

Assignment-03

①

$$(\sim P \vee Q) \wedge R \rightarrow S \vee (\sim R \wedge Q)$$

□ I_1 : P is true, Q is true, R is false, S is true.

Here,

Rule 2 gives $\sim P$ as False.

Rule 5 gives $(\sim P \vee Q)$ as True.

Rule 4 gives $(\sim P \vee Q) \wedge R$ as False.

Rule 2 gives $\sim R$ as True.

Rule 3 gives $\sim R \wedge Q$ as True.

Rule 4 gives $S \vee (\sim R \wedge Q)$ as True.

Rule 7 gives $(\sim P \vee Q) \wedge R \rightarrow S \vee (\sim R \wedge Q)$ as True.

So, the statement I_1 is True.

□ I_2 : P is true, Q is false, R is true, S is true.

Here, Rule 2 gives $\sim P$ as False.

Rule 5 gives $\sim P \vee Q$ as False.

Rule 3 gives $(\sim P \vee Q) \wedge R$ as False.

Rule 2 gives $\sim R$ as False.

Rule 3 gives $\sim R \wedge Q$ as False.

Rule 4 gives $S \vee (\sim R \wedge Q)$ as True.

Rule 7 gives $(\neg P \vee Q) \wedge R \rightarrow S \vee (\neg R \wedge Q)$ as True.

So, the statement I_2 is True. \square

(2)

$$S1: (P \wedge Q) \vee \sim(P \wedge Q)$$

P	Q	$(P \wedge Q)$	$\sim(P \wedge Q)$	$(P \wedge Q) \vee \sim(P \wedge Q)$
F	F	F	T	T
F	T	F	T	T
T	F	F	T	T
T	T	T	F	T

So, S1 is valid because it is true for every interpretation

$$S2: (P \vee Q) \rightarrow (P \wedge Q)$$

P	Q	$P \vee Q$	$P \wedge Q$	$\sim(P \vee Q)$	$(P \vee Q) \rightarrow (P \wedge Q)$
F	F	F	F	T	T
F	T	T	F	F	F
T	F	T	F	F	F
T	T	T	T	F	T

So, S2 is satisfiable because there is some interpretation for which it is true.

$$S_3: (P \wedge Q) \rightarrow R \vee \sim Q$$

P	Q	R	$P \wedge Q$ (X)	$\sim Q$	$R \vee \sim Q$ (Y)	$\sim X$	$X \rightarrow Y$
F	F	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	T	F	F	F	F	T	T
F	T	T	F	F	T	T	T
T	F	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	T	F	T	F	F	F	F
T	T	T	T	F	T	F	T

Let's assume,

$$P \wedge Q = X$$

$$R \vee \sim Q = Y$$

Here, S_3 is satisfiable because there is some interpretation for which it is true.

$$S_4: (P \vee Q) \wedge (P \vee \sim Q) \vee P$$

P	Q	$P \vee Q$ (X_1)	$\sim Q$	$P \vee \sim Q$ (X_2)	$X = X_1 \wedge X_2$	$X \vee P$
F	F	F	T	T	F	F
F	T	T	F	F	F	F
T	F	T	T	T	T	T
T	T	T	F	T	T	T

So, S_4

Let's assume,

$$P \vee Q = X_1$$

$$P \vee \sim Q = X_2$$

So, S_4 is satisfiable because there is some interpretation for which it is true.

$$S_5: P \rightarrow Q \rightarrow \sim P$$

P	Q	$\sim P$	$\sim Q$	$Q \rightarrow \sim P$	$P \rightarrow Q \rightarrow \sim P$
F	F	T	T	T	T
F	T	T	F	T	T
T	F	F	T	T	T
T	T	F	F	F	F

So, S_5 is satisfiable because there is some interpretation for which it is true.

$$S_6: P \vee Q \wedge \sim P \vee \sim Q \wedge P$$

P	Q	$\sim P$	$\sim Q$	$P \vee Q$ (X_1)	$\sim P \vee \sim Q$ (X_2)	$X = X_1 \wedge X_2$	$X \wedge P$
F	F	T	T	F	T	F	F
F	T	T	F	T	T	T	F
T	F	F	T	T	T	T	T
T	T	F	F	T	F	F	F

Let's assume,

$$P \vee Q = X_1$$

$$\sim P \vee \sim Q = X_2$$

So, S_6 is satisfiable because there is some interpretation for which it is true.

Meaning of Statements

☐ If the earth moves round the sun or the sun moves round the earth, then Copernicus might be a mathematician but wasn't an astronomer.

⇒ Let's assume,

P = The earth moves round the sun.

Q = The sun moves round the earth.

R = Copernicus is a mathematician.

S = Copernicus was an astronomer.

So, the sentence will be: $P \vee Q \rightarrow R \wedge \sim S$

☐ In spite of having French nationality, B. Russel was a critic of imperialism, then either he was not a bachelor or he was a universal lover.

⇒ Let's assume,

P = have french nationality.

Q = ^{B. Russel was} a critic of imperialism.

R = He was a bachelor.

S = He was a universal lover.

So, the sentence will be: $P \rightarrow Q \rightarrow (\sim R \vee S)$.

Examples of Predicate Logic (Page-12)

* $\forall x \exists y (\text{LOVES}(x, y))$

\Rightarrow Everyone loves someone.

* $\forall x (\text{HANDSOME}(x) \Rightarrow \exists y (\text{LOVES}(y, x)))$

\Rightarrow Every handsome people are loved by some people.

* All men are mortal.

$\Rightarrow \forall x (\text{Man}(x) \Rightarrow \text{Mortal}(x))$

* Noone likes hartal.

$\Rightarrow \forall x (\sim \text{LIKES}(x, \text{HARTAL}))$

* Everyone taking AI will pass their exam.

$\Rightarrow \forall x (\text{TAKING}(x, \text{AI}) \rightarrow \text{PASS}(x))$

* Every race has a winner.

$\Rightarrow \forall x (\text{RACE}(x) \rightarrow \exists y (\text{WINNER}(y, x)))$

* Sajjad likes everyone who is tall.

$\Rightarrow \forall x (\text{TALL}(x) \rightarrow \text{LIKES}(\text{Sajjad}, x))$

* Rita doesn't like anyone who prefers arguments.

$\Rightarrow \forall x (\text{PREFERS ARGUMENT}(x) \rightarrow \sim \text{LIKES}(\text{Rita}, x))$

* There is something small and slimy on the table.

$\Rightarrow \exists x (\text{SMALL}(x) \wedge \text{SLIMY}(x) \wedge \text{ON}(x, \text{table}))$