MA378: Assignment 2 (Version 2.0)

Deadline: 13:00, Wednesday 20 March.

Your solutions must be clearly written, and neatly presented. You can submit an electronic copy, through blackboard, or a hard copy. If submitting a hard copy, do so at the 1pm lecture in the 20th. Staple pages of the hard-copy, and write your name/ID at the topi of each *page. Marks will be given for quality and clarity of exposition ([15 MARKS]). Usual collaboration policy applies.*

Chapter 2: Piecewise Polynomial Interpolation

Exer 2.4 [20 MARKS] Take $f(x) = \ln(x)$, $x_0 = 1$, $x_N = 2$. What value of N would you have to take to ensure that $|\ln(x) - S(x)| \leqslant 10^{-4}$ for all $x \in [1, 2]$, where S is the natural cubic spline interpolant to f.

Exer 2.6 [20 MARKS] Suppose that S is a natural cubic spline on $[0, 2]$ with

$$S(x) = \begin{cases} 3x + a(1-x)^3 + bx^3, & \text{for } 0 \leqslant x < 1, \\ c(2-x) - (2-x)^3 + d(x-1)^3, & \text{for } 1 \leqslant x \leqslant 2. \end{cases}$$

Find a, b, c, and d.

Chapter 3: Numerical Integration

Exer 1.1 [10 MARKS] (*For simplicity, you may assume that the quadrature rule is integrating* f *on the interval* $[-1, 1]$.) Let q_0, q_1, ..., q_N be the quadrature weights for the Newton-Cotes rule $Q_N(f)$. Show that $q_i = q_{N-i}$ for $i = 0, \dots N$.

Exer 3.5 [20 MARKS] Consider the rule (which is not, strictly speaking, a Newton-Cotes rule):

$$R(f) = q_0 f(\frac{1}{3}) - f(\frac{1}{2}) + q_2 f(\frac{3}{4})$$

for approximating $\int_0^1 f(x)dx$.

(a) Determine values of q_0 and q_2 that ensure this rule has precision 2.

(b) What is the maximum precision of $R(\cdot)$ with the values of q_1 and q_2 that you have determined?

(c) Why is this not, strictly speaking, a Newton-Cotes rule?

Exer 5.2 [15 MARKS]

(i) Using the Inner Product

$$(f, g) := \int_0^1 f(x)g(x)dx,$$

find $\widetilde{p}_0(x)$, $\widetilde{p}_1(x)$, $\widetilde{p}_2(x)$ and $\widetilde{p}_3(x)$.

(ii) Find the zeros of $\widetilde{p}_2(x)$ and call them x_0 and x_1. Construct a quadrature rule for $\int_0^1 f(x)dx$ taking these as the quadrature points, and the weights as the integrals to the corresponding Lagrange polynomials.

Notes:

- An earlier version of this exercise had a typo in Part (ii), stating that the quadrature was on $[-1, 1]$.
- You can compute the weights using any method you like; although they are defined as the integrals of the relevent Lagrange polynomials, that is not the only way to compute them.