MA211 – **Problem Set 4**

Q16.1 For each of the following differential equations, find a recurrence relation for the coefficients of the power series solution, and write out the solution up to the x^5 term.

(a) $y'' + xy = 0$.

(b) $y'' + x^2 y = 0$.

(c) $y'' - 2xy' + y = 0$.

(d) $y'' - 2xy' + y = 0, \quad y(0) = 1, y'(0) = -1$

(e) $y'' - xy' = 0, \qquad y(0) = 0, y'(0) = 2$

Q16.2 Which of the following statements is true? Why?

$$\int_a^b |f(x)|dx \leqslant \left| \int_a^b f(x)dx \right|,$$

or

$$\left| \int_a^b f(x)dx \right| \leqslant \int_a^b |f(x)|dx.$$

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Q17.1 By finding a suitable substitution, evaluate the following integrals:

(i) $\displaystyle\int \frac{1+x}{\sqrt{1+x}}dx.$ (ii) $\displaystyle\int e^{(2x-2)}dx.$

(iii) $\displaystyle\int \frac{\sin(1/x)}{x^2}dx.$ (iv) $\displaystyle\int e^{\sin(x)}\cos(x)dx$

Q17.2 Use a suitable substitution to show that

$$\int \frac{1}{\tan(x)}dx = \ln|\sin(x)|.$$

Q17.3 Evaluate the following definite integrals:

(i) $\displaystyle\int_0^4 \frac{x^3}{\sqrt{x^2+1}}dx.$ (ii) $\displaystyle\int_1^{\sqrt{e}} \frac{\sin\left(\pi\ln(x)\right)}{x}dx.$

(iii) $\displaystyle\int_e^{e^2} \frac{1}{x\ln(x)}dx.$

Q17.4 Evaluate the following integrals:

(i) $\displaystyle\int xe^{x^2}dx.$ (ii) $\displaystyle\int \frac{\cos(x)}{4+\sin^2(x)}dx.$

(iii) $\displaystyle\int e^{2x}\sin(e^{2x})dx$ (iv) $\displaystyle\int \frac{\ln(x)}{x}dx$

(v) $\displaystyle\int \frac{e^x+1}{e^x-1}dx.$

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Q18.1 Recall that if we are differentiating the product of two functions u and v then $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$. Use that $\displaystyle\int \frac{d}{dx}(uv)dx = uv$ to deduce the formula for integration by parts.

Q18.2 Using *Integration by parts*, evaluate the following integrals

(i) $\displaystyle\int x\cos(x)dx.$ (ii) $\displaystyle\int \big(\ln(x)\big)^2 dx.$

(iii) $\displaystyle\int x\tan^{-1}(x)dx.$ (iv) $\displaystyle\int x^2\tan^{-1}(x)dx.$

(v) $\displaystyle\int (x+3)e^{2x}dx.$

Q18.3 Evaluate the following definite integrals

(i) $\displaystyle\int_1^2 \ln(x)dx$

(ii) $\displaystyle\int_1^2 \frac{\ln(x)}{x}dx$

(iii) $\displaystyle\int_{\pi/6}^{\pi/2} \frac{x}{\sin^2(x)}dx.$

Hint: if $f(x) = \dfrac{\cos(x)}{\sin(x)}$, *what is* $f'(x)$*?*

Q18.4 Using *Integration by parts to answer the following questions*

(i) Evaluate $\displaystyle\int x^2e^x dx.$

(ii) Evaluate $\displaystyle\int x^5e^{x^2}dx.$ (*Hint: first use a substitution, then use the answer to part (i)*).

(iii) Evaluate $\displaystyle\int e^x\sin(x)dx.$

(iv) Let $I_n = \displaystyle\int_0^1 x^n e^x dx.$ Show that $I_n + nI_{n-1} = e.$

(v) Evaluate $\displaystyle\int \sin\big(\ln(x)\big)dx.$

Q18.5 Evaluate the following:

(i) $\displaystyle\int \frac{1}{x(x^2-1)}dx$ (ii) $\displaystyle\int \frac{x^3+2}{x^3-1}dx$

(iii) $\displaystyle\int \frac{2x+1}{x^2+4x+4}dx$ (iv) $\displaystyle\int_2^3 \frac{3x^3+1}{x^3-2x^2+x}dx$