

Since $n \rightarrow \infty$, it is continuous set of random variables

As there is a uniform distribution between 0 and θ ,

the p.d.f. $f(x) = \text{constant}$.

Since $\int_0^\theta f(x) dx = 1 \Rightarrow f(x) = \underline{\underline{1/\theta}}$

$$P(a \leq X \leq b) = \int_a^b \frac{1}{\theta} dx$$

Given: $P(X < \epsilon) \leq \frac{E(X)}{\epsilon}$

$$\left[\begin{aligned} E(X) &= \int_0^\theta x f(x) dx \\ &= \theta/2 \end{aligned} \right]$$

$$P(X_{n-13} < \theta - \epsilon) \leq \frac{\theta/2}{\theta - \epsilon}$$

$$P(X_{n-13} < \theta - \epsilon) \leq 0 \quad \forall \epsilon > 0$$

$$\text{As } \epsilon \rightarrow \infty, \frac{\theta}{2(\theta - \epsilon)} \rightarrow 0$$

Hence, $P(X_{n-13} < \theta - \epsilon) = 0$ since inequality holds $\forall \epsilon > 0$.

Also, $P(X_{n-13} > \theta + \epsilon) = 0$ since the distribution of X is

Hence, combining them

from 0 to θ .
and $\epsilon > 0$

$$\lim_{n \rightarrow \infty} P(\underline{|X_{n-13} - \theta| > \epsilon}) = 0 \quad \left(\text{Hence, } X_{n-13} \text{ converges to } \theta \text{ as } n \rightarrow \infty \right)$$