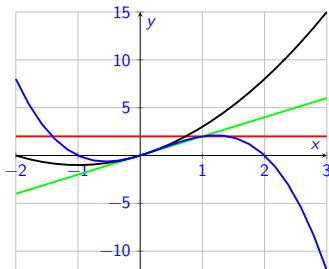


More About Functions

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17 September, 2025



The function of today's class is to learn about:

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|--|---|
| 1 The function of today's class: | 4 Graphical Representation |
| 2 Functions: notation <ul style="list-style-type: none">■ Domain of a function | 5 A Catalog of Functions <ul style="list-style-type: none">■ Linear functions |
| 3 4 Ways to Represent a Function | 6 Polynomials |
| | 7 Exercises |

For more, see Sections 1.1 and 1.2 of [https://math.libretexts.org/Bookshelves/Calculus/Calculus_\(OpenStax\)/01%3A_Functions_and_Graphs](https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/01%3A_Functions_and_Graphs)

Functions: notation

Recall: This section is all about **functions**, which a “rule” for mapping inputs to outputs.

1. Writing $f : A \rightarrow B$ means the inputs come from the set A , and the outputs come from the set B . (A **set** is just a collection of things).
2. A is called the **domain**, and B is called the **co-domain**.
3. $y = f(x)$ means “ x gets mapped to y according to the rule defined by f ”. We sometimes also say “ y is the image of x ”.
4. The subset of B that contains all the images of the things in A is called the **range** of f .
5. When we write $x \in A$ we mean “ x is an element of A ”, or “ x belongs to A ”.

Often, the domain of a function is not explicitly stated.
In such a case the following **Domain Convention** applies.

The **domain** of a function f is the set of all numbers x for which $f(x)$ *makes sense* and gives a *real-number output*.

Example

1. Find the subset of \mathbb{R} that is the **domain** of $f_1(x) = \frac{1}{x^2 - x}$.

Example

Find the subset of \mathbb{R} that is the **domain** of the function $f_2(x) = \sqrt{x+2}$.

Example

Given the function $f_3(x) = 3x^2 + 1$, find the largest subset of \mathbb{R} that is the domain of f_3 . What is the corresponding **range**?

Example

Identify the domain (in \mathbb{R}) and range of

$$f_4(x) = \sqrt{(x+4)(3-x)}$$

Example

Identify the domain and range of $f_5(x) = \frac{1}{x}$.

4 Ways to Represent a Function

A function can be represented in different ways:

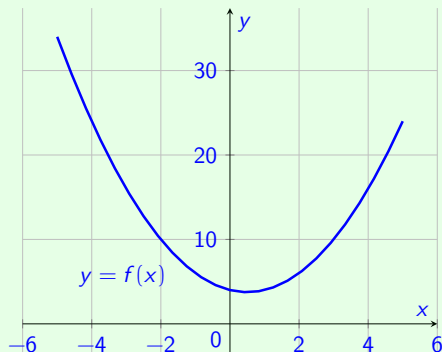
1. **verbally** (by a description in *words*);
2. **numerically** (as a *table* of values);
3. **visually** (as a *graph*);
4. **algebraically** (by an explicit *formula*).

Often it is possible, and useful, to go from one way to another.

Graphical Representation

Graph \rightarrow Table

A common way to *visualize* a function $f: X \rightarrow \mathbb{R}$ is its *graph* in the x, y -plane. In this example, $f(x) = x^2 - x + 4$.



x	$f(x)$
-4	24
-2	10
0	4
2	6
4	16

A Catalog of Functions

There are many *different types of functions* that can be used to *model relationships* between objects in the *real world*.

The most common types of functions (in MA140) are:

- ▶ *Linear Functions,*
- ▶ *Polynomial Functions,*
- ▶ *Power Functions,*
- ▶ *Rational Functions,*
- ▶ *Algebraic Functions,*
- ▶ *Trigonometric Functions,*
- ▶ *Exponential Functions,*
- ▶ *Logarithms.*

Linear functions have formulae such as $f(x) = mx + c$, where m and c are some given numbers.

It is often represented graphically as a straight line of slope m through the point $(0, c)$.

Polynomials

A **polynomial function** (or just **polynomial**) is a function of the form

$$y = f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0, \quad x \in \mathbb{R},$$

where a_0, a_1, \dots, a_n are real numbers called the **coefficients** of the polynomial.

The number n is called the **degree** of the polynomial.

There are special names for polynomials of low degree:

Exercise 1.2.1

Identify the largest possible subset of \mathbb{R} that could be the domain and range of these functions:

1. $f(x) = (x - 4)^2 + 5$

2. $f(x) = \sqrt{3x + 2} - 1$

3. $f(x) = 3/(x - 2)$.

(See Example 1.1.2 of the textbook).