MA378: Assignment 2(Version 2.0)

Deadline: 5pm, Monday 20 March.

Your solutions must be clearly written, and neatly presented. You can submit an electronic copy, through blackboard, or a hard copy. If submitting a hard copy, please do so at the 10am lecture in the 10th. Also, make sure pages should be stapled together. Marks will be given for quality and clarity of exposition. Usual collaboration policy applies.

Note: a new version of this was posted on 3rd March because the original one included questions for which tutorial notes were posted to Blackboard.

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Chapter 2: Piecewise Polynomial Interpolation

Exer 3.2 Let $f(x) = \ln(\mathbf{x^2}) - \mathbf{x^4}$. Let l and S be the piecewise linear and Hermite cubic spline interpolants (respectively) to f on $n+1$ equally spaced points $1 = x_0 < x_1 < \cdots < x_N = 2$. What value of n would you have to take to ensure that

(i) $\max_{1 \leqslant x \leqslant 2} |f(x) - l(x)| \leqslant 10^{-\mathbf{6}}$?

(ii) $\max_{1 \leqslant x \leqslant 2} |f(x) - S(x)| \leqslant 10^{-\mathbf{6}}$?

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Chapter 3: Numerical Integration

Exer 1.1 (*For simplicity, you may assume that the quadrature rule is integrating* f *on the interval* $[-1, 1]$.) Let $q_0, q_1, \ldots, q_n$ be the quadrature weights for the Newton-Cotes rule $Q_n(f)$. Show that $q_i = q_{n-i}$ for $i = 0, \ldots n$.

Exer 3.5 Consider the rule (which is not, strictly speaking, a Newton-Cotes rule):

$$R(f) = q_0 f(1/3) - f(\frac{1}{2}) + q_2 f(\frac{3}{4})$$

for approximating $\int_0^1 f(x)dx$.

(a) Determine values of q_0 and q_2 that ensure this rule has precision 2.

(b) What is the maximum precision of $R(\cdot)$ with the values of q_1 and q_2 that you have determined?

Exer 3.4 Determine the precision of the following schemes for estimating $\int_0^1 f(x)dx$.

(i) $Q(f) = f(\frac{1}{2})$.

(ii) $Q(f) = \frac{1}{4}f(0) + \frac{3}{4}f(\frac{2}{3})$.

(iii) $Q(f) = \frac{3}{2}f(\frac{1}{3}) - 2f(\frac{1}{2}) + \frac{3}{2}f(\frac{\mathbf{2}}{\mathbf{3}})$.

Exer 3.5 Derive a 3-point Gaussian Quadrature Rule to estimate $\int_{-1}^1 f(x)dx$. *Hint:* $x_1 = 0$.