2425-MA140 Engineering Calculus

# Week 11: review
# Practice paper


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# MA140 Practice Paper 2024–2025

**See Canvas... 2425-MA140... Modules... Week 11 for more**

## About these questions

This set of questions are provided to help you prepare for the MA140 Semester 1 exam. Information on the similarlity, and differences, between it and the actual exam will be given in class in Week 11. Part of Questions 1 and 2 will be covered in tutorials; Parts of Questions 3-5 will be covered in Lectures. Answers to all questions will be posted during Study Week.

# Q1 I

Q1(a) Express $\dfrac{10x-27}{5x^2-25x+30}$ as partial fractions.

# Q1 II

Q1(b) Let $f(x) = \dfrac{x^2 - 2x - 8}{x^2 - x - 12}$. For each of the following, evaluate the limit, or determine that it does not exist.

(i) $\lim_{x \to -3} f(x)$

(ii) $\lim_{x \to 4} f(x)$

(iii) $\lim_{x \to \infty} f(x)$

Q1(c) Let $f(x) = x^{-2}(2 - e^x - e^{-x})$, $g(x) = -x^2 - 1$ and $h(x) = x^2 - 1$. You may assume that $g(x) \leqslant f(x) \leqslant h(x)$ for all $x$ in the region $[-2, 2]$.

(i) Use the Squeeze Theorem to determine $\lim_{x \to 0} f(x)$.

(ii) Explain why you can't use the Squeeze Theorem to determine $\lim_{x \to 1} f(x)$.

# Q1 III

Q1(d) Evaluate the limit $\lim_{\theta \to 0} \dfrac{2\sin(\theta)}{\theta + 3\tan(\theta)}$.

# Q1 IV

Q1(e) Let $f(x) = \begin{cases} a/x & x < 2 \\ 3 + bx & x \geqslant 2 \end{cases}$.

Find values of $a$ and $b$ for which both $f(x)$ and $f'(x)$ are continuous at $x = 2$.

# Q2 I

Q2(a) Differentiate $f(x) = x^2 e^{-3x} \sin(4x)$, with respect to $x$

# Q2 II

Q2(b) Differentiate $f(\theta) = (\sin(3\theta) + 1)(3\theta + 1)^{-1}$ with respect to $\theta$.

# Q2 III

Q2(c) Differentiate $f(x) = \ln(\cos(x^2)$ with respect to $x$.

# Q2 IV

Q2(d) Let $f(x) = 10\ln(x) + e^{-10x}$. Find $f'(x)$, $f''(x)$, and $f'''(x)$.

# Q2 V

Q2(e) Use the Inverse Power Rule, to fund the derivative, with respect to $x$, of $y = \cos^{-1}(x)$.

# Q2 VI

Q2(f) Find the equation of the tangent to the curve implicitly defined by

$$2x^2 + y^2 = 3,$$

at the point $(x, y) = (1, 1)$.

# Q3 I

Q3(a) Let $f(x) = 2 - \ln(x^2 + 9)$.

(i) Find all critical point(s) of $f$. For each, determine if it corresponds to a has a local minimum of $f$, a local maximum of $f$, or neither.

(ii) Determine regions of the domain where $f$ is increasing, and the regions where it is decreasing.

(iii) Find all point(s) of inflection of $f$.

Q3(b) Use logarithmic differentiation to evaluate the derivative of $y = e^{\sin(x)} \cos(x)$.

# Q3 III

Q3(c) Use L'Hôpital's rule to evaluate $\lim_{x\to 0} \frac{1-e^x}{x}$.

# Q3 IV

Q3(d) A sheet of cardboard measures 3cm by 8cm. You can make an open top box is made by cutting four identical squares from the four corners of the sheet, and folding up the flaps on each side.

(i) What size square should you remove to maximise the *volume* of the box? What is the resulting volume?

(ii) What size square should you remove to maximise the *surface area* of the box?

# Q4 I

Q4(a) Evaluate the following integrals.

(i) $\int x^2 \sin(x^3)\, dx$.

(ii) $\int \ln(x) x^{-4}\, dx$

# Q4 II

(iii) $\displaystyle\int_{-1}^{0} e^{1/x}/x^2\,dx$

(iv) $\displaystyle\int_{0}^{\infty} \frac{1}{1+x^2}\,dx.$

# Q4 III

Q4(b) Let $F(x) = \int_{x}^{3x} f(t)\,dt$.

(i) Explain why, for any function $f(t)$, we get $F(0) = 0$.

(ii) Let $f(t) = t^2$. Use the Fundamental Theorem of Calculus to evaluate $F'(x)$ as a polynomial.

# Q5 I

Q5(a) Compute the area of the region bounded above by curve $f(x) = e^x$, below by $g(x) = x^2$, to the left by $x = 0$ and to the right by $x = 2$.

# Q5 II

Q5(b) Compute the **volume** of the solid of revolution formed by rotating the curve $y = \sqrt{2x+1}$, between $x = 1$ and $x = 2$ about the $x$-axis. Give your answer to three decimal places.

# Q5 III

Q5(c) Compute the **surface area** of the solid of revolution formed by rotating the curve $y = 2\sqrt{x}$, between $x = 0$ and $x = 1$ about the $x$-axis. Give your answer to three decimal places.

# Q5 IV

Q5(d) Calculate the arc length of the graph of $f(x) = x^{3/2}$, over the interval $[0, 1]$. Give your answer to three decimal places.