CS4423: Assignment 2: Part 1 - Tutorial Sheet ANS with solutions

Tutorial sheet for working on in classes

These exercises are similar Q5–Q9 on Homework Assignment 2: Part 1. They can be covered in class by the tutor. For the actual assignment, you need to answer the questions at `www.niallmadden.ie/2425-CS4423/CS4423-HW2-1.pdf`

Some background: *The Network Laplacian*

Graph Laplacian. There are many ways to represent a network as a matrix, such as the adjacency matrix. Another is the *Laplacian*, $L = (l_{ij})$. For a network $G = (X, E)$ of order n with nodes labelled $1, 2, \ldots, n$, L is the square $n \times n$ matrix with entries

$$l_{ij} = \begin{cases} \deg(i) & i = j \\ -1 & \{i, j\} \in X \\ 0 & \text{otherwise} \end{cases}$$

For example, if $G = K_3$, then

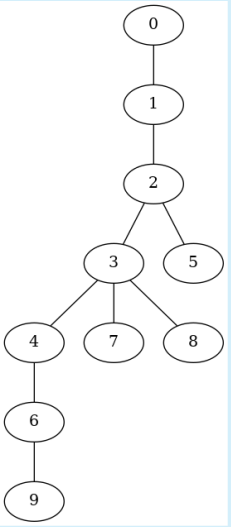
$$L = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

(a) Let G_1 be the tree on the nodes $\{0, 1, 2, \ldots, 9\}$ with Laplacian matrix

$$L = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 2 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}$$

Sketch G_1.

Answer:

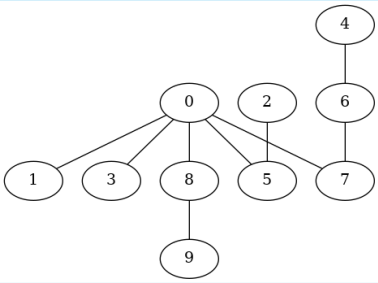


(b) Give the Prüfer code for G_1.

Answer: [1, 2, 2, 3, 3, 3, 4, 6]

(c) Sketch the tree, T_2, on the nodes $\{0, 1, \ldots, 9\}$ that has as its Prüfer code $[0, 5, 0, 6, 0, 7, 0, 8]$.

Answer: First step is to write down the degree sequence, which should be $[5, 1, 1, 1, 1, 2, 2, 2, 2, 1]$. Then proceed to make the dege list. The tree should look like:



(d) Give the order in which the nodes of T_2 would be visited if it is traversed by **Depth First Search**.

Answer: [0, 8, 9, 7, 6, 4, 5, 2, 3, 1]

(e) Give the order in which the nodes of T_2 would be visited if it is traversed by *Breadth First Search*.

Answer: [0, 1, 3, 5, 7, 8, 2, 6, 9, 4]