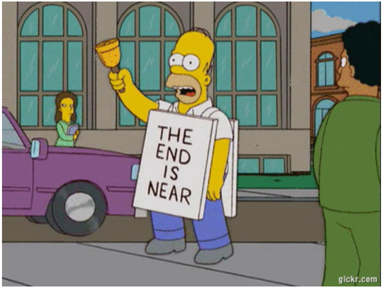
§4 Course review

MA385 – Numerical Analysis 1

28 November, 2019



The end...

MA385 is four (or five) main components

(0 Preliminaries: Taylor's Theorem.

(1 Solving nonlinear equations.

(2 Solving initial value differential equations.

(3 Linear algebra (linear systems of equations, norms and eigenvalues)

(4 Implementation and investigation of numerical method in Matlab.

Assessment is based on

- two assignments, each worth 10%;
- three labs (weighted 3%, 3% and 4%), worth 10$;
- and the class test, worth 10%;
- and final exam contributes the remaining **60%** of your MA385 grade.

It exam has **4** questions, each worth 20 marks. The number of marks for each section is indicated.
Answer 3 correctly for full marks.

There is be one question on each of the topics (1)–(3), and one on MATLAB.

Given $f : \mathbb{R} \to \mathbb{R}$, find $\tau \in [a, b]$ such that $f(\tau) = 0$.

We now know how to solve this

- using interval **bisection**;
- the **Secant** method;
- **Newton's** method;
- **fixed point** iteration.

We also know:

- a sufficient (but not necessary) condition for a solution to exist;
- what the ***order of convergence*** of a method is;
- for each method, how to prove it converges (subject to certain assumptions);
- how to determine the order of convergence experimentally;
- the Fixed Point and Contraction Mapping Theorems.

1. Solving Nonlinear Equations.

Don't leave home without....

- knowing about Taylor polynomials and remainders;
- being able to state, motivate, and use these methods;
- knowing the terminology (order of convergence, fixed points, contraction, etc.);
- knowing which method is best in which situation;
- being able to prove convergence.

What you don't have to know

- precise assumptions needed for convergence of Newton's and Secant method (will be given in the exam if needed);
- anything about the Black-Scholes equation, Julia sets and roots of unity (won't come up).

Solve the differential equation $y(t_0) = y_0, \quad \dfrac{dy}{dt} = f(t, y)\ t > t_0.$

We now know:

- what is meant by a ***Lipschitz condition***, and how to check that one is satisfied;
- how to derive and use **Euler's** method;
- the definitions of the **global** and **truncation** errors, **consistency** and convergence;
- the conditions on the RK-2 method's parameters for it to be 2nd-order;
- how to show a given RK method has the correct order of convergence for a simple linear problem;
- we can summarise an RK method as a tableau;

- how to apply Euler's method to a system of IVPs;
- how to write a high-order problem as a system of 1^{st}-order IVPs.

Don't leave home without...

- being able to state and use Euler's Method.
- being able to show how it, and a formula for its truncation error, can be derived from a Taylor series;
- for any **one-step** method, knowing how to show it is consistent, and how it relates to a Taylor series.

What you don't have to know:

- Picard's Theorem (will be given if needed);
- the definition of the truncation error for an arbitrary one-step method (will be given if needed);
- formulae for any RK-2, RK-3 or RK-4 method (will be given if needed);;
- implicit methods;
- systems and higher-order problems;
- finite difference methods for PDEs (i.e., the heat equation).

Solve the linear system of equations $A\boldsymbol{x} = b$ where $A \in \mathbb{R}^{n \times n}$

We now know...

- how to relate systems of equations to matrix-vector equations;
- a good way and a bad way to compute $\det(A)$;
- that Gaussian Elimination and row reduction is effectively the same as ***LU*-factorisation**;
- all about **triangular** matrices;
- how construct the LU-factorisation of A, and prove it exists;
- how to use LU-factorisation and back-substitution to solve $A\boldsymbol{x} = \boldsymbol{b}$;
- that the computational cost is $\mathcal{O}(n^3)$;
- all about vector and matrix norms;

- how to derive useful formulae for $\|A\|_1$, $\|A\|_2$, and $\|A\|_\infty$; how to show they are **consistent**.
- how the **condition number**, $\kappa(A)$, relates to errors in solving $Ax = b$;
- **Gerschgorin's** theorems for estimating eigenvalues, and can prove the first one.

Don't leave home without being able to ....

- explain triangular matrices, matrix partitioning, principal sub-matrices;
- establish properties of the product and inverse of triangular matrices;
- show the existence of the LU-factorisation;
- derive (with justification) the formulae for L and U;
- write down the LU-factorisation of a given matrix;
- use the LU-factorisation to solve a linear system;
- explain vector and matrix norms, and consistency of matrix norms;
- relate the vector norms $\|x\|_1$, $\|x\|_2$ and $\|x\|_\infty$ to each other.
- prove the formulae for $\|A\|_1$, $\|A\|_2$, and $\|A\|_\infty$.
- state, prove and apply Gerschgorin's 1st Theorem.

- state and apply Gerschgorin's 2nd Theorem.

What you don't have to know

- Permutations/pivoting.
- Computer representation of numbers

4. Implementation in Matlab

We now know

- The fundamentals of MATLAB programming: defining vectors and matrices; basic arithmetic, including element-by-element calculations; plotting, including log-log plots; iterating using `for` and `while` loops; conditionals (`if`), using function handles; output with `fprintf`;
- How to implement key numerical algorithms such as interval bisection, Newton's method, the secant method, Euler's method, (explicit) Runge-Kutta methods.
- How to interpret the results generated by test problems to establish (experimentally) rates of convergence of certain methods.

What you will be asked to do in the question on Implementation/MATLAB *will* include how to implement (as a Matlab code-snippet) **one** of the following algorithms

- Interval bisection;
- The Secant Method
- Newton's Method;
- Euler's Method
- A specified RK-2 or RK-3 method

You may also be expected to explain/interpret the output generated by a MATLAB program.

But not: MATLAB implementation of LU-Factorisaion and related topics.

AMA!