MA385: Tutorials 2+3

These exercises are for Tutorials 2 and 3 (Weeks 6 and 7). You do not have to submit solutions to these *questions. However, you do have to submit solutions to related questions on Assignment 1*

Q1. Suppose that we have a fixed point method $x_{k+1} = g(x_k)$ which we know to be converges to fixed point of g, denoted τ. Show that, if $g'(\tau) = g''(\tau) = 0$, then convergence of the method is at least Order **3**.

Q2. About 2,000 years ago, in Alexandria (Egypt), Hero proposed the following iterative method for estimating $\sqrt{n}$ for any $n > 0$:

$$x_{k+1} = \frac{x_k}{2} + \frac{n}{2x_k}. \tag{1}$$

(a) If this is a fixed point method, what is g?

(b) For the method to (provably) work we need to determine if there is a region around $\sqrt{n}$ for which it is a contraction. First show that $1 \leqslant g(x) \leqslant n$ for all $x \in [1, n]$. Then determine a region around $x = \sqrt{n}$ for which $|g'(x)| < 1$.

(c) Show that it is equivalent to Newton's Method, for a suitably defined function f, where $f(\sqrt{n}) = 0$.

(d) Show that it converges (at least) quadratically (i.e., with Order 2).

(e) Does it converge cubically (i.e., with Order 3)?

Q3. Edmund Halley is famous for analysing the orbit of the comet which is now named after him. Another of his discoveries is the following method for solving nonlinear equations:

$$x_{k+1} = x_k - \frac{2f(x_k)f'(x_k)}{2\big(f'(x_k)\big)^2 - f(x_k)f''(x_k)}. \tag{2}$$

Write down the associated Fixed Point method for estimating $\sqrt{2}$. Show that this is the same as the method given by $g_3(x)$ in Lab 1.

(Extra: if you really want, you can show that $g_3'(\sqrt{2}) = g_3''(\sqrt{2}) = 0$, but it is a little tedious).