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Bisection Method

PTO

ID: 17201026

Bisection method:

Bisection method is a numerical method that used to find the root of a nonlinear equation $f(x)=0$ was the bisection method.

since, the method is based on finding the root between two points, the method falls under the category of ~~bracket~~ bracketing methods.

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Theorem :

An equation $f(x) = 0$, where $f(x)$ is a real continuous function, has at least one root between x_l and x_u if $f(x_l)\,f(x_u) < 0$.

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Algorithm for the bisection method:

Suppose to find the root of the equation $f(x)=0$ are,

1. Choose x_l and x_u as two guesses for the root such that $f(x_l)\, f(x_u) \leq 0$, or in other words, $f(x)$ changes sign between x_l and x_u

2. Estimate the root, x_m, of the equation $f(x)=0$ as the mid-point between x_l and x_u as, $x_m = \frac{x_l + x_u}{2}$

3. Now check the following

a) If $(x_l)\, f(x_m) < 0$, then the root lies between x_l and x_m; then $x_l = x_l$ and $x_u = x_m$

b) If $f(x_l)\, f(x_m) > 0$, then the root lies between x_m and x_u; then $x_l = x_m$ and $x_u = x_u$

c) If $f(x_l)\, f(x_m) = 0$, then the root is x_m

stop the algorithm.

4. find the new estimate of the root

$$x_m = \frac{x_l + x_u}{2}$$

Find. the absolute relative approximate error as

$$|\epsilon_a| = \left| \frac{x_m^{new} - x_m^{old}}{x_m^{new}} \right| \times 100$$

where x_m^{new} = estimated root

x_m^{old} = estimated root from previous iteration

5. Compare the absolute relative approximate error $|\epsilon_a|$ withe the pre-specified relati error tolerance ϵ_s. If $|\epsilon_a| > \epsilon_s$, then go to step 3, else stop.

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Advantages of bisection method :

a) The bisection method is always convergent. This method is guaranteed to converge.

b) As iterations are conducted, the interval gets halved. So one can guarantee the error in the solution of the equation.

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Drawbacks of Bisection method:

a) The convergence of the bisection method is slow as it is simply based on halving the interval

b) If one of the initial guesses is closer to the root, it will take larger number of iterations to reach the root.

c) If a function $f(x)$ is such that it just touches the x-axis (in the figure) such as

$$f(x) = x^2 = 0$$

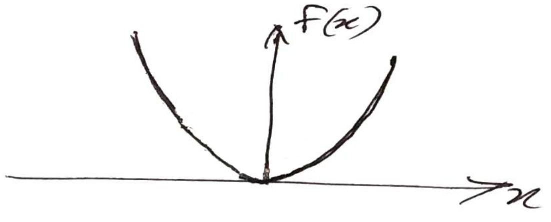

figure: The equation $f(x) = x^2 = 0$ has single root at $x=0$ that cannot be bracketed

it will be unable to find the lower guess, x_l, and upper guess x_u, such that $f(x_l)f(x_u) < 0$

d) for functions $f(x)$ where there is a singularity, the bisection method may conerge on the singularity.