MA313 : Linear Algebra 1 ("Linear Algebra for Data Science")

Week 1: Introduction to MA313 and to Vector Spaces

Dr Niall Madden

6 and 9 September, 2022

Image taken from the Burren College of Art Logo

Outline

MA313
Week 1: Introduction to MA313 and to Vector Spaces

Start of ...

PART 1: All about MA313

Lecturer: **Dr Niall Madden** (he/him),
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The best way to contact me is by email.

https://commons.wikimedia.org/wiki/File:%C3%81ras_de_Br%C3%BAn.jpg

This module is taken by about 20 students in

- 3rd Arts: 3BA1 and 3CMS1
- 4th Arts: 4BCS1, 4BDA1, 4BMU1
- 3rd Science: 3BS9
- Visiting student (perhaps).
- Anyone else?

This group has different backgrounds. So please complete this form to help me understand:
https://forms.office.com/r/Me6nmgBk5R

While I'll try to take that into account, please let me know if I am incorrectly assuming your prior knowledge.

This is *Linear Algebra 1*: a mathematics module focused on topics of

- *vector spaces*
- *linear transformations*
- *orthogonality*

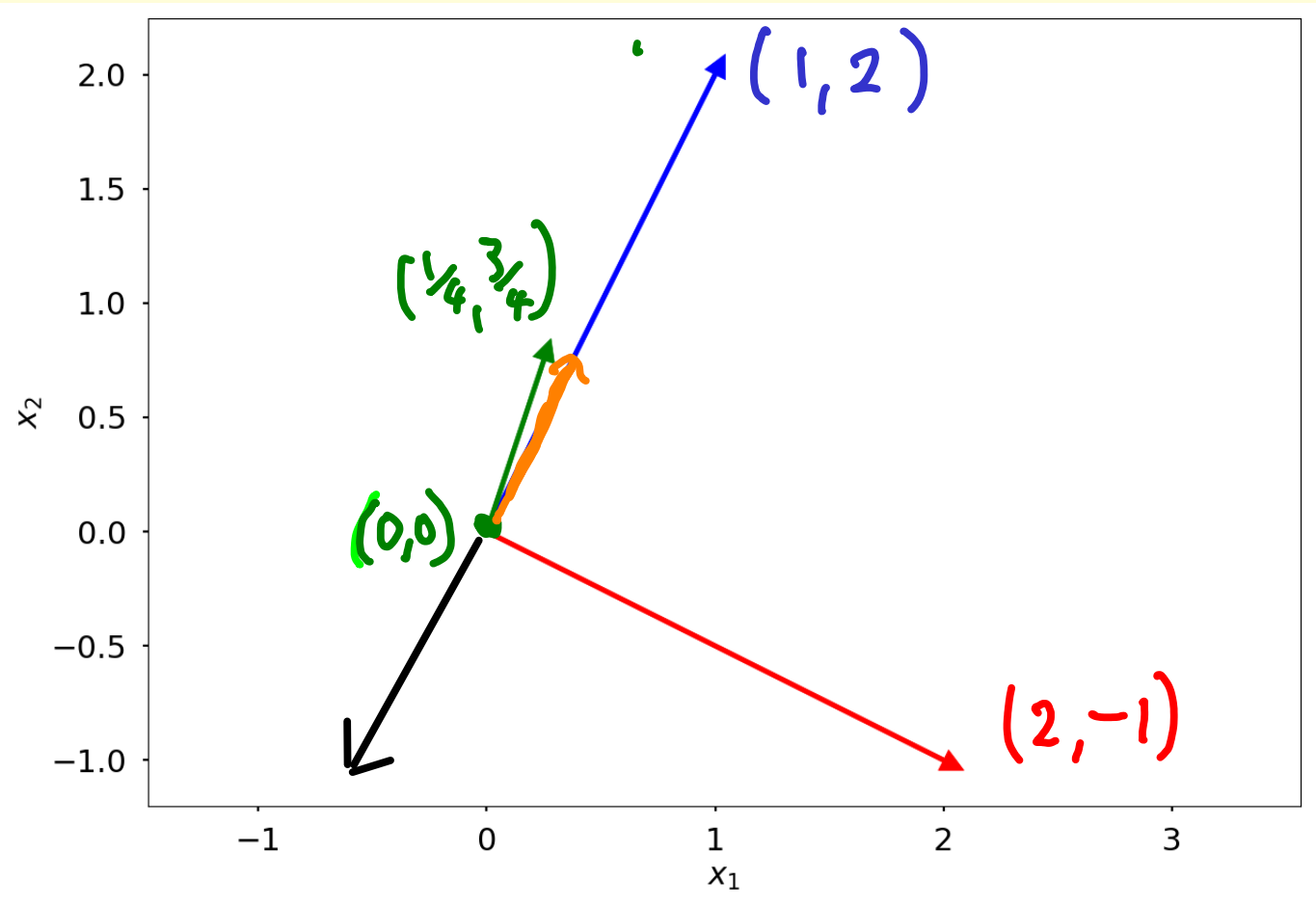
I've subtitled it "Linear Algebra for Data Science", because most of the applications we'll look come from data science. These include

- determining how much *independent information* is in a data set, and explaining what that means.
- finding concise ways of expressing data sets, which reveal some intrinsic information.
- ***fitting*** mathematical functions to data.

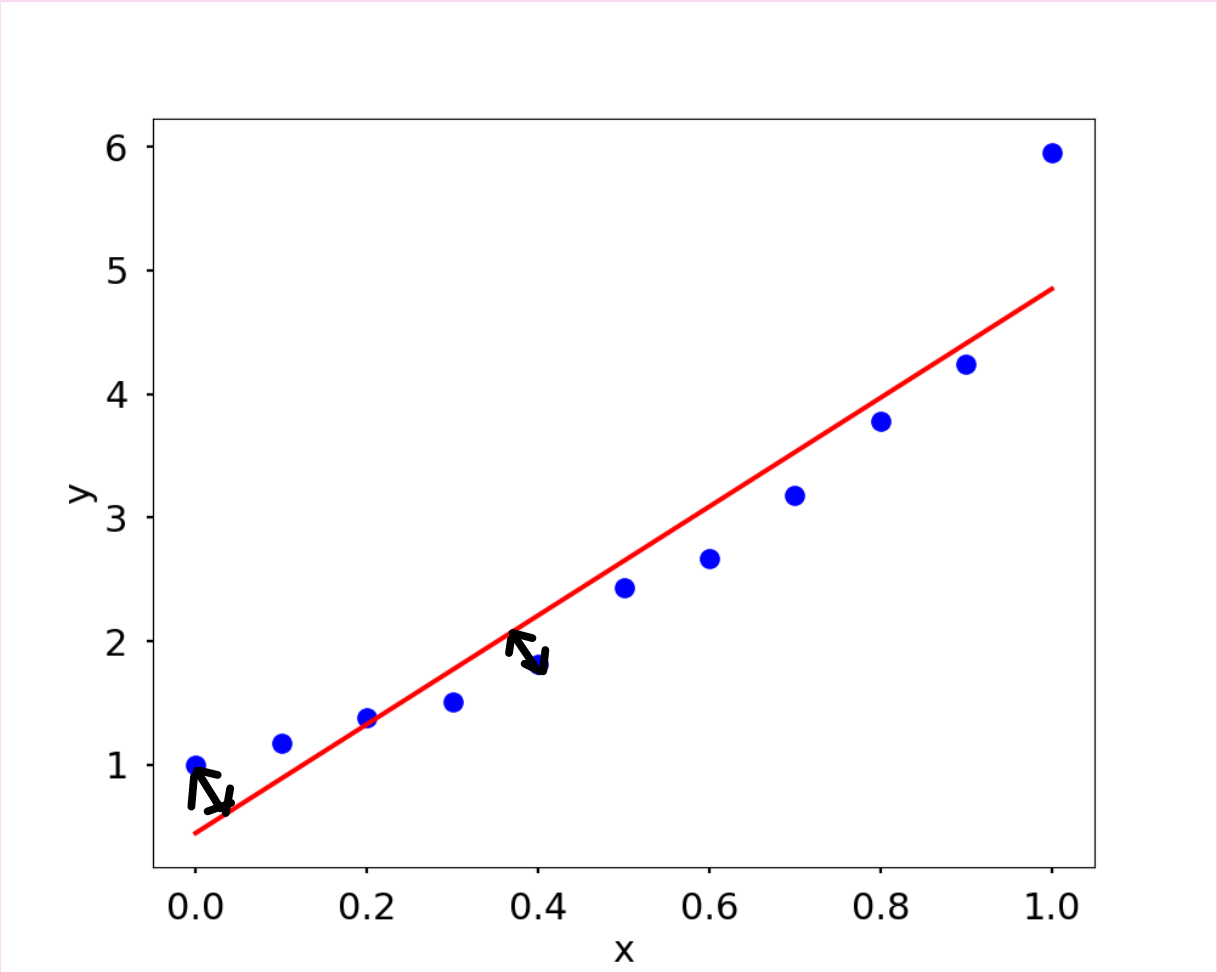
Along the way, we'll learn about the following ideas:

1. **Vectors**, and collections of vectors: *vector spaces*
2. **Subspaces** and how to identify them.
3. Combining vectors: **Linear combinations**
4. **Spans**, Spanning sets, **Linear Independence** and **Bases**.
5. **Dimension** of vector spaces; **Rank** and Nullity of matrices.
6. The so-called "**fundamental subspaces** associated with a matrix, A: the **Column space** and **row space** of A, and the **Null space** of a A and of A^T.
7. Reduced **Row Echelon Form**
8. **Linear Transformations**, and their link to matrices.
9. **Coordinate vectors** and coordinate mappings
10. **Inner products**, and angles between vectors.
11. **Orthogonality**, the theorem of Pythogoras.
12. **Length** (norm) of a vector, and the distance between two vectors.
13. **Orthogonality** and Least squares.

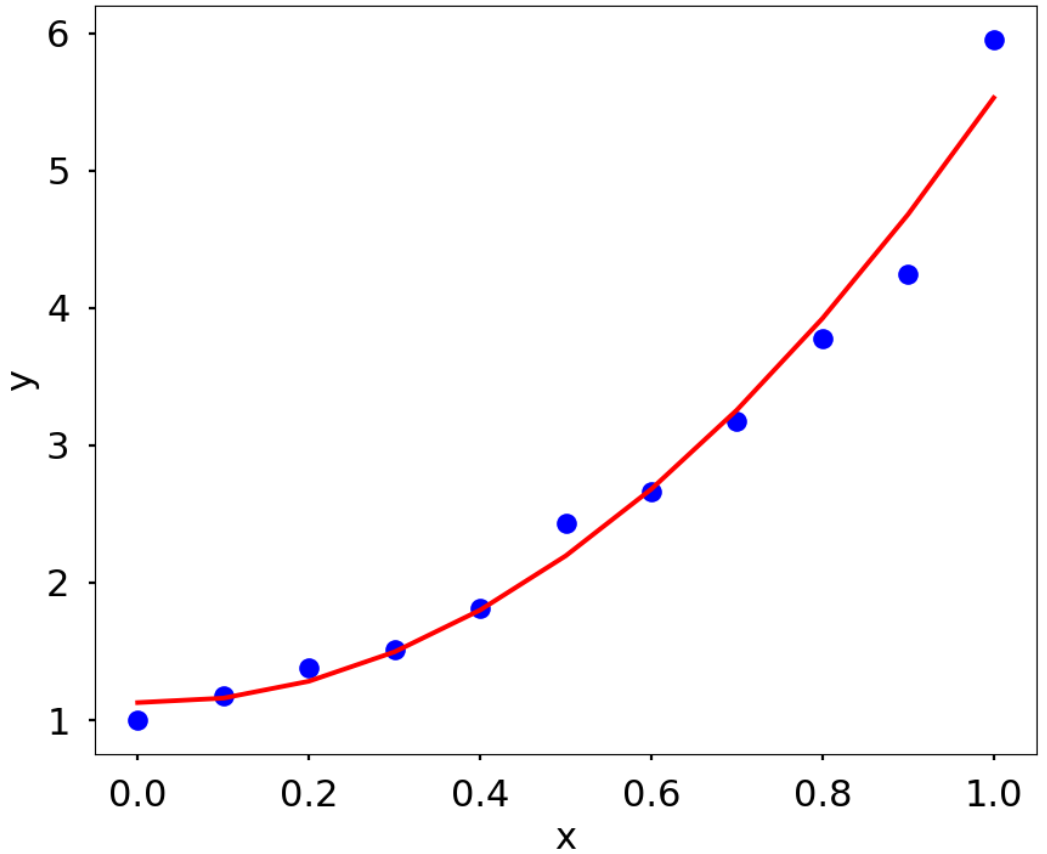
Example: which pair of vectors are most "similar"?



What line best fits a data set?



What "curve" best fits a data set?



Lectures: Tuesday, 13.00-13.50 in AC202
Friday, 12.00-12.50 in AC214.

Tutorials: Need to find a time! More of this presently.

Format: All classes are **in-person**. If live-streaming or recording is requested, we can discuss. Should it happen that we need to go online (e.g., if I get COVID), we'll use Blackboard Collaborate.

Blackboard: At `http://NUIGalway.BlackBoard.com`

- Slides from lectures, including, where appropriate, annotated ones.
- Announcements;
- Grade centre;
- Access to assignments;
- etc...

Work load: 5 ECTS (60 is the typical yearly total for a full-time programme)
24 lectures, all in Semester 1
Roughly 120 hours of student effort time.

Lecture materials: Slides for each week's classes will be available for download in advance of the Tuesday lecture.
These contain the main definitions, ideas, and examples, as well as exercises that are of a similar style and standard as those on the final exam.
When slides are annotated in class, the annotated version will be posted at the end of the week. (If I forget, a gentle reminder is welcome!).

SUMS: The School of Maths provides a free drop-in centre called

SUMS: Support for Undergraduate Maths Students.

SUMS opens from **2pm to 5pm, Monday to Friday**, from Monday of Week 3. For more information, see http://www.maths.nuigalway.ie/sums/

Devices: The use of portable electronic devices during class is *encouraged*. For example, you might want to use it to check Wikipedia, or access the textbook.
Be aware that these can be distracting to other students. Please be considerate.

Your achievement in MA313 will be assessed as follows:

Final exam: 50%. 2 hour written exam.

Online assignments: 20%. Four WeBWorK assignments. They will be open for at least 5 working days. Multiple attempts can be made. Scoring (right/wrong) is provided immediately for most questions.

Written assignment: 10%. There will be one written homework assignment. Questions on this will more closely resemble "exam questions" that the WeBWorK assignments can.

Communication skills: 20%. Completed jointly with MA335 (Algebraic Structures). You'll write an essay on a agreed topic. If taking MA335, you'll also give presentation. (If not, presentation is optional).

Tentative schedule of deadlines:

Assignment 1 (WeBWorK) - end of Week 2 (16 September)

Assignment 2 (WeBWorK) - end of Week 4 (30 September)

Assignment 3 (Written) - end of Week 6 (14 October)

Assignment 4 (WeBWorK) - end of Week 8(?) (28 October)

Assignment 5 (WeBWorK) - end of Week 11(?) (18 ~~September~~ December.)

Essay : details to be confirmed.

These dates can be flexible, if discussed in advance.

Homework!

Verify that you can access the homework system by trying the "Demo" assignment. Link is on `Blackboard/Assignments`.

Tutorials will start in Week 3. When suits?

	Mon	Tue	Wed	Thu	Fri
9 – 10					
10 – 11					
11 – 12					
12 – 1					Lecture
1 – 2		Lecture			
2 – 3					
3 – 4					
4 – 5					

There is no required textbook for MA313.

However, several are recommended:

- David Lay, Steven Lay, and Judith McDonald: *Linear algebra and its applications*. Free online access available: https://search.library.nuigalway.ie/permalink/f/1pmb9lf/353GAL_ALMA_DS51920676300003626. *Quite extensive, and has lots of examples and exercises. But aimed at an electronic engineering audience. You are strongly recommended to review Chapters 1 and 2. We'll start at Chapter 4.*
- Shaina Race Bennet: Linear Algebra for Data Science with examples in R. *Free and open, and with the right emphasis. Though we won't use R.* (maybe...).

Lecture notes will have most of the material needed, but the text is great for providing more examples, different explanations, and exercises.

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PART 2: Mathematical Preliminaries

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In spite of the fact that this module is called "**Linear Algebra 1**", everyone here has taken at least one previous module on linear algebra: MA203 or MA283. And linear algebra features in all first year mathematics modules. So you all know the basic idea of

- Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$. Examples:

$$u = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad v = \begin{bmatrix} -3.14 \\ 62 \end{bmatrix}, \quad u, v \in \mathbb{R}^2.$$

$\mathbb{R}^2$ is 2 dimensional space.

- Matrices: rectangular tables of numbers. Examples:

$$w = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad y = \begin{bmatrix} -52.1 \\ 17 \\ 1/2 \end{bmatrix} \quad w, y \in \mathbb{R}^3$$

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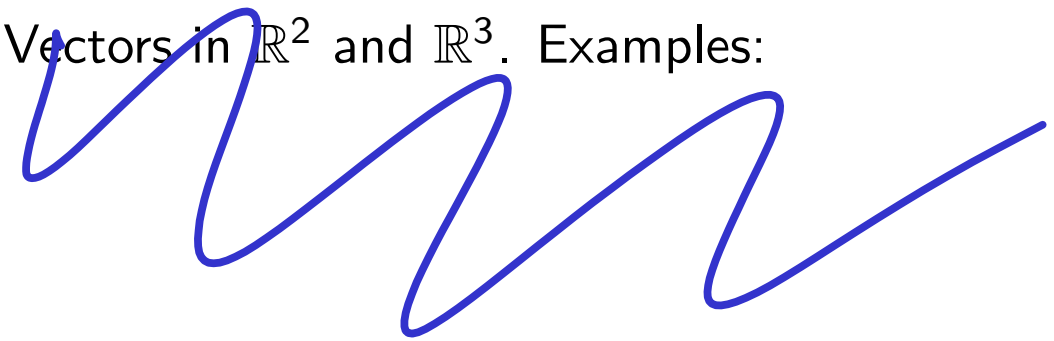

So a vector is an $n \times 1$ matrix.

- Matrices: rectangular tables of numbers. Examples:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, \quad \begin{bmatrix} -0.1 & 0.2 & \pi \\ \pi & 10 & 100 \\ 5 & 6 & 0 \end{bmatrix}$$

Stop here Tuesday.