Simultaneous Linear equation.

1// Gausin Elemenation :

$[A]\,[x] = [e]$ → Right hand side

$[A]$ → coefficent

$[x]$ → unknowns → বের করব

⇒ 1. Forward Elimination, 2. Back substitution

pitfall : 1. Divisiom by zero, 2. Round of error

To minimize pitfalls used portial pivoting

2//

$$\begin{bmatrix} U & U & U \\ 0 & U & V \\ 0 & 0 & V \end{bmatrix}$$

— Determinant

3// LU Decomposition: $[A]\,[x] = [c]$

$[A] = [L]\,[U]$ → Decompose

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \qquad \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$[L]\,[Z] = [c]$

$[U]\,[X] = [Z]$

$[X]$ → unknowns.

4 Gauss - Sidel Method :

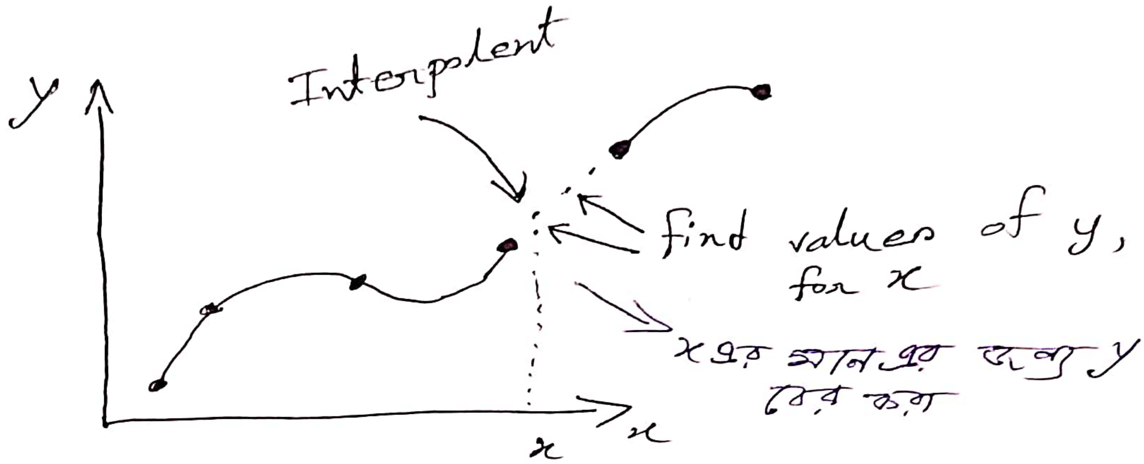
benifits : 1. Control round off error
2. If the physics of the problem are understood, a close initial guess can be made, decresing the number of iterations needed.
3. Control divided by zero problem.
4. iteration শুরু করা যায় এবং অনেক Iteration এর মাধ্যমে error কমানো যায়।

pitfall :
1. Converge নাও হতে পারে। (diagonally dominant)

সূত্র : Rewrite equation, put intially guessed value.

Absolute error : $|\epsilon_a|_i = \left| \frac{x_i new - x_i old}{x_i new} \right| \times 100$

Interplation



4// Direct method.

linear interpolation 2 point নিব,
quadric interpolation 3 point নিব,
cubic interpolation 4 point নিব।

$$V(t) = \underbrace{\overbrace{a_0 + a_1 t}^{2\text{ point}} + a_2 t^2}_{3\text{ point}} + a_3 t^3 \rightarrow 4\text{ point}$$

Rule: Gaus elimenation,

5// Newton divided:

⇒ Linear interpolation:

$$V(t) = b_0 + b_1 (t - t_0)$$

$$b_0 = V(t_0) \quad , \quad b_1 = \frac{V(t_1) - V(t_0)}{t_1 - t_0}$$

⇒ Quadratic Interpolation:

$$f_2(x) = b_0 + b_1(x - x_0) + b_2 (x - x_0)(x - x_1)$$

$$b_2 = \frac{\frac{f(x_2)-f(x_1)}{x_2-x_1} - \frac{f(x_1)-f(x_0)}{x_1-x_0}}{x_2-x_0}$$

⇒ Tree shape:

২য় step এ $b = \dfrac{v(t_1)-v(t_0)}{t_1-t_0}$

benifits:

1. Compact than Direct method (tree shap)
2. Solve devision by zero problem

6// Lagrangian Interpolation:

⇒ Linear interpolation: $V(t) = \sum_{i=0}^{1} L_i(t)\, v(t_i)$

$$\therefore V_1 t = \underbrace{L_0(t)}_{\text{weight}}\, V(t_0) + L_1(t)\, V(t_1)$$

$$L_0(t) = \prod_{\substack{j=0 \\ j\neq 0}}^{1} \frac{t-t_j}{t_i-t_j} = \frac{t-t_0}{t_1-t_0}$$

$$L_1(t) = \prod_{\substack{J=0 \\ j\neq 1}}^{1} \frac{t-t_j}{t_i-t_j} = \frac{t-t_0}{t_1-t_0}$$

Cubic : (4 points)

$$v(t) = L_0(t)\,v(t_0) + L_1(t)\,v(t_1) + L_2(t)\,v(t_2) + L_3(t)\,v(t_3)$$

$n-1=n$ [illegible] 4

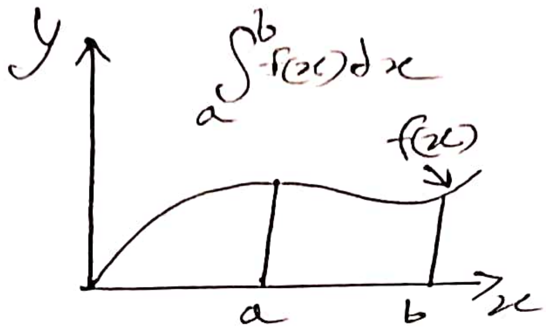
$$L_0(t) = \prod_{\substack{j=0 \\ j\neq 0}}^{3} \frac{t-t_j}{t_0-t_j} = \left(\frac{t-t_1}{t_0-t_1}\right)\left(\frac{t-t_2}{t_0-t_2}\right)\left(\frac{t-t_3}{t_0-t_3}\right)$$

$$L_1(t) = \prod_{\substack{J=0 \\ J\neq 1}}^{3} \left(\frac{t-t_0}{t_1-t_0}\right)\left(\frac{t-t_2}{t_1-t_2}\right)\left(\frac{t-t_3}{t_1-t_3}\right)$$

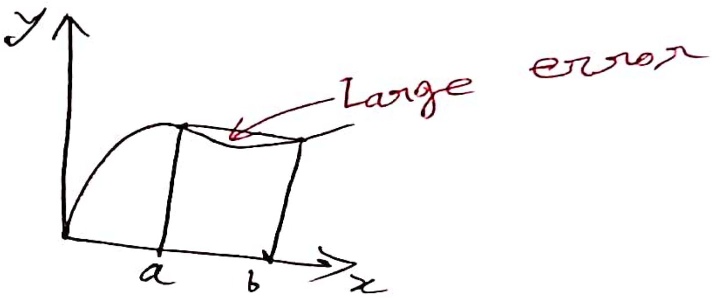
$$L_2(t) = \prod_{\substack{j=0 \\ J\neq 2}}^{3} \left(\frac{t-t_j}{t_2-t_j}\right)\left(\frac{t-t_1}{t_2-t_1}\right)\left(\frac{t-t_3}{t_2-t_3}\right)$$

$$L_3(t) = \prod_{\substack{j=0 \\ j\neq 3}}^{3} \left(\frac{t-t_j}{t_3-t_j}\right)$$

Integration :

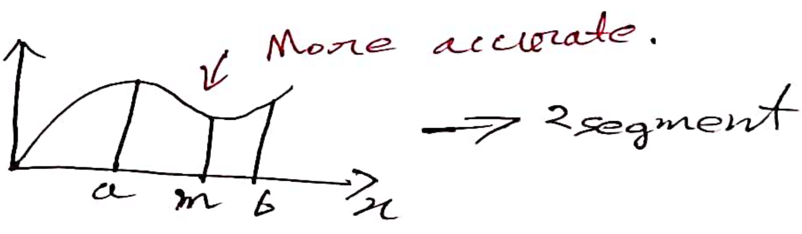


7 Trapezoidal :



अतः $\int_a^b f(x)dx \simeq (b-a)\left[\frac{f(a)+f(b)}{2}\right]$

8 Multisegment :



$$f(x) = \left(\frac{b-a}{2n}\right)\left[f(a) + 2\left\{\sum_{i=1}^{n-1} f(a+ih)\right\} + f(b)\right]$$

segment

$$\boxed{h = \frac{b-a}{n}}$$

9 Simpson's 1/3 :

$$\int_a^b f_2(x)\,dx = \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$$\boxed{h = \frac{b-a}{2}}$$

10 Multisegment simpson's : minimize the error

$$f(x) = \underbrace{\frac{b-a}{3n}}_{\text{segment}}\left[f(t_0) + 4\sum_{\substack{i=1 \\ i=\text{odd}}}^{n-1} f(t_i) + 2\sum_{\substack{i=2 \\ i=\text{even}}}^{n-2} f(t_i) + f(t_n)\right]$$

11 Gauss Quadrature : (Higher point)

$$\int_{-1}^{1} \qquad \int_a^b f(x)\,dx = \frac{b-a}{2}\int_{-1}^{1} f\Big(\underbrace{\frac{b-a}{2}t + \frac{b+a}{2}}_{x}\Big)\,dt$$

limit changed to −1 to 1

$$\text{উদা:}\int_a^b f(x) = \overbrace{\underbrace{c_1 f(x_1) + c_2 f(x_2)}_{\text{2 point}} + c_3 f(x_3)}^{\text{3 point}}$$

12 Linear regression:

best fit find করা

$$y = a_0 + a_1 x$$

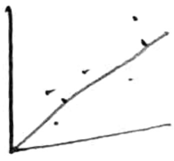
$$a_1 = \frac{n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

x	y	x^2	xy

$$a_0 = \bar{y} - a_1 \bar{x}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} \qquad \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

13 মূল বিন্দু গামী রেখা →



$$y = a_1 x$$

$$a_1 = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} = \frac{\bar{y}}{\bar{x}}$$

Ordinary differential

14 Euler's method:

temparature

সূত্র : $y_1 = y_0 + f(x_0, y_0)\, h$

$\quad\quad\quad i+1 \quad\quad i \quad\quad\quad i \quad i \quad$ → step size

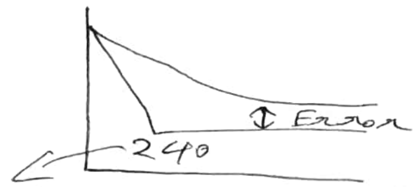
Example: $f(t, \theta) = \theta^4$, $\theta(0) = 1200k$

(θ → y, 0 → t_0, $1200k$ → θ_0)

find temparature $t = 480$

step, $h = 240$

$$\theta_1 = \theta_0 + f(t_0, \theta_0)\, h$$



step size কম দিলে error কম।

15 Runge kutta 2nd order:

(Heun's) $f(t, \theta) = \theta^4$, $\theta(0) = 1200k$

more accurate

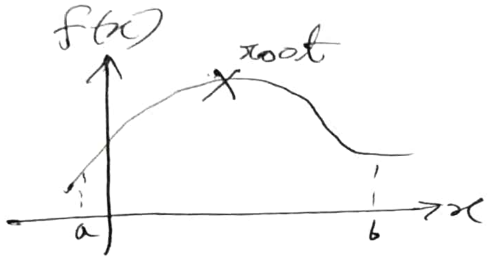
সূত্র $\theta_{i+1} = \theta_i + \left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right) h$

or $y_{i+1} = y_i + \left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right) h$ → step

$$k_1 = f(t_0, \theta_0)$$

$$k_2 = f(t_0 + h,\ \theta_0 + k_1 h)$$

(16) Golden section Search Method



root find

$\varepsilon = \dfrac{\text{final}}{\text{interval}}$ ↳ সর্বনিম্ন যেই interval ওর মাঝেই থাকবে।

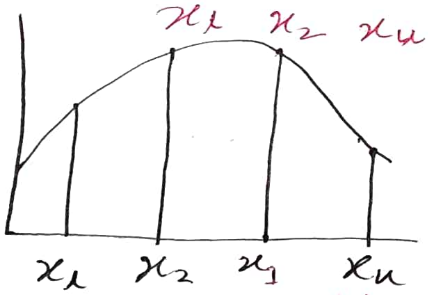
Iteration → করতে করতে আসতেছে।

The Equal Interval method is inefficient when ε is small

The Golden section search method divides the search more efficiently closing in on the optima in fewer iterations.

if, $f(x_1) > f(x_2)$ $\quad x_l \quad x_2 \quad x_u \longrightarrow$ calculate x_1

Golden ratio = 0.618



if, $f(x_1) < f(x_2)$: $\quad x_l \quad x_1 \quad \cancel{x_u} \longrightarrow$ calculate x_2

$$x_1 = x_l + 0.618\,(x_u - x_l) =$$

$$x_2 = x_u - 0.618\,(x_u - x_l) =$$

$$x_u - x_l = \ldots\ldots \quad < \text{or} > \varepsilon$$