(III) Sources of Error

two types of error, 1) Round off error
2) Truncation error

Round of error :

Round off error is error created due to approximate representation of numbers.

example, $\frac{1}{3}$ may be represented as

0.3333333.....

Now the round of error in this case is

$$\frac{1}{3} - 0.333 = 0.00033......$$

$$= 0.00\overline{033}$$

there are other numbers that cannot be represented exactly, π, $\sqrt{2}$

Id: 17201026

Truncation error:

Truncation error is defined as the error caused by truncating or approximating a methematical proceedure.

example:

a series (Maclaurin):

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots$$

there infinite number of terms.

But When calculating e^x, only a finite number of terms are used.

Now if we uses 3 terms to calculate,

$$e^x \approx 1 + x + \frac{x^2}{2!}$$

Now the truncation error,

$$= e^x - \left(1 + x + \frac{x^2}{2!}\right)$$

$$= \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\cdots$$

Id: 17201026

control truncation error:

We can, use the concept of relative approximate error to see how many terms need to be considered.

Let, calculating $e^{1.2}$ using Maclarin series,

$$e^{1.2} = 1+1.2 + \frac{1.2^2}{2!} + \frac{1.2^3}{3!} + \dots$$

suppose, we want the absolute approximate error to be less than 1%.

Now if we take 2 terms in series

$|\epsilon_a|\% = 54.546\%$

formula
$$|\epsilon_a|\% = \left|\frac{CA-PA}{CA}\right| * 100\%$$

if we tak 3 terms,

$|\epsilon_a|\% = 24.658\%$

⋮

like these if we take 6 terms

$|\epsilon_a|\% = 0.62550\% < 1\%$

Q. calculate trancation error

$$f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$f(x) = 6x^2$, $f'(3) = ?$, $\Delta x = 0.2$

Approx value :

$$f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(3) \approx \frac{f(3+0.2) - f(3)}{0.2}$$

$$\approx \frac{6(3.2)^2 - 6(3)^2}{0.2}$$

$$\approx 37.2$$

actual/True value :

$f(x) = 6x^2$

$f'(x) = 12x$

$f'(3) = 12 \times 3 = 36$

$\therefore$ Trancation error $= 36 - 37.2$ (True − Aprox)

$= -1.2$

$$\text{Relative error} = \frac{\text{Trancation error}}{\text{True value}} = \frac{-1.2}{36}$$

$= 0.0333$

Binary Representation

Base-10,

Suppose, 256.79 → Base 10 এর নাম্বার কে Represent করেছে

$$256.79 = 2\times10^{2}+5\times10^{1}+6\times10^{0}+7\times10^{-1}+9\times10^{-2}$$

(0-9) Digits

Decimal point

শতক এর ঘর

দশক এর ঘর

Now,

Binary number : Base-2

(0-1)

Suppose a number $(1011.0011)_2$

radix point

$$(1011.0011)_2 = 1\times2^{3}+0\times2^{2}+1\times2^{1}+1\times2^{0}+1\times2^{0}+0\times2^{-1}+0\times2^{-2}+1\times2^{-3}+1\times2^{-4}$$

$$= (11.1875)_{10}$$

Fractional Decimal Number to Binary

$(11.1875)_{10} = (?.?)_2$

Radix point

	Quotient	Remiander
11/2	5	$a_0 = 1$
5/2	2	$a_1 = 1$
2/2	1	$a_2 = 0$
1/2	0	$a_3 = 1$

$(11)_{10} = (a_3, a_2, a_1, a_0)$

$= (1011)_2$

	Number	After decimal	Before decimal
.1875 ×2	.375	0.375	$0 = a_{-1}$
0.375 ×2	0.750	0.750	$0 = a_{-2}$
0.75×2	1.50	0.500	$1 = a_{-3}$
0.5×2	1.00	0.00	$1 = a_{-4}$

$\therefore (.1875)_{10} = (.0011)_2$

$\therefore (11.1875)_{10} = (1011.0011)_2$

Another approach : 2 এর Power এ ভাঙ্গাব

$(11.1875)_{10} = (?.?)_2$

highest power which $2^3 = 8 < 11$

$11 - 8 = 3$

$2^1 = 2 < 3$

$(11)_{10} = 2^{3} + 3$

$= 2^3 + 2^1 + 1$

what is the largest power of 2? which is part of 11

$= 2^3 + 2^1 + 2^0$

1

নাই তাই 0

$= \underline{1} \times 2^3 + \underline{0} \times \underline{2^2} + \underline{1} \times 2^1 + \underline{1} \times 2^0$

এখানে 1 কারণ

$\rightarrow 2^3$ আছে

$= (1011)_2$

Now $\Rightarrow (.1875)_{10} = 2^{-3} + (0.1875 - 0.125)$

$= 2^{-3} + 0.0625$

$= 2^{-3} + 2^{-4}$

$= 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}$

$= (0.0011)_2$

$2^{-1} = 0.5 > .1875$

$2^{-2} = 0.25 > .1875$

$2^{-3} = 0.125 < .1875$

2^{-3} part of $.1875$

$2^{-4} = 0.0625$

$$\therefore (11.1875)_{10} = (1011.0011)_2$$