

2425-MA140 Engineering Calculus

Week 11, Lecture 3 (L33)
Practice Paper

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Tutorials

There is a change to the tutorial plan for this week:

- ▶ Thursday at 11am: Teams 9 and 10 attend a tutorial in ENG-3035. Teams 7, 8, 11, and 12's tutorial is Aras Moyola MY129.
- ▶ Friday: no MA140 tutorials!
- ▶ Irish tutorial: no change

Assignments

- ▶ Grades for **Assignment 6** (as “Assignment-6-combined”) and **Assignment 7** have been posted to Canvas.
- ▶ **Assignment 8**: grades will be posted tomorrow.
- ▶ **CA total (from 33)** will be posted by Monday.

Today is all about the Practice Paper:

1 Q1

2 Q2

3 Q3

4 Q4

5 Q5

6 $t \rightarrow \infty$

Q1(a) Express $\frac{10x - 27}{5x^2 - 25x + 30}$ as partial fractions.

Q1(b) Let $f(x) = \frac{x^2 - 2x - 8}{x^2 - x - 12}$. For each of the following, evaluate the limit, or determine that it does not exist.

- (i) $\lim_{x \rightarrow -3} f(x)$
- (ii) $\lim_{x \rightarrow 4} f(x)$
- (iii) $\lim_{x \rightarrow \infty} f(x)$

Q1 Q1 III

Q1(c) Let $f(x) = x^{-2}(2 - e^x - e^{-x})$, $g(x) = -x^2 - 1$ and $h(x) = x^2 - 1$. You may assume that $g(x) \leq f(x) \leq h(x)$ for all x in the region $[-2, 2]$.

- (i) Use the Squeeze Theorem to determine $\lim_{x \rightarrow 0} f(x)$.
- (ii) Explain why you can't use the Squeeze Theorem to determine $\lim_{x \rightarrow 1} f(x)$.

Q1(d) Evaluate the limit $\lim_{\theta \rightarrow 0} \frac{2 \sin(\theta)}{\theta + 3 \tan(\theta)}$.

Q1(e) Let $f(x) = \begin{cases} a/x & x < 2 \\ 3 + bx & x \geq 2 \end{cases}$.

Find values of a and b for which both $f(x)$ and $f'(x)$ are continuous at $x = 2$.

Q2(a) Differentiate $f(x) = x^2 e^{-3x} \sin(4x)$, with respect to x

Q2(b) Differentiate $f(\theta) = (\sin(3\theta) + 1)(3\theta + 1)^{-1}$ with respect to θ .

Q2(c) Differentiate $f(x) = \ln(\cos(x^2))$ with respect to x .

Q2(d) Let $f(x) = 10 \ln(x) + e^{-10x}$. Find $f'(x)$, $f''(x)$, and $f'''(x)$.

Q2(e) Use the Inverse Power Rule, to find the derivative, with respect to x , of $y = \cos^{-1}(x)$.

Q2(f) Find the equation of the tangent to the curve implicitly defined by

$$2x^2 + y^2 = 3,$$

at the point $(x, y) = (1, 1)$.

Q3(a) Let $f(x) = 2 - \ln(x^2 + 9)$.

- (i) Find all critical point(s) of f . For each, determine if it corresponds to a local minimum of f , a local maximum of f , or neither.
- (ii) Determine regions of the domain where f is increasing, and the regions where it is decreasing.
- (iii) Find all point(s) of inflection of f .

Q3(b) Use logarithmic differentiation to evaluate the derivative of $y = e^{\sin(x)} \cos(x)$.

Q3(c) Use L'Hôpital's rule to evaluate $\lim_{x \rightarrow 0} \frac{1 - e^x}{x}$.

Q3(d) A sheet of cardboard measures 3cm by 8cm. You can make an open top box is made by cutting four identical squares from the four corners of the sheet, and folding up the flaps on each side.

- (i) What size square should you remove to maximise the *volume* of the box? What is the resulting volume?
- (ii) What size square should you remove to maximise the *surface area* of the box?

Q4(a) Evaluate the following integrals.

(i) $\int x^2 \sin(x^3) dx.$

(ii) $\int \ln(x) x^{-4} dx$

$$(iii) \int_{-2}^{-1} e^{1/x}/x^2 dx$$

$$(iv) \int_0^{\infty} \frac{1}{1+x^2} dx.$$

Q4 Q4 III

Q4(b) Let $F(x) = \int_x^{3x} f(t) dt$.

- (i) Explain why, for any function $f(t)$, we get $F(0) = 0$.
- (ii) Let $f(t) = t^2$. Use the Fundamental Theorem of Calculus to evaluate $F'(x)$ as a polynomial.

Q5(a) Compute the area of the region bounded above by curve $f(x) = e^x$, below by $g(x) = x^2$, to the left by $x = 0$ and to the right by $x = 2$.

Q5(b) Compute the **volume** of the solid of revolution formed by rotating the curve $y = \sqrt{2x + 1}$, between $x = 1$ and $x = 2$ about the x -axis. Give your answer to three decimal places.

Q5(c) Compute the **surface area** of the solid of revolution formed by rotating the curve $y = 2\sqrt{x}$, between $x = 0$ and $x = 1$ about the x -axis. Give your answer to three decimal places.

Q5(d) Calculate the arc length of the graph of $f(x) = x^{3/2}$, over the interval $[0, 1]$. Give your answer to three decimal places.

$$t \rightarrow \infty$$

THE END!

Thanks for taking MA140, and for your time, energy and patience.
Hope you learned something (and enjoyed it). I did!

Best of luck!