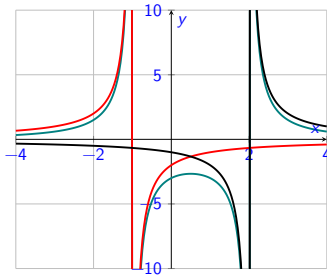


# Partial Fractions

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University of Galway

Tuesday, 23 September, 2025



# Outline

## 1 News!

- Tutorials
- Assignments
- Class tests

## 2 Partial Fractions

- 4 cases
- Case 1
- Case 2
- Case 3
- Case 4

## 3 Towards Limits

## 4 Exercises

Slides are on canvas, and at  
[https://www.niallmadden.ie/  
2526-MA140/](https://www.niallmadden.ie/2526-MA140/)



Tutorials start **this** week. The schedule is:

Teams	Time	Venue	Leader
1, 2	Tuesday 15:00	ENG- <b>2003</b>	ST
3, 4	Tuesday 15:00	ENG- <b>2034</b>	JM
9, 10	Thursday 11:00	ENG- <b>2002</b>	ST
11, 12	Thursday 11:00	ENG- <b>3035</b>	JM
5, 6	Friday 13:00	Eng- <b>2002</b>	ST
7, 8	Friday 13:00	Eng- <b>2035</b>	JM

*Rang teagaisc trí Ghaeilge* (Irish tutorial): Dé Máirt (Tuesday) 15:00, Áras na Gaeilge 221.

- ▶ There is currently a “practice” assignment open. See <https://universityofgalway.instructure.com/courses/46734/assignments/128373>
- ▶ During tutorials, the tutor will solve some similar questions. You can access the **tutorial sheet** at [https://universityofgalway.instructure.com/courses/46734/files/2842617?module\\_item\\_id=925893](https://universityofgalway.instructure.com/courses/46734/files/2842617?module_item_id=925893). You can also access this through the Canvas page: Modules... Tutorial Sheets.
- ▶ **Assignment 1** will be due 5pm, Monday 5 October.

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Also: try the exercises at the end of each set of lecture slides: they are similar in style and standard to exam questions.

There are two class test planned for this module:

- ▶ MCQ format;
- ▶ both worth 10% of the final grade;
- ▶ Test 1: **Tuesday, 14 October** (Week 5)
- ▶ Test 2: **Tuesday, 18 November** (Week 10)
- ▶ Contact Niall if you have any concerns, or wish to avail of alternative arrangements, as provided by LENS reports.

# Partial Fractions

**Rational Functions** have the general form  $f(x) = \frac{p(x)}{q(x)}$ , where  $p(x)$  and  $q(x)$  are polynomials.

An (proper) rational function can often be written as a sum of simpler ones: **partial fractions**.

For example

$$\frac{8x - 12}{x^2 - 2x - 3}$$

can be written as

$$\frac{3}{x - 3} + \frac{5}{x + 1}$$

**Check:** (*next slide*)

# Partial Fractions

# Partial Fractions

**Note:** Any polynomial (with real coefficients) can be factorised fully into the product of

- ▶ linear
- ▶ and irreducible quadratic factors.

**Examples:**



We get different combinations of factors in the denominator. Let's look at **four cases**, and how to find the partial fractions in each case.

## The four cases

1. Denominator has **linear factors to the power of 1**
2. Denominator has **factors to the power greater than 1 (i.e repeated linear factors)**.
3. Denominator has **irreducible quadratic factors**.
4. Denominator has **irreducible quadratic factors to power greater than 1**.

**Case 1:** Linear factors to the power of 1 in the denominator.

We have **two methods** to find  $A$  and  $B$ .

**Method 1:** Comparing coefficients

### Example

$$\frac{3x}{(x-1)(x+2)}$$



**Method 2:** Substituting specific values for  $x$ .

**Example**

Write  $\frac{8x - 12}{x^2 - 2x - 3}$  as sum of partial fractions.

(2) Linear factors to the power greater than 1 in the denominator, (i.e repeated linear factors).

If  $(x - \alpha)^k$  appears in the denominator, it will give rise to the following terms:

$$\frac{A_1}{x - \alpha} + \frac{A_2}{(x - \alpha)^2} + \dots + \frac{A_k}{(x - \alpha)^k}$$

**Example**

Find  $A$ ,  $B$  and  $C$  such that

$$\frac{3x+1}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$$

(Note: we'll find that  $A = 5/9$ ,  $B = 4/3$  and  $C = -5/9$ ).





**(3) Irreducible quadratic factors.**

Irreducible quadratic factors can not be factorised using real numbers, e.g.  $x^2 + x + 1$ .

An irreducible quadratic factor  $ax^2 + bx + c$  gives rise to partial fractions of the form

$$\frac{Ax + B}{ax^2 + bx + c}.$$

**Example 2.34 from textbook**

If one writes

$$\frac{5x}{(x^2 + x + 1)(x - 2)} = \frac{Ax + B}{x^2 + x + 1} + \frac{C}{x - 2}$$

then we find  $A = 10/7$ ,  $B = 5/7$  and  $C = 10/7$ .

**(4)** Irreducible quadratic factors to power greater than 1.

Each repeated irreducible quadratic factor  $(ax^2 + bx + c)^k$  in the denominator will give rise to

$$\frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{A_kx + B_k}{(ax^2 + bx + c)^k}.$$

These can be done in a similar way to the previous case. But the calculations are pretty messy, so we won't even try!

## Towards Limits

When we were considering the domain of a function, we looked at those  $x$ -values for which the function was not defined.

### Example

$$f(x) = \frac{x^2 - 2}{x - 1}$$

$$g(x) = \frac{x^2 - 1}{x - 1}$$

Neither  $f$  nor  $g$  are defined at  $x = 1$ .

**But what happens if  $x$  gets very closed to 1?**

$x$	0.900	0.990	0.999	1	1.001	1.010	1.100
$f(x)$							
$g(x)$							

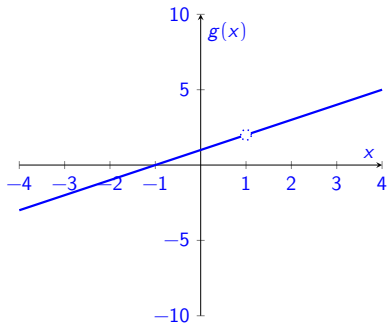
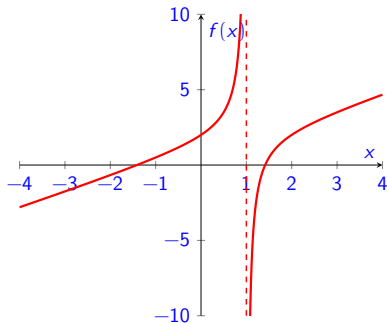
Let's look at the graphs of  $f$  and  $g$ .

# Towards Limits

## Example

$$f(x) = \frac{x^2 - 2}{x - 1}$$

$$g(x) = \frac{x^2 - 1}{x - 1}$$



## Towards Limits

In the previous example, we saw that, although neither  $f$  nor  $g$  was defined at  $x = 1$ , they behaved very differently as  $x$  approaches 1.

To discuss this we'll need the concept of a **limit** which, roughly, relates to the value of function as it **approaches** a point (but not actually at that point).

$$\lim_{x \rightarrow a} f(x) = L$$

The concept of a limit is a prerequisite for a proper understanding of calculus and numerical methods.

# Exercises

## Exercise 2.1.1

Find the constants  $A$ ,  $B$  and  $C$ , so that

$$\frac{2x + 1}{(x - 2)(x + 1)(x - 3)} = \frac{A}{x - 2} + \frac{B}{x + 1} + \frac{C}{x - 3}$$

## Exercises

### Exercise 2.1.2

Express the following as partial fractions.

1.  $\frac{6}{x^2 - x - 2}$

2.  $\frac{2x - 1}{x^2 - x - 2}$

3.  $\frac{x - 1}{(x + 1)(x^2 - x - 2)}$

4.  $\frac{x}{x^2 + 2x + 1}$

5.  $\frac{1}{x^3 - 1}$

## Exercises

### Exercise 2.1.3 (MA140 Exam, 24/25)

Express  $\frac{3x + 1}{x^2 - x - 2}$  as partial fractions.