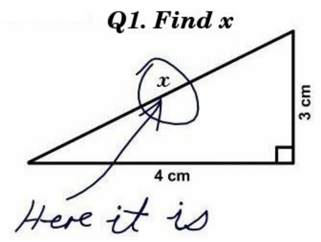
MA211
Lecture 12: Class Test
Wed 16 October 2008



Q1. Use $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$ and $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$ to show that
$$\cosh^2 x - \sinh^2 x = 1.$$

Q2. Write down the general solution to the following differential equations:

(i) $25y'' - 20y' + 4y = 0.$

(ii) $y'' + y' - 12y = 0$

Q3. Find values of b and c such that $y(x) = \cosh(2x)$ is a solution to the differential equation:
$$y'' + by' + cy = 0.$$

Solutions

Q1

Use $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$ and $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$ to show that
$$\cosh^2 x - \sinh^2 x = 1.$$

$\cosh^2(x) = \frac{1}{4}(e^x + e^{-x})(e^x + e^{-x}) = \frac{1}{4}(e^{2x} + 2 + e^{-2x})$
$\sinh^2(x) = \frac{1}{4}(e^x - e^{-x})(e^x - e^{-x}) = \frac{1}{4}(e^{2x} - 2 + e^{-2x})$

So $\cosh^2(x) - \sinh^2(x) =$
$\frac{1}{4}\left[e^{2x} + 2 + e^{-2x} - e^{-2x} + 2 - e^{-2x}\right] = 1$

Solutions

Q2 (i)

Write down the general solution to the following differential equation: $25y'' - 20y' + 4y = 0.$

The auxilliary equation is $25R^2 - 20R + 4 = 0$
It's solutions are $R = \dfrac{20 \pm \sqrt{400-400}}{50} = 2/5$

Since there is only one root,

$y(x) = Ae^{2x/5} + Bxe^{2x/5}$

Solutions

Q2 (ii)

Write down the general solution to the following differential equation: $y'' + y' - 12y = 0$

The aux equation is $R^2 + R - 12 = 0$
The discriminant $D = b^2 - 4ac = 1 + 48 = 49 > 0$.
So there are 2 distinct real roots
$R_1 = \dfrac{-b + \sqrt{b^2-4ac}}{2a} = \dfrac{-1+7}{2} = 3,\quad R_2 = \dfrac{-1-7}{2} = -4$

So
$y(x) = Ae^{3x} + Be^{-4x}$

(Note, the left hand side of the Auxillary Equation is easily factorised, so $(R-3)(R+4) = 0$)

Solutions

Q3

Find values of b and c such that $y(x) = \cosh(2x)$ is a solution to the differential equation:
$$y'' + by' + cy = 0.$$

$y = \cosh(2x)$ so $y'(x) = 2\sinh(2x)$
and $y''(x) = 4\cosh(2x)$.
Substitute into the DE to get
$4\cosh(2x) + 2b\sinh(2x) + c\cosh(2x) = 0$
This gives $b = 0$ and $c = -4$.