Assignment

I am in Section A and my ID is odd. (17101007)

(a) Transition Probability matrix for the given seenario-

$$\begin{array}{c} \\ C \\ S \\ Gr \end{array}\begin{array}{c} \begin{array}{ccc} C & S & Gr \end{array} \\ \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix} \end{array}$$

Here,

C = cheerful.

S = so-so.

Gr = glum.

(b) Let's assume,

On any given day probability for the nex day is P.

∴ Probability 6 days from now will be: P^6

$$P^6 = P^{4+2} = P^4 \times P^2$$

$$P^2 = P^{1+1} = P^1 \times P^1 = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix} \times \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.39 & 0.39 & 0.22 \\ 0.33 & 0.37 & 0.30 \\ 0.29 & 0.35 & 0.36 \end{bmatrix}$$

Now,

$$P^4 = P^{2+2} = P^2 \times P^2 = \begin{bmatrix} 0.39 & 0.39 & 0.22 \\ 0.33 & 0.37 & 0.30 \\ 0.29 & 0.35 & 0.36 \end{bmatrix} \times \begin{bmatrix} 0.39 & 0.39 & 0.22 \\ 0.33 & 0.37 & 0.30 \\ 0.29 & 0.35 & 0.36 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3446 & 0.3734 & 0.282 \\ 0.3378 & 0.3706 & 0.2916 \\ 0.333 & 0.3686 & 0.2984 \end{bmatrix}$$

$$P^6 = P^4 \times P^2 = \begin{bmatrix} 0.3446 & 0.3734 & 0.282 \\ 0.3378 & 0.3706 & 0.2916 \\ 0.333 & 0.3686 & 0.2984 \end{bmatrix} \times \begin{bmatrix} 0.39 & 0.39 & 0.22 \\ 0.33 & 0.37 & 0.30 \\ 0.29 & 0.35 & 0.36 \end{bmatrix}$$

$$= \begin{array}{c} \\ C \\ S \\ Gr \end{array} \begin{array}{c} \begin{array}{ccc} C & S & Gr \end{array} \\ \begin{bmatrix} 0.3394 & 0.3713 & 0.2894 \\ 0.3386 & 0.3709 & 0.2905 \\ 0.3380 & 0.3707 & 0.2913 \end{bmatrix} \end{array}$$

So, If Zaman is glum (Gr) today, the probability that he will be cheerful 6 days from now is 0.3380.

(c) The probability that Zaman will be cheerful (C) 10,000 days from now is calculated below-

Given,

$$\begin{array}{c} \\ x_0 \leftarrow C \\ x_1 \leftarrow S \\ x_2 \leftarrow Gr \end{array} \begin{array}{c} \begin{array}{ccc} C^{+0} & S^{+1} & Gr^{+2} \end{array} \\ \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix} \end{array}$$

Here, $x_0 + x_1 + x_2 = 1$ ——— (i)

$$x_0 = 0.5x_0 + 0.3x_1 + 0.2x_2 \text{ — (ii)}$$

$$x_1 = 0.4x_0 + 0.4x_1 + 0.3x_2 \text{ — (iii)}$$

Solving equation (i), (ii) and (iii) we get,

$x_0 \approx 0.33871$, $x_1 \approx 0.370968$, $x_2 \approx 0.290323$.

So, the probability that Zaman will be cheerful 10,000 days from now is 0.33871