

Given

$$f(x) = \text{Uniform}(0, \theta)$$

And the ordered statistics

$$X_{(1)} \leq X_{(2)} \leq X_{(3)} \leq \dots \leq X_{(n)}, \quad X_i \sim \text{Uniform}(0, \theta)$$

To simplify, to Define

$$Y_i = \frac{X_i}{\theta}$$

$$\therefore Y_i \sim \text{Uniform}(0, 1), \quad X_{(n-13)} = \theta Y_{(n-13)}$$

Now, let a fix $\epsilon > 0$. Consider

$$Y_{(n-13)} \leq 1 - \epsilon \quad [\text{At least 14 of } Y_i < 1 - \epsilon]$$

But for one Y_i ,

$$P(Y_i \leq 1 - \epsilon) = 1 - \epsilon \quad \left[P(Y_i \leq 1 - \epsilon) = \int_0^{1-\epsilon} dx = 1 - \epsilon \right]$$

Since each Y_i has a probability of $1 - \epsilon$ of being below $1 - \epsilon$ and each of them is Independent. We can count define another experiment to count success and failures of being above or below $1 - \epsilon$ which is exactly the Bernoulli binomial experiment.

\therefore Number of samples below $1 - \epsilon$ is:

$$\text{Binomial}(n, 1 - \epsilon)$$

As $n \rightarrow \infty$, this expected number is:

$$n(1 - \epsilon) \rightarrow \infty$$

\therefore with $P(Y_i) \rightarrow 1$, more than 14 samples exceed $1 - \epsilon$.

$$\therefore P(Y_{(n-13)} \leq 1 - \epsilon) \rightarrow 0$$

As this holds for all $\epsilon > 0$,

$$Y_{(n-13)} \xrightarrow{n \rightarrow \infty} 1$$

$$\Rightarrow \boxed{X_{(n-13)} \xrightarrow{n \rightarrow \infty} \theta} \quad \square$$