MA385: Tutorial 4

These exercises are for Tutorial 4 (Week 10). You do not have to submit solutions to these *questions.*

Q1. For the following functions show that they satisfy a Lipschitz condition on the corresponding domain, and give an upper-bound for L:

(i) $f(t, y) = 2yt^{-4}$ for $t \in [1, \infty)$,

(ii) $f(t, y) = 1 + t\sin(ty)$ for $0 \leqslant t \leqslant 2$.

Q2. Suppose we use Euler's method to find an approximation for $y(2)$, where y solves

$$y(1) = 1, \qquad y' = (t - 1)\sin(y).$$

(i) Give an upper bound for the global error taking $n = 4$ (i.e., $h = 1/4$).

(ii) What n should you take to ensure that the global error is no more that 10^{-3}?

Q3. Here is the tableau for a three stage Runge-Kutta method:

$$\begin{array}{r||lll} \alpha_1 & & & \\ \alpha_2 & \beta_{21} & & \\ \alpha_3 & \beta_{31} & \beta_{32} & \\ \hline\hline & b_1 & b_2 & b_3 \end{array} = \begin{array}{c||lll} 0 & & & \\ \alpha_2 & 1/2 & & \\ 1 & \beta_{31} & 2 & \\ \hline\hline & 1/6 & b_2 & 1/6 \end{array}$$

(i) Use that the method is consistent to determine b_2.

(ii) The method is exact when used to compute the solution to

$$y(0) = 0, \quad y'(t) = 2t, \ t > 0.$$

Use this to determine α_2.

(iii) The method should agree with an appropriate Taylor series for the solution to $y'(t) = \lambda y(t)$, up to terms that are $\mathcal{O}(h^3)$. Use this to determine β_{31}.