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# Segregation in a Galton Board

**J G Benito[1], A M Vidales[1] and I Ippolito[2]**
1 INFAP-CONICET, Departamento de Física, Universidad Nacional de San Luis, 5700 San Luis, Argentina.
2 GMP, CONICET y Facultad de Ingeniería, Universidad de Buenos Aires, 1063 Buenos Aires, Argentina

E-mails: jbenito@unsl.edu.ar, avidales@unsl.edu.ar, iippoli@fi.uba.ar

**Abstract.** This work deals with a numerical study of the problem of separation of particles with different elastic properties. The separation procedure uses a Galton Board which consist in a bidimensional system of obstacles arranged in a tiangular lattice. Disks of equal diameters but different elastic properties are launched from the top of the device. The Galton Board is commonly used for mixing particles, but here, we intend to find special conditions under which one can use it as a segregating device. We introduce a mixture of particles and generate, through simulations, different conditions to favor the segregation process based on the different elastic coefficients of the particles. We inspect which is the best configuration of size, density of obstacles and wall separation to favor the separations of particles. Our results prove that the Galton Board can be used as a segregation device under certain conditions.

## 1. Introduction

The problem of mixing and segregation in granular materials has been the subject of renewed and intense interest in recent years because of its great importance to the industry [1-3].

The bi-dimensional Galton Board (BGB) is a system of small pins forming a regular triangular lattice. A typical sketch of a board is shown in figure 1. There are plenty of examples in the literature of much reserch done using this set up to the interesting dynamics present in it. Those examples range from charged particles scattering to dispersive flow of glass beads [4-7].

Usually, the BGB is arranged in a vertical position and experiments consist in launching small disks (compared to the spacing between pins of the lattice) from the top row of the array and collecting them at the bottom part of the system by a set of collectors.

The implementation of the BGB like a particle mixer has been developed and successfully proved. Bruno *et al.* [8-11] studied experimentally and numerically the dynamics of gravity driven particles going through an ordered array of obstacles. Collisions with obstacles make particles to diffuse, and the diffusive regime is attained after a few number of collisions. Due to this process, particles are forced to mix. They also analyzed the feasibility and efficiency of the mixture of particles through the BGB. They developed an analytical method to predict the relative composition of a mixture as a function of the radii ratio of the particles in the mixture.

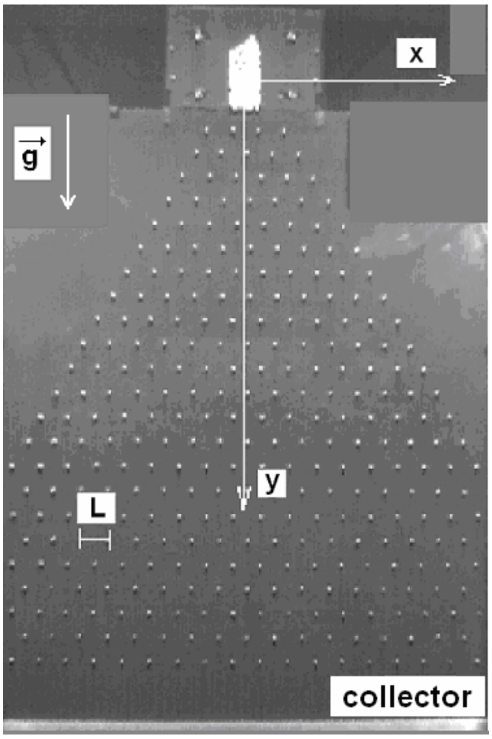


**Figure 1.** A typical bi-dimensional Galton Board device. At the top, there is a container with particles ready to be released. The spacing between pins, L, is indicated as well as the coordinates and gravity direction.

Recently, Benito *et al*. [12-14] have studied and characterized the behavior of the exit distributions of same species of particles falling down through a BGB.

They found a change in the behavior of these distributions as the spacing W between BGB walls is reduced. They observed a crossover in the behavior of the exit distributions changing from a Gaussian-like to a uniform-like shape. They determined a simple parabolic relationship that holds when searching for the necessary separation between lateral walls in order to obtain a uniform exit distribution of percolating particles. As a second stage, they have studied experimentally and numerically the influence of lateral walls on the mixing capability of a BGB, finding that this separation is crucial in a mixing process, and obtaining, for small separations W, a homogeneous mixture.

All above inspires the following question: can this particle mixer be used in a reversal way?. In other words, can it be used to segregate particles originally mixed?. For this reason, the present paper will focus on the problem of particle separation through the BGB.

## 2. Numerical model

Simulations were driven to reproduce the geometrical features of BGB device used by Bruno *et al*. [8-11] and Benito *et al*. [12-14]. We define sites (representing the pins of the board) arranged in a triangular lattice and keeping experimental distances.

Disks of the same size like those of the experiment are launched from the center of the top of the table. Trajectories followed by the disks were calculated in this way: a disk falls down by gravity until it encounters a pin in its trajectory. Then, and depending on the relative positions of the centres, the small disk will roll over the pin to the right (the *x*-coordinate of its centre is to the right of that of the pin) or to the left (the *x*-coordinate of its centre is to the left of that of the pin). At this stage, we have to decide which will be the new x-position of the particle, i.e., we have to introduce the effect of bouncing without calculating any force (we are not interested here in the dynamics of the problem).

Thus, we just choose a random number uniformly distributed between 0 and 1 (ran(0, 1)) and put the particle at the corresponding x-position given by:

$$x_p = x_{pin} + aR_{pin} + aR_p + a\delta d\, ran(0,1) \qquad (1)$$

Here, $x_{pin}$ is the $x$-coordinate of the centre of the pin; $R_{pin}$ is the radius of the pin (the same for all of them in the lattice), $R_p$ is the particle radius, d is the distance between two neighboring pins minus two times their radius, δ is a factor to account for the hardness of the disks and, finally, $a$ is a factor equal to +1(−1) if the $x$-coordinate of the centre of the particle is to the right (left) of that of the pin. Figure 2 depicts the details.

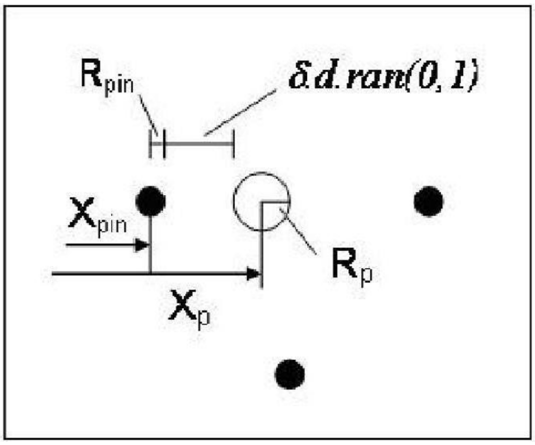


**Figure 2.** Schematic representation showing how we get the new $x$-position of the disk in simulations. this case relates to $a$ = +1 in Equation 1.

We use parameters δ = 0.55, 0.6 and 1 simulating different hardness of the disks. This selection is totally arbitrary and these parameters will just represent some real bouncing parameters: the smaller the value of δ, the smaller the hardness of the percolating particle and thus, the smaller the lateral displacement.

After a new position is attained by the particle, a new calculation for the trajectory of the particle is performed, following the procedure explained above, and so on.

After the particle left the last row of the BGB, its final $x$-position is recorded to build the exit distribution histogram. In simulations, we have the advantage to know exactly the final position of each particle and thus one can classify these data to build a histogram as desired. Walls are consider sufficiently far appart to avoid colisions to a minimum.

Under these conditions, we begin a typical run by dropping through the BGB a binary mixture composed of $10^5$ disks of each species. At this stage, it is worth mentioning that the effect of throwing disks one at a time in simulations is not relevant for the final results obtained. In fact, this has been demonstrated experimentally by Bruno *et al*. [8-11].

In a typical BGB experiment, while the discs are falling, they collide inelastically with the obstacles and other falling discs, losing energy and changing their local velocity direction. Bruno *et al*. [8-11] analyzed the influence of the collective interactions, i.e., the influence of other falling particles on a tracer disc. They studied the velocity distribution of tracer particle and compared it to that obtained when the discs are released one by one and they found no significant influence. Even more, they performed numerical simulations of the spatial frequency of disc–disc and disc–obstacle collisions. They have seen that the spatial frequency of disc–obstacle collisions remains constant in average during all the chute and disc–disc collisions occur mainly close to the entrance where the particles are clustered together. This is the region of high disc–disc interaction density corresponding to the first four or five rows of obstacles. Afterwards, disc–disc collisions became less probable when the particles have sufficiently spread due to the presence of obstacles. Thus, the system will behave as a single particle. For this reason, the results presented and discussed in simulations will refer to the one particle-at-a-time problem.

## 3. Segregation mechanism

The mechanism of separation through BGB is based on the fact that we use particles with different elastic properties.

At the end of a typical experiment, the final exit distribution shows a good degree of mixture in the central collectors, but in the lateral ones we only find particles belonging to the set of higher parameter $\delta$. This is due to the fact that they can difuse far appart, compared to those with lower hardness, thus exploring more remote regions of the table.

Using this property, we extract the disks on the sides of the device not mixed with other species. The remaining mixture is re-injected into a new BGB, thus beginning a new cycle. This reinjection can be performed in two ways.

### *3.1. Design with funnel*

In this design, the remaining mixture is redirected toward the middle of the new BGB simulating a funnel dynamics. Disks fall again through the BGB and we obtain the new final *x*-position of each reinserted disk. As explained above, those particles located in laterals collectors belonging to one species and not mixed with the other, will be extracted, while the remaining mixture will be redirected again by means of a funnel to a new BGB, beginning another new cycle. This process is repeated *n* times.

For simplicity we define a new parameter $\Delta$ as the ratio between the parameters $\delta$ of the disks composing the binary mixture:

$$\Delta = \frac{\delta_1}{\delta_2} \tag{2}$$

Here, the bouncing parameter $\delta_2$ is always the higher value of the two parameters. In our experiences, $\delta_2$ is always equal to one.

Two different mixtures were tried. One with parameter $\Delta$ = 0.55, and the other with $\Delta$ = 0.6. Figure 3 shows the results obtained. We have plotted the cumulative number of particles separated from the original mixture per number of cycles *n*.

In both cases, we note that after a certain number of cycles $n = N$, the species of disks initially mixed are completely separated.

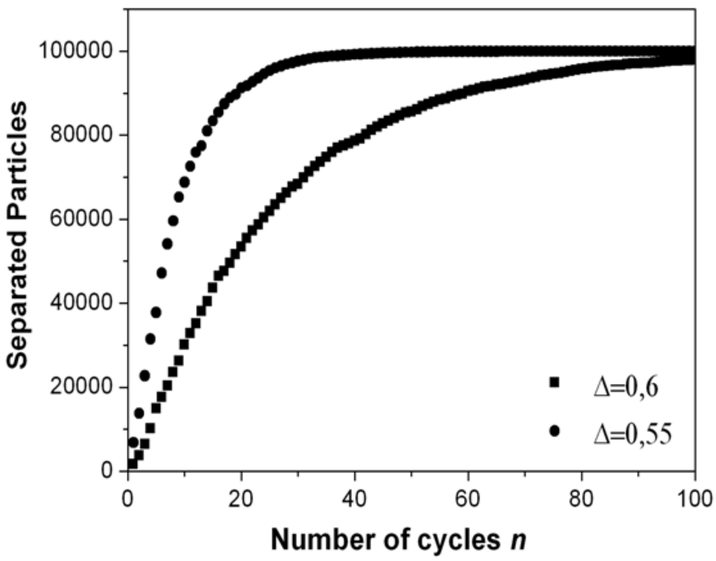


**Figure 3.** Number of accumulative disks separated with the funnel design vs. number of cycles for mixtures with $\Delta$=*0.6* and *0.55*.

The number of cycles required for this depends on the parameter Δ of the mixture. The lower the parameter, the lower the number of cycles *N*.

For the mixture with Δ = 0.55, the number of cycles *N* needed to completely segregate the particles is equal 35, while for the case Δ = 0.6 , it is approximately equal to 100.

### *3.2. Design without funnel*

In this design, the reinjection of the remaining mixture comming from the first BGB is performed by retaining the final *x*-position of each disk for the reinjection to the next BGB. In other words, the initial position of each disk reinserted in a new cycle is equal to the final position obtained in the previous one. No funnel effect is present here.

In figure 4 we compare the results obtained with both designs in the case of a binary mixture with Δ =0.6 and composed of $10^5$ disks of each species.

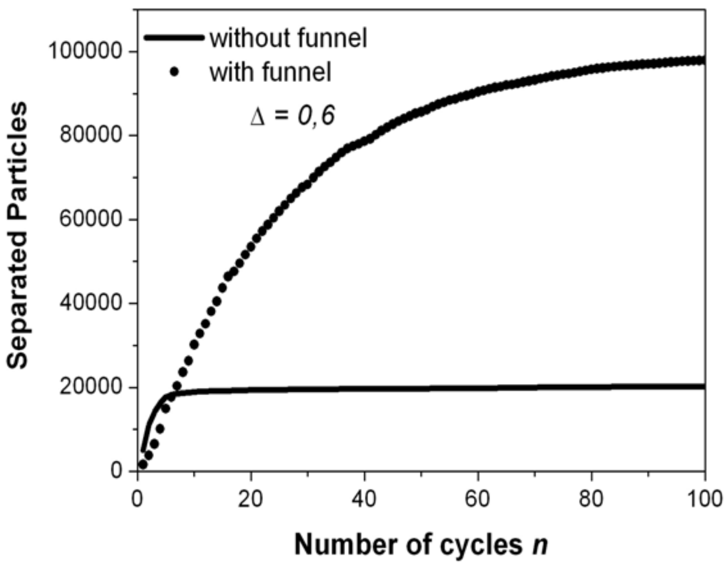


**Figure 4.** Accumulative number of separated disks vs. number of cycles for designs with and without funnel and Δ=0.6

We note that, after a few cycles, the design without funnel is unable to continue separating particles meaning that the algorithm fails to find disks with higher δ not mixed with the other species in the lateral collectors. This is due to border effects, as indeed it was recently proved by Benito *et al*. [14]. In that work, it was proved that, both experimental and numerically, the existence of lateral walls in a BGB is crucial to obtain a homogeneous mixture of disks. They performed experiments with particles with two different bouncing properties studying the exit distributions and the mixing indexes as the separation between lateral walls was varied. It was found that the mixing region changes as walls become closer. It goes from a normal distribution to a uniform one which is better achieved when particles with similar hardness properties are mixed. As separation between walls decreases, the mixing is more efficiently fulfilled and more homogeneous. Thus, in our present design without funnel, the disks that are reinjected into a new BGB (retaining their previous final x-coordinates values), can explore further regions and notice after a few cycles the presence of laterals walls. Under these conditions, after approximately 5 cycles, we always get a mixture (enhanced by the presence of walls), and disks can not be separated anymore.

### *3.3. Design duo*

Despite the inability to separate particles for the design without funnel, we note that, for the first cycles, this design separates a greater amount of disks compared with the funnel design. This is clearly indicated by the knee in figure 4 corresponding to the design without funnel. For this reason, we will try to avoid the above mentioned border problem, designing a new device that will segregate particles comming from a combination of the two designs analyzed previously.

Basically, we want to maintain the higher segregation capacity of the design without funnel, but also preventing an excessive diffusion of the disks when they move down the board. This double effect is obtained by building the following "design duo". First, it consists of a series of four BGB without funnel. Then we insert a BGB with funnel, which is followed by a new series of four BGB without funnel. After this, another BGB with funnel is inserted, and so on.

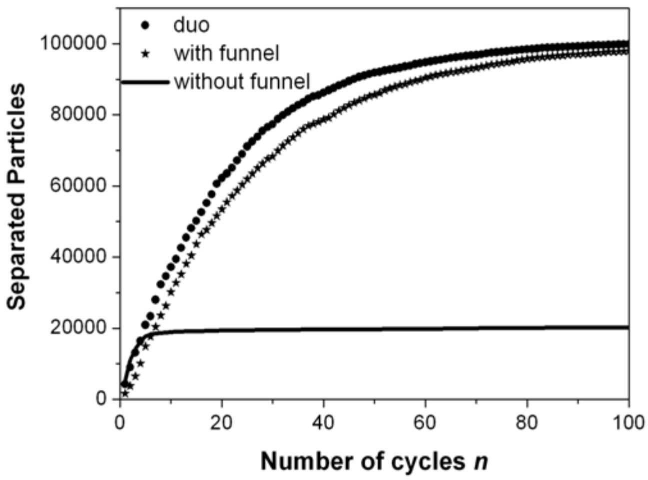


**Figure 5.** Cumulative number of disks separated vs. number of cycles for designs duo, with funnel and without funnel for mixtures with Δ=0.6.

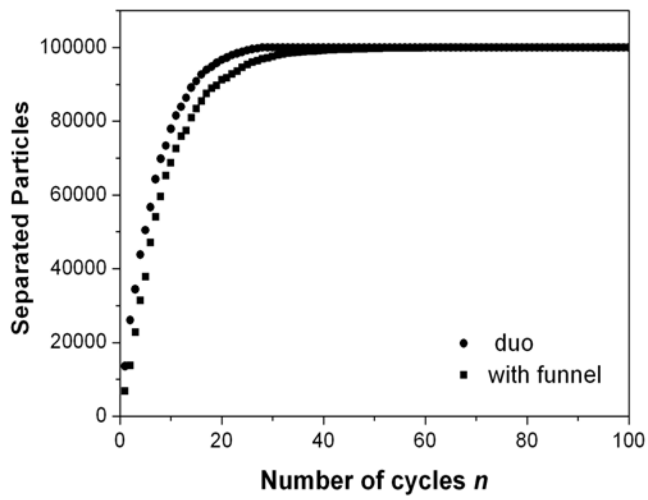


**Figure 6.** Cumulative number of separated disks vs. number of cycles for designs duo and with funnel and Δ=0.55.

Figure 5 shows the results for the three designs: duo, with funnel and without funnel, for a mixture with $\Delta$=0.6. The amount of disks separated with the duo design is always higher than those corresponding to the previous designs.

A similar behavior is observed in figure 6, for a mixture with $\Delta$=0.55. Again, we observe that the duo design has the greatest ability to keep apart disks. In this case, we only show duo and funnel designs because the poor performance obtained for the design without funnel.

In this way, the new design leaves aside the effects of lateral walls, thus obtaining a better mechanism for particles segregation.

## 4. Effect of the obstacle density

In order to check whether a different obstacle density in BGB can even improve the segregation obtained by the duo design, we have performed a set of runs changing the density of pins in the central part of the board. The aim of this change is to provoke more diffusion and eventually more segregation among particles of different species.

In figure 7, we present the results obtained for a the density of obstacles in the central part of the table which is twice the density of the rest. The fraction of the total length of the table occupied by higher density of obstacles is 0.11, i.e., 11% of the total area of pins. We found that there is no improvement due to this change of density, as shown in the figure.

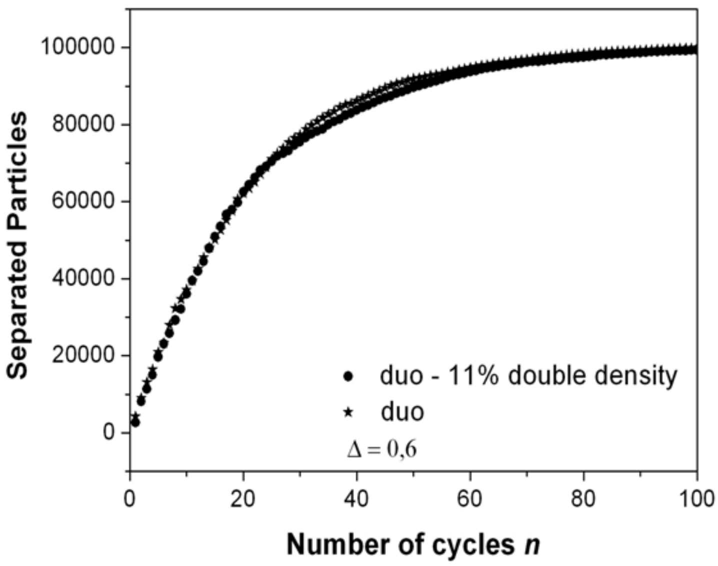


**Figure 7.** Accumulated number of separated disks vs. number of cycles for designs duo and duo with 11% double density of pins in central part of the table and $\Delta$= 0.6.

We have further increased the fraction area of higher density of obstacles to 14.5% and still obtained the same results. Figure 8 shows the comparison to the original duo design and we see no improvements.

As clearly shown in the above figures, the pin density change does not make better segregation of particles. The increment of obstacles affects identically the behavior of both species and, thus, the probability of finding mixed particles in the collectors is the same as for the case of uniform density tables.

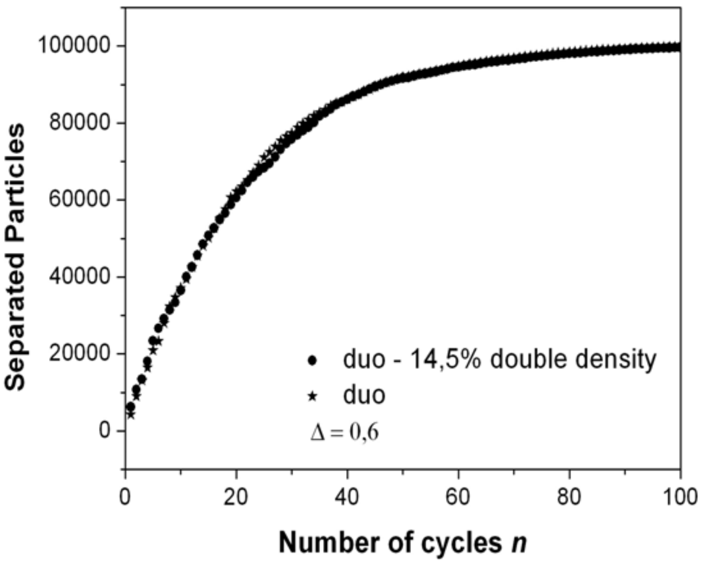


**Figure 8.** Accumulated number of separated disks vs. number of cycles for designs duo and duo with 14.5% double density of pins in central part of the table and $\Delta = 0.6$.

## 5. Conclusions

In this paper, we have shown that a BGB can be used to separate particles with different bouncing properties. This result is totally novel taking into account that these kind of apparatus is commonly used to mix particles and has never been tried in the opposite way.

In particular, different designs were tried to search for the best configuration of BGBs to achieve the desired segregation effect.

We have found that a design with funnel notably improves the segregation capability of a series of BGBs. Nevertheless, the “without” funnel design proved to be better at the first cycles of injection, but its performance was too poor for further cycling due to border effects.

The combination of those effects, gave rise to the duo design. This last design was found to be the best one to separate particles under the conditions studied in the present work.

We attempt to improve even more the results obtained with the duo design by increasing the density of pins in the central part of the BGB. It was proven that no effect is produced by this new configuration.

It still remains to be established the relationship between $\Delta$ and N through a systematic study as the one performed here for different bouncing relationships.

In the same way, it would be useful to determine the number of lateral collectors that can be drained in each cycle of a series of BGBs to assure a given degree of purity in the separated particles.

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