MA211 Problem Set 3

Note Title 04/12/2008

Q1 Solve the IVP

$$y'' + 4y' + 5y = 0 \qquad y(0) = 0, \quad y'(0) = 1$$

Soln : First find the general solution

The auxillary equation is $R^2 + 4R + 5 = 0$

This has 2 complex Root $(D = 16 - 20 < 0)$

$$R = -2 \pm i \qquad = k \pm i\omega$$

So

$$y = e^{-2x}\left(A\cos(x) + B\sin(x) \right)$$

Now use the initial conditions $y(0) = 0$ & $y'(0) = 1$ to find A & B

$$y(0) = 1(A + 0) = 0 \qquad \text{so} \qquad A = 0$$

$$y'(x) = \frac{d}{dx}\left[Be^{-2x}\sin(x) \right]$$

$$= B\left[(-2e^{-2x})\sin(x) - e^{-2x}\cos(x) \right]$$

So $y'(0) = B = 1$ so $B = 1$

ANS: $y = e^{-2x}\sin(x)$

2. Find general solution to the following differential equations:

(i) $y'' - 6y' + 9y = 3x^2$.

(ii) $2y'' + 5y' - 3y = e^x + x$.

(i) $y = h + u$ where h is the general solution to $h'' - 6h' + 9h = 0$

& u is a particular solution to $u'' - 6u' + 9u = 3x^2$

To solve for h: Auxillary Eqn is $R^2 - 6R + 9 = 0$

$\Rightarrow (R-3)(R-3) = 0$

this has just one solution: $R = 3$. So

$h = Ae^{-3x} + Bxe^{-3x}$

Next find $u = q_0 + q_1 x + q_2 x^2$ that solves $u'' - 6u + 9u = 3x^2$

$u' = q_1 + 2q_2 x$, $u'' = 2q_2$ So we need

$2q_2 - 6(q_1 + 2q_2 x) + 9(q_0 + q_1 x + q_2 x^2) = 3x^2$

Gathering the coefs of x^2, x^1 & x^0, we get

$x^2(9q_2) + x(9q_1 - 12q_2) + (9q_0 - 6q_1 + 2q_2) = 3x^2$

So $9q_2 = 3 \Rightarrow q_2 = 1/3$.

Next $9q_1 - 12q_2 = 0 \Rightarrow q_1 = \frac{1}{9}(4) = \frac{4}{9}$.

and $9q_0 - 6q_1 + 2q_2 = 0 \Rightarrow q_0 = \frac{1}{9}\left(\frac{8}{3} - \frac{2}{3}\right) = \frac{2}{9}$

So $u = 2/9 + \frac{4}{9}x + \frac{1}{3}x^2$

ANS: $y = h + u = Ae^{-3x} + Bxe^{-3x} + 2/9 + \frac{4}{9}x + \frac{1}{3}x^2$

Q2 (ii) $2y'' + 5y' - 3y = e^x + x$

$y = h + u + v$ where h is the general solution to

$2h'' + 5h' - 3h = 0$

and u & v are particular solutions to

$2u'' + 5u' - 3u = e^x$ & $2v'' + 5v' - 3v = x$,

To solve for h: $D = b^2 - 4ac = 25 + 24 = 49 > 0$. So there are 2 real roots: $R = \frac{1}{2}$ and $R = -3$

So $h = Ae^{x/2} + Be^{-3x}$

Next find a particular soln to $2u'' + 5u' - 3u = e^x$

u has the form $u = Me^x$, so $u' = u'' = Me^x$

substituting into the DE gives

$2Me^x + 5Me^x - 3Me^x = e^x$. So $M = \frac{1}{4}$

$u = \frac{1}{4}e^x$

Next, find $v = q_0 + q_1 x$ that solves $2v'' + 5v' - 3v = x$

$\Rightarrow$ $2(0) + 5(q_1) - 3(q_0 + q_1 x) = x$

So $q_1 = -\frac{1}{3}$ & $q_0 = -\frac{1}{3} \cdot \frac{5}{3} = -\frac{5}{9}$.

Giving $v = -\frac{5}{9} - \frac{1}{3}x$

Ans: $y = h + u + v = Ae^{x/2} + Be^{-3x} + \frac{1}{4}e^x - \frac{5}{9} - \frac{1}{3}x$