MA140: Engineering Calculus

Week 4: Exercises (and answers)

This is a collection of exercises from this week's MA140 lectures. You don't have to submit solutions for these, but you should work through them. Some may be similar to questions on the final exam.

1 Week 04, Lecture 1

2 Week 04, Lecture 2

3 Week 04, Lecture 3

Exercises 4.1.1 (Based on Q2(a), 2019/2020)

Use the (limit) definition of a derivative to differentiate the function $f(x) = x^2 + 2$.

Answer: Outline: By definition $f'(x) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$. In this case, $f'(x) = \lim_{h\to 0} \frac{(x+h)^2+2-(x^2+2)}{h} = \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2-2}{h} = \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} 2x + h = 2x$

Exercise 4.1.2

Use the (limit) definition of a derivative to show that the derivative of $f(x) = \cos(x)$ is $f'(x) = -\sin(x)$.

Answer: Outline: $f'(x) = \lim_{h\to 0} \frac{\cos(x+h)-\cos(x)}{h} = \lim_{h\to 0} \frac{-2\sin(\frac{2x+h}{2})\sin(h/2)}{h} = -\lim_{h\to 0} \sin(\frac{2x+h}{2})\frac{\sin(h/2)}{h/2} = -\big(\sin(\frac{2x}{2})\big)\big(1\big)$.

Exercises 4.2.1 (Based on Q2(a), 2023/2024)

Find the derivative of $f(x) = \dfrac{\sin(x)}{\sqrt{x}}$.

Answer: $x^{-1/2}\cos(x) - \sin(x)x^{-3/2}/2.$

Exercise 4.2.2 (Based on Q2(b), 2019/2020

Find the derivative of $f(x) = \dfrac{x^2 + x - 2}{x^3 + 6}$.

Answer: $\dfrac{-x^4 - 2x^3 + 6x^2 + 12x + 6}{(x^3 + 6)^2}$

Exercise 4.3.1

Find the derivative of

1. $f(x) = x^3 \cos(x^2)$ Ans: $3x^2 \cos(x^2) - 2x^4 \sin(x^2)$
2. $f(x) = \tan\left(\sin^2(x^4)\right)$

   Ans: $8x^3 \sin(x^4)\cos(x^4)(1 + \tan^2(\sin^2(x^4))$

Exercise 4.3.2

Show that $\dfrac{d}{dx}(\cos^{-1} x) = \dfrac{-1}{\sqrt{1-x^2}}$.