MA313 : Linear Algebra I

Review: Sample Exam Questions

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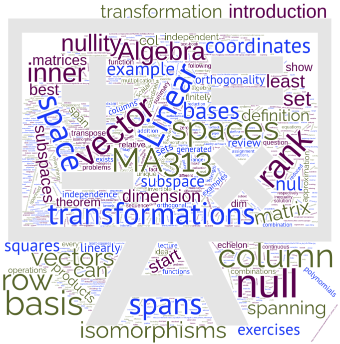

Q1. Q1(a) [5 Marks] Give an example of a 3-dimensional subspace of a 5-dimensional vector space.

Ans: $\mathbb{R}^5$ is a 5-dimensional vector space.

$$W = \text{span}\left\{ \underbrace{\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\\0\end{bmatrix}}_{\text{3 standard basis functions}} \right\}$$ is a subspace of $\mathbb{R}^5$.

Also $\dim(W) = 3$ since the 3 vectors form a basis (they are linearly independent).

Alt: $\mathbb{P}^4 = \text{span}\{1, t, t^2, t^3, t^4\}$ has dimension 5.

Q1(b) [5 Marks] Find vectors $u, v, w \in V$ with $V = \mathrm{Span}\{u, v, w\}$, where V is the subspace of $\mathbb{R}^4$ consisting of all vectors of the form

$$\begin{bmatrix} a+b-2c \\ 3a \\ c-b \\ 3a-12c \end{bmatrix}$$

for $a, b, c \in \mathbb{R}$.

$$\begin{bmatrix} a+b-2c \\ 3a \\ c-b \\ 3a-12c \end{bmatrix} = \begin{bmatrix} a \\ 3a \\ 0 \\ 3a \end{bmatrix} + \begin{bmatrix} b \\ 0 \\ -b \\ 0 \end{bmatrix} + \begin{bmatrix} -2c \\ 0 \\ c \\ -12c \end{bmatrix}$$

$$= a \underbrace{\begin{bmatrix} 1 \\ 3 \\ 0 \\ 3 \end{bmatrix}}_{u} + b \underbrace{\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}}_{v} + c \underbrace{\begin{bmatrix} -2 \\ 0 \\ 1 \\ -12 \end{bmatrix}}_{w}.$$

so $V = \mathrm{span}\left\{ \overset{u}{\begin{bmatrix} 1 \\ 3 \\ 0 \\ 3 \end{bmatrix}}, \overset{v}{\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}}, \overset{w}{\begin{bmatrix} -2 \\ 0 \\ 1 \\ -12 \end{bmatrix}} \right\}$

Q1(c) [10 Marks] Let

$$A = \begin{bmatrix} -3 & -9 & 1 & 2 \\ -4 & 12 & 1 & 0 \\ 2 & -6 & 1 & 2 \\ -1 & 3 & 2 & 2 \end{bmatrix} \quad \text{and} \quad x = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

Determine, with justification, if $x \in \operatorname{Nul} A$, and if $x \in \operatorname{Col} A$.

Hint: $rref([A|b]) = \begin{bmatrix} 1 & 0 & 0 & 0 & -2/3 \\ 0 & 1 & 0 & 0 & 1/9 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 5/2 \end{bmatrix}$

If $x \in \text{Null}(A)$, then $Ax = 0$. Here

$$Ax = \begin{bmatrix} -3 & -9 & 1 & 2 \\ -4 & 12 & 1 & 0 \\ 2 & -6 & 1 & 2 \\ -1 & 3 & 2 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -9-9+0+0 \\ -12+12+0+0 \\ 6-6+0+0 \\ 3-3+0+0 \end{bmatrix} = \begin{bmatrix} -18 \\ 0 \\ 0 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

So x is <u>not</u> in Null(A),

Q1(c) [10 Marks] Let

$$A = \begin{bmatrix} -3 & -9 & 1 & 2 \\ -4 & 12 & 1 & 0 \\ 2 & -6 & 1 & 2 \\ -1 & 3 & 2 & 2 \end{bmatrix} \qquad \text{and} \qquad x = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

Determine, with justification, if $x \in \operatorname{Nul} A$, and if $x \in \operatorname{Col} A$.

Hint: $rref([A|b]) = \begin{bmatrix} 1 & 0 & 0 & 0 & -2/3 \\ 0 & 1 & 0 & 0 & 1/9 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 5/2 \end{bmatrix}$

If $x \in Col(A)$ then $Ab = x$ for some vector b.

We can apply row reduction to solve for this b

$$[A|b] \Rightarrow \left[\begin{array}{cccc|c} -3 & -9 & 1 & 2 & 3 \\ -4 & 12 & 1 & 0 & 1 \\ 2 & -6 & 1 & 2 & 0 \\ -1 & 3 & 2 & 2 & 0 \end{array}\right]$$

Apply row reduction

...

Augmented Matrix.

Q1(c) [10 MARKS] Let

$$A = \begin{bmatrix} -3 & -9 & 1 & 2 \\ -4 & 12 & 1 & 0 \\ 2 & -6 & 1 & 2 \\ -1 & 3 & 2 & 2 \end{bmatrix} \qquad \text{and} \qquad x = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

Determine, with justification, if $x \in \operatorname{Nul} A$, and if $x \in \operatorname{Col} A$.

Hint: $rref([A|b]) = \begin{bmatrix} 1 & 0 & 0 & 0 & -2/3 \\ 0 & 1 & 0 & 0 & 1/9 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 5/2 \end{bmatrix}$

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$$\left[\begin{array}{cccc|c} -3 & -9 & 1 & 2 & 3 \\ -4 & 12 & 1 & 0 & 1 \\ 2 & -6 & 1 & 2 & 0 \\ -1 & 3 & 2 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 3 & -1/3 & -2/3 & -1 \\ 0 & 0 & -1/3 & -8/3 & -3 \\ 0 & -12 & 5/3 & 8/2 & 2 \\ x & x & x & x & x \end{array}\right]$$

$$\text{etc} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2/3 \\ 0 & 1 & 0 & 0 & 1/9 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 5/3 \end{array}\right]$$

So x is in col (A),

Q1(d) [5 Marks] Decide (with justification) whether

$$H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2; x^2 + y^2 \leq 0 \right\},$$

is a subspace of $\mathbb{R}^2$.

Eg $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \notin H$ since $1^2 + 2^2 = 5 \not\leq 0$

But $\begin{bmatrix} 0 \\ 0 \end{bmatrix} \in H$ since $0^2 + 0^2 = 0 \leq 0$ ✓.

Note: for any $x, y \in \mathbb{R}$, $x^2 \geq 0$ & $y^2 \geq 0$ with $x^2 = 0$ only if $x = 0$ & $y^2 = 0$ only if $y = 0$.

So the only vector in H is the zero vector.

But it <u>is</u> a subspace of H!

Q2. Q2(a) Find the dimension of the subspace

$$H = \left\{ \begin{bmatrix} 2p-2q \\ 2p+3q \\ 2q \\ 5r \end{bmatrix} : p, q, r \in \mathbb{R} \right\}$$

of $\mathbb{R}^4$. [5 marks]

First write a spanning set for H.

$$\begin{bmatrix} 2p-2q \\ 2p+3q \\ 2q \\ 5r \end{bmatrix} = p \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix} + q \begin{bmatrix} -2 \\ 3 \\ 2 \\ 0 \end{bmatrix} + r \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix}.$$

So

$$H = \operatorname{span} \left\{ \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix} \right\}.$$

Q2. Q2(a) Find the dimension of the subspace

$$H = \left\{ \begin{bmatrix} 2p - 2q \\ 2p + 3q \\ 2q \\ 5r \end{bmatrix} : p, q, r \in \mathbb{R} \right\}$$

of $\mathbb{R}^4$. [5 marks]

Dim H is the number of linearly indep vectors in

$$\left\{ \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix} \right\}.$$

So check if they are linearly indep: Can we solve

$$\begin{aligned} 2p - 2q + 0(r) &= 0 \\ 2p + 3q + 0(r) &= 0 \\ 0(p) + 2q + 0(r) &= 0 \\ 0(p) + 0(q) + 5(r) &= 0. \end{aligned}$$

?

Q2. Q2(a) Find the dimension of the subspace

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of $\mathbb{R}^4$. [5 marks]

$$\begin{aligned} 2p - 2q + 0(r) &= 0 \\ 2p + 3q + 0(r) &= 0 \\ 0(p) + 2q + 0(r) &= 0 \\ 0(p) + 0(q) + 5(r) &= 0. \end{aligned}$$

?

The 4th equation, $5r = 0$, gives $r = 0$.
The 3rd equation, $2q = 0$, gives $q = 0$
And the 1st eqn gives $2p - 2q = 0 \Rightarrow 2p = 0$
$\Rightarrow p = 0$. So the only solution is $p = q = r = 0$.

Q2. Q2(a) Find the dimension of the subspace

$$H = \left\{ \begin{bmatrix} 2p - 2q \\ 2p + 3q \\ 2q \\ 5r \end{bmatrix} : p, q, r \in \mathbb{R} \right\}$$

of $\mathbb{R}^4$. [5 marks]

This means the 3 vectors are linearly independent. So $\dim(H) = 3$.

Q2(b) [10 marks] Show that $\mathcal{B} = \left(\begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 8 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ -2 \end{bmatrix} \right)$ is a basis of $\mathbb{R}^3$.

Moreover, find the coordinate vector of $y = \begin{bmatrix} -10 \\ -5 \\ 4 \end{bmatrix}$ relative to $\mathcal{B}$.

(Check = solution is $(42, -22, -13)^T$)

Idea: transform the Augmented matrix

$$\left[\begin{array}{ccc|c} -1 & -5 & 6 & -10 \\ 5 & 8 & 3 & -5 \\ 0 & 1 & -2 & 4 \end{array}\right]$$

into reduced row echelon form:

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 42 \\ 0 & 1 & 0 & -22 \\ 0 & 0 & 1 & -13 \end{array}\right]$$

So the coordinate vector is $\begin{bmatrix} 42 \\ -22 \\ -13 \end{bmatrix}$

check: $(42)\begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix} - 22\begin{bmatrix} -5 \\ 8 \\ 1 \end{bmatrix} - 13\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -42 + 110 - 78 \\ \text{etc} \end{bmatrix} = \begin{bmatrix} -10 \\ \text{etc} \end{bmatrix}$

Q2(c) Find bases of the null space and the column space of

$$A = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 5 & 13 & 3 & 5 \\ 1 & -1 & -3 & 1 \end{bmatrix}.$$

[10 marks]

Write A in reduced row echelon form

$$\begin{bmatrix} 1 & 0 & -2 & 1 \\ 5 & 13 & 3 & 5 \\ 1 & -1 & -3 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 13 & 13 & 0 \\ 0 & -1 & -1 & 0 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 13 & 13 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

To find basis for col (A), we now see that Cols 1 and 2 are linearly independent.

Q2(c) Find bases of the null space and the column space of

$$A = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 5 & 13 & 3 & 5 \\ 1 & -1 & -3 & 1 \end{bmatrix}.$$

[10 marks]

So $\text{col}(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 13 \\ -1 \end{bmatrix} \right\}$.

To gen $\text{nul}(A)$, we want $\begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} a - 2c + d \\ b + c \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. So d & c are "free". set $b = -c$ and $a = 2c - d$.

Q2(c) Find bases of the null space and the column space of

$$A = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 5 & 13 & 3 & 5 \\ 1 & -1 & -3 & 1 \end{bmatrix}.$$

[10 marks]

$a = 2c - d$ and $b = -c$.

So any vector of the form $\begin{bmatrix} 2c-d \\ -c \\ c \\ d \end{bmatrix} \in \text{nul}(A)$,

$$\text{Nul}(A) = \text{span}\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

Q3. Q3(a) Explain the meaning of the following statement: *Linear transformations can be regarded as generalisations of matrices.* [5 marks]

Q3(b)
(i) What is the largest possible rank of an 20×10 matrix?
(ii) If the null space of a 12×4 matrix A is 2-dimensional, what is the dimension of its column space?
(iii) Give an example of a 4×3 matrix A with nullity $A = 2$. [5 marks]

See text book & notes.

Q3. Q3(a) Explain the meaning of the following statement: *Linear transformations can be regarded as generalisations of matrices.* [5 marks]

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(iii) Give an example of a 4×3 matrix A with nullity $A = 2$. [5 marks]

An $m \times n$ matrix has m rows & n cols.

$$A = \underbrace{\begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}}_{n \text{ cols}} \; \begin{matrix} m \\ \text{rows} \end{matrix}$$

Column space of A is the space spanned by the cols of A. The Rank of A is the dimension of the column space.

Q3. Q3(a) Explain the meaning of the following statement: *Linear transformations can be regarded as generalisations of matrices.* [5 marks]

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(iii) Give an example of a 4×3 matrix A with nullity $A = 2$. [5 marks]

The max rank of any $m \times n$ matrix is n.

So (i) Answer is 10.

(ii) Recall the Rank Nullity Thm from Week 7

$$\underbrace{\text{rank}(A)}_{\text{dim of col space}} + \underbrace{\text{nullity}(A)}_{\text{dim Null space}} = n$$

Here $n = 4$, nullity $(A) = 2$, so dim of col space is 2.

Q3. Q3(a) Explain the meaning of the following statement: *Linear transformations can be regarded as generalisations of matrices.* [5 marks]

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(iii) Give an example of a 4×3 matrix A with nullity $A = 2$. [5 marks]

(iii) Idea: here $n=3$, so, if rank $(A) = 1$, then nullity $(A) = 2$.

So, eg,

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

Q3(c) ~~Briefly indicate how vector spaces and linear transformations arise in signal processing.~~ [5 marks]

Q3(d) Recall that $\mathbb{P}_n$ denotes the vector space of polynomials $p(t)$ of degree at most n. Find the matrix of the linear transformation

$$T : \mathbb{P}_3 \to \mathbb{P}_3, \quad p(t) \mapsto 3p(t) - 2p'(t) + p''(t)$$

relative to the basis $(1, t, t^2, t^3)$ of $\mathbb{P}_3$. [10 marks]

p	p'	p''	$3p - 2p' + p''$
1	0	0	3
t	1	0	$3t - 2$
t^2	$2t$	2	$3t^2 - 4t + 2$
t^3	$3t^2$	$3t$	$3t^2 - 6t^2 + 3t$.

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ -2 & 3 & 0 & 0 \\ 2 & -4 & 3 & 0 \\ 0 & 3 & -6 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 + 3t \\ 2 - 4t + 3t^2 \\ 3t - t^2 + 3t^3 \end{bmatrix}$$

So the Matrix is
$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ -2 & 3 & 0 & 0 \\ 2 & -4 & 3 & 0 \\ 0 & 3 & -6 & 3 \end{bmatrix}$$

$\boxtimes$

Q4. Q4(a) Give an example of a vector in $\mathbb{R}^3$, none of whose entries are zero and is length 3, or explain why this cannot be done.
~~Give an example of a vector in $\mathbb{R}^3$, none of whose entries are zero and is length 0, or explain why this cannot be done.~~ [5 marks]

A vector in $\mathbb{R}^3$ has the form $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $a, b, c \in \mathbb{R}$.

The lenght of a vector, v, is $\|v\| = \sqrt{v_1^2 + v_2^2 \dots + v_n^2}$

So, we want a, b, c, so that $\sqrt{a^2 + b^2 + c^2} = 3$.

So $a^2 + b^2 + c^2 = 9$. Eg

(i) $a = b = c = \sqrt{3}$

or

(ii) $a = 1, \ b = 2, \ c = 2$

Q4. Q4(a) Give an example of a vector in $\mathbb{R}^3$, none of whose entries are zero and is length 3, or explain why this cannot be done.
Give an example of a vector in $\mathbb{R}^3$, none of whose entries are zero and is length 0, or explain why this cannot be done. [5 marks]

we would need $a \neq 0$, $b \neq 0$, $c \neq 0$ with

$$a^2 + b^2 + c^2 = 0.$$

But since these are real numbers, so
if $a > 0$ then $a^2 > 0$, similarly for b, c.

So, this is not possible.

(note $\|v\| = 0 \iff v_1 = v_2 = \cdots = v_n = 0$).

Q4(b) Find the orthogonal projection of $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ onto the line passing through $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and the origin in $\mathbb{R}^2$. [5 marks]

That is, find the orthogonal projection of $v = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

onto $W = \text{span}\left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix} \right\}$.

Let $u = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Set $\hat{v} = \dfrac{v \cdot u}{u \cdot u} u = \dfrac{\begin{bmatrix} 3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 4 \end{bmatrix}}{\begin{bmatrix} 1 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 4 \end{bmatrix}} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \dfrac{7}{17} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

Ans: $\hat{v} = \begin{bmatrix} 7/17 \\ 28/17 \end{bmatrix}$.

Recall : $(AB)^T = B^T A^T$.

Q4(c) Explain the meaning of the following statement: *Orthogonal matrices preserve angles.* [5 marks]

(i) A is orthogonal if $A^T = A^{-1}$, ie $A^T A = I$.

(ii) cos of the Angle between u & v is $\dfrac{u \cdot v}{\|u\| \cdot \|v\|}$

Then the angle between Au & Av is

$$\frac{(Au) \cdot (Av)}{\|Au\| \cdot \|Av\|}.$$

But $u \cdot v = u^T v$.

So $(Au) \cdot (Av) = u^T A^T A v = u^T v = u \cdot v$.

Also $\|v\| = \sqrt{v^T v}$. So $\|Au\| = \sqrt{u^T A^T A u} = \sqrt{u^T u} = \|u\|$

So $\dfrac{(Au) \cdot (Av)}{\|Au\| \cdot \|Av\|} = \dfrac{u \cdot v}{\|u\| \cdot \|v\|}$.

Q4(d) Find a least-squares solution of the system $Ax = b$, where

$$A = \begin{bmatrix} 3 & -2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 50 \\ 0 \\ -25 \end{bmatrix}.$$

What is the length of the residual? [10 marks]

To "solve" $Ax = b$,

solve $A^T A \hat{x} = A^T b \Rightarrow \begin{bmatrix} 3 & 2 & 1 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 0 \\ -25 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 14 & -3 \\ -3 & 6 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} 125 \\ -125 \end{bmatrix}.$ Solve this.

Q4(d) Find a least-squares solution of the system $Ax = b$, where

$$A = \begin{bmatrix} 3 & -2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 50 \\ 0 \\ -25 \end{bmatrix}.$$

What is the length of the residual? [10 marks]

$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -55/3 \end{bmatrix}$. The residual is $A\hat{x} - b$.

Here $A\hat{x} - b = \begin{bmatrix} 3 & -2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -55/3 \end{bmatrix} - \begin{bmatrix} 50 \\ 0 \\ -25 \end{bmatrix} = \begin{bmatrix} 5/3 \\ -25/3 \\ 35/3 \end{bmatrix}$.

Then length of this is

$$\sqrt{(5/3)^2 + (-25/3)^2 + (35/3)^2} = 14.434.$$