MA140: Engineering Calculus

# Week 3: Exercises (and answers)

*This is a collection of exercises from this week's MA140 lectures. You don't have to submit solutions for these, but you should work through them. Some may be similar to questions on the final exam.*

## Exercise 3.1.1 (from 2023/24 Q1(b))

Evaluate

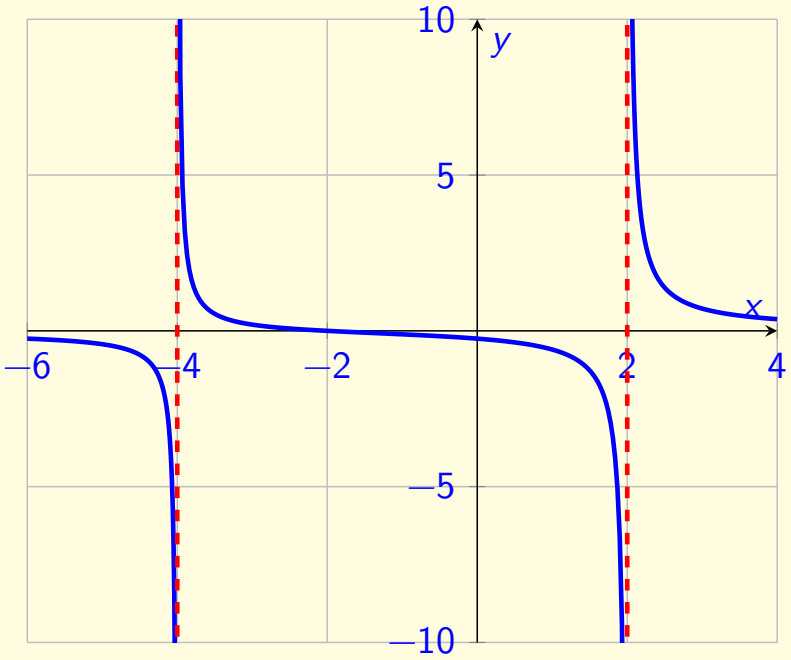
$$\lim_{\theta \to 0} \frac{2\sin(\theta)}{\theta + 3\tan(\theta)}$$

Ans: $1/3$

## Exercise 3.2.1

Find all the vertical asymptotes of $f(x) = \dfrac{x+2}{x^2+2x-8}$.

**Answer:** There are two asymptotes: one at $x = -4$ and the other at $x = 2$



## Exercises 3.3.1 (Based on Q1(a), 23/24)

Let $g(x) = \begin{cases} 3 & x \leqslant 0 \\ 2x+1 & 0 < x < 1 \\ x^2 & x \geqslant 1. \end{cases}$

(i) Sketch the graph of $g(x)$ on the interval $[-3, 4]$, making use of the empty and full circle notation. Answer not (yet) included

(ii) Compute $\lim_{x \to 1^-} g(x)$ and $\lim_{x \to 1^+} g(x)$. Is $g$ continuous at $x = 1$. If not, classify the type of discontinuity.

$\lim_{x \to 1^-} g(x) = 3$ and $\lim_{x \to 1^+} g(x) = 1$. This is a **jump** discontinuity

## Exercise 3.3.2

For what values of $b$ and $c$ is $f(x) = \begin{cases} x^2 + 1 & x \leqslant -1 \\ x + b & -1 < x < 1 \\ cx^2 & x \geqslant 1. \end{cases}$

continuous at $x = -1$ and $x = 1$? $b = 3,\ c = 4$

## Exercise 3.3.3 (23/24 Q(1)(c)(ii))

Use the IVT to show that the equation $x^3 - 3x + 1 = 0$ has three solutions in the range $-2 < x < 2$.

Ans: Check the sign of $f$ at $x = -2$, $x = -1$, $x = 1$ and $x = 2$