MA140: Engineering Calculus

Lecture 1: Introduction; Numbers, Notation, Functions

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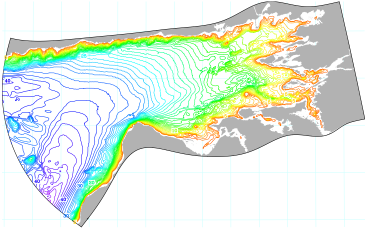
17 September, 2024



This version of the slides are by Niall Madden, but are adapted from original notes by Dr Kirsten Pfeiffer.

Outline

MA140 - Engineering Calculus is a Semester 1 module on calculus and its applications to engineering.

We'll covers several major topics in calculus:

- Functions
- Limits, Continuity, Intermediate Value Theorem
- Differentiation;
- Logarithms;
- Basic properties of integrals;
- Fundamental Theorem of Calculus;
- Techniques of integration: substitution, integration by parts, partial fractions and the Logarithm Rule.

These tools will help us tackle various engineering problems, such as those involving rates of change, maxima and minima, areas and volumes, . . .

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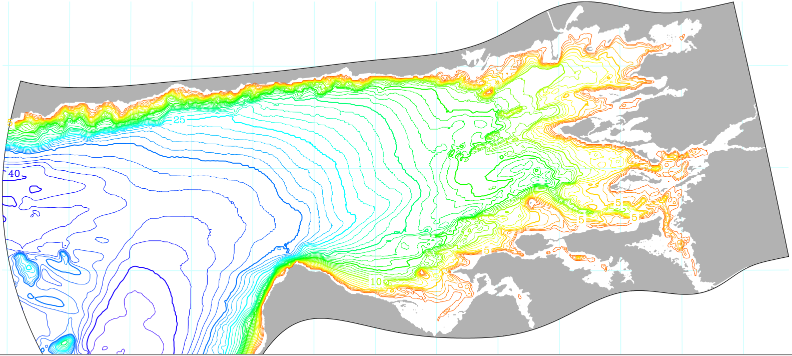
Web: `https://www.niallmadden.ie`

My area of research is **numerical analysis** and **computational mathematics**. It varies from (trying to) prove theorems, to collaborating with other scientists, engineers, and medics.

For example, right now I am working on a project with Drs Indiana Olbert and Alexander Shchepetkin from Civil Engineering, on trying to improve some computational models of Galway Bay. Here is an image from a paper we are working on (Galway Bay model bottom topography: credit AS).



There are about 280 students enrolled in MA140, across all Engineering Disciplines:

- Civil
- Biomedical
- Electrical and Electronic
- Mechanical
- Electronic and Computer
- Energy Systems
- Undenominated
- *anyone else?*

	Mon	Tue	Wed	Thu	Fri
9 – 10					
10 – 11		**Lecture**	**Lecture**	**Lecture**	
11 – 12				**Tutorial**	
12 – 1					
1 – 2					**Tutorial**
2 – 3					
3 – 4		**Tutorial**			
4 – 5					

- You should attend all three lectures.
- Tutorials start next week (TBC). You should attend one per week: your session will be assigned to you by the School of Engineering.
- Also: **SUMS** ("Support for Undergraduate Mathematics and Statistics"). FREE Drop-In service on campus: also available online. `universityofgalway.ie/public-sites/s-u-m-s/`.

Assessment

Assessment

- Online Assignments (together worth 33%)
- End-of-term Exam (worth 67%)

There will be 8 assignments through the semester, starting from Week 3 - so there will be a task to complete and submit almost every week. **Deadlines will always be Friday at 5pm**.

Recommended Texts

- Modern Engineering Mathematics, by G. James (Prentice Hall). This is freely available through the library at https://search.library.nuigalway.ie/permalink/f/3b1kce/TN_cdi_askewsholts_vlebooks_9780273742517
- I'll update the reading list as I cover each section.

Recommended Online-Resources

- Irish Mathematics Learning Support Network (IMLSN) Resources: https://www.imlsn.ie/index.php/resources-index. This is particularly useful if there is some concept that you need to revise.
- Paul's Online notes https://tutorial.math.lamar.edu/ Great range of "Cheat Sheets".
- Helping Engineers Learn Mathematics (HELM) Workbooks.
- Check these and other online resources on the SUMS website: universityofgalway.ie/public-sites/s-u-m-s/resources

Learning materials

Canvas

The on-line content for the course will be hosted at https://universityofgalway.instructure.com/courses/35693

There you'll find:

- Announcements (1 per week) → Eg Deadlines.
- Information (where, when, what)
- These slides, posted in advance.
- Links to assignments
- Grades
- Etc

The lecture slides contain most of the course material.

They are arranged by lecture, will be posted before the lecture.

The slides contain most of the main ideas and examples. However, they are "gappy", with extra details added during the class. The annotated versions will also be posted to canvas (after a day or two).

MA313
Lecture 1: Introduction; Numbers, Notation, Functions

Start of ...

Section 3: Numbers

This short section is about sets of numbers, culminating with the set of real numbers, $\mathbb{R}$. For more, see Chapter 1 of "Modern Engineering Calculus (James)

In a mathematics course, you'll encounter various different sets of **numbers**. The most familiar should be:

- $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ **Natural Numbers**
- $\mathbb{N}_0 = \{0, 1, 2, 3, 4, \ldots\}$ Natural Numbers with zero.
- $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ **Integers**
- $\mathbb{Q} = \{\frac{a}{b} \mid a, b \in \mathbb{Z}$ and $b \neq 0\}$ **Rational Numbers**

e.g. $54 \in \mathbb{N}$, $54 \in \mathbb{Z}$, $54 \in \mathbb{Q}$.

$-35 \in \mathbb{Z}$, $-35 = \frac{-35}{1} \in \mathbb{Q}$.

$\frac{7}{4} \in \mathbb{Q}$ $0.1 = \frac{1}{10} \in \mathbb{Q}$

But there are many important numbers, such as π and $\sqrt{2}$ that cannot be written as a fraction: they are *not* rational.

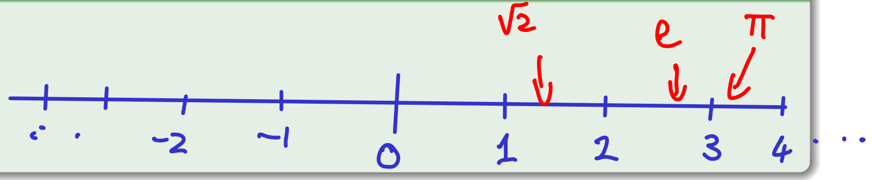
We say such numbers are **irrational**:

- $\pi \notin \mathbb{Q}$
- $e \notin \mathbb{Q}$
- $\sqrt{2} \notin \mathbb{Q}$

Look up proof that $\sqrt{2}$ is irrational.

However, these numbers *do* exist on a number line. (Note: $\sqrt{2} \approx 1.414213...$, and $e \approx 2.7182818$).

Number Line



The ~~real~~ numbers, ~~$\mathbb{R}$~~ $\mathbb{R}$

For MA140, the most important set of numbers is *the reals*, denoted $\mathbb{R}$. It is the set of all points on the number line. Roughly, if

- $\mathbb{N}$ is the set we use for counting;
- $\mathbb{Z}$ is the set of whole ("entire") numbers;
- And $\mathbb{Q}$ is used for ratios
- Then $\mathbb{R}$ is set use for (positive and negative) quantities: speed, height, weight, volume, etc, etc.

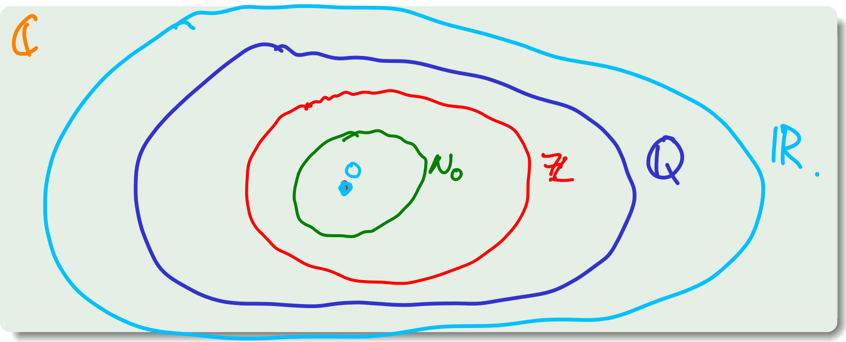
We mention, in passing, that there is another important set: $\mathbb{C}$, the set of **complex numbers**.

If $c \in \mathbb{C}$ we can write

$$c = a + ib, \quad a, b \in \mathbb{R}, \quad \mathbf{i} = \sqrt{-1}$$

However, they are not so important for MA140, so we'll pass on for now.

We can represent our number system visually as follows:



or write

$\mathbb{N} \subseteq \mathbb{N}_0 \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$

where $\subseteq$ means **"is subset of"**.

$\le$

$\subseteq$ "subset".

$\mathbb{R} \setminus \mathbb{Q}$ are the **irrational numbers**.

- "$\setminus$" means **"less"** or **"without"**;
- "$\in$" means **"is element of"**; eg $3 \in \mathbb{N}$
- "$\forall$" means **"for all"**;
- "$\exists$" means **"exists"**;
- "!" can mean **"unique"** or **"factorial"**, depending on the context.

e.g. $\exists!\, a \in \mathbb{Z}$ s.t. $a + (-3) = 0$.

"There exists a unique a in the integers which gives zero when added to -3"

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Start of ...

Section 4: Functions

The single most important concept in MA140 is that of a **function***.*
For more, see Chapter 2 of "Modern Engineering Calculus (James).

Functions

Question!

What is a <u>function</u> in Mathematics?

Take a minutes to answer the question.

Use your own words, or a picture. Don't look it up!

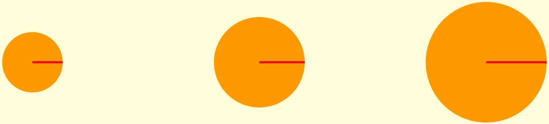
"Something with inputs and outputs"

"Arrows pointing from one thing to another"

⋮

Functions

Functions arise whenever one quantity *depends* on another.

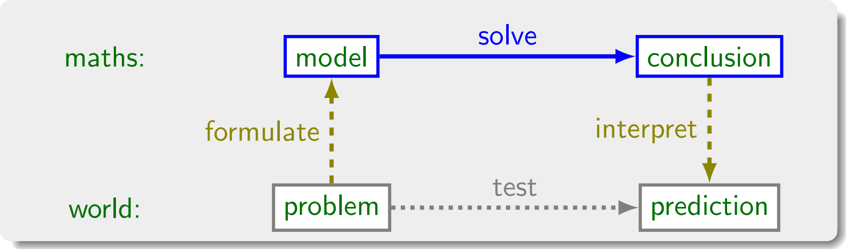


- The *area* A of a circle *depends* on the *radius* r of the circle.
- The *rule* connecting r and A is given by the *equation* $A = \pi r^2$.

This *formula* assigns to each positive number r one value of A. We say: "A **is a function of** r", and write $A(r) = \pi r^2$.

Here, A is called the **dependent variable** and r is called the **independent variable**: A depends on r.

- A **mathematical model** is a *mathematical description* (by means of a function, equation) of a *real-world phenomenon*.
- The model helps to *understand* the phenomenon, and perhaps to make *predictions* about future behavior.



- A good model *simplifies reality* to permit *mathematical calculations*, while being *sufficiently accurate* to provide *valuable conclusions*.
- Be aware of the *limitations* of the chosen model.

A **function** is a *rule* that maps an element of one set to another unique element of another set. "every element is mapped to one value"

Eg: 1 a mapping from students to ID numbers

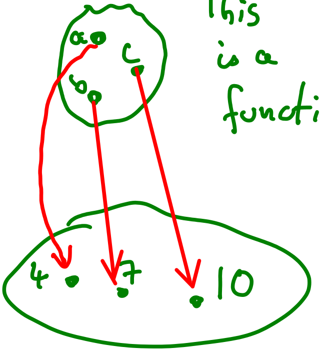
2 a mapping from MA140 students to Eircodes.

3 a function $f(x) = 2x$

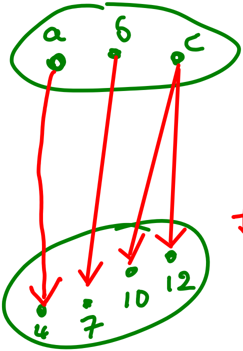
4 $f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$ Eg $f(-3) = 3$, $f(3) = 3$

ie $f(x) = |x|$.

A **function** is a *rule* that maps an element of one set to another unique element of another set.



This is a function



Not a function!!

Finished here 17 Sep.

We represent a function symbolically in two ways, either

$$f : x \to y$$

or

$$y = f(x)$$

Here x is in the set of X (or $x \in X$), and y is in the set of Y (or $y \in Y$).

If f is a function from X to Y...

- The set X is called the **domain** of the function.
- The set Y is called the **codomain**.
- When we write $y = f(x)$, we say "x" is the **argument** of the function.
- When $y = f(x)$ for some $x \in X$, y is said to be the **image** of x under f.
- The set of all images $y = f(x), x \in X$, is called the **range** (or **image set**) of f.

- While we could have functions between any pair of sets (e.g., a function from students in this class to their ID numbers), usually X and Y are *sets of numbers*.
- It is not necessary for all elements y of the codomain Y to be images under f.
- One element $y \in Y$ can serve as value $f(x)$ for several $x \in X$.

A function can be represented in different ways:

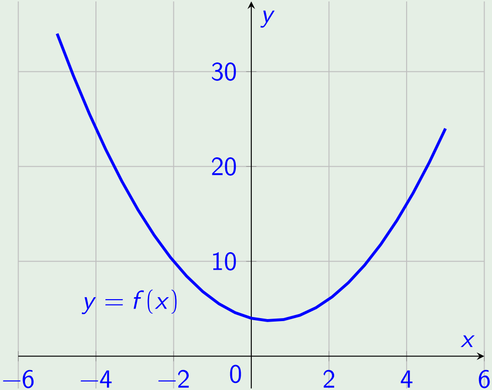
1. **verbally** (by a description in *words*);
2. **numerically** (as a *table* of values);
3. **visually** (as a *graph*);
4. **algebraically** (by an explicit *formula*).

Often it is possible, and useful, to go from one way to another.

Graphical Representation

Graph → Table

A common way to *visualize* a function $f\colon X \to \mathbb{R}$ is its *graph* in the x, y-plane.



x	$f(x)$
-4	24
-2	10
0	4
2	6
4	16

Often, the domain of a function is not expilicitly stated.
In such a case the following **Domain Convention** applies.

The **domain** of a function f is the set of all numbers x for which $f(x)$ *makes sense* and gives a *real-number output*.

Example

1. Find the domain D of $f(x) = \frac{1}{x^2 - x}$.

Example

Find the domain of the function $f(x) = \sqrt{x+2}$.

Example

Identify the domain, codomain and range of

1. $f(x) = 3x^2 + 1$
2. $f(x) = \sqrt{(x+4)(3-x)}$
3. $g(x) = \frac{1}{x}$

Solution

Example

Identify the domain, codomain and range of

1. $f(x) = 3x^2 + 1$
2. $f(x) = \sqrt{(x+4)(3-x)}$
3. $g(x) = \frac{1}{x}$

Solution ctd.

Example

Identify the domain, codomain and range of $f_1(x) = 3x^2 + 1$

Example

Identify the domain, codomain and range of
$f_2(x) = \sqrt{(x+4)(3-x)}$ $g(x) = \frac{1}{x}$

Example

$f_3(x) = \frac{1}{x}$