MA211 – **Problem Set 1**

Q1.1 ♣ Go to to the library. Find where they keep the calculus books. Choose any three. Find the section where they introduce the concept of a **limit** of a function at a point. Write down the *definition of a limit* they provide, *their explanation of what it means*, and *one example.*

Rank the books in order of how useful you think they are.

Q2.2 The study of what we call "Calculus" is said to have been started by *Isaac Newton* and *Gottfried von Leibniz*. Find out when and where they lived, and their major mathematical discoveries.

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Q2.1 Show that $\sqrt{2}$ is *not* a rational number. That is, show that there is no pair on integers a and b such that a and b have no common divisors and $(a/b)^2 = 2$.

Hint: See Proposition 2.6 in Smith's *Introductory Mathematics.*

Q2.2 What sets are usually represented by the symbols $\mathbb{R}$, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{C}$?
For each one, determine which of the others it is a subset of.

Q2.3 For each of the following functions, give the largest possible subset of $\mathbb{R}$ that can be the domain and range:

(i) $f(t) = 1/(1+t)$ (ii) $f(x) = \sqrt{9-x^2}$
(iii) $f(x) = \cos(x)$ (iv) $f(t) = \sin(5t-2)$
(v) ♣ $f(x) = 1 + \dfrac{1}{1-x^2}$ (vi) $f(x) = e^x$.

Q2.4 For each of the following functions, determine if it is even, odd, or neither.

(i) ♣ $\dfrac{x}{x^2+1}$ (ii) $\dfrac{x^2}{x^4+1}$
(iii) ♣ $x|x|$ (iv) $\dfrac{t^3+3t}{t^4-3t^2+4}$
(v) $2+x^2+x^4$

Q2.5 Are the trigonometric functions sin, cos and tan even, odd, or neither?

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Q3.1 Give an example of a function:

(i) $f: \mathbb{Z} \to \mathbb{N}$ that is onto but *not* one-to-one.
(ii) $f: \mathbb{N} \to \mathbb{N}$ that is one-to-one, but not onto.

Q3.2 Find subsets X and Y of the real numbers such that the functions $f: X \to Y$ are *invertible* (i.e., both *one-to-one* and *onto*) for

(i) $f(x) = \sin(x)$ (ii) $f(x) = \cos(x^2)$

Q3.3 Show *carefully* that

(i) $\lim_{x\to 4} 3x - 7 = 5$. (ii) $\lim_{x\to 2} \left(\dfrac{x}{2} + 3\right)$ is 4

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Q4.1 Use the Squeeze theorem to answer the following questions.

(i) Find $\lim_{x\to 0} f(x)$ if f is a function such that
$$2 - x^2 \le f(x) \le 2\cos(x),$$
(ii) If $\lim_{t\to 0} |f(t)| = 0$, show that $\lim_{x\to 0} f(t) = 0$.
(iii) Show that $\lim_{x\to 0} x^2 \cos(2/x) = 0$.

Q4.2 Calculate the derivatives of the following functions from 1st principles:

(i) ♣ $f(x) = \dfrac{1}{3}x^3$
(ii) $f(x) = x^n$ for any $x \in \mathbb{N}$.
(iii) $f(x) = x^{-n}$ for any $x \in \mathbb{N}$.

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Q5.1 (i) ♣ Working from 1st principles, show that $\dfrac{d}{dx}\sin(x) = \cos(x)$.

Hints:
- $\lim_{\theta\to 0} \dfrac{\sin(\theta)}{\theta} = 1$, and $\lim_{\theta\to 0} \dfrac{\cos(\theta)-1}{\theta} = 0$.
- $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$.

(ii) Use the "Quotient Rule" and the fact that $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$ to find the derivative of $\tan(x)$ with respect to x.

Q5.2 Use the product and quotient rules to evaluate the derivatives (with respect to x) of the following functions

(i) $f(x) = xe^x$, (ii) $f(x) = \dfrac{x^3}{1-x^2}$
(iii) $f(x) = x^2\sin(x)$

Q5.3 Use the *Chain Rule* to evaluate the derivative (with respect to x) of each the following functions:

(i) $\sin(x^2)$. (ii) $\cos(k^2+x^2)$.
(iii) $\dfrac{1}{\sqrt[3]{x^2+x+1}}$ (iv) $\dfrac{x}{(x^4+1)^3}$
(v) xe^{-kx}

Q5.4 Use the Product Rule and Chain Rule together to deduce the Quotient Rule.

Q5.5 Use *l'Hospital's Rule* to evaluate the limits:

(i) ♣ $\lim_{x\to 0} \dfrac{\sin(x)}{x}$ (ii) $\lim_{x\to 0} \dfrac{x^2+1}{x+1}$
(iii) $\lim_{x\to 0} \dfrac{1-\cos(x)}{x^2}$

Submit *carefully* written solutions to the problems marked ♣ no later than **11am, Monday Oct 6th**.