MA140: Engineering Calculus

Week 2: Collection of Exercises

And answers!

This is a collection of exercises from this week's MA140 lectures. You don't have to submit solutions for these, but you should work through them. Some may be similar to questions on the final exam.

Exercise 2.1.1

Find the constants A, B and C, so that

$$\frac{2x+1}{(x-2)(x+1)(x-3)} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{x-3}$$

Answer: $A = -5/3$, $B = -1/12$, $C = 7/4$

Exercise 2.1.2

Express the following as partial fractions.

1. $\dfrac{6}{x^2-x-2}$ Ans: $\frac{2}{x-2}+\frac{-2}{x+1}$
2. $\dfrac{2x-1}{x^2-x-2}$ Ans: $\frac{1}{x-2}+\frac{1}{x+1}$
3. $\dfrac{x-1}{(x+1)(x^2-x-2)}$ Ans: $\frac{1}{9(x-2)}+\frac{2}{3(x+1)^2}-\frac{1}{9(x+1)}$
4. $\dfrac{x}{x^2+2x+1}$ Ans: $\frac{1}{x+1}-\frac{1}{(x+1)^2}$
5. $\dfrac{1}{x^3-1}$ Ans: $\frac{1}{3(x-1)}+\frac{1-x}{3(x^2+x+1)}$

Exercise 2.2.1

Evaluate the following limits

(a) $\lim_{x \to \frac{1}{2}} \dfrac{x - \frac{1}{2}}{x^2 - \frac{1}{4}}$ Ans: 1

(b) $\lim_{x \to -4} \dfrac{x^2 + 3x - 4}{x^2 + x - 12}$ Ans: 5/7

Exercise 2.3.1

(From 2023/2024 MA140 exam, Q1(a)) Evaluate the limit

$$\lim_{x \to 4} \frac{x-4}{(\sqrt{x}-2)(x+9)}$$

Ans: 4/13