CSE 8803 CDS, Spr 2024, Georgia Tech

CSE 8803 CDS: Homework 4

Due Mar 20,'24 (11:59 pm ET) on Gradescope

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In this homework, we will explore computing Lyapunov vectors and exponents. We will consider the three-variable Lorenz '63 system, which was introduced as a reduced order model for atmospheric convection. Let $x = [a, b, c]$ be a phase point, and $a(x)$ is the first component/coordinate at x. The system is given by the ODEs:

$$\frac{d\varphi^t(x)}{dt} = v(\varphi^t(x)) = \begin{bmatrix} \sigma(b-a) \\ a(\rho - c) - b \\ ab - \beta c \end{bmatrix} \circ \varphi^t(x). \tag{1}$$

Fix $\sigma = 10, \beta = 8/3$ and $\rho = 28$, which are values at which the system is known to be chaotic.

I (2 points) Plot the Lorenz attractor. Hint: time integrate the equations starting with a random initial condition and plot the result

II (5 points) Compute the three Lyapunov exponents. Write a code snippet, explaining each line.

III (5 points) Are the following statements true or false for the Lorenz '63 system? Provide justification.

(a) The adjoint covariant Lyapounov vector is the same as the covariant Lyapunov vector for the top LE

(b) The top (backward) Lyapunov vector is always covariant

(c) There is a zero Lyapunov exponent for the ODE system

(d) The stable adjoint Lyapunov vector is perpendicular to the unstable Lyapunov vector

(e) There is a dense set of points on the attractor that result in different LEs than the ones computed

IV (10 points) Compute and plot the unstable adjoint Lyapunov vector. Provide a code snippet/algorithm.