MA140: Engineering Calculus

Lecture 2: Functions

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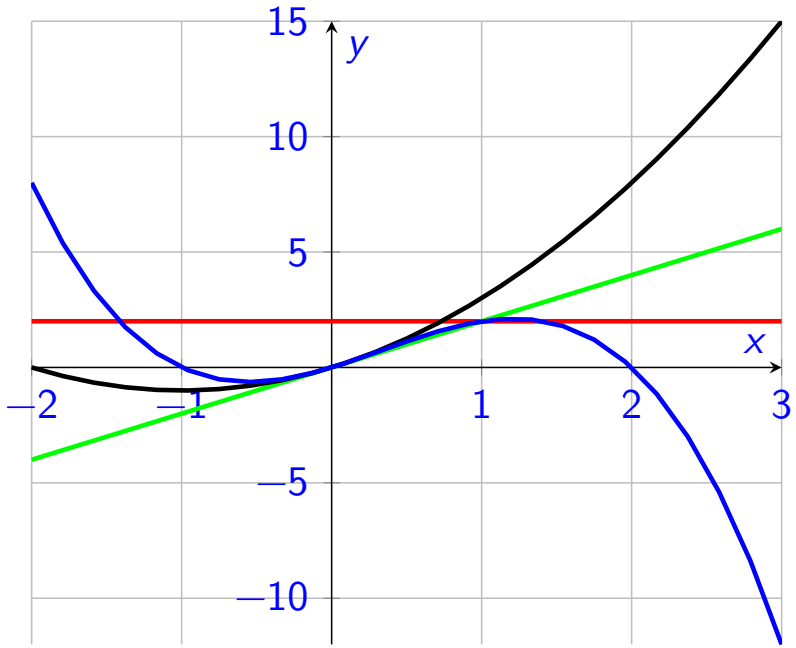


This version of the slides are by Niall Madden, but are adapted from original notes by Dr Kirsten Pfeiffer.

Outline

For more, see Chapter 2 of *Modern Engineering Mathematics*:
https://search.library.nuigalway.ie/permalink/f/3b1kce/TN_cdi_askewsholts_vlebooks_9780273742517

MA313
Lecture 2: Functions

Start of ...

Section 1: Functions

The single most important concept in MA140 is that of a **function**.
For more, see Chapter 2 of "Modern Engineering Calculus (James).

We represent a function symbolically in two ways, either

$$f : x \to y$$

or

$$y = f(x)$$

Here x is in the set of X (or $x \in X$), and y is in the set of Y (or $y \in Y$).

If f is a function from X to Y...

- The set X is called the **domain** of the function.
- The set Y is called the **codomain**.
- When we write $y = f(x)$, we say "x" is the **argument** of the function.
- When $y = f(x)$ for some $x \in X$, y is said to be the **image** of x under f.
- The set of all images $y = f(x), x \in X$, is called the **range** (or **image set**) of f.

- While we could have functions between any pair of sets (e.g., a function from students in this class to their ID numbers), usually X and Y are *sets of numbers*.
- It is not necessary for all elements y of the codomain Y to be images under f.
- One element $y \in Y$ can serve as value $f(x)$ for several $x \in X$.

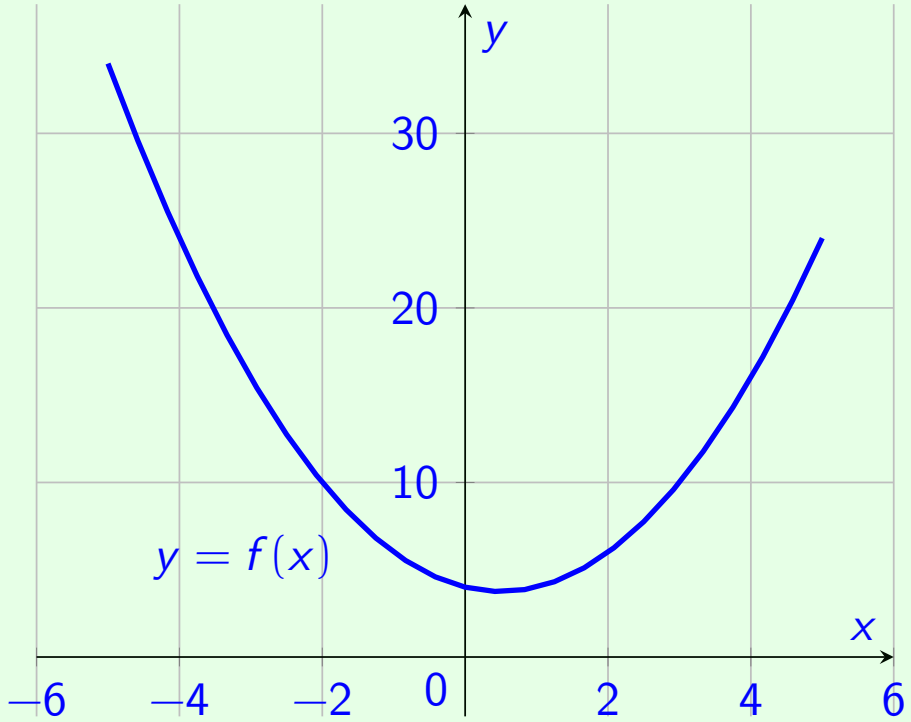
A function can be represented in different ways:

1. **verbally** (by a description in *words*);
2. **numerically** (as a *table* of values);
3. **visually** (as a *graph*);
4. **algebraically** (by an explicit *formula*).

Often it is possible, and useful, to go from one way to another.

Graph $\to$ Table

A common way to *visualize* a function $f : X \to \mathbb{R}$ is its *graph* in the x, y-plane. In this example, $f(x) = x^2 - x + 4$.



x	$f(x)$
-4	24
-2	10
0	4
2	6
4	16

Often, the domain of a function is not expilicitly stated.
In such a case the following **Domain Convention** applies.

The **domain** of a function f is the set of all numbers x for which $f(x)$ *makes sense* and gives a *real-number output*.

Example

1. Find the subset of $\mathbb{R}$ that is the **domain** of $f(x) = \dfrac{1}{x^2 - x}$.

Example

Find the subset of $\mathbb{R}$ that is the **domain** of the function $f(x) = \sqrt{x+2}$.

Example

Given the function $f_1(x) = 3x^2 + 1$, find the largest subset of $\mathbb{R}$ that is the domain of f_1. What is the corresponding **range**?

Example

Identify the domain (in $\mathbb{R}$) and range of
$f_2(x) = \sqrt{(x+4)(3-x)}$

Example

Identify the domain and range of $f_3(x) = \frac{1}{x}$.

A Catalog of Functions

There are many *different types of functions* that can be used to *model relationships* between objects in the *real world*.

The most common types of functions (in MA140) are:

- *Linear* Functions,
- *Polynomial* Functions,
- *Power* Functions,
- *Rational* Functions,
- *Algebraic* Functions,
- *Trigonometric* Functions,
- *Exponential* Functions,
- *Logarithms*.

Linear functions have formulae such as $f(x) = mx + c$, where m and c are some given numbers.

It is often represented graphically as a straight line of slope m through the point $(0, c)$.

Polynomials

A **polynomial function** (or just **polynomial**) is a function of the form

$$y = f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0, \quad x \in \mathbb{R},$$

where $a_0, a_1, \ldots, a_n$ are real numbers called the **coefficients** of the polynomial.
The number n is called the **degree** of the polynomial.

Exercises

Exercise 1.2.1

Identify the largest possible subset of $\mathbb{R}$ that could be the domain of $f_2(x) = \sqrt{(x+4)(3-x)}$ if the co-domain is $\mathbb{R}$. What is the range?