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Assignment-02

Answer to the question no: 1

Here, Total no. of balls = 15

Let, Z denote the number of red balls selected respectively.

$$\begin{aligned} & \text{(a)} \\ P(X=0|Y=3) &= \frac{P(X=0, Y=3)}{P(Y=3)} \\ &= \frac{P(X=0, Y=3, Z=4)}{P(Y=3)} \\ &= \frac{\frac{7!}{0!3!4!} \left(\frac{4}{15}\right)^0 \cdot \left(\frac{6}{15}\right)^3 \cdot \left(\frac{5}{15}\right)^4}{\frac{7!}{3!4!} \left(\frac{6}{15}\right)^3 \cdot \left(\frac{9}{15}\right)^4} \\ &= \frac{0.0277}{0.2903} \\ &= 0.0954. \end{aligned}$$

(b)

$$P(X=3|Y=3) = \frac{P(X=3, Y=3)}{P(Y=3)}$$

$$= \frac{P(X=3, Y=3, Z=1)}{P(Y=3)}$$

$$= \frac{\frac{7!}{3!3!1!} \left(\frac{4}{15}\right)^3 \cdot \left(\frac{6}{15}\right)^3 \cdot \left(\frac{5}{15}\right)^1}{\frac{7!}{3!4!} \left(\frac{6}{15}\right)^3 \cdot \left(\frac{9}{15}\right)^4}$$

$$= \frac{0.0566}{0.2903}$$

$$= 0.19497$$

(c)

$$P(X=4|Y=3) = \frac{P(X=4, Y=3)}{P(Y=3)}$$

$$= \frac{P(X=4, Y=3, Z=0)}{P(Y=3)}$$

$$= \frac{\frac{7!}{4!3!0!} \left(\frac{4}{15}\right)^4 \cdot \left(\frac{6}{15}\right)^3 \cdot \left(\frac{5}{15}\right)^0}{\frac{7!}{3!4!} \left(\frac{6}{15}\right)^3 \cdot \left(\frac{9}{15}\right)^4}$$

$$= \frac{0.0113}{0.2903}$$

$$= 0.0389$$

(d)

$$E[X|Y=1] = \sum_i i \cdot P\{X=i|Y=1\}$$

Here, i represents the number of white balls.

$$i = 0, 1, 2, 3, 4$$

$$\therefore P\{X=0|Y=1\} = \frac{P(X=0, Y=1)}{P(Y=1)}$$

$$= \frac{P(X=0, Y=1, Z=6)}{P(Y=1)}$$

$$\frac{\frac{7!}{0!1!6!} \left(\frac{4}{15}\right)^0 \cdot \left(\frac{6}{15}\right)^1 \cdot \left(\frac{5}{15}\right)^6}{\frac{7!}{1!6!} \left(\frac{6}{15}\right)^1 \cdot \left(\frac{9}{15}\right)^6}$$

$$= \frac{0.0038}{0.1306}$$

$$= 0.0291$$

$$P\{X=1|Y=1\} = \frac{P(X=1, Y=1)}{P(Y=1)}$$

$$= \frac{P(X=1, Y=1, Z=5)}{P(Y=1)}$$

$$= \frac{\frac{7!}{1!1!5!} \left(\frac{4}{15}\right)^1 \cdot \left(\frac{6}{15}\right)^1 \cdot \left(\frac{5}{15}\right)^5}{\frac{7!}{1!6!} \left(\frac{6}{15}\right)^1 \cdot \left(\frac{9}{15}\right)^6}$$

$$= \frac{0.0184}{0.1306}$$

$$= 0.1409$$

$$P\{X=2|Y=1\} = \frac{P\{X=2, Y=1\}}{P(Y=1)}$$

$$= \frac{P(X=2, Y=1, Z=4)}{P(Y=1)}$$

$$= \frac{\frac{7!}{2!1!4!} \left(\frac{4}{15}\right)^2 \cdot \left(\frac{6}{15}\right)^1 \cdot \left(\frac{5}{15}\right)^4}{\frac{7!}{1!6!} \left(\frac{6}{15}\right)^1 \cdot \left(\frac{9}{15}\right)^6}$$

$$= \frac{0.0369}{0.1306}$$

$$= 0.2825$$

$$P\{X=3|Y=1\} = \frac{P(X=3, Y=1)}{P(Y=1)}$$

$$= \frac{P(X=3, Y=1, Z=3)}{P(Y=1)}$$

$$= \frac{\frac{7!}{3!1!3!} \left(\frac{4}{15}\right)^3 \cdot \left(\frac{6}{15}\right)^1 \cdot \left(\frac{5}{15}\right)^3}{\frac{7!}{1!6!} \left(\frac{6}{15}\right)^1 \cdot \left(\frac{9}{15}\right)^6}$$

$$= \frac{0.0393}{0.1306}$$

$$= 0.3009$$

$$P\{X=4|Y=1\} = \frac{P(X=4, Y=1)}{P(Y=1)}$$

$$= \frac{P(X=4, Y=1, Z=2)}{P(Y=1)}$$

$$= \frac{\frac{7!}{4!1!2!} \left(\frac{4}{15}\right)^4 \cdot \left(\frac{6}{15}\right)^1 \cdot \left(\frac{5}{15}\right)^2}{\frac{7!}{1!6!} \left(\frac{6}{15}\right)^1 \cdot \left(\frac{9}{15}\right)^6}$$

$$= \frac{0.0236}{0.1306}$$

$$= 0.1807$$

∴

$$E[X|Y=1] = 0 \times P\{X=0|Y=1\} + 1 \times P\{X=1|Y=1\} \\ + 2 \times P\{X=2|Y=1\} + 3 \times P\{X=3|Y=1\} \\ + 4 \times P\{X=4|Y=1\}$$

$$= (0 \times 0.0291) + (1 \times 0.1409) + (2 \times 0.2825) + (3 \times 0.3009) \\ + (4 \times 0.1807)$$

$$= 2.3314$$

(Ans)