MA438: Mathematics of Decision Making II

Module Outline

Semester 2, 2022/2023

Who and When

Lecturers

Dr Niall Madden (Niall.Madden@UniversityOfGalway.ie)
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Schedule

Monday 13.00 and Tuesday at 11.00 om AdB-1020.
We'll also have lab and tutorials session, starting in Week 3.
Time TBC.

Syllabus

This course will look at some mathematical principles and algorithms that can support decisions of particular types. Our syllabus will have four chapters, each devoted to a theme in linear algebra and/or combinatorial optimization.

The algorithmic aspects of each section will be investigated in labs, using the MATLAB platform.

1. Low-rank approximation and the singular value decomposition

Given a (large) matrix A, how can we find a matrix B that is "near" to A but whose rank is at most equal to some specified k? The ***rank*** of a matrix is the dimension of its row space (or column space).

Decision problems:

- You have a large data set. You want to summarise it. What is the "best" summary of a given size?
- You want to identify some key features in a blurry image from an imperfect recording process. How do you distinguish the desired features from the blurry noise? How do you remove the noise? How do you know if you've removed too much of it?

2. Matching and vertex covers in graphs

A ***matching*** in a graph assignment to (some) vertices of a partner, in such a way that all partnered pairs are adjacent to each other, and every vertex has at most one partner. A *perfect* matching is one in which every vertex has a partner.

Questions in the theory of matchings include determining the maximum number of vertices that can be involved in a matching in a given graph, and devising methods for finding a matching that attains this bound. In a case where the edges in a graph carry weights that correspond to cost in some sense, one may be interested in finding a perfect (or good) matching of minimal total cost.

Many matching problems are of particular interest in bipartite graphs, where variants for example might a single vertex in one part to have multiple "partners" in the other.

Application: You are running a large-scale event with different activities at different venues. Each venue is potentially suitable for certain activities, but in each case there is a cost involved in equipping the venue. How do you assign the activities to the venues so that the total cost is minimized?

3. Nonnegative matrices

Matrices that arise from data and from dynamical processes often naturally have the property that their entries are positive, or at least not negative. Fortunately, non-negative matrices have special and interesting properties that have a big impact in these application areas.
Foremost among these is the Perron-Frobenius Theorem, which says that every square matrix with positive entries has a “dominant” positive real eigenvalue.
Also of importance is the characterisation of matrices whose *inverse* is non-negative.

4. Introduction to matroid theory

Sometimes an optimal solution to a decision problem can be constructed through a process of doing "what is best right now" at each step, without considering possible consequences. Problems that admit such solutions are relatively manageable, and one well-known example is the problem of constructing a spanning tree of minimal weight in a connected weighted graph. This can be achieved by repeating the step of adding an edge of minimal weight from all available edges, subject to not creating a cycle, until a tree is obtained. Algorithms of this type are referred to as "greedy". In this chapter we will explore the concept of a matroid, which connects to both matrices and graphs, and provides a framework for the applicability of greedy algorithms.

Assessment

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The assessment for this module will consist of:

- Three written assignments;
- Three lab assignments.
- Each written assignment will be paired with a lab assignment; each pair will contribute **25%** to your overall grade for MA438.
- One project in which students will produce an “artefact” (such as a video or poster, but there are lots of other possibilities), on a topic of interest that connects to the themes of the course, and a report on that artefact. Together, these contribute 25% to your grade.

Projects will be arranged from Week 7. You’ll be supported in selecting a topic at labs and tutorials. Group projects are encouraged.