Numerically accurate formulation of implicit turbulent bottom stress in an ocean model with barotropic-baroclinic mode splitting

Boundary and Interior Layers, 2024

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Ollscoil na Gaillimhe University of Galway

Foras na Mara Marine Institute

The team

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The overarching problem

We aim to identify and address two issues impacting on modern oceanic codes

(i) the need to include bottom drag into implicit solver for vertical viscosity (and reconcile complications this induces)

(ii) the relationship between the drag coefficient, r_D and computed velocity in the bottom cell.

Bottom drag plays an important role in dissipating tides, and becomes one of the dominant forces in tidal bays and estuaries. One of such places is Galway Bay, Ireland, where tidally induced currents can reach the speed of 2 m/sec, posing challenges in hydrodynamic modeling, essentially due to the interference of different algorithms, which need to work in concert, but originally were not thought to be this way.

The focus of the talk

We want to be able to represent bottom drag in a reasonable way on **coarse grid**.

- Propose a formula for the bottom drag;
- Consider convergence (numerically!) of the model for an Ekman problem
- Present results of a model of Galway Bay.

For more detail, see niallmadden.ie/docs/BAIL2024-AFS.pdf

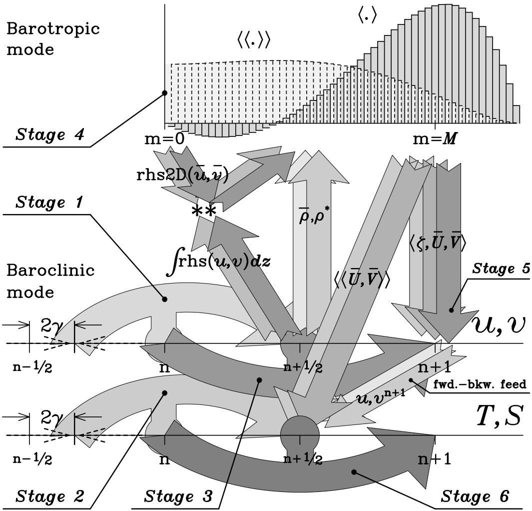
Regional Ocean Modeling System (ROMS)

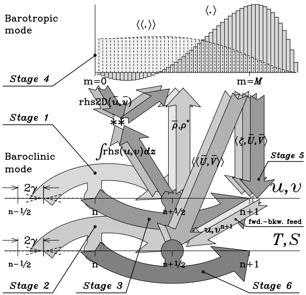
ROMS is a free-surface, terrain-following, primitive equations ocean model; see [Shchepetkin and McWilliams, 2003, Shchepetkin and McWilliams, 2005]. Also `https://www.myroms.org`.

It is rather complex... some (numerical analysis) highlights

- 2D and 3D equations are time-discretized using a third-order accurate predictor (Leap-Frog) and corrector (Adams-Molton) time-stepping algorithm;
- The default stencil uses centered, second-order finite differences on a staggered vertical grid.
- In the horizontal, the primitive equations are evaluated using boundary-fitted, orthogonal curvilinear coordinates on a staggered Arakawa C-grid.

ROMS time-stepping alg with barotropic mode splitting





- Arched horizontal arrows indicate progression of slow-time (baroclinic) variables: u, v, T, S.
- 4 ascending arrows represent computation of 3D$\rightarrow$ Barotropic Mode (BM) forcing terms: vertically integrated r.h.s. terms for 3D momentum equations and 2-way vertically averaged densities, $\overline{\rho}$ and ρ_*.
- 5 descending arrows represent updated barotropic fluxes (vertically integrated velocities), which are 2-way averaged in fast time.
- Vertical integrals of intermediate values $u^{n+1/2}, v^{n+1/2}$ (computed during predictor stage) are forced from the respective barotropic values at the previous time step n.

Issue 1: The split-explicit time stepping procedure for an oceanic model (e.g. ROMS, [Shchepetkin and McWilliams, 2005]) implies that the barotropic mode system (vertical integrals of horizontal velocities, coupled with contribution of free-surface elevation into vertical integrals of pressure-gradient terms) is solved separately from the rest of the model using **smaller time step**.

This leads to significant computational savings, because the **large number of short time steps are applicable only for 2D part** of the whole 3D model.

However, this also results in more complicated code, carefully designed to avoid numerical errors and instability.

Issue 2: At the same time, vertical processes—mixing of tracers, viscous exchange of momenta, and, recently, vertical advection (only where it is strictly unavoidable: [Shchepetkin, 2015])–are treated implicitly, but only in a one-dimensional manner resulting in a simple and efficient solver.

Issue 3: Also require parameterization of vertical profile of turbulent mixing coefficient along with kinematic stress bottom boundary condition, which is of the no-slip type, but nonlinear in nature due to the fact that both bottom drag coefficient and vertical viscosity profile depend on the magnitude of the current ([Soulsby, 1983]).

Problem: the barotropic mode needs to know the **bottom drag terms** in advance (which can be computed only within the 3D part of the code), but when done, the result barotropic mode calculation adjusts the horizontal velocity components in the 3D mode, compromising both the no-slip boundary conditions and the consistency of bottom stress with vertical viscosity profile.

The plan

In this study we propose modification to existing algorithms to eliminate the splitting errors associated with bottom drag.
Special attention is also paid to a **bottom boundary condition**, capable of dealing with under-resolved boundary layer in vertical mixing parameterization scheme.

Why bottom drag term does not naturally fit into this data flow and requires special attention?

The **bottom drag** term relates to the net force applied to the fluid in bottom-most grid box Δz_1 with velocity inside it u_1,

$$\frac{\partial}{\partial t}(\Delta z_1 \cdot u_1) + \cdots = -r_D \cdot u_1$$

thus is to be interpreted in finite-volume sense.

The **drag coefficient**, r_D, may be constant, i.e., a finite-difference or finite-volume approximation based on constant viscosity, no-slip boundary condition and assumption about velocity profile (hence its derivative at the bottom), or it may depend on velocity itself (nonlinear, as it happens in turbulent mixing schemes).

The situation when $r_D \Delta t/\Delta z_1 > 1$, where Δt is the baroclinic time step, requires implicit treatment of vertical viscosity, including bottom b.c.: it attempts to remove more momentum from grid-box Δz_1 than is available there, leading to oscillations and instability.

During the coupling stage, barotropic-mode corrections to the 3D-mode u and v components are applied uniformly throughout the entire vertical column, which unavoidably alters the bottom-most u_1, v_1, thus distorting the no-slip bottom b.c.

Recomputing bottom drag term after the barotropic corrections are applied restores the no-slip bottom b.c., but distorts the state of approximate non-divergence of barotropic fluxes.

Using Ekman spiral test problem to illustrate mode splitting error associated with bottom drag

Wind-driven Ekman problem for a layer of finite depth, h,

$$\frac{\partial u}{\partial t} - fv = \frac{\partial}{\partial z}\left(A\frac{\partial u}{\partial z}\right), \qquad \frac{\partial v}{\partial t} + fu = \frac{\partial}{\partial z}\left(A\frac{\partial v}{\partial z}\right),$$

subject to wind stress forcing, $A\frac{\partial u}{\partial z}\Big|_{z=0} = \tau_x\,, \quad A\frac{\partial v}{\partial z}\Big|_{z=0} = \tau_y\,,$ at surface

and no-slip b.c. $u\big|_{z=-h} = 0\,, \quad v\big|_{z=-h} = 0$ at bottom $z = -h$.

We are interested in the stationary solution with constant vertical viscosity A. That simplifies to

$$\mathrm{i}f(u+\mathrm{i}v) = A\partial^2_{zz}(u+\mathrm{i}v).$$

The general solution is

$$u+\mathrm{i}v = (\mathcal{U}_+ + \mathrm{i}\mathcal{V}_+)e^{+\sigma z} + (\mathcal{U}_- + \mathrm{i}\mathcal{V}_-)e^{-\sigma z},$$

$$\text{where} \quad \sigma = \frac{1+\mathrm{i}}{\sqrt{2}}\sqrt{\frac{f}{A}}, \quad \text{hence} \quad \sigma^2 = \mathrm{i}\sqrt{\frac{f}{A}},$$

The four constants, $\mathcal{U}_+, \mathcal{V}_+, \mathcal{U}_-, \mathcal{V}_-$, are to be determined to satisfy surface and bottom boundary condition.

No-slip boundary condition at $z = -h$ requires

$$(\mathcal{U}_+ + \mathrm{i}\mathcal{V}_+)e^{-\sigma h} + (\mathcal{U}_- + \mathrm{i}\mathcal{V}_-)e^{+\sigma h} = 0.$$

hence

$$u+\mathrm{i}v = (\mathcal{U}+\mathrm{i}\mathcal{V})\sinh\big(\sigma(z+h)\big)$$

leaving only two unknowns.

Applying the surface boundary condition

$$A\partial_z(u+iv)\Big|_{z=0} = A\sigma(\mathcal{U}+i\mathcal{V})\cosh\big(\sigma(z+h)\big)\Big|_{z=0} = \tau_x + i\tau_y$$

yields

$$\mathcal{U}+i\mathcal{V} = \frac{\tau_x + i\tau_y}{A\cdot\sigma\cdot\cosh(\sigma h)},$$

and finally

$$u+iv = \frac{\tau_x + i\tau_y}{A\sigma\cosh(\sigma h)}\sinh\big(\sigma(z+h)\big).$$

(Etc, further simplifications are possible, and one still needs to separate real and imaginary parts).

Now introduce Ekman depth: $h_E = \sqrt{A/f}$.

- $h/h_E < 1 \rightarrow$ "shallow" Ekman layer.
- $h/h_E > 1 \rightarrow$ "deep" Ekman layer.

If $h/h_E \gg 1$ this "asymptotes" to the classical Ekman spiral in infinitely deep barotropic fluid, with 45^0 angle to the right between velocity direction at surface and wind stress.

Vertical integration yields

$$\mathrm{i}f \int_{z=-h}^{z=0} (u + \mathrm{i}v)\,dz = \mathrm{i}fh\big(\bar{x} + \mathrm{i}\bar{v}\big)$$

$$= \mathrm{i}f \frac{\tau_x + \mathrm{i}\tau_y}{A\sigma\cosh(\sigma h)} \frac{\cosh\big(\sigma h\big) - 1}{\sigma} = \overbrace{\tau_x + \mathrm{i}\tau_y}^{\text{surface wind stress}} \underbrace{- \frac{\tau_x + \mathrm{i}\tau_y}{\cosh(\sigma h)}}_{\text{bottom drag}},$$

which means that, from the point of view of split barotropic mode, the stationary solution is a 3-way balance of net Coriolis force, surface wind stress, and bottom drag.

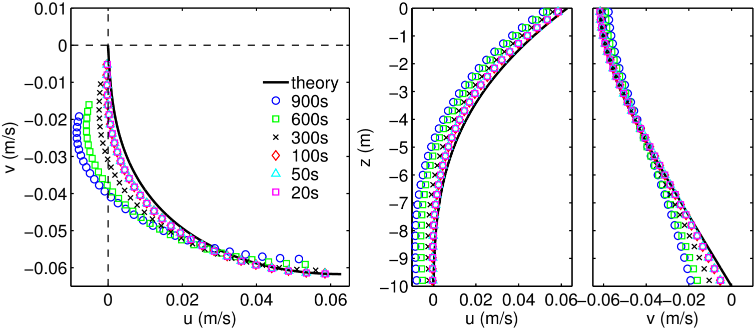
A split-explicit model, like ROMS, must be able to achieve this stationary state as the result of time stepping under steady wind forcing.

Wind-driven Ekman layer in finite-depth layer of fluid: test problem for ROMS (courtesy of Yusuke Uchiyama)

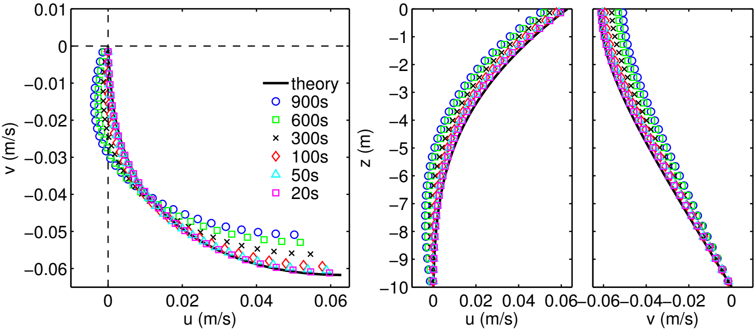
- $h = 10m$
- surface wind stress $u_* = 6 \times 10^{-2} m/s$ ($\approx 5m/s$ wind)
- $f = 10^{-4}$
- $A_v = 2 \times 10^{-3} m^2/s$
- non-slip b.c. at $z = -h$,
- $N = 30$; everything is constant in time,

Run from rest state until stationary solution is reached.

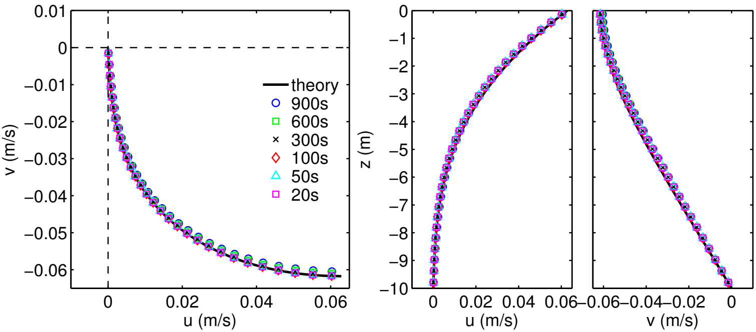
Case 1: Explicit, CFL-limited, bottom drag term computed **before** Barotropic Mode (BM) for **both** r.h.s. 3D and for BM forcing **(⇒ no splitting error)**; implicit step for vertical viscosity **after** with bottom drag excluded (⇒ undisturbed coupling of 2D and 3D); however needs $r_D < \Delta z_{\text{bottom}}/\Delta t_{3D}$ for stability



Case 2: Unlimited drag **before** BM applies for BM forcing **only**; implicit vertical viscosity **after** with drag included into implicit vertical solver **(⇒ no-slip bottom b.c. is OK)**, however the drag term is recomputed relative to what BM got before as input (⇒ splitting error)



Case 3: Bottom drag term is computed as a part of implicit vertical viscosity step **before** BM and for **both** 3D and BM forcing (more below).



Nonlinear bottom drag: a modeller prospective

The model needs

$$\Delta z_1 \frac{u_1^{n+1} - u_1^n}{\Delta t} = A_{3/2} \frac{u_2^{n+1} - u_1^{n+1}}{\Delta z_{3/2}} - r_D u_1^{n?} \qquad r_D = ?$$

where u_1 is understood in finite-volume sense

$$u_1 = \frac{1}{\Delta z_1} \int_{\text{bottom}}^{\text{bottom}+\Delta z_1} u\left(z'\right)\, dz'.$$

From physics STRESS $= F(u)$, $\quad F =?$

Denote: $u_\star$ is sheer velocity, and $z_\star$ is roughness length.

"Duality" of u_*: it controls *both* bottom stress and vertical viscosity profile

$$\text{STRESS} = u_*^2 \qquad \text{and} \qquad A = A(z) = \kappa u_* \cdot (z_* + z) \qquad z \to 0$$

constant-stress boundary layer

$$A(z) \cdot \partial_z u = \text{STRESS} = const = u_*^2,$$

$$\kappa u_* \, (z_* + z) \, \partial_z u = u_*^2$$

and hence

$$u(z) = \frac{u_*}{\kappa} \ln \left(1 + \frac{z}{z_*} \right)$$

So now,

$$u_1 = \frac{u_*}{\kappa}\left[\left(\frac{z_*}{\Delta z_1}+1\right)\ln\left(1+\frac{\Delta z_1}{z_*}\right)-1\right] \qquad \text{hence} \qquad u_* = \kappa u_1/[...]$$

and

$$-r_D \cdot u_1 = -\kappa^2\,|u_1|\underbrace{\left[\left(\frac{z_*}{\Delta z_1}+1\right)\ln\left(1+\frac{\Delta z_1}{z_*}\right)-1\right]^{-2}}_{f(\cdot)^{-2}}u_1.$$

Considering computing $f\left(\Delta z_1/z_*\right) = f(x)$, as $x \to 0$. As a Taylor expansion

$$f(x) = \left(1+\frac{1}{x}\right)\ln(1+x) - 1 = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^n}{n(n+1)} = \frac{x}{2}-\frac{x^2}{6}+\frac{x^3}{12}-\frac{x^4}{20}+\ldots$$

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From that

$$r_D = \kappa^2 |u_1| \Bigg/ \left[\left(\frac{z_*}{\Delta z_1} + 1 \right) \ln \left(1 + \frac{\Delta z_1}{z_*} \right) - 1 \right]^2 .$$

In the **well-resolved** asymptotic limit, i.e., $\Delta z_1 / z_* \ll 1$, this behaves as

$$r_D \sim 4\kappa^2 |u_1| \frac{z_\star^2}{\Delta z_1^2}.$$

Then

$$u(z) = \frac{u_*}{\kappa} \ln \left(1 + \frac{z}{z_*} \right) \sim \frac{u_*}{\kappa} \cdot \frac{z}{z_*} \qquad \text{hence} \quad u_1 = \frac{u_*}{\kappa} \cdot \frac{\Delta z_1}{2z_*},$$

resulting in

$$r_D \sim \kappa^2 u_* t \frac{2z_*}{\Delta z_1} = \frac{A_{\text{bottom}}}{\Delta z_1 / 2}$$

in line with no-slip with laminar viscosity.

In the **unresolved** case, $\Delta z_1/z_* \gg 1$, $\qquad r_D \sim \kappa^2 \, |u_1| \Big/ \ln^2\left(\dfrac{\Delta z_1}{z_*}\right)$

known as "log-layer"

This gives a smooth transition between resolved and unresolved limits, and avoids introduction of *ad hoc* "reference height" z_a, e.g., Soulsby (1995) formula.

Turbulent Ekman spiral: Convergence with respect to vertical resolution

The relationship between the magnitude of velocity at the bottom most grid box

$$u_* = r_D = |\mathbf{u}_1| \cdot \kappa^2 \Bigg/ \left[\left(\frac{z_*}{\Delta z_1} + 1 \right) \ln \left(1 + \frac{\Delta z_1}{z_*} \right) - 1 \right]^2$$

is radically different from commonly used in other models, e.g., in Princeton ocean Model POM,

$$u_* r_D = C_D \cdot |\mathbf{u}_1|$$

where

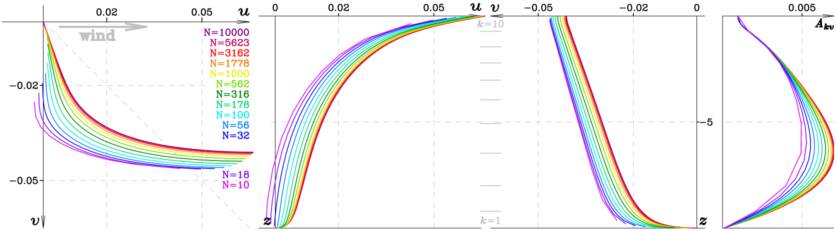
$$C_D = \max \left\{ C_{D\,\mathsf{min}},\ \min \left\{ \left[\kappa \Bigg/ \ln \left(\frac{\Delta z_1/2}{z_*} \right) \right]^2,\ C_{D\,\mathsf{max}} \right\} \right\}$$

canonically restricting $C_{D\,\mathsf{min}} = 0.0025 \leqslant C_D \leqslant 1 = C_{D\,\mathsf{max}}$.

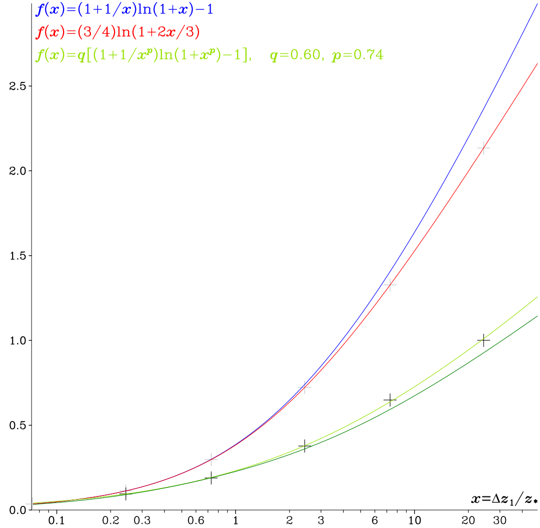
Very similar expressions appear in many modern codes—COAWST, CROCO, Rutgers ROMS, GETM, and others.

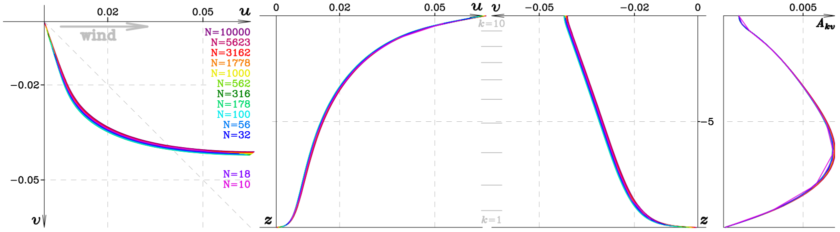
A convergence study for turbulent Ekman spiral test problem with respect to vertical resolution, for $N \to \infty$

$$r_D = |\mathbf{u}_1| \cdot \kappa^2 \Big/ \left[(1 + 1/x + 1) \ln (1 + x) - 1 \right]^2 \quad \text{where} \quad x = \Delta z_1 / z_*$$



Alternatives for representing $f(x)$



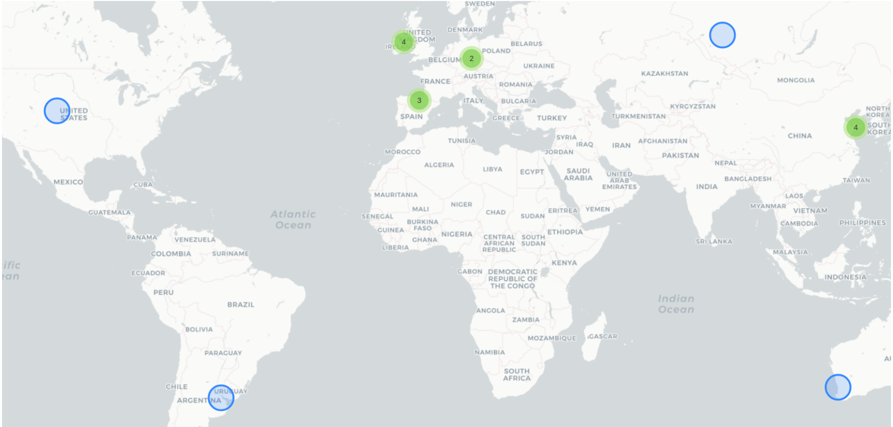


Convergence study for turbulent Ekman spiral problem for $N \to \infty$ using $f = f(x)$ from the previous slide.

Computational Model of Galway Bay

We'll finish with some results show the model applied to Galway Bay.

- Where is Galway Bay?
- Why is it interesting?
- Why is Galway Bay interesting?
- Model set up.
- Movies!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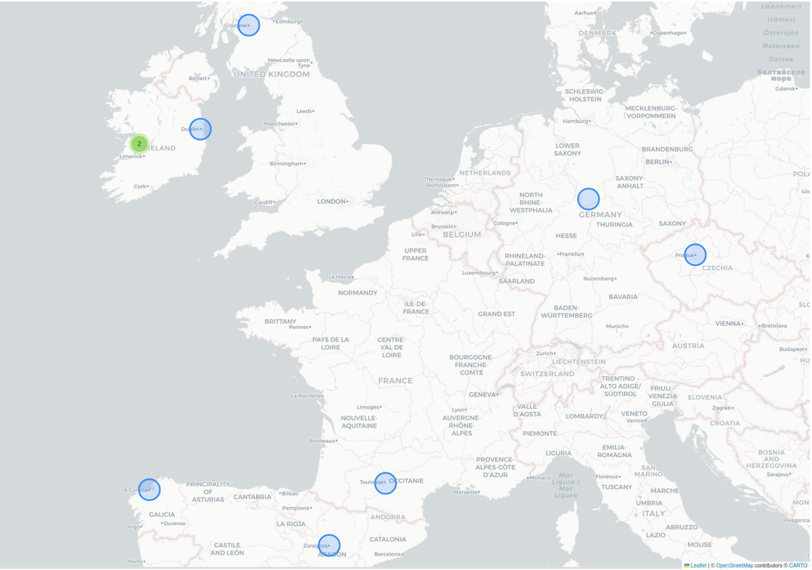
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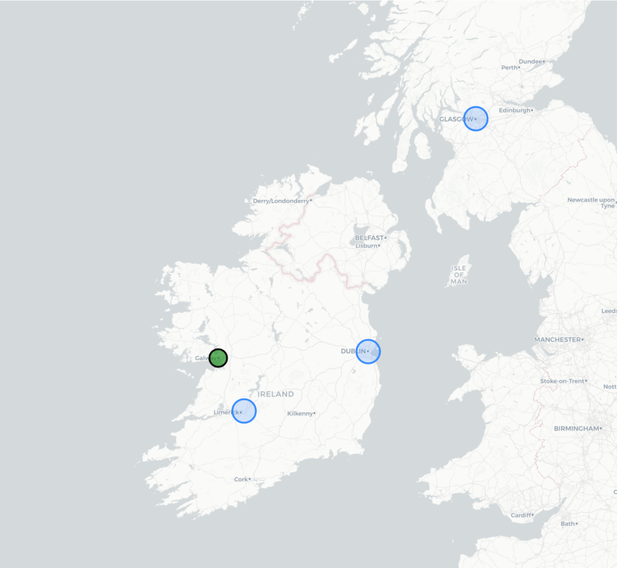
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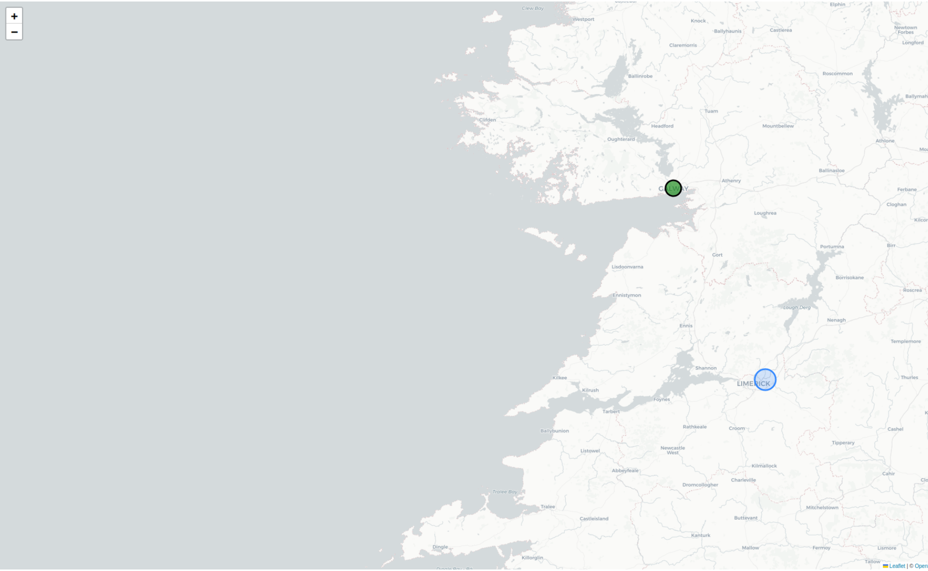
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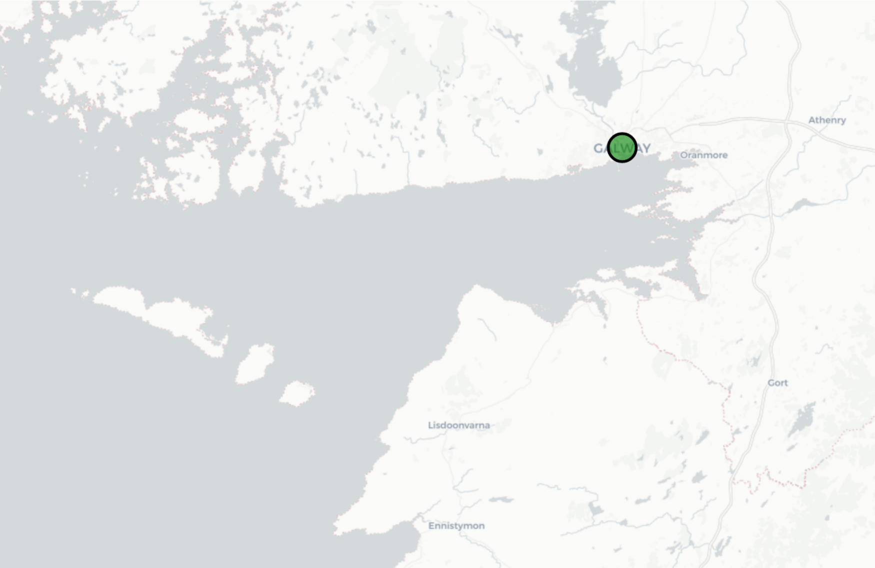
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About Galway

- Galway Bay takes its name from the city of Galway (which in turn takes its name from a large river that runs through it).
- It is university city since 1845, with a river-side campus: its Mathematics and Engineering schools date from its inception.
- We hope to host the Layer Phenomena Workshop in April 2025, ...

Shameless advertisement: 21st Annual Workshop on Numerical Methods for Problems with Layer Phenomena, April 2024

- Will take place on the campus of University of Galway.
- Takes place over 1.5 or 2 days, depending on the number of talks.
- No registration fee!
- Watch NA-Digest or `https://www.niallmadden.ie/`

Why Galway Bay?

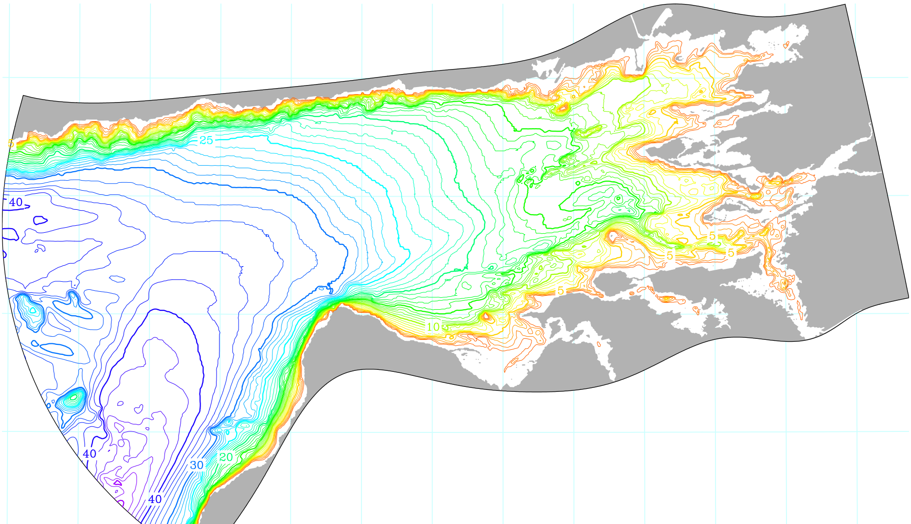
From a modelling perspective, the bay has some interesting features.

- It is a large bay (roughly 40km $\times$ 20km).
- Various islands at the entrance of the bay, and along the north shore.
- Significant discharge of fresh water from various rivers; *submarine groundwater discharge in karst region*.
- Tidal races and other phenomena that are challenging to model.
- Quite a lot of data (SmartBay: continuous oceanographic and environmental data in near-real time).

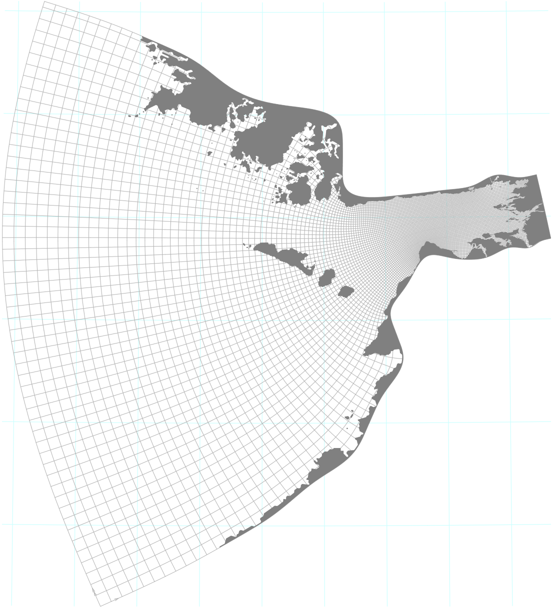
Galway Bay model setup

- ROMS code (UCLA branch)
- Orthogonal curvilinear grid in horizontal directions,
- Variable grid size $50 < \Delta x_{i,j} < 450$; locally $\Delta x_{i,j} = \Delta y_{i,j}$.
- For simulations shown, mesh has $1150 \times 322 \times 32$ ($\sim 12 \times 10^3$ DoFs).
- Time step for 3D mode: $\texttt{dt} = 20\,sec$
- Mode splitting ratio: $\texttt{ndtfast} = 14$
- Implicit bottom drag for both 3D and 2D.
- data input every hour, extracted from Marine Institute North-Eastern Atlantic model.

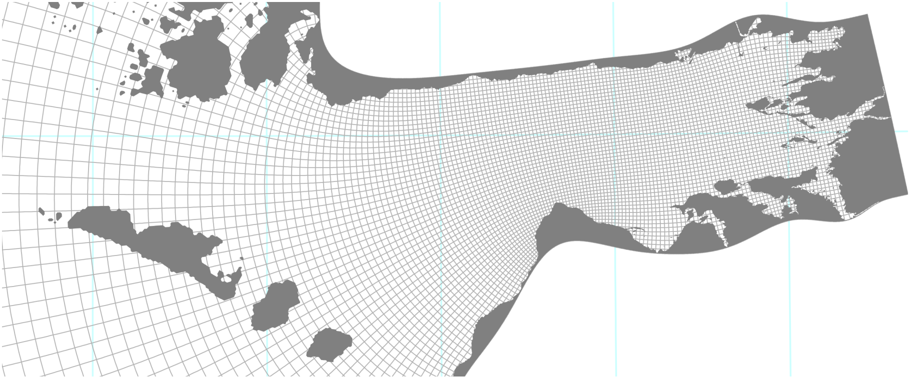
Galway Bay model bottom topography (inner bay only)



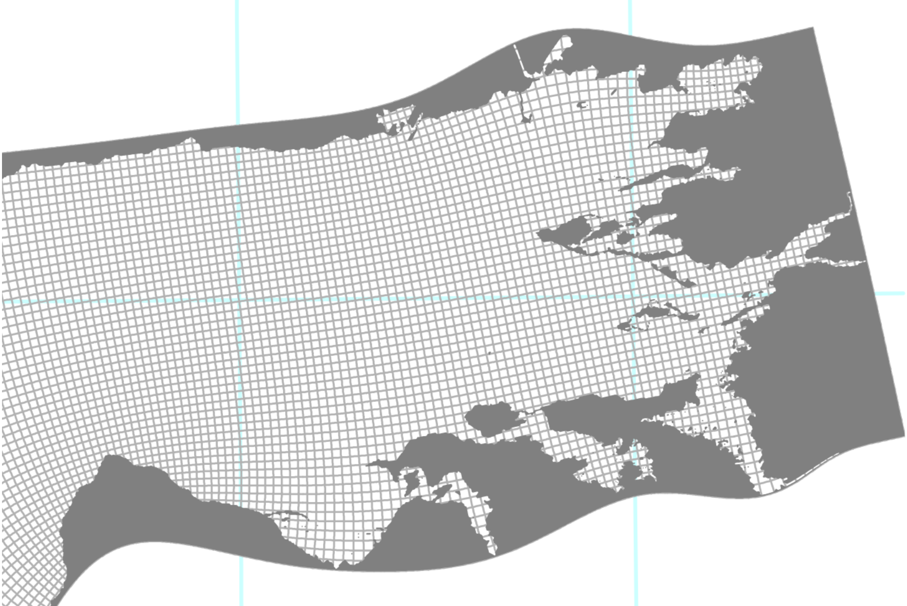
The finite difference grid (coarsened): full model



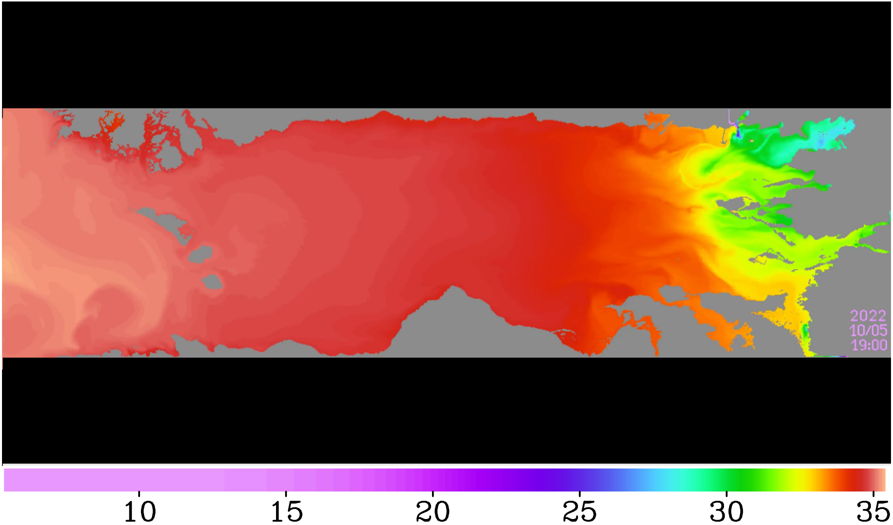
The finite difference grid (coarsened): inner bay



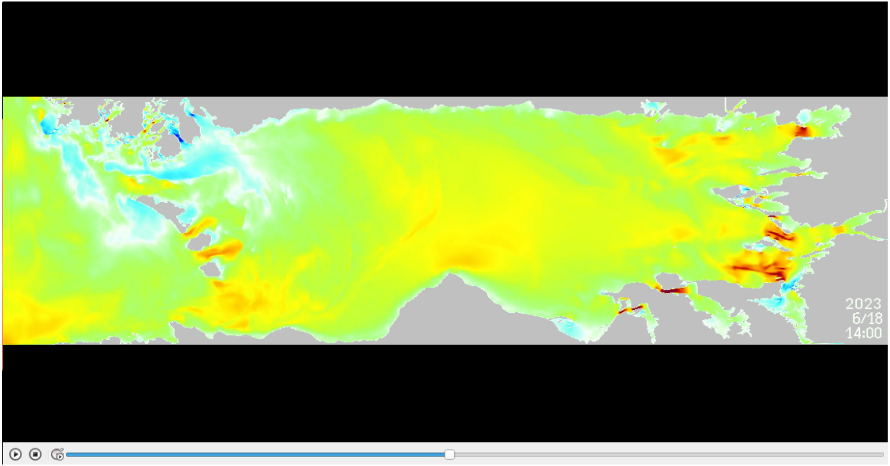
The finite difference grid (coarsened): western end



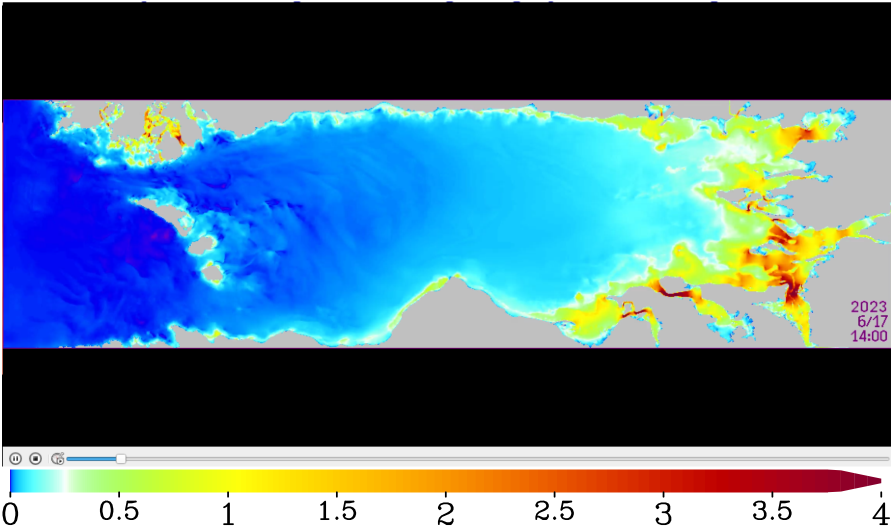
Example: salinity in Galway Bay, predicted by ROMS



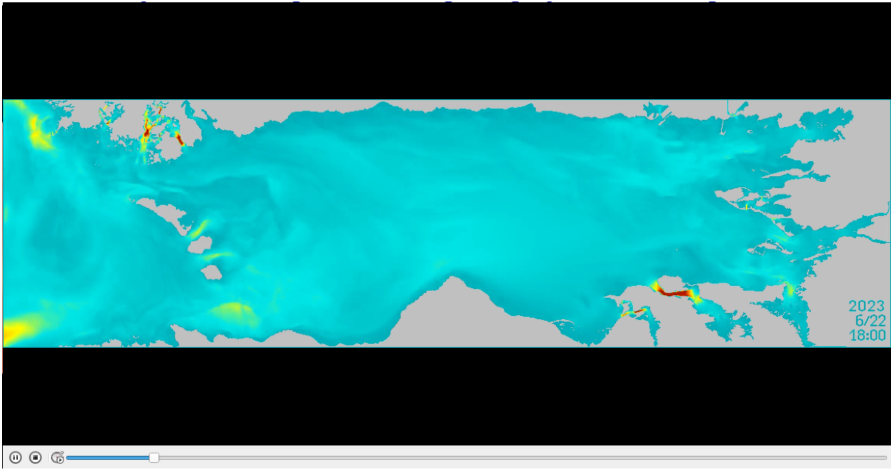
Surface velocities



$r_D \Delta_t / \Delta z_1$



Maximum velocity in a water column



An aside... maximum surface velocity is observed on the south of the bay, between the "island" of Aughinish, and mainland (it was truly an island between 1755 and 1811).

Implicit treatment of $-\Delta t \cdot r_D \cdot u_1^{n+1}$ term: should include it into implicit solver for vertical viscosity terms. However this interferes with Barotropic Mode (BM) splitting:

- Bottom drag can be computed only from full 3D velocity, not from the vertically averaged velocities alone.
- Barotropic Mode must know the bottom drag term *in advance* as a part of 3D→2D forcing for consistency of splitting. This places computing vertical viscosity before BM. But, later, when BM corrects the vertical mean of 3D velocities, the consistency of (no-slip like) bottom boundary condition is impacted.
- Current ROMS practice is to split bottom drag term from the rest of vertical viscosity computation. This limits the time step (or r_D itself) by the explicit stability constraint.
- Possible only in corrector-coupled and Generalized FB variants of ROMS kernels

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