CSE 8803 CDS, Spr 2024, Georgia Tech

## CSE 8803 CDS: Homework 3

Due Feb 24,'24 (11:59 pm ET) on Gradescope

Cite any sources and collaborators; do not copy. See syllabus for policy.

## Problem 1

This problem is about closed orbits in two-dimensional flows. Read about the Poincaré-Bendixson theorem, which gives conditions for the existence of closed orbits.

Part I 7.3.1 (5 points), 7.3.2 (3 points) Strogatz

Part II 7.3.4 a) (1 point)

Part III 7.3.4 b) (2 points). Is V a Lyapunov function? (1 point)

Part IV 7.3.9 a) (3 points), b) (2 points), c) (4 points)

## Problem 2

Here we will study oscillators, which we will express as flows in two dimensions. Consider the van der Pol oscillator:  $d^2x/dt^2 + \mu(x^2 - 1)dx/dt + x = a$ .

- Let  $\mu < 0$ . Find the fixed points and determine their stability via linear stability analysis (1 point)
- Determine Lyapunov stability of the fixed points (2 points)
- Part c of 7.5.6 Strogatz (3 points)
- Part d of 7.5.6 Strogatz (4 points)