

Given, a random variable  $X \in (0, \theta)$

$\Rightarrow X$  is a continuous random variable

So for this  $X$  its PDF  $= f(x) = \begin{cases} \frac{1}{\theta} & \text{for } x \in (0, \theta) \\ 0 & \text{otherwise} \end{cases}$

(In the question it is given that  $X$ 's PDF  $= \frac{1}{\theta}$  for  $x \in (1, \theta)$  but I think that's a typing mistake because

we know that  $\int_{-\infty}^{\infty} f(x) dx = 1 = \int_{-\infty}^0 0 dx + \int_0^{\theta} \frac{1}{\theta} dx + \int_{\theta}^{\infty} 0 dx$ .)

We are given a sample of  $n$  i.i.d and the order statistics are

$$X_1 \leq X_2 \leq \dots \leq X_k \leq \dots \leq X_n$$

So  $X_1$  is sample minimum.

$X_n$  is sample maximum

$X_{n-1}$  is  $(n-1)^{\text{th}}$  sample ~~max~~ smallest value.

Since  $n$  is very large,  $n-1$  is very close to  $n$  and  $X_n$  should be very close to  $\theta$  so  $X_{n-1}$  is also expected to be very close to  $\theta$  as  $n \rightarrow \infty$ .

using convergence in probability:-

we say  $X_{n-1} \xrightarrow{\text{converges}} \theta$  if  $P(|X_{n-1} - \theta| > \epsilon) = 0 \quad \forall \epsilon > 0$   
as  $n \rightarrow \infty$

$$\text{also } X_{n-1} \leq X_n \leq \theta$$

$$\text{so } \theta - X_{n-1} > \epsilon$$

$$\text{and } X_{n-1} < \theta - \epsilon$$