

Given

$$f(x) = \text{Uniform}(0, \theta)$$

And the ordered statistics

$$X_{(1)} \leq X_{(2)} \leq X_{(3)} \leq \dots \leq X_{(n)}, \quad X_i \sim \text{Uniform}(0, \theta)$$

To Simplify, let Define

$$Y_i = \frac{X_i}{\theta}$$

$$\therefore Y_i \sim \text{Uniform}(0, 1), \quad X_{(n-13)} = \theta Y_{(n-13)}$$

Now, let a fix  $\varepsilon > 0$ . Consider

$$Y_{(n-13)} \leq 1 - \varepsilon \quad [\text{At least } 14 \text{ of } Y_i < 1 - \varepsilon]$$

But for one  $Y_i$ ,

$$P(Y_i \leq 1 - \varepsilon) = 1 - \varepsilon \quad [P(Y_i \leq 1 - \varepsilon) = \int_0^{1-\varepsilon} dx = 1 - \varepsilon]$$

Since each  $Y_i$  has a probability of  $1 - \varepsilon$  of being below  $1 - \varepsilon$  and each of them is Independent. We can count define another experiment to count success and failures of being above or below  $1 - \varepsilon$  which is exactly the Bernoulli binomial experiment.

$\therefore$  Number of samples below  $1 - \varepsilon$  is:

$$\text{Binomial}(n, 1 - \varepsilon)$$

As  $n \rightarrow \infty$ , this expected number is:

$$n(1 - \varepsilon) \rightarrow \infty$$

$\therefore$  with  $P(Y_i) \rightarrow 1$ , more than 14 samples exceed  $1 - \varepsilon$ .

$$\therefore P(Y_{(n-13)} \leq 1 - \varepsilon) \rightarrow 0$$

As this holds for all  $\varepsilon > 0$ ,  $\theta > 0$

$$Y_{(n-13)} \xrightarrow{n \rightarrow \infty} 1$$

$$\Rightarrow \boxed{X_{(n-13)} \xrightarrow{n \rightarrow \infty} \theta}$$