2324-MA378: Sample Exercises for Class Test in Week 8:

1. *Let p_n be the polynomial of degree n that interpolates the function f at the distinct points $\{x_0, x_1, \dots, x_N\}$. State Cauchy's Theorem for $f(x) - p_n(x)$. (You do* not *have to prove it).*

2. *Suppose that S is a natural cubic spline on $[0, 2]$ with*

$$S(x) = \begin{cases} -x + 2(1-x) + a(1-x)^3 + \frac{2}{3}x^3, & \text{for } 0 \le x < 1, \\ b(2-x) + c(2-x)^3 + d(x-1)^3, & \text{for } 1 \le x \le 2. \end{cases}$$

   Find a, b, c, and d.

3. *Suppose that S is the cubic spline interpolant to $f(x) = xe^{-x}$ on the $N+1$ equally spaced points $\{x_0 = 0 < x_1 < \dots < x_N = 2\}$. We know that*

$$\|f - S\| := \max_{0 \le x \le 2} |f - S| \le \frac{5h^4}{384} \max_{0 \le x \le 2} |f^{(4)}(x)|,$$

   where $h = 2/N$.
   What value of N should one take to ensure that $\|f - S\|$ is no more than 10^{-8}.

4. *Suppose that S is the natural cubic spline interpolant to a function g on $[-1, 1]$. If*

$$\max_{-1 \le x \le 1} |g(x) - S(x)| = 0,$$

   what can we say about g?