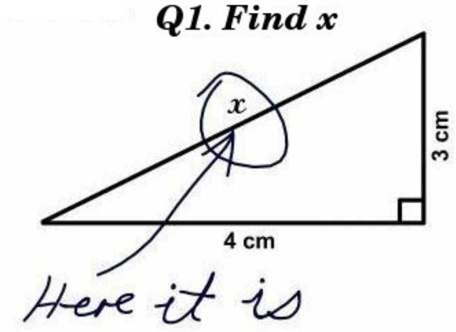
MA211
Lecture 12: Class Test

Wed 16 October 2008



Q1. Use $\cosh(x) = \frac{1}{2}\left(e^x + e^{-x}\right)$ and $\sinh(x) = \frac{1}{2}\left(e^x - e^{-x}\right)$ to show that

$$\cosh^2 x - \sinh^2 x = 1.$$

. . . .

Q2. Write down the general solution to the following differential equations:

(i) $25y'' - 20y' + 4y = 0.$

(ii) $y'' + y' - 12y = 0$

. . . .

Q3. Find values of b and c such that $y(x) = \cosh(2x)$ is a solution to the differential equation:

$$y'' + by' + cy = 0.$$

Solutions

Q1

Use $\cosh(x) = \frac{1}{2}\left(e^x + e^{-x}\right)$ and $\sinh(x) = \frac{1}{2}\left(e^x - e^{-x}\right)$ to show that

$$\cosh^2 x - \sinh^2 x = 1.$$

$$\cosh^2(x) = \tfrac{1}{4}(e^x + e^{-x})(e^x + e^{-x}) = \tfrac{1}{4}(e^{2x} + 2 + e^{-2x})$$

$$\sinh^2(x) = \tfrac{1}{4}(e^x - e^{-x})(e^x - e^{-x}) = \tfrac{1}{4}(e^{2x} - 2 + e^{-2x})$$

So $\cosh^2(x) - \sinh^2(x) =$

$$\tfrac{1}{4}\left[e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}\right] = 1$$

Solutions

Q2 (i)

Write down the general solution to the following differential equation: $25y'' - 20y' + 4y = 0.$

The auxillary equation is $25R^2 - 20R + 4 = 0$

It's solutions are $R = \dfrac{20 \pm \sqrt{400-400}}{50} = 2/5$

Since there is only one root,

$$y(x) = Ae^{2x/5} + Bxe^{2x/5}$$

Solutions

Q2 (ii)

Write down the general solution to the following differential equation: $$y'' + y' - 12y = 0$$

The aux equation is $R^2 + R - 12 = 0$

The discriminant $D = b^2 - 4ac = 1 + 48 = 49 > 0$.

So there are 2 distinct real roots

$$R_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-1+7}{2} = 3, \quad R_2 = \frac{-1-7}{2} = -4$$

So

$$y(x) = A e^{3x} + B e^{-4x}$$

(Note, the left hand side of the Auxillary Equation is easily factorised, so $(R-3)(R+4) = 0$)

Solutions

Q3

Find values of b and c such that $y(x) = \cosh(2x)$ is a solution to the differential equation:

$$y'' + by' + cy = 0.$$

$y = \cosh(2x)$ so $y'(x) = 2\sinh(2x)$

and $y''(x) = 4\cosh(2x)$.

Substitute into the DE to get

$$4\cosh(2x) + b\sinh(2x) + c\cosh(2x) = 0$$

This gives $b = 0$ and $c = -4$.