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Cite as: AIP Conference Proceedings **2293**, 330004 (2020); https://doi.org/10.1063/5.0026582
Published Online: 25 November 2020

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Numerical Study of Galton Board Experiment via Discrete Element Method Simulation

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Abstract. Galton board is an experimental device invented by Francis Galton in 1873. A particle thrown into the Galton board moves down due to gravity and bounces off pegs along its path down. Galton claimed it to be a random system, in which each particle has equal probability of moving to the left or right side of each peg it bounces off. It means that the final distribution of particles in the bottom part of the device shall approximate the shape of the binomial curve or Gaussian distribution. In this work, the behaviour of particles when moving through the Galton board using the discrete element numerical method was studied. The objective was to verify if the result of the simulation with calibrated input parameters of the contact model shows binomial distribution of particles in the bins in the bottom part of the Galton board as in the case of real experiments with various constructions and versions of Galton boards.

INTRODUCTION

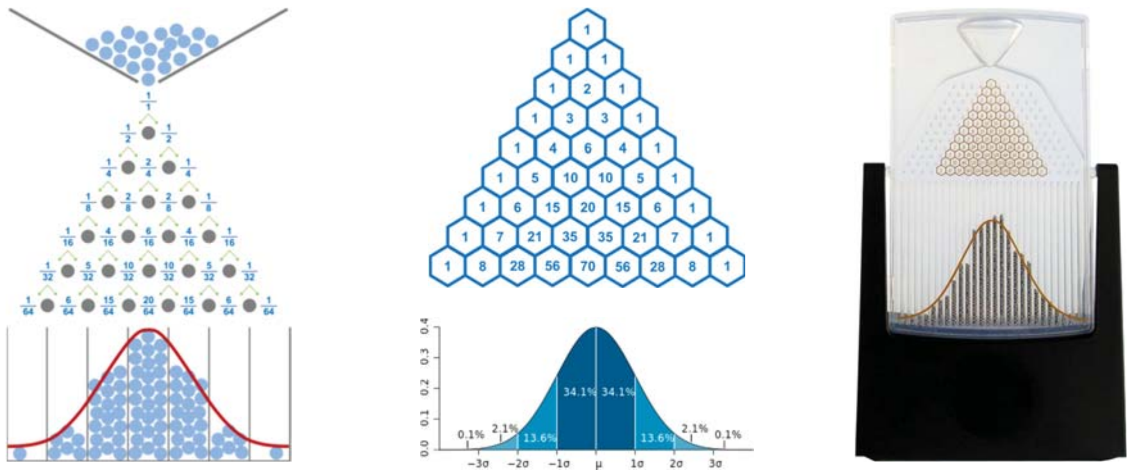
Random walk can be described as certain simplification of phenomena commonly observed in, for example, chemistry and physics. It is a process in which an object moves either left or right with certain probability in each step. To make it simple let us assume that the steps are equally long and the object moves along the straight line. The probability of finding the object in a certain distance from the centre leads to binomial distribution. Simple experimental representation of binomial distribution of such random walk is provided by the Galton board. The Galton board is an experimental device invented by Francis Galton in 1873. It consists of two vertical transparent boards interleaved with rows of hexagonally arranged pegs as shown in Figure 1.

The upper part of the Galton board consists of a hopper, through which particles are brought to the peg area. The bottom part, on the other hand, consists of several narrow bins where particles accumulate [1]. A particle thrown into the Galton board moves down due to gravity and bounces off the pegs along its path down. Galton claimed it to be a random system, in which each particle has equal probability of moving to the left or right side of each peg it bounces off. It means that the final distribution of particles in the bottom part of the device shall approximate the shape of the binomial curve or Gaussian distribution [2]. The importance of normal distribution lies in the fact that it relatively faithfully simulates various commonly encountered distributions. The curve always has one global maximum, which is also equal to the average value. Gaussian distribution can be parametrized using mean μ and standard deviation or dispersion σ. If we have a distribution which is normal with standard deviation σ, it must be held that 68 % of values can be found in the interval $\langle \mu-\sigma, \mu+\sigma \rangle$. Thus 68 % of the values differ from the mean by the maximum of one standard deviation. About 95 % and 99.7 % of the values must then lie in the interval $\langle \mu-2\sigma, \mu+2\sigma \rangle$ and $\langle \mu-3\sigma, \mu+3\sigma \rangle$, respectively, as shown in Figure 1b.

In this work, the behaviour of particles when moving through the Galton board using the discrete element numerical method was studied. The objective was to verify if the result of the simulation with calibrated input

International Conference of Numerical Analysis and Applied Mathematics ICNAAM 2019
AIP Conf. Proc. 2293, 330004-1–330004-4; https://doi.org/10.1063/5.0026582
Published by AIP Publishing. 978-0-7354-4025-8/$30.00

parameters of the contact model shows binomial distribution of particles in the bins in the bottom part of the Galton board as in the case of real experiments with various constructions and versions of Galton boards.



(a) (b) (c)

FIGURE 1. (a) Galton board scheme, (b) Pascal's triangle and Gaussian distribution, (c) Real Galton board model

MATERIAL AND METHODS

For the numerical model, the virtual material in the form of wooden beads with the diameter of 6 mm was created. The real particular material was characterized at the laboratories of the Bulk Solids Centre[1], where using several basic properties (angle of repose, friction parameters, coefficient of restitution, flow properties, etc.) a virtual twin with real particle properties was created and calibrated. In industrial applications, the coefficient of restitution is a parameter, which usually has no great influence on the correctness of the numerical model. In this particular case, however, it presents the dominant input data, and therefore special attention was paid to calibration of the coefficient of restitution. To determine the input values of the coefficient of restitution of the used particular material, the double pendulum method, which was described by Hlosta et al. (2018), was used [3]. Also general calibration procedure via static angle of repose was performed for material properties verification [4].

The DEM model used in this work employs the soft-sphere (SSDEM) method originally developed by Cundall and Strack [5]. In this method, the particles in contact are able to withstand small deformations. These deformations are then used in calculations of the forces acting between particles. The translation and rotary movement can be described by integration of the Newton equation of motion and its equivalent:

$$m_i \frac{dv_i}{dt} = \sum \left(F_{ij}^n + F_{ij}^t\right) + m_i g \tag{1}$$

$$I_i \frac{d\omega_i}{dt} = \sum \left(RV_i \times F_{ij}^t - \tau_{ij}^r\right) \tag{2}$$

where m_i, I_i, v_i and ω_i stand for weight, the moment of inertia, the speed of movement, and angular speed of rotation of particle i. F_{ij}^n and F_{ij}^t stand for normal and tangential force induced due to the contact of particle i with particle j in the relevant time step. RV_i stands for reaction vector between the centre of particle i and the contact point where force F_{ij}^t acts.

Tsuji et al. (1992) [6] designed a nonlinear contact model as a result of adapting the original model designed by Cundall and Strack. This currently frequently used Hertz-Mindlin model is described by the following equations:

$$F_{ij}^n = \left(-k_n \delta_{ij}^{n\,3/2} - \eta_n \left(v_{ij}^{sh} \cdot n_{ij}\right) n_{ij}\right) \tag{3}$$

[1] https://bsc.vsb.cz

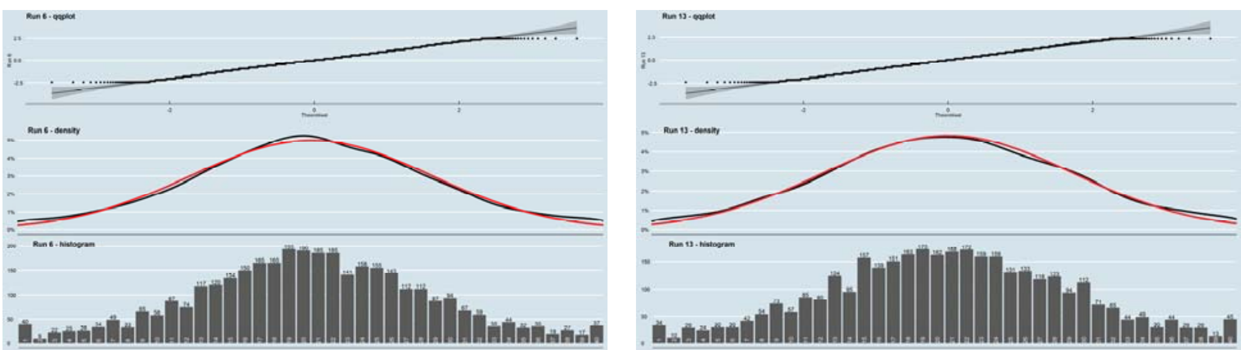
$$F_{ij}^{t} = \left(-k_t \delta_{ij}^{t} - \eta_t v_{ij}^{t}\right) \tag{4}$$

The calculations are performed in discrete time steps. Between each time step, the particles move at speed and acceleration calculated in the previous time step. These trajectories are used to calculate the position of particles in the next step. The behaviour of discrete elements (particles) depends on the parameters the value of which shall be included in the input setting. The basic input values include particle radius *R*, particle density *ρ*, Young modulus *E* (shear elasticity modulus *G*), and interaction coefficients [7].

RESULTS

A virtual Galton board, which contains 20 horizontal rows of pegs in the upper part and 40 distribution bins in the bottom part, was created. The particles were generated in a consecutive order in the vertical axis of the board above the first row of pegs. A total of 3500 particles were generated, and their distribution in the bottom part of the Galton board was monitored. In this way, 16 DEM simulations were performed altogether and subsequently statistically evaluated. To verify that it is normal distribution, several verification tests as part of the R software were carried out. These tests included, for example, Cramér-von Mises test [8], Anderson-Darling test [9], and Jarque-Bera test [10].

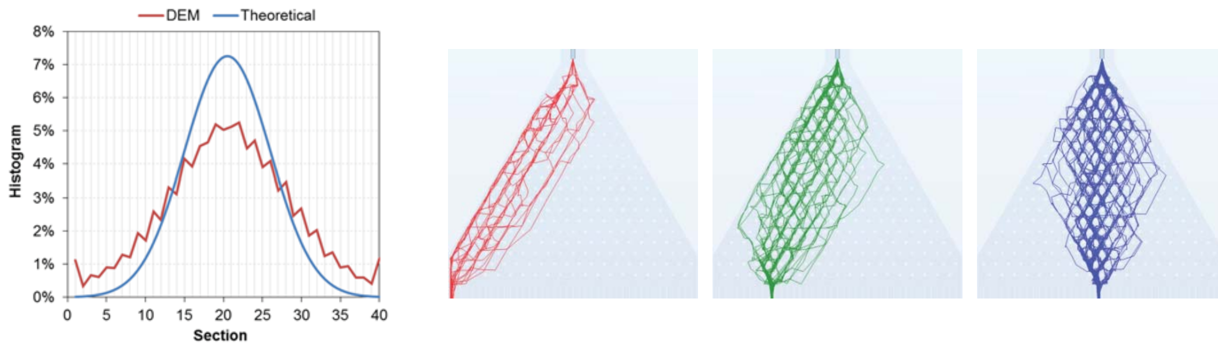
The only test at least partially verifying that it is normal distribution in five cases was Jarque-Bera test. The other tests did not verify normal distribution in either simulation. The outputs of the R software for the best and worst equality in Jarque-Bera test can be seen in Figure 1. As can be seen in Figure 3, the biggest problem faced are the outside bins, which should contain the least number of particles, whilst being by far higher than the neighbouring bins. Quite a few tests cannot cope with this fact. This could have been avoided if we had removed the outside bins from both sides during testing. However, this would lead to a distortion of the results.



(a) (b)

FIGURE 2. Statistical evaluation of the DEM simulation for a) 6th test, b) 13th test

The high number of particles in the outside bins number 1 and 40 could stem for various reasons whether it is the influence of the Galton board geometry itself, different simplifications of the contact model, and the particle interpretation or minor inaccuracy of the values of the calibrated input parameters of Hertz-Mindlin contact model. In Figure 3a, the final distribution of particles in simulation no. 16 is shown. The DEM simulation provides the advantage of applying particle tracking. Using this tracking, the particle trajectories, which ended up in bins no. 1, 10, and 20 (see Figure 3) at the end of the experiment, were represented. Tracking of particles showed that the virtual particles which immediately get to the lateral wall of the Galton board at the beginning of simulation find it hard to get back to its centre. Moreover, they are most likely to end their path in one of the outside bins. In addition, the question of generating the particles into the system plays its role here. The particles in the simulations were generated in the consecutive order individually in the device axis while in real experiments they are poured all at once from the hopper. The movement of a group of particles in full volume can eliminate their rebound to the outside part of the Galton board. The fact that the analysis of the obtained data did not verify normal distribution of particles in bins no. 1 to 40 in the bottom part of the Galton board at all leads to the idea that this simple experiment can be used as a calibration experiment for making the input parameters of the discrete element method more accurate. These parameters include material properties, friction parameters, the coefficient of restitution, and the shape of particles. In case of creating digital twins in the form of the real and virtual Galton board, this experiment could be ranked among the commonly used DEM calibrated experiments such as creation of static and dynamic angle of repose or the duration of discharge of material from the discharging hopper.



a) b) c) d)

FIGURE 3. a) Theoretical and simulation data, Tracking - b) Section no.1, c) Section no.10, d) Section no.20

CONCLUSION

16 repetitions of DEM simulations with throwing beads into the Galton board were performed. The statistical tests verified that the movement of particles through the board describes random walk with binomial distribution. However, none of the tests expressly verified normal distribution of particles. It was mainly due to the high number of particles in the outside bin no. 1 and 40. The main causes can be found in the geometry of the Galton board, in simplifications brought about by simulation, or inaccuracy of the input parameters of the DEM model. After further testing of different geometries of the Galton boards, this simple device can thus be considered as a calibration experiment for making the input data more accurate and for calibration of virtual materials for the discrete element method.

ACKNOWLEDGMENTS

This paper was conducted within the framework of the projects: LO1404 – Sustainable development of ENET Centre and project SP2019/115: Research in optimized design of transport and storage processes using test equipment and simulation techniques.

REFERENCES

1. A. A. M. Daud, Commun. Nonlinear. Sci. **19**, 3476-3491 (2014).
2. M. Leitner, M. P. Leubner and Z. Vörös. Physica A. **390**, 1248-1257 (2011).
3. J. Hlosta, D. Žurovec, J. Rozbroj, Á. Ramírez-Gómez, J. Nečas and J. Zegzulka. Chem. Eng. Res. Des. **135**, 222-233 (2018).
4. L. Jezerská, M. Žídek, D. Gelnar, B. Palkovská, J. Zegzulka and M. Čermák, "DEM model calibration based on mechanical-physical properties", in *International Conference of Numerical Analysis and Applied Mathematics (ICNAAM 2018)*, AIP Conference Proceedings 2116, 320006 (2019), https://doi.org/10.1063/1.5114328
5. P.A. Cundall and O.D.L. Strack. Géotechnique. **29**, 47-65 (1979).
6. Y. Tsuji, T. Tanaka and T. Ishida. Powder Technol. **71**, 239-250 (1992).
7. J. Hlosta and M. Čermák, "DEM-CFD Numerical Model of Laboratory Scale Fluidized Bed Coffee Roaster", in *International Conference of Numerical Analysis and Applied Mathematics (ICNAAM 2018)*, AIP Conference Proceedings 2116, 320004 (2019), https://doi.org/10.1063/1.5114326
8. H. Cramér. Scandinavian Actuarial Journal. **1**, 13-74, (1928).
9. T. W. Anderson, D. A. Darling. Annals of Mathematical Statistics. **23,** 193–212 (1952).
10. C. M. Jarque, A.K. Bera. Economics Letters. **6** (3), 255–259 (1980).