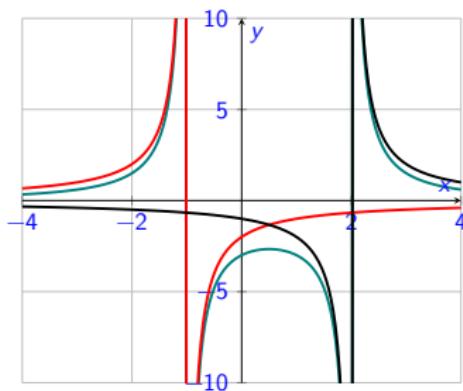


Partial Fractions

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University of Galway

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The parts of today's class:

1 News!

- Tutorials
- Assignments
- Class tests

2 Partial Fractions

- 4 cases
- Case 1
- Case 2
- Case 3
- Case 4

- 3 Towards Limits
- 4 Exercises

Slides are on canvas, and at
<https://www.niallmadden.ie/>
2526-MA140/



Tutorials start **this** week. The schedule is:

Teams	Time	Venue	Leader
1, 2	Tuesday 15:00	ENG- 2003	ST
3, 4	Tuesday 15:00	ENG- 2034	JM
9, 10	Thursday 11:00	ENG- 2002	ST
11, 12	Thursday 11:00	ENG- 3035	JM
5, 6	Friday 13:00	Eng- 2002	ST
7, 8	Friday 13:00	Eng- 2035	JM

Rang teagaisc trí Ghaeilge (Irish tutorial): Dé Máirt (Tuesday) 15:00, Áras na Gaeilge 221.

- ▶ There is currently a “practice” assignment open. See
<https://universityofgalway.instructure.com/courses/46734/assignments/128373>
 - ▶ During tutorials, the tutor will solve some similar questions. You can access the **tutorial sheet** at
https://universityofgalway.instructure.com/courses/46734/files/2842617?module_item_id=925893. You can also access this through the Canvas page: Modules... Tutorial Sheets.
 - ▶ **Assignment 1** will be due 5pm, Monday 5 October.
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Also: try the exercises at the end of each set of lecture slides: they are similar in style and standard to exam questions.

There are two class test planned for this module:

- ▶ MCQ format;
- ▶ both worth 10% of the final grade;
- ▶ Test 1: **Tuesday, 14 October** (Week 5)
- ▶ Test 2: **Tuesday, 18 November** (Week 10)
- ▶ Contact Niall if you have any concerns, or wish to avail of alternative arrangements, as provided by LENs reports.

Partial Fractions

Rational Functions have the general form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials.

An (proper) rational function can often be written as a sum of simpler ones: **partial fractions**.

For example

$$\frac{8x - 12}{x^2 - 2x - 3}$$

can be written as

$$\frac{3}{x - 3} + \frac{5}{x + 1}$$

Check: (*next slide*)

Partial Fractions

Partial Fractions

Note: Any polynomial (with real coefficients) can be factorised fully into the product of

- ▶ linear
- ▶ and irreducible quadratic factors.

Examples:

We get different combinations of factors in the denominator. Let's look at **four cases**, and how to find the partial fractions in each case.

The four cases

1. Denominator has **linear factors to the power of 1**
2. Denominator has **factors to the power greater than 1 (i.e repeated linear factors)**.
3. Denominator has **irreducible quadratic factors**.
4. Denominator has **irreducible quadratic factors to power greater than 1**.

Case 1: Linear factors to the power of 1 in the denominator.

We have **two methods** to find A and B .

Method 1: Comparing coefficients

Example

$$\frac{3x}{(x - 1)(x + 2)}$$

Method 2: Substituting specific values for x .

Example

Write $\frac{8x - 12}{x^2 - 2x - 3}$ as sum of partial fractions.

(2) Linear factors to the power greater than 1 in the denominator, (i.e repeated linear factors).

If $(x - \alpha)^k$ appears in the denominator, it will give rise to the following terms:

$$\frac{A_1}{x - \alpha} + \frac{A_2}{(x - \alpha)^2} + \dots + \frac{A_k}{(x - \alpha)^k}$$

Example

Find A , B and C such that

$$\frac{3x+1}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$$

(Note: we'll find that $A = 5/9$, $B = 4/3$ and $C = -5/9$).

(3) Irreducible quadratic factors.

Irreducible quadratic factors can not be factorised using real numbers, e.g. $x^2 + x + 1$.

An irreducible quadratic factor $ax^2 + bx + c$ gives rise to partial fractions of the form

$$\frac{Ax + B}{ax^2 + bx + c}.$$

Example 2.34 from textbook

If one writes

$$\frac{5x}{(x^2 + x + 1)(x - 2)} = \frac{Ax + B}{x^2 + x + 1} + \frac{C}{x - 2}$$

then we find $A = 10/7$, $B = 5/7$ and $C = 10/7$.

(4) Irreducible quadratic factors to power greater than 1.

Each repeated irreducible quadratic factor $(ax^2 + bx + c)^k$ in the denominator will give rise to

$$\frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \dots + \frac{A_kx + B_k}{(ax^2 + bx + c)^k}.$$

These can be done in a similar way to the previous case. But the calculations are pretty messy, so we won't even try!

Towards Limits

When we were considering the domain of a function, we looked at those x -values for which the function was not defined.

Example

$$f(x) = \frac{x^2 - 2}{x - 1}$$

$$g(x) = \frac{x^2 - 1}{x - 1}$$

Neither f nor g are defined at $x = 1$.

But what happens if x gets very close to 1?

x	0.900	0.990	0.999	1	1.001	1.010	1.100
$f(x)$							
$g(x)$							

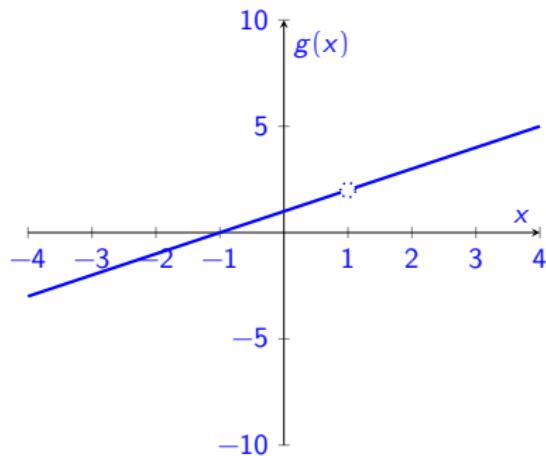
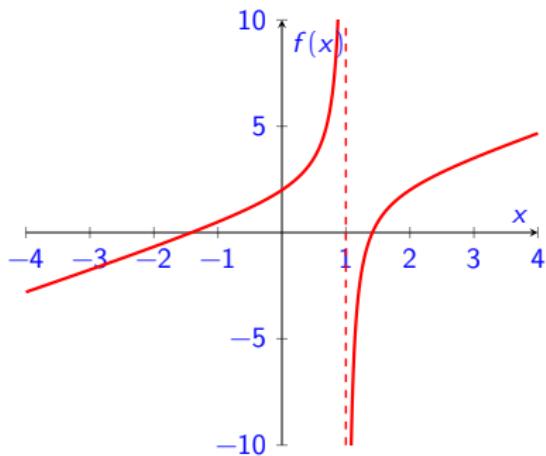
Let's look at the graphs of f and g .

Towards Limits

Example

$$f(x) = \frac{x^2 - 2}{x - 1}$$

$$g(x) = \frac{x^2 - 1}{x - 1}$$



Towards Limits

In the previous example, we saw that, although neither f nor g was defined at $x = 1$, they behaved very differently as x approaches 1.

To discuss this we'll need the concept of a **limit** which, roughly, relates to the value of function as it **approaches** a point (but not actually at that point).

$$\lim_{x \rightarrow a} f(x) = L$$

The concept of a limit is a prerequisite for a proper understanding of calculus and numerical methods.

Exercises

Exercise 2.1.1

Find the constants A , B and C , so that

$$\frac{2x+1}{(x-2)(x+1)(x-3)} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{x-3}$$

Exercises

Exercise 2.1.2

Express the following as partial fractions.

$$1. \frac{6}{x^2 - x - 2}$$

$$2. \frac{2x - 1}{x^2 - x - 2}$$

$$3. \frac{x - 1}{(x + 1)(x^2 - x - 2)}$$

$$4. \frac{x}{x^2 + 2x + 1}$$

$$5. \frac{1}{x^3 - 1}$$

Exercises

Exercise 2.1.3 (MA140 Exam, 24/25)

Express $\frac{3x + 1}{x^2 - x - 2}$ as partial fractions.