2. The expression for true error in calculating the derivative of $\sin(2x)$ at $x=\frac{\pi}{4}$ by using the approximate expression $f'(x)\approx\frac{f(x+h)-f(x)}{h}$ is

$f(x)=\sin(2x)$

$\rightarrow x$

(A) $\frac{h-\cos(2h)-1}{h}$

(B) $\frac{h-\cos(h)-1}{h}$

(C) $\frac{1-\cos(2h)}{h}$

(D) $\frac{\sin(2h)}{h}$

Solution

The correct answer is (C).

Exact answer :

$$f(x)=\sin(2x)$$
$$f'(x)=2\cos(2x)$$
$$f'\left(\frac{\pi}{4}\right)=2\cos\left(2\frac{\pi}{4}\right)$$
$$=0$$

$\frac{d}{dx}\sin(2x)$
$=2\cos(2x)$

Approximate Solution $\rightarrow x=\frac{\pi}{4}$

$$f'(x)\approx\frac{f(x+h)-f(x)}{h}$$
$$f(x)=\sin(2x)$$
$$f'(x)\cong\frac{\sin(2(x+h))-\sin(2x)}{h}$$
$$=\frac{\sin(2x)\cos(2h)+\cos(2x)\sin(2h)-\sin(2x)}{h}$$
$$f'\left(\frac{\pi}{4}\right)\cong\frac{\sin\left(2\frac{\pi}{4}\right)\cos(2h)+\cos\left(2\frac{\pi}{4}\right)\sin(2h)-\sin\left(2\frac{\pi}{4}\right)}{h}$$
$$=\frac{\sin\left(\frac{\pi}{2}\right)\cos(2h)+\cos\left(\frac{\pi}{2}\right)\sin(2h)-\sin\left(\frac{\pi}{2}\right)}{h}$$
$$=\frac{(1)\cos(2h)+(0)\sin(2h)-1}{h}$$
$$=\frac{\cos(2h)-1}{h}$$

$E_t=$ True Value – Approximate Value

$$\therefore E_t=0-\frac{\cos(2h)-1}{h}=\frac{1-\cos(2h)}{h}$$ (Ans.)

4. The relative approximate error at the end of an iteration to find the root of an equation is 0.004%. The least number of significant digits we can trust in the solution is

(A) 2
(B) 3
(C) 4
(D) 5

Solution

The correct answer is (C).

If $|\in_a| \le 0.5 \times 10^{2-m}$, then at least m significant digits are correct in the answer.

Given $|\in_a| = |\,0.004\,| = 0.004\%$

$0.004 \le 0.5 \times 10^{2-m}$

5

m is at least 1, as $0.004 \le 0.5 \times 10^{2-1}$, that is, $0.004 \le 5$, is true,

0.5

m is at least 2, as $0.004 \le 0.5 \times 10^{2-2}$, that is, $0.004 \le 0.5$, is true,

0.05

m is at least 3, as $0.004 \le 0.5 \times 10^{2-3}$, that is, $0.004 \le 0.05$, is true,

0.005

m is at least 4, as $0.004 \le 0.5 \times 10^{2-4}$, that is, $0.004 \le 0.005$, is true,

0.0005

m is at ***not*** at least 5, as $0.004 \le 0.5 \times 10^{2-5}$, that is, $0.004 \le 0.0005$, is ***not*** true,

So the least number of significant digits correct in my answer is 4.

Alternative solution

$$|\in_a| \le 0.5 \times 10^{2-m}$$

$$|\,0.004\,| \le 0.5 \times 10^{2-m}$$

$$\frac{0.004}{0.5} \le 10^{2-m}$$

$$0.008 \le 10^{2-m}$$

Taking log of both sides

$$\log_{10}(0.008) \le \log_{10}\left(10^{2-m}\right)$$

$$-2.0969 \le 2 - m$$

$$m \le 2 + 2.0969$$

$$m \le 4.0969$$

Since m can only be an integer, $m \le 4$.

So the least number of significant digits correct in my answer is 4.