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Newton-Raphson Method:

Newton-Raphson-Method, based on the principle that if the initial guess of the root of $f(x)=0$ is at x_i, then if one draws the tangent to the curve at $f(x_i)$, the point x_{i+1} where the tangent crosses the x-axis is an improved estimate of the root

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

The Newton Raphson method of finding roots of nonlinear equations falls unter the category of open methods

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Algorithm:

equation $f(x)=0$, find root;

1. Evaluate $f'(x)$ symbolically

2. Use an initial guess of the root, x_i, to estimate the new value of the root, x_{i+1}, as,

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

3. Find the absolute relative approximate error $|\epsilon_a|$ as, $|\epsilon_a| = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \times 100$

4. Compare the absolute relative approximate error with the pre-specified relative error tolerance, ϵ_s. If $|\epsilon_a| > \epsilon_s$, then go to Step 2, else stop the algorithm.

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Drawbacks:

1. Divergence at inflection points:

If the selection of the initial guess or an iterated value of the root turns out to be close to the inflection point of the function f(x) in the equation $f(x)=0$, Newton-Raphson method may start diverging away from the root.

2. Division by zero:

for the equation,

$$f(x) = x^3 - 0.03x^2 + 2.4\times10^{-6} = 0$$

the Newton-Raphson method reduces to

$$x_{i+1} = x_i - \frac{x_i^3 - 0.03x_i^2 + 2.4\times10^{-6}}{3x_i^2 - 0.06x_i}$$

for $x_0 = 0$ or $x_0 = 0.02$, division by zero occurs.

for an initial guess close to 0.02 such as $x_0 = 0.01999$, one may avoid division by zero, but then the denominator in the formula is a small number.

3. Oscillations near local maximum and minimum:

Results obtained from the Newton-Raphson method may oscillate about the local maximum or minimum without converging on a root but converging on the local maximum or minimum.

4. Root jumping:

In some case where the function $f(x)$ is oscillating and has a number of roots, one may choose an initial guess close to a root. However, the guesses may jump and ~~converge~~ converge to some other root.

Advantages :

1. It converges at faster than a linear rate, so that it is more rapidly convergent than the bisection method.

2. Requires only one guess of point to find the root.