

```
In [ ]: import numpy as np
import matplotlib.pyplot as plt
import scipy.optimize as opt
from scipy.optimize import newton
from scipy.optimize import minimize
from scipy.optimize import minimize_scalar
from scipy.optimize import fsolve
plt.style.use('seaborn-whitegrid')
```

<ipython-input-1-5c8774f982ff>:3: MatplotlibDeprecationWarning: The seaborn styles shipped by Matplotlib are deprecated since 3.6, as they no longer correspond to the styles shipped by seaborn. However, they will remain available as 'seaborn-v0.8-<style>'. Alternatively, directly use the seaborn API instead.
plt.style.use('seaborn-whitegrid')

```
In [ ]: def power_scarcity_utility(quantity, price, A, gamma, B):

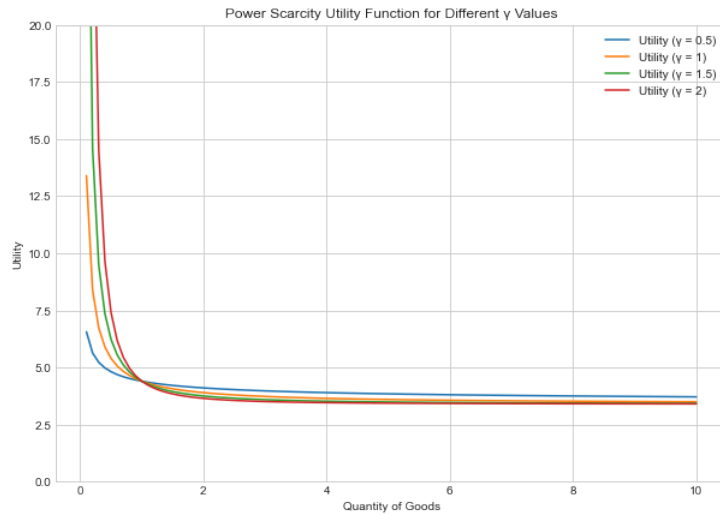
    utility_from_quantity = A / np.power(quantity, gamma)
    utility_from_price = B * np.log(price)
    return utility_from_quantity + utility_from_price

A = 1
B = 1
price_veblen = 30
gamma_values = [0.5, 1, 1.5, 2]

quantities = np.linspace(0.1, 10, 100)
plt.figure(figsize=(10, 7))
for gamma in gamma_values:
    utilities = power_scarcity_utility(quantities, price_veblen, A, gamma, B)
    plt.plot(quantities, utilities, label=f'Utility (γ = {gamma})')

plt.title('Power Scarcity Utility Function for Different γ Values')
plt.xlabel('Quantity of Goods')
plt.ylabel('Utility')
plt.legend()
plt.grid(True)
plt.ylim(0, 20)

plt.show()
```



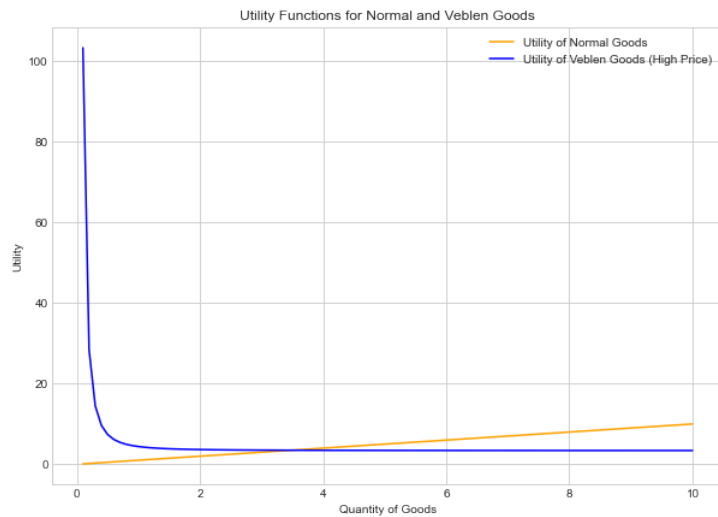
```
In [ ]: def normal_goods_utility(quantity, alpha):
    return quantity**alpha

alpha_normal = 1
price_veblen = 30
gamma = 2

utilities_normal = normal_goods_utility(quantities, alpha_normal)
utilities_veblen = power_scarcity_utility(quantities, price_veblen, A, gamma, B)
```

```
plt.figure(figsize=(10, 7))
plt.plot(quantities, utilities_normal, label='Utility of Normal Goods', color='orange')
plt.plot(quantities, utilities_veblen, label='Utility of Veblen Goods (High Price)', color='blue')
plt.title('Utility Functions for Normal and Veblen Goods')
plt.xlabel('Quantity of Goods')
plt.ylabel('Utility')
plt.legend()
plt.grid(True)

plt.show()
```



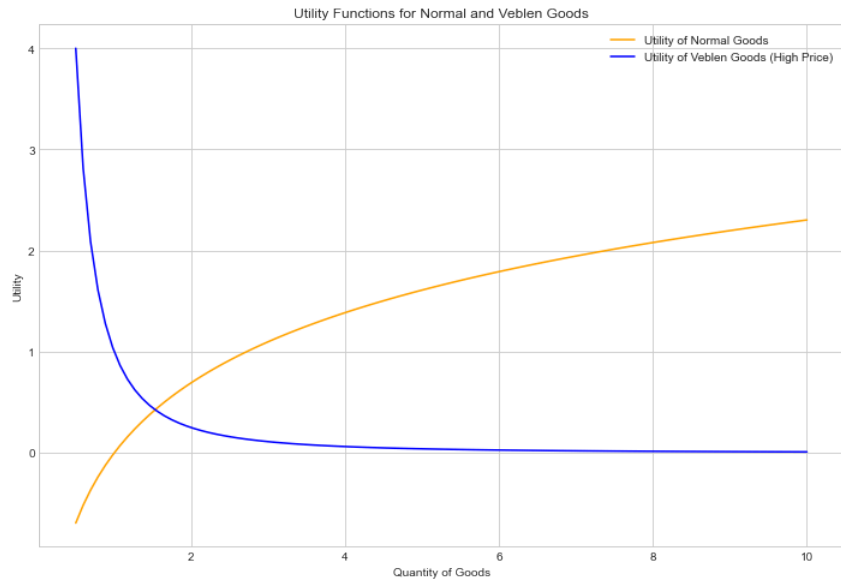
```
In [ ]: def normal_goods_utility2(quantity):
        return np.log(quantity)

price_veblen = 1
gamma = 2

quantities = np.linspace(0.5, 10, 100)
utilities_normal = normal_goods_utility2(quantities)
utilities_veblen = power_scarcity_utility(quantities, price_veblen, A, gamma, B)

plt.figure(figsize=(12, 8))
plt.plot(quantities, utilities_normal, label='Utility of Normal Goods', color='orange')
plt.plot(quantities, utilities_veblen, label='Utility of Veblen Goods (High Price)', color='blue')

plt.title('Utility Functions for Normal and Veblen Goods')
plt.xlabel('Quantity of Goods')
plt.ylabel('Utility')
plt.legend()
plt.grid(True)
plt.show()
```



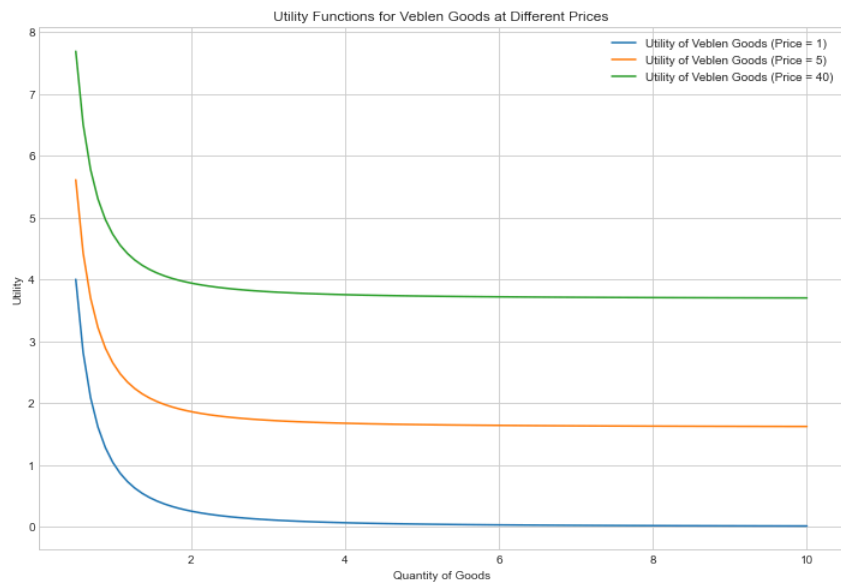
```
In [ ]: prices = [1, 5, 40]

plt.figure(figsize=(12, 8))

for price in prices:
    utilities_veblen = power_scarcity_utility(quantities, price, A, gamma, B)
    plt.plot(quantities, utilities_veblen, label=f'Utility of Veblen Goods (Price = {price})')

plt.title('Utility Functions for Veblen Goods at Different Prices')
plt.xlabel('Quantity of Goods')
plt.ylabel('Utility')
plt.legend()
plt.grid(True)

plt.show()
```



```
In [ ]: def veblen_engel_curve(income, price_sensitivity):
    return price_sensitivity * income
```

```
def normal_engel_curve(income, price_sensitivity):
    return price_sensitivity * income

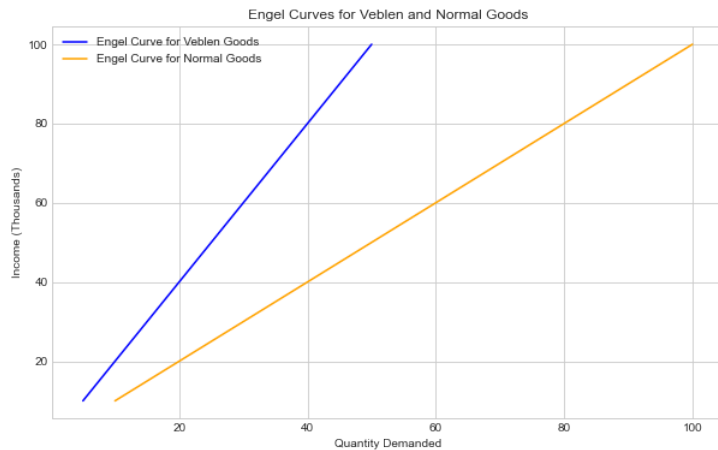
price_sensitivity_veblen = 0.5
price_sensitivity_normal = 1

income_levels = np.linspace(10, 100, 50)

quantities_demanded_veblen = veblen_engel_curve(income_levels, price_sensitivity_veblen)
quantities_demanded_normal = normal_engel_curve(income_levels, price_sensitivity_normal)

plt.figure(figsize=(10, 6))
plt.plot(quantities_demanded_veblen, income_levels, label='Engel Curve for Veblen Goods', color='blue')
plt.plot(quantities_demanded_normal, income_levels, label='Engel Curve for Normal Goods', color='orange')
plt.ylabel('Income (Thousands)')
plt.xlabel('Quantity Demanded')
plt.title("Engel Curves for Veblen and Normal Goods")
plt.legend()
plt.grid(True)

plt.show()
```



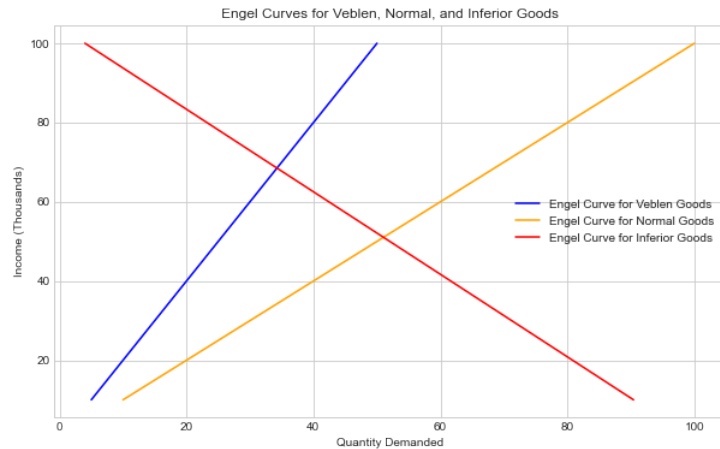
```
In [ ]: def inferior_engel_curve(income, base_quantity, income_sensitivity):
    return base_quantity - income_sensitivity * income

base_quantity_inferior = 100
income_sensitivity_inferior = 0.96

quantities_demanded_inferior = inferior_engel_curve(income_levels, base_quantity_inferior, income_sensitivity_inferior)

plt.figure(figsize=(10, 6))
plt.plot(quantities_demanded_veblen, income_levels, label='Engel Curve for Veblen Goods', color='blue')
plt.plot(quantities_demanded_normal, income_levels, label='Engel Curve for Normal Goods', color='orange')
plt.plot(quantities_demanded_inferior, income_levels, label='Engel Curve for Inferior Goods', color='red')
plt.ylabel('Income (Thousands)')
plt.xlabel('Quantity Demanded')
plt.title("Engel Curves for Veblen, Normal, and Inferior Goods")
plt.legend()
plt.grid(True)

plt.show()
```



```
In [ ]: def utility_derivative_x(x, *args):
    income, price1, price2 = args
    y = (income - price1 * x) / price2
    if y <= 0:
        return np.inf
    return 1 - price1 / (income - price1 * x)

def mydemand(income, price1, price2):
    start_x = max(0.1, income / (2 * price1))
    try:
        optimal_x = newton(utility_derivative_x, start_x, args=(income, price1, price2))
        optimal_y = (income - price1 * optimal_x) / price2
        if optimal_y <= 0:
            raise ValueError("Non-positive y computed, invalid result.")
    except RuntimeError:
        return np.nan, np.nan
    return optimal_x, optimal_y

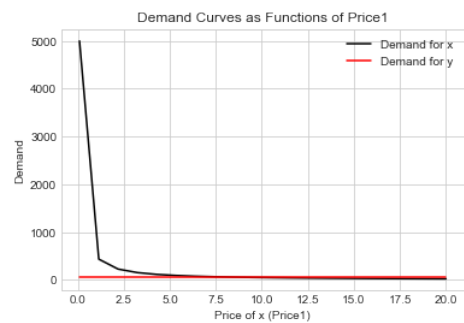
income = 1000
price2 = 7

prices = np.linspace(0.1, 20, 20)
demandx = np.zeros(20)
demandy = np.zeros(20)

for i in range(20):
    demandx[i], demandy[i] = mydemand(income, prices[i], price2)

plt.plot(prices, demandx, '-k', label='Demand for x')
plt.plot(prices, demandy, '-r', label='Demand for y')

plt.xlabel('Price of x (Price1)')
plt.ylabel('Demand')
plt.title('Demand Curves as Functions of Price1')
plt.legend()
plt.grid(True)
plt.show()
```



```

In [ ]: x = np.linspace(0.1, 10, 100)

y1 = np.zeros_like(x)
y2 = np.zeros_like(x)
y3 = np.zeros_like(x)
y4 = np.zeros_like(x)

def myutility(x, y):
    return x + np.log(y)

def our_newton_solver(funcname, startvalue, arglist):
    current = startvalue
    for _ in range(1000):
        fval = funcname(current, arglist)
        grad = (funcname(current + 0.5 * 1e-5, arglist) - funcname(current - 0.5 * 1e-5, arglist)) * 1e+5
        if abs(fval) < 1e-8:
            break
        current = current - fval / grad
    return current

def indiff1(y, mylist):
    x, util = mylist
    ut = myutility(x, y)
    return ut - util

def indiff(x, util):
    return our_newton_solver(indiff1, 1, [x, util])

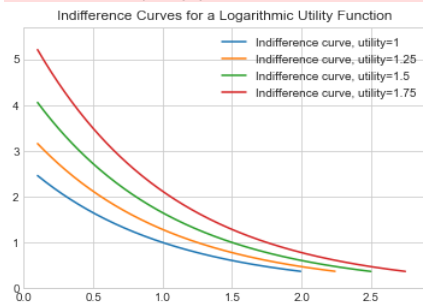
for i, element in enumerate(x):
    y1[i] = indiff(element, 1)
    y2[i] = indiff(element, 1.25)
    y3[i] = indiff(element, 1.5)
    y4[i] = indiff(element, 1.75)

plt.plot(x, y1, label='Indifference curve, utility=1')
plt.plot(x, y2, label='Indifference curve, utility=1.25')
plt.plot(x, y3, label='Indifference curve, utility=1.5')
plt.plot(x, y4, label='Indifference curve, utility=1.75')

plt.ylim(0, 5.7)
plt.xlim(0, 2.9)
plt.title('Indifference Curves for a Logarithmic Utility Function')
plt.legend(loc='upper right')
plt.show()

```

<ipython-input-9-3096e1e6b66c>:16: RuntimeWarning: invalid value encountered in log
 return x + np.log(y)



```

In [ ]: utility_levels = [5, 7, 9, 12]

x_values = np.linspace(0.1, 10, 100)

plt.figure(figsize=(10, 6))

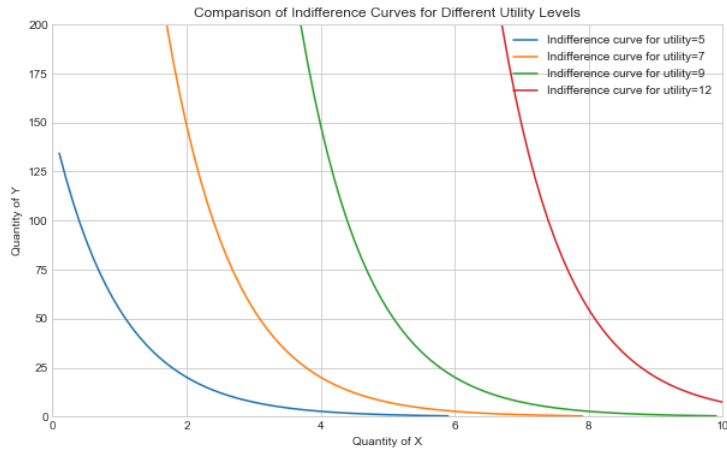
for util in utility_levels:
    ys = np.array([indiff(x, util) for x in x_values])
    plt.plot(x_values, ys, label=f'Indifference curve for utility={util}')

plt.xlabel('Quantity of X')
plt.ylabel('Quantity of Y')
plt.title('Comparison of Indifference Curves for Different Utility Levels')

```

```
plt.ylim(0, 200)
plt.xlim(0, 10)
plt.legend()
plt.grid(True)
plt.show()
```

```
<ipython-input-9-3096e1e6b66c>:16: RuntimeWarning: invalid value encountered in log
return x + np.log(y)
```



```
In [ ]: def budget_constraint(x, price_x, price_y, income):
        return (income - price_x * x) / price_y

def objective(x, price_x, price_y, income):
    y = budget_constraint(x, price_x, price_y, income)
    return -myutility(x, y)

price_x = 200
price_y = 7
income = 1000

result = minimize(objective, x0=1, args=(price_x, price_y, income), bounds=[(0, income / price_x)])
optimal_x = result.x[0]
optimal_y = budget_constraint(optimal_x, price_x, price_y, income)

utility_at_optimal_bundle = myutility(optimal_x, optimal_y)

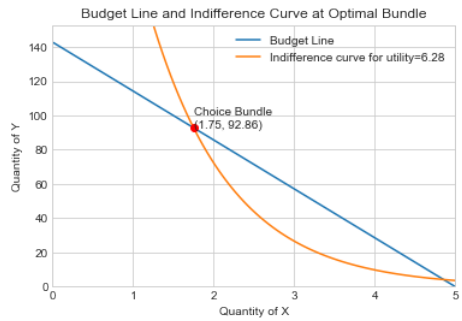
x = np.linspace(0, income / price_x, 100)
plt.plot(x, budget_constraint(x, price_x, price_y, income), label='Budget Line')

indiff_y = np.exp(utility_at_optimal_bundle - x)
plt.plot(x, indiff_y, label=f'Indifference curve for utility={utility_at_optimal_bundle:.2f}')

plt.plot(optimal_x, optimal_y, 'ro')
plt.text(optimal_x, optimal_y, f'Choice Bundle\n({optimal_x:.2f}, {optimal_y:.2f})')

plt.ylim(0, budget_constraint(0, price_x, price_y, income) + 10)
plt.xlim(0, income / price_x)
plt.xlabel('Quantity of X')
plt.ylabel('Quantity of Y')
plt.title('Budget Line and Indifference Curve at Optimal Bundle')
plt.legend()
plt.grid(True)
plt.show()
```

```
<ipython-input-11-ca0d77e28963>:7: RuntimeWarning: divide by zero encountered in log
return x + np.log(y)
```



```
In [ ]: def budget_constraint2(x, price_x, price_y, income):
    return (income - price_x * x) / price_y

def objective2(x, price_x, price_y, income):
    y = budget_constraint(x, price_x, price_y, income)
    if y <= 0:
        return np.inf
    return -myutility(x, y)

result = minimize(objective2, x0=0.5 * income / price_x, bounds=[(0, income / price_x)], args=(price_x, price_y, income))
optimal_x = result.x[0]
optimal_y = budget_constraint2(optimal_x, price_x, price_y, income)

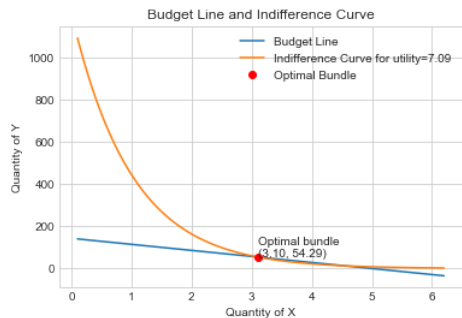
utility_at_optimal_bundle = myutility(optimal_x, optimal_y)

x_values = np.linspace(0.1, optimal_x * 2, 100)

y_budget = budget_constraint2(x_values, price_x, price_y, income)
y_indiff = np.exp(utility_at_optimal_bundle - x_values)

plt.plot(x_values, y_budget, label='Budget Line')
plt.plot(x_values, y_indiff, label=f'Indifference Curve for utility={utility_at_optimal_bundle:.2f}')
plt.plot(optimal_x, optimal_y, 'ro', label='Optimal Bundle')
plt.text(optimal_x, optimal_y, f'Optimal bundle\n({optimal_x:.2f}, {optimal_y:.2f})')
plt.xlabel('Quantity of X')
plt.ylabel('Quantity of Y')
plt.title('Budget Line and Indifference Curve')
plt.legend()
plt.grid(True)

plt.show()
```



```
In [ ]: def utility_of_xf(xf, income=1000, price_xf=10, price_y=7):
    y = (income - price_xf * xf) / price_y
    if xf <= 0 or y <= 0:
        return -np.inf
    return np.log(xf) + np.log(y)

income = 1000
price_xf = 10
price_y = 7

result = minimize_scalar(lambda xf: -utility_of_xf(xf, income, price_xf, price_y), bounds=(0, income / price_xf), method='bounded')
```



```

if result.success:
    xf_optimal = result.x
    y_optimal = (income - price_xf * xf_optimal) / price_y
    utility_optimal = utility_of_xf(xf_optimal, income, price_xf, price_y)
else:
    raise ValueError("Optimization failed.")

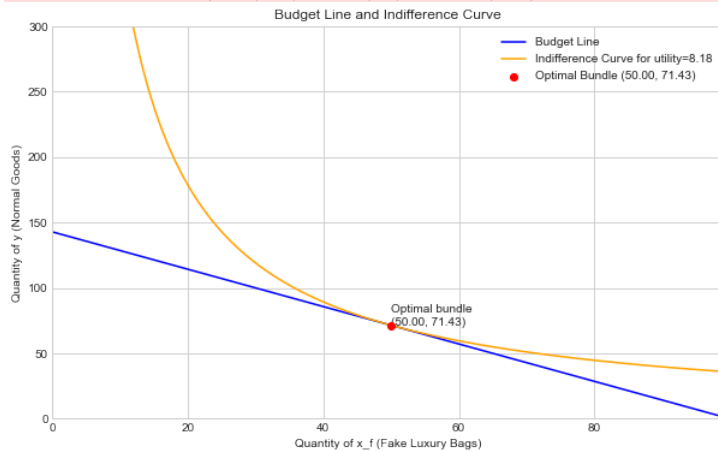
xf_values = np.linspace(0, income / price_xf, 100)
y_values_budget_line = (income - price_xf * xf_values) / price_y

indifference_curve_y = np.exp(utility_optimal - np.log(xf_values))

plt.figure(figsize=(10, 6))
plt.plot(xf_values, y_values_budget_line, label='Budget Line', color='blue')
plt.plot(xf_values, indifference_curve_y, label='Indifference Curve for utility={:.2f}'.format(utility_optimal), color='orange')
plt.scatter([xf_optimal], [y_optimal], color='red', zorder=5, label='Optimal Bundle ({:.2f}, {:.2f})'.format(xf_optimal, y_optimal))
plt.text(xf_optimal, y_optimal, f'Optimal bundle\n({xf_optimal:.2f}, {y_optimal:.2f})')
plt.xlabel('Quantity of x_f (Fake Luxury Bags)')
plt.ylabel('Quantity of y (Normal Goods)')
plt.title('Budget Line and Indifference Curve')
plt.ylim(0, 300)
plt.xlim(0, 99)
plt.legend()
plt.grid(True)
plt.show()

```

<ipython-input-13-b326d229ccb>:33: RuntimeWarning: divide by zero encountered in log
indifference_curve_y = np.exp(utility_optimal - np.log(xf_values))

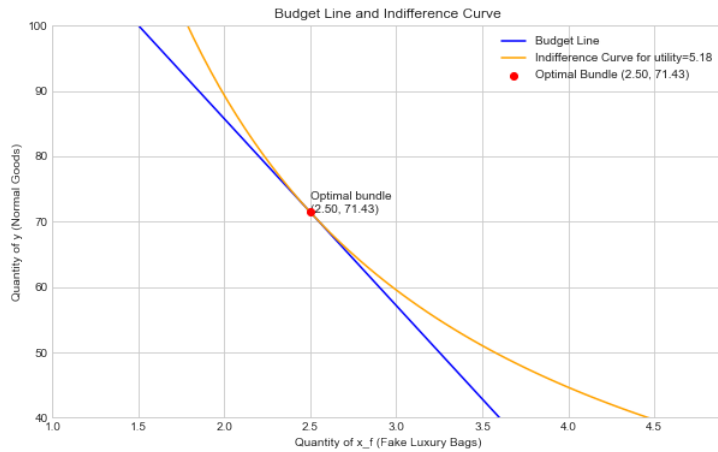


```

In [ ]: plt.figure(figsize=(10, 6))
plt.plot(xf_values, y_values_budget_line, label='Budget Line', color='blue')
plt.plot(xf_values, indifference_curve_y, label='Indifference Curve for utility={:.2f}'.format(utility_optimal), color='orange')
plt.scatter([xf_optimal], [y_optimal], color='red', zorder=5, label='Optimal Bundle ({:.2f}, {:.2f})'.format(xf_optimal, y_optimal))
plt.text(xf_optimal, y_optimal, f'Optimal bundle\n({xf_optimal:.2f}, {y_optimal:.2f})')
plt.xlabel('Quantity of x_f (Fake Luxury Bags)')
plt.ylabel('Quantity of y (Normal Goods)')
plt.title('Budget Line and Indifference Curve')
plt.ylim(40, 100)
plt.xlim(1, 4.9)
plt.legend()
plt.grid(True)
plt.show()

```

<ipython-input-14-6553b775d538>:33: RuntimeWarning: divide by zero encountered in log
indifference_curve_y = np.exp(utility_optimal - np.log(xf_values))



```
In [ ]: income = 1000
price_xf = 10
price_y = 7
utility_optimal = 8.18
utility_fixed = 7.09

xf_values = np.linspace(0, income / price_xf, 100)
y_values_budget_line = (income - price_xf * xf_values) / price_y

indifference_curve_optimal_y = np.exp(utility_optimal - np.log(xf_values))
indifference_curve_fixed_y = np.exp(utility_fixed - np.log(xf_values))

def find_intersection(utility, income, price_xf, price_y):
    return fsolve(lambda xf: np.exp(utility - np.log(xf)) - ((income - price_xf * xf) / price_y), income / (price_xf + price_y))

xf_intersection_optimal = find_intersection(utility_optimal, income, price_xf, price_y)[0]
y_intersection_optimal = (income - price_xf * xf_intersection_optimal) / price_y

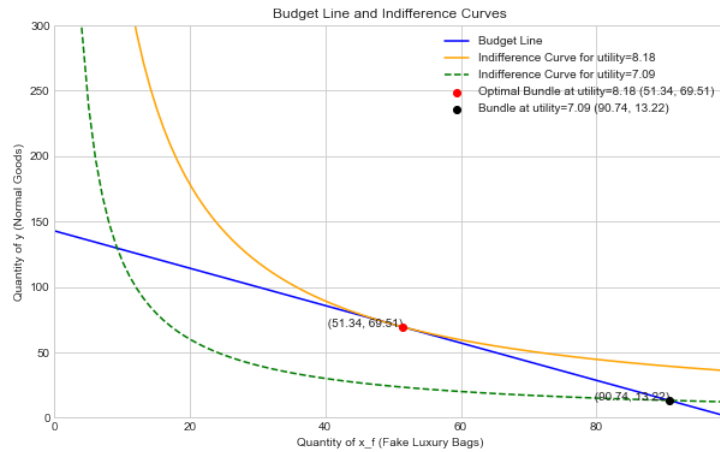
xf_intersection_fixed = find_intersection(utility_fixed, income, price_xf, price_y)[0]
y_intersection_fixed = (income - price_xf * xf_intersection_fixed) / price_y

plt.figure(figsize=(10, 6))
plt.ylim(0, 300)
plt.xlim(0, 99)
plt.plot(xf_values, y_values_budget_line, label='Budget Line', color='blue')
plt.plot(xf_values, indifference_curve_optimal_y, label='Indifference Curve for utility=8.18', color='orange')
plt.plot(xf_values, indifference_curve_fixed_y, 'g--', label='Indifference Curve for utility=7.09', color='green')
plt.scatter(xf_intersection_optimal, y_intersection_optimal, color='red', zorder=5, label=f'Optimal Bundle at utility=8.18 ({xf_intersection_optimal:.2f}, {y_intersection_optimal:.2f})')
plt.scatter(xf_intersection_fixed, y_intersection_fixed, color='black', zorder=5, label=f'Bundle at utility=7.09 ({xf_intersection_fixed:.2f}, {y_intersection_fixed:.2f})')
plt.text(xf_intersection_optimal, y_intersection_optimal, f' ({xf_intersection_optimal:.2f}, {y_intersection_optimal:.2f})', ha='right')
plt.text(xf_intersection_fixed, y_intersection_fixed, f' ({xf_intersection_fixed:.2f}, {y_intersection_fixed:.2f})', ha='right')
plt.xlabel('Quantity of x_f (Fake Luxury Bags)')
plt.ylabel('Quantity of y (Normal Goods)')
plt.title('Budget Line and Indifference Curves')
plt.legend()
plt.grid(True)
plt.show()
```

<ipython-input-15-8187e501991d>:16: RuntimeWarning: divide by zero encountered in log
indifference_curve_optimal_y = np.exp(utility_optimal - np.log(xf_values))

<ipython-input-15-8187e501991d>:17: RuntimeWarning: divide by zero encountered in log
indifference_curve_fixed_y = np.exp(utility_fixed - np.log(xf_values))

<ipython-input-15-8187e501991d>:36: UserWarning: color is redundantly defined by the 'color' keyword argument and the fmt string "g--" (-> color='g'). The keyword argument will take precedence.
plt.plot(xf_values, indifference_curve_fixed_y, 'g--', label='Indifference Curve for utility=7.09', color='green') # Dotted line



```
In [ ]: def myutility(x, y):
    if x <= 0 or y <= 0:
        return -np.inf
    return x + np.log(y)

def myutility_derivative(x, income, price1, price2):
    y = (income - price1 * x) / price2
    if y <= 0:
        return np.inf
    return 1 - (price1 / price2) / y

def mydemand(income, price1, price2):
    start_x = 3

    optimal_x = newton(myutility_derivative, start_x, args=(income, price1, price2))

    if optimal_x < 0 or (income - price1 * optimal_x) < 0:
        return np.nan, np.nan

    optimal_y = (income - price1 * optimal_x) / price2
    return optimal_x, optimal_y

income = 1000
price1 = 200
price2 = 7

optimal_x, optimal_y = mydemand(income, price1, price2)
print(f"Optimal values: x ≈ {optimal_x:.2f}, y ≈ {optimal_y:.2f}")

Optimal values: x ≈ 4.00, y ≈ 28.57
```

```
In [ ]: def myutility(x, y):
    if x <= 0 or y <= 0:
        return -np.inf
    return np.log(x) + np.log(y)

def myutility_derivative(x, income, price1, price2):
    y = (income - price1 * x) / price2
    if x <= 0 or y <= 0:
        return np.inf
    return 1/x - (price1 / price2) / y

def mydemand(income, price1, price2):
    start_x = 15
    optimal_x = newton(myutility_derivative, start_x, args=(income, price1, price2))

    optimal_y = (income - price1 * optimal_x) / price2
    return optimal_x, optimal_y

income = 5000
price1 = 200
price2 = 7
```

```
optimal_x, optimal_y = mydemand(income, price1, price2)
print(f"Optimal values: x ≈ {optimal_x:.2f}, y ≈ {optimal_y:.2f}")
```

Optimal values: x ≈ 12.50, y ≈ 357.14

```
In [ ]: income = 1000
new_price_x = 200
price_y = 7
utility_level = 5.18 # The given utility level

def find_x_on_indifference_curve(x):
    y = np.exp(utility_level - np.log(x))
    return new_price_x * x + price_y * y - income

initial_guess_x = 50

new_optimal_x = newton(find_x_on_indifference_curve, initial_guess_x)
new_optimal_y = np.exp(utility_level - np.log(new_optimal_x))

print(f"New optimal bundle on the same indifference curve: x = {new_optimal_x:.2f}, y = {new_optimal_y:.2f}")
```

New optimal bundle on the same indifference curve: x = 2.68, y = 66.39

```
In [ ]: income = 1000
old_price_x = 10
price_y = 7
utility_level = 5.18

initial_guess_x = 50

old_optimal_x = newton(find_x_on_indifference_curve, initial_guess_x)
old_optimal_y = np.exp(utility_level - np.log(old_optimal_x))

print(f"Old optimal bundle on the same indifference curve: x = {old_optimal_x:.2f}, y = {old_optimal_y:.2f}")
```

Old optimal bundle on the same indifference curve: x = 98.74, y = 1.80

```
In [ ]: def difference_in_indirect_utilities(income_to_be_found, longlist):
    income1 = longlist[0]
    px1 = longlist[1]
    py1 = longlist[2]
    px2 = longlist[3]
    py2 = longlist[4]
    return my_indirect_utility(income1, px1, py1) - my_indirect_utility(income_to_be_found, px2, py2)

a = our_newton_solver(difference_in_indirect_utilities, 1.01, [1000, 200, 7, 10, 7])
print(a)
```

223.60679774997763

```
In [ ]: x = np.linspace(0.1, 100, 1000)
plt.plot(x, (1000 - 200 * x) / 7)
plt.plot(x, (223.6069 - 10 * x) / 7)

def mydemand(income, price1, price2):
    x = our_newton_maximizer(myutility_included_budget, 0.1, [income, price1, price2])
    y = (income - price1 * x) / price2
    return x, y

utility_level = my_indirect_utility(1000, 200, 7)
y1 = np.zeros((1000, 1))
i = 0
for element in x:
    y1[i] = indiff(x[i], utility_level)
    i += 1

plt.plot(x, y1)

dems1 = mydemand(1000, 200, 7)
dems3 = mydemand(223.6069, 10, 7)

plt.scatter(dems1[0], dems1[1], s=50, alpha=0.5, color='purple')
plt.scatter(dems3[0], dems3[1], s=50, alpha=0.5, color='purple')
plt.text(dems1[0], dems1[1], f' ({dems1[0]:.2f}, {dems1[1]:.2f})')
plt.text(dems3[0], dems3[1], f' ({dems3[0]:.2f}, {dems3[1]:.2f})', ha='left',)

plt.plot((dems1[0], dems1[0]), (0, dems1[1]), '--')
plt.plot((0, dems1[0]), (dems1[1], dems1[1]), '--')
```

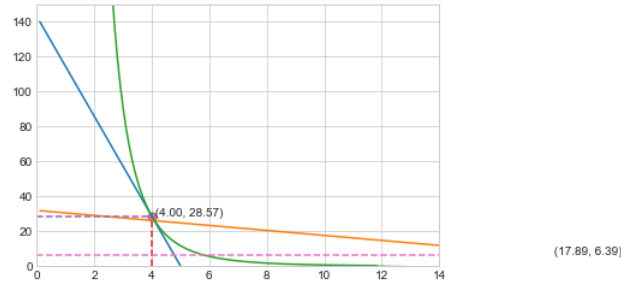
```
plt.plot((dems3[0], dems3[0]), (0, dems3[1]), '--')
plt.plot((0, dems3[0]), (dems3[1], dems3[1]), '--')

plt.ylim(0, 150)
plt.xlim(0,14)

print(utility_level)
```

1.7795169323944573

```
<ipython-input-412-9c3aea300cbe>:2: RuntimeWarning: invalid value encountered in log
return 0.8*np.log(x)+0.2*np.log(y)
```



So, doing the Slutsky decomposition, the demand change in x of a substitution effect from 2.4999999996769455 to 11.18034505680577 (the relative price of x and y has changed)

```
In [ ]: x_new = 11.18
        y_new = 15.97

MRS_new = -y_new / x_new
print(f"The Marginal Rate of Substitution at the new bundle is: {MRS_new:.4f}")
```

The Marginal Rate of Substitution at the new bundle is: -0.1436

```
In [ ]: def myutility_ql(x,y):
        return np.log(x)+np.log(y)

def myutility_ql_included_budget(x,mylist):
    income=mylist[0]
    price1=mylist[1]
    price2=mylist[2]
    y = (income-price1*x)/price2
    return -myutility_ql(x,y)

def mydemand_ql(income,price1,price2):
    x_res=opt.minimize(myutility_ql_included_budget,1,[income,price1,price2])
    x=x_res['x']
    y = (income-price1*x)/price2
    return x,y

def myindutility_ql(income,price1,price2):
    x,y = mydemand_ql(income,price1,price2)
    return myutility_ql(x,y)

def myutility(x,y):
    return 0.8*np.log(x)+0.2*np.log(y)

def myutility_included_budget(x,mylist):
    income=mylist[0]
    price1=mylist[1]
    price2=mylist[2]
    y = (income-price1*x)/price2
    return -myutility(x,y)

def mydemand(income,price1,price2):
    x_res=opt.minimize(myutility_included_budget,1,[income,price1,price2])
    x=x_res['x']
    y = (income-price1*x)/price2
    return x,y

def myindutility(income,price1,price2):
    x,y = mydemand(income,price1,price2)
    return myutility(x,y)

prices = np.linspace(0.1,500,1000)
```

```

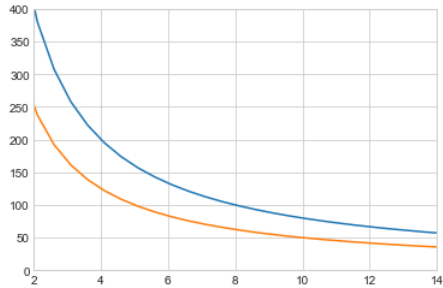
dem1 = np.zeros(1000)
dem2 = np.zeros(1000)

for i in range(1000):
    dem1[i],y=mydemand(1000,prices[i],7)
    dem2[i],y=mydemand_ql(1000,prices[i],7)

plt.ylim(0, 400)
plt.xlim(2,14)
plt.plot(prices,dem1)
plt.plot(prices,dem2)

```

Out[]: [<matplotlib.lines.Line2D at 0x7f7df010bd30>]



```

In [ ]: prices = np.linspace(0.1,500,1000)
dem1 = np.zeros(1000)

for i in range(1000):
    dem1[i],y=mydemand_ql(1000,prices[i],7)

ypos = 200
text = "old price"
plt.hlines(y=ypos, xmin=2, xmax=20, color = 'black', linestyle="dotted")

plt.plot((dems1[0], dems1[0]), (200, dems1[1]), 'r--')
plt.plot((200, dems1[0]), (dems1[1], dems1[1]), 'r--')

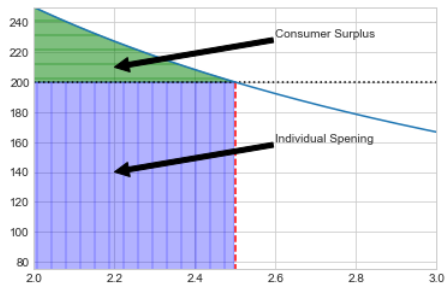
plt.ylim(75, 250)
plt.xlim(2,3)
plt.plot(dem1,prices)

plt.fill_between(dem1, prices, 200, where = (prices > 200), color = 'green', alpha = 0.5, hatch = '-')
plt.fill_between(dem1, 200, where = (dem1<2.5), color = 'blue', alpha = 0.3, hatch = '|')

plt.annotate('Consumer Surplus', xy=(2.2, 210), xytext=(2.6, 230), arrowprops=dict(facecolor='black'))
plt.annotate('Individual Spening', xy = (2.2, 140), xytext = (2.6,160), arrowprops=dict(facecolor='black'))

```

12.18726155267223



```

In [ ]: prices = np.linspace(0.1,500,1000)
dem2 = np.zeros(1000)

for i in range(1000):
    dem2[i],y=mydemand(1000,prices[i],7)

ypos = 200
text = "old price"
plt.hlines(y=ypos, xmin=0, xmax=30, color = 'black', linestyle="dotted")

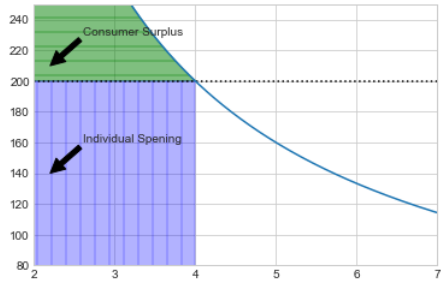
```

```
plt.ylim(80, 250)
plt.xlim(2,7)
plt.plot(dem2,prices)

plt.fill_between(dem2, prices, 200, where = (prices > 200), color = 'green', alpha = 0.5, hatch = '-')
plt.fill_between(dem2, 200, where = (dem2<4), color = 'blue', alpha = 0.3, hatch = '|')

plt.annotate('Consumer Surplus', xy=(2.2, 210), xytext=(2.6, 230), arrowprops=dict(facecolor='black'))
plt.annotate('Individual Spening', xy = (2.2, 140), xytext = (2.6,160), arrowprops=dict(facecolor='black'))
```

Out[]: Text(2.6, 160, 'Individual Spening')



```
In [ ]: def demand_function(ai, bi, ci, P, Qj):
    return ai - bi * P + ci * Qj

ai = 500
bi = 1
ci = -0.0005

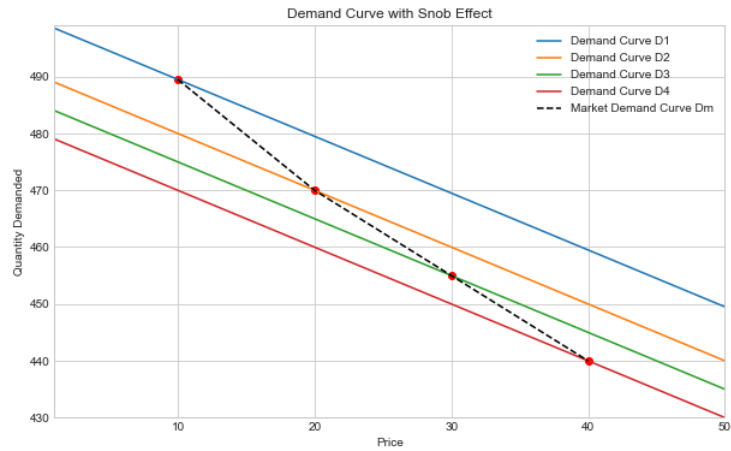
price_range = np.linspace(1, 50, 100)
Qj_values = [1000, 20000, 30000, 40000]
demand_curves = [demand_function(ai, bi, ci, price_range, Qj) for Qj in Qj_values]

plt.figure(figsize=(10, 6))
for i, demand_curve in enumerate(demand_curves):
    plt.plot(price_range, demand_curve, label=f'Demand Curve D{i+1}')

equilibrium_prices = [10, 20, 30, 40]
equilibrium_quantities = [demand_function(ai, bi, ci, P, Qj) for P, Qj in zip(equilibrium_prices, Qj_values)]
plt.scatter(equilibrium_prices, equilibrium_quantities, color='red')
plt.plot(equilibrium_prices, equilibrium_quantities, color='black', linestyle='--', label='Market Demand Curve Dm')

plt.xlabel('Price')
plt.ylabel('Quantity Demanded')
plt.title('Demand Curve with Snob Effect')
plt.legend()
plt.grid(True)

plt.ylim(430, 499)
plt.xlim(1,50)
plt.show()
```



```
In [ ]: def supply(p):
    return 10 * p

def demand(p):
    return 1000 - 10 * p

def demand_ls(p):
    return 700 - 10 * p

prices = np.linspace(0, 100, 100)

supply_quantities = supply(prices)
demand_quantities = demand(prices)
demand_loss = demand_ls(prices)

plt.plot(supply_quantities, prices, label='Supply')
plt.plot(demand_quantities, prices, label='Demand')
plt.plot(demand_loss, prices, label='Demand Neg Ext.')

plt.scatter(500, 50, color='red', zorder=5)
plt.text(500, 50, "A")
plt.scatter(500, 20, color='grey', zorder=5)
plt.text(500, 20, "B")
plt.scatter(350, 35, color='blue', zorder=5)
plt.text(350, 35, "C")

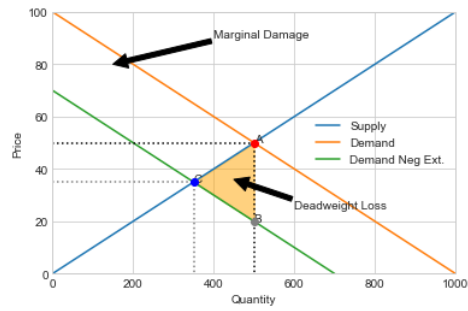
plt.xlabel('Quantity')
plt.ylabel('Price')

plt.vlines(x=500, ymin=-5, ymax=50, color='black', linestyle="dotted")
plt.hlines(y=50, xmin=0, xmax=500, color='black', linestyle="dotted")

plt.vlines(x=350, ymin=-5, ymax=35, color='grey', linestyle="dotted")
plt.hlines(y=35, xmin=0, xmax=350, color='grey', linestyle="dotted")

plt.fill_between([350, 500], [35, 20], [35, 50], color='orange', alpha=0.5)

plt.annotate('Marginal Damage', xy=(150, 80), xytext=(400, 90), arrowprops=dict(facecolor='black'))
plt.annotate('Deadweight Loss', xy=(450, 36), xytext=(600, 25), arrowprops=dict(facecolor='black'))
plt.legend()
plt.xlim(0, 1000)
plt.ylim(0, 100)
plt.show()
```

In []:

```
def supply(p):
    return 10 * p

def supply_ls(p):
    return 10 * p - 300

def demand(p):
    return 1000 - 10 * p

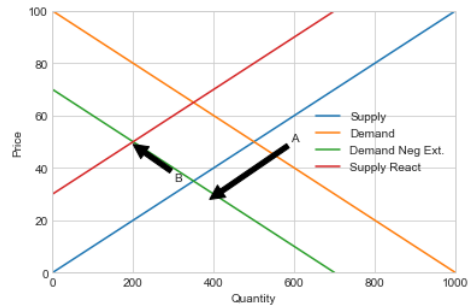
def demand_ls(p):
    return 700 - 10 * p

supply_quantities = supply(prices)
demand_quantities = demand(prices)
demand_loss = demand_ls(prices)
supply_loss = supply_ls(prices)

plt.plot(supply_quantities, prices, label='Supply')
plt.plot(demand_quantities, prices, label='Demand')
plt.plot(demand_loss, prices, label='Demand Neg Ext.')
plt.plot(supply_loss, prices, label='Supply React')

plt.xlabel('Quantity')
plt.ylabel('Price')

plt.legend()
plt.xlim(0, 1000)
plt.ylim(0, 100)
plt.show()
```



```
In [ ]: import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import newton
from scipy.optimize import minimize
from scipy.optimize import minimize_scalar

def our_newton_maximizer(funcname,startvalue,arglist):
    current=startvalue
    fval = funcname(current,arglist)
    grad = (funcname(current+0.5*1e-5,arglist)-funcname(current-0.5*1e-5,arglist))*1e+5
    secgrad1 = (funcname(current+0.5*1e-5+0.5*1e-5,arglist)-funcname(current-0.5*1e-5+0.5*1e-5,arglist))*1e+5
    secgrad2 = (funcname(current+0.5*1e-5-0.5*1e-5,arglist)-funcname(current-0.5*1e-5-0.5*1e-5,arglist))*1e+5
    secderiv = (secgrad1-secgrad2)*1e+5
    while (abs(grad)>1e-8):
        current = current - grad/secderiv
        fval = funcname(current,arglist)
        grad = (funcname(current+0.5*1e-5,arglist)-funcname(current-0.5*1e-5,arglist))*1e+5
        secgrad1 = (funcname(current+0.5*1e-5+0.5*1e-5,arglist)-funcname(current-0.5*1e-5+0.5*1e-5,arglist))*1e+5
        secgrad2 = (funcname(current+0.5*1e-5-0.5*1e-5,arglist)-funcname(current-0.5*1e-5-0.5*1e-5,arglist))*1e+5
        secderiv = (secgrad1-secgrad2)*1e+5

    return current
```

```
In [ ]: def myutility(x,y):
    return 0.8*np.log(x)+0.2*np.log(y)

def myutility_included_budget(x,mylist):
    income=mylist[0]
    price1=mylist[1]
    price2=mylist[2]
    y = (income-price1*x)/price2
    return myutility(x,y)

def mydemand(income,price1,price2):
    x=our_newton_maximizer(myutility_included_budget,0.1,[income,price1,price2])
    y = (income-price1*x)/price2
    return x,y
```

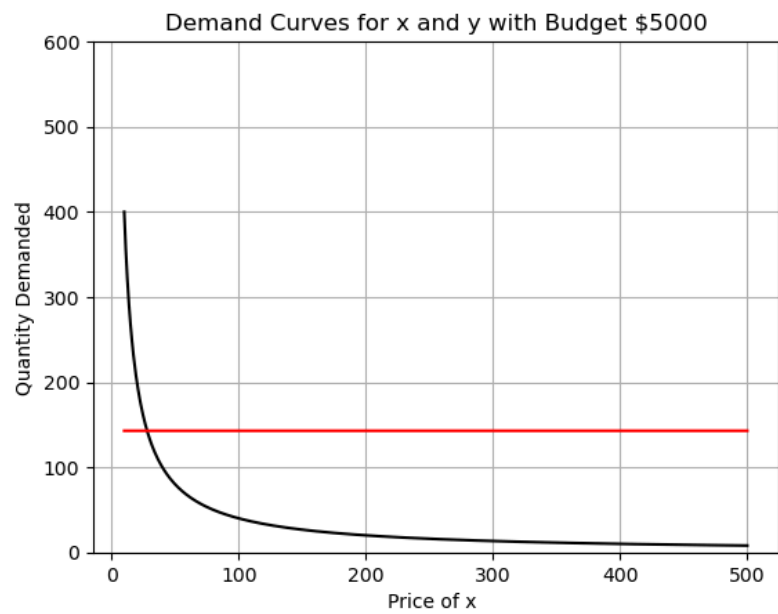
```
In [ ]: x,y = mydemand(5000,200,7)
print(x)
print(y)
```

```
20.00000025516103
142.8571355668277
```

```
In [ ]: prices = np.linspace(10, 500, 800)
demandx = np.zeros(800)
demandy = np.zeros(800)

for i in range(800):
    demandx[i], demandy[i] = mydemand(5000, prices[i], 7)

plt.plot(prices, demandx, '-k', label='Demand for x')
plt.plot(prices, demandy, '-r', label='Demand for y')
plt.xlabel('Price of x ')
plt.ylabel('Quantity Demanded')
plt.title('Demand Curves for x and y with Budget $5000')
plt.ylim([0,600])
plt.grid(True)
plt.show()
```



```
In [ ]: y1 = np.zeros_like(x)
y2 = np.zeros_like(x)
y3 = np.zeros_like(x)
y4 = np.zeros_like(x)

def myutility(x, y):
    return 0.8*np.log(x) + 0.2*np.log(y)

def our_newton_solver(funcname, startvalue, arglist):
    current = startvalue
    for _ in range(100):
        fval = funcname(current, arglist)
        grad = (funcname(current + 0.5 * 1e-5, arglist) - funcname(current - 0.5 * 1e-5, arglist)) * 1e+5
        if abs(fval) < 1e-8:
            break
        current = current - fval / grad
    return current

def indiff1(y,mylist):
    x=mylist[0]
    util=mylist[1]
    ut=myutility(x,y)
    return ut - util

def indiff(x,util):
    return our_newton_solver(indiff1,1,[x,util])

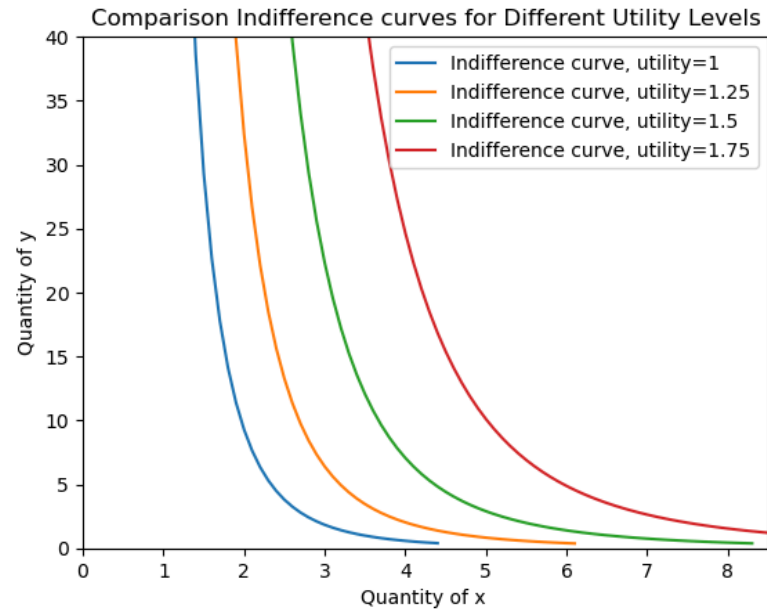
x=np.linspace(0.1,10,100)
i=0
for element in x:
    y1[i]=indiff(x[i],1)
    y2[i]=indiff(x[i],1.25)
    y3[i]=indiff(x[i],1.5)
    y4[i]=indiff(x[i],1.75)
    i +=1

plt.plot(x,y1,label='Indifference curve, utility=1')
plt.plot(x,y2,label='Indifference curve, utility=1.25')
```

```
plt.plot(x,y3,label='Indifference curve, utility=1.5')
plt.plot(x,y4,label='Indifference curve, utility=1.75')
plt.ylim(0,40)
plt.xlim(0,8.5)
plt.title('Comparison Indifference curves for Different Utility Levels')
plt.xlabel('Quantity of x ')
plt.ylabel('Quantity of y')
plt.legend(loc='upper right')
```

```
/var/folders/n6/rx4l4kx50rb_qrr2fpwdsf_00000gn/T/ipykernel_34901/3978256092.py:7: RuntimeWarning: invalid value encountered in log
return 0.8*np.log(x) + 0.2*np.log(y)
```

Out[]: <matplotlib.legend.Legend at 0x130e897d0>



```
In [ ]: def my_indirect_utility(income,price1,price2):
        x,y=mydemand(income,price1,price2)
        return myutility(x,y)
```

```
In [ ]: my_indirect_utility(5000,200,7)
```

Out[]: 3.3889548448285574

```
In [ ]: x=np.linspace(0.1,100,100)
plt.plot(x,(5000-200*x)/7)

utility_level = my_indirect_utility(5000,200,7)
y=np.zeros((100,1))
i=0
for element in x:
    y[i]=indiff(x[i],utility_level)
    i+=1

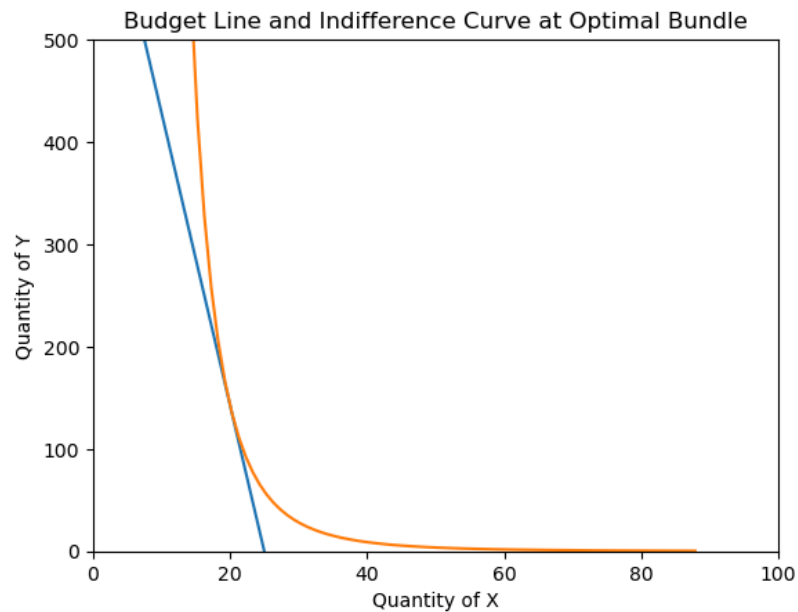
plt.plot(x,y)
plt.xlabel('Quantity of X')
plt.ylabel('Quantity of Y')
plt.title('Budget Line and Indifference Curve at Optimal Bundle')
plt.ylim(0,500)
plt.xlim(0,100)
```

```

/var/folders/n6/rx4l4kx50rb_qrr2fpwdsf_00000gn/T/ipykernel_34901/3978256092.py:16: RuntimeWarning: divide by zero encountered in scalar divide
  current = current - fval / grad
/var/folders/n6/rx4l4kx50rb_qrr2fpwdsf_00000gn/T/ipykernel_34901/3978256092.py:13: RuntimeWarning: invalid value encountered in scalar subtract
  grad = (funcname(current + 0.5 * 1e-5, arglist) - funcname(current - 0.5 * 1e-5, arglist)) * 1e+5
/var/folders/n6/rx4l4kx50rb_qrr2fpwdsf_00000gn/T/ipykernel_34901/3978256092.py:7: RuntimeWarning: invalid value encountered in log
  return 0.8*np.log(x) + 0.2*np.log(y)

```

Out[]: (0.0, 100.0)



```

In [ ]: def difference_in_indirect_utilities(income_to_be_found, longlist):
        income1 = longlist[0]
        px1 = longlist[1]
        py1 = longlist[2]
        px2 = longlist[3]
        py2 = longlist[4]
        return my_indirect_utility(income1, px1, py1) - my_indirect_utility(income_to_be_found, px2, py2)

```

```

In [ ]: a = our_newton_solver(difference_in_indirect_utilities, 80, [5000, 200, 7, 500, 7])
        print(a)

```

10406.915056692358

```

In [ ]: x = np.linspace(0.1, 100, 100)

plt.plot(x, (5000 - 200 * x) / 7)
plt.plot(x, (10406.9151 - 500 * x) / 7)
plt.plot(x, (5000 - 500 * x) / 7)

utility_level = my_indirect_utility(5000, 200, 7)
utility_level2 = my_indirect_utility(5000, 500, 7)
y1 = np.zeros((100, 1))
y2 = np.zeros((100, 1))
i = 0
for element in x:
    y1[i] = indiff(x[i], utility_level)
    y2[i] = indiff(x[i], utility_level2)
    i += 1

plt.plot(x, y1)
plt.plot(x, y2)

```

```

plt.ylim(0,350)
plt.xlim(0,35)

dems1=mydemand(5000,200,7)
dems2=mydemand(5000,500,7)
dems3=mydemand(10406.9151,500,7)

plt.xlabel('Quantity of X')
plt.ylabel('Quantity of Y')
plt.scatter(dems1[0],dems1[1],s=50,alpha=0.5,color='black')
plt.scatter(dems2[0],dems2[1],s=50,alpha=0.5,color='black')
plt.scatter(dems3[0],dems3[1],s=50,alpha=0.5,color='black')

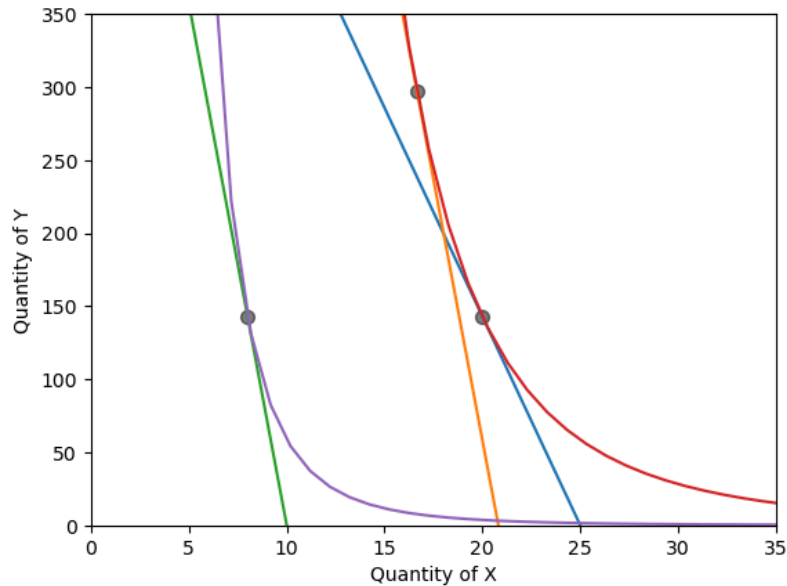
```

```

/var/folders/n6/rx4l4kx50rb_qrr2fpwdsf_00000gn/T/ipykernel_34901/3978256092.py:16: RuntimeWarning: divide by zero encountered in scalar divide
current = current - fval / grad
/var/folders/n6/rx4l4kx50rb_qrr2fpwdsf_00000gn/T/ipykernel_34901/3978256092.py:13: RuntimeWarning: invalid value encountered in scalar subtract
grad = (funcname(current + 0.5 * 1e-5, arglist) - funcname(current - 0.5 * 1e-5, arglist)) * 1e+5
/var/folders/n6/rx4l4kx50rb_qrr2fpwdsf_00000gn/T/ipykernel_34901/3978256092.py:7: RuntimeWarning: invalid value encountered in log
return 0.8*np.log(x) + 0.2*np.log(y)

```

Out[]: <matplotlib.collections.PathCollection at 0x14392d190>



In []: `print(f'So, doing the Slutsky decomposition, the demand change in x from {dems1[0]} to {dems2[0]}' consists of a substitution effect from {dems1[0]} to {dems3[0]} (the relative price of x and y has changed) and an income effect of from {dems3[0]} to {dems2[0]} (the consumer got richer and wants to consume more of x).`

So, doing the Slutsky decomposition, the demand change in x from 20.00000025516103 to 8.00000000064376' consists of a substitution effect from 20.00000025516103 to 16.65106396043938 (the relative price of x and y has changed) and an income effect of from 16.65106396043938 to 8.00000000064376 (the consumer got richer and wants to consume more of x).

Demand for Luxury Goods Across Different Prices

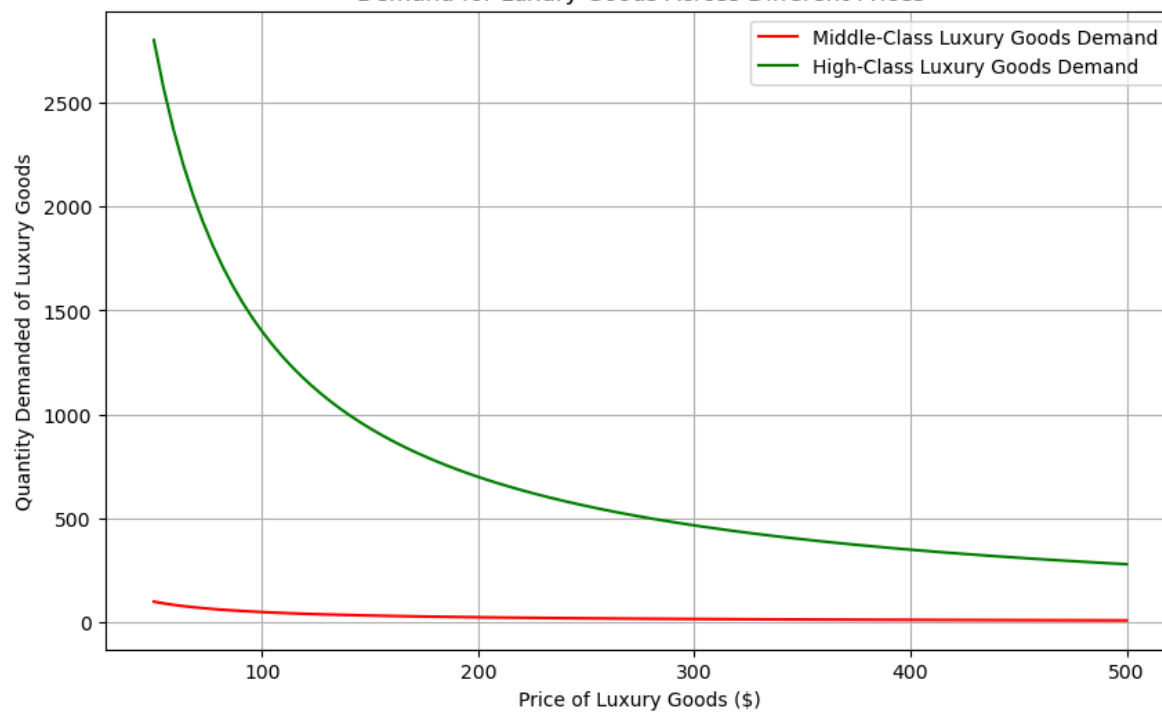
```
In [ ]: import numpy as np
import matplotlib.pyplot as plt

def demand_goods(alpha, I, p_x, p_y):
    x = alpha * I / p_x
    y = (1 - alpha) * I / p_y
    return x, y

# Parameters
alpha_M = 0.9
beta_H = 0.3
I_M = 50_000
I_H = 200_000
p_x = 20
luxury_price_range = np.linspace(50, 500, 100)
x_M, y_M = [], []
x_H, y_H = [], []
for p_y in luxury_price_range:
    x, y = demand_goods(alpha_M, I_M, p_x, p_y)
    x_M.append(x)
    y_M.append(y)
    x, y = demand_goods(beta_H, I_H, p_x, p_y)
    x_H.append(x)
    y_H.append(y)

plt.figure(figsize=(10, 6))
plt.plot(luxury_price_range, y_M, label='Middle-Class Luxury Goods Demand', color='red')
plt.plot(luxury_price_range, y_H, label='High-Class Luxury Goods Demand', color='green')
plt.title('Demand for Luxury Goods Across Different Prices')
plt.xlabel('Price of Luxury Goods ($)')
plt.ylabel('Quantity Demanded of Luxury Goods')
plt.legend()
plt.grid(True)
plt.show()
```

Demand for Luxury Goods Across Different Prices



Luxury Goods for Demand vs income

```
In [ ]: def demand_luxury_goods(alpha, I, p_y):
        return (1 - alpha) * I / p_y

alpha_M = 0.9
beta_H = 0.3
p_y = 150

middle_class_incomes = [50000, 75000, 100000]
high_class_incomes = [200000, 300000, 500000]

middle_class_demands = [demand_luxury_goods(alpha_M, income, p_y) for income in middle_class_incomes]
high_class_demands = [demand_luxury_goods(beta_H, income, p_y) for income in high_class_incomes]

# Plotting the demands
plt.figure(figsize=(10, 6))
plt.plot(middle_class_incomes, middle_class_demands, label='Middle-Class Luxury Goods Demand', marker='o')
plt.plot(high_class_incomes, high_class_demands, label='High-Class Luxury Goods Demand', marker='o')
plt.title('Luxury Goods Demand vs. Income')
plt.xlabel('Income ($)')
plt.ylabel('Quantity Demanded of Luxury Goods')
plt.legend()
plt.grid(True)
plt.show()
```


Luxury Goods Demand vs. Income



```
In [ ]: def demand_luxury_goods(alpha, I, p_y, essential_expenses):
    disposable_income = I - essential_expenses
    return max((1 - alpha) * disposable_income / p_y, 0)

alpha_M = 0.9
beta_H = 0.3
p_y = 150
p_x = 50

essential_expenses_M = [25000, 35000, 40000]
essential_expenses_H = [40000, 50000, 80000]

middle_class_incomes = [50000, 75000, 100000]
high_class_incomes = [200000, 300000, 500000]

middle_class_demands = [demand_luxury_goods(alpha_M, income, p_y, expenses) for income, expenses in zip(middle_class_incomes, essential_expenses_M)]
high_class_demands = [demand_luxury_goods(beta_H, income, p_y, expenses) for income, expenses in zip(high_class_incomes, essential_expenses_H)]

print("Middle-Class Luxury Goods Demand:")
for income, demand in zip(middle_class_incomes, middle_class_demands):
    print(f"Income: ${income}, Essential Expenses: ${essential_expenses_M[middle_class_incomes.index(income)]}, Demand for Luxury Goods: {demand:.2f} units")

print("\nHigh-Class Luxury Goods Demand:")
for income, demand in zip(high_class_incomes, high_class_demands):
    print(f"Income: ${income}, Essential Expenses: ${essential_expenses_H[high_class_incomes.index(income)]}, Demand for Luxury Goods: {demand:.2f} units")
```

Middle-Class Luxury Goods Demand:

Income: \$50000, Essential Expenses: \$25000, Demand for Luxury Goods: 16.67 units

Income: \$75000, Essential Expenses: \$35000, Demand for Luxury Goods: 26.67 units

Income: \$100000, Essential Expenses: \$40000, Demand for Luxury Goods: 40.00 units

High-Class Luxury Goods Demand:

Income: \$200000, Essential Expenses: \$40000, Demand for Luxury Goods: 746.67 units

Income: \$300000, Essential Expenses: \$50000, Demand for Luxury Goods: 1166.67 units

Income: \$500000, Essential Expenses: \$80000, Demand for Luxury Goods: 1960.00 units

Luxury Goods by Social Class Affordability Histogram

```
In [ ]: def demand_luxury_goods(alpha, I, p_y):
        disposable_income = I - (alpha * I)
        return disposable_income / p_y

alpha_M = 0.8
beta_H = 0.2
p_y_middle = 100
p_y_high = 500

income_middle_class = 100000
income_high_class = 500000

demand_middle = demand_luxury_goods(alpha_M, income_middle_class, p_y_middle)
demand_high = demand_luxury_goods(beta_H, income_high_class, p_y_high)

classes = ['Middle-Class', 'High-Class']
demands = [demand_middle, demand_high]
prices = [p_y_middle, p_y_high]

# Plotting
fig, ax = plt.subplots()
bar1 = ax.bar(classes[0], demands[0], color='blue', label=f'Lower-end Luxury (Price: ${p_y_middle})')
bar2 = ax.bar(classes[1], demands[1], color='green', label=f'High-end Luxury (Price: ${p_y_high})')

ax.set_ylabel('Units of Luxury Goods Affordable')
ax.set_title('Affordable Luxury Goods by Social Class')
ax.legend()

# Adding data labels
ax.bar_label(bar1, labels=[f'{demands[0]:.2f} units'], padding=3)
ax.bar_label(bar2, labels=[f'{demands[1]:.2f} units'], padding=3)

plt.show()
```

