MA211 – **Problem Set 3**

Q13.1 Find general solutions to the following differential equations:

(a) $y'' + y' - 2y = 1$.
(b) $y'' - 6y' + 9y = x$.
(c) $y'' - 2y' = x^2 + 4$.
(d) $y'' = 4x^3$.

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Q14.1 Find general solutions to the following non-homogeneous differential equations:

(a) $y'' + y' - 2y = e^{-x}$.
(b) $y'' + y' - 2y = 3e^{x}$.
(c) $y'' + 5y' + 6y = 4e^{-2x}$.
(d) $-3y'' + 3y' - y = \frac{1}{2}e^{-x/2}$.

Q14.2 Suppose the solution to $ah'' + bh' + ch = 0$, where $D = b^2 - 4ac > 0$, so h is of the form $h = Ae^{R_1 x} + Be^{R_2 x}$.

Show that, if u is a *particular* solution to $au'' + bu' + cu = Ke^{R_1 x}$, then $u = \dfrac{K}{\sqrt{D}} x e^{R_1 x}$.

Q14.3 Find general solutions to the following differential equations:

(a) $y'' - y = \cos(x)$.
(b) $y'' + y' - 2y = 5\sin(-2x)$.

Q14.4 Here is an alternative approach to solving problems of the type in Q6. Recall the Euler Formula: $e^{ix} = \cos(x) + i\sin(x)$ where $i = \sqrt{-1}$. If the right-hand side of the equation involves a cos or a sin, choose the particular solution to be of the form $u = Ae^{ix}$, solve for A, and take the real or imaginary parts of u as appropriate.

Example: *Find the particular solution to* $u'' + u' - 2u = 5\sin(-2x)$.
Solution: Assume $u = Ae^{-2ix}$.
So $u' = -2iAe^{-2ix}$ and $u'' = -4Ae^{-2ix}$.
Substituting back into the DE we get

$$-4Ae^{-2ix} - 2iAe^{-2ix} - 2e^{-2ix} = Ae^{-2ix}.$$

This gives $A = \frac{5}{-6-2i} = -\frac{3}{4} + \frac{1}{4}i$. Thus $u = Ae^{-2x} = \big(-\frac{3}{4} + \frac{1}{4}i\big)\big(\cos(2x) + i\sin(2x)\big)$. Since $\sin(2x)$ is the imaginary part of e^{2ix}, the particular solution we are looking for is the imaginary part of Ae^{-2x}, that is:

$$\frac{1}{4}\cos(-2x) - \frac{3}{4}\sin(-2x) = \frac{1}{4}\cos(2x) + \frac{3}{4}\sin(2x)$$

Use this approach to solve $y'' - y = \cos(x)$.

Q14.5 Find general solutions to the following differential equations:

(a) $y'' + 4y' + y = e^{x} + \cos(x)$.
(b) $y'' + y' - 2y = 2 + 2\sin(x)$.

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Q15.1 Find general solutions to the following differential equations:

(a) $y'' - 2y' + y = x + 1 + \sin(x)$
(b) $y'' - 4y = x\sin(2x)$.
(c) $y'' + 4y = 5xe^{-x}$.

HOMEWORK

1. Solve the initial value problem:

$$y'' + 4y' + 5y = 0; \quad y(0) = 0,\ y'(0) = 1.$$

2. Find general solution to the following differential equations:

(i) $y'' - 6y' + 9y = 3x^2$.
(ii) $2y'' + 5y' - 3y = e^{x} + x$.

Submit your solutions to Questions **1** and **2** no later than noon, **Friday 7$^{\text{th}}$ of Nov**.

Solutions should be *carefully written*. If they are one more than one page, then the pages should be stappled together.