Annotated slides

2526-MA140: Week 01, Lecture 2 (L02)

More About Functions

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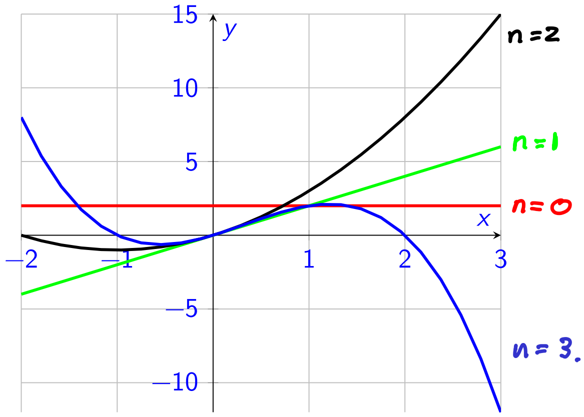
17 September, 2025



Outline

For more, see Sections 1.1 and 1.2 of https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/01%3A_Functions_and_Graphs

Functions: notation $\subseteq$

Recall: This section is all about **functions**, which a "rule" for mapping inputs to outputs.

1. Writing $f : A \to B$ means the inputs come from the set A, and the outputs come from the set B. (A **set** is just a collection of things).
2. A is called the **domain**, and B is called the **co-domain**.
3. $y = f(x)$ means "x gets mapped to y according to the rule defined by f". We sometimes also say "y is the image of x".
4. The subset of B that contains all the images of the things in A is called the **range** of f.
5. When we write $x \in A$ we mean "x is an element of A", or "x belongs to A".

6. $C \subseteq B$ means "C is a subset of B"
ie, everything in C is also in B.

Often, the domain of a function is not expilicitly stated.
In such a case the following **Domain Convention** applies.

The **domain** of a function f is the set of all numbers x for which $f(x)$ *makes sense* and gives a *real-number output*.

where $f: \mathbb{R} \to \mathbb{R}$

Example

1. Find the subset of $\mathbb{R}$ that is the **domain** of $f_1(x) = \dfrac{1}{x^2 - x}$.

f_1 is defined for all $x \in \mathbb{R}$, except where it leads to division by 0, ie where $x^2 - x = 0$.
That is, $x(x-1) = 0$. meaning $x = 0$ or $x = 1$. So the domain of f is $\mathbb{R}$ except $x=0$, $x=1$.

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Example

1. Find the subset of $\mathbb{R}$ that is the **domain** of $f_1(x) = \dfrac{1}{x^2 - x}$.

So the answer is $\mathbb{R}$ except $x=0$ & $x=1$
we can write this in several different ways
a) $\mathbb{R} / \{0, 1\}$
b) $x \in (-\infty, 0) \cup (0,1) \cup (1, \infty)$
"Union".

Note:
$x \in (a, b)$
$\Rightarrow a < x < b$
$x \in [a,b] \Rightarrow$
$a \leq x \leq b$

Example

Find the subset of $\mathbb{R}$ that is the **domain** of the function $f_2(x) = \sqrt{x+2}$.

Since $\sqrt{x+2}$ is a real number only if

$x+2 \geq 0$, we have

$x \geq -2$.

Ans: $x \geq -2$

Equivalently: $x \in [-2, \infty)$

[-2, infinity)

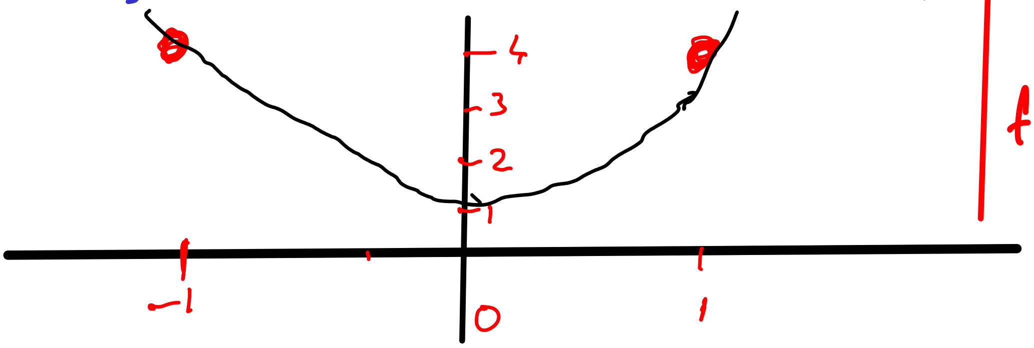
Example

Given the function $f_3(x) = 3x^2 + 1$, find the largest subset of $\mathbb{R}$ that is the domain of f_3. What is the corresponding **range**?

The domain is all of $\mathbb{R}$, ie $(-\infty, \infty)$.

Also, for any $x \in \mathbb{R}$, $x^2 \geq 0$

So $f_3(x) \geq 1$ for all $x \in \mathbb{R}$.



$f(-1)$
$= 3(-1)^2+1$
$= 4$

$f(0) = 1$

$f(1) = 4$

Example

Identify the domain (in $\mathbb{R}$) and range of
$f_4(x) = \sqrt{(x+4)(3-x)}$

For x to be in the domain of f_4 we need $(x+4)(3-x) \geq 0$

So, Either both $x+4 \leq 0$ & $3-x \leq 0$ (a)
or $x+4 \geq 0$ & $3-x \geq 0$ (b)

(a) Need $x \leq -4$ & $x \geq 3$.
There are none!

(b) $x \geq -4$ & $x \leq 3$

Ans: Domain is
$-4 \leq x \leq 3$
or $x \in [-4, 3]$

Example

Identify the domain (in $\mathbb{R}$) and range of $f_4(x) = \sqrt{(x+4)(3-x)}$

For the range: since $(x+4)(3-x) \geq 0$ for all x in the domain

then $\sqrt{(x+4)(3-x)} \geq 0$ too

However (and I got this bit wrong in class!) (x+4)(3-x) takes its maximum at the midpoint between x=-4 and x=3. That is,

at x=-1/2. So, in fact $f(x) \leq \sqrt{(7/2)(7/2)} = 7/2 = 3.5$

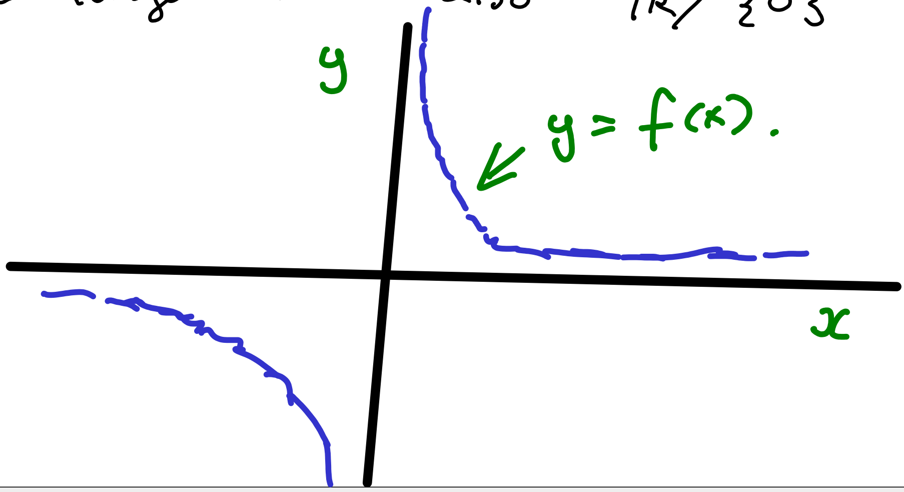
(Sorry about that!!)

Example

Identify the domain and range of $f_5(x) = \dfrac{1}{x}$.

The domain is $\mathbb{R}/\{0\} = (-\infty, 0) \cup (0, \infty)$.

The Range is .... also $\mathbb{R}/\{0\}$



4 Ways to Represent a Function

A function can be represented in different ways:

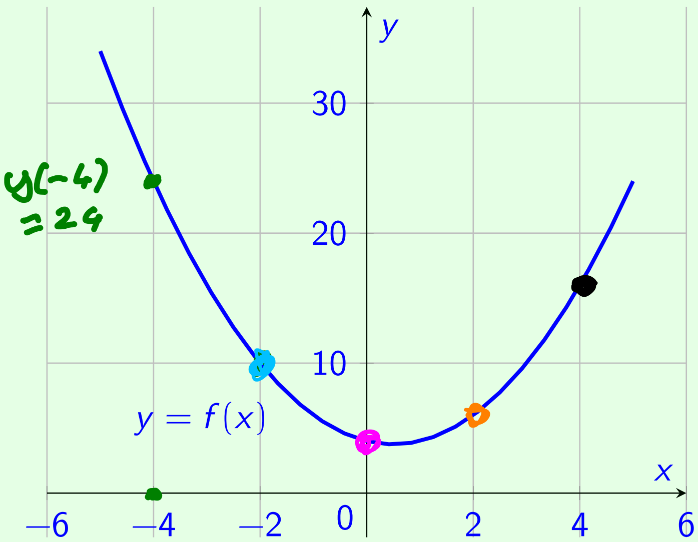
1. **verbally** (by a description in *words*);
2. **numerically** (as a *table* of values); (only at certain points)
3. **visually** (as a *graph*);
4. **algebraically** (by an explicit *formula*).

Often it is possible, and useful, to go from one way to another.

Graphical Representation

Graph $\rightarrow$ Table

A common way to *visualize* a function $f\colon X \to \mathbb{R}$ is its *graph* in the x, y-plane. In this example, $f(x) = x^2 - x + 4$.



x	$f(x)$
-4	24
-2	10
0	4
2	6
4	16

A Catalog of Functions

There are many *different types of functions* that can be used to *model relationships* between objects in the *real world*.

The most common types of functions (in MA140) are:

- *Linear* Functions,
- *Polynomial* Functions,
- *Power* Functions,
- *Rational* Functions,
- *Algebraic* Functions,
- *Trigonometric* Functions,
- *Exponential* Functions,
- *Logarithms*.

Also :
Constant functions!

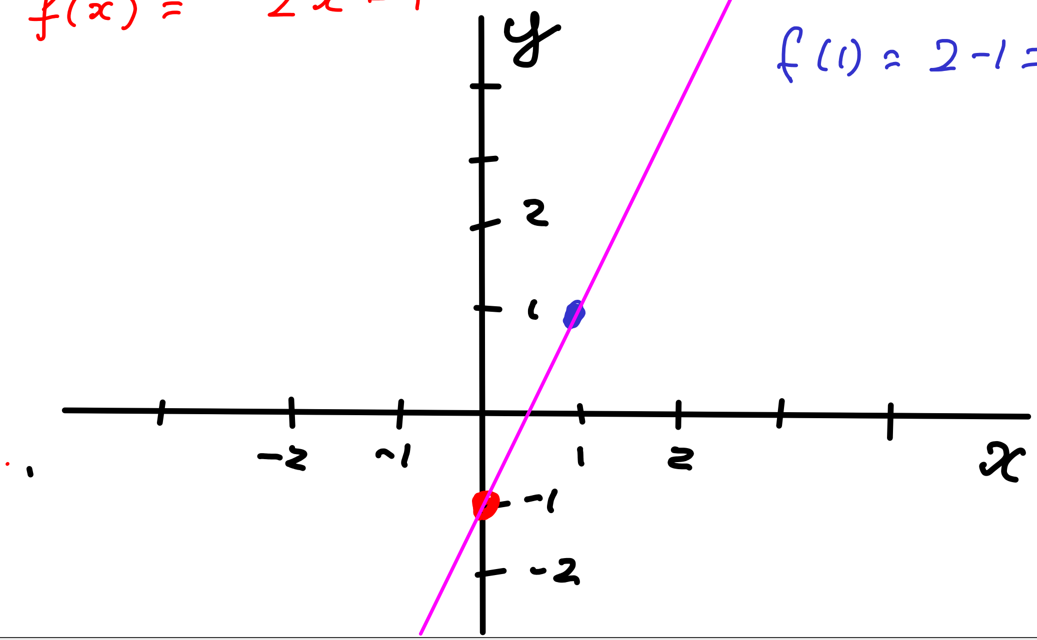
Linear functions have formulae such as $\boxed{f(x) = mx + c,}$ where m and c are some given numbers.

It is often represented graphically as a straight line of slope m through the point $(0, c)$.

Eg $f(0) = -1$

$f(x) = 2x - 1$

$f(1) = 2 - 1 = 1$



Polynomials

Polynomials

A **polynomial function** (or just **polynomial**) is a function of the form

Note $x^0 = 1$

$$y = f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + \underline{a_1 x^1 + a_0}, \quad x \in \mathbb{R},$$

where $a_0, a_1, \ldots, a_n$ are real numbers called the **coefficients** of the polynomial.
The number n is called the **degree** of the polynomial.

There are special names for polynomials of low degree:

If $n=0$, then $f(x)$ is constant

If $n=1$, $f(x)$ is Linear

$n=2 \Rightarrow f(x)$ is quadratic, $n=3 \Rightarrow f$ is cubic.

Exercises

Exercise 1.2.1

Identify the largest possible subset of $\mathbb{R}$ that could be the domain and range of these functions:

1. $f(x) = (x-4)^2 + 5$
2. $f(x) = \sqrt{3x+2} - 1$
3. $f(x) = 3/(x-2)$.

(See Example 1.1.2 of the textbook).

Exercise 1.2.2

Sketch the graphs of

(i) $y = 5x^2 - 7$

(ii) $y = x^2 - 4x + 3$

(iii) $y = x^3 - 6x^2 - 11x - 6$