Binomial Probability Distribution

- A fixed number of observations (trials), n
  - e.g., 20 tosses of a coin
- Binary random variable
  - e.g., Head or tail in coin toss
  - Often called as success or failure
  - Prob of success is p, and prob of failure is $1-p$
- Constant probability for each observation

Binomial example

- Take the example of 5 coin tosses

- What's the probability that you flip exactly 3 heads in 5 coin tosses?

Binomial distribution

- Solution:
- One way to get exactly 3 heads: HHHTT

- What's the probability of this exact arrangement?
  - P(heads) x P(heads) x P(heads) x P(tails) x P(tails) $= (1/2)^3 \times (1/2)^2$
- Another way to get exactly 3 heads: THHHT
  - Probability of this exact outcome $= (1/2) \times (1/2)^3 \times (1/2) = (1/2)^3 \times (1/2)^2$

Binomial distribution

- In fact, $(1/2)^3 \times (1/2)^2$ is the probability of each unique outcome that has exactly 3 heads and 2 tails
- So, the overall probability of 3 heads and 2 tails is: $(1/2)^3 \times (1/2)^2 + (1/2)^3 \times (1/2)^2 + (1/2)^3 \times (1/2)^2 +$ ….. for as many unique arrangements as there are

- But how many are there??

$\binom{5}{3}$ ways to arrange 3 heads in 5 trials

Outcome	Probability
THHHT	$(1/2)^3 \, x \, (1/2)^2$
HHHTT	$(1/2)^3 \, x \, (1/2)^2$
TTHHH	$(1/2)^3 \, x \, (1/2)^2$
HTTHH	$(1/2)^3 \, x \, (1/2)^2$
HHTTH	$(1/2)^3 \, x \, (1/2)^2$
THTHH	$(1/2)^3 \, x \, (1/2)^2$
HTHTH	$(1/2)^3 \, x \, (1/2)^2$
HHTHT	$(1/2)^3 \, x \, (1/2)^2$
THHTH	$(1/2)^3 \, x \, (1/2)^2$
HTHHT	$(1/2)^3 \, x \, (1/2)^2$

10 arrangements $x \, (1/2)^3 \, x \, (1/2)^2$

The probability of each unique outcome (note: they are all equal)

$_5C_3 = 5!/3!2! = 10$

$\therefore$ **P(3 heads and 2 tails) =** $\binom{5}{3}$ ***x P(heads)^3 x P(tails)^2***

= *10 x $(\frac{1}{2})^{5}$=31.25%*

Binomial distribution, generally

Note the general pattern emerging → if you have only two possible outcomes (call them 1/0 or yes/no or success/failure) in *n* independent trials, then the probability of exactly *X* "successes"=

n = number of trials

$$\boxed{\binom{n}{X} p^{X} (1-p)^{n-X}}$$

X = # successes out of *n* trials

p = probability of success

1-p = probability of failure