MA140: Engineering Calculus

Week 5: Exercises (and answers)

This is a collection of exercises from this week's MA140 lectures. You don't have to submit solutions for these, but you should work through them. Some may be similar to questions on the final exam.

1 Week 05, Lecture 1

2 Week 05, Lecture 2

3 Week 05, Lecture 3

Exercise 5.1.1

Find the equation of the tangent to the curve defined by $x^2 - y^2 = 16$ at the point $(5, 3)$.

Answer: $y = (5x - 16)/3$

Exercise 5.2.1 [2019 exam, Q2(b)(i)]

Differentiate $f(x) = e^{\sin(x)} \cos x$. **ANS** $f'(x) = e^{\sin(x)} \big(\cos^2(x) - \sin(x) \big)$

Exercise 5.2.2 [2023 exam, Q2(a)(i)]

Differentiate $f(x) = x e^{\sin(x)}$. **ANS** $f'(x) = e^{\sin(x)} \big(1 + x \cos(x) \big)$

Exercise 5.2.3

Let $f(x) = x^2 e^x$. Find $f'(x)$, $f''(x)$ and $f'''(x)$.

Answer: $f'(x) = e^x(x^2 + 2x)$, $f''(x) = e^x(x^2 + 4x + 2)$, $f'''(x) = e^x(x^2 + 6x + 6)$.

Exercise 5.3.1 : 23/24 Exam, Q3(a)

Let $f(x) = \ln(x^2 + 1)$.

(i) Find all critical point(s) of f and determine whether f has a local minimum, local maximum or neither.

> **Answer:** There is one critical point: at $x = 0$. And $f''(0) = 2 > 0$, so this corresponds to a local minimum.

(ii) Determine the interval on which f is increasing. ANS $x > 0$

(iii) Determine the interval on which f is decreasing. ANS $x < 0$

(iv) Find all point(s) of inflection of f, justifying your answer.

> **Answer:** $x = -1$ and $x = 1$ are inflection points, since at these points $f''(x) = 0$.