2526-MA140 Engineering Calculus

Week 05, Lecture 2
Implicit Differentiation; Exponential Functions

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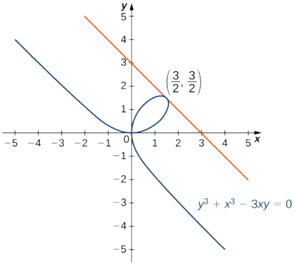


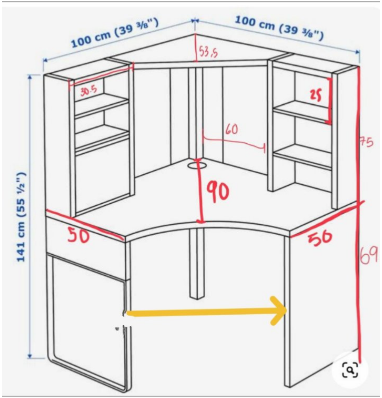
Assessment Schedule for the Rest of the Semester

- Week 6 (next week): **Assignment 3** due 17:00, Monday, 20 Oct.
- Week 7: **Assignment 4** (which just opened) due 17:00, Tuesday, 28 Oct.
- Week 8: **Assignment 5** due 17:00, Monday, 3 Nov.
- Week 9: **Assignment 6** due 17:00, Monday, 10 Nov.
- Week 10: **Assignment 7** due 17:00, Monday, 17 Nov.
- Week 10: **Class Test 2** 10:00, Tuesday 18 Nov.
- Week 11: **Assignment 8** due 17:00, Monday, 24 Nov.

About the online assignments

- The purpose of the assignments is to maintain engagement and develop confidence.
- When you submit an answer, you get immediate feedback on if your answer is correct or not.
- If not, you usually have up to 4 more attempts (except for "true/false" type questions).
- Each assignment is worth (only) about 1.25% of your final grade.
- Most assignments have 6 questions, so each is worth about 0.2%.
- Tutorial sheets are based on the same assignments. Solutions to those are posted after the deadline (usually).
- Collaboration is encouraged. Engagement with SUMS is especially encouraged.
- The system is very well established, and bugs are rare. However, integration with Canvas is not perfect. When I allow you to see your "live" grade, it sends a message saying the assignment has been graded.

Remember "Olive's desk"?

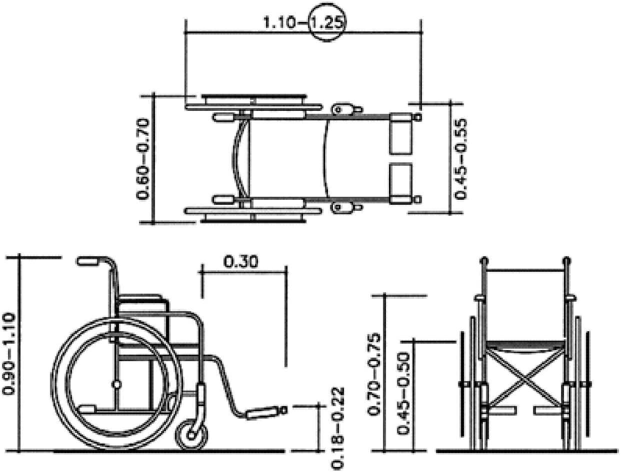


Source: IKEA catalogue

Last week, I told you that "Olive" was thinking of buying this "MICKE" corner desk unit in IKEA. Her (wheel)chain is 55cm wide. Is the sitting region of the desk indicated by the yellow line, wide enough?

1. What do you think the answer is?
2. But actually..

Remember "Olive's desk"?



Source: *https://www.un.org/esa/socdev/enable/designm/AD5-02.htm*

Today, in Engineering Calculus...

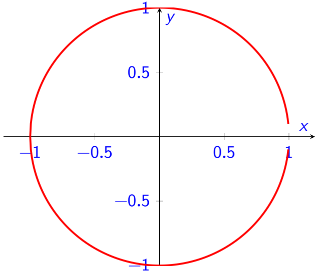
See also: Sections 3.8 (Implicit Differentiation) and 3.9 (Derivatives of Exponential and Logarithmic Functions) of **Calculus** by Strang & Herman: https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)

Implicit differentiation

Last week, we introduced the idea of an *implicitly defined function*:

- **Explicit**: given a value of x, we have a formula for computing the (single) corresponding value of y;
- **Implicit**: the formula relates the variables, without giving an explicit value of one (y) in terms of the other (x).

Classic example: $x^2 + y^2 = 1$. For any pair (x, y) we can check if it is on the curve described by the equation.



Implicit differentiation

Since **implicit equations** define curves, we can use **implicit differentiation**, for example to find a tangent to an implicitly defined curve.

Method:

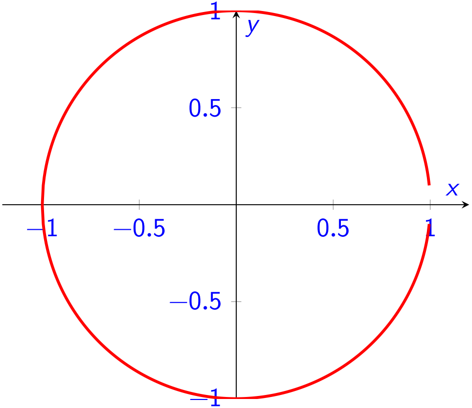
1. Differentiate both size of the equation, with respect to x, keeping in mind that y is a function of x, using the **Chain Rule** where needed.
2. Solve for dy/dx.

Implicit differentiation

If y is defined by $x^2 + y^2 = 1$, find $\frac{dy}{dx}$.

Implicit differentiation

Now we know that if $x^2 + y^2 = 1$, then $\dfrac{dy}{dx} = -\dfrac{x}{y}$. We can check that this relates to the slope of the tangents to this curve at various places:



Implicit differentiation

> Find the tangent to the curve $x^2+y^2=25$, at the point $(3,-4)$.

(*Details here were added after class*).
First check that the point $(x,y)=(3,-4)$ is on the curve:
$(3)^2+(-4)^2=9+16=25$, as required.
Now differentiate:

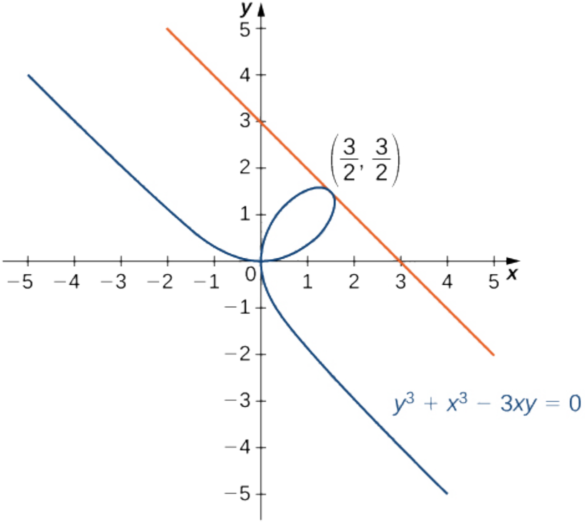
$$\frac{d}{dx}\left[x^2+y^2\right]=\frac{d}{dx}[25] \qquad \implies \frac{d}{dx}\left[x^2\right]+\frac{d}{dx}\left[y^2\right]=0$$
$$\implies 2x+2y\frac{dy}{dx}=0 \qquad \implies \frac{dy}{dx}=-\frac{x}{y}.$$

So the slope of the tangent to the curve at $x_1=3$ and $y_1=-4$ is $M=-x/y=3/4$.
The equation of the tangent comes from $y-y_1=M(x-x_i)$. For our values of $x_1=3$, $y_1=-4$ and $M=3/4$, we get
$y=(-4)+(3/4)(x-3)$. That is $\boxed{y=\frac{3}{4}x-\frac{25}{4}}$.

Implicit differentiation

Find the tangent to the curve $y^3 + x^3 - 3xy = 0$, at the point $(3/2, 3/2)$.

Implicit differentiation



Exponential functions

Earlier in this course we met functions such as $y = x^2$; this is a **power** function.

Now we consider **exponential functions**, such as $\boxed{y = 2^x}$.

Such functions occur in many applications. For example: if I invest €100 with an annual interest rate of 20%, then after x years, I will have €$100 \times (1.2)^x$. **Why?**

Exponential functions

Exponential functions grow quite fast: if my investment is indeed worth $f(x) = 100 \times (1.2)^x$ euros after x years, then...

- After 1 year, I have €120
- After 10 years, I have €619.17
- After 20 years, I have €3,833.80
- After 25 years, I have €9,539.60
- After 50 years, and 190 days, I'll be a millionaire!

Properties of exponents

Here I remind you of some properties of exponents that you should already know: for any positive numbers a and b, and for all x and y,

1. $b^x b^y = b^{x+y}$
2. $\frac{b^x}{b^y} = b^{x-y}$
3. $(b^x)^y = b^{xy}$
4. $(ab)^x = a^x b^x$
5. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

There is more information on this in Section 1.5 of the textbook (Exponential and Logarithmic Functions). ($\longleftarrow$ link to book, which does not work in annotated notes)

Exercises

Exercise 5.2.1

Find the derivative of

1. $f(x) = x^3 \cos(x^2)$
2. $f(x) = \tan^3\left(\sin^2(x^4)\right)$

Exercise 5.2.2

Show that $\dfrac{d}{dx}(\cos^{-1} x) = \dfrac{-1}{\sqrt{1-x^2}}$.

Exercise 5.2.3

Find the equation of the tangent to the curve defined by $x^2 - y^2 = 16$ at the point $(5, 3)$.