

CS319: Scientific Computing (with C++)

CS319 Lab 5: Numerical Integration in 1D

Week 7 (27+28 Feb, 2025)

Goals:

- ▶ Implement Simpson's Rule.
- ▶ Implement Boole's Rule.
- ▶ Use a Jupyter Notebook to verify its convergence.

You don't have to submit your work this week: we'll develop these ideas some more next week, for integration in two dimensions.

Sections

1 Review

2 Simpson's Rule

3 Boole's Rule

4 Next week...

1. Review

1. Review your notes from Week 4 and (especially) Week 6 on **Numerical Integration**.
2. Download and run `00CompareRules.cpp` from Week 6. That has implementations of both the Trapezium and Simpson's Rule. It tests them for $f(x) = e^x$ and for values of N that the user enters.
3. Download and run `01CheckConvergence.cpp` from Week 6. It computes the error for the Trapezium rule applied to this problem for $N = 4, 8, 16, \dots, 2014$.
4. Upload the `CS319-Week06-notebook.ipynb` Jupyter notebook from Week 7 to a Jupyter server, such as <https://cloudjupyter.universityofgalway.ie>. Pay particular attention to the final section ("Analysing the Quadrature Data") which shows how to verify that, if the error for the Trapezium Rule is

$$E_n \approx CN^{-q},$$

then, for this problem, $C \approx 0.143$ and $q \approx 2.0$.

2. Simpson's Rule

1. Adapt `01CheckConvergence.cpp` from Week 6 so that Simpson's Rule is also implemented (see `Quad2()` from `00CompareRules.cpp`).
2. Use it to compute estimates of the error for Simpson's Rule for various values of N .
3. We suppose again that

$$E_n \approx CN^{-q},$$

Adapt the Jupyter Notebook to find C and q for Simpson's Rule.

3. Boole's Rule

1. George Boole (from Cork) developed a quadrature rule. Read about it at https://en.wikipedia.org/wiki/Boole's_rule
2. Adapt your C++ code so that it also implements the Composite Boole Rule. Note that, in the Wikipedia page, N is used to represent the number of points, whereas we use it to represent the number of intervals. Therefore, in our notation, the formula works for any N that is a multiple of 4.
3. Suppose that Boole's Rule's error is roughly

$$E_n \approx CN^{-q},$$

Adapt the Jupyter Notebook to find C and q for Boole's Rule.

4. **Tip:** Boole's method should be more accurate than Simpson's. However, that means it can only be used for relatively small values of N , before round-off error begins to dominate.

4. Next week...

In next week's lab we'll extend some of these methods to 2D problems. In that setting we'll be interested, not just in the accuracy of the method, but also the **efficiency** too.

In addition, we'll compare some of these methods with a Monte Carlo approach.