MA385: Tutorial 1

These exercises are for Tutorial 1 (Week 4). You do not have to submit this work. However, you can expect similar questions on the final exam.

In the following questions, take

$$f(x) = x - \cos(x) \tag{1}$$

It may be useful to know that, if $f(\tau) = 0$ then $\tau \approx 0.7390851332$.

Q1. Show that there is a solution to $f(x) = 0$ in the interval $[0, 1]$.

Q2. Suppose we want to implement the Bisection Method for this problem, with $x_0 = 0$ and $x_1 = 1$. If the sequence generated is denoted $\{x_0, x_1, x_2, \dots\}$, what is the minimum number of iterations needed to ensure that $|x_k - \tau| \leqslant 10^{-3}$?

Q3. Is it possible to use Theorem 1.5.2 to determine that Newton's method, applied to f in (1) will converge for any choice of $x_0 \in [0, 1]$? Can it be used to determine that Newton's method will converge for *any* $x_0 \in \mathbb{R}$?

Q4. Take $x_0 = 1$. Use Newton's method x_1, x_2, and x_3 as estimates for solutions to $x - \cos(x) = 0$. (If possible, evaluate the corresponding values of $f(x_k)$, and $|\tau - x_k|$, to convince yourself that the method is converging) .

Q5. Let's suppose we don't know τ, only that it is located between $x = 0$ and $x = 1$. Taking $x_0 = 1$, give upper bounds for $|\tau - x_1|$, $|\tau - x_2|$, and $|\tau - x_3|$ using the *Newton Error Formula*. How does this compare with the corresponding bounds for the Bisection Method, and with the actual errors?