MA385 Class Test

When: 15:00 Thursday, 30 October 2025.
Duration: 40 minutes.
Instructions: Answer all questions in the answer book provided
Don't forget to write your name and ID number on the answer book.
Rules No notes or books allowed. No calculators allowed.
Don't sit beside anybody else.

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1. Suppose we have a function g and points a and b, such that $a \leq g(x) \leq b$ for all $x \in [a, b]$.

   (a) What does it mean for g to be a *contraction* on $[a, b]$?

   (b) We know that, if g is a contraction on $[a, b]$, then it has a fixed point in $[a, b]$. Show that that fixed point is unique.

   (c) Suppose we know that a particular *Fixed Point* iteration, $x_{k+1} = g(x_k)$, converges at least linearly to the fixed point $\tau = g(\tau)$. Use a Taylor Series to show that, if $g'(\tau) = 0$, then, in fact, it converges at least quadratically.

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2. (a) State *Newton's method* for solving the nonlinear equation $f(x) = 0$ *for some* $x \in [a, b]$.

   (b) Explain how Newton's Method can thought of as *Fixed Point* iteration $x_{k+1} = g(x_k)$. That is, *what is* g?

   (c) Assuming Newton's method converges, show that it does so at least quadratically.

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3. (a) State *Euler's method* for solving initial value problems:

$$y(t_0) = y_0 \qquad \text{and } y'(t) = f(t, y) \text{ for } t > t_0.$$

   (b) Show how Euler's method can be motivated by using a Taylor Series Expansion.

   (c) Give an example of an Initial Value problem for which Euler's method would give the exact solution (i.e., with no error). Justify your answer with the Taylor Series you used to motivate Euler's method.