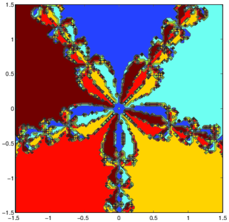
§0. Introduction to MA385; Taylor's Theorem

§0.1 Introduction

MA385 – Numerical Analysis 1

September 2025



0. Outline

1. Welcome to MA385!

This is a Semester 1, upper level module on **numerical analysis**.

You may be taking this course if you are studying

1. Financial Mathematics and Economics (x45?)
2. Mathematics (?)
3. Applied Mathematics (3?)
4. Mathematical Science (8+6)
5. Computer Science (4)
6. Visiting student (2)

1. Welcome to MA385!

Lecturer: **Dr Niall Madden** (he/him)

Addressed: Niall (pronounced "Knee"–"al" #StartsWithAName)

School: Mathematical and Statistical Sciences, University of Galway.

Office: Room AdB-1013, Arás de Brún

Email: Niall.Madden@UniversityOfGalway.ie. This is the best way to contact me. When you do so, please include "MA385" in the subject line. It can also be helpful to include your ID number.

Web: https://www.niallmadden.ie

2. Assessment

- Written Homework Assignment 1 (10%)
- In-class test (10%)
- 3 computer labs (10%)
- A 2-hour exam in December (70%).

3. Schedule

	Mon	Tue	Wed	Thu	Fri
9 – 10	Lecture				
10 – 11					
11 – 12					
12 – 1					
1 – 2					
2 – 3					
3 – 4				Lecture	
4 – 5					
5 – 6					

- Tutorials start Week 3.
- Tutorial runs every week we **don't** have labs.
- Labs schedule TBC.

4. Text books

Süli and Mayers, An Introduction to Numerical Analysis. Available from the library at https://search.library.nuigalway.ie/permalink/f/3b1kce/TN_cdi_askewsholts_vlebooks_9781139636902

Its scope is almost perfect for the course, especially for those students also taking MA378. Other good references include

- G.W. Stewart, **Afternotes on Numerical Analysis**. Not as formal as Süli and Mayers.
- James F Epperson, **An Introduction to Numerical Methods and Analysis**. Somewhat more introductory.
- Toby Driscoll: **Fundamentals of Numerical Computation by Tobin** (https://fncbook.com/). Excellent examples, in Julia, Python and MATLAB.

However, check the start of each section for more precise references. Also, see "Library Reading List" on Canvas.

5. Online Resources

The online content for the course will be hosted on Canvas at https://universityofgalway.instructure.com/courses/46945

- Links to lecture notes; though, actually, these will be hosted at https://www.niallmadden.ie/2526-MA385/ (File are hosted here just because it has an easier work-flow for me; you should not notice any difference if you access from Canvas).
- The Reading List
- Assignments
- Grades
- **Announcements**

These slides are an almost-complete record of what was covered in class.

If notes are added/annotated to the slides in class, they will be posted a day or two later (send Niall a gentle nudge if needed).

However, some details will be worked out on the whiteboard: you should transcribe these.

Note: *There are probably many typos in these notes. None are deliberate. Please help me, and the rest of the class, by pointing them out so I can fix them, no matter how small.*

6. What is Numerical Analysis?

Numerical analysis is the

(a) design,
(b) analysis and
(c) implementation of

numerical methods that yield **exact or approximate** solutions to mathematical problems.

It does not involve long, tedious calculations. We won't (usually) implement Newton's Method by hand, or manually do the arithmetic of Gaussian Elimination, etc.

. . . .

If you would like a more thorough discussion on the nature and history of numerical analysis, read Nick Trefethen's essay on Numerical Analysis from the Princeton Companion to Mathematics, 2008.

6. What is Numerical Analysis?

(a) Design of a numerical method is perhaps the most interesting; it's often about finding a clever way of swapping the problem for one that is easier to solve, but has the same or similar solution. If the two problems have the same solution, then the method is **exact**. If they are similar (but not the same), then it is **approximate**.

6. What is Numerical Analysis?

(b) Analysis is the mathematical part; it usually culminates in proving a theorem that tells us (at least) one of the following.

- **The method will work**: that our algorithm will yield the solution we are looking for;
- **How much effort is required**
- If the method is approximate, determine **how close the approximate solution is to the real one**. A description of this aspect of the course, to quote Epperson, is being "*rigorously imprecise* or *approximately precise*".

(c) The **implementation** is generating solutions with the algorithms, that is, it is the programming part. We'll study this in labs.

7. Topics

1. Root-finding, solving non-linear equations, and optimization. That, and Section 2, depend heavily on Taylor's theorem, so we'll start with that.
2. Initial value ordinary differential equations.
3. Numerical Linear Algebra 1: solving systems of linear equations;
4. Numerical Linear Algebra 2: estimating eigenvalues and eigenvectors.

We also see how these methods can be applied to "real world" problems, including ones in Financial Mathematics.

7. Topics

However, although we'll do some computation, and applications will be key to our motivation: this is a **mathematics** module:

- We'll study when methods can be guaranteed to succeed, and when they might fail;
- We'll study their **accuracy** and **precision** (and why those two terms mean different things).
- We'll **state and prove theorems** concerning properties of numerical methods.

When you have successfully completed this course, you will be able to demonstrate your factual knowledge of the core topics

- root-finding,
- solving ODEs,
- solving linear systems of equations and
- estimating eigenvalues of matrices,

using appropriate mathematical syntax and terminology.
You will be able to describe the fundamental principles of the concepts (e.g., Taylor's Theorem) underpinning Numerical Analysis. You will apply these principles to design algorithms for solving mathematical problems, and discover the properties of these algorithms.

You will also see how to implement these algorithms, and adapt the codes for more general problems, and for different techniques. (In Python, but we won't learn Python, per se.)

8. Mathematical Preliminaries

Anyone who can remember their first and second years of analysis and algebra should be well-prepared for this module. Students who know a little about initial value differential equations will find Section 2 somewhat easier than those who haven't.

If it has been a while since you studied basic calculus, you will find it very helpful to revise the following:

- the Intermediate Value Theorem;
- Rolle's Theorem and The Mean Value Theorem;
- **Taylor's Theorem**,
- and the triangle inequality: $|a+b| \leq |a| + |b|$.

You'll find them in any good text-book, e.g., Appendix 1 of Süli and Mayers.

8. Mathematical Preliminaries

You'll also find it helpful to recall some basic linear algebra, particularly relating to eigenvalues and eigenvectors. Consider the statement:

"all the eigenvalues of a real symmetric matrix are real".

If are unsure what the meaning of any of the terms used, or if you didn't know that it's true, you should have a look at a book on Linear Algebra.

9. Why take this course?

- It is classical: some of the ideas are over 2,000 years old. Some more recent (1950s).
- It is very modern: for each section, we can point to current research on the development and applications in these methods. Usually, these references will be no more than a year or two old.
- It is really interesting (in my completely unbiased opinion).
- It mixes theory and application in a very natural way.
- It is very likely that all of you will find references and uses of this material in future years.