MA385/MA530: Class Test
Thursday, 24 October 2019

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1. Write out the Taylor Polynomial of degree 4, about $a = 0$, for $f(x) = 1/(1 + x)$.

   Give an expression for the remainder.
   Give an upper bound for the remainder when $x = 1$, and when $x = 0.1$. [25 Marks]

2. State *Newton's method* for solving the nonlinear equation $f(x) = 0$. [10 Marks]

   Use a Taylor's series to show that if $f(\tau) = 0$, and Newton's method generates a sequence of approximations $\{x_0, x_1, \dots\}$, then

$$\tau - x_{k+1} = -\frac{1}{2}(\tau - x_k)^2 \frac{f''(\eta_k)}{f'(x_k)}, \qquad \text{for some } \eta_k \in [x_k, \tau]. \tag{1}$$

   [30 Marks]

3. Suppose we wish to find a solution to

$$x + \frac{1}{x+1} = 2,$$

   in the interval $[1, 2]$.

   (a) Show that a solution to this problem exists. [10 Marks]

   (b) Say we take $x_0 = 1$. Use the Newton Error formula to give an upper bound for the error $|\tau - x_1|$ [25 Marks]