Project 2

Consider steady-state heat conduction in an isotropic rectangular region of dimension *4a* by *4a* (see Fig. 1). The origin of the x and y coordinates is taken at the lower left corner. The boundary *x=0* is insulated, the boundaries *y=0* and *x=4a* are maintained at zero temperature. The boundary *y=4a* is maintained at $T=T_0\cos(\pi x/8a)$. The heat transfer is governed by

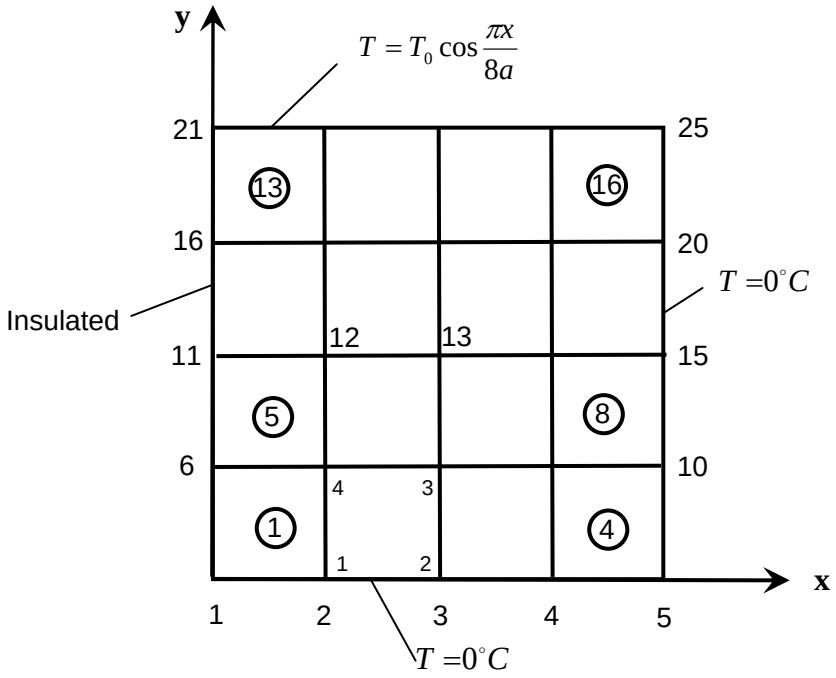
$$-k\nabla^2 T = f$$

with zero internal heat generation (*f=0*).

1. Develop the finite element model of the equation above.
2. Identify the element coefficient matrices for linear rectangular and right triangular elements.
3. Write the finite element equations associated with nodes 13, 16, and 19 for the rectangular elements shown in Fig. 1a and triangular elements shown in Fig. 1b.
4. Write the specified primary and secondary variables at all boundary nodes shown in Fig. 1a and Fig. 1b.
5. Develop a general computer code using the finite element method with linear rectangular elements and right triangular elements and run the code for the meshes shown in Figs. 1a and 1b to determine the temperature distribution.
6. Compare the nodal temperatures $T(x,y)/T_0$ obtained using rectangular elements and triangular elements shown in Figs. 1a and 1b in a table with the analytical solution

$$T(x,y) = \frac{T_0 \sinh(\pi y/8a)\cos(\pi x/8a)}{\sinh(\pi/2)}$$

7. Develop a computer code using the finite element method with arbitrary unstructured linear triangular elements, as it is shown in Fig. 1c, run the code with the mesh shown in Fig. 1b first to check the code and to see whether you obtain the same results as those in Step 5. Then run the code with the mesh shown in Fig. 1c (attached data file). Plot the results with temperature contour lines for the meshes shown in Figs. 1a, 1b and 1c.



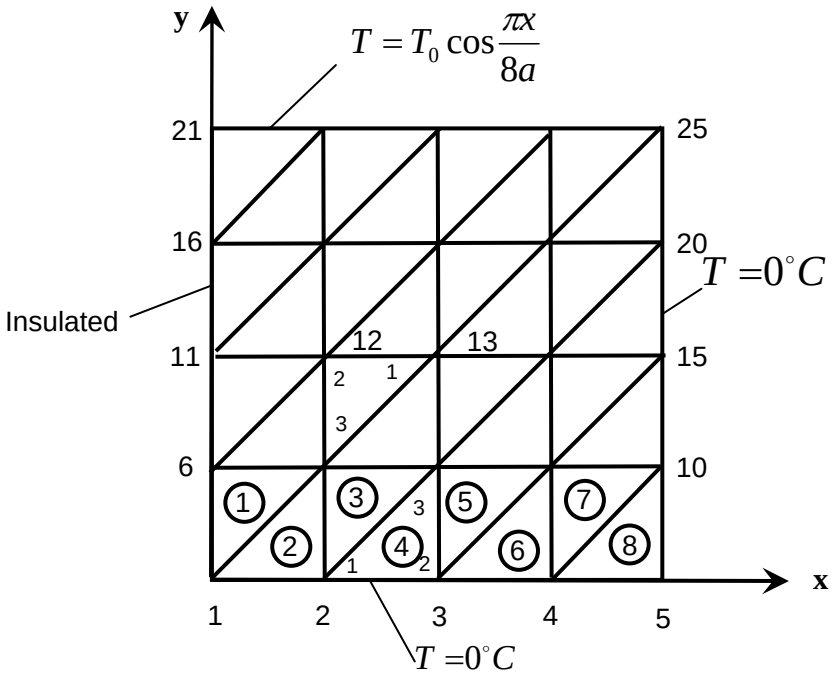
$f = 0 W/m^3$

$k = 25 W/(m \bullet {}^\circ C)$

$a = 1m$

$T_0 = 100^\circ C$

Fig. 1a



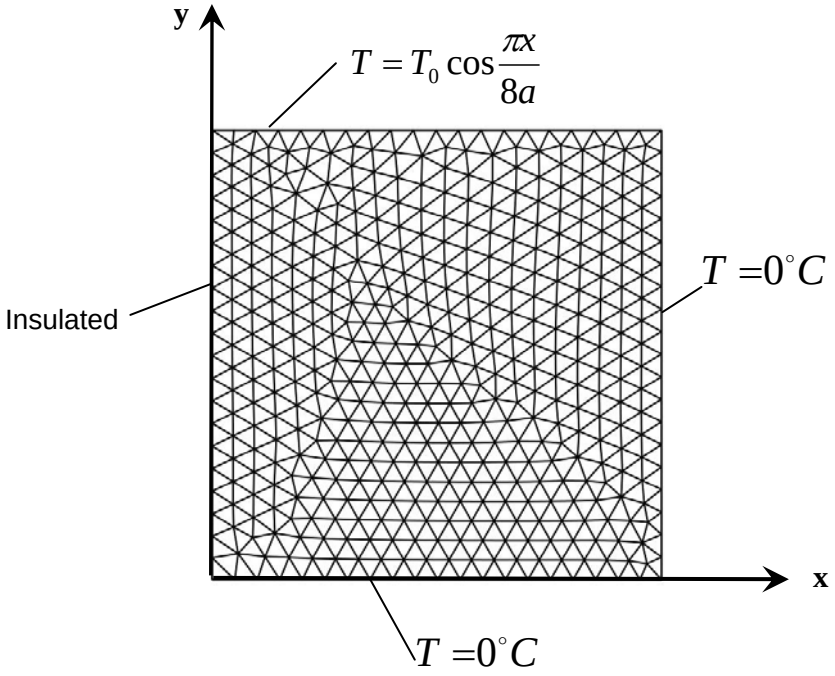
$f = 0W/m^3$

$k = 25W/(m\bullet^\circ C)$

$a = 1m$

$T_0 = 100^\circ C$

Fig. 1b



$f = 0W/m^3$

$k = 25W/(m\bullet^\circ C)$

$a = 1m$

$T_0 = 100^\circ C$

Fig. 1c

Note:

The attached data file, domain.data is the input data file for the Step 7 described above. The format of the input data file is as follows:

Card 1: Title Line
Card 2:

- Number of elements: **nelem** **(integer)**
- Number of node points: **npoin** **(integer)**

Card 3: Title Line
Cards 4.1 – 4.nelem:

- Element number: **ielem** **(integer)**
- Global nodal points corresponding to each element: **intmat(1:3)** **(3* integer)**

Card 5: Title Line
Card 6.1 – 6.npoin:

- Node number: **ipoin** **(integer)**
- Coordinates of nodal point: **coord(1:2)** **(2*real)**

Please prepare an input data file for the mesh shown in Fig. 1b using the same format as that in attached data file, so that you can run both data sets to determine the temperature distribution for the meshes shown in Fig. 1b and 1c.

Please also note that the attached data file has one additional real number and one additional integer number in each line after the coordinates. Please just ignore these two numbers when you write a routine to read the input data file.