University of Asia Pacific

Department of Computer Science and Engineering

Mid Term Examination: Spring-2020 Section-A

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Answer to the question no:1(a)

$N = (007)\,\%\,100 + 1728 = 7 + 1728 = 1735$

$W = \sum_{1 \leqslant K \leqslant K} (3K+4) + \sum K^3 \leqslant n \leqslant N$

$= \sum_{1 \leqslant K \leqslant K-1} 3K + \sum_{1 \leqslant K \leqslant K-1} 4 + \sum_m K^3 \leqslant Km \leqslant N$

$= 3\frac{(K-1)(K-1+1)}{2} + 4(K-1) + \sum_m K^2 \leqslant m \leqslant \frac{N}{K}$

$= \frac{1}{2}(K-1)(3K+8) + \sum_m \left[m \in \left[K^2 \cdots \frac{N}{K}\right]\right]$

$= \frac{1}{2}(K-1)(3K+8) + \left\lfloor \frac{N}{K} \right\rfloor - \lceil K^2 \rceil + 1$

$= \cancel{\frac{1}{2}(12A)} = \frac{1}{2}(12-1)(3\times 12+8) + \left\lfloor \frac{1735}{12} \right\rfloor - \lceil (12)^2 \rceil + 1$

$= \frac{1}{2} \times 11 \times 44 + \lfloor 144.58 \rfloor - 144 + 1$

$= 242 + 144 - 144 + 1$

$= 243$

(Ans)

Answer to the question no: 1(b)

Double tower of Hanoi:

Recursive equation:

$$D_n = 2D_{n-2} + 2$$

$N = (007 \% 6) + 5 = 1 + 5 = 6$

$$D_6 = 2D_{6-2} + 2$$
$$= 2D_4 + 2$$
$$= 2(2D_{4-2} + 2) + 2$$
$$= 2(2D_2 + 2) + 2$$
$$= 2(2(2D_{2-2} + 2) + 2) + 2$$
$$= 2(2(2D_0 + 2) + 2) + 2$$
$$= 2(2(2 \times 0 + 2) + 2) + 2$$
$$= 2(2 \times 2 + 2) + 2$$
$$= 14$$

(Ans)

Answer to the question no: 2(a)

$N = (007+2)\ \%\ 10 + 20 = (9\ \%\ 10) + 20 = 29$

$q = (007\ \%\ 4) + 3 = 3+3 = 6$

Total no of people $N = 29$, 6th person is eliminated.

$\therefore D = \left\lceil \frac{6D}{5} \right\rceil$

$D = 1$

while $D <= 2N$

$D <= 58$:

$D = \left\lceil \frac{6D}{5} \right\rceil$

$D = \left\lceil \frac{6}{5} \cdot 1 \right\rceil = 2$

$D = \left\lceil \frac{6}{5} \cdot 2 \right\rceil = 3$

$D = \left\lceil \frac{6}{5} \cdot 3 \right\rceil = 4$

$D = \left\lceil \frac{6}{5} \cdot 4 \right\rceil = 5$

$D = \left\lceil \frac{6}{5} \cdot 5 \right\rceil = 6$

$D = \left\lceil \frac{6}{5} \cdot 6 \right\rceil = 8$

$D = \left\lceil \frac{6}{5} \cdot 8 \right\rceil = 10$

$D = \left\lceil \frac{6}{5} \cdot 10 \right\rceil = 12$

$D = \left\lceil \frac{6}{5} \cdot 12 \right\rceil = 15$

$D = \left\lceil \frac{6}{5} \cdot 15 \right\rceil = 18$

$D = \left\lceil \frac{6}{5} \cdot 18 \right\rceil = 22$

$D = \left\lceil \frac{6}{5} \cdot 22 \right\rceil = 27$

$D = \left\lceil \frac{6}{5} \cdot 27 \right\rceil = 33$

$D = \left\lceil \frac{6}{5} \cdot 33 \right\rceil = 40$

$D = \left\lceil \frac{6}{5} \cdot 40 \right\rceil = 48$

$D = \left\lceil \frac{6}{5} \cdot 48 \right\rceil = 58$

$\therefore J_6(29) = 6N + 1 - D$

$= 6 * 29 + 1 - 58$

$= 117$

(Ans)

Answer to the question no: 2(b)

Recursive equation for finding GCD of two numbers:

$$gcd(m,n) = gcd(n \% m, m)$$

$N_1 = (007 \% 100) + 50 = 57$

$N_2 = N_1 + 1000 = 57 + 1000 = 1057$

$$\begin{aligned} gcd(N_1, N_2) &= gcd(57, 1057) \\ &= gcd(31, 57) \\ &= gcd(26, 31) \\ &= gcd(5, 26) \\ &= gcd(1, 5) \\ &= gcd(0, 1) \\ &= 1 \quad [\because gcd(0, n = n \rightarrow \text{best case}] \end{aligned}$$

Ans

Answer to the question no: 4

$N_1 = (007 \% 3) + 1 = 2$

$N_2 = (007 \% 4) + 2 = 3 + 2 = 5$

$\therefore C_0 = 2$

$(n-2) C_n = (n+1) C_{n-1} + 5(n-2)(n^2-1)$ — (i)

(i) $* S_n \Rightarrow$

$S_n (n-2) C_n = S_n (n+1) C_{n-1} + 5 S_n (n-2)(n^2-1)$ — (ii)

$S_n (n+1) = S_{n-1} (n-3)$

$$S_n = \frac{(n-3)}{(n+1)} S_{n-1}$$

$$= \cancel{\frac{(n-3)}{(n+1)}} \cdot \cancel{\frac{(n-6)}{(n+2)}} \cdot \cancel{S_{n-2}}$$

$$= \frac{(n-3)}{(n+1)} \cdot \frac{(n-4)}{n} \cdot S_{n-2}$$

$$= \frac{(n-3)(n-4)\ldots 3 \cdot 2 \cdot 1}{(n+1) \cdot n \cdot (n-1) \ldots 4 \cdot 3} S_1$$

$$= \frac{2}{(n+1) \cdot n \cdot (n-1)(n-2)}$$

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From (II)

$$\frac{2}{(n+1).n.(n-1)(n-2)}(n-2)C_n = \frac{2}{(n+1).n.(n-1)(n-2)}(n+1)C_{n-1}$$

$$+ 5\frac{2}{(n+1).n.(n+1)(n-2)}(n-2)(n^2-1)$$

$$\frac{1}{n(n^2-1)}C_n = \frac{1}{n(n-1)(n-2)}C_{n-1} + \frac{5}{n(n^2-1)}(n^2-1)$$

$$\frac{1}{n(n^2-1)}C_n = \frac{1}{n(n-1)(n-2)}C_{n-1} + \frac{5}{n} \text{ --- (III)}$$

Let,

$$\frac{C_n}{n(n^2-1)} = Q_n \ ; \ \frac{C_{n-1}}{(n-1)\{(n-1)^2-1\}} = Q_{n-1}$$

From (III) ⇒

$$Q_n = Q_{n-1} + \frac{5}{n}$$

$$Q_n = \sum_{K=1}^{n} \frac{5}{K} + 2$$

$$= 5\sum_{K=1}^{n} \frac{1}{K} + 2$$

$$= 5 \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right) + 2$$

$$= 5H(n) + 2$$

$$\frac{C_n}{n(n^2-1)} = 5H(n) + 2$$

$$C_n = 5H(n) \cdot n(n^2-1) + 2n(n^2-1)$$

$$\therefore C_n \approx 5\log(n) \cdot n(n^2-1) + 2n(n^2-1) \quad \left[\because H(n) \approx \log(n)\right]$$

(Ans)

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