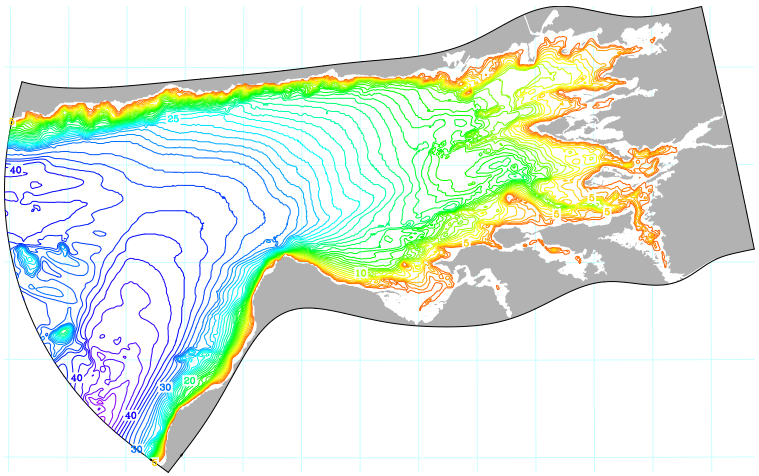
MA140: Engineering Calculus

Lecture 1: Introduction; Numbers, Notation, Functions

Dr Niall Madden
School of Mathematical and Statistical Sciences
University of Galway

17 September, 2024



This version of the slides are by Niall Madden, but are adapted from original notes by Dr Kirsten Pfeiffer.

Outline

1. Introductions
   - Welcome to MA140!
   - About me
   - About you
   - Schedule
   - Assessment
2. Learning materials
   - Texts and Resources
   - Canvas
3. Numbers and Notation
   - Numbers
   - The real numbers
   - The complex numbers
   - Notation
4. Functions
   - Mathematical Models
   - Functions as mappings

MA140 - Engineering Calculus is a Semester 1 module on calculus and its applications to engineering.

We'll covers several major topics in calculus:

- Functions
- Limits, Continuity, Intermediate Value Theorem
- Differentiation;
- Logarithms;
- Basic properties of integrals;
- Fundamental Theorem of Calculus;
- Techniques of integration: substitution, integration by parts, partial fractions and the Logarithm Rule.

These tools will help us tackle various engineering problems, such as those involving rates of change, maxima and minima, areas and volumes, ...

Lecturer: **Dr Niall Madden** (he/him)

Addressed: Niall (pronounced "Knee"-"al" `#StartsWithAName`)

School: Mathematical and Statistical Sciences, University of Galway.

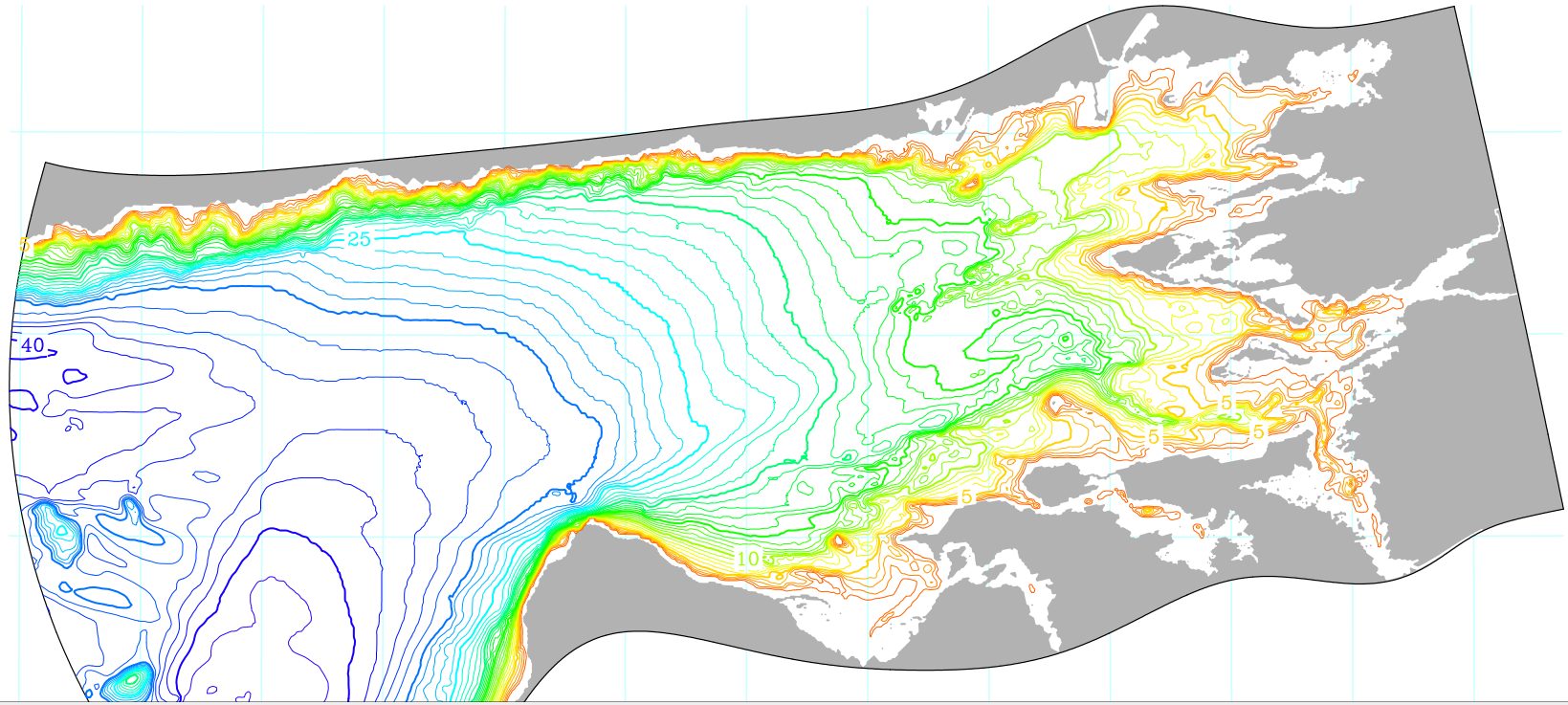
Office: Room AdB-1013, Arás de Brún

Email: `Niall.Madden@UniversityOfGalway.ie`. This is the best way to contact me. When you do so, please include "`MA140`" in the subject line. It can also be helpful to include your ID number.

Web: `https://www.niallmadden.ie`

My area of research is **numerical analysis** and **computational mathematics**. It varies from (trying to) prove theorems, to collaborating with other scientists, engineers, and medics.

For example, right now I am working on a project with Drs Indiana Olbert and Alexander Shchepetkin from Civil Engineering, on trying to improve some computational models of Galway Bay. Here is an image from a paper we are working on (Galway Bay model bottom topography: credit AS).



There are about 280 students enrolled in MA140, across all Engineering Disciplines:

- Civil
- Biomedical
- Electrical and Electronic
- Mechanical
- Electronic and Computer
- Energy Systems
- Undenominated
- *anyone else?*

	Mon	Tue	Wed	Thu	Fri
9 – 10					
10 – 11		**Lecture**	**Lecture**	**Lecture**	
11 – 12				**Tutorial**	
12 – 1					
1 – 2					**Tutorial**
2 – 3					
3 – 4		**Tutorial**			
4 – 5					

- You should attend all three lectures.
- Tutorials start next week (TBC). You should attend one per week: your session will be assigned to you by the School of Engineering.
- Also: **SUMS** ("Support for Undergraduate Mathematics and Statistics"). FREE Drop-In service on campus: also available online. `universityofgalway.ie/public-sites/s-u-m-s/`.

Assessment

Assessment

- Online Assignments (together worth 33%)
- End-of-term Exam (worth 67%)

There will be 8 assignments through the semester, starting from Week 3 - so there will be a task to complete and submit almost every week. **Deadlines will always be Friday at 5pm**.

Recommended Texts

- Modern Engineering Mathematics, by G. James (Prentice Hall). This is freely available through the library at https://search.library.nuigalway.ie/permalink/f/3b1kce/TN_cdi_askewsholts_vlebooks_9780273742517
- I'll update the reading list as I cover each section.

Recommended Online-Resources

- Irish Mathematics Learning Support Network (IMLSN) Resources: https://www.imlsn.ie/index.php/resources-index. This is particularly useful if there is some concept that you need to revise.
- Paul's Online notes https://tutorial.math.lamar.edu/ Great range of "Cheat Sheets".
- Helping Engineers Learn Mathematics (HELM) Workbooks.
- Check these and other online resources on the SUMS website: universityofgalway.ie/public-sites/s-u-m-s/resources

Learning materials — Canvas

The on-line content for the course will be hosted at https://universityofgalway.instructure.com/courses/35693
There you'll find:

- Announcements (1 per week)
- Information (where, when, what)
- These slides, posted in advance.
- Links to assignments
- Grades
- Etc

The lecture slides contain most of the course material.

They are arranged by lecture, will be posted before the lecture.

The slides contain most of the main ideas and examples. However, they are "gappy", with extra details added during the class. The annotated versions will also be posted to canvas (after a day or two).

MA313
Lecture 1: Introduction; Numbers, Notation, Functions

Start of ...

Section 3: Numbers

This short section is about sets of numbers, culminating with the set of real numbers, $\mathbb{R}$. For more, see Chapter 1 of "Modern Engineering Calculus (James)

In a mathematics course, you'll encounter various different sets of **numbers**. The most familiar should be:

- $\mathbb{N} = \{1, 2, 3, 4, ...\}$ **Natural Numbers**
- $\mathbb{N}_0 = \{0, 1, 2, 3, 4, ...\}$
- $\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$ **Integers**
- $\mathbb{Q} = \{\frac{a}{b} \mid a, b \in \mathbb{Z} \text{ and } b \neq 0\}$ **Rational Numbers**

e.g.

But there are many important numbers, such as π and $\sqrt{2}$ that cannot be written as a fraction: they are *not* rational.

We say such numbers are **irrational**:

- $\pi \notin \mathbb{Q}$
- $e \notin \mathbb{Q}$
- $\sqrt{2} \notin \mathbb{Q}$

However, these numbers *do* exist on a number line. (Note: $\sqrt{2} \approx 1.414213...$, and $e \approx 2.7182818$).

Number Line

The real numbers, $\mathbb{R}$

For MA140, the most important set of numbers is *the reals*, denoted $\mathbb{R}$. It is the set of all points on the number line. Roughly, if

- $\mathbb{N}$ is the set we use for counting;
- $\mathbb{Z}$ is the set of whole ("entire") numbers;
- And $\mathbb{Q}$ is used for ratios
- Then $\mathbb{R}$ is set use for (positive and negative) quantities: speed, height, weight, volume, etc, etc.

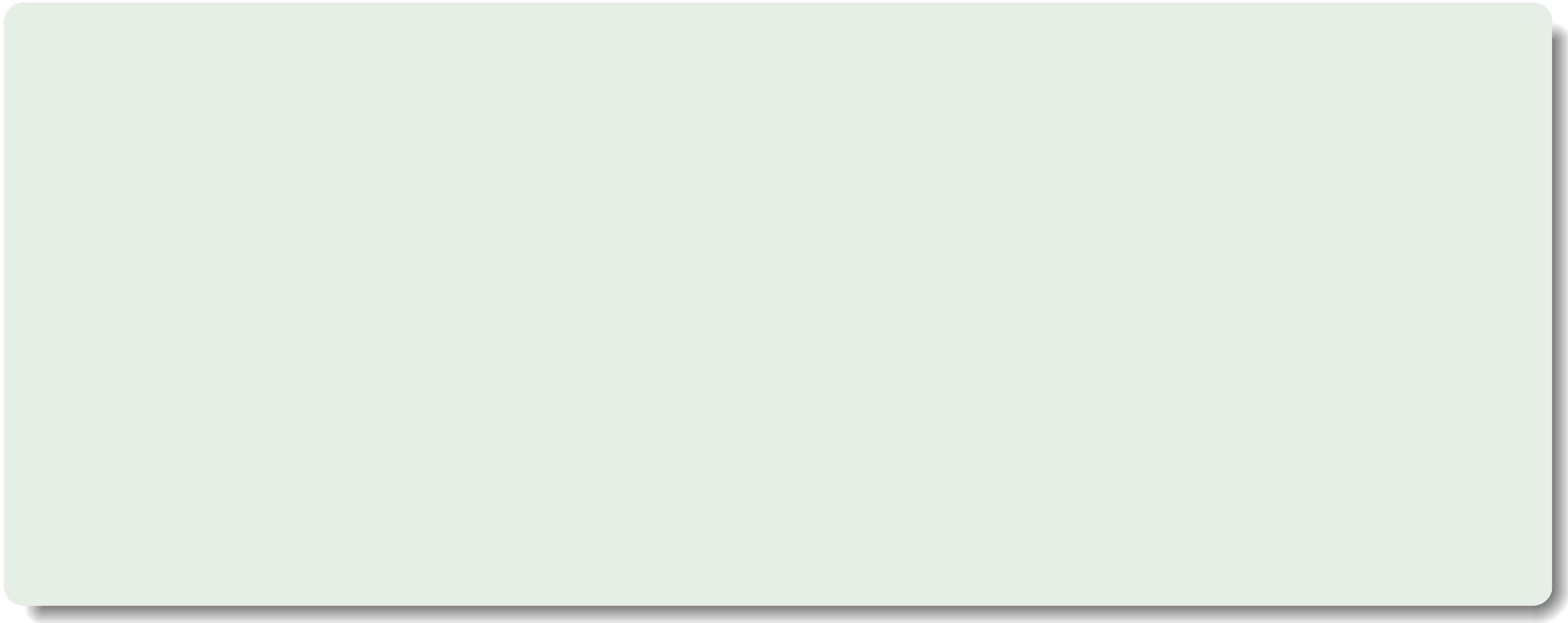
We mention, in passing, that there is another important set: $\mathbb{C}$, the set of **complex numbers**.

If $c \in \mathbb{C}$ we can write

$$c = a + ib, \quad a, b \in \mathbb{R}, \quad \mathbf{i} = \sqrt{-1}$$

However, they are not so important for MA140, so we'll pass on for now.

We can represent our number system visually as follows:



or write

$$\mathbb{N} \subseteq \mathbb{N}_0 \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$$

where $\subseteq$ means **"is subset of"**.

$\mathbb{R} \setminus \mathbb{Q}$ are the **irrational numbers**.

- "$\setminus$" means **"less"** or **"without"**;
- "$\in$" means **"is element of"**;
- "$\forall$" means **"for all"**;
- "$\exists$" means **"exists"**;
- "$!$" can mean **"unique"** or **"factorial"**, depending on the context.

e.g.

MA313
Lecture 1: Introduction; Numbers, Notation, Functions

Start of ...

Section 4: Functions

The single most important concept in MA140 is that of a **function**.
For more, see Chapter 2 of "Modern Engineering Calculus (James).

Functions

Question!

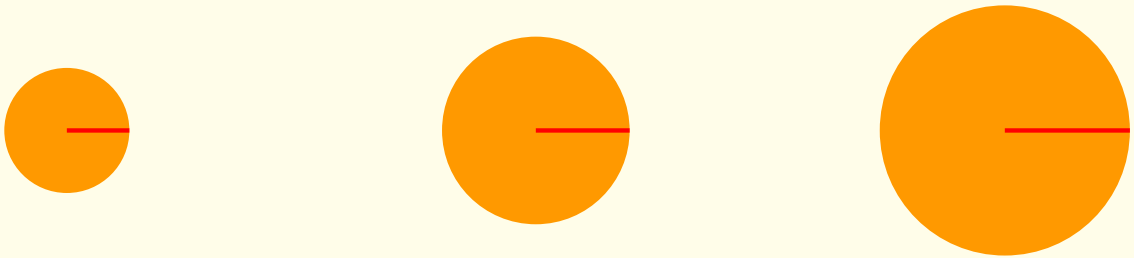
What is a function in Mathematics?

Take a minutes to answer the question.

Use your own words, or a picture. Don't look it up!

Functions

Functions arise whenever one quantity *depends* on another.

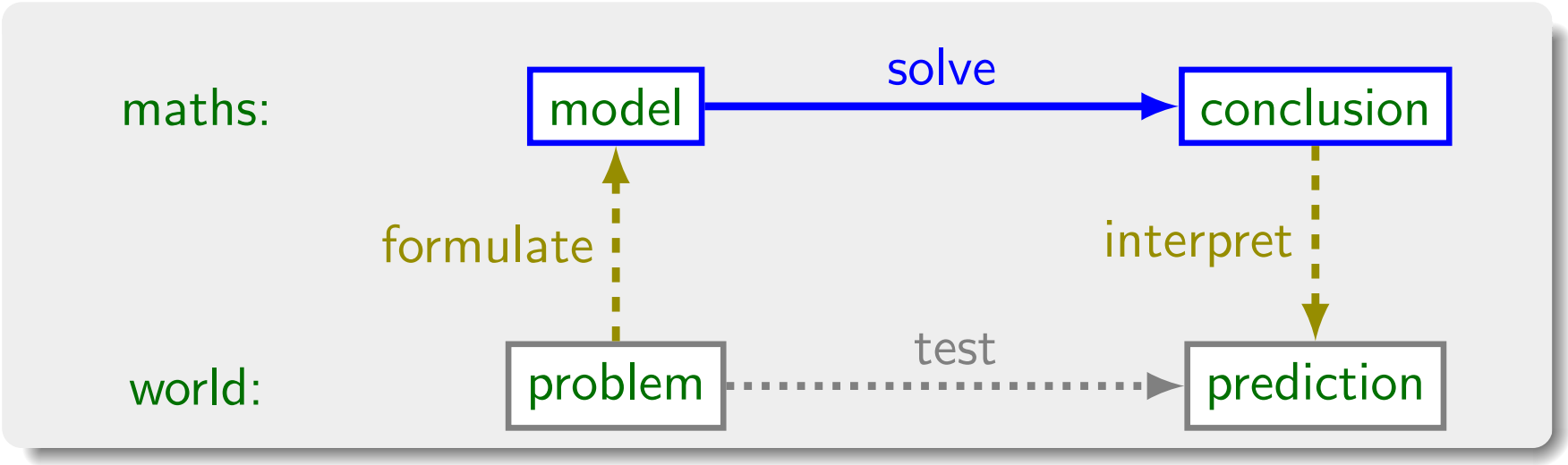


- The *area* A of a circle *depends* on the *radius* r of the circle.
- The *rule* connecting r and A is given by the *equation* $A = \pi r^2$.

This *formula* assigns to each positive number r one value of A. We say: "A **is a function of** r", and write $A(r) = \pi r^2$**.**

Here, A is called the **dependent variable** and r is called the **independent variable**: A depends on r.

- A **mathematical model** is a *mathematical description* (by means of a function, equation) of a *real-world phenomenon*.
- The model helps to *understand* the phenomenon, and perhaps to make *predictions* about future behavior.



- A good model *simplifies reality* to permit *mathematical calculations*, while being *sufficiently accurate* to provide *valuable conclusions*.
- Be aware of the *limitations* of the chosen model.

A **function** is a *rule* that maps an element of one set to another unique element of another set.