Computing to understand complex systems

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Links

1. Computational Mathematics Activity Group Website: https://sites.gatech.edu/compmath
2. Activity group materials including code and handouts: https://github.com/ni-sha-c/CompMath

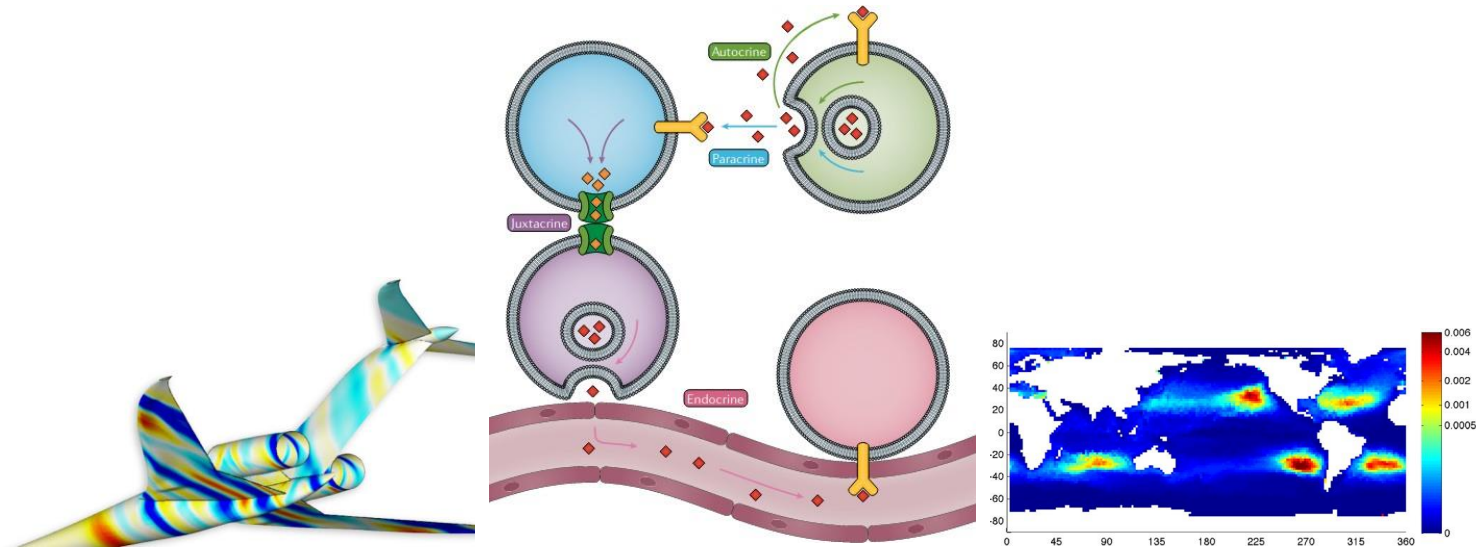
1. Makecode for CircuitPlayground Express https://makecode.adafruit.com/
2. Circuit Playground Express Tutorials https://makecode.adafruit.com/tutorials
3. Robert May's paper on population dynamics https://www.nature.com/articles/269471a0
4. Steven Strogatz's book on nonlinear dynamics https://www.stevenstrogatz.com/books/nonlinear-dynamics-and-chaos-with-applications-to-physics-biology-chemistry-and-engineering

Today’s goal: interactive learning!

1. What engineering and scientific questions are they interested in?
2. How do they construct and use mathematical models to answer those questions?
3. How do they build and use machine learning tools for those questions?
4. What background does one need for these questions?
5. What are related career options for someone in their field?
6. What does the day-to-day of that researcher entail?

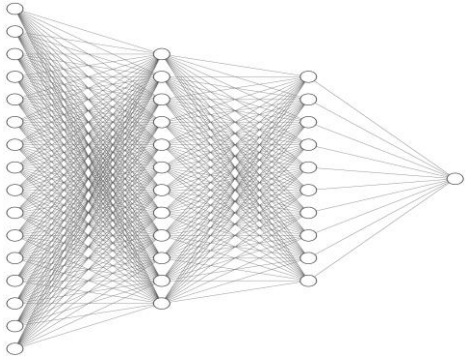
Computational dynamics + statistics

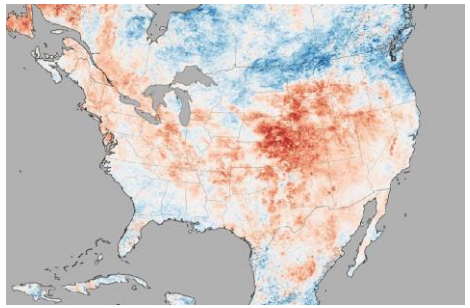
- **Goal**: To answer fundamental scientific questions around complex dynamics



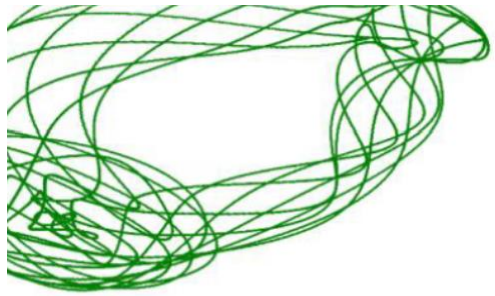
Left: Aftosmis et al 2015; Center: Armingol et al 2021; Right: Froyland et al 2014

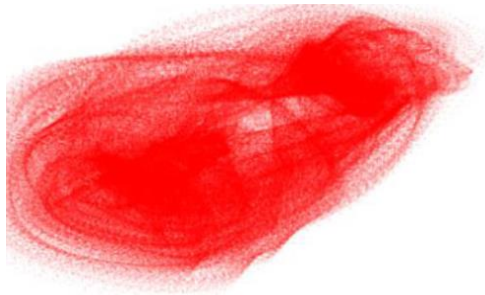
Application areas



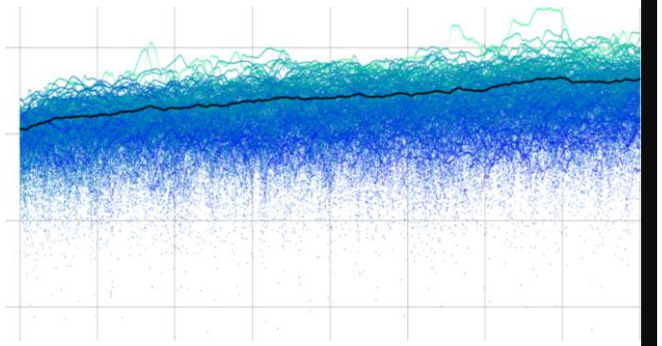


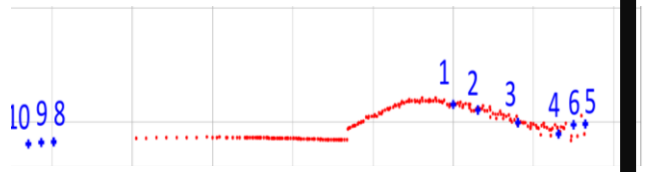
Information from numerical models





Computational methods development

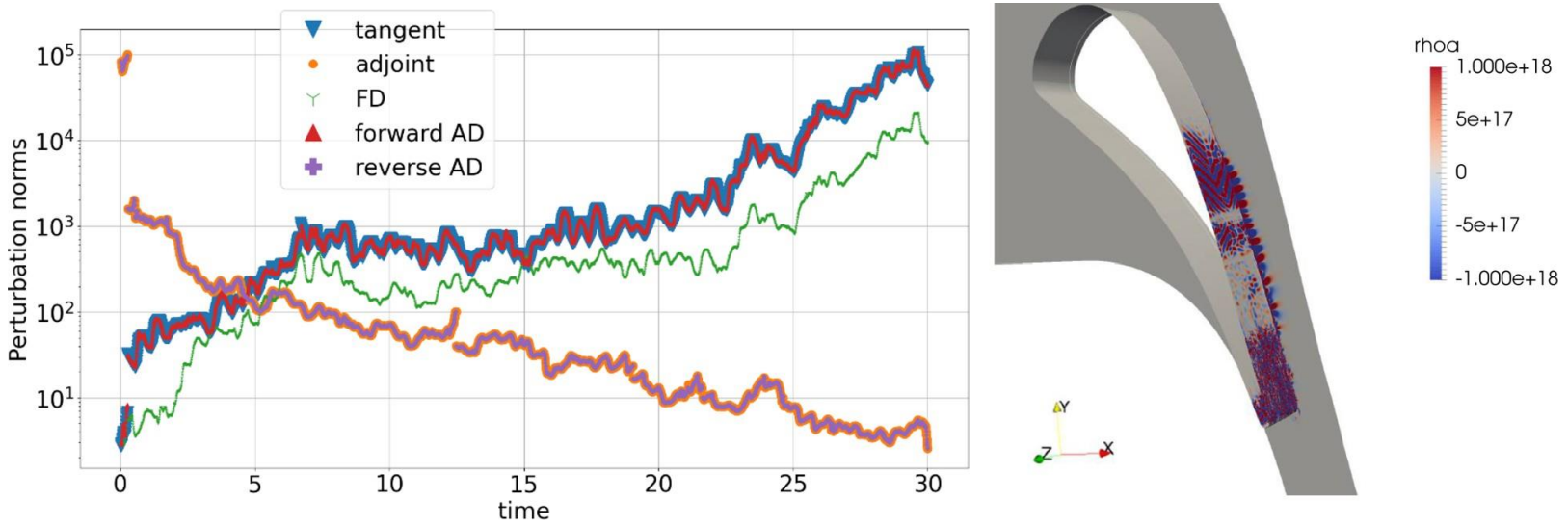




- Optimization and design
- Sensitivity analysis and control
- Order reduction
- Data assimilation

Insights obtained from rigorous algorithms and analyses

Computing with **chaotic** systems

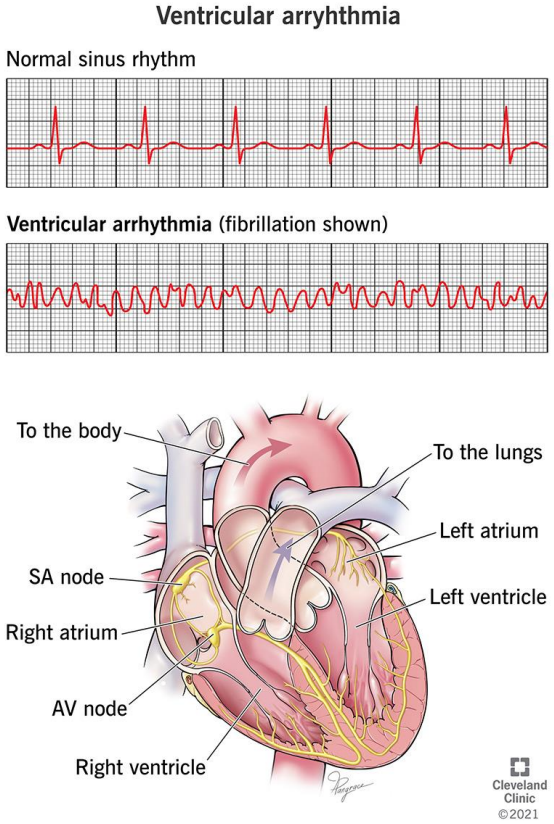


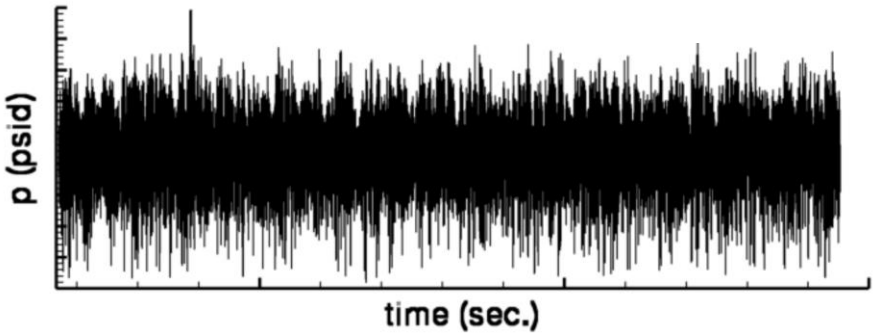
Goal: Tractable computation of **linear response**

$$\frac{d}{ds} \lim_{N \to \infty} \langle J \rangle_N , \tag{1}$$

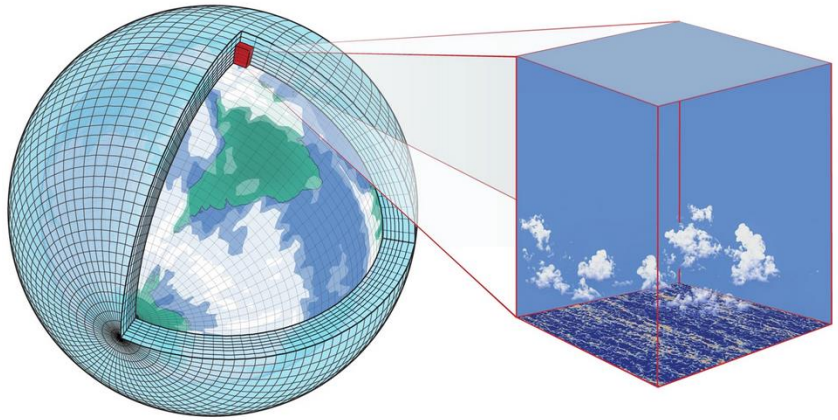
s: Design input, $\langle J \rangle_N$:N-time average of an observable J.
Downstream applications: optimization, data assimilation, model selection, mechanistic understanding etc.

Computing **sensitivities** will help with UQ, optimization





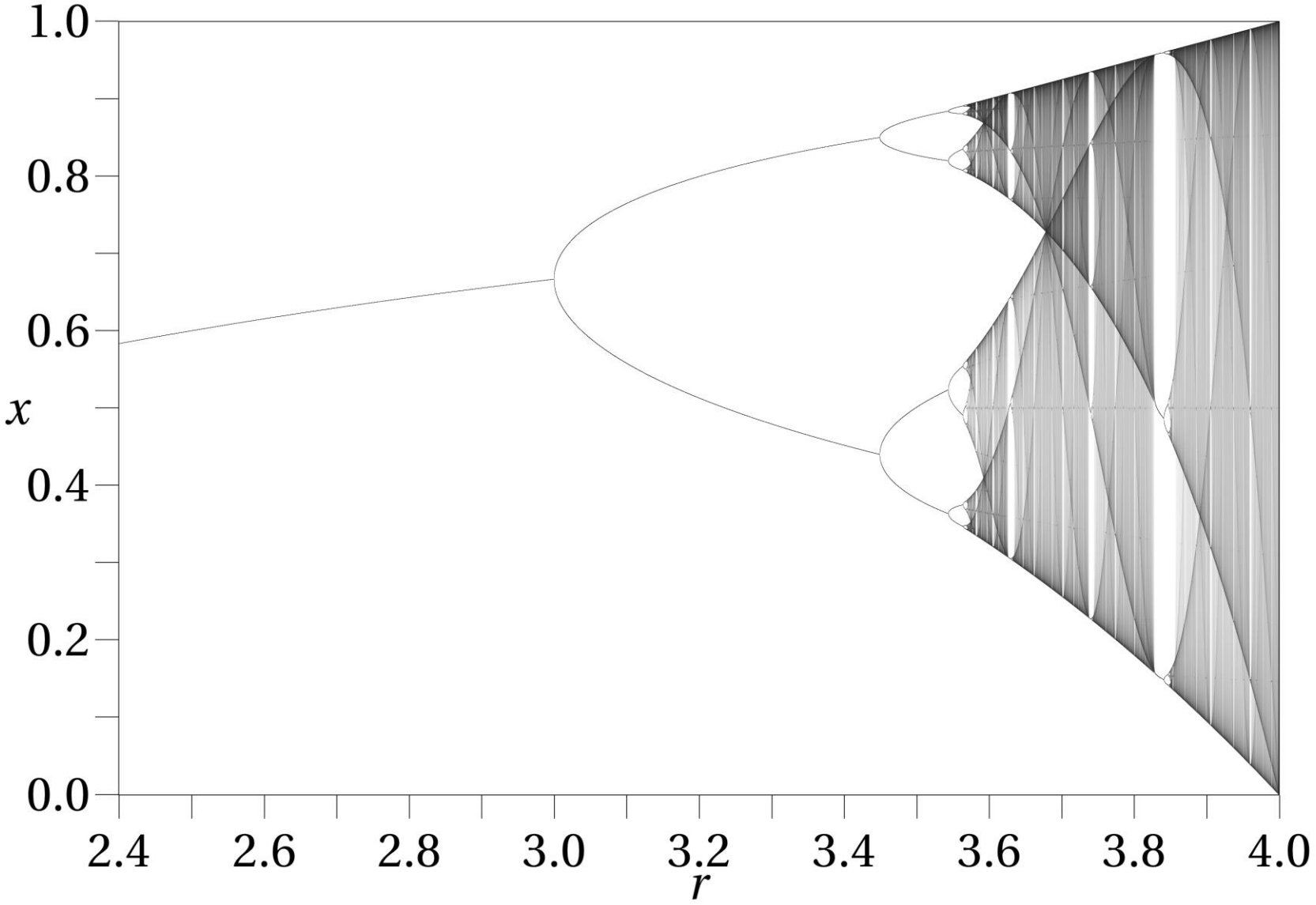
Alter et al 2015



Schneider, T., Teixeira, J., Bretherton, C. et al. 2017

Today: the logistic map

$$\varphi(x) = r\,x\,(1-x), \qquad x \in (0,1)$$



Today: the logistic map

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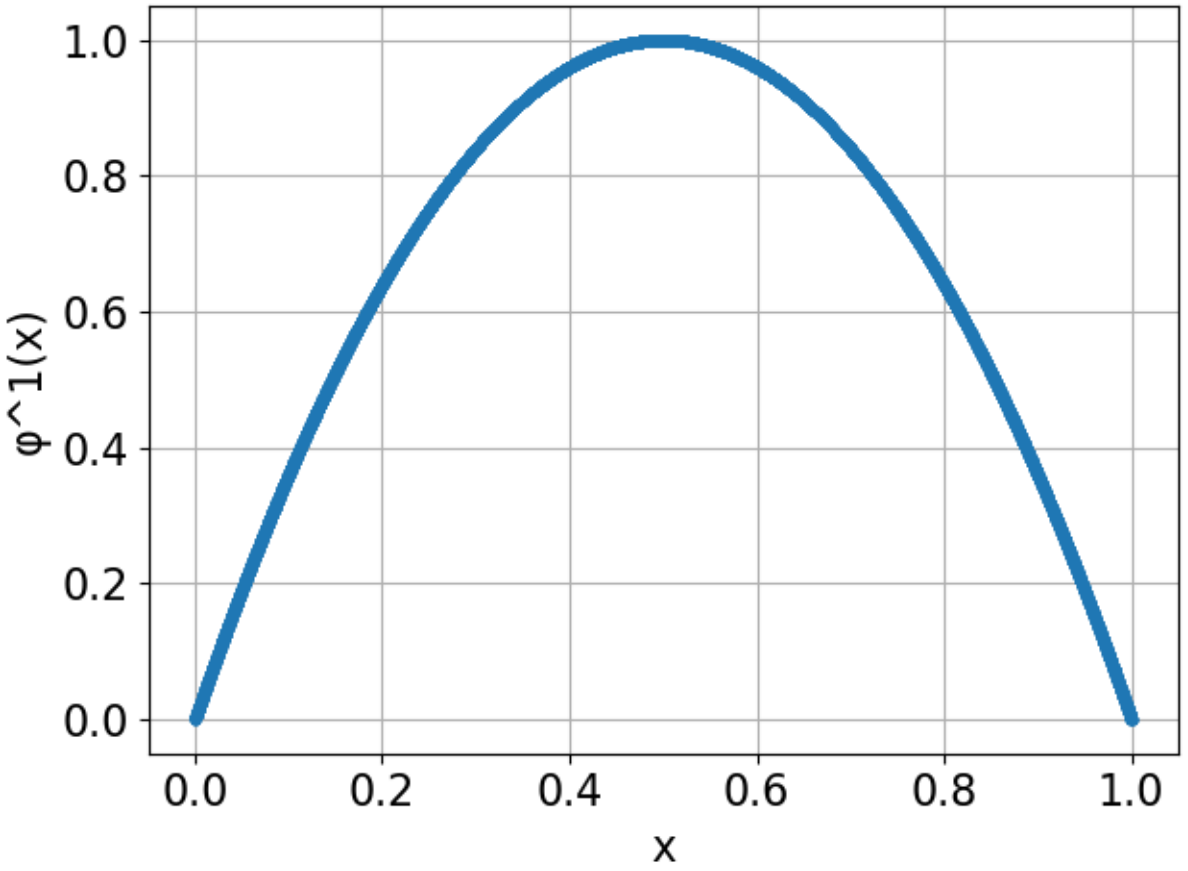
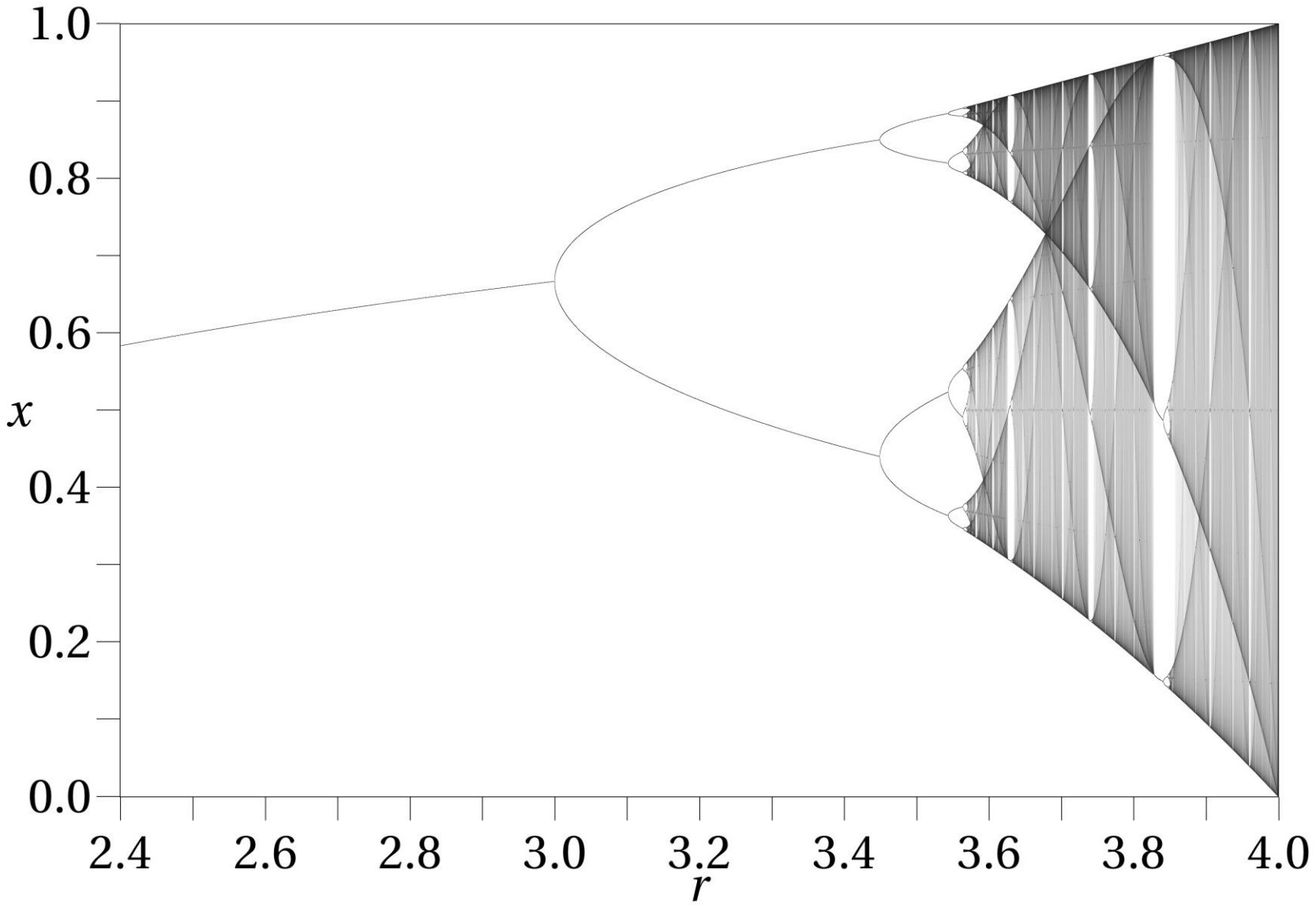
Orbit: $x, \varphi(x), \varphi^2(x), \varphi^3(x), \varphi^4(x), \ldots, \varphi^{10}(x), \ldots, \varphi^{1000}(x), \ldots$

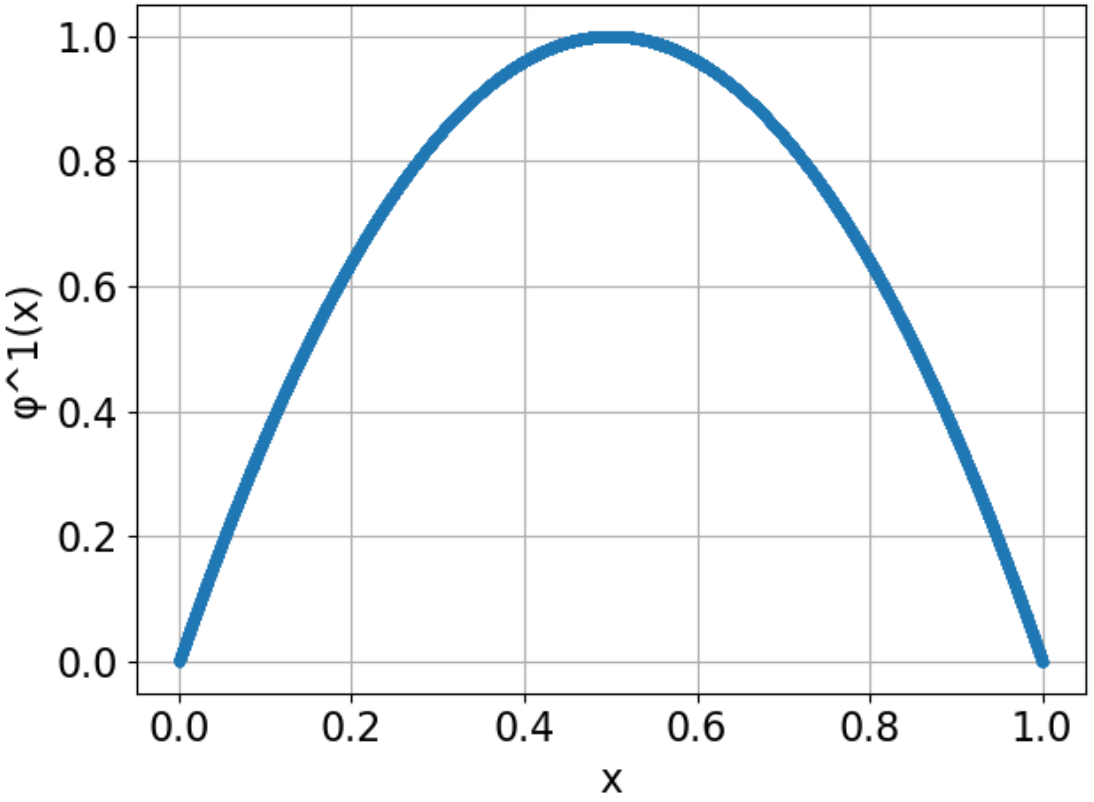


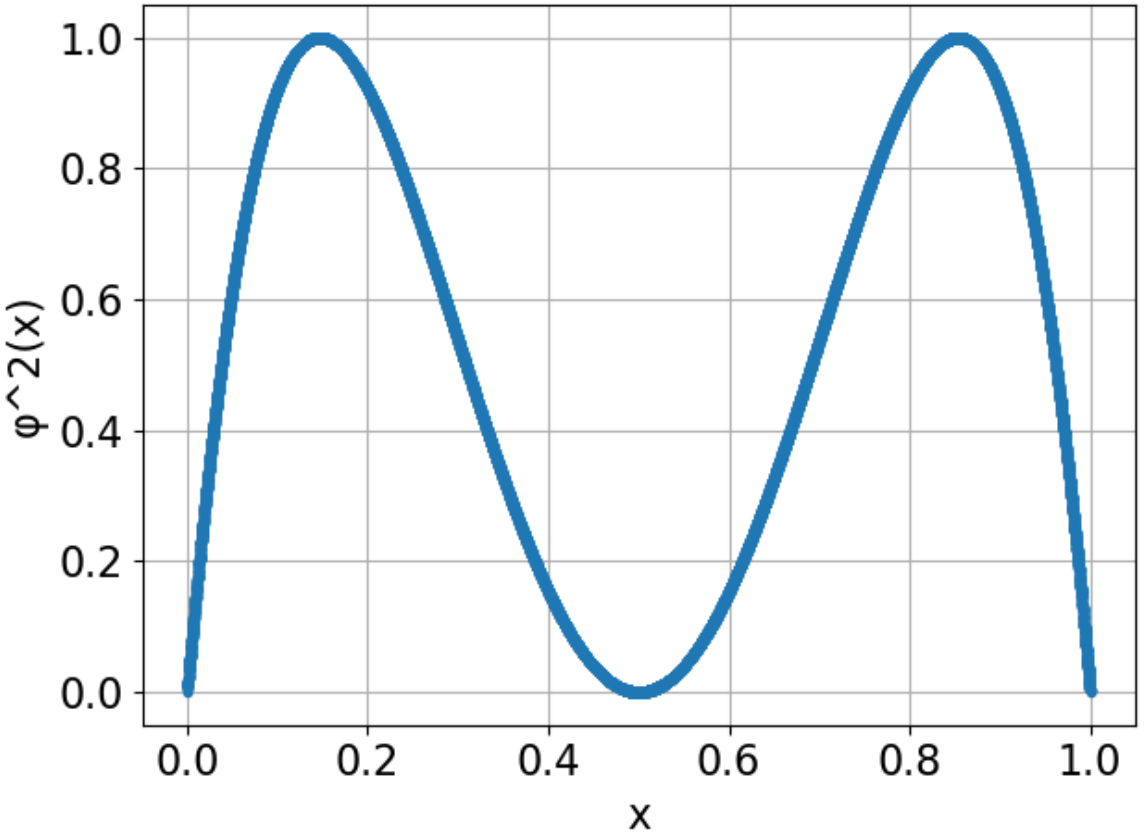
Bifurcations: qualitatively different behavior of orbits as parameters are varied



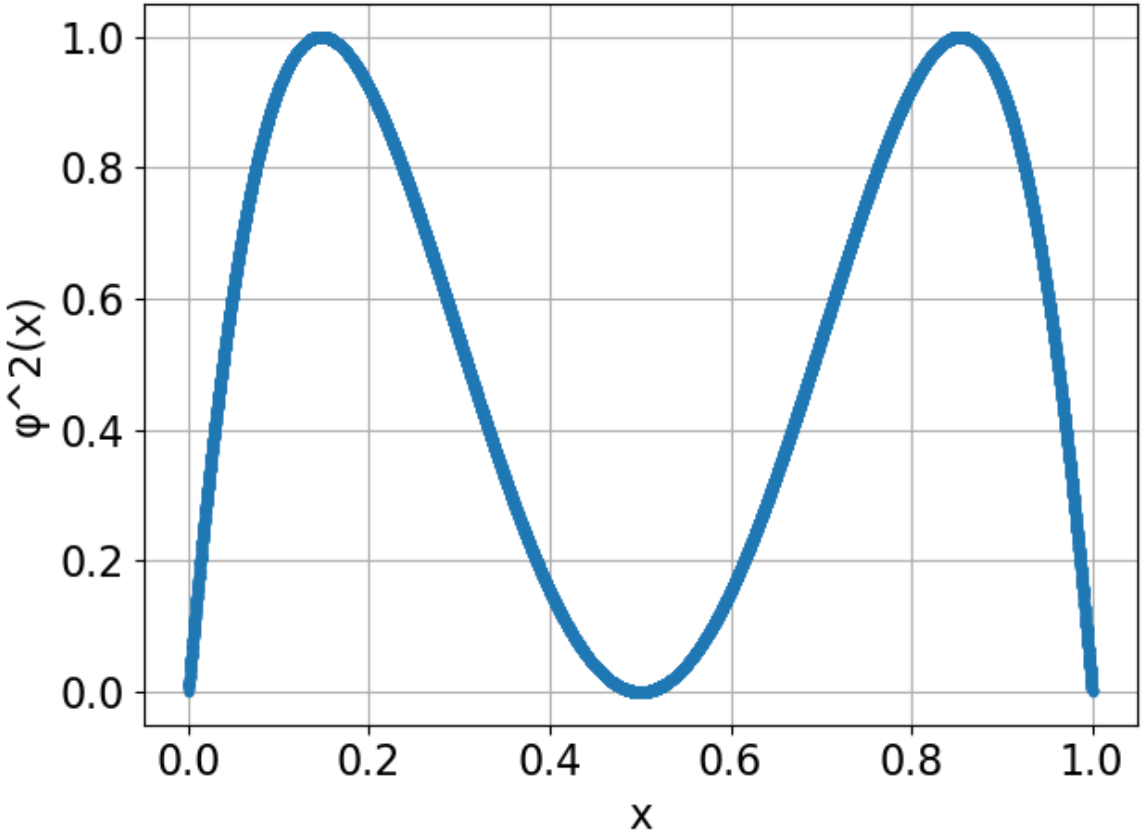
- Introduced chaos to the world of ecology
- Model of competition within a population
- Number of competing interactions quadratic
- X refers to percent of maximum population
- Fix rate r = 4
- Question: at a given time, is the population as likely to be < 50 percent as > 50 percent?

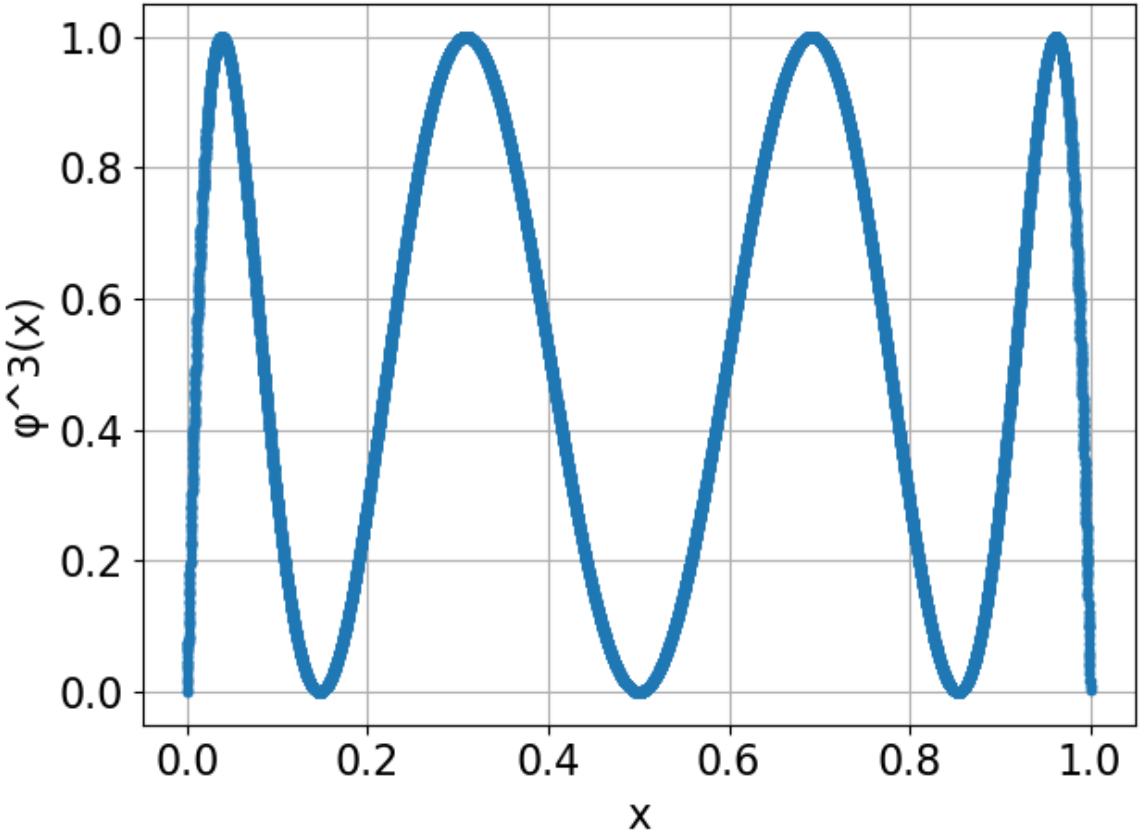
Evolution of ensembles under the dynamics



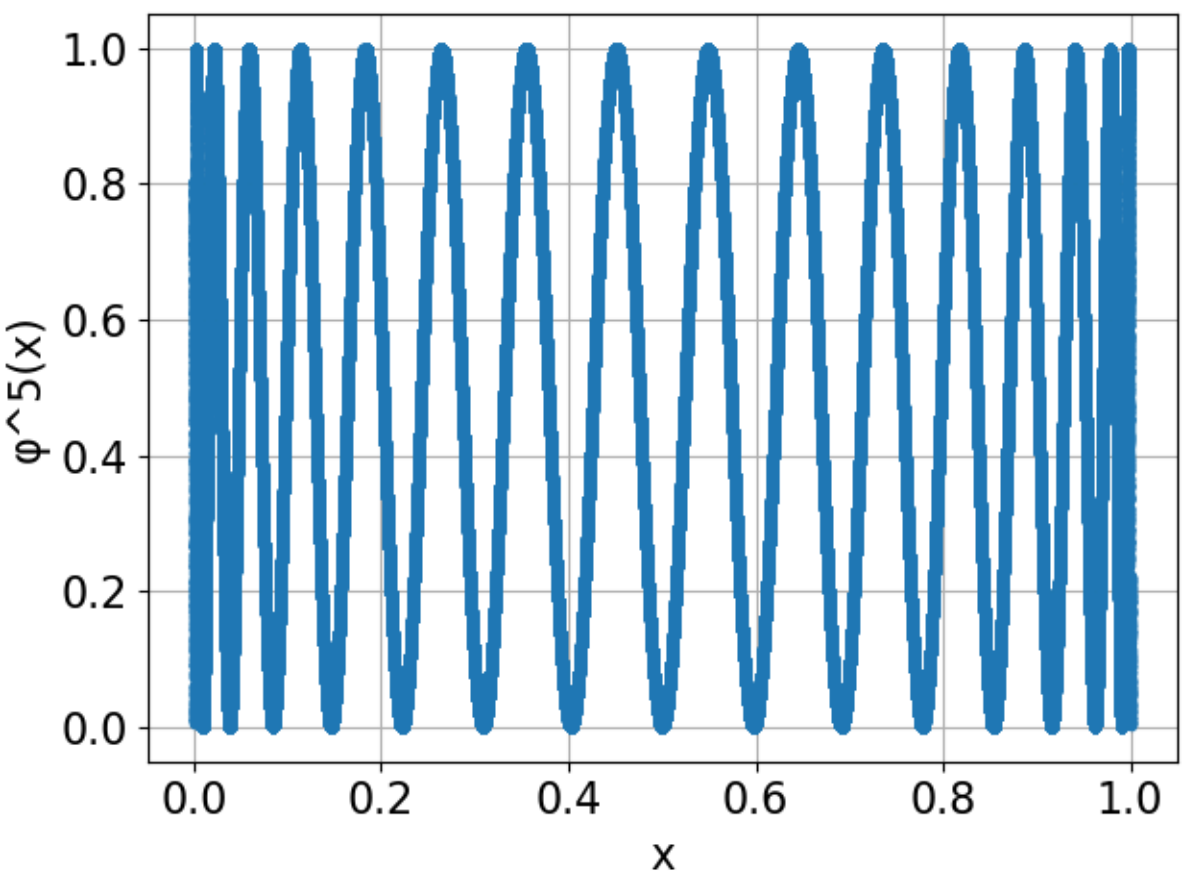


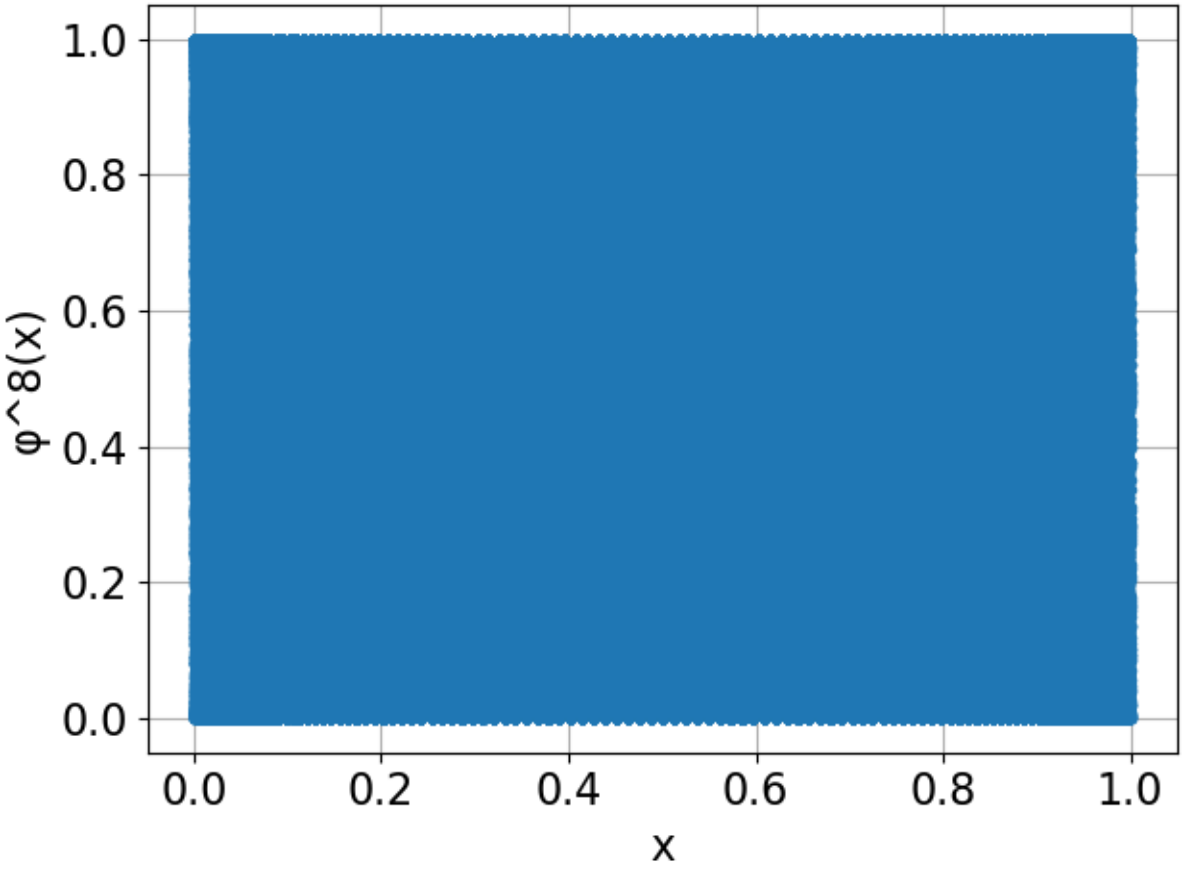
Long-term dynamical behavior of orbits





Chaotic systems: deterministic but appear random





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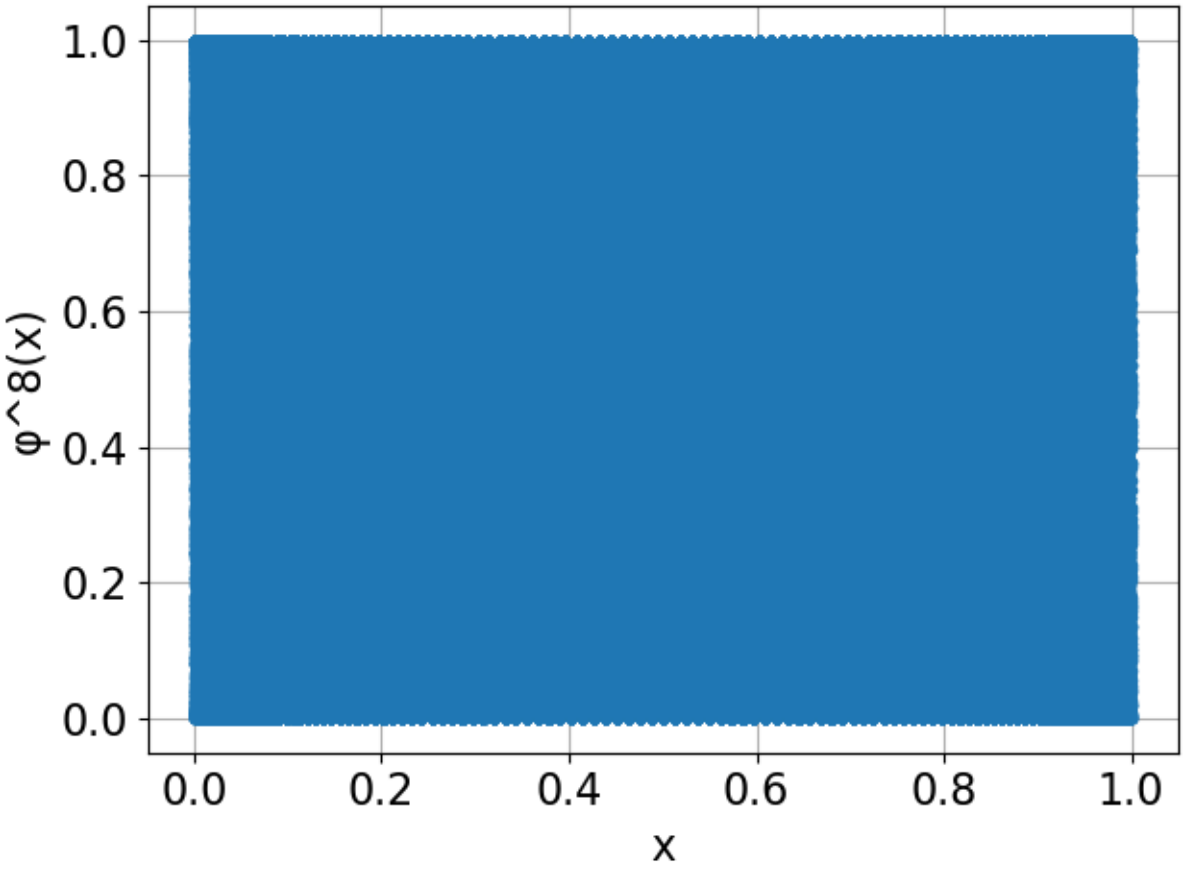
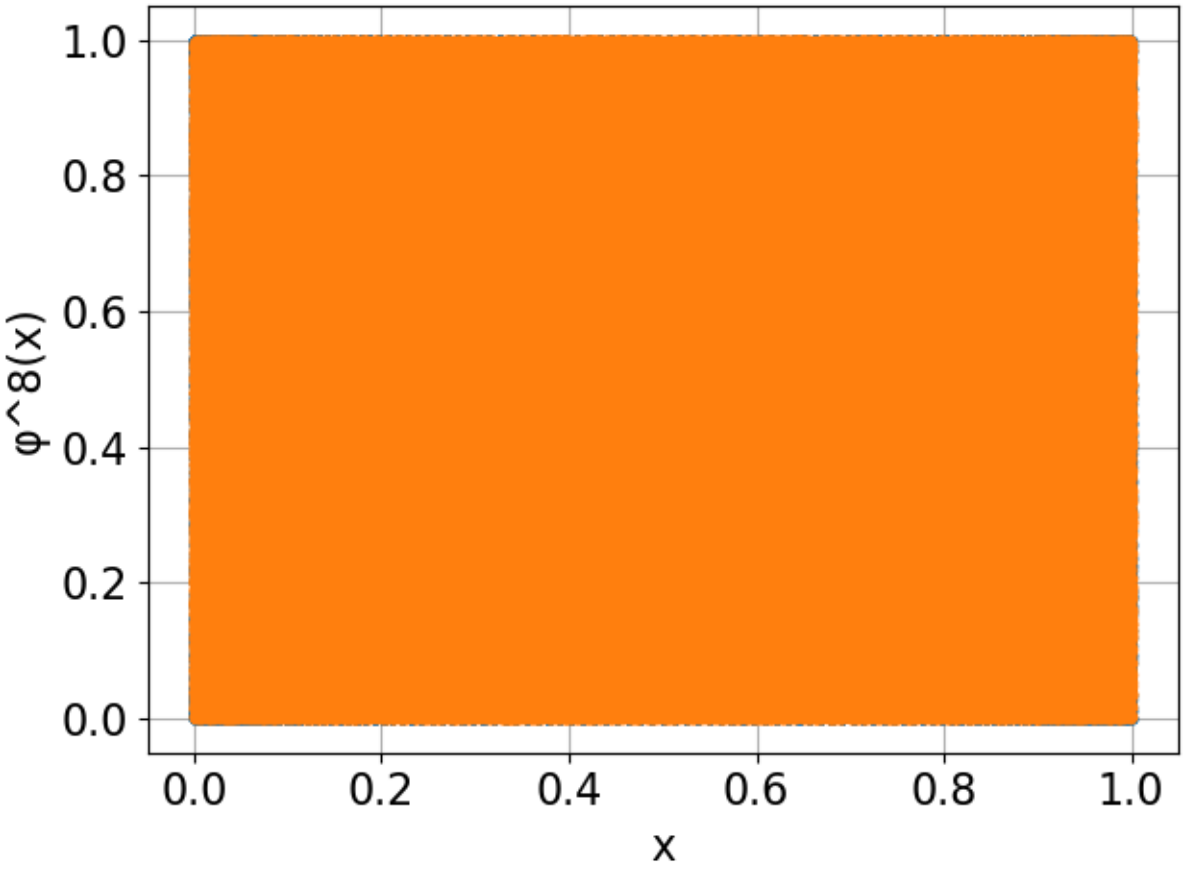
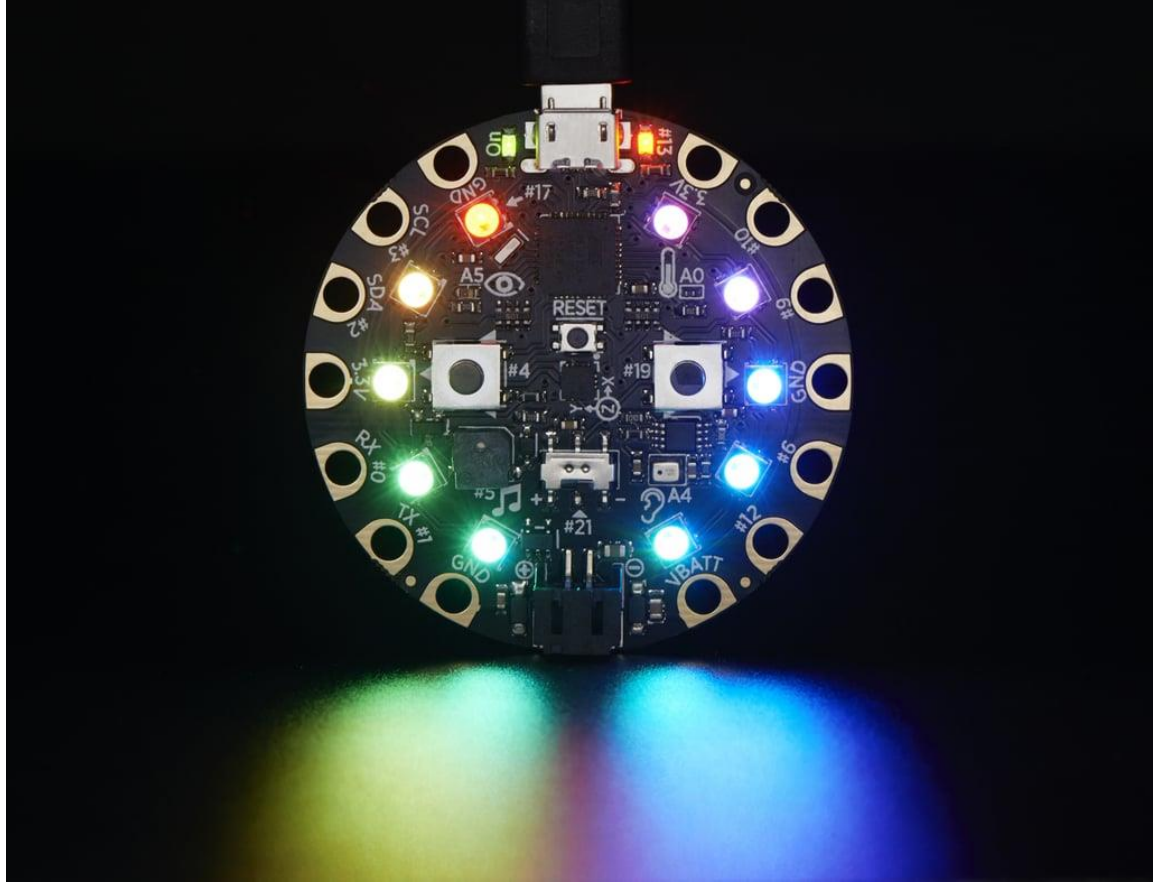




Circuit Playground Express kit

1. This is a small easy-to-program electronics board with built-in sensors and LEDs
2. We will use Javascript editor: https://makecode.adafruit.com
3. Write program
4. Connect board to laptop through USB
5. Remove from laptop, connect to battery and see results



Adafruit's resources

Main manual: https://cdn-learn.adafruit.com/downloads/pdf/adafruit-circuit-playground-express.pdf

Hundreds of examples on makecode.adafruit.com

Can also use CircuitPython IDEs

Question 1: At any time, is x equally likely to be less than and greater than 0.5?

Question 2: Is $x < 0.25$ equally likely as $0.25 < x < 0.5$?

See or hear your answer!

Be creative!

Question 1: At any time, is x equally likely to be less than and greater than 0.5?

Question 2: Is x < 0.25 equally likely as 0.25 < x < 0.5?

Answer 1: Yes, equally likely!
Answer 2: No, less than 0.25 is more likely!