2323-MA378: **Sample Exercises** for Class Test in Week 7

Name: ______________________

ID Number: ______________________

Programme: ______________________

1. (`10 marks`) Let p_n be the polynomial of degree n that interpolates the function f at the distinct points $\{x_0, x_1, \dots, x_N\}$. State Cauchy's Theorem for $f(x) - p_n(x)$. (You do *not* have to prove it).

2. (`40 marks`) Suppose that S is a natural cubic spline on $[0, 2]$ with

$$S(x) = \begin{cases} -x + 2(1-x) + a(1-x)^3 + \frac{2}{3}x^3, & \text{for } 0 \le x < 1, \\ b(2-x) + c(2-x)^3 + d(x-1)^3, & \text{for } 1 \le x \le 2. \end{cases}$$

   Find a, b, c, and d.

3. (`40 marks`) Suppose that S is the cubic spline interpolant to $f(x) = xe^{-x}$ on the $N+1$ equally spaced points $\{x_0 = 0 < x_1 < \cdots < x_N = 2\}$. We know that

$$\|f - S\|_\infty := \max_{0 \le x \le 2} |f(x) - S(x)| \le \frac{5h^{\mathbf{4}}}{384} \max_{0 \le x \le 2} |f^{(4)}(x)|,$$

   where $h = 2/N$.

   What value of N should one take to ensure that $\|f - S\|$ is no more than 10^{-8}.

4. (`10 marks`) Suppose that S is the natural cubic spline interpolant to a function g on $[-1, 1]$. If

$$\max_{-1 \le x \le 1} |g(x) - S(x)| = 0,$$

   what can we say about g?