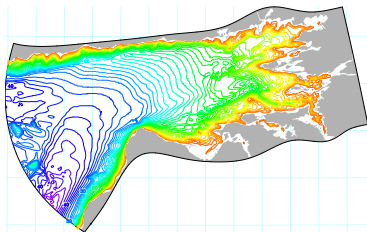


## Introduction; Functions

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University of Galway

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*These slides are by Niall Madden,  
with some material adapted from  
notes by Dr Kirsten Pfeiffer.*

# Outline

## 1 Introductions

- Welcome to MA140!
- About me
- About you
- Schedule
- Assessment

## 2 Learning materials

- Texts and Resources
- Canvas

## 3 Functions

- Functions as mappings
- Notation and terminology
- The real numbers
- Domain, range, image, ...
- 4 Ways to Represent a Function

## 4 Graphical Representation

- Domain of a function

## 5 Exercises

**MA140 - Engineering Calculus** is a Semester 1 module on calculus and its applications to engineering.

We'll covers several major topics in calculus:

- ▶ Functions
- ▶ Limits, Continuity, Intermediate Value Theorem
- ▶ Differentiation;
- ▶ Logarithms;
- ▶ Integration;
- ▶ Fundamental Theorem of Calculus;
- ▶ Applications, including solids of rotation, and centres of mass.

These tools will help us tackle various engineering problems, such as those involving rates of change, maxima and minima, areas and volumes, . . .

**Lecturer:** Dr Niall Madden (he/him)

**Addressed:** Niall (pronounced “Knee”- “al” #StartsWithAName)

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**Office:** Room AdB-1013, Arás de Brún

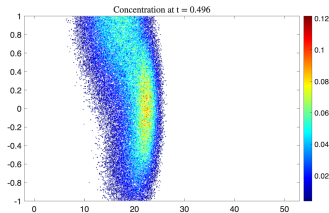
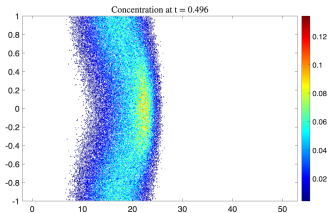
**Email:** [Niall.Madden@UniversityOfGalway.ie](mailto:Niall.Madden@UniversityOfGalway.ie). This is the best way to contact me. When you do so, **please** include “MA140” in the subject line. It can also be helpful to include your ID number.

**Web:** <https://www.niallmadden.ie>

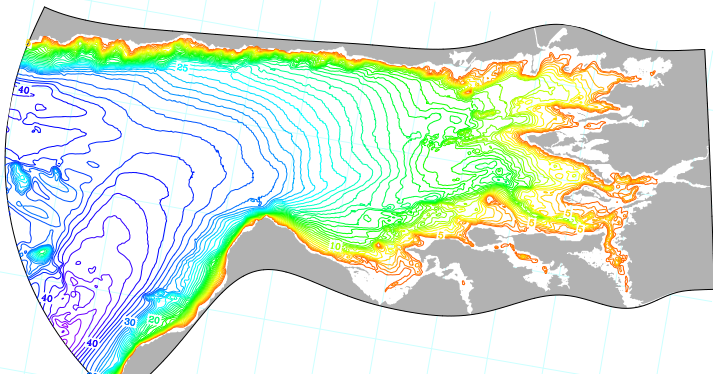
Also: do avail of the time before and, especially, after class, to have a quick chat. if the co-domain is  $\mathbb{R}$ . What is the range?

My area of research is **numerical analysis** and **computational mathematics**. It varies from (trying to) prove theorems, to collaborating with other scientists, **engineers** (mostly), and medics.

Specifically, I work on the maths of solving fluids equations, and huge linear systems, by computer methods. The maths can involve answering such questions as: **can we guarantee that our numerical error is less than our model error?**



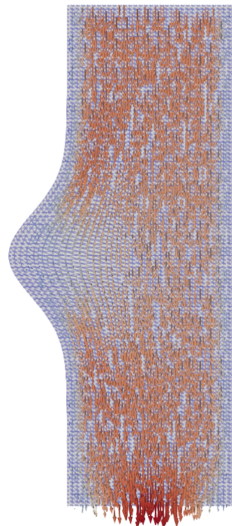
For example, I am working on a project with Drs Indiana Olbert and Alexander Shchepetkin (Civil Engineering), trying to improve some computational models of Galway Bay. Here's an image from a paper we are working on, on the Storm Éowyn tidal surge. (Galway Bay topography: credit AS).



I also work on other topics related to this module:

- ▶ mathematical models of aortic flows, with Niamh Hynes (Biomedical Engineering/Surgery), Kevin Moerman (Mechanical Eng), and Sean Tobin (your tutor!).
- ▶ Solute dispersal in channels, with Nanda Poddar (Maths).

I'll tell you more about these projects, and how they related to MA140 later in the semester.



There are about 280 students enrolled in MA140, across all Engineering Disciplines:

- ▶ Civil
- ▶ Biomedical
- ▶ Electrical and Electronic
- ▶ Mechanical
- ▶ Electronic and Computer
- ▶ Energy Systems
- ▶ Undenominated
- ▶ *anyone else?*



	Mon	Tue	Wed	Thu	Fri
9 – 10					
10 – 11		Lecture	Lecture	Lecture	
11 – 12				Tutorial	
12 – 1					
1 – 2					Tutorial
2 – 3					
3 – 4		Tutorial			
4 – 5					

- ▶ You should attend all three lectures.
- ▶ Tutorials start next week (TBC). You should attend one per week: your session will be assigned to you by the School of Engineering.
- ▶ Also: **SUMS** (“Support for Undergraduate Mathematics and Statistics”). FREE Drop-In service on campus: also available online. [universityofgalway.ie/public-sites/s-u-m-s/](http://universityofgalway.ie/public-sites/s-u-m-s/).

## Assessment

### Assessment

- ▶ Short weekly online assignments (together worth 10%). There will be 8 of these through the semester, starting from Week 3 - so there will be a task to complete and submit almost every week. **Deadlines will always be Monday at 5pm.**
- ▶ Two in-class tests (together worth 20%). Will schedule these by Thursday, but most likely Tuesdays of Weeks 4 and 8.
- ▶ A short project at the end of semester (3%)
- ▶ End-of-term Exam (worth 67%)

## Recommended Texts

Our primary reference is **Calculus** by Strang and Herman. It is free and open, and available from [https://math.libretexts.org/Bookshelves/Calculus/Calculus\\_\(OpenStax\)](https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax))

Also good: **Modern Engineering Mathematics**, by G. James (Prentice Hall). This is freely available through the library at [https://search.library.nuigalway.ie/permalink/f/3b1kce/TN\\_cdi\\_askewsholts\\_vlebooks\\_9780273742517](https://search.library.nuigalway.ie/permalink/f/3b1kce/TN_cdi_askewsholts_vlebooks_9780273742517)

I'll update the reading list as I cover each section.

## Recommended Online-Resources

- ▶ Irish Mathematics Learning Support Network (IMLSN)

Resources:

<https://www.imlsn.ie/index.php/resources-index>.

This is particularly useful if there is some concept that you need to revise.

- ▶ Paul's Online notes <https://tutorial.math.lamar.edu/>  
Great range of “Cheat Sheets”.
- ▶ Helping Engineers Learn Mathematics (HELM) Workbooks.
- ▶ Check these and other online resources on the SUMS website:  
[universityofgalway.ie/public-sites/s-u-m-s/resources](http://universityofgalway.ie/public-sites/s-u-m-s/resources)

The on-line content for the course will be hosted at <https://universityofgalway.instructure.com/courses/46734>

There you'll find:

- ▶ Announcements (1 per week)
- ▶ Information (where, when, what)
- ▶ These slides, posted in advance.
- ▶ Links to assignments
- ▶ Grades
- ▶ Etc

## **The lecture slides contain most of the course material.**

They are arranged by lecture, will be posted before the lecture.

The slides contain most of the main ideas and examples. However, they are “gappy”, with extra details added during the class. The annotated versions will also be posted to canvas (after a day or two).

## Functions

The single most important concept in MA140 is that of a **function**. For more, see [Chapter 1 of Strang's Calculus](#):

*“Calculus is the mathematics that describes changes in functions. We define polynomial, rational, trigonometric, exponential, and logarithmic functions. We review how to evaluate these functions, and show the properties of their graphs”.*

### Definition (Function)

A **function**,  $f$  consists of a set of *inputs*, a set of *outputs*, and a **rule** for assigning each input to exactly one output.

- ▶ The set of inputs is called the **domain** of the function.
- ▶ The set of outputs is called the **range** of the function.

**Examples:**



- ▶ When we write  $f : X \rightarrow Y$ , we mean that “ $f$  maps values from the set  $X$  to the set  $Y$ ”.
- ▶ We write the rule for that mapping as  $y = f(x)$ .

## The Real Numbers, $\mathbb{R}$

In almost every example we'll consider, in MA140,  $f$  will be a mapping from one **real** number to another.

The set of **real** numbers, denoted  $\mathbb{R}$ , are the numbers used to represent continuous quantities, such as length, speed, time and temperature.

- ▶ **continuous** means that there are no gaps, or “jumps”: as a car accelerates from, say, 0m/s to 20m/s, at some point it will have taken on every speed between these values; where it has had .
- ▶ real numbers can be positive or negative.

If  $f$  is a function from  $X$  to  $Y$ ...

- ▶ The set  $X$  is called the **domain** of the function.
- ▶ The set  $Y$  is called the **codomain**.
- ▶ When we write  $y = f(x)$ , we say “ $x$ ” is the **argument** of the function.
- ▶ When  $y = f(x)$  for some  $x \in X$ ,  $y$  is said to be the **image** of  $x$  under  $f$ .
- ▶ The set of all images  $y = f(x), x \in X$ , is called the **range** (or **image set**) of  $f$ .

- ▶ While we could have functions between any pair of sets (e.g., a function from students in this class to their ID numbers), usually  $X$  and  $Y$  are *sets of numbers*.
- ▶ It is not necessary for all elements  $y$  of the codomain  $Y$  to be images under  $f$ .
- ▶ One element  $y \in Y$  can serve as value  $f(x)$  for several  $x \in X$ .

A function can be represented in different ways:

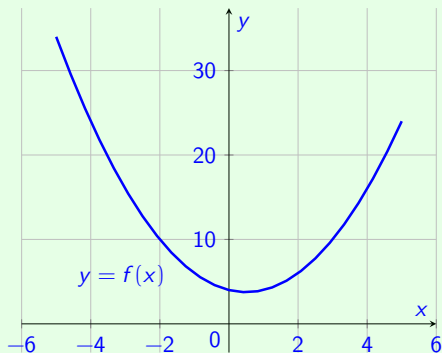
1. **verbally** (by a description in *words*);
2. **numerically** (as a *table* of values);
3. **visually** (as a *graph*);
4. **algebraically** (by an explicit *formula*).

Often it is possible, and useful, to go from one way to another.

# Graphical Representation

## Graph $\rightarrow$ Table

A common way to *visualize* a function  $f: X \rightarrow \mathbb{R}$  is its *graph* in the  $x, y$ -plane. In this example,  $f(x) = x^2 - x + 4$ .



$x$	$f(x)$
-4	24
-2	10
0	4
2	6
4	16

Often, the domain of a function is not explicitly stated.  
In such a case the following **Domain Convention** applies.

The **domain** of a function  $f$  is the set of all numbers  $x$  for which  $f(x)$  *makes sense* and gives a *real-number output*.

### Example

1. Find the subset of  $\mathbb{R}$  that is the **domain** of  $f(x) = \frac{1}{x^2 - x}$ .

**Example**

Find the subset of  $\mathbb{R}$  that is the **domain** of the function  $f(x) = \sqrt{x+2}$ .



**Example**

Given the function  $f_1(x) = 3x^2 + 1$ , find the largest subset of  $\mathbb{R}$  that is the domain of  $f_1$ . What is the corresponding **range**?

**Example**

Identify the domain (in  $\mathbb{R}$ ) and range of

$$f_2(x) = \sqrt{(x+4)(3-x)}$$

**Example**

Identify the domain and range of  $f_3(x) = \frac{1}{x}$ .

## Exercise 1.1.1

Identify the largest possible subset of  $\mathbb{R}$  that could be the domain and range of these functions:

1.  $f(x) = (x - 4)^2 + 5$

2.  $f(x) = \sqrt{3x + 2} = 1$

3.  $f(x) = 3/(x - 2)$ .

(See Example 1.1.2 of the textbook).