2526-MA140: Week 01, Lecture 1 (L01)

Introduction; Functions

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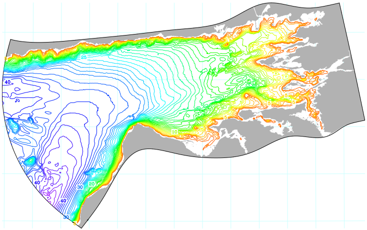

These slides are by Niall Madden, with some material adapted from notes by Dr Kirsten Pfeiffer.

Outline

MA140 - Engineering Calculus is a Semester 1 module on calculus and its applications to engineering.

We'll covers several major topics in calculus:

- Functions
- Limits, Continuity, Intermediate Value Theorem
- Differentiation;
- Logarithms;
- Integration;
- Fundamental Theorem of Calculus;
- Applications, including solids of rotation, and centres of mass.

These tools will help us tackle various engineering problems, such as those involving rates of change, maxima and minima, areas and volumes, . . .

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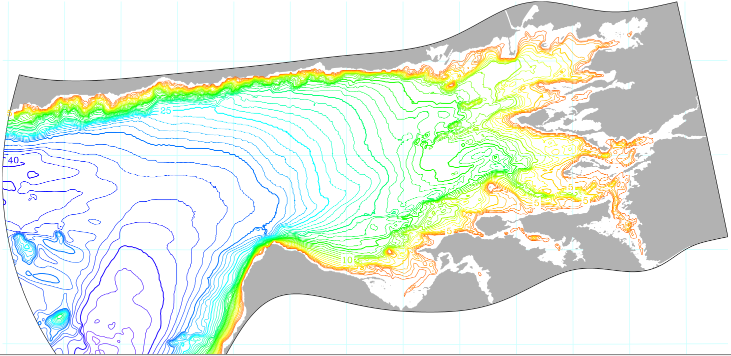
Web: `https://www.niallmadden.ie`

My area of research is **numerical analysis** and **computational mathematics**. It varies from (trying to) prove theorems, to collaborating with other scientists, **engineers** (mostly), and medics.

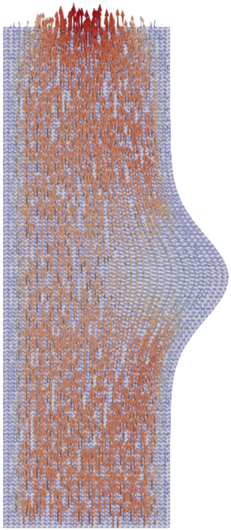
For example, I am working on a project with Drs Indiana Olbert and Alexander Shchepetkin (Civil Engineering), trying to improve some computational models of Galway Bay. Here's an image from a paper we are working on, on the Storm Éowyn tidal surge. (Galway Bay topography: credit AS).



I also work on other topics related to this module:

- mathematical models of aortic flows, with Niamh Hynes (Biomedical Engineering/Surgery), Kevin Moerman (Mechanical Eng), and Sean Tobin (your tutor!).
- Solute dispersal in channels, with Nanda Poddar (Maths).

I'll tell you more about these projects, and how they related to MA140 later in the semester.



There are about 280 students enrolled in MA140, across all Engineering Disciplines:

- Civil
- Biomedical
- Electrical and Electronic
- Mechanical
- Electronic and Computer
- Energy Systems
- Undenominated
- *anyone else?*

	Mon	Tue	Wed	Thu	Fri
9 – 10					
10 – 11		**Lecture**	**Lecture**	**Lecture**	
11 – 12				**Tutorial**	
12 – 1					
1 – 2					**Tutorial**
2 – 3					
3 – 4		**Tutorial**			
4 – 5					

- You should attend all three lectures.
- Tutorials start next week (TBC). You should attend one per week: your session will be assigned to you by the School of Engineering.
- Also: **SUMS** ("Support for Undergraduate Mathematics and Statistics"). FREE Drop-In service on campus: also available online. `universityofgalway.ie/public-sites/s-u-m-s/`.

Assessment

Assessment

- Short weekly online assignments (together worth 10%). There will be 8 of these through the semester, starting from Week 3 - so there will be a task to complete and submit almost every week. **Deadlines will always be Monday at 5pm**.
- Two in-class tests (together worth 20%)
- A short project at the end of semester (3%)
- End-of-term Exam (worth 67%)

Recommended Texts

Our primary reference is **Calculus** by Strang and Herman. It is free and open, and available from https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)

Also good: **Modern Engineering Mathematics**, by G. James (Prentice Hall). This is freely available through the library at https://search.library.nuigalway.ie/permalink/f/3b1kce/TN_cdi_askewsholts_vlebooks_9780273742517
I'll update the reading list as I cover each section.

Recommended Online-Resources

- Irish Mathematics Learning Support Network (IMLSN) Resources: https://www.imlsn.ie/index.php/resources-index. This is particularly useful if there is some concept that you need to revise.
- Paul's Online notes https://tutorial.math.lamar.edu/ Great range of "Cheat Sheets".
- Helping Engineers Learn Mathematics (HELM) Workbooks.
- Check these and other online resources on the SUMS website: universityofgalway.ie/public-sites/s-u-m-s/resources

The on-line content for the course will be hosted at https://universityofgalway.instructure.com/courses/46734
There you'll find:

- Announcements (1 per week)
- Information (where, when, what)
- These slides, posted in advance.
- Links to assignments
- Grades
- Etc

The lecture slides contain most of the course material.

They are arranged by lecture, will be posted before the lecture.

The slides contain most of the main ideas and examples. However, they are “gappy”, with extra details added during the class. The annotated versions will also be posted to canvas (after a day or two).

Functions

The single most important concept in MA140 is that of a **function**. For more, see Chapter 1 of Strang's Calculus:

> *"Calculus is the mathematics that describes changes in functions. We define polynomial, rational, trigonometric, exponential, and logarithmic functions. We review how to evaluate these functions, and show the properties of their graphs".*

Definition (Function)

A **function**, f consists of a set of *inputs*, a set of *outputs*, and a **rule** for assigning each input to exactly one output.

- The set of inputs is called the **domain** of the function.
- The set of outputs is called the **range** of the function.

Examples:

- When we write $f : X \to Y$, we mean that "f *maps values from the set* X *to the set* Y".
- We write the rule for that mapping as $y = f(x)$.

The Real Numbers, $\mathbb{R}$

In almost every example we'll consider, in MA140, f will a mapping form one **real** number to another.

The set of **real** numbers, denoted $\mathbb{R}$, are the numbers use to represent continuous quantities, such as length, speed, time and temperature.

- **continuous** means that there are no gaps, or "jumps": as a car accelerates from, say, 0m/s to 20m/s, at some point it will have taken on every speed between these values; where it has had .
- real numbers can be positive or negative.

If f is a function from X to Y...

- The set X is called the **domain** of the function.
- The set Y is called the **codomain**.
- When we write $y = f(x)$, we say "x" is the **argument** of the function.
- When $y = f(x)$ for some $x \in X$, y is said to be the **image** of x under f.
- The set of all images $y = f(x), x \in X$, is called the **range** (or **image set**) of f.

- While we could have functions between any pair of sets (e.g., a function from students in this class to their ID numbers), usually X and Y are *sets of numbers*.
- It is not necessary for all elements y of the codomain Y to be images under f.
- One element $y \in Y$ can serve as value $f(x)$ for several $x \in X$.

A function can be represented in different ways:

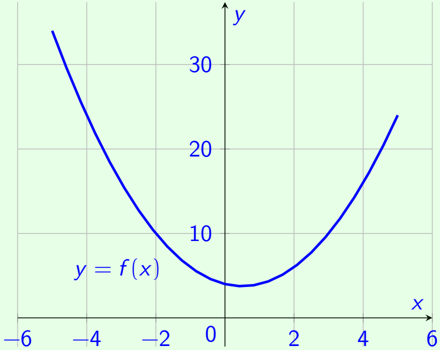
1. **verbally** (by a description in *words*);
2. **numerically** (as a *table* of values);
3. **visually** (as a *graph*);
4. **algebraically** (by an explicit *formula*).

Often it is possible, and useful, to go from one way to another.

Graphical Representation

Graph $\to$ Table

A common way to *visualize* a function $f\colon X \to \mathbb{R}$ is its *graph* in the x, y-plane. In this example, $f(x) = x^2 - x + 4$.



x	$f(x)$
-4	24
-2	10
0	4
2	6
4	16

Often, the domain of a function is not expilicitly stated.
In such a case the following **Domain Convention** applies.

The **domain** of a function f is the set of all numbers x for which $f(x)$ *makes sense* and gives a *real-number output*.

Example

1. Find the subset of $\mathbb{R}$ that is the **domain** of $f(x) = \dfrac{1}{x^2 - x}$.

Example

Find the subset of $\mathbb{R}$ that is the **domain** of the function $f(x) = \sqrt{x+2}$.

Example

Given the function $f_1(x) = 3x^2 + 1$, find the largest subset of $\mathbb{R}$ that is the domain of f_1. What is the corresponding **range**?

Example

Identify the domain (in $\mathbb{R}$) and range of
$f_2(x) = \sqrt{(x+4)(3-x)}$

Example

Identify the domain and range of $f_3(x) = \dfrac{1}{x}$.

Exercises

Exercise 1.1.1

Identify the largest possible subset of $\mathbb{R}$ that could be the domain and range of these functions:

1. $f(x) = (x-4)^2 + 5$
2. $f(x) = \sqrt{3x+2} = 1$
3. $f(x) = 3/(x-2)$.

(See Example 1.1.2 of the textbook).