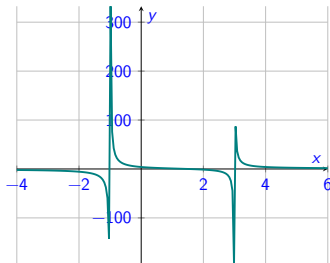


# Polynomials and Rational Functions

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# Today we rationalize the following topics:

## 1 News!

- Tutorials
- Tutorial sheet

## 2 Polynomials (again)

- Linear
- Quadratic
- Sketching polynomials

## 3 Rational Functions

- Long division

## 4 Exercises

See also Sections 1.2 and 7.4(!)  
of [https://math.libretexts.org/Bookshelves/Calculus/Calculus\\_\(OpenStax\)/01%3A\\_Functions\\_and\\_Graphs](https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/01%3A_Functions_and_Graphs)

Slides are on canvas, and at  
<https://www.niallmadden.ie/2526-MA140/>



Tutorials start next week. Here is the schedule:

- ▶ Teams 1+2: Tuesday 15:00 ENG-**2003**
- ▶ Teams 3+4: Tuesday 15:00 ENG-**2034**
- ▶ Teams 11+12: Thursday 11:00 ENG-**2002**
- ▶ Teams 9+10: Thursday 11:00 ENG-**3035**
- ▶ Teams 5+6: Friday 13:00 Eng-**2002**
- ▶ Teams 7+8: Friday 13:00 Eng-**2035**

Note: I think the schedule is correct. If there are any changes, you'll be informed on Canvas.

Would you be interested to taking a tutorial through Irish? (Show of hands?) If so, please fill out this form:

<https://forms.office.com/e/13kQHhwG8K>

You don't have to complete a graded assignment next week. However, this is a “practice” one available. See <https://universityofgalway.instructure.com/courses/46734/assignments/128373>

During tutorials, the tutor will solve some similar questions. You can access the **tutorial sheet** at [https://universityofgalway.instructure.com/courses/46734/files/2842617?module\\_item\\_id=925893](https://universityofgalway.instructure.com/courses/46734/files/2842617?module_item_id=925893). You can also access this through the Canvas page: Modules... Tutorial Sheets.

The Tutorial Sheet has questions that are nearly identical to your own version.

# Polynomials (again)

Yesterday, we saw that...

## Polynomials

**Polynomials** are functions of the form

$$y = f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0, \quad x \in \mathbb{R},$$

where  $a_0, a_1, \dots, a_n$  are real numbers called the **coefficients** of the polynomial. The number  $n$  is called the **degree** of the polynomial.

**Examples:**

**Example: Linear Polynomial**

A polynomial of degree  $n = 1$  is called “linear”. Its graph is a straight line. E.g.  $y = x - 1$  is a **linear** polynomial.

**Example: quadratic**

$x^2 - 2x - 3$  is a **quadratic** polynomial: it has degree  $n = 2$ .

There are many occasions when we want to **factorise** such quadratics, meaning we write them as the product of a pair of linear polynomials.

For example, we can **factorise**  $x^2 - 2x - 3$  as

$$x^2 - 4x + 3 = (x - 3)(x + 1)$$

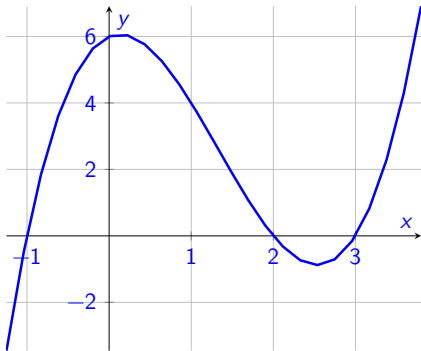
It is important to note that not all quadratic polynomials can be factorised as two linear polynomials. Such quadratics are called **irreducible**.

For example,  $x^2 + 1$  is irreducible.



**Example**

$y = x^3 - 4x^2 + x + 6$  is a **cubic** function with degree  $n = 3$ .



**Fact**

A polynomial function of grade  $n$  has **up to**  $n-1$  turning points (“bends”).

**Examples:**

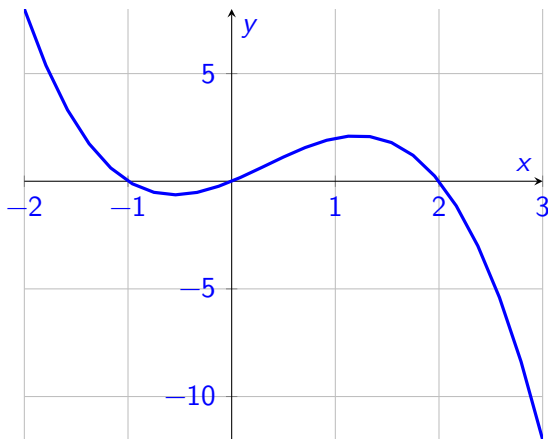
When sketching the graph of a function, we first find the **intercepts**:

- ▶ The **y-intercept** is where the graph of the function cuts the  $y$ -axis: found by letting  $x = 0$ .
- ▶ The **x-intercepts** are where the function's graph cuts the  $x$ -axis. These points are also called the **roots** (or **zeros**). To find them, set  $y$  equal to zero and solve for  $x$ .

**Example**

Sketch the graph of  $y = -x^3 + x^2 + 2x$

Actual plot of  $y = -x^3 + x^2 + 2x$



# Rational Functions

**Rational Functions** have the general form

$$f(x) = \frac{p(x)}{q(x)},$$

where  $p(x)$  and  $q(x)$  are polynomials.

- ▶ If degree of  $p(x) < \text{degree of } q(x)$ ,  
 $f(x)$  is called a **strictly proper rational function**.
- ▶ If degree of  $p(x) = \text{degree of } q(x)$ ,  
 $f(x)$  is called a **proper rational function**.
- ▶ If degree of  $p(x) > \text{degree of } q(x)$ ,  
 $f(x)$  is called an **improper rational function**.

# Rational Functions

An improper or proper rational function can always be expressed as a polynomial plus a strictly proper rational function, for example by algebraic division.

## Example

$$\frac{4x^3 + 4x^2 + 4}{x^2 - 3} = 4x + 4 + \frac{12x + 16}{x^2 - 3}$$

For the previous example, we can work this out ourselves using **Long Division** to divide numerator by denominator:



**Example 2.30 from text book**

Use long division to show that

$$\frac{3x^4 + 2x^3 - 5x^2 + 6x - 7}{x^2 - 2x + 3} = 3x^2 + 8x + 2 - \frac{14x + 13}{x^2 - 2x + 3}$$

## Exercise 1.3.1

Sketch the graphs of

(i)  $y = 5x^2 - 7$

(ii)  $y = x^2 - 4x + 3$

(iii)  $y = x^3 - 6x^2 - 11x - 6$