CS4423: Class Test (6 March 2025) ANS with solutions

- You have 45 minutes to complete this test. It ends at **14:45**. **Answer all 3 questions.**
- **Write your answers on the test paper**
- You may not use any notes, or consult with any other person during the class.

Name: ______________________________ **ID Number:** ______________________

Q1. [8 MARKS] For each of the following, select one of the proposed answers by circling it. Explanations are not required. In all cases G represents a *simple*, *undirected* graph: $G = (X, E)$ with node set X, and edge set E.

(i) [1 MARK]Which of the following is $|X|$? ANS Order

A: Order of G B: Size of G C: Diameter of G

(ii) [1 MARK]G is *connected* if and only if there is a path between every pair of nodes: *True or False*? ANS A: True

A: True B: False C: Neither

(iii) [1 MARK]Suppose that G is a tree with 10 nodes. How many edges does G have? ANS A: 9

A: 9 B: 10 C: 11

(iv) [1 MARK]Is the complete bipartite graph $K_{n,1}$ a tree? ANS A: Yes

A: Yes B: No C: Only when n is odd.

(v) [1 MARK]Let C_n be the cycle graph on n nodes. Suppose that the diameter of C_n is 3. Which of the following is true? ANS C: 6 or 7

A: n must be 6. B: n must be 7. C: n can be either 6 or 7.

(vi) [1 MARK]Let A be the adjacency matrix of G. Suppose that $I + A + A^2 > 0$. Which one of the following statements are true? ANS B

A: G is not connected. B: All shortest paths in G are of length at *most* 2

C: All shortest paths in G are of length at *least* 2.

(vii) [1 MARK]Let G_1 be the graph on $\{0, 1, 2, 3, 4\}$ with edges $0-1$, $0-2$, $1-2$, $1-3$, $2-4$, $3-4$. Let H and L be the complement and line graphs of G_1, respectively. Which one of the following statements is true? ANS C

A: H is isomorphic to G_1 B: H is isomorphic to L C: H is a path graph.

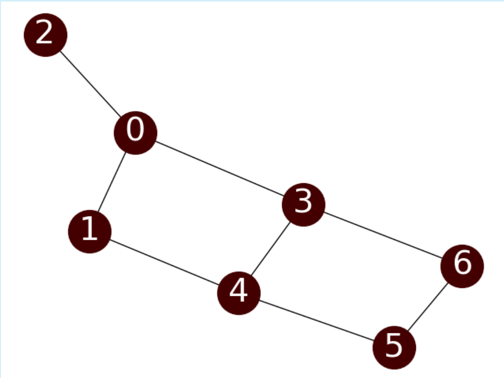
(viii) [1 MARK]Let G_1 be the graph in the previous question. Does G_1 have the same order as its line graph? ANS B: No

A: Yes B: No

Q2. [8 MARKS] Let G_2 be the graph on the nodes $\{0, 1, 2, 3, 4, 5, 6\}$, with edges $0-1$, $0-2$, $0-3$, $1-4$, $3-4$, $3-6$, $4-5$, and $5-6$.

(a) [2 MARKS] Give a sketch of the graph, G_2.

Answer:



(b) [2 MARKS] Is this G_2 bipartite? If so, list parts X_1 and X_2. If not, give an example of a path of odd length.

Answer: Yes, G_2 is a bipartite graph. Parts are $X_1 = \{0, 4, 6\}$ and $x_2 = \{1, 2, 3, 5\}$.

(c) [2 MARKS] Write down the adjacency matrix, A_2, of G_2;

Answer:

$$A_2 = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

(d) [2 MARKS] Explain *briefly* why there is no permutation matrix P such that $P^T A_2 P$ has the structure:

$$P^T A_2 P = \begin{pmatrix} A_{11} & \mathbf{0} \\ \mathbf{0}^T & A_{22} \end{pmatrix}$$

where A_{11} and A_{22} are square matrices, and $\mathbf{0}$ is a zero matrix.

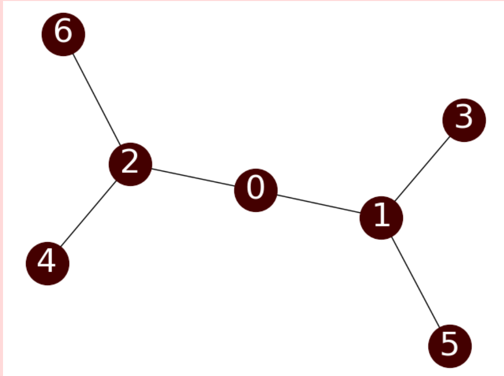
Answer: There is no such P, since G_2 is connected.

Q3. [8 MARKS]

(a) [2 MARKS] Let G_3 be the tree on the nodes $\{0, 1, 2, 3, 4, 5, 6\}$ with edges $0-1$, $0-2$, $1-3$, $1-5$, $2-4$, $2-6$. Compute the Prüfer code for G_3.

Answer: $[1, 2, 1, 0, 2]$.

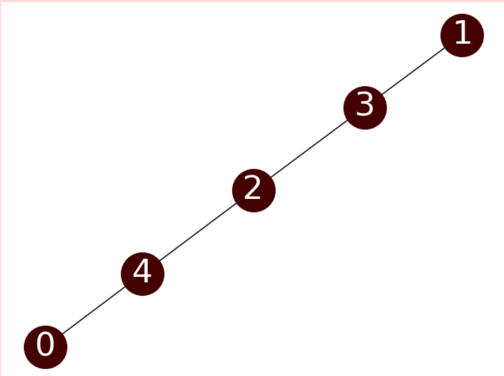
Also, though not asked, it may be helpful to sketch G_3:



(b) [2 MARKS] Determine the tree on the nodes $\{0, 1, 2, 3, 4\}$ which has Prüfer code $(4, 3, 2)$. (You can give your answer as an edge list, or with a sketch).

Answer: Edges are $0-4$, $1-3$, $2-3$, $2-4$.

Sketch:



(c) [2 MARKS] List the nodes of G_3 in the order they would be traversed by the **breadth-first search** (BFS) algorithm, starting at node 6.

Answer: 6-2-0-4-1-3-5

(d) [2 MARKS] Let G_4 be the **graph** on the nodes $\{0, 1, 2, 3\}$ with edges $0-1$, $0-2$, $0-3$, $1-2$. Compute the *closeness* centrality of all 4 nodes.

Answer:
Un-normalised: $c_0^C = 1/3, c_1^C = 1/4, c_2^C = 1/4, c_3^C = 1/5$.
Normalised: $C_0^C = 1, C_1^C = 3/4, C_2^C = 3/4, C_3^C = 3/5$.
(Full marks for either)