Answer: 1(a)

$$\begin{array}{c c} & \begin{matrix} C & A & E \end{matrix} \\ \begin{matrix} C \\ A \\ E \end{matrix} & \begin{bmatrix} 0.8 & 0.19 & 0.01 \\ 0.6 & 0.34 & 0.06 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

1(b)

$$P^5 = P^{2+3} = P^2 \times P^3$$

$$P^2 = P^{1+1} = P^1 \times P^1 = \begin{bmatrix} 0.8 & 0.19 & 0.01 \\ 0.6 & 0.34 & 0.06 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.8 & 0.19 & 0.01 \\ 0.6 & 0.34 & 0.06 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.754 & 0.2166 & 0.0294 \\ 0.684 & 0.2296 & 0.0864 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P^3 = P^{2+1} = P^2 \times P^1 = \begin{bmatrix} 0.754 & 0.2166 & 0.0294 \\ 0.684 & 0.2296 & 0.0864 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.8 & 0.19 & 0.01 \\ 0.6 & 0.34 & 0.06 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.733 & 0.217 & 0.0499 \\ 0.685 & 0.208 & 0.107 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore P^5 = P^2 \times P^3 = \begin{bmatrix} 0.754 & 0.2166 & 0.0294 \\ 0.684 & 0.2296 & 0.0864 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.733 & 0.217 & 0.0499 \\ 0.685 & 0.208 & 0.107 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{array}{c c} & \begin{array}{ccc} C & A & E \end{array} \\ \begin{array}{c} C \\ A \\ E \end{array} & \begin{bmatrix} 0.7012 & 0.2086 & 0.0902 \\ 0.6587 & 0.1961 & 0.1451 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Ans: 0.1451

(c)

$$x_0 + x_1 + x_2 = 1 \quad \text{——— (I)}$$

$$x_0 = 0.8x_0 + \overset{0.6}{\cancel{0.19}}x_1 + 0 \times x_2 \quad \text{— (II)}$$

$$x_1 = 0.19x_0 + 0.34x_1 + 0 \times x_2 \quad \text{— (III)}$$

$$x_0 \approx 0 \quad , x_1 \approx 0 \; , \; x_2 = 1$$

✓

2(b)

Binomial.

(c)

$X=$

(a)

$^{n}C_{i}\ p^{i}(1-p)^{n-i}$

$^{3}C_{1} \times (0.3)^{1} \times (1-0.3)^{3-1}$

3(a)

Book 1 $\rightarrow$ 2hrs ✗

2 $\longrightarrow$ 5hours ✓

3 — 3 hours. ✗

X = time needed to find the answer and write it in the script 'DONE'.

Y = # of book

$$P\{Y=1\} = P\{Y=2\} = P\{Y=3\} = \frac{1}{3}$$

$$E[X|Y=1] = 2 + E[X]$$

$$E[X|Y=2] = 5$$

$$E[X|Y=3] = 3 + E[X]$$

$$\therefore E[X] = E[X|Y=1] \cdot P\{Y=1\} + E[X|Y=2] \cdot P\{Y=2\} + E[X|Y=3] \cdot P\{Y=3\}$$

$$\Rightarrow E[X] = (2+E[X]) \times \frac{1}{3} + 5 \times \frac{1}{3} + (3+E[X]) \times \frac{1}{3}$$

$$\Rightarrow 3E[X] = 2 + E[X] + 5 + 3 + E[X]$$

$E[X] = 10$

3(b) (Example-2)

$\lambda_1 = 1$ $\lambda_2 = 1$

x = # of votes Lamia got.

y = # of votes Tasnima got.

$$P\{X=39 \mid X+Y=60\} = \frac{P\{X=39, X+Y=60\}}{P\{X+Y=60\}}$$

$$= \frac{P\{X=39, Y=60-X=60-39\}}{P\{X+Y=60\}}$$

$$= \frac{P\{X=39, Y=21\}}{P\{X+Y=60\}}$$

$$= \frac{P\{X=39\} \cdot P\{Y=21\}}{P\{X+Y=60\}}$$

$$= \frac{\dfrac{e^{-\lambda_1}\lambda_1^{39}}{39!} \times \dfrac{e^{-\lambda_2}\lambda_2^{21}}{21!}}{\dfrac{e^{-(\lambda_1+\lambda_2)}\cdot(\lambda_1+\lambda_2)^{60}}{60!}}$$

$$= \frac{\dfrac{e^{-\lambda_1}\cdot\lambda_1^{39}}{39!} \times \dfrac{e^{-\lambda_2}\lambda_2^{21}}{21!}}{\dfrac{e^{-\lambda_1}\cdot e^{-\lambda_2}\cdot(\lambda_1+\lambda_2)^{60}}{60!}}$$

$$= \frac{\cancel{e^{-\lambda_1}}\lambda_1^{39}}{39!} \times \frac{\cancel{e^{-\lambda_2}}\lambda_2^{21}}{21!} \times \frac{60!}{\cancel{e^{-\lambda_1}}\cancel{e^{-\lambda_2}}\cdot(\lambda_1+\lambda_2)^{60}}$$

$$= \frac{60!}{39!\,21!}\cdot\frac{\lambda_1^{39}}{(\lambda_1+\lambda_2)^{39}}\cdot\frac{\lambda_2^{21}}{(\lambda_1+\lambda_2)^{21}}$$

$$= {}^{60}C_{39}\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^{39}\cdot\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{21}$$

$$= {}^{60}C_{39}\cdot\left(\frac{1}{1+1}\right)^{39}\cdot\left(\frac{1}{1+1}\right)^{21}$$

$$= 0.006.$$

4(a) বই Page-13

2W
7B

5W
6B

Let, W = White ball selected.

H = the coin comes up head and from 1st basket.

$$P(H|W) = \frac{P(W|H)\,P(H)}{P(W|H).P(H) + P(W|H').P(H')}$$

$$= \frac{\frac{2}{9} \times \frac{1}{2}}{\frac{2}{9} \times \frac{1}{2} + \frac{5}{11} \times \frac{1}{2}} = \frac{22}{67}$$

Scanned with CamScanner

4(b)

E = We have already searched file 1 but did not find the letter 'm'

$\alpha_i \rightarrow 60\% = 0.6$

$$P(\text{File } 1 \mid E) = \frac{P(E \mid F_1) \cdot P(F1)}{P(E \mid F_1) \cdot P(F_1) + P(E \mid F_2) \cdot P(F_2) + P(E \mid F_3) \cdot P(f_3)}$$

$$= \frac{(1-\alpha_1) \cdot \cancel{P(F_1)} \frac{1}{3}}{(1-\alpha_1) \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3}}$$

$$= \frac{(1-0.6) \frac{1}{3}}{(1-0.6) \cdot \frac{1}{3} + \frac{1}{3} + \frac{1}{3}}$$

$$= \frac{\frac{0.4}{3}}{\frac{0.4}{3} + \frac{1}{3} + \frac{1}{3}}$$

$$= \frac{0.4}{3} \times \frac{3}{2.4}$$

$= 0.167$ ✓

OR

4(a)

P → she knows the answer

1−p → " guess the answer.

q → guess করা ansr correct → $\frac{1}{m}$.

$$P(P|q) = \frac{P(P).P(Q)}{P(Q)}$$

$$= \frac{p.(1-p)}{1-p} = p$$

K → Know   C → correct

K' → don't Know

$$P(K|C)? = \frac{P(C|K)\cdot P(K)}{P(C|K).P(K)+P(C|K').P(K')}$$

$$= \frac{1\times P}{1\times p + \frac{1}{m}\times(1-p)}$$

4(b)

~~1st basket~~ $\rightarrow$ ~~2W~~

$\boxed{\frac{7}{800}}$ Concrete math book

$\boxed{\frac{13}{1000}}$ Probability book

$X \rightarrow$ No. of mispelled words

$Y \rightarrow$

$$\left(\frac{7}{800} \times \frac{1}{2}\right) + \left(\frac{13}{1000}\right) \times \frac{1}{2}$$