2526-MA378: Assignment 1

Information on the deadline, weighting, collaboration policy, submission process, etc, is on Canvas. Direct link: https://universityofgalway.instructure.com/courses/46941/assignments/140160

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Q1. (Based on the 22/23 exam paper) Let $\{x_0, x_1, x_2\} := \{1, 3, 5\}$ be a set of interpolation points. Write down the formulae for the associated Lagrange Polynomials, $\{L_0, L_1, L_2\}$, and give a rough sketch of them, clearly indicating their values at the interpolation points.

Suppose we define $q(x) := 1 - L_0(x) - L_1(x) - L_2(x)$. Show that, in fact, $q(x) \equiv 0$, for **all** real numbers x.

Q2. Let $f(x) = x^{3/2}$. Give the Lagrange form of p_2, the polynomial interpolant to f at at $\{x_0, x_1, x_2\} = \{1, 3, 5\}$. Use Cauchy's Theorem to give an upper bound for $|f(4) - p_2(4)|$. How does this compare with the actual error?

Q3. Let $f(x) = x^{3/2}$ again. Write down the formula for the linear spline, l, which interpolates f at $\{x_0, x_1, x_2\} = \{1, 3, 5\}$.
Use the relevant theorem from Section 2.1 (Linear Splines) to give an upper bound for $|f(4) - l(4)|$. How does this compare with the actual error?