MA 211 Problem Set 1 p1/2

Note Title 08/10/2008

Some notes on solutions to selected problems.

2.3 (v) $f(x) = 1 + \frac{1}{1-x^2}$ is defined for all x except when $1 - x^2 = 0$ i.e., when $x = 1$ or -1. So the domain is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ (equiv: $\mathbb{R} / \{-1, 1\}$

The Range is all of $\mathbb{R}$

2.4 (i)

A function f is $\begin{cases} \text{even} & \text{if } f(-x) = f(x) \\ \text{odd} & \text{if } f(-x) = -f(x) \\ \text{neither} & \text{otherwise} \end{cases}$

Here $f(x) = \frac{x}{x^2+1}$.

so $f(-x) = \frac{-x}{(-x)^2+1} = -\frac{x}{x^2+1} = -f(x)$

So f is odd

(iii) $f(x) = x|x|$ so $f(-x) = -x|-x| = -x|x|$

So f is odd.

4.2 (i) Let $f(x) = \frac{1}{3}x^3$. Find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\frac{1}{3}(x+h)^3 - \frac{1}{3}x^3}{h} = \frac{1}{3}\left[\lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}\right]$

$= \frac{1}{3} \lim_{h \to 0} \left[3x^2 + 3xh + h^2\right] = x^2$

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5.1 We need to calculate

$$\lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$\sin(x+h) = \sin(x)\cos(h) + \cos(x)\sin(h)$

So $\lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h} =$

$\sin(x) \lim_{h \to 0} \frac{\cos(h) - 1}{h} + \cos(x) \lim_{h \to 0} \frac{\sin(h)}{h}$

$= \cos(x)$ because $\lim_{h \to 0} \frac{\cos(h) - 1}{h} = 0$

and $\lim_{h \to 0} \frac{\sin(h)}{h} = 1$

5.5 l'Hopital's Rule tells us that

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{f'(x)}{g'(x)}.$$

So $\lim_{x \to 0} \frac{\sin(x)}{x} = \lim_{x \to 0} \frac{\cos(x)}{1} = 1.$