Lab 4: Graph in MATLAB (MA438)

[From Week 7, Feb 2023]

Table of Contents

Graphs in MATLAB

The simplest way to define a graph in MATLAB is by using the graph() function, with two integer arrays, a and b , as arguments: the resulting graph will have edges between $a(i)$ and $b(i)$, for i=1:length(a).

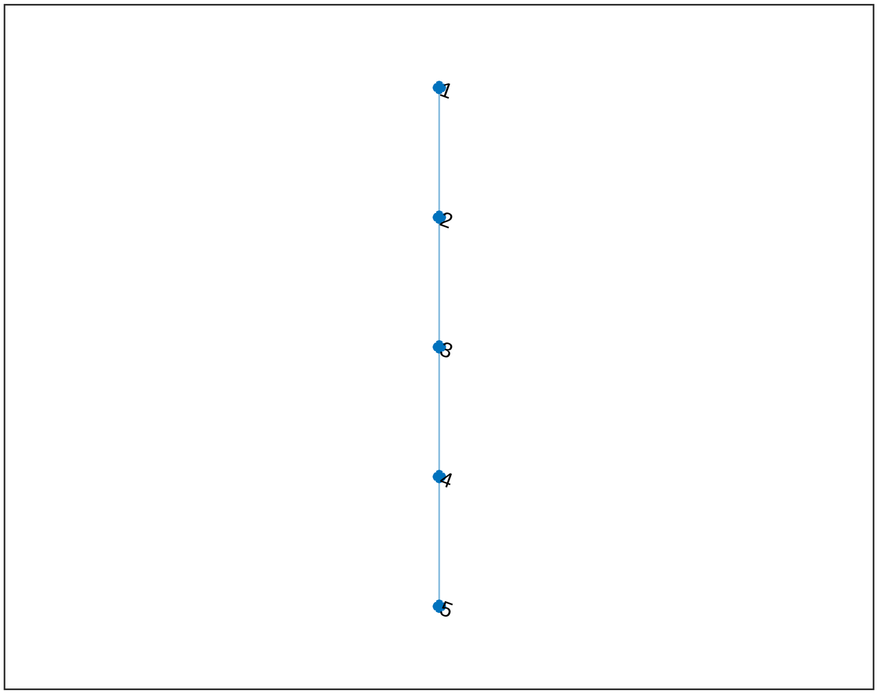
Let's start with P_n, the path graph on n vertices

```
n = 5;
G = graph(1:n-1, 2:n)
```

```
G =
  graph with properties:

    Edges: [4×1 table]
    Nodes: [5×0 table]
```

```
plot(G)
```



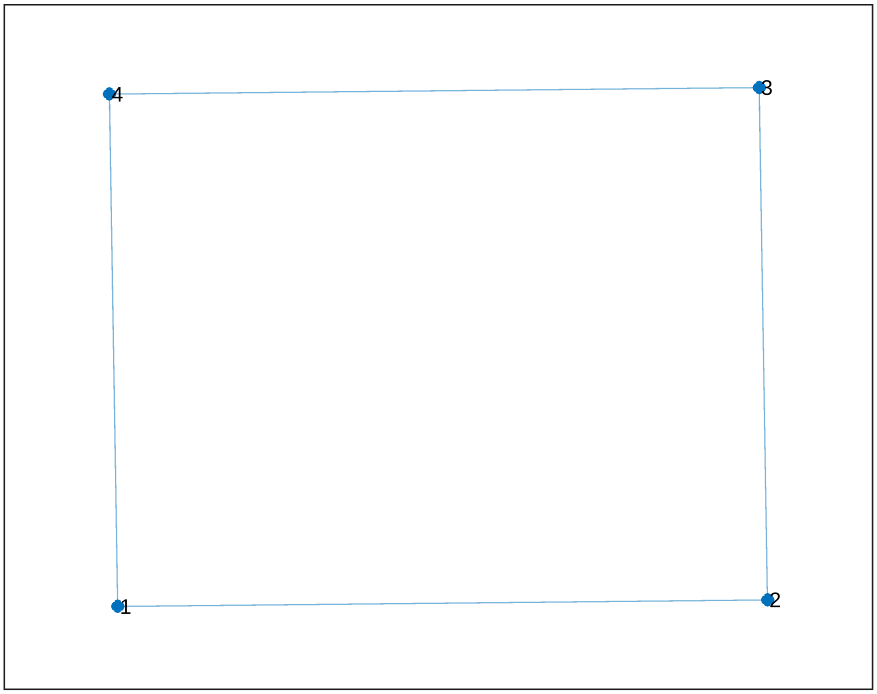
For example, here is a cycle graph, C_4

```
a = [1,2,3,4];
b = [2,3,4,1];
C4 = graph(a,b)
```

```
C4 =
  graph with properties:

    Edges: [4×1 table]
    Nodes: [4×0 table]
```

```
plot(C4)
```



Adjacency Matrix

Alternatively, we can define the graph using its adjacency matrix. If a graph has n vertices, then its adjacency matrix is the $n \times n$ matrix, A, with $a_{\text{ij}} = \begin{cases} 1 & \text{if there is an edge between vertices } i \text{ and } j \\ 0 & \text{otherwise} \end{cases}$

When G is not directed, A is symmetric.

For example, K_4, the complete graph on 4 vertices has the adjacency matrix:

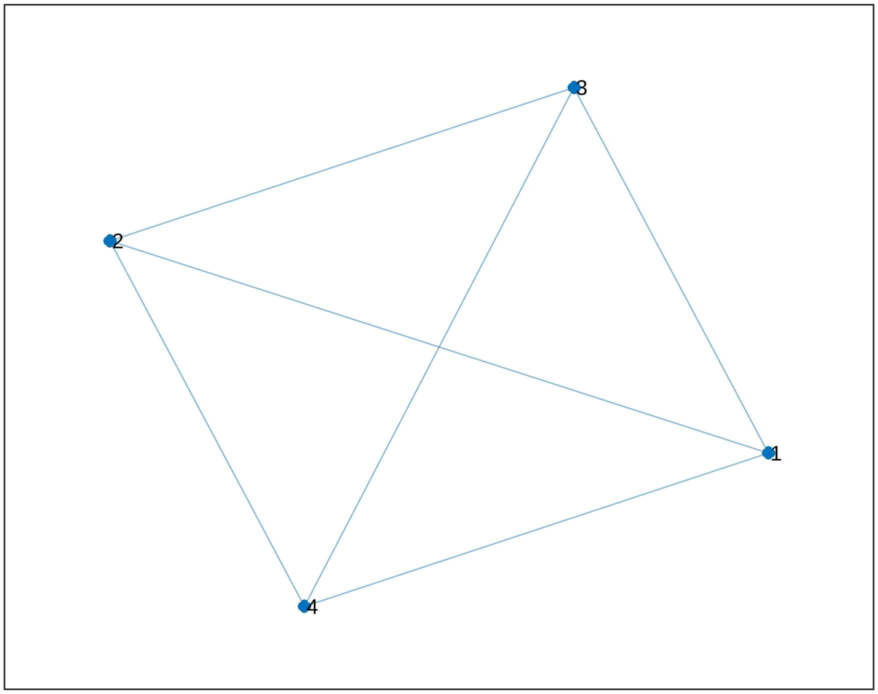
```
A4 = ones(4) - diag(ones(1,4))
```

```
A4 = 4×4
     0     1     1     1
     1     0     1     1
     1     1     0     1
     1     1     1     0
```

The graph of a matrix

You can pass A to the graph function to create the associated graph.

```
K4 = graph(A4);
plot(K4)
```

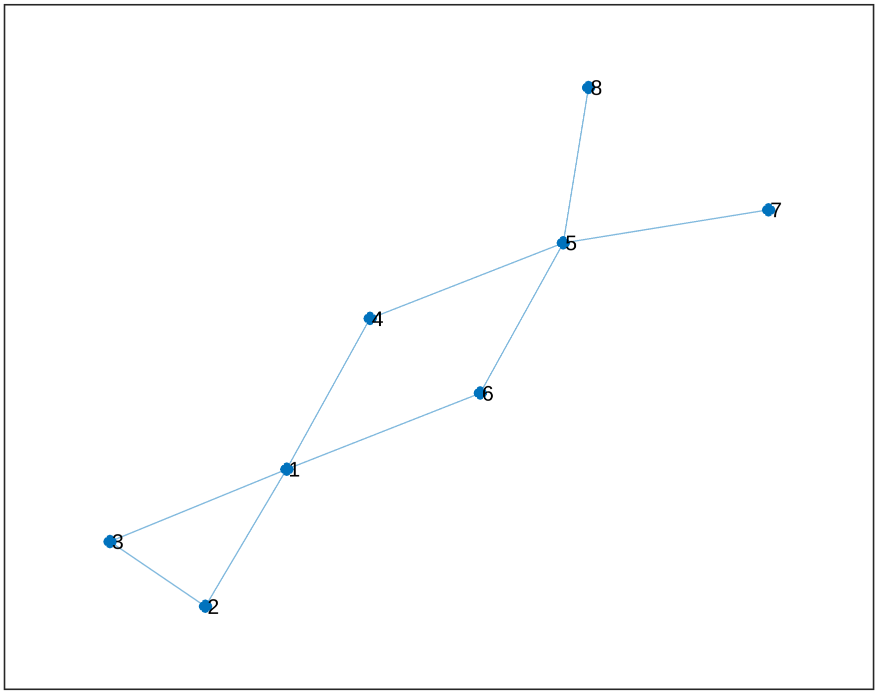


Another example:

```
A = zeros(8);
A(1,2)=1; A(1,4)=1; A(1,6)=1;
A(1,3)=1; A(2,3)=1; A(4,5) = 1; A(5,6)=1;
A(5,7)=1; A(5,8)=1;
A=A+A'  % make A symmetric, since this is non-directed
```

```
A = 8×8
     0     1     1     1     0     1     0     0
     1     0     1     0     0     0     0     0
     1     1     0     0     0     0     0     0
     1     0     0     0     1     0     0     0
     0     0     0     1     0     1     1     1
     1     0     0     0     1     0     0     0
     0     0     0     0     1     0     0     0
     0     0     0     0     1     0     0     0
```

```
G8 = graph(A);
plot(G8)
```



If we have a graph, we can also extract the Adjacency Matrix. By default, this is "sparse". so we will convert to full.

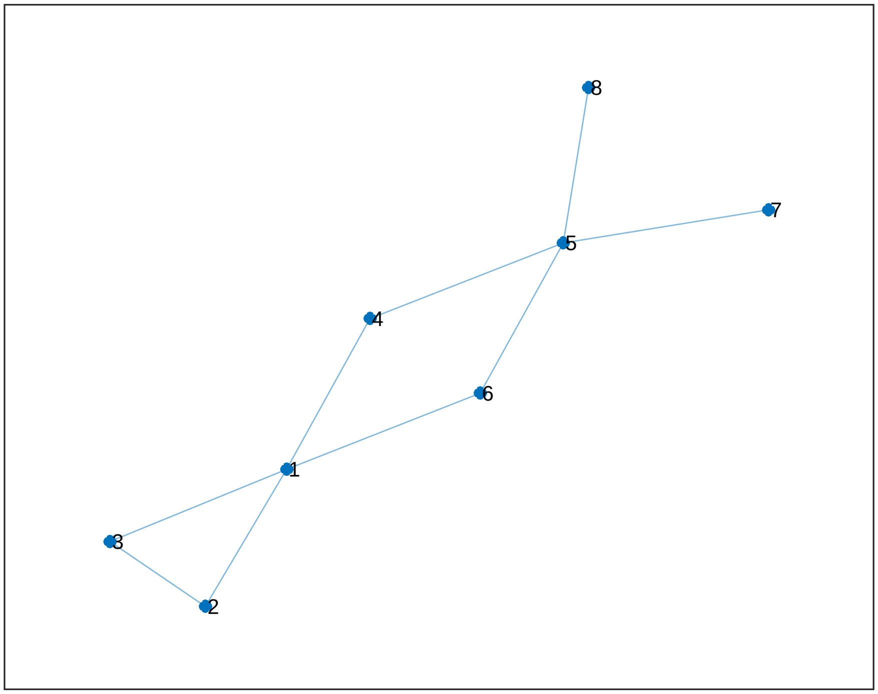
```
A4 = full(C4.adjacency)
```

```
A4 = 4×4
     0     1     0     1
     1     0     1     0
     0     1     0     1
     1     0     1     0
```

Computing with the Adjacency matrix

The adjacency matrix has many applications. A simple one is the the row (or column) suns of A give you the degree of each vertex:

```
plot(G8)
```




```
d=sum(A)
```

```
d = 1×8
     4     2     2     2     4     2     1     1
```

```
for i=1:length(d)
   fprintf("Vertex %d has degree %d\n", i, d(i))
end
```

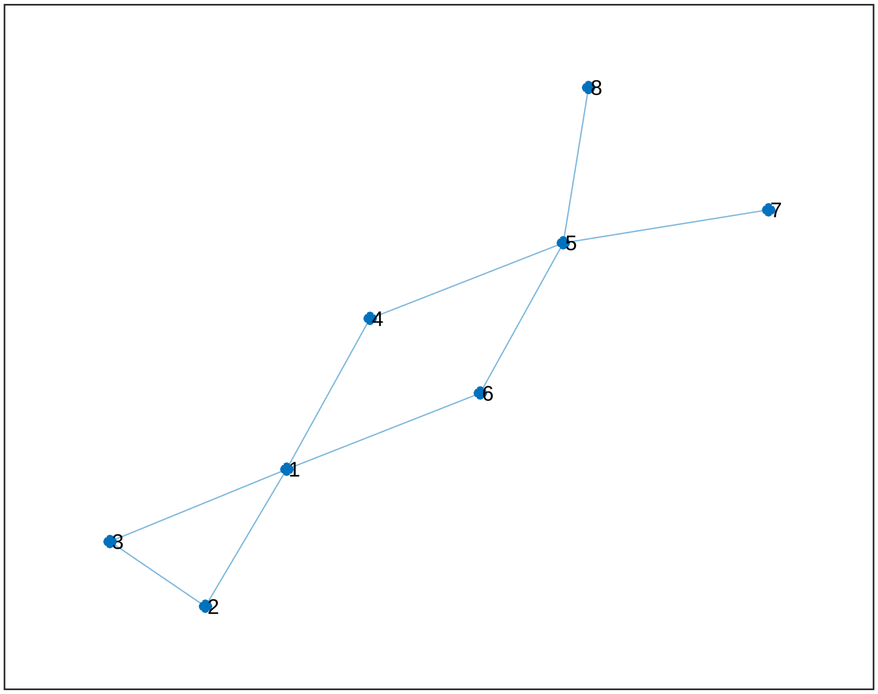
```
Vertex 1 has degree 4
Vertex 2 has degree 2
Vertex 3 has degree 2
Vertex 4 has degree 2
Vertex 5 has degree 4
Vertex 6 has degree 2
Vertex 7 has degree 1
Vertex 8 has degree 1
```

Similarly, we can count the number of edges in the graph: it is half the sum of all the entries in the graph.

Here is another useful property. Set $e^{(i)}$ to be the vector where $e_i^{(i)} = 1$, and all other entries are zero; that is, $e^{(i)}$ is the ith row of the identity matrix. Now compute $v = Ae^{(i)}$. Then $v_j = 1$ if and only if i is a neightbour of j in G.

A consequence of this is that, if $B = A^k$, then b_{ij} is the number of paths of length k between vertex i and vertex j:

```
plot(G8)
```




```
B=A^3
```

```
B = 8×8
     2     5     5     6     0     6     2     2
     5     2     3     1     2     1     0     0
     5     3     2     1     2     1     0     0
     6     1     1     0     6     0     0     0
     0     2     2     6     0     6     4     4
     6     1     1     0     6     0     0     0
     2     0     0     0     4     0     0     0
     2     0     0     0     4     0     0     0
```

Graphs as Objects

Graphs are examples of *objects* in MATLAB. The full ramifications of that is beyond this module. But, roughly, it means, that, unlike, say, a matrix, it as properties that we can check and change. Try these:

```
Number_of_edges = C4.numedges
```

```
Number_of_edges = 4
```

```
C4.numnodes
```

```
ans = 4
```

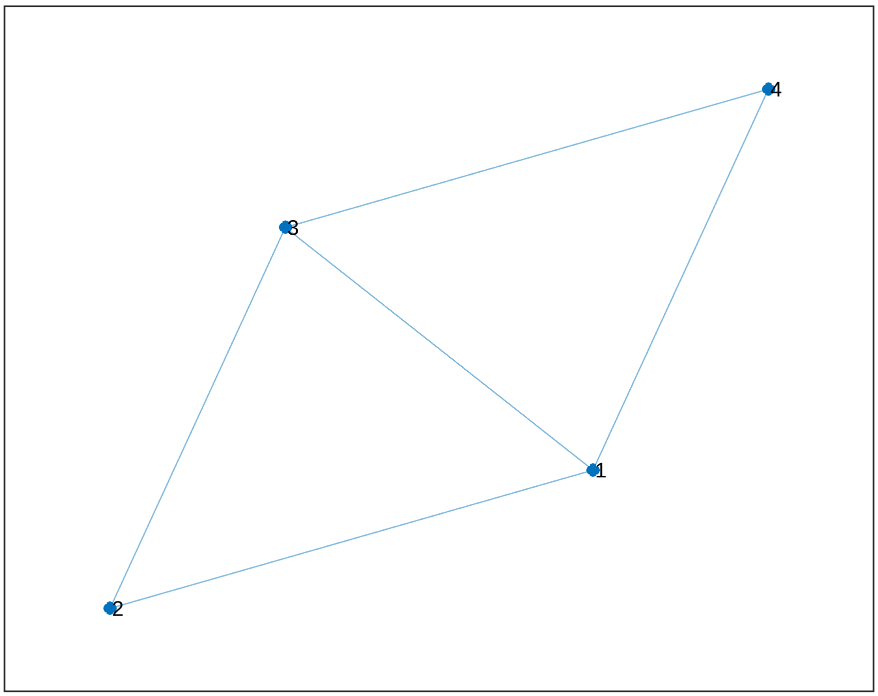
```
C4.Edges
```

```
ans = 4×1 table
```

	EndNodes	
1	1	2
2	1	4
3	2	3
4	3	4

And these:

```
G=C4.addedge(1,3);
plot(G)
```



There are various built-in functions for graphs (though fewer than you might think). For example, the `minspantree()` function returns a minimum spanning tree for the graph. Here is an example of using it along with `highlight()`.

```
M34 = K34.minspantree()
```

```
M34 =
  graph with properties:

    Edges: [6×1 table]
    Nodes: [7×0 table]
```

```
h = plot(K34, 'EdgeColor','green', 'LineWidth',2, 'MarkerSize',10)
```

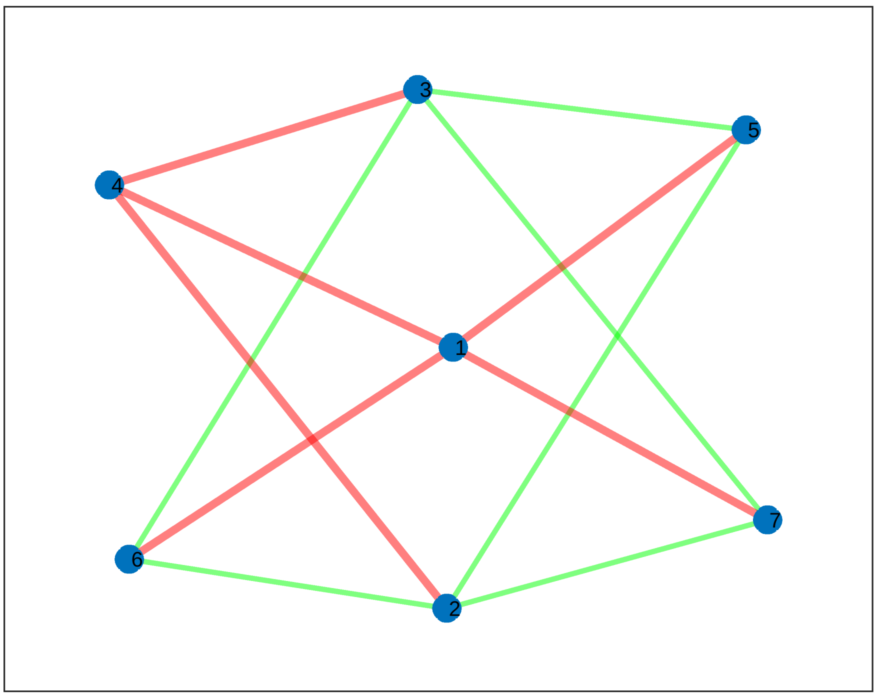
```
h =
```

```
GraphPlot with properties:

     NodeColor: [0 0.4470 0.7410]
    MarkerSize: 10
        Marker: 'o'
     EdgeColor: [0 1 0]
     LineWidth: 2
     LineStyle: '-'
     NodeLabel: {'1'  '2'  '3'  '4'  '5'  '6'  '7'}
     EdgeLabel: {}
         XData: [0.0782 0.0448 -0.1121 -1.7761 1.6592 -1.6689 1.7749]
         YData: [0.0050 -1.2084 1.2042 0.7596 1.0166 -0.9797 -0.7973]
         ZData: [0 0 0 0 0 0 0]

Show all properties
```

```
h.highlight(M34, 'EdgeColor','red', 'LineWidth',3)
```

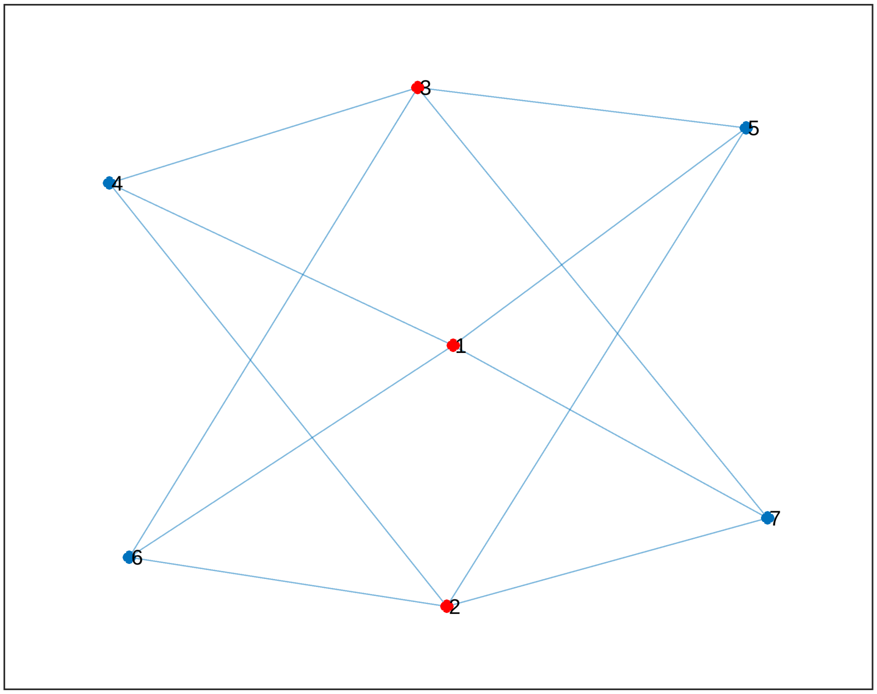


Bipartite Graphs

From Rachel's lectures, you know what a bipartite graph is. Let's construct $K_{3,4}$. We'll let the parts be $\{1,2,3\}$ and $\{4,5,6,7\}$. We'll make it with a for-loop (though there are oth er ways).

```
K34 = graph(); % empty graph
for i=1:3
   for j=4:7
      K34=K34.addedge(i,j);
   end
end
```

```
h=plot(K34);
highlight(h, [1:3], 'NodeColor','red')
```



Here the `highlight()` function is used to distinguish some vertices. We can also do this for edges, which will prove very useful.

The adjacency matrix of a bipartite graph can be revealing, at least of the vertices are ordered in a certain way,

```
full(K34.adjacency())
```

```
ans = 7×7
     0     0     0     1     1     1     1
     0     0     0     1     1     1     1
     0     0     0     1     1     1     1
     1     1     1     0     0     0     0
     1     1     1     0     0     0     0
     1     1     1     0     0     0     0
     1     1     1     0     0     0     0
```

Determining if a graph is bipartite is not too hard: one can use a breadth-first search. We'll look at that another time.

Directed Graphs

To make a directed grap, use the `digraph()` function. Example: let's make a random graph with 6 vertices, and 12 edges

```
D = digraph();
```

```
D = addnode(D, 6)
```

```
D =
  digraph with properties:

    Edges: [0×1 table]
    Nodes: [6×0 table]
```

```
while(D.numedges < 10)
   a = randi(6,1,1);
   b = randi(6,1,1);
   if ((a ~= b) && (findedge(D,a,b)==0))
      D = D.addedge(a,b);
   end
end
plot(D)
```