

Given, a random variable $X \in (0, \theta)$

$\Rightarrow X$ is a continuous random variable

so for this X , its PDF = $f(x) = \begin{cases} 1/\theta & \text{for } x \in (0, \theta) \\ 0 & \text{otherwise} \end{cases}$

In the question it is given that X 's PDF = $\frac{1}{\theta}$ for $x \in (1, \theta)$
but I think that's a typing mistake because

we know that $\int_{-\infty}^{\infty} f(x)dx = 1 = \int_0^{\theta} dx + \int_{\theta}^{\infty} dx + \int_0^0 dx$

We are given a sample of n i.i.d and the order statistics are

$$x_1 \leq x_2 \leq \dots \leq x_k \leq \dots \leq x_n$$

so x_1 is sample minimum.

x_n is sample maximum

x_{n-13} is $(n-13)^{th}$ sample ~~not~~ smallest value.

Since n is very large, $n-13$ is very close to n and
 x_n should be very close to θ so x_{n-13} is also expected
to very close θ as $n \rightarrow \infty$.

using convergence in probability:-

we say $x_{n-13} \xrightarrow{\text{converges}} \theta$ if $P(|x_{n-13} - \theta| > \epsilon) = 0$ for $\epsilon > 0$
as $n \rightarrow \infty$

also $x_{n-13} \leq x_n \leq \theta$

so ~~$\theta - x_{n-13} > \epsilon$~~

and $x_{n-13} < \theta - \epsilon$