Chapter 0 (Taylor's Theorem) and Chapter 1 (Solving nonlinear equations)

Please submit carefully written solutions to the following exercises: Exercises 0.2 1.8, 1.10, 1.11, and 1.15.

Exercise 0.2 ($\star$ Homework problem). *Write out the Taylor polynomial at* x, *about* $a = 0$, *of degree 5 for* $f(x) = \sin(x)$. *How does its derivative compare to the corresponding Taylor polynomial for* $f(x) = \cos(x)$*?*

Exercise 1.8 ($\star$ Homework problem). *Write down the equation of the line that is tangential to the function* f *at the point* x_k. *Give an expression for its zero. Hence show how to derive Newton's method.*

Exercise 1.10 ($\star$ Homework problem). *(i) Let* q *be your student ID number. Find* k *and* m *where* $k - 2$ *is the remainder on dividing* q *by 4, and* $m - 2$ *is the remainder on dividing* q *by 6.*

(ii) Show how Newton's method can be applied to estimate the postive real number $m^{1/k}$. *That is, state the nonlinear equation you would solve, and give the formula for Newton's method, simplified as much as possible.*

(iii) Do three iterations by hand of Newton's Method for this problem, taking $x_0 = 1$.

Exercise 1.11 ($\star$ Homework problem). *Suppose we want to apply Newton's method to solving* $f(x) = 0$ *where* f *is such that* $|f''(x)| \leqslant 10$ *and* $|f'(x)| \geqslant 2$ *for all* x. *How close must* x_0 *be to* τ *for the method to converge?*

Exercise 1.15 ($\star$ Homework problem). *Show that* $g(x) = \ln(2x + 1)$ *is a contraction on* $[1, 2]$. *Give an estimate for* L. *(Hint: Use the Mean Value Theorem).*