# Linear Algebra I - Assignment 4 (make-up version)

Q1 [25 Marks] For each of the following sets, $H_1, H_2, \dots H_6$, state whether or not they are subspaces of $M_{3\times 2}$, the space of $3\times 2$ matrices with real entries. If not, explain why.

(a) $H_1 = \left\{ \begin{bmatrix} x_1 & x_2 \\ 0 & x_3 \end{bmatrix} : x_1, x_2, x_3 \in \mathbb{R} \right\}$

(b) $H_2 = \left\{ \begin{bmatrix} x_1 & x_2 \\ 0 & 0 \\ 0 & x_3 \end{bmatrix} : x_1 = x_2 = x_3 = 0 \right\}$

(c) $H_3 = \left\{ \begin{bmatrix} x_1 & 0 \\ 0 & 0 \\ 0 & x_2 \end{bmatrix} : x_1, x_2 \in \mathbb{R} \right\}$

(d) $H_4 = \left\{ \begin{bmatrix} x_1 & 0 \\ 0 & 0 \\ 0 & x_2 \end{bmatrix} : x_1, x_2 \in \mathbb{R}, x_1 + x_2 = 0 \right\}$

(e) $H_5 = \left\{ \begin{bmatrix} x_1 & x_2 \\ 0 & x_3 \\ 0 & 0 \end{bmatrix} : x_1, x_2, x_3 \in \mathbb{R}, x_1 x_2 = 0 \right\}$

(f) $H_6 = \mathrm{Span} \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \right\}$

*Tip: in Week 2 we saw that, in order to verify that $H$ is a subspace of a real vector space $V$, we have to check:*

- That every element of $H$ is also an element of $V$;
- That the zero vector in $V$ is also in $H$;
- If $u, v \in H$ then $u + v \in H$.
- If $u \in H$ then $cu \in H$ for any scalar $c \in \mathbb{R}$.

Q2 [20 Marks] Let

$$A = \begin{bmatrix} 2 & 2 & -3 \\ 4 & -6 & -1 \\ 2 & -2 & -1 \end{bmatrix} \quad \text{and} \quad x = \begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix}.$$

Determine, with justification, if $x \in \mathrm{Nul}\, A$, and if $x \in \mathrm{Col}\, A$.

Q3 [20 Marks] Find the dimension of the subspace

$$H = \left\{ \begin{bmatrix} 4p - 2q \\ 2p - q \\ q + 2r \\ p + r \end{bmatrix} : p, q, r \in \mathbb{R} \right\}$$

of $\mathbb{R}^4$ and give a basis for it.

Q4 [20 Marks]

(a) What is the largest possible rank of an $6 \times 6$ matrix?

(b) What is the smallest possible rank of an $6 \times 6$ matrix?

(c) If the null space of a $6 \times 8$ matrix $A$ is 1-dimensional, what are the dimensions of its column space, of its row space, and of its left null space?

(d) Give an example of a $4 \times 4$ matrix $A$ of rank 2.

(e) Give an example of a matrix that has $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ in its null space, and $y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in its column space.

[15 Marks] for clarity and correctness of exposition and presentation.