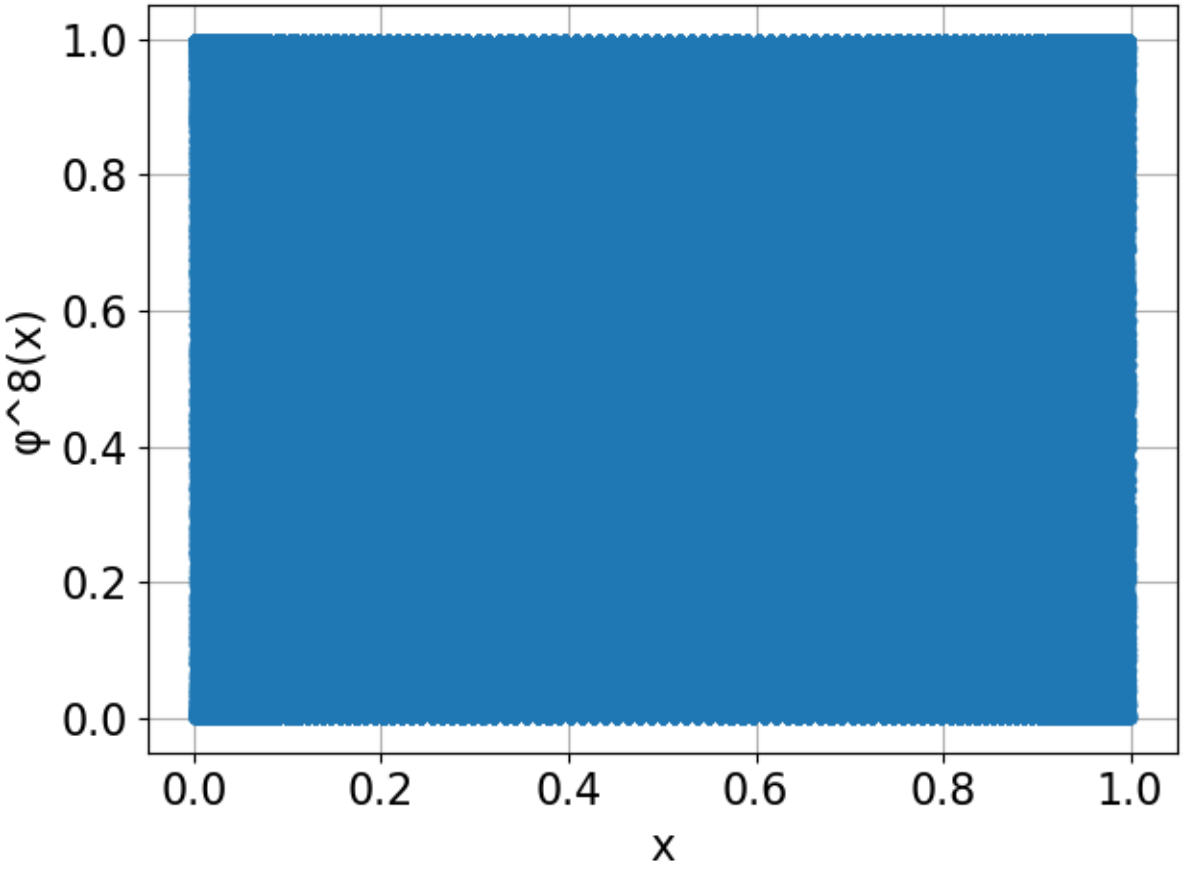
Long-term dynamical behavior of orbits





Chaotic systems: deterministic but appear random





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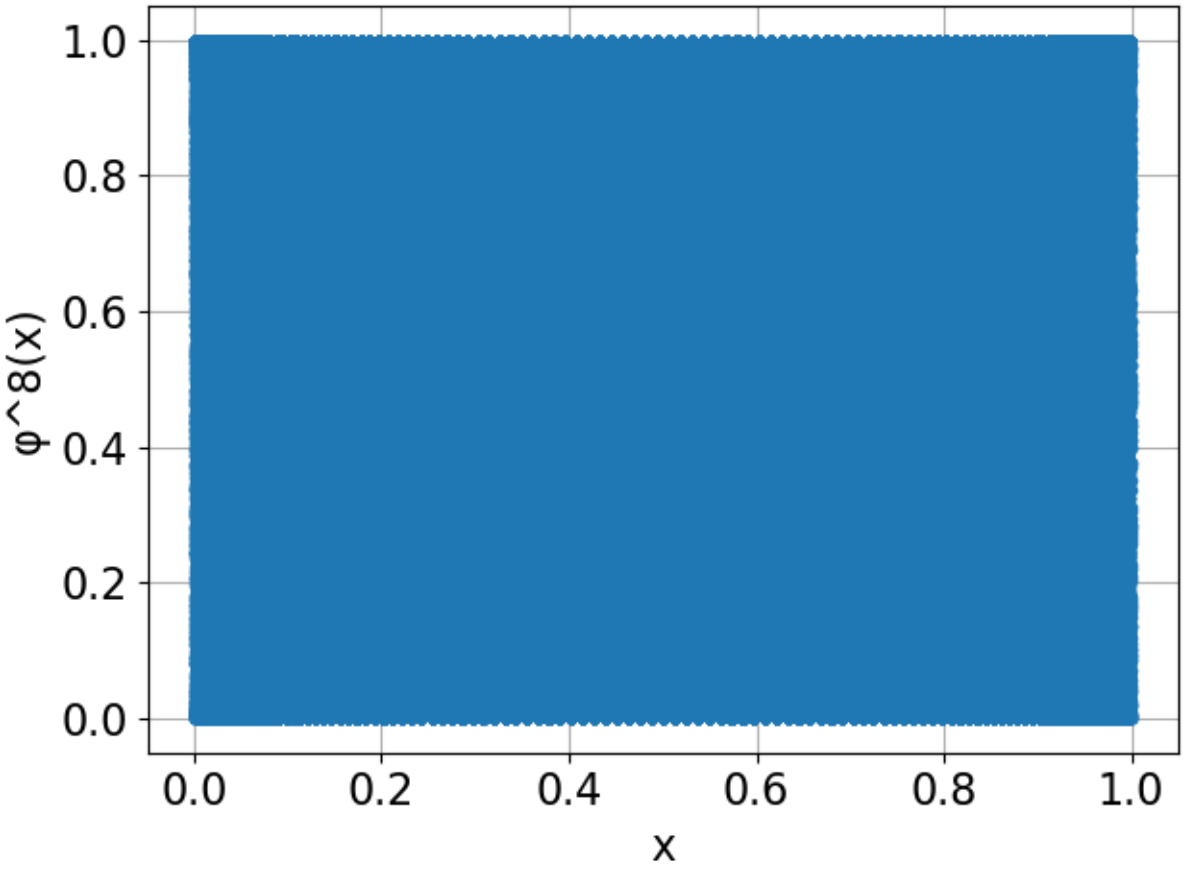
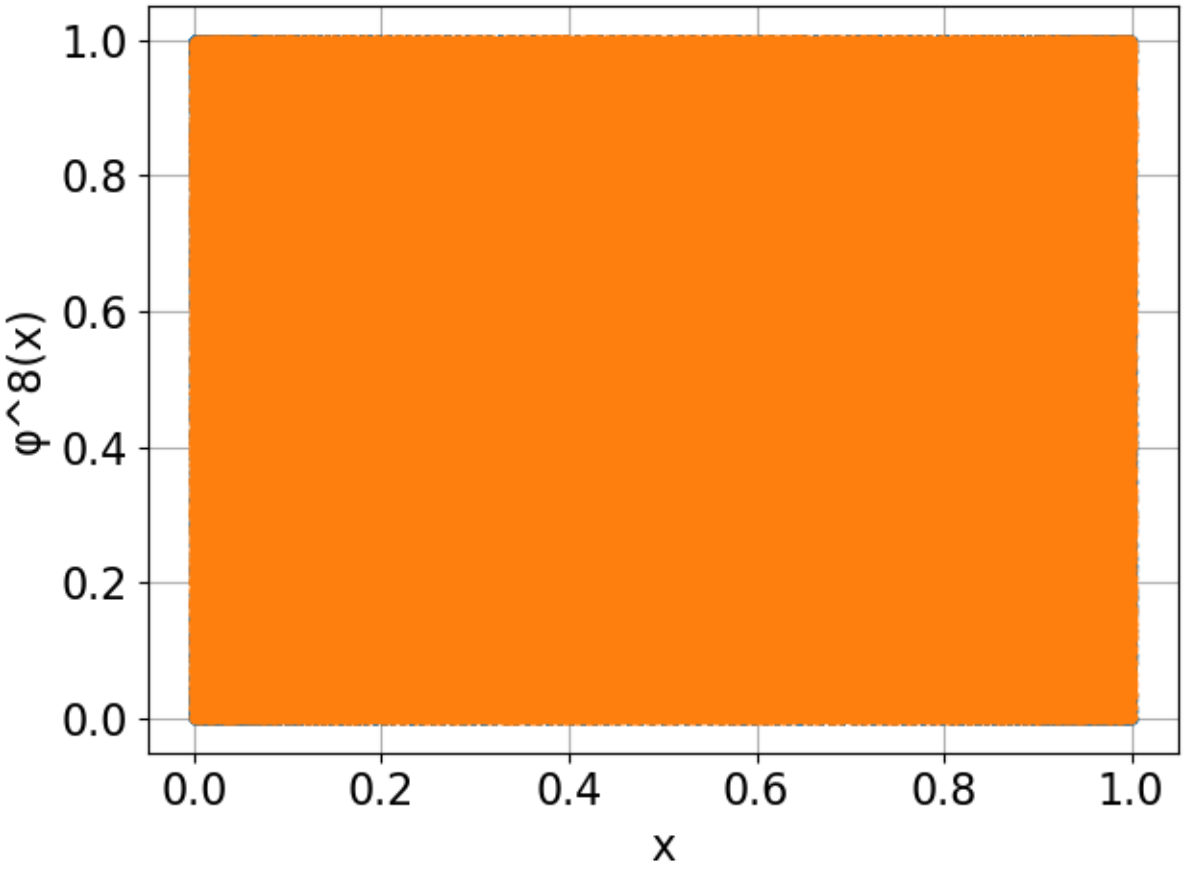
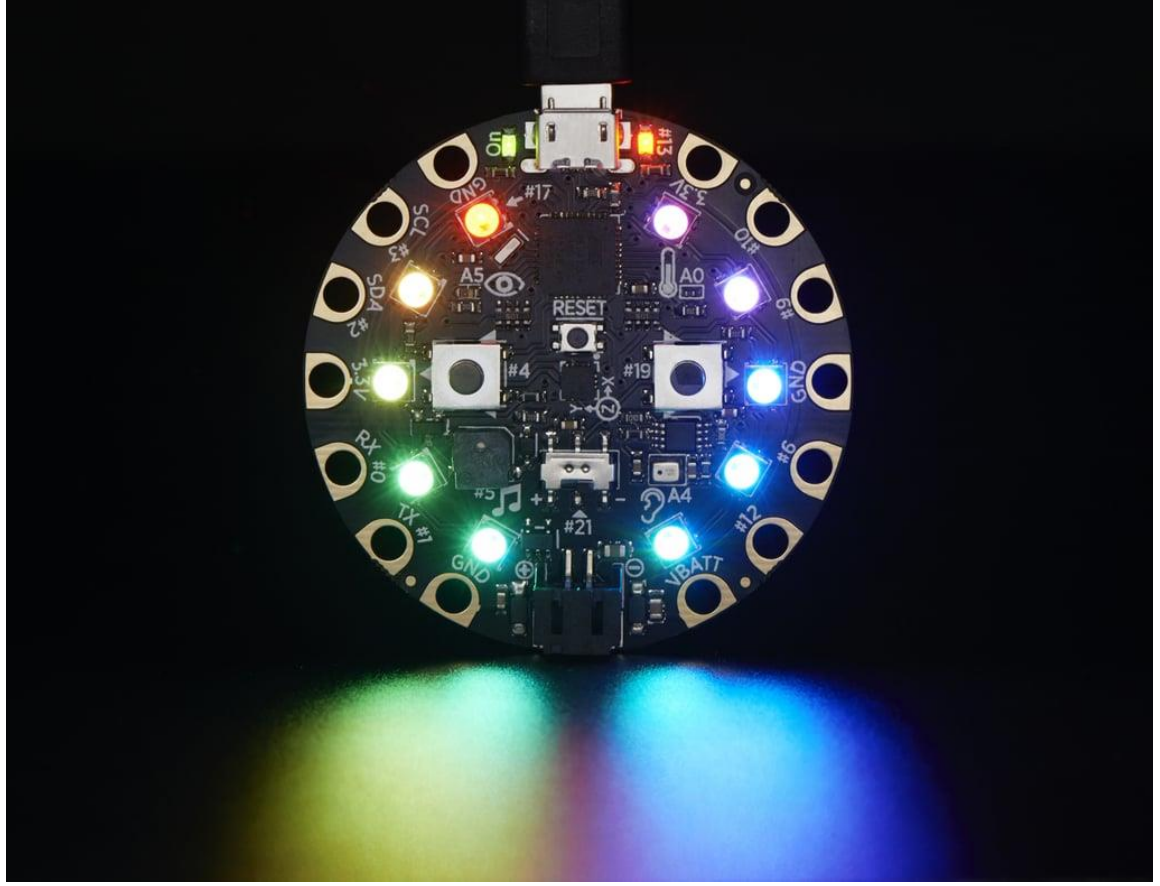




Circuit Playground Express kit

1. This is a small easy-to-program electronics board with built-in sensors and LEDs
2. We will use Javascript editor: https://makecode.adafruit.com
3. Write program
4. Connect board to laptop through USB
5. Remove from laptop, connect to battery and see results



Adafruit's resources

Main manual: https://cdn-learn.adafruit.com/downloads/pdf/adafruit-circuit-playground-express.pdf

Hundreds of examples on makecode.adafruit.com

Can also use CircuitPython IDEs

Question 1: At any time, is x equally likely to be less than and greater than 0.5?

Question 2: Is $x < 0.25$ equally likely as $0.25 < x < 0.5$?

See or hear your answer!

Be creative!

Question 1: At any time, is x equally likely to be less than and greater than 0.5?

Question 2: Is $x < 0.25$ equally likely as $0.25 < x < 0.5$?

Answer 1: Yes, equally likely!
Answer 2: No, less than 0.25 is more likely!