2526-MA140: Week 01, Lecture 3 (L03)

Polynomials and Partial Fractions

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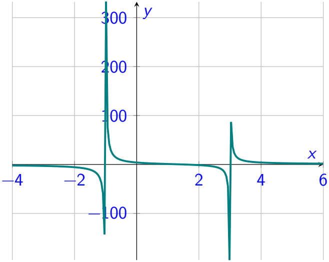


Outline

For more, see Sections 1.2 and 7.4(!) of https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/01%3A_Functions_and_Graphs

Tutorials start next week. Here is the schedule:

- Teams 1+2: Tuesday 15:00 ENG-**2003**
- Teams 3+4: Tuesday 15:00 ENG-**2034**
- Teams 11+12: Thursday 11:00 ENG-**2002**
- Teams 9+10: Thursday 11:00 ENG-**3035**
- Teams 5+6: Friday 13:00 Eng-**2002**
- Teams 7+8: Friday 13:00 Eng-**2035**

Note: I think the schedule is correct. If there are any changes, you'll be informed on Canvas.

Would you be interested to taking a tutorial through Irish? (Show of hands?) If so, please fill out this form: https://forms.office.com/e/13kQHhwG8K

You don't have to complete a graded assignment next week. However, this is a "practice" one available. See https://universityofgalway.instructure.com/courses/46734/assignments/128373

During tutorials, the tutor will solve some similar questions. You can access the tutorial sheet at https://universityofgalway.instructure.com/courses/46734/files/2842617?module_item_id=925893. You can also access this through the Canvas page: Modules... Tutorial Sheets.

Polynomials (again)

Yesterday, we saw that...

Polynomials

Polynomials are functions of the form

$$y = f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0, \quad x \in \mathbb{R},$$

where $a_0, a_1, ..., a_n$ are real numbers called the **coefficients** of the polynomial. The number n is called the **degree** of the polynomial.

Examples:

Example: Linear Polynomial

A polynomial of degree $n = 1$ is called "linear". Its graph is a straight line. E.g. $y = x - 1$ is a **linear** polynomial.

Example: quadratic

$x^2 - 2x - 3$ is a **quadratic** polynomial: it has degree $n = 2$.

There are many occasions when we want to **factorise** such quadratics, meaning we write them as the product of a pair of linear polynomials.

For example, we can **factorise** $x^2 - 2x - 3$ as

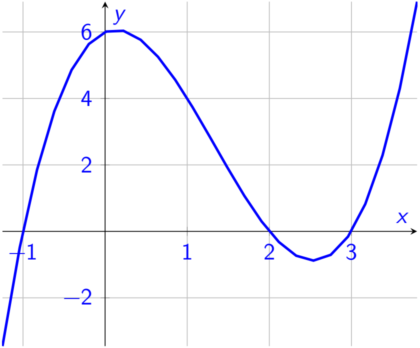
$$x^2 - 4x + 3 = (x - 3)(x + 1)$$

It is important to note that not all quadratic polynomials can be factorised as two linear polynomials. Such quadratics are called **irreducible**.

For example, $x^2 + 1$ is irreducible.

Example

$y = x^3 - 4x^2 + x + 6$ is a **cubic** function with degree $n = 3$.



Fact

A polynomial function of grade n has **up to** $n-1$ turning points ("bends").

Examples:

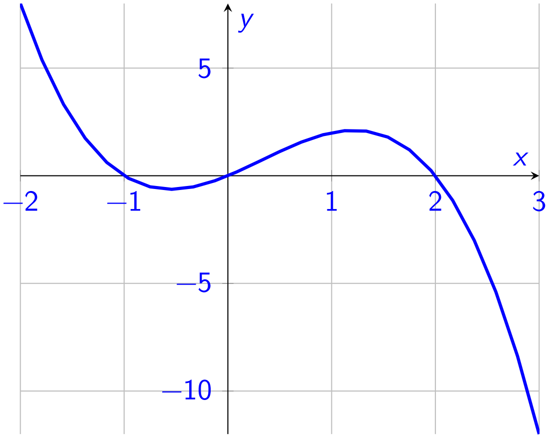
When sketching the graph of a function, we first find the **intercepts**:

- The y-**intercept** is where the graph of the function cuts the y-axis: found by letting $x = 0$.
- The x-**intercepts** are where the function's graph cuts the x-axis. These points are also called the **roots** (or **zeros**). To find them, set y equal to zero and solve for x.

Example

Sketch the graph of $y = -x^3 + x^2 + 2x$

Actual plot of $y = -x^3 + x^2 + 2x$



Rational Functions

Rational Functions have the general form

$$f(x) = \frac{p(x)}{q(x)},$$

where $p(x)$ and $q(x)$ are polynomials.

- If degree of $p(x) <$ degree of $q(x)$, $f(x)$ is called a **strictly proper rational function**.
- If degree of $p(x) =$ degree of $q(x)$, $f(x)$ is called a **proper rational function**.
- If degree of $p(x) >$ degree of $q(x)$, $f(x)$ is called an **improper rational function**.

Rational Functions

An improper or proper rational function can always be expressed as a polynomial plus a strictly proper rational function, for example by algebraic division.

Example

$$\frac{4x^3 + 4x^2 + 4}{x^2 - 3} = 4x + 4 + \frac{12x + 16}{x^2 - 3}$$

For the previous example, we can work this out ourselves using **Long Division** to divide numerator by denominator:

Example 2.30 from text book

Use long division to show that

$$\frac{3x^4 + 2x^3 - 5x^2 + 6x - 7}{x^2 - 2x + 3} = 3x^2 + 8x + 2 - \frac{14x + 13}{x^2 - 2x + 3}$$

Partial Fractions

A (proper) rational function can often be written as a sum of simpler ones: **partial fractions.**

For example

$$\frac{8x-12}{x^2-2x-3}$$

can be written as

$$\frac{3}{x-3}+\frac{5}{x+1}$$

Check:

Partial Fractions

Note: Any polynomial (with real coefficients) can be factorised fully into the product of

- linear
- and irreducible quadratic factors.

We get different combinations of factors in the denominator. Let's look at **four cases**, and how to find the partial fractions in each case.

Partial Fractions

The four cases

1. Linear factors to the power of 1 in the denominator.
2. Linear factors to the power greater than 1 in the denominator, (i.e repeated linear factors).
3. Irreducible quadratic factors.
4. Irreducible quadratic factors to power greater than 1.

Partial Fractions

(1) Linear factors to the power of 1 in the denominator.

Example

$$\frac{3x}{(x-1)(x+2)}$$

Partial Fractions

We have **two methods** to find A and B.

Method 1: Comparing coefficients

Partial Fractions

Method 2: Substituting specific values for x.

Partial Fractions

Example

Write $\dfrac{8x-12}{x^2-2x-3}$ as sum of partial fractions.

Exercises

Exercise 1.3.1

Sketch the graphs of

(i) $y = 5x^2 - 7$

(ii) $y = x^2 - 4x + 3$

(iii) $y = x^3 - 6x^2 - 11x - 6$

Exer 1.3.2

Find the constants A, B and C, so that

$$\frac{2x+1}{(x-2)(x+1)(x-3)} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{x-3}$$