MA385/MA530: Sample questions for Class Test, October 2019

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1. Match each of the following functions, f, with p: its (truncated) Taylor polynomial approximation about $a = 0$.

   (a) $f(x) = e^{x^2}$; (b) $f(x) = e^{-6x}$; (c) $f(x) = \cos(2x)$; (d) $f(x) = \sqrt{4+x}$.

   (i) $p(x) = 1 - 6x + 18x^2 - 36x^4 - \frac{324}{6}x^5$.

   (ii) $p(x) = 2 + \frac{1}{4}x - \frac{1}{64}x^2 + \frac{1}{512}x^3$.

   (iii) $p(x) = 1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{15}x^6$.

   (iv) $p(x) = 1 + x^2 + \frac{1}{2}x^4 + \frac{1}{6}x^6$.

2. State *Newton's method* for solving the nonlinear equation $f(x) = 0$.
   Use a Taylor's series to show that if $f(\tau) = 0$, and Newton's method generates a sequence of approximations $\{x_0, x_1, \dots\}$, then

$$\tau - x_{k+1} = -\frac{1}{2}(\tau - x_k)^2 \frac{f''(\eta_k)}{f'(x_k)}, \qquad \text{for some } \eta_k \in [x_k, \tau]. \tag{1}$$

3. Consider the following 2nd-order Runge-Kutta (RK2) method:

$$\Phi(t_i, y_i; h) = \frac{1}{4}f(t_i, y_i) + \frac{3}{4}f\big(t_i + \frac{2}{3}h, y_i + \frac{2}{3}hf(t_i, y_i)\big).$$

   (i) Show that it is consistent.

   (ii) Use it to estimate the solution to the initial value problem

$$y(1) = 1, \qquad y' = 1 + t + y/t \quad \text{for } t > 1,$$

   taking $n = 2$ time steps.