2425-MA140 Engineering Calculus

Week 2, Lecture 1
Partial Fractions

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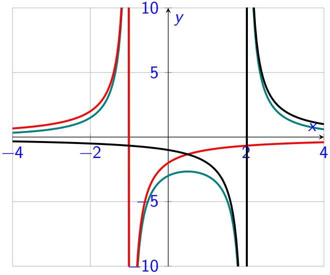

This version of the slides are by Niall Madden, but are adapted from original notes by Dr Kirsten Pfeiffer.

Outline

1 News!
- Tutorials
- Assignments

2 Partial Fractions
- Case 1
- Case 2
- Case 3
- Case 4
- Exercises

For more, see Section 2.5 (Rational Functions) of *Modern Engineering Mathematics*:
https://search.library.nuigalway.ie/permalink/f/3b1kce/TN_cdi_askewsholts_vlebooks_9780273742517

Tutorials start **this** week. The schedule is:

- Teams 1+2: Tuesday 15:00 ENG-**2003**
- Teams 3+4: Tuesday 15:00 ENG-**2034**
- Teams 9+10: Thursday 11:00 ENG-**2002**
- Teams 11+12: Thursday 11:00 ENG-**3035**
- Teams 5+6: Friday 13:00 Eng-**2002**
- Teams 7+8: Friday 13:00 Eng-**2035**

If you are interested to taking a tutorial through Irish, please complete this survey: http://tinyurl.com/suirbhe1

- There is currently a “practice” assignment open. See https://universityofgalway.instructure.com/courses/35693/assignments/94873
- During tutorials, the tutor will solve some similar questions. You can access the tutorial sheet at https://universityofgalway.instructure.com/courses/35693/files/2023552?module_item_id=650912
- A new assignment will open by tomorrow...

. . . .

In addition, each week I’ll post a set of exercises related to the material covered. You don’t have to submit your work for these, but you should try them: they are similar in style and standard to exam questions.

Partial Fractions

Rational Functions have the general form $f(x) = \dfrac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials.

An (proper) rational function can often be written as a sum of simpler ones: **partial fractions.**

For example

$$\frac{8x-12}{x^2-2x-3}$$

can be written as

$$\frac{3}{x-3}+\frac{5}{x+1}$$

In order to do this, we try to **factorize** the denominator.

Partial Fractions

Note: Any polynomial (with real coefficients) can be factorised fully into the product of

- linear
- and irreducible quadratic factors.

We get different combinations of factors in the denominator. Let's look at **four cases**, and how to find the partial fractions in each case.

Partial Fractions

The four cases

1. Linear factors to the power of 1 in the denominator.
2. Linear factors to the power greater than 1 in the denominator, (i.e repeated linear factors).
3. Irreducible quadratic factors.
4. Irreducible quadratic factors to power greater than 1.

(1) Linear factors to the power of 1 in the denominator.

Example

$$\frac{3x}{(x-1)(x+2)}$$

There are **two methods** for finding A and B.

Method 1: Comparing coefficients

Method 2: Substituting specific values for x.

Example

Write $\dfrac{8x-12}{x^2-2x-3}$ as sum of partial fractions.

(2) Linear factors to the power greater than 1 in the denominator, (i.e repeated linear factors).

If $(x-\alpha)^k$ appears in the denominator, it will give rise to the following terms:

$$\frac{A_1}{x-\alpha}+\frac{A_2}{(x-\alpha)^2}+...+\frac{A_k}{(x-\alpha)^k}$$

Example

Find A, B and C such that

$$\frac{3x+1}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$$

(Note: we'll find that $A = 5/9$, $B = 4/3$ and $C = -5/9$).

(3) Irreducible quadratic factors.

Irreducible quadratic factors can not be factorised using real numbers, e.g. $x^2 + x + 1$.

An irreducible quadratic factor $ax^2 + bx + c$ gives rise to partial fractions of the form

$$\frac{Ax + B}{ax^2 + bx + c}.$$

Example 2.34 from textbook

If one writes

$$\frac{5x}{(x^2 + x + 1)(x - 2)} = \frac{Ax + B}{x^2 + x + 1} + \frac{C}{x - 2}$$

then we find $A - 10/7$, $B = 5/7$ and $C = 10/7$.

(4) Irreducible quadratic factors to power greater than 1.

Each repeated irreducible quadratic factor $(ax^2 + bx + c)^k$ in the denominator will give rise to

$$\frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \dots + \frac{A_kx + B_k}{(ax^2 + bx + c)^k}.$$

These can be done in a similar way to the previous case. But the calculations are pretty messy, so we won't even try!

Exercise 2.1.1

Find the constants A, B and C, so that

$$\frac{2x+1}{(x-2)(x+1)(x-3)} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{x-3}$$

Exercise 2.1.2

Express the following as partial fractions.

1. $\dfrac{6}{x^2-x-2}$
2. $\dfrac{2x-1}{x^2-x-2}$
3. $\dfrac{x-1}{(x+1)(x^2-x-2)}$
4. $\dfrac{x}{x^2+2x+1}$