MA140: Engineering Calculus

Week 1, Lecture 3: Polynomials and Partial Fractions

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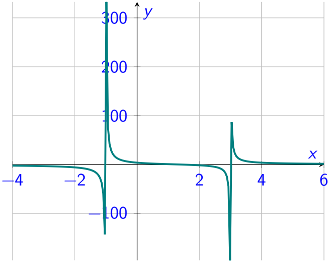

This version of the slides are by Niall Madden, but are adapted from original notes by Dr Kirsten Pfeiffer.

Outline

For more, see Sections 2.4 (Polynomials) 2.5 (Rational Functions) of *Modern Engineering Mathematics*:
https://search.library.nuigalway.ie/permalink/f/3b1kce/TN_cdi_askewsholts_vlebooks_9780273742517

News!

Tutorials start next week. Here is the schedule:

- Teams 1+2: Tuesday 15:00 ENG-**2003**
- Teams 3+4: Tuesday 15:00 ENG-**2034**
- Teams 9+10: Thursday 11:00 ENG-**2002**
- Teams 11+12: Thursday 11:00 ENG-**3035**
- Teams 5+6: Friday 13:00 Eng-**2002**
- Teams 7+8: Friday 13:00 Eng-**2035**

Note: I think the schedule is correct, but the venues are not confirmed... An announcement will be posted to Canvas on Monday confirming.

Would you be interested to taking a tutorial through Irish? If so, please complete this survey: `https://tinyurl.com/suirbhe1`

News!

You don't have to complete a graded assignment next week. However, this is a "practice" one available. See https://universityofgalway.instructure.com/courses/35693/assignments/94873

During tutorials, the tutor will solve some similar questions. You can access the tutorial sheet at https://universityofgalway.instructure.com/courses/35693/files/2023552?module_item_id=650912

Functions (again)

Recall...

Yesterday, we learned that

- A **function** is a rule for mapping from elements of one set (the domain) to elements of another (the codomain).
- When we write $y = f(x)$, we say "x" is the **argument** of the function.
- When $y = f(x)$ for some $x \in X$, y is said to be the **image** of x under f.
- The set of all images $y = f(x), x \in X$, is called the **range** of f.

Polynomials

A **polynomial function** (or just **polynomial**) is a function of the form

$$y = f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0, \quad x \in \mathbb{R},$$

where $a_0, a_1, ..., a_n$ are real numbers called the **coefficients** of the polynomial.
The number n is called the **degree** of the polynomial.

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Polynomials

Example: linear

$y = x$ is a **linear** polynomial with degree $n = 1$.

Polynomials

Example: quadratic

$x^2 - 2x - 3$. is a **quadratic** polynomial with degree $n = 2$.

There are many occasions when we want to **factorise** such quadratics, meaning we write them as the product of a pair of linear polynomials.

For example, we can **factorise** $x^2 - 2x - 3$ as

$$x^2 - 4x + 3 = (x - 3)(x + 1)$$

Polynomials

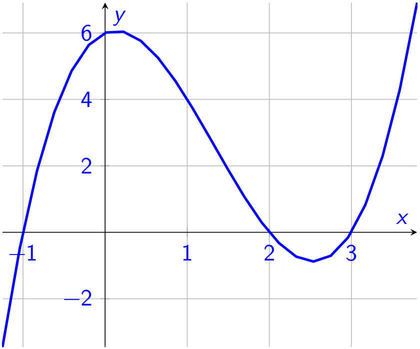
It is important to note that not all quadratic polynomials can be factorised as two linear polynomials. Such quadratics are called **irreducible**.

For example, $x^2 + 1$ is irreducible.

Polynomials

Example

$y = x^3 - 4x^2 + x + 6$ is a **cubic** function with degree $n = 3$.



Fact: A polynomial function of grade n has **up to** $n-1$ truning points ("bends").

Examples:

Break Time

During the break, think and talk about what you might do to sketch the graph of

$$y = -x^3 + x^2 + 2x$$

- To sketch the graph, first find the **intercepts**:
  - The **y-intercepts** can be found by letting $x = 0$.
  - The **x-intercepts** are called the **roots** (or **zeros**).
    To find the roots, set y equal to zero and solve for x.
- You don't have to use the same scale on the x- and on the y-axis.
- Do not use graph paper.

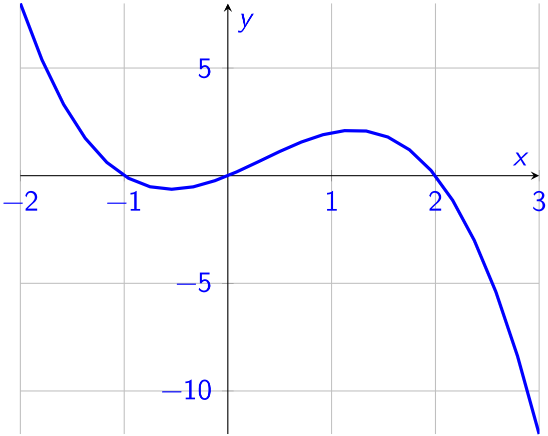
Example

Sketch the graph of

$$y = -x^3 + x^2 + 2x$$

How to sketch $y = -x^3 + x^2 + 2x$

Actual plot of $y = -x^3 + x^2 + 2x$



Exercise 1.3.1

Sketch the graphs of

(i) $y = 5x^2 - 7$

(ii) $y = x^2 - 4x + 3$

(iii) $y = x^3 - 6x^2 - 11x - 6$

Rational Functions

Rational Functions have the general form

$$f(x) = \frac{p(x)}{q(x)},$$

where $p(x)$ and $q(x)$ are polynomials.

- If degree of $p(x) <$ degree of $q(x)$,
  $f(x)$ is called a **strictly proper rational function**.
- If degree of $p(x) =$ degree of $q(x)$,
  $f(x)$ is called a **proper rational function**.
- If degree of $p(x) >$ degree of $q(x)$,
  $f(x)$ is called an **improper rational function**.

Rational Functions

An improper or proper rational function can always be expressed as a polynomial plus a strictly proper rational function, for example by algebraic division.

Example

$$\frac{4x^3 + 4x^2 + 4}{x^2 - 3} = 4x + 4 + \frac{12x + 16}{x^2 - 3}$$

For the previous example, we can work this out ourselves using **Long Division** to divide numerator by denominator:

Example 2.30 from text book

Use long division to show that

$$\frac{3x^4 + 2x^3 - 5x^2 + 6x - 7}{x^2 - 2x + 3} = 3x^2 + 8x + 2 - \frac{14x + 13}{x^2 - 2x + 3}$$

Partial Fractions

A (proper) rational function can often be written as a sum of simpler ones: **partial fractions.**

For example

$$\frac{8x - 12}{x^2 - 2x - 3}$$

can be written as

$$\frac{3}{x - 3} + \frac{5}{x + 1}$$

We verified this in class. Next week, we see how to compute partial fractions?