MF6013/MF4056 - Computational Finance II

Assignment 1 – Monte Carlo valuation of a bond option under the Vasicek model

The Vasicek model of the short rate is given by

$$dr(t) = \alpha(b - r(t))dt + \sigma dW(t), \quad t \in [0, S]$$

where W is a standard Brownian motion. For the entirety of the assignment, set $\alpha = 1$, $b = 0.03$, $\sigma = 0.2$.

1. By generating exact trajectories of r over a suitably long interval with the parameters above for a range of values of r_0, illustrate the asymptotic properties

   $$\lim_{t\to\infty} \mathbb{E}[r(t)] = b; \quad \lim_{t\to\infty} \mathrm{Var}[r(t)] = \sigma^2/2\alpha.$$

   Justify your choice of interval. Were any of your samples negative?

2. Produce an empirical distribution (as a 3d plot) of the bivariate Gaussian pair $\left(r(T), \int_0^T r(s)ds\right)$ when $r_0 = 0.04$ and $T = 0.5$. Does the distribution change significantly if you vary r_0?

3. Let $T < S$. Under the Vasicek model, the price of a zero-coupon bond (ZCB) with face-value \$1 and maturity S at any time $t \in [0, S]$ is given by $P(r, t, S)$, where r is the observed value of the short rate at time t. Moreover

   $$P(r, t, S) = e^{A(t,S) - B(t,S)r}$$

   where

   $$B(t, S) = \frac{1}{\alpha}\left(1 - e^{-\alpha(S-t)}\right);$$
   $$A(t, S) = (B(t, S) - (S - t))\left(b - \frac{\sigma^2}{2\alpha^2}\right) - \frac{\sigma^2}{4\alpha}B(t, S)^2.$$

   The time $t = 0$ price of a call option with strike K and expiry $T < S$ written on this ZCB is given by

   $$\mathbb{E}\left[e^{-\int_0^T r(s)ds}\left(P(r, T, S) - K\right)^+\right].$$

   Using exact sampling of $\left(r(T), \int_0^T r(s)ds\right)$, produce a Monte Carlo valuation (with confidence interval) of this option when $r_0 = 0.04$, $K = 1$, $T = 0.5$, $S = 1$.

(15 marks)

Submission instructions are overleaf.

Submission instructions:

- Submit your assignment in pairs by uploading through the module Canvas page before 5pm on Friday, March 14, 2025.
- Late submissions without acceptable documentation will be awarded a mark of zero.
- Your submission should consist of a single `.ipynb` file containing all explanation, examples and executable code as a single report.
- Marks will be given for the clarity of your presentation.