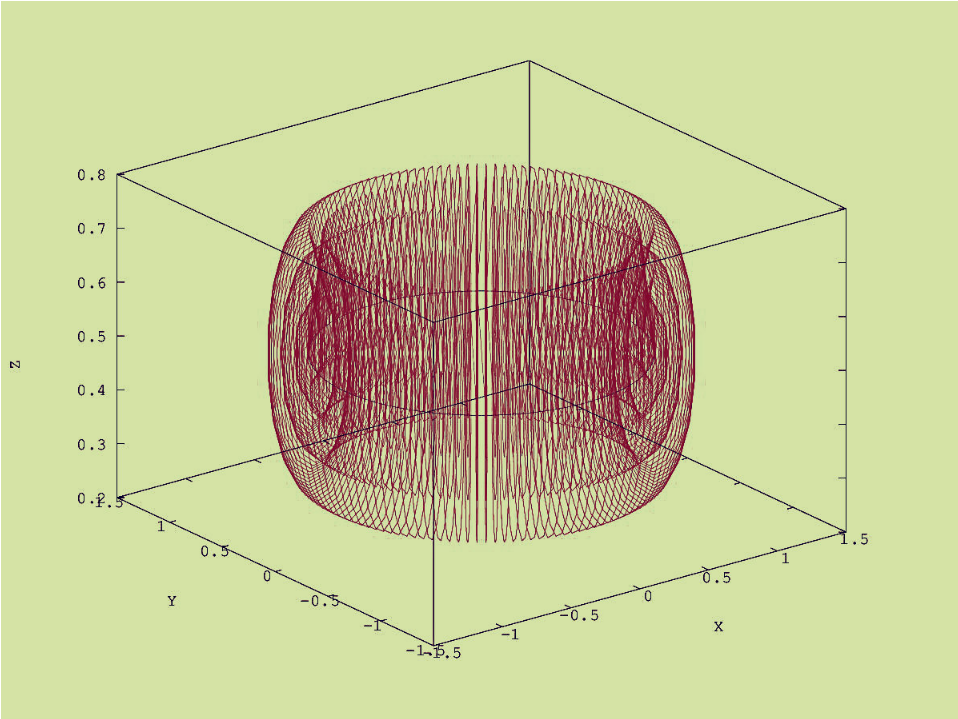

Figure 1: Vortex Torus with ring axis in blue

Simulation of Toroidal Vortex Ring(From Ghoniem's paper[1])

0.1 Geometry and Initial Vorticity Distribution

Here we look at a toroidal vortex ring with an initial vorticity distribution modeled by a third order Gaussian function:

$$\omega(\mathbf{x}) = \frac{1}{a\sigma^2} exp\left(-r_{\mathbf{x}}^3/\sigma^3\right)\mathbf{e}_\theta \tag{1}$$

The vorticity is along the direction $\mathbf{e}_\theta$ which is tangential to the ring axis(shown in the figure). The ring axis is the circle, $x^2+y^2=R^2$ on the plane $z=z_i$. The magnitude of the total vorticity is independent of θ where (ρ,θ,z) are represent coordinates in the cylindrical coordinate system.

The cross section of the ring, which is circular, has a radius, $\sigma = 0.275R$. In the figure, $R=1$ and $z_i=0.5$.

In a particular cross section,

- r is the radial distance measured from the centre of the core of the ring or the centre of a cross section. For all $\mathbf{x}$ within the ring, $0 \le r_{\mathbf{x}} \le \sigma$.

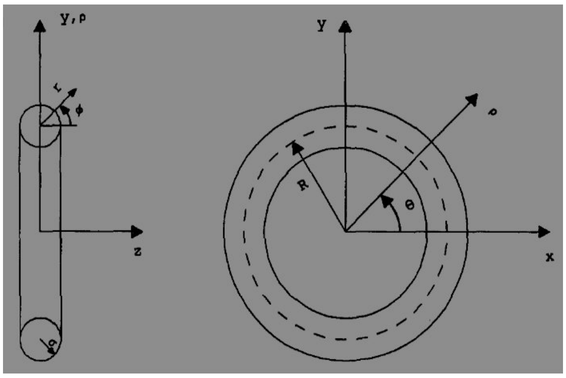

Figure 2: From [?]

- ϕ is the angle measured, at every cross section, from the line passing through the centre of the cross section and parallel to the z axis.

0.2 Discretization of the Ring

The toroidal ring is divided into many small vortex tubes. Here, we make an equi-spaced mesh found in the third position on the third row of Fig.11 b of the paper[1]. The ring is discretized at N_c cross sections and each cross-section is discretized at N_per_θ locations in total.

Of these N_per_θ locations, N_θ points are at each radial location and there are N_r radial locations per cross section.

In the code, $N_r = 3$ and N_θ is a multiple of 6. Hence, there are $N\_per\_\theta = 1 + 6 + 12 + 18 = 37$ elements (including one at the centre) in total per cross section.

Therefore, the ring is divided into $N_c \times N\_per\_\theta$ vortex tubes in total. In the code, $N_c = 120$. Hence, we have 4440 vortex elements which are part of N_per_θ different vortex tubes.

0.3 Evaluating Strength Vectors

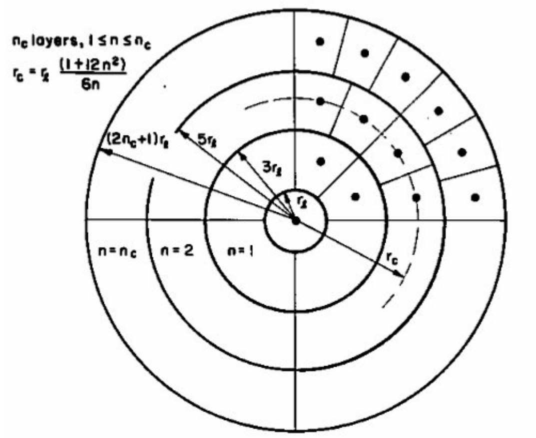

Figure 3: From [2]

Bibliography

[1] *Omar M. Knio and Ahmed F. Ghoniem*,Numerical Study of a Three Dimensional Vortex Method, Journal of Computational Physics,1990

[2] *G.S. Wincklemans and A. Leonard* ,Contributions to Vortex Particle Methods for the Computation of Three-Dimensional Incompressible Flows, Journal of Computational Physics, 1993.