2526-MA140: Week 03, Lecture 1 (L04)

Partial Fractions

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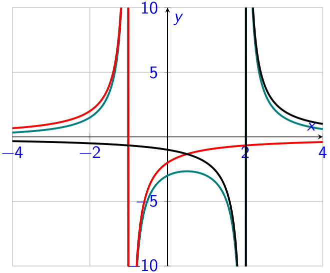
Tuesday, 23 September, 2025



Outline

Slides are on canvas, and at

https://www.niallmadden.ie/2526-MA140/

Tutorials start **this** week. The schedule is:

- Teams 1+2: Tuesday 15:00 ENG-**2003**
- Teams 3+4: Tuesday 15:00 ENG-**2034**
- Teams 9+10: Thursday 11:00 ENG-**2002**
- Teams 11+12: Thursday 11:00 ENG-**3035**
- Teams 5+6: Friday 13:00 Eng-**2002**
- Teams 7+8: Friday 13:00 Eng-**2035**

In addition, *rang teagaisc trí Ghaeilge* (Irish tutorial): Dé Máirt (Tuesday) 15:00, Áras na Gaeilge 221.

- There is currently a "practice" assignment open. See https://universityofgalway.instructure.com/courses/46734/assignments/128373
- During tutorials, the tutor will solve some similar questions. You can access the **tutorial sheet** at https://universityofgalway.instructure.com/courses/46734/files/2842617?module_item_id=925893. You can also access this through the Canvas page: `Modules`... `Tutorial Sheets`.
- **Assignment 1** will be due 5pm, Monday 5 October.

. . . .

Also: try the exercises at the end of each set of lecture slides: they are similar in style and standard to exam questions.

There are two class test planned for this module:

- MCQ format;
- both worth 10% of the final grade;
- Test 1: **Tuesday, 14 October** (Week 5)
- Test 2: **Tuesday, 18 November** (Week 9)
- Contact Niall if you have any concerns, or wish to avail of alternative arrangements, as provided by LENS reports.

Partial Fractions

Rational Functions have the general form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials.

An (proper) rational function can often be written as a sum of simpler ones: **partial fractions.**

For example

$$\frac{8x-12}{x^2-2x-3}$$

can be written as

$$\frac{3}{x-3} + \frac{5}{x+1}$$

Check:

Partial Fractions

Note: Any polynomial (with real coefficients) can be factorised fully into the product of

- linear
- and irreducible quadratic factors.

We get different combinations of factors in the denominator. Let's look at **four cases**, and how to find the partial fractions in each case.

The four cases

1. Linear factors to the power of 1 in the denominator.
2. Linear factors to the power greater than 1 in the denominator, (i.e repeated linear factors).
3. Irreducible quadratic factors.
4. Irreducible quadratic factors to power greater than 1.

Case 1: Linear factors to the power of 1 in the denominator.

We have **two methods** to find A and B.

Method 1: Comparing coefficients

Example

$$\frac{3x}{(x-1)(x+2)}$$

Method 2: Substituting specific values for x.

Example

Write $\dfrac{8x-12}{x^2-2x-3}$ as sum of partial fractions.

(2) Linear factors to the power greater than 1 in the denominator, (i.e repeated linear factors).

If $(x-\alpha)^k$ appears in the denominator, it will give rise to the following terms:

$$\frac{A_1}{x-\alpha}+\frac{A_2}{(x-\alpha)^2}+...+\frac{A_k}{(x-\alpha)^k}$$

Example

Find A, B and C such that

$$\frac{3x+1}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$$

(Note: we'll find that $A = 5/9$, $B = 4/3$ and $C = -5/9$).

(3) Irreducible quadratic factors.

Irreducible quadratic factors can not be factorised using real numbers, e.g. $x^2 + x + 1$.

An irreducible quadratic factor $ax^2 + bx + c$ gives rise to partial fractions of the form

$$\frac{Ax + B}{ax^2 + bx + c}.$$

Example 2.34 from textbook

If one writes

$$\frac{5x}{(x^2 + x + 1)(x - 2)} = \frac{Ax + B}{x^2 + x + 1} + \frac{C}{x - 2}$$

then we find $A - 10/7$, $B = 5/7$ and $C = 10/7$.

(4) Irreducible quadratic factors to power greater than 1.

Each repeated irreducible quadratic factor $(ax^2 + bx + c)^k$ in the denominator will give rise to

$$\frac{A_1 x + B_1}{ax^2 + bx + c} + \frac{A_2 x + B_2}{(ax^2 + bx + c)^2} + \ldots + \frac{A_k x + B_k}{(ax^2 + bx + c)^k}.$$

These can be done in a similar way to the previous case. But the calculations are pretty messy, so we won't even try!

Towards Limits

When we were considering the domain of a function, we looked at those x-values for which the function was not defined.

Example

$$f(x) = \frac{x^2 - 2}{x - 1} \qquad g(x) = \frac{x^2 - 1}{x - 1}$$

Neither f nor g are defined at $x = 1$.

But what happens if x gets very closed to 1?

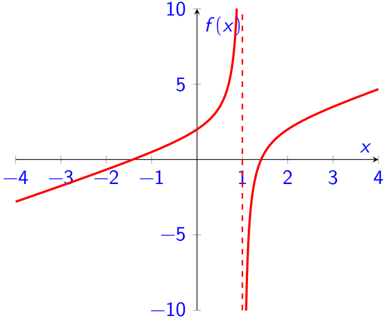
x	0.900	0.990	0.999	1	1.001	1.010	1.100
$f(x)$							
$g(x)$							

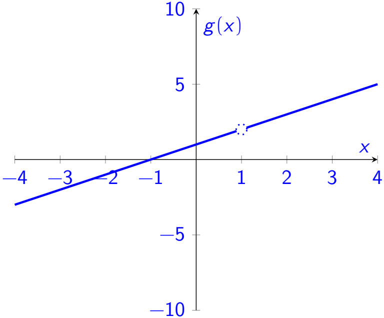
Let's look at the graphs of f and g.

Towards Limits

Example

$$f(x) = \frac{x^2 - 2}{x - 1} \qquad g(x) = \frac{x^2 - 1}{x - 1}$$





Towards Limits

In the previous example, we saw that, although neither f nor g was defined at $x = 1$, they behaved very differently as x approaches 1.

To discuss this we'll need the concept of a **limit** which, roughly, relates to the value of function as it **approaches** a point (but not actually at that point).

$$\lim_{x \to a} f(x) = L$$

The concept of a limit is a prerequisite for a proper understanding of calculus and numerical methods.

Exercises

Exercise 2.1.1

Find the constants A, B and C, so that

$$\frac{2x+1}{(x-2)(x+1)(x-3)} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{x-3}$$

Exercises

Exercise 2.1.2

Express the following as partial fractions.

1. $\dfrac{6}{x^2 - x - 2}$
2. $\dfrac{2x - 1}{x^2 - x - 2}$
3. $\dfrac{x - 1}{(x + 1)(x^2 - x - 2)}$
4. $\dfrac{x}{x^2 + 2x + 1}$
5. $\dfrac{1}{x^3 - 1}$

Exercises

Exercise 2.1.3 (MA140 Exam, 24/25)

Express $\dfrac{3x+1}{x^2-x-2}$ as partial fractions.