MA385 Part 5: Review

5.1: Review

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1. Outline

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MA385 has four main components

(1) Solving nonlinear equations.

(2) Solving initial value differential equations.

(3) Linear algebra 1: solving linear systems of equations.

(4) Linear algebra 2: norms and condition numbers.

We also studied the implementation and investigation of numerical method in Python.

1. Outline

Assessment is based on

- one assignment, worth 10%;
- three labs (weighted 3%, 3% and 4%), worth 10%;
- and the class test, worth 10%;
- and final exam contributes the remaining **70%** of your MA385 grade.

2. The final exam

- It exam has **5** questions.
- Two questions are based on Section 1, with one each on Sections 2, 3 and 4.
- You should attempt all 5.
- The number of marks for question, and each section is indicated. The total may vary from question to question.

Python does not feature at all on the paper.

Given $f : \mathbb{R} \to \mathbb{R}$, find $\tau \in [a, b]$ such that $f(\tau) = 0$.

We now know how to solve this

- using interval **bisection**;
- the **Secant** method;
- **Newton's** method;
- **fixed point iteration** method.

3. Section: 1. Solving Nonlinear Equations

We also know:

- a sufficient (but not necessary) condition for a solution to exist;
- what the **order of convergence** of a method is;
- for each method, how to prove it converges (subject to certain assumptions);
- how to determine the order of convergence experimentally (in Lab 1);
- the Fixed Point and Contraction Mapping Theorems.

Take time to revise:

- Taylor Series and remainders.
- Each of the methods: be able to state, motivate, and use the.
- Review terminology (order of convergence, fixed points, contraction, etc.);
- Be aware of which method is best in which situation;
- The major theorems in the section.

- You should be able to motivate all of the methods. The proofs of convergence of Newton's method and the Secant method will not feature.
- The exam will not feature anything about the Black-Scholes equation, Julia sets and roots of unity.

Solve the differential equation $y(t_0) = y_0$ and $\frac{dy}{dt} = f(t, y)$ $t > t_0$.

We now know:

- what is meant by a **Lipschitz condition**, and how to check that one is satisfied;
- how to derive and use **Euler's** method;
- the definitions of the **global** and **truncation** errors, **consistency** and convergence;
- the conditions on the RK-2 method's parameters for it to be 2nd-order;

4. Section 2: Initial Value Problems

- how to show a given RK method has the correct order of convergence for a simple linear problem;
- we can summarise an RK method as a tableau;
- how to apply Euler's method to a system of IVPs;
- how to write a high-order problem as a system of 1^{st}-order IVPs.

You should revise the following

- How to state and use Euler's Method.
- Show how Euker's Method, and a formula for its truncation error, can be derived from a Taylor series;
- for any **one-step** method, knowing how to show it is consistent, and how it relates to a Taylor series.

What you don't have to know:

- Picard's Theorem (will be given if needed);
- the definition of the truncation error for an arbitrary one-step method (will be given if needed);
- formulae for any **RK-2**, RK-3 or RK-4 method (will be given if needed);
- implicit methods;
- systems and higher-order problems;
- finite difference methods for PDEs (i.e., the heat equation).

5. Section 3: Solving Linear Systems

Solve the linear system of equations $A\mathbf{x} = b$ where $A \in \mathbb{R}^{n\times n}$

We now know...

- how to relate systems of equations to matrix-vector equations;
- a good way and a bad way to compute $\det(A)$;
- that Gaussian Elimination and row reduction is effectively the same as LU-**factorisation**;
- all about **triangular** matrices;
- how construct the LU-factorisation of A, and prove it exists;
- how to use LU-factorisation and back-substitution to solve $A\mathbf{x} = \mathbf{b}$;
- that the computational cost is $\mathcal{O}(n^3)$;

5. Section 3: Solving Linear Systems Revise

Take time to review:

- triangular matrices, matrix partitioning, principal sub-matrices;
- properties of the product and inverse of triangular matrices;
- the existence of the LU-factorisation;
- the derivation of the formulae for L and U;
- writing down the LU-factorisation of a given matrix;
- using the LU-factorisation to solve a linear system;

Topics that are not examinable include:

- Permutations/pivoting.
- Computer representation of numbers

6. Section 4: Norms and condition numbers

We now know...

- all about vector norms, especially the 1-, 2-, and ∞-norms. And we know about Cauchy-Schwarz.
- how to derive useful formulae for the matrix norms $\|A\|_1$, $\|A\|_2$, and $\|A\|_\infty$;
- how to show they are **consistent**.
- how the **condition number**, $\kappa(A)$, relates to errors in solving $Ax = b$;
- **Gerschgorin's** theorems for estimating eigenvalues, and can prove the first one.

6. Section 4: Norms and condition numbers Revise

You should revise

- Definition of norms; how to show function $\mathbb{R}^n \to \mathbb{R}$ is indeed a norm.
- How to prove the formula for the three matrix norms of interest.
- Gerschgorin's first theorem (statement, proof and application), but not the second.

Any questions?