4th Annual

Stokes Modelling Workshop

National University of Ireland, Galway

6 - 9 June 2017

Supported by MI-NET, Mathematics for Industry Network COST Action

Stokes Applied Mathematics Cluster
School of Mathematics, Statistics and Applied Mathematics
and
NUI Galway SIAM Student Chapter

Organising Committee:

Michel Destrade, Roberto Galizia, Paul Greaney, Niall Madden, Martin Meere, Robert Mangan
Petri Piiroinen, Eoghan Staunton, Michael Welby, Giuseppe Zurlo

Welcome

Dear participant,

Welcome to 2017 Stokes Modelling Workshop. This is our 4th annual workshop, and the largest to date. The series was orininally devised as a means of introducing undergradute students to collaborative research and real-world problem solving, usually guided by staff and research students from the School of Mathematics, Statistics, and Applied Mathematics. These were largely organised by the students members of our SIAM Chapter.

This year, for the first time, we are funded, in part, by MI-NET, the *Mathematics for Industry Network* COST Action. This has allowed us to extend the workshop in a number of ways:

- Involve industry partners who have proposed problems that you will be working on this week;
- Bring skilled and experienced mentors to NUI Galway to support teams;
- Widen the participation to involve graduate students from near and far.

Nonetheless, the main core principles of our first three workshops remain in place: we welcome undergraduate and graduate participants from different disciplines, encourage creative thinking, clear communication, and supportive team work.

We are delighted to welcome our mentors, Art, Doireann, John, Luca and Tuoi, and look forward to working with them, and learning from them, over the coming week.

We are grateful to our industrial partners from IBM Research, Medtonic, and MET Eireann for providing problems of industrial and societal relevance, and for being present (in person or virtually) to present these problems.

Ostensibly, our collective goal will be to optimally pack catheters, place rainfall radar equipment, model rivers converging, visualise 3D objects using 2D images, and get ketchup out of a bottle. More fundamentally, our aims are to raise awareness of the potential of mathematical modelling in industry and society, to challenge ourselves to think creatively, and to apply our mathematical skills and knowledge and to real-world problems.

We hope to create a relaxed environment, where everyone can contribute their talents and skills, and work together collaboratively.

We would like to express our gratitude to the School of Mathematics, Statistics and Applied mathematics who, through the Stokes Applied Mathematics Cluster have provided support for undergraduate participants, to the NUI Galway SIAM Chapter, and, of course, MI-NET.

Dr Niall Madden (Head of the Stokes Applied Mathematics Cluster)

On behalf of the organising committee.

Participants

Name	Institution	Name	Institution
Al Baydli, Daher	NUI Galway	Marshall, Christine	NUI Galway
Allen, James	University of Surrey	Martin, Richard	NUI Galway
Alssaedi, Faiza	NUI Galway	Mary, Rose	IT Carlow
Atan, Reduan	NUI Galway	Mayah, Faik	NUI Galway
Burke, Richard	NUI Galway	Moran, Claire	NUI Galway
Cloherty, Philip	NUI Galway	Okeke, Saviour	IT Carlow
Collins, Jack	NUI Galway	Onaga, Tomokatsu	University of Kyoto
Colson, David	NUI Galway	Phoenix, Anna	NUI Galway
Conroy Broderick, Hannah	NUI Galway	Qiu, Songkai	NUI Galway
Cormican, John	NUI Galway	Rawle, Noeleen	NUI Galway
Donnelly, Cliona	NUI Galway	Regan, Brian	NUI Galway
Dubovskaya, Alina	University of Limerick	Samantray, Suman	NUI Galway
Galizia, Roberto	NUI Galway	Sheil, Ashley	Maynooth University
Geraghty, Samuel	NUI Galway	Smyth, David	NUI Galway
Greaney, Paul	NUI Galway	Staunton, Eoghan	NUI Galway
Hanley, Tracey	NUI Galway	Suleiman, Issah Nazif	Univeristy of Verona
Hill, Aoife	NUI Galway	Urbas, Szymon	NUI Galway
Hill, Róisín	NUI Galway	Walsh, Dylan	NUI Galway
Kamperis, Sam	Oxford Brookes	Walsh, Shane	UCD
Mangan, Robert	NUI Galway	Wisdom, Adrian	NUI Galway

Trainers

Name	Institution
John Donohue	MACSI, University of Limerick
Artur Gower	University of Manchester
Luca Manzari	Stockholm's Royal Institute of Technology
Fergus McAuliffe	iCRAG – University College Dublin
Doireann O'Kiely	University of Oxford
Thi Ngoc Tuoi Vo	MACSI, University of Limerick

Industry partners

Name	Institution
Colm Clancy,	MET Eireann
David Clarke	Medtronic
Jack O'Callaghan	Medtronic
Seshu Tirupathi	IBM Research

Stokes Modelling Workshop, 6-9 June, NUI Galway: Draft Schedule

	Tuesday, 6 June	Wednesday, 7 June	Thursday, 8 June	Friday, 9 June
09:00	**Registration** from 9.15 Foyer, Arts Millennium Building	**Tea/Coffee,** THB-G011	*Work on Problems*	*Work on Problems*
09:30		**Fergus McAuliffe: Science Communication Workshop** THB-G010 (Library Building) Break 10.45-11.00		
10:00	**Problem Presentations, AM150** Seshu Tirupathi, IBM; David Clarke, Jack O'Callaghan, Medtronic; Luca Manzari, KTH; Colm Clancy, MET Eireann; Doireann O'Kiely, Oxford.			
10:30				
11:00			**Tea/Coffee**, ADB-G021	
11:30			**John Donohue:** *Protection effects in predator-prey systems* ***Luca Manzari:*** *Look but don't touch!Toward non-contact measurement techniques and solutions for the chatacterization of anisotropic, viscoelastic materials*	
12:00	**Tea/Coffee**, ADB-G021	*Work on Problems*		
12:30	Group Allocation / Meet with Mentors			
13:00	**Lunch** An Bialann	**Lunch** An Bialann	**Lunch** An Bialann	**Lunch** An Bialann
13:30				
14:00	*Work on Problems*	**Tuoi Vo:** *Mathematical models of drug release from drug-eluting stents* ***Doireann O'Kiely:*** *mathematical models for glass manufacture (with asymptotics + FEniCS)*	*Work on Problems*	**Team Presentations**
14:30				
15:00				
15:30	***Tea/Coffee,*** *AdB-G021*	***Tea/Coffee,*** *AdB-G021*	***Tea/Coffee,*** *AdB-G021*	
16:00	*Work on Problems*	*Work on Problems*	*Work on Problems*	**Closing**
16:30				
17:00				
17:30				
18:30			**Networking Dinner** The Brasserie, Eglington St.	

Practicalities

Problem allocation: Presentations of the modelling problems will take place from 10:00am in AM150. After that, you will have the opportunity to indicate which problems you would like to work on; based on that, we'll allocate everyone to teams. Please make sure you complete your Problem Preference Form and return it to the organisers.

Wifi For Wifi Access, connect to `NUIGWifi`.
User ID: `9876001T`
Password: `htbps4729`

Lunch: Most participants will have received vouchers valued at €7 which can be redeemed at An Bhialann.

Networking Dinner: There will be a Workshop Dinner for participants on Thursday at 6.45 in The Brasserie, Eglington St (to be confirmed). The cost of food (but not drinks) will be covered by the organisers.

Problem 1
Shallow Water Equations for River Flows

Seshu Tirupathi, IBM Research

Shallow water equations describing unsteady open channel flows are typically used to model river flows. Conservative form of the equations are difficult to numerically formulate and apply in real test cases (Eq. 1 in [?]). On the other hand, the non conservative form of the equations are easier to formulate numerically (Eq. 3 in [?]). There have been various attempts to handle both these formulations. It is important to note that the one dimensional model decreases the computational cost significantly to model river flows to a reasonable accuracy as compared to a more comprehensive 2D resolution. The problem becomes tricky when there is confluence of river segments/channels (Fig. 1). Work has been done to handle merging of flows from different

Figure 1: Confluence of rivers, the Mosel flows into the Rhine at Koblenz (From wikipedia (By Holger Weinandt))

channels into a single channel by using a 2D formulation at the junction.

For this workshop, can you comment on some or all of the following aspects (assuming continuous flow):

1. In general, are there any disadvantages of using the non-conservative form of the equations even if the flow is continuous?

2. For flow at a junction, is it possible to come up with internal boundary conditions/constraints without the necessity to solve the 2D shallow water equations? How do the boundary conditions compare with a 1D-2D coupling at the junction? (reference: Fig. 2)

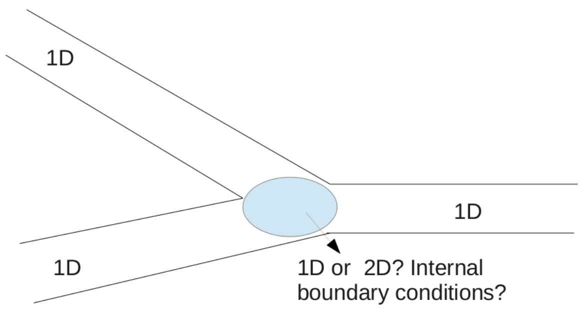


Figure 2:

3. How do the internal boundary conditions/constraints compare when the flow merges at a cross section? (reference: Fig. 3)

Problem 1

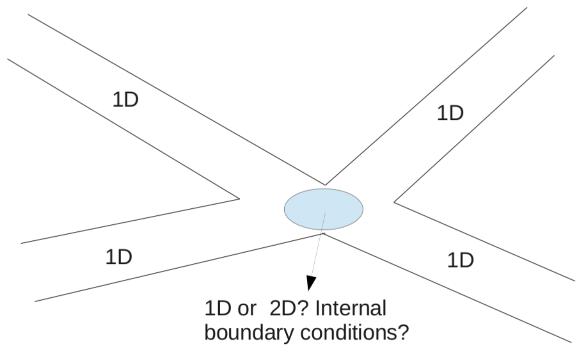


Figure 3:

4. Can you come up with a rigorous analysis for questions 1−3 assuming rectangular/trapezoidal/parabolic cross sections?

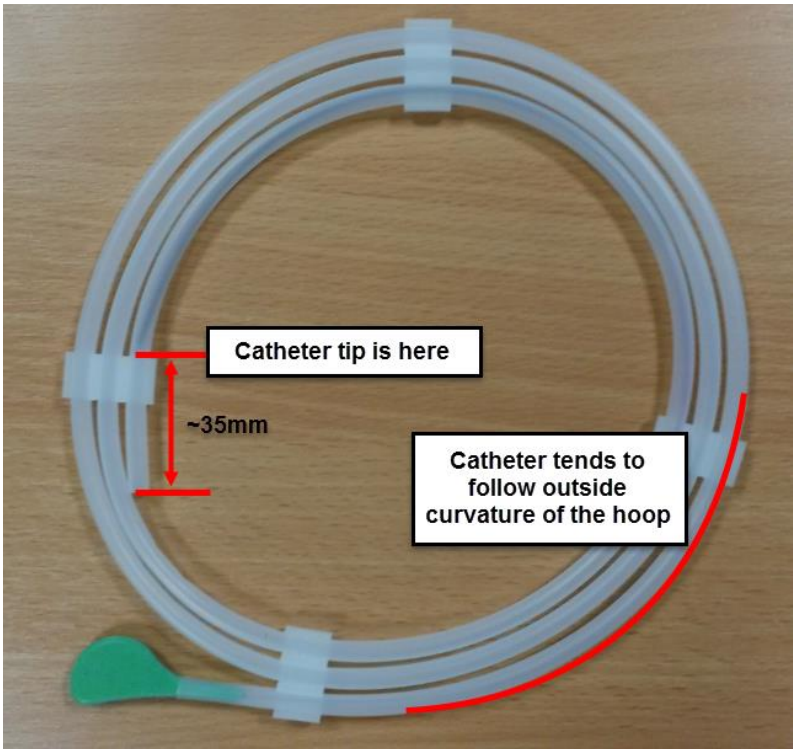
5. What other novel methods can you come up with to handle merging flows while keeping the formulation limited to 1D?

Problem 2

Feasibility of creating Mathematical Model to determine hoop dimensions

In Medtronic we manufacture catheters (long flexible tubes of small diameter) which are packaged in a protective hoop. These hoops are manufactured from long plastic extruded tubes which are curved into a spiral shape. We would like to know if it is possible to develop a mathematical model which can help us in determining the required hoop length for a given catheter length.

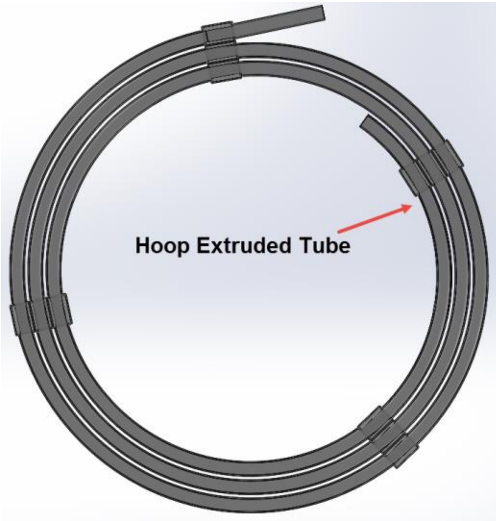
At present we specify the length of the extruded tube used to make the hoop based on the length of the catheter but since in practice the catheter does not travel down the center line of the hoop extrusion, it does not need to be quite this long. For example, in the image below, there is 1515mm of catheter within the hoop (which is 1525mm long) and the catheter tip is approximately 35mm from the end of the tubing.

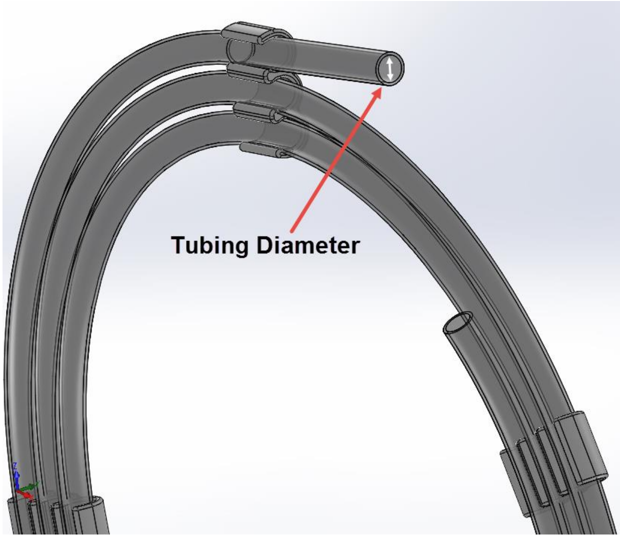


There are a couple of key dimensions which will constrain what we can do in terms of hooping product:

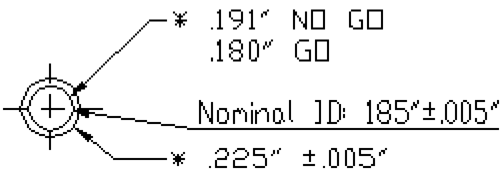
1. **The inner diameter of the plastic tubing** used to create the hoop. This needs to be big enough to accommodate the catheter but small enough to ensure that the catheter is reasonably well constrained during distribution & storage. It also impacts what is possible in terms of hoop wind OD.

Problem 2

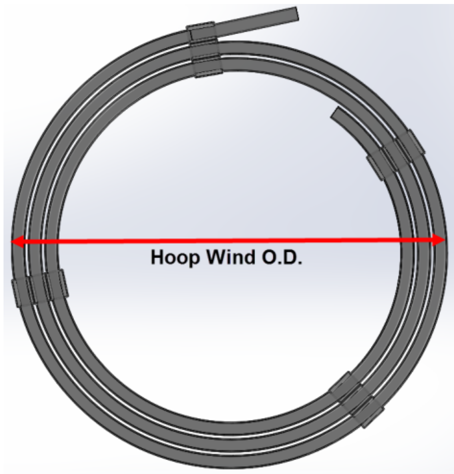




Typical tubing dimensions: I.D.: 0.185", O.D.: 0.225"



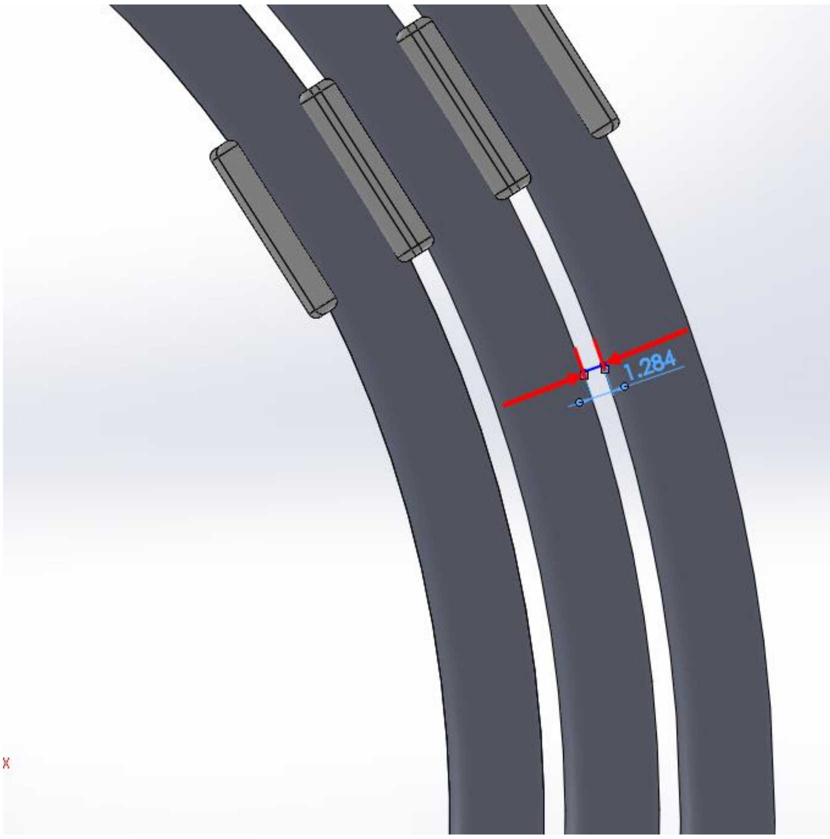
2. **The wind O.D. of the hoop** – the hoop is contained within a pouch, which is in turn contained within a cardboard carton so the wind diameter will dictate the size of the pouch and cardboard shelf carton. In order to ensure that the catheter packaging is as user friendly as possible we do not want to have too large a hoop wind O.D.



Typical hoop wind O.D.: 7.25"

Problem 2

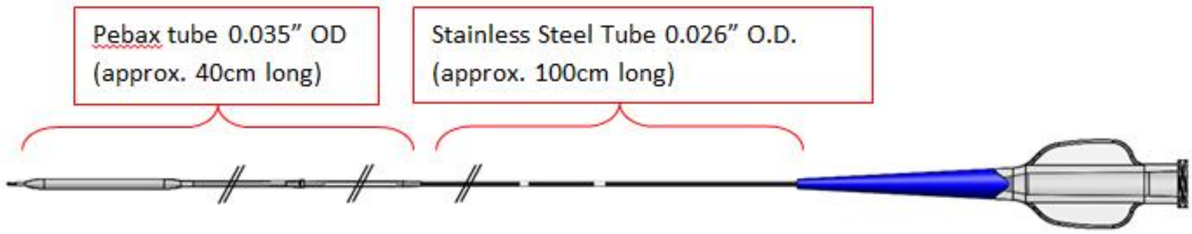
3. **The distance between each wrap of the tube extrusion** – this is dictated by the dimensions of the clips used to hold the wind in place. As you can see from below, this is typically around 1.3mm

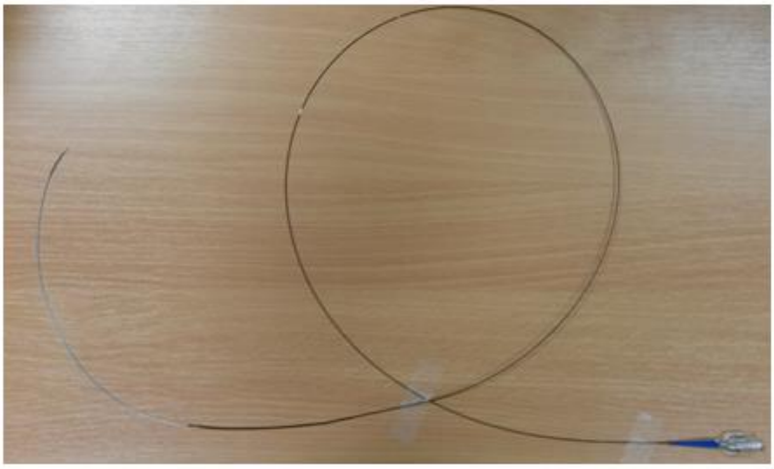


Problem 2

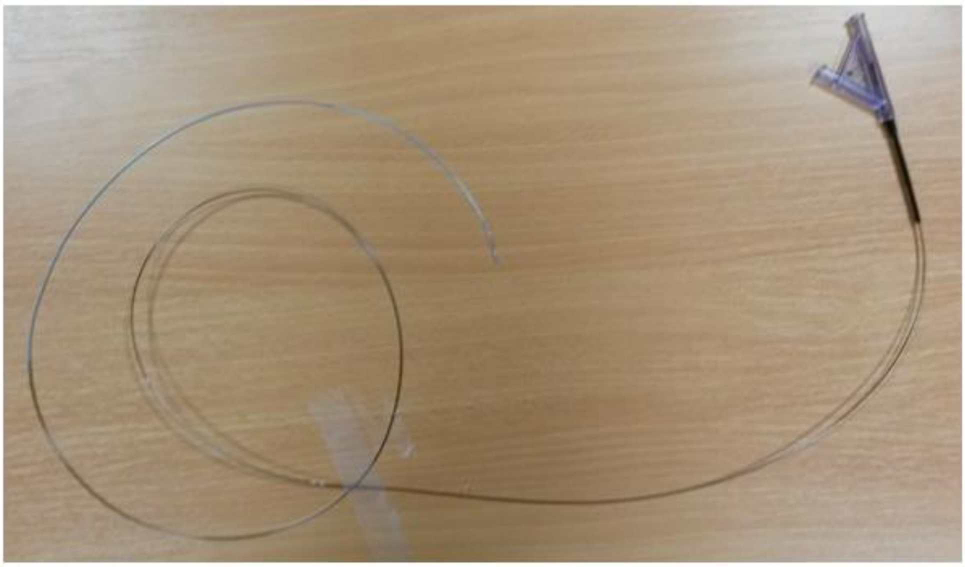
Description of catheters

Type A:



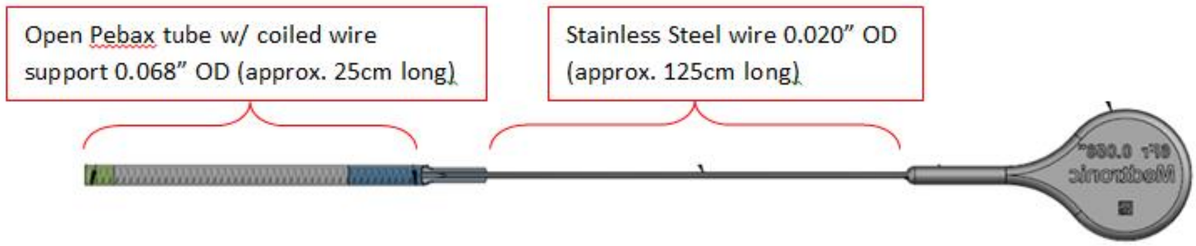


Type B:



Problem 2

Type C:



Problem 3
Mapping objects in 3D videos using a single camera

Luca Manzari – manzari@kth.se

A mandatory introduction

In the era of camera-enabled smartphones and self-driving cars, computer vision is a hot topic. Advancements in the field allowed for the use of cameras in non-contact measurement systems in place of strain gauges and accelerometers. In order to estimate depth – just like humans need two eyes! – two cameras are needed.

Stereopsis, the science of 3-dimensional perception, deals with mapping points between the real world and the cameras image planes. Other techniques take then care of the object recognition in between different images, in order to know what went where.

In many applications (e.g. microscopy, macro photography) only one camera is available, and no additional means of estimating depth can be used.

The main idea to explore

In the field of visual arts, photographers use many shallow-depth-of-field shots and stack them together to get an extended-focus image.

Figure 1: Focus bracket of a Tachinid fly. The rightmost image results from the composition of the two other images. Source: Wikipedia

What if the depth information could then be extracted by the shallow-depth-of-field shots? If the outcome of a phenomenon is deterministic, such phenomenon could be video recorded many times, each time focusing at a different plane. The in-focus parts of each image would then be detected, and the geometry of the object could be reconstructed in 3D for the whole observation period.

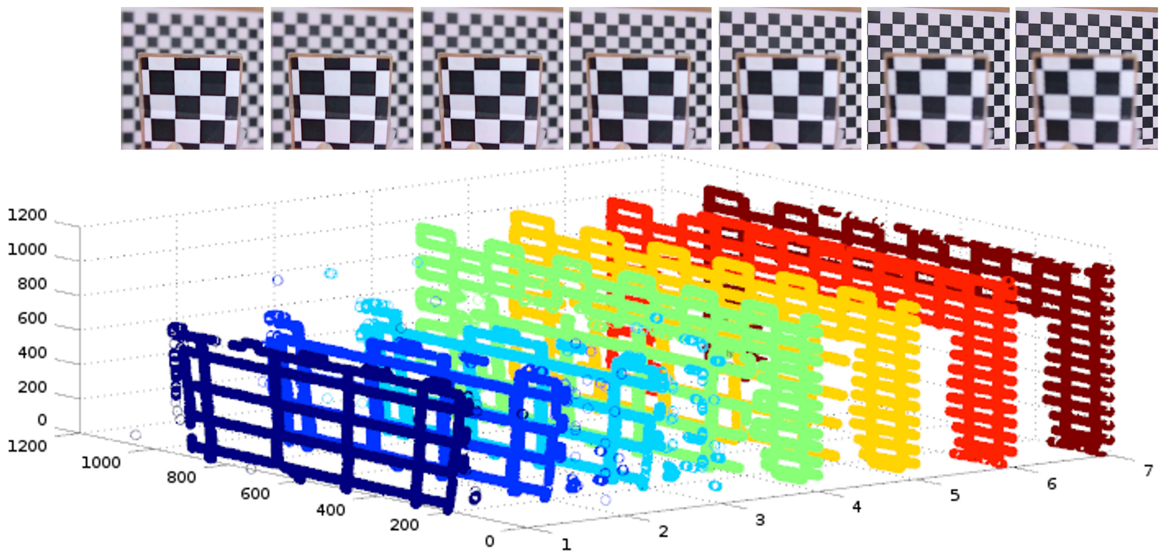


Figure 2: Same scene focused at different planes, and extraction of the points that are in focus. Own work.

Problem 4

Designing an optimal weather radar network for Ireland

Weather radar are a valuable source of information for national meteorological services. They play a crucial role in the monitoring and forecasting of rainfall and weather systems.

The weather radar work by transmitting pulses of microwave energy. The waves are scattered and reflected by precipitation such as rain, snow or hail. By measuring the returning echoes, the radar can “see” this precipitation. The radar transmitters rotate through 360 degrees on the horizontal plane, repeated over a number of elevation angles, to build up a three-dimensional picture.

Currently Met Éireann operates two radar, located at Dublin and Shannon airports.

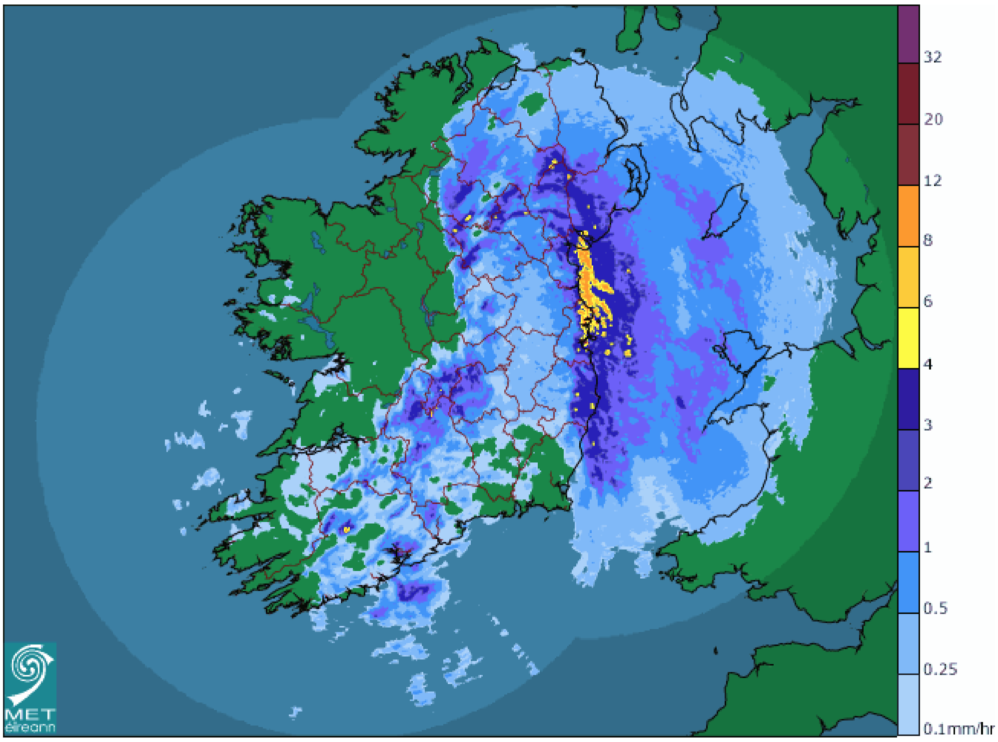


Fig.1: Radar coverage from Dublin and Shannon airports (http://www.met.ie/news/display.asp?ID=411)

While this network appears at first sight to provide adequate coverage for Ireland, there are some shortcomings. The radar often report echoes from nearby mountain ranges (such as the Wicklow or Mourne mountains), mistaking the terrain for precipitation. Blocking by mountains can also create a radar shadow on the far side, a region in which nothing is detected. In addition, distance from the radar is an issue. Measurements are accurate up to ~100km, with a rapid decline in performance beyond this, and the distance between Shannon and Dublin is more then 200km.

Ideally, therefore, we would have a network consisting of many more radar covering the whole country sufficiently, to overcome these issues and allow for loss of coverage due to breakdowns etc.

However, given the high costs involved in building and maintaining such a network, the problem is to model an optimal network for Ireland such that

1. a minimal number of radar towers are needed, and
2. their locations are chosen to maximise radar coverage

Problem 4

Some further information

- A brief introduction to radar meteorology may be found here:
  http://www.met.ie/news/display.asp?ID=411

- As mentioned above, the radar performance declines with distance from the tower, with a maximum range of 240 km. The decline is not linear, but can be better modelled with a cubic function.
  [See, for example, Section 5.2 of the following Technical Note, which also provides a useful background and introduction to radar: http://edepositireland.ie/handle/2262/70547]

- As well as distance and the position of mountain ranges, a further consideration might be to ensure reliable coverage in areas where we can expect the most rainfall; see Fig. 2 below from http://www.met.ie/climate-ireland/rainfall.asp

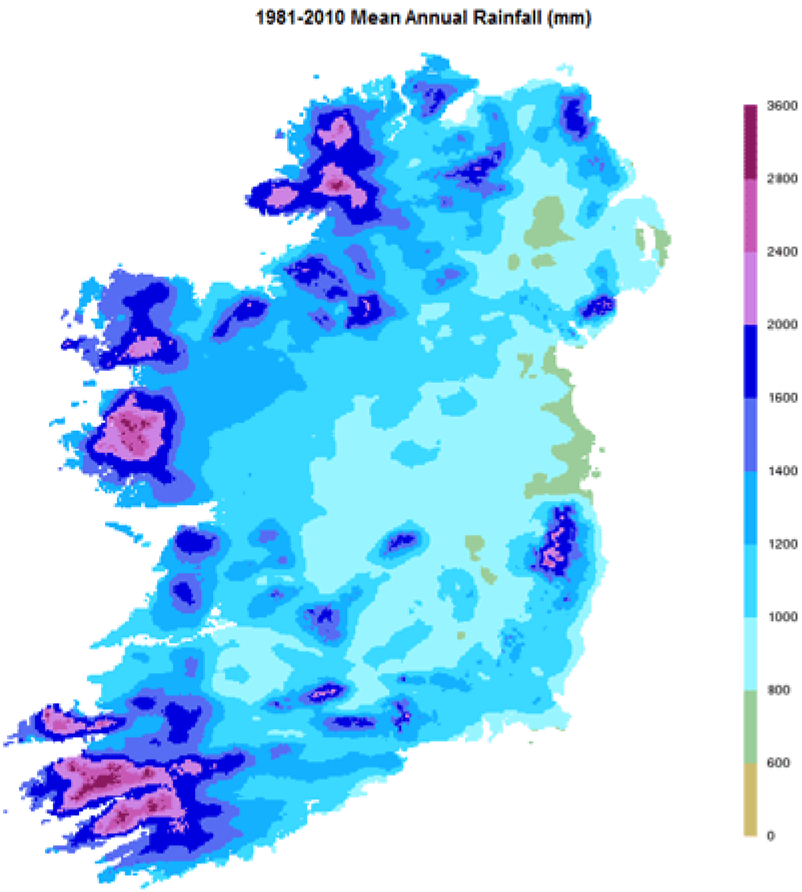


Fig. 2: Rainfall climatology of Ireland.

- An example of the denser UK network may be seen here:
  http://www.metoffice.gov.uk/binaries/content/gallery/mohippo/images/research/weather/observations/radar-systems/radar_graphic5th.jpg

Problem 5

The role of rheology in everyday fluid flow

From a mathematical modelling viewpoint, the simplest fluids have either negligible viscosity (inviscid flow) or a constant viscosity that dominates over inertial effects (Stokes flow). However, many fluids in our homes display interesting or challenging behaviour due to their non-Newtonian rheology. The viscosity of honey changes when it is heated, ketchup does not flow from the bottle unless it is shaken or tapped, and certain fluids such as cornstarch in water and shampoo are known as "shear thickening" and "shear thinning".

Mathematical models for non-Newtonian fluids must take into account their rheology. Modelling challenges could include:

- What is the optimal strategy for getting ketchup out of the bottle and onto your burger? Is a glass or plastic bottle better? How do you control the amount of ketchup delivered? What about shampoo?

- A spoon or finger can move through a cornflour–water mixture slowly with little impediment, but move too quickly and the mixture hardens, so that the spoon becomes stuck. Similarly, popular science shows sometimes include a presenter or audience participant running across the surface of a cornflour–water pool, but if the runner stops they will sink. What is the mechanism that allows for these two apparently disparate behaviours? Can a mathematical model determine the criteria for which a runner will sink into the pool or bounce of the surface?