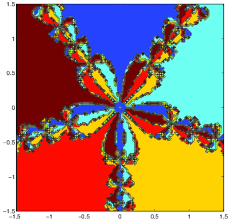
§0. Introduction to MA385; Taylor's Theorem

§0.1 Introduction

MA385/MA530 – Numerical Analysis 1

September 2019



This is a Semester 1, upper level module on ***numerical analysis***.

You may be taking this course if you are studying Mathematics, Applied Mathematics, Mathematical Science, Computer Science, or are a visiting student.

A small number of you are taking this as "MA530", but we'll usually just go by the MA385 code.

The basic information for the course is as follows:

Lecturer: Dr Niall Madden (he/him),
School of Mathematics, Statistics, and Applied Mathematics, NUI Galway.
My office is in room ADB-1013, Arás de Brún
Email: Niall.Madden@NUIGalway.ie

Lectures: 9am **Monday** and 3pm **Thursday** in AC201.

Tutorial/Lab: To begin during Week 3. **Homework:** To determine the times, you'll need to share your schedule with me...

Assessment

- Written Homework Assignment 1 (10%)
- In-class test in Week 6 [tbc] (10%)
- Written Homework Assignment 2 (10%)
- 3 Computer labs (10%)
- A 2-hour exam in December (60%).

Text books

Main text: **Süli and Mayers, *An Introduction to Numerical Analysis***. Available from the library at 519.4 MAY, and the college book-shop. The scope of the book is almost perfect for the course, especially for those students taking both semesters. ***You should buy this book.*** Other useful books include

- G.W. Stewart, ***Afternotes on Numerical Analysis***. *The full text is freely available online to NUI Galway users!* Not as formal as Süli and Mayers.
- Cleve Moler, ***Numerical Computing with MATLAB***. The emphasis is on the implementation of algorithms in Matlab, but the techniques are well explained and there are some nice exercises. Also, it is freely available online.
- James F Epperson, ***An Introduction to Numerical Methods and Analysis***. There are five copies in the library at 519.4.
- Stoer and Bulirsch,*Introduction to Numerical Analysis*.

Web site

The on-line content for the course will be hosted at NUIGalway.BlackBoard.com and at http://www.maths.nuigalway.ie/MA385. There you'll find various pieces of information, including these notes/slides, problem sets, announcements, etc.

If you are registered for MA385, you should be automatically enrolled onto the blackboard site. If you are enrolled in MA530, please send an email to me.

These notes are a synopsis of the course material. My aim is to provide these in 4 sections, and always in advance of the class. They contain most of the main remarks, statements of theorems, results and **exercises**. However, they will not contain proofs of theorems, examples, solutions to exercises, etc.

You should try to bring these notes to class.
It will make it easier to follow the lecture.

Please help!

These notes contain numerous typos and errors. None are deliberate. Please help me, and the rest of the class, by pointing them out so I can fix them.

Numerical analysis is the

(a) design,
(b) analysis and
(c) implementation of

numerical methods that yield ***exact or approximate*** solutions to mathematical problems.

It does not involve long, tedious calculations. We won't (usually) implement Newton's Method by hand, or manually do the arithmetic of Gaussian Elimination, etc.

. . . .

If you would like a more thorough discussion on the nature and history of numerical analysis, read Nick Trefethen's essay on Numerical Analysis from the Princeton Companion to Mathematics, 2008.

(a) ***Design*** of a numerical method is perhaps the most interesting; it's often about finding a clever way of swapping the problem for one that is easier to solve, but has the same or similar solution. If the two problems have the same solution, then the method is ***exact***. If they are similar (but not the same), then it is ***approximate***.

(b) ***Analysis*** is the mathematical part; it usually culminates in proving a theorem that tells us (at least) one of the following.

- ***The method will work***: that our algorithm will yield the solution we are looking for;
- ***How much effort is required***
- If the method is approximate, determine ***how close the approximate solution is to the real one***. A description of this aspect of the course, to quote Epperson, is being "*rigorously imprecise* or *approximately precise*".

(c) The ***implementation*** is generating solutions with the algorithms, that is, it is the programming part. We'll study this in labs.

0. We'll start with a review of Taylor's theorem. It is central to the algorithms of the following sections.
1. Root-finding and solving non-linear equations.
2. Initial value ordinary differential equations.
3. Matrix Algorithms: solving systems of linear equations, and estimating eigenvalues and eigenvectors.

We also see how these methods can be applied to so-called "real world" problems, including ones in Financial Mathematics.

When you have successfully completed this course, you will be able to demonstrate your factual knowledge of the core topics

- root-finding,
- solving ODEs,
- solving linear systems of equations and
- estimating eigenvalues of matrices,

using appropriate mathematical syntax and terminology.

Moreover, you will be able to describe the fundamental principles of the concepts (e.g., Taylor's Theorem) underpinning Numerical Analysis. Then, you will apply these principles to design algorithms for solving mathematical problems, and discover the properties of these algorithms.

You will learn how to use Matlab to implement these algorithms, and adapt the codes for more general problems, and for different techniques.

Anyone who can remember their first and second years of analysis and algebra should be well-prepared for this module. Students who know a little about initial value differential equations will find Section 2 somewhat easier than those who haven't.

If it has been a while since you studied basic calculus, you will find it very helpful to revise the following:

- the Intermediate Value Theorem;
- Rolle's Theorem and the The Mean Value Theorem;
- **Taylor's Theorem**,
- and the triangle inequality: $|a + b| \leq |a| + |b|$.

You'll find them in any good text-book, e.g., Appendix 1 of Süli and Mayers.

You'll also find it helpful to recall some basic linear algebra, particularly relating to eigenvalues and eigenvectors. Consider the statement:

> *"all the eigenvalues of a real symmetric matrix are real".*

If are unsure what the meaning of any of the terms used, or if you didn't know that it's true, you should have a look at a book on Linear Algebra.

Why take this course?

Many industry and academic environments require graduates who can solve real-world problems using a mathematical model, but these models can often only be resolved using numerical methods. To quote one Financial Engineer: "We prefer approximate (numerical) solutions to exact models rather than exact solutions to simplified models".

Another expert, who leads a group in fund management with DB London, when asked "what sort of graduates would you hire", the list of specific skills included

- A programming language and a 4th-generation language such as MATLAB (or R).
- Numerical Analysis.