Annotated slides from Tuesday

MA313 : Linear Algebra 1 (“Linear Algebra for Data Science”)

Week 1: Introduction to MA313 and to Vector Spaces

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6 and 9 September, 2022

Image taken from the Burren College of Art Logo

Slides created by Niall Madden, with some content by Tobias Rossmann. If you reuse them, please do so with credit.

This module is taken by about 20 students in

- 3rd Arts: 3BA1 and 3CMS1
- 4th Arts: 4BCS1, 4BDA1, 4BMU1
- 3rd Science: 3BS9
- Visiting student (perhaps).
- Anyone else?

This group has different backgrounds. So please complete this form to help me understand: `https://forms.office.com/r/Me6nmgBk5R` While I'll try to take that into account, please let me know if I am incorrectly assuming your prior knowledge.

Updates:

- Almost everyone knows R! So I'll sprinkle a little through the semester.
- People want videos. So people get (old) videos.

Tutorials will start in Week 3. When:
https://forms.office.com/r/0ya9Bp8qBU

	Mon	Tue	Wed	Thu	Fri
9 – 10					
10 – 11					
11 – 12			???		???
12 – 1					Lecture
1 – 2		Lecture			
2 – 3					
3 – 4					
4 – 5					

So far, least worst times are:

- Monday at 13.00 (7/11)
- Tuesday at 15.00 (7/11)
- Wednesday at 11.00 (8/11) or 14.00 (9/11)
- Thursday at 12.00 (8/11)
- Friday at 11.00 (9/11)

Part 2: Mathematical Preliminaries

- How matrix-vector multiplication works: Examples (including identifying rows and columns):

Eg
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix} = x \begin{bmatrix} a \\ c \end{bmatrix} + y \begin{bmatrix} b \\ d \end{bmatrix}$$

$$\underset{A}{\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax+by+cz \\ dx+ey+fz \\ gx+hy+iz \end{bmatrix} = x \begin{bmatrix} a \\ d \\ g \end{bmatrix} + y \begin{bmatrix} b \\ e \\ h \end{bmatrix} + z \begin{bmatrix} c \\ f \\ i \end{bmatrix}$$

row 2 of A is $\begin{bmatrix} d & e & f \end{bmatrix}$, column 3 of A is $\begin{bmatrix} c \\ f \\ i \end{bmatrix}$

(Note: can think of these as vectors].

Part 2: Mathematical Preliminaries

- How matrix-vector multiplication works: Examples (including identifying rows and columns):

$$\underbrace{\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + iz \end{bmatrix} = x \begin{bmatrix} a \\ d \\ g \end{bmatrix} + y \begin{bmatrix} b \\ e \\ h \end{bmatrix} + z \begin{bmatrix} c \\ f \\ i \end{bmatrix}$$

We'll often split a matrix into colums, here denoted $a_1 = \begin{bmatrix} a \\ d \\ g \end{bmatrix}$ $a_2 = \begin{bmatrix} b \\ e \\ h \end{bmatrix}$ $a_3 = \begin{bmatrix} c \\ f \\ i \end{bmatrix}$.

So $A = \left[a_1 \mid a_2 \mid a_3 \right]$.

Also, a_{ij} is the entry in row i, col j.

Eg $a_{12} = b$.

Part 2: Mathematical Preliminaries

Exercises

1. Read Sections 2.1, 2.2 and 2.3 of the text-book.
2. Verify that you can access the homework system by trying the "Demo" assignment, which also checking your matrix algebra skills. Link is on `Blackboard/Assignments`.
3. Complete the survey at https://forms.office.com/r/Me6nmgBk5R by 5pm Thursday, 8 September 2022.

Part 2: Mathematical Preliminaries

There are some other concepts relating to vectors and matrices that you should know from previous courses:

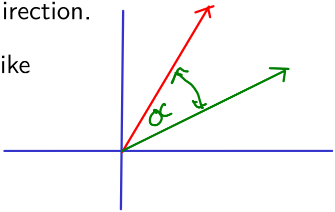
- determinants of (square) matrices
- eigenvalues of matrices, and corresponding eigenvectors;
- transpose of a vector or matrix; symmetric matrix.
- solving linear systems of equations with Gaussian Elimination; row reduction (to row reduced echelon form).

MA313
Week 1: Introduction to MA313 and to Vector Spaces

Start of ...

PART 3: The big idea

When we first study *vectors*, we are taught to think of them as points in space, or as lines of a particular length and direction.



Then we can think of operations on vectors like

- adding vectors
- changing the length of a vector
- rotating a vector
- calculating the angle between two vectors.

The intuition that we gain from this is very valuable, but also limiting.

This entire module is based around the idea of the **abstract** definition of a vector space.

This concept of **abstraction** is central to modern mathematics. The idea is to strip away parts of the concept that just come from our intuition, so that we can see the real essence of the object.

Image taken from the Burren College of Art Logo

We'll do this with the idea of a vector: try to distil what makes a vector a vector in Euclidean space, so that we can look at lots of other examples.

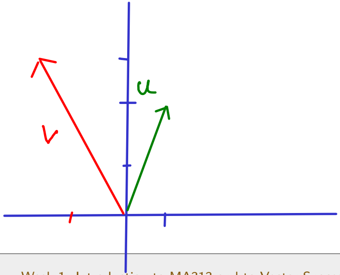
Suppose we have three vectors in $\mathbb{R}^2$, called u, v and w.

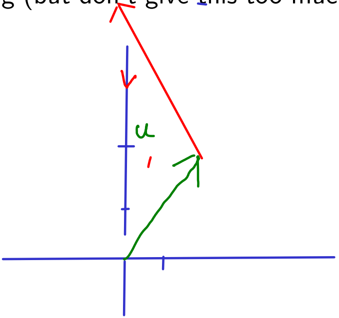
$u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $v = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ $w = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

- We can add any pair of them (in any order):

$u + v = \begin{bmatrix} 0 \\ 5 \end{bmatrix} = v + u$

- Adding them has a geometric meaning (but don't give this too much importance):





- Add all three of them (in any order):

$$u + v + w = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

- Multiply any one of them by a scalar (i.e., something that is "just a number", and not a vector).

$$4u = 4\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

- Multiplying by zero is particularly important.

$$0\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$ the zero vector.

Apart from the geometry bit, everything we've said about vectors in $\mathbb{R}^2$ is true in $\mathbb{R}^3$ or $\mathbb{R}^4$ or $\mathbb{R}^{2021}$, or $\mathbb{R}^n$.

Example: $\mathbb{R}^4$ includes the vectors

$$u = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \qquad v = \begin{bmatrix} 1.5 \\ -7 \\ 3 \\ 4.5 \end{bmatrix}, \quad u + v = \begin{bmatrix} 2.5 \\ -5 \\ 6 \\ 8.5 \end{bmatrix}$$

"componentwise addition".

$$(\tfrac{1}{2})u = \begin{bmatrix} 1/2 \\ 1 \\ 3/2 \\ 2 \end{bmatrix}$$

"Scaler multiplication".

You will also have met vectors of complex numbers. E.g., $\mathbb{C}^2$ and $\mathbb{C}^3$.

Eg

$$v = \begin{bmatrix} 3+2i \\ 3-2i \end{bmatrix} \in \mathbb{C}^2$$

is an element of.

Here $i = \sqrt{-1}$

But there are lots of other collection of things that seem have obey the same rules, about how we can add them and multiply them by scalars, etc. Matrices are an obvious example, but my favourite is a collection of polynomials of degree at most n, when we'll call $\mathbb{P}_n$.

Example

The set $\mathbb{P}^3$, set of all cubic polynomials.

Eg $1 + 2x + 3x^2 + 4x^3 \in \mathbb{P}^3$.

So too are $-\frac{1}{2}x + 53x^3 \in \mathbb{P}^3$

and even. $2x + x^2$.

And $x^3 \in \mathbb{P}^3$

But not, say $2x^3 + x^4$ or $x^{-1} + x^3$.

THE BIG IDEA

So, our plan is to identify first the properties that are common to all examples so far, and then the properties that are common to **collections** of these vectors.

Finished here Friday.