CS4423-W04-Jupyter

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This Jupyter notebook, and PDF and HTML versions, can be found at https://www.niallmadden.ie/2425-CS4423/#Week04

This notebook was written by Niall Madden, adapted from notebooks by Angela Carnevale.

2.1 Modules for this notebook

Today, we'll default to lime-coloured nodes. For more options, see https://xkcd.com/color/rgb/

[1]:
```
import networkx as nx
import numpy as np
opts = { "with_labels": True, "node_color": 'xkcd:lime' } # show labels; lime␣
↪noodes
```

2.2 Our small affiliation network

We built an affiliation network based on data you provided. The network had two types of nodes: * People (usually referred to as *Actors*) * Programmes (here a programme is an example of a *focus*;

so these nodes are *foci*).

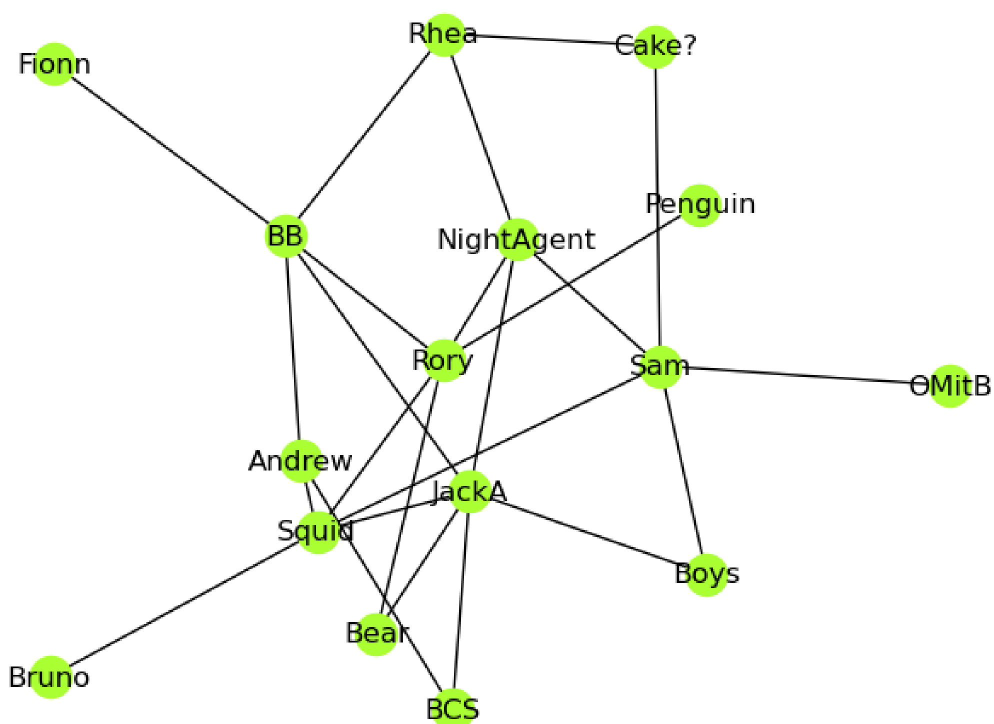
We took a subgraph with just 7 people. The data for that is in a file called `CS4423-7.txt`.

[2]:
```
!cat CS4423-7.txt
```

```
Fionn BB
Rhea BB NightAgent Cake?
Rory BB Penguin Squid Bear NightAgent
Bruno Squid
JackA BB Squid Bear Boys NightAgent BCS
Sam OMitB Squid Boys Cake? NightAgent
Andrew BB Squid BCS
```

We can easily build a network from this file, and draw it:

[3]:
```
G7 = nx.read_adjlist('CS4423-7.txt')
nx.draw(G7, **opts)
```



Let's check the graphs basic properties:

[4]:
```
print(f"G7 is {G7.order()} nodes and {G7.size()} edges")
```

```
G7 is 16 nodes and 24 edges
```

Unfortunately, this graph can be a little tricky to work with, unless we change it a little. This is not least because `networkx` does not automatically order the nodes the way we would like. In this example, here is how they are ordered:

[5]:
```
print(list(G7.nodes()))
```

```
['Fionn', 'BB', 'Rhea', 'NightAgent', 'Cake?', 'Rory', 'Penguin', 'Squid',
'Bear', 'Bruno', 'JackA', 'Boys', 'BCS', 'Sam', 'OMitB', 'Andrew']
```

We can see that Nodes 0, 2, 5, 9, 10, 13 and 15 are the "people" nodes. We could build a permutation matrix from this, but will leave that for another time.

2.3 Bipartite graphs in networkx

Since affiliation networks (and, more generally, bipartite graphs) are so important in Network Theory, `networkx` comes with various tools for working with them.

In fact, `networkx` comes with sub-module, `bipartite` for working with these graphs.

For example, it has a tool for verifying that a graph is, indeed, bipartite:

[6]:
```
print(f"G7 is bipartite: {nx.bipartite.is_bipartite(G7)}")
K33 = nx.complete_bipartite_graph(3,3)
K5 = nx.complete_graph(5)
print(f"K33 is bipartite: {nx.bipartite.is_bipartite(K33)}")
print(f"K5 is bipartite: {nx.bipartite.is_bipartite(K5)}")
```

```
G7 is bipartite: True
K33 is bipartite: True
K5 is bipartite: False
```

One of the key methods is the `sets` function, which tries to compute the parts of the graph, which it returns as a `tuple`:

[7]:
```
top, bottom = nx.bipartite.sets(G7)
print(f"Set 1: {top}")
print(f"Set 2: {bottom}")
```

```
Set 1: {'Fionn', 'Rhea', 'Sam', 'JackA', 'Rory', 'Andrew', 'Bruno'}
Set 2: {'BCS', 'BB', 'NightAgent', 'Boys', 'Cake?', 'OMitB', 'Bear', 'Penguin',
'Squid'}
```

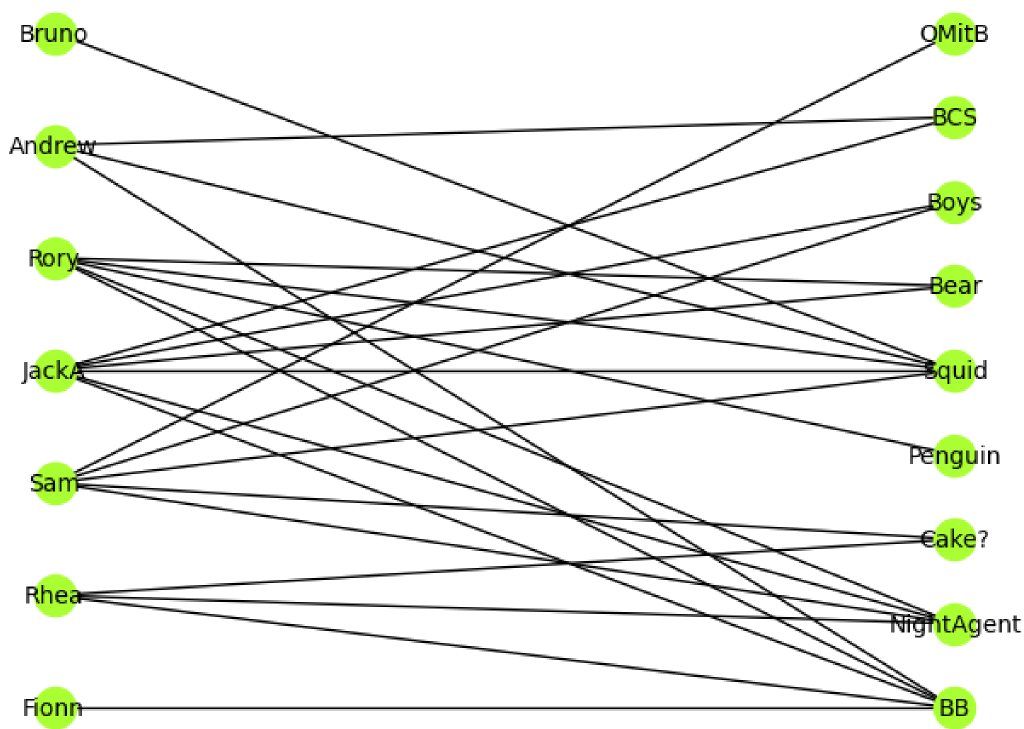
Since we can see that Set 1 represents the people, and Set 2 the programmes, let's give them suitable names:

[8]:
```
Actors = top; Foci = bottom;
```

2.3.1 Drawing

We can use this information, for example, to compute good positions for drawing the graph:

[9]:
```
positions = nx.bipartite_layout(G7, Actors) # compute the positions
nx.draw(G7, **opts, font_size=10, pos=positions)
```



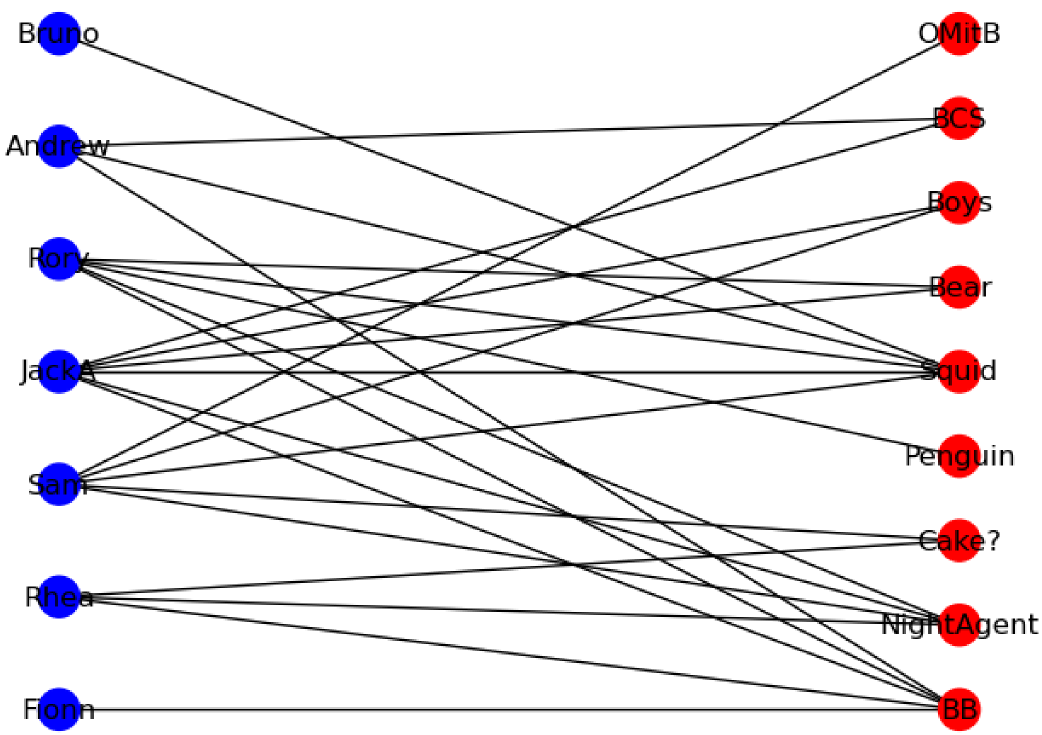
2.3.2 2-colouring

We'll use this to make the *node colouring* which is a list of colours, corresponding the the list of nodes.

[10]:
```
Nodes = list(G7.nodes())
G7_colours = ['b' if node in top else 'r' for node in Nodes]
print(G7_colours)
```

```
['b', 'r', 'b', 'r', 'r', 'b', 'r', 'r', 'r', 'b', 'b', 'r', 'r', 'b', 'r', 'b']
```

[11]:
```
nx.draw(G7, node_color=G7_colours, pos=positions, with_labels=True)
```



2.4 Adjacency Matrices

We know the adjacency matrix A of a bipartite graph G, with respect to a suitable ordering/permutation of the nodes (V_1 first, then V_2), has the form of a 2×2-block matrix,

$$A = \begin{pmatrix} 0 & C \\ C^T & 0 \end{pmatrix}$$

where the blocks on the diagonal consist entirely of zeros, as there are no edges between nodes belonging to the same part.

However, without the right ordering, we don't see this nice structure..

Let's look at the adjacency matrix for G_7:

[12]:
```
A = nx.adjacency_matrix(G7).toarray()
print(A)
```

```
[[0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
 [1 0 1 0 0 1 0 0 0 0 1 0 0 0 0 1]
 [0 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0]
 [0 0 1 0 0 1 0 0 0 0 1 0 0 1 0 0]
 [0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0]
```

```
 [0 1 0 1 0 0 1 1 1 0 0 0 0 0 0 0]
 [0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]
 [0 0 0 0 0 1 0 0 0 1 1 0 0 1 0 1]
 [0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0]
 [0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]
 [0 1 0 1 0 0 0 1 1 0 0 1 1 0 0 0]
 [0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0]
 [0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1]
 [0 0 0 1 1 0 0 1 0 0 0 1 0 0 1 0]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]
 [0 1 0 0 0 0 0 1 0 0 0 0 1 0 0 0]]
```

Here is one way to get a version of the graph with a “nice” ordering:

[13]:
```
H=nx.Graph()
H.add_nodes_from(Actors) # first add ther Actor nodes
H.add_nodes_from(Foci)   # then the Foci nodes
```

Check the order:

[14]:
```
print(H.nodes())
```

```
['Fionn', 'Rhea', 'Sam', 'JackA', 'Rory', 'Andrew', 'Bruno', 'BCS', 'BB',
'NightAgent', 'Boys', 'Cake?', 'OMitB', 'Bear', 'Penguin', 'Squid']
```

Then copy the edges from G_7

[15]:
```
H.add_edges_from(G7.edges())     # Now add the edges
```

Let’s check the adjacency matrix for H

[16]:
```
A = nx.adjacency_matrix(H).toarray()  # matrix  for H
print(A)
```

```
[[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 1]
 [0 0 0 0 0 0 0 1 1 1 1 0 0 1 0 1]
 [0 0 0 0 0 0 0 0 1 1 0 0 0 1 1 1]
 [0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 1]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]
 [0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0]
 [1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0]
 [0 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0]
 [0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0]
 [0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
 [0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0]
 [0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0]
 [0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0]
 [0 0 1 1 1 1 1 0 0 0 0 0 0 0 0 0]]
```

2.4.1 The Biadjacency Matrix

Bipartite graphs have a special matrix representation called the **Biadjacency Matrix**. If $|V_1| = r$ and $|V_2| = s$, then it is a $r \times s$ matrix, B, where $b_{ij} = 1$ if there is an edge between Node i in V_1 and Node j in V_2.

We can compute it as follows:

[17]:
```
B = nx.bipartite.biadjacency_matrix(H, Actors, Foci).toarray()
print(B)
```

```
[[0 1 0 0 0 0 0 0 0]
 [0 1 1 0 1 0 0 0 0]
 [0 0 1 1 1 1 0 0 1]
 [1 1 1 1 0 0 1 0 1]
 [0 1 1 0 0 0 1 1 1]
 [1 1 0 0 0 0 0 0 1]
 [0 0 0 0 0 0 0 0 1]]
```

2.5 Projections

We learned just a while ago that the adjacency matrix of the projection of a bipartite graph G is related the top-left, or bottom-right, non-zero block of A^2

[18]:
```
print(A@A)
```

```
[[1 1 0 1 1 1 0 0 0 0 0 0 0 0 0 0]
 [1 3 2 2 2 1 0 0 0 0 0 0 0 0 0 0]
 [0 2 5 3 2 1 1 0 0 0 0 0 0 0 0 0]
 [1 2 3 6 4 3 1 0 0 0 0 0 0 0 0 0]
 [1 2 2 4 5 2 1 0 0 0 0 0 0 0 0 0]
 [1 1 1 3 2 3 1 0 0 0 0 0 0 0 0 0]
 [0 0 1 1 1 1 1 0 0 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 2 2 1 1 0 0 1 0 2]
 [0 0 0 0 0 0 0 2 5 3 1 1 0 2 1 3]
 [0 0 0 0 0 0 0 1 3 4 2 2 1 2 1 3]
 [0 0 0 0 0 0 0 1 1 2 2 1 1 1 0 2]
 [0 0 0 0 0 0 0 0 1 2 1 2 1 0 0 1]
 [0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 1]
 [0 0 0 0 0 0 0 1 2 2 1 0 0 2 1 2]
 [0 0 0 0 0 0 0 0 1 1 0 0 0 1 1 1]
 [0 0 0 0 0 0 0 2 3 3 2 1 1 2 1 5]]
```

However, we can also get these blocks from the Biadjacency matrix, B. The top left is BB^T, and bottom right is B^TB:

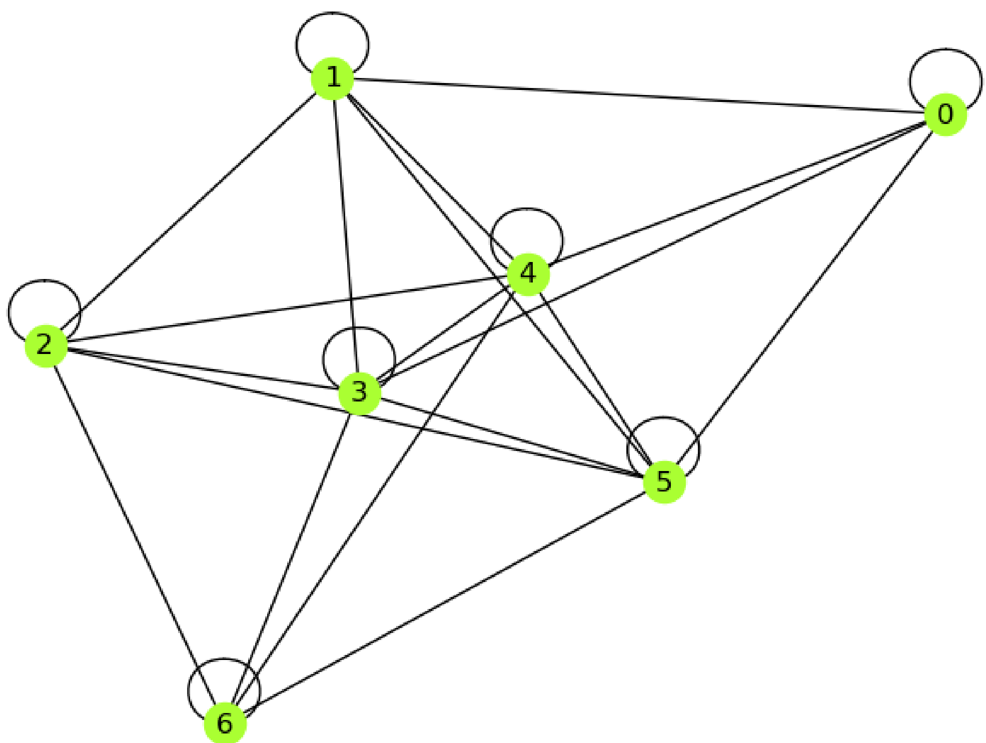
[19]:
```
C = B@B.T
print(C)
```

```
[[1 1 0 1 1 1 0]
```

```
 [1 3 2 2 2 1 0]
 [0 2 5 3 2 1 1]
 [1 2 3 6 4 3 1]
 [1 2 2 4 5 2 1]
 [1 1 1 3 2 3 1]
 [0 0 1 1 1 1 1]]
```

However, this is not an adjacency matrix of a (simple) graph:

[20]:
```
G7_1 = nx.from_numpy_array(C)
nx.draw(G7_1, **opts)
```



But we can convert it to one:

[21]:
```
C[C>0]=1 # set everything to 0 or 1
print(C)
```

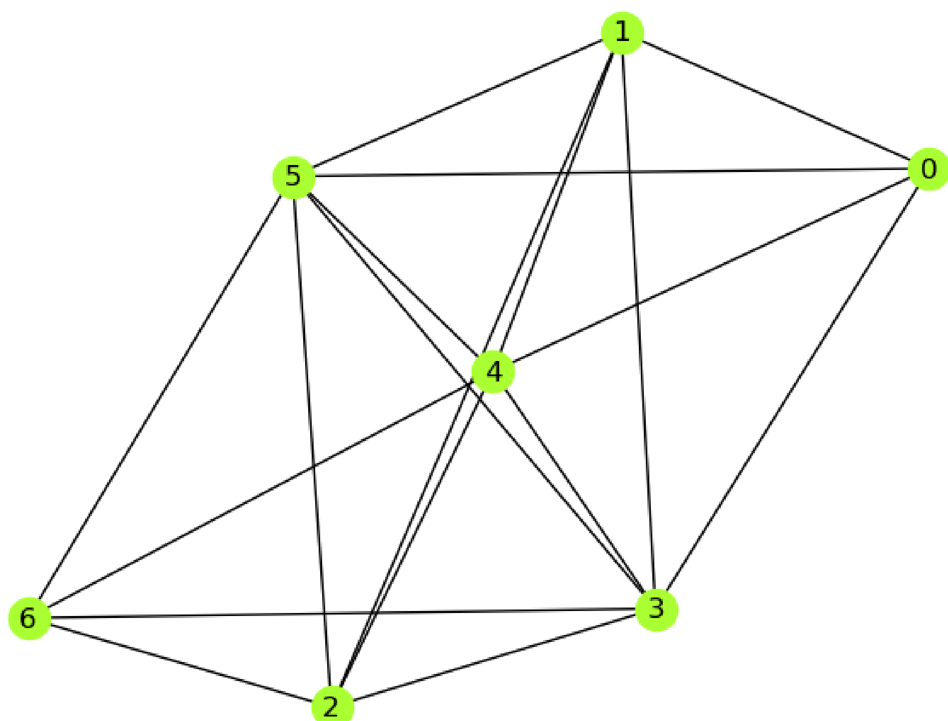
```
[[1 1 0 1 1 1 0]
 [1 1 1 1 1 1 0]
 [0 1 1 1 1 1 1]
 [1 1 1 1 1 1 1]
 [1 1 1 1 1 1 1]
```

```
 [1 1 1 1 1 1 1]
 [0 0 1 1 1 1 1]]
```

[22]:
```
np.fill_diagonal(C,0)
print(C)
```

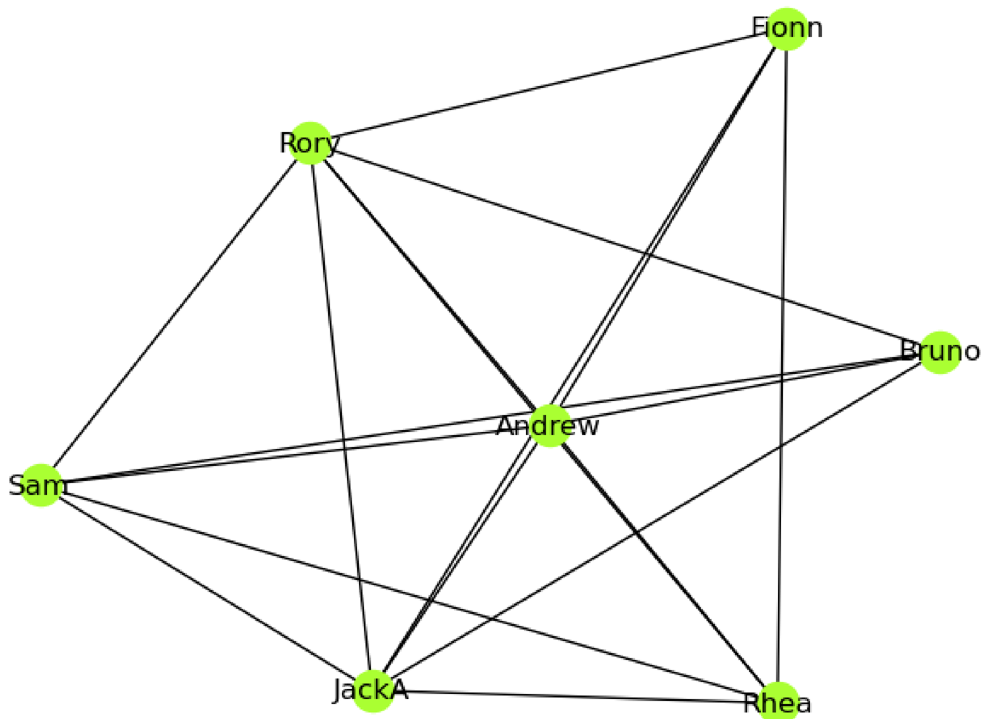
```
[[0 1 0 1 1 1 0]
 [1 0 1 1 1 1 0]
 [0 1 0 1 1 1 1]
 [1 1 1 0 1 1 1]
 [1 1 1 1 0 1 1]
 [1 1 1 1 1 0 1]
 [0 0 1 1 1 1 0]]
```

[23]:
```
G7_2 = nx.from_numpy_array(C)
nx.draw(G7_2, **opts)
```



r we could have used the `networkx` function `projected_graph` (taking input a bipartite graph and one of the two sets of vertices) does this for us:

[24]:
```
XX = nx.projected_graph(H, Actors)
nx.draw(XX, **opts)
```



Finished here Thursday