A Unified Theory of Comparative Risk

1. Comparative Risk

A theory of the *comparative risk relation* gives conditions under which a proposition is more risky than another proposition. Let 'R(p)' be an operator which takes sentences as inputs and outputs sentences of the form 'the risk that p'.

Aims of the talk:

A1 To argue univocal theories of the comparative risk relation are extensionally inadequate.

A2 To offer an explanation of why: extant theories take the risk of a proposition to be a univariate function, but the risk of a proposition is a multivariable function.

A3 To outline and defend a novel, extensionally adequate theory: the *unified theory of comparative risk.*

2. Comparative Risk: by Example

Worse Odds & *Better Odds*:

> An evil scientist has rigged a bomb to explode if ticket #65 wins the lottery. The odds that ticket #65 wins the lottery is n.
>
> An evil scientist has rigged a bomb to explode if ticket either #65 or #66 wins the lottery. The odds that either ticket wins the lottery is m.

Closer Risk & *Farther Risk*:

> An evil scientist has rigged a bomb to explode if ticket #65 wins the lottery. The odds that ticket #65 wins the lottery is n.
>
> An evil scientist has rigged a bomb to explode if, (a) the weakest team remaining in the FA cup beats the best team in the cup by ten goals, (b) the weakest horse in the field at the Grand National wins the race by at least 2000 meters, and (c) the Queen of England spontaneously chooses to speak a complete sentence of Polish during her next public speech. The odds of a, b, and c occurring jointly is n. (Pritchard, 2015, p. 441)

3. Extant Theories of Comparative Risk

3.1. The Probabilistic/Proportional Theory

The risk of p is a function of the probability/modal frequency of p. A non-probabilistic alternative would take the risk of p to be a function of the proportion of worlds at which p is true. We'll focus on the former.

Two variants:

3.1.1 Unconditional

R(p) > R(q) if, and only if:

> Prob(p) > Prob (q)

A less controversial theory uses conditional probabilities. Perhaps the risk of a proposition is conditional on some body of evidence E:

3.1.2 Conditional

R(p) >R(q) if, and only if:

> Prob($p \mid E$) > Prob($q \mid E$)

The Good:

Is extensionally adequate across contrast cases where the only relevant variance appears to be probabilistic, e.g., *Worse Odds* & *Better Odds*.

The Bad:

Is extensionally inadequate across equiprobable contrast cases, e.g., *Closer Risk* & *Farther Risk*.

3.2 Two Modal Theories

3.2.1 The Closeness Theory

The risk of p is a function of the closeness of the closest world at which p is true. (Pritchard, 2015, 2016, 2017)

R(p) > R(q) if, and only if:

> the closest world w at which p is true is closer than the closest world w' at which q is true.

3.2.2 The Normic Theory

The risk of p is a function of the normalcy of the most normal world at which p is true. (Ebert, Smith & Durbach, 2019)

R(p) > R(q) if, and only if:

the most normal world w at which p is true is more normal than the most normal world at which q is true.

The Good:

Are extensionally adequate across equiprobable contrast cases, e.g., *Closer Risk* & *Farther Risk*.

The Bad:

(a) Are extensionally inadequate across contrast cases where the only relevant variance appears to be probabilistic, e.g., *Worse Odds* & *Better Odds*.

(b) Suffer from the *ideal world problem*. This the problem that there exists a world at which all propositions must be assigned a maximal risk score. On the closeness view this is the actual world; on the normic view this is any of the most normal worlds.

(c) Validate *Checklist Reasoning*:

The risk of p is low
The risk of q is low
therefore
The risk of $p \vee q$ is low.

(d) Potentially allow mutually exclusive propositions to be assigned a maximal risk value. This depends on whether the proponents opt for weak or strong centering[1] i.e., on whether they accept that there there exists a unique most close/normal world.

3.3 Pluralist Variants

In light of the above, Ebert, Smith & Durbach (2019) recommend a pluralist theory according to which there exist at least two distinct conceptions of risk, a probabilistic and normic conception. We could also have a probabilistic and closeness conception.

On our view, pluralism should be avoided if possible: better to unify the cases under a single theory. We'll now argue this is possible.

4. A Unified Theory of Comparative Risk

4.1. Diagnosing the Extensional Inadequacy

Why are extant univocal theories inadequate? Our answer: because they take the risk of a proposition to be a univariable function. E.g., they take the risk of a proposition to be a function of *only* the closeness/normalcy or *only* of the probability of the proposition. What is needed is a *multivariate* function. More precisely, the risk of a proposition is a function of *both* the frequency of that proposition across worlds *and* the closeness of those worlds. This allows us to accommodate the following two observations:

1. All else being equal, increasing the probability of a proposition increases the risk of that proposition.

2. All else being equal, increasing the modal closeness (/normalcy) of a proposition increases the risk of that proposition.

4.2. The Unified Theory: A Model

The model is a tuple $<W, R, D, V, S>$, which is built as follows:

We start with a *frame*, that is, an ordered pair $<W, R>$ where W is a set and R is a binary relation over W. Informally, W is conceived of as the set of possible worlds and R as an accessibility relation between possible worlds.

We add a *distance measure* D, which is a function from worlds to the unit interval $[0, 1]$. Informally, D assigns a value to worlds which represents their distance from the actual world.

We add a *function* V which takes as arguments worlds and maps to sets of atomic formulas, informally conceived as the atomic formulas that are true at the argument world.

We add a *function* S maps the range of D to the half open unit interval $(0, 1]$. Informally, S is conceived as assigning a *risk value* to each world.

The composite function $S \circ D(w)$ first maps worlds to the unit interval, which assigns the distance of the argument world to the actual world, and then maps the output values to the half open unit interval, specifying the risk value of the argument world.

There are four restrictions on S:

Boundedness
$$\forall w \in W, S \circ D(w) > 0 \ \wedge \ S \circ D(w) \leq 1$$

Normalisation
$$\forall w \in W, w \vDash p \rightarrow \sum_{w \vDash p \in W} S \circ D(w) = 1$$

Additivity
$$\neg \exists w \vDash (p \ \wedge \ q) \in W \rightarrow \sum_{w \vDash p \vee q \in W} S \circ D(w)$$
$$= \sum_{w \vDash p \in W} S \circ D(w) + \sum_{w \vDash q \in W} S \circ D(w)$$

Closeness
$$\forall w \in W, D(w^1) < D(w^2) \rightarrow S \circ D(w^1) > S \circ D(w^2)$$
$$\wedge \, D(w^1) = D(w^2) \rightarrow S \circ D(w^1) = S \circ D(w^2)$$

Important: S assigns *risk values* to worlds, *not* propositions. The riskiness of propositions is determined by what we call *risk scores*. The risk score of p is obtained by summing the risk values of all worlds in which p is derivable. The risk of p is straightforwardly equated with the risk score of p. That is:

[1](Cf. Lewis 1973, sec. 1.7)

$$R(p) = \sum_{w \vDash p \in W} S \circ D(w)$$

On this view:

R(p) > R(q) if, and only if:

$$\sum_{w \vDash p \in W} S \circ D(w) > \sum_{w \vDash q \in W} S \circ D(w)$$

As a matter of mathematical necessity, the risk score of p $\geq$ 0 and $\leq$ 1. A proposition's riskiness is equated with its risk score. The risk score 1 indicates maximal risk, 0 indicates no risk.

4.3. An Example

In this toy model there are five possible worlds: @–w^4. Every world is accessible from @. We'll consider four propositions: p, q, r, s:

World	Risk value	p value	q value	r value	s value
@	0.4	1	1	0	0
w^1	0.2	1	1	1	0
w^2	0.2	0	1	0	0
w^3	0.1	0	1	0	1
w^4	0.1	0	1	0	0

<u>Risk scores:</u>

$R(p) = 0.4 + 0.2 = 0.6$

$R(q) = 0.4 + 0.2 + 0.2 + 0.1 + 0.1 = 1$

$R(r) = 0.2$

$R(s) = 0.1$

Benefits of the view:

1. The unified theory does not suffer from the ideal world problem. This is because that there is no world w such that: if p is true at w then the risk score of $p = 1$.

Proof: Let w be the ideal world. By the law of excluded middle, w is either a p or a not-p world. In the above model, the risk score of p is 0.6 and the risk score of $\neg p$ is 0.4. Since either p or $\neg p$ is true at w, the fact that a proposition is true at w does not make it maximally risky.

2. The unified theory invalidates checklist reasoning.

Proof: Consider r and s. The risk scores of both r and s are low (0.2 and 0.1 respectively). But the risk score of the disjunction of r and s is not low (0.3). So checklist reasoning is invalidated on this view.

3. All else remaining equal, if the probability of a proposition increases, the risk increases. All else remaining equal, if the worlds at which a risk proposition is true become closer, the risk increases. For this reason the theory is extensionally adequate with respect to both sets of contrast case.

4. Two mutually exclusive propositions cannot receive the maximal risk score 1, unlike the normic theory.

Proof: Let p and q be mutually exclusive and the risk score of p = 1. If the risk score of p = 1 then p is true at all worlds. Since p and q are mutually exclusive there are no worlds at which q is true. So q's risk score is 0, and hence q is not maximally risky. So if two propositions are mutually exclusive, it is false that both are maximally risky.

Normalcy, or similarity?

The model is understood in terms of closeness, i.e., similarity from the actual world. But it could also be understood in terms of normalcy, substituting the actual world for the most normal world, and distance for comparative normalcy. The model allows for both, but we hold that it should not be understood in terms of normalcy, at least insofar as normalcy is understood by Smith (2016).

According to Smith, the normalcy of p is determined by the extent to which p calls for a 'special explanation'. Here is an example that puts pressure on that idea:

> *Spaghetti Noodle*: If you were to try to break a spaghetti noodle, it would likely break into three or more pieces. This calls out for a special explanation (scientists have historically been mystified by this fact). Hence, it should be highly abnormal by the lights of this conception of normalcy. But clearly the risk of the noodle breaking into three pieces is high.

References

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