2425-MA140 Engineering Calculus

Week 11: Tutorials
Practice paper: Q1+Q2

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MA140 Practice Paper 2024–2025

DRAFT VERSION: Questions 4 and 5 will be added later

About these questions

This set of questions are provided to help you prepare for the MA140 Semester 1 exam. Information on the similarlity, and differences, between it and the actual exam will be given in class in Week 11. Part of Questions 1 and 2 will be covered in tutorials; Parts of Questions 3-5 will be covered in Lectures. Answers to all questions will be posted during Study Week.

Q1(a) Express $\dfrac{10x-27}{5x^2-25x+30}$ as partial fractions.

Q1(b) Let $f(x) = \dfrac{x^2 - 2x - 8}{x^2 - x - 12}$. For each of the following, evaluate the limit, or determine that it does not exist.

(i) $\lim_{x \to -3} f(x)$

(ii) $\lim_{x \to 4} f(x)$

(iii) $\lim_{x \to \infty} f(x)$

Q1(c) Let $f(x) = x^{-2}(2 - e^x - e^{-x})$, $g(x) = -x^2 - 1$ and $h(x) = x^2 - 1$. You may assume that $g(x) \leqslant f(x) \leqslant h(x)$ for all x in the region $[-2, 2]$.

(i) Use the Squeeze Theorem to determine $\lim_{x\to 0} f(x)$.

(ii) Explain why you can't use the Squeeze Theorem to determine $\lim_{x\to 1} f(x)$.

Q1(d) Evaluate the limit $\lim_{\theta \to 0} \dfrac{2\sin(\theta)}{\theta + 3\tan(\theta)}$.

Q1(e) Let $f(x) = \begin{cases} a/x & x < 2 \\ 3 + bx & x \geqslant 2 \end{cases}$.

Find values of a and b for which both $f(x)$ and $f'(x)$ are continuous at $x = 2$.

Q2(a) Differentiate $f(x) = x^2 e^{-3x} \sin(4x)$, with respect to x

Q2(b) Differentiate $f(\theta) = (\sin(3\theta) + 1)(3\theta + 1)^{-1}$ with respect to θ.

Q2(c) Differentiate $f(x) = \ln(\cos(x^2))$ with respect to x.

Q2(d) Let $f(x) = 10\ln(x) + e^{-10x}$. Find $f'(x)$, $f''(x)$, and $f'''(x)$.

Q2(e) Use the Inverse Power Rule, to fund the derivative, with respect to x, of $y = \cos^{-1}(x)$.

Q2(f) Find the equation of the tangent to the curve implicitly defined by

$$2x^2 + y^2 = 3,$$

at the point $(x, y) = (1, 1)$.