《Analysis of Financial Time Series》

Time：（Tu）10:10-12:00 （Fri）11:10-12:00
Classroom：理 Sci 4009-1

Textbook : Analysis of Financial Time Series, 3rd Edition
Author : Ruey S. Tsay
https://faculty.chicagobooth.edu/ruey.tsay/teaching/fts3

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Financial time series and their characteristics

- What is financial time series analysis?
- Theory and practice of asset valuation (資產評價) over time.
- Different from other T.S. analysis?
- Highly related, but with some added uncertainty.
- For example, FTS must deal with the ever-changing business & economic environment and the fact that **volatility** is not directly observed.

Objective of the course

- Provide some basic knowledge of financial time series data.
- Introduce some statistical tools & econometric models useful for analyzing these series.
- Gain empirical experience in analyzing FTS.
- Introduce high-frequency finance.
- Study methods for assessing market risk, credit risk, and expected loss.
- Analyze high-dimensional asset returns.

Daily log returns of Apple stock from 2000 to 2007

- 大約有 $8 \times 252 = 2016$ 筆日資料。

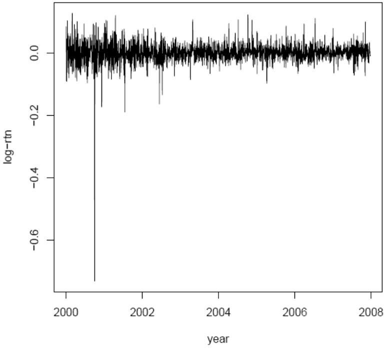


Figure: Daily log returns of Apple stock.

Quarterly earnings per share (季每股盈餘)of Johnson & Johnson, $1960 \sim 1980$.

- 大約有 $21 \times 4 = 84$ 筆季資料。
- Seasonal time series useful in
  - earning forecasts.
  - pricing weather related derivatives (衍生性商品)(e.g. energy).
  - modeling intraday behavior of asset returns.

Figure: Quarterly earnings per share of Johnson and Johnson.

Earning Per Share (EPS; 每股盈餘)

- EPS：the portion of a company's profit allocated to each outstanding share of common stock (普通股), serves as an indicator of a company's profitability (獲利指標).
- $EPS = \frac{Net\ Income - \ Dividends\ on\ Preferred\ Stock}{Average\ outstanding\ shares}$
  $= (\text{稅後純益 - 發給優先股股東的利息})/\text{發行股數}$
- Net Income (稅後純益) = (公司年度獲利) − (應繳稅額)。
- outstanding shares (發行股數) 與公司資本額相關。 Eg. 若公開發行公司股票的每股面額為 10 元，則發行股數 = 公司資本額 /10。

US monthly interest rates, 1953.04 $\sim$ 2009.02

- 美國 10 年期及 1 年期的月利率：約 12×56 筆月資料。
- Fixed income (固定收益)：每期固定發放利息，且於期末收回本金的投資商品。
- Relations between the two series? Term structure of interest rates.

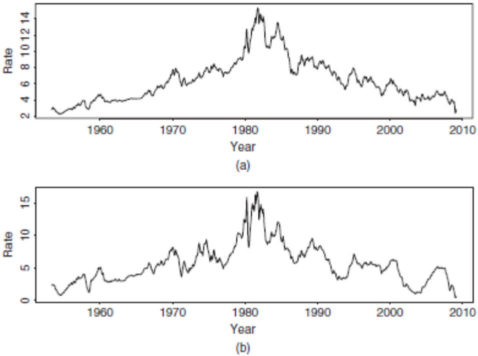


Figure 1.5 Time plots of monthly U.S. interest rates from April 1953 to February 2009: (a) 10-year Treasury constant maturity rate and (b) 1-year maturity rate.

Exchange rate between US Dollar vs Japanese Yen, 1971.1.4 $\sim$ 2009.3.20

- 約 18 天 (1 月) + 22 天 (2 月) + 252 天 $\times$ 38 年 = 9616 筆。
- Hedging (避險)。

Figure: Daily Exchange Rate: Yen vs Dollar.

World Time Zone

- Foreign exchange market 24 hr (匯率 24 小時均有交易)。

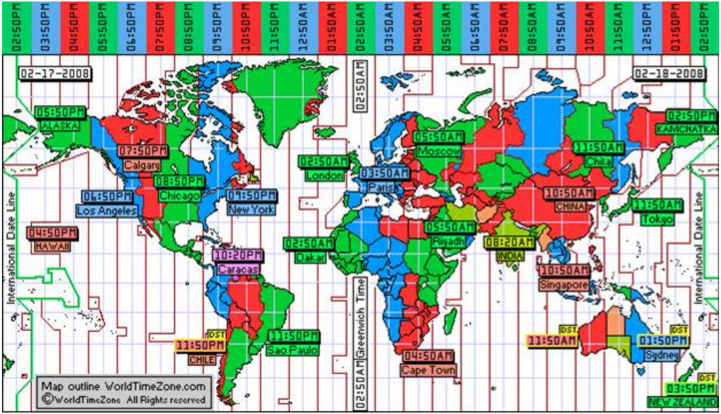
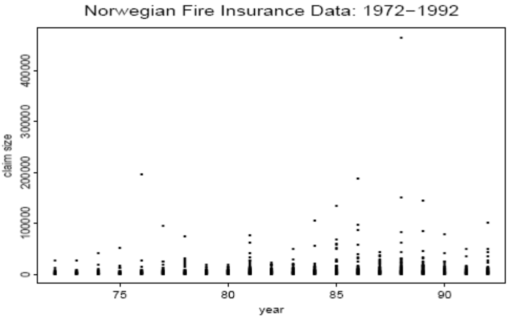


Figure: World Time Zone.

Claim sizes of the Norwegian fire insurance, $1972 \sim 1992$

- Yearly values of Norwegian(挪威) fire insurance claims ($\times 1000$ Krone(挪威克郎貨幣)) that exceeded 500 from 1972 to 1992.
- 1 Norwegian Krone (NOK) $=$ 3.15 Taiwan Dollar (TWD) (2021.02.11 exchange rate)



High frequency Tick-by-tick trading data of Boeing stock, 2005.12.5.

symbol	date	time	price	size
BA	05DEC2005	9:31:10	69.4500	60700
BA	05DEC2005	9:31:11	69.4500	100
BA	05DEC2005	9:31:11	69.4500	200
...				
BA	05DEC2005	9:31:11	69.4500	2500
BA	05DEC2005	9:31:11	69.4500	100
BA	05DEC2005	9:31:11	69.4500	100
BA	05DEC2005	9:31:12	69.4500	1600
BA	05DEC2005	9:31:12	69.4500	1500
...				
BA	05DEC2005	9:31:18	69.4500	100
BA	05DEC2005	9:31:19	69.4500	100

Price and Return

- Why "return" not "price"?
- 大部分的財務研究都是探討報酬率 (return) 卻不探討價格的兩大主因 (two reasons)：
  - 報酬率提供一般投資者關於投資機會的完整訊息且報酬率的計算與使用的貨幣無關 (scale-free)
  - 報酬率較價格具有較佳的統計性質 (better statistical properties)，such as stationarity (平穩性).

One-period Simple Asset Returns

- Let P_t be the price of an asset at time t, and assume no dividend (股利 or 股息).
- One-period simple return
  - Gross return:

$$1 + R_t = \frac{P_t}{P_{t-1}} \text{ or } P_t = P_{t-1}(1 + R_t).$$

  - Simple return:

$$R_t = \frac{P_t}{P_{t-1}} - 1 = \frac{P_t - P_{t-1}}{P_{t-1}}.$$

- Time interval is important when talking about return!
- Default of this book is one year $\Longrightarrow$ one-period $=$ one year.

Multiperiod Simple Asset Returns

- The k-period simple gross return

$$\begin{aligned} 1 + R_t(k) &= \frac{P_t}{P_{t-k}} \\ &= \frac{P_t}{P_{t-1}} \times \frac{P_{t-1}}{P_{t-2}} \times \cdots \times \frac{P_{t-k+1}}{P_{t-k}} \\ &= (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1}). \end{aligned}$$

- The k-period simple net return is

$$\begin{aligned} R_t(k) &= \frac{P_t}{P_{t-k}} - 1 \\ &= (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1}) - 1. \end{aligned}$$

Example of simple return

Suppose the daily closing prices of a stock are

Day	1	2	3	4	5
Price	37.84	38.49	37.12	37.60	36.30

1. What is the simple return from day 1 to day 2?

Ans $R_2 = \frac{38.49 - 37.84}{37.84} = 0.017.$

2. What is the simple return from day 1 to day 5?

Ans $R_5(4) = \frac{36.30 - 37.84}{37.84} = -0.041.$

3. Verify that $1 + R_5(4) = (1 + R_2)(1 + R_3) \ldots (1 + R_5)$.

- 給定 k 期報酬率之下，求平均的單期報酬率

$$P_t = P_{t-k}(1 + \text{Annualized}[R_t(k)])^k$$

- Annualized (average) return (年內的月平均報酬率), $k = 12$。
- Since

$$\frac{P_t}{P_{t-k}} = 1 + R_t(k) = (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1}),$$

$$1 + \text{Annualized}[R_t(k)] = \left[\prod_{j=0}^{k-1}(1 + R_{t-j})\right]^{1/k}.$$

$$\text{Annualized}[R_t(k)] = \exp\left(\frac{1}{k}\sum_{j=0}^{k-1}\ln(1 + R_{t-j})\right) - 1$$

$$\approx \exp\left(\frac{1}{k}\sum_{j=0}^{k-1}R_{t-j}\right) - 1 \approx \frac{1}{k}\sum_{j=0}^{k-1}R_{t-j}$$

(When x is small, $\ln(1 + x) \approx x$, $\exp(x) \approx 1 + x$)

- Recall

$$\text{Annualized}[R_t(k)] = (\frac{P_t}{P_{t-k}})^{\frac{1}{k}} - 1$$

$$= \left[\prod_{j=0}^{k-1}(1+R_{t-j})\right]^{1/k} - 1 \quad \approx \frac{1}{k}\sum_{j=0}^{k-1} R_{t-j}$$

E.g. 去年一月投資價值為 10 萬的資產，今天一月初資產價值為 11 萬。則平均每個月的簡單報酬率為：

$$\left(\frac{11}{10}\right)^{1/12} - 1 = 0.00797414$$

其值近似於 12 個單月簡單報酬率的算數平均。

Continuously compounding (連續複利)

- Illustration of the power of continuously compounding.
- $$A = C \cdot \lim_{k\to\infty} \left(1 + \frac{r}{k}\right)^{kn} = C \cdot \exp[r \times n]$$
  - r is the interest rate per annum
  - C is the initial capital, k is the number of payment per year
  - n is the number of years.
- Interest rate $= 10\%$ per annum

Type	#(payment)	利率	Net
Annual	1	0.1	$\$1.10000$
Semi-Annual	2	0.05	$\$1.10250$
Quarterly	4	0.025	$\$1.10381$
Monthly	12	0.0083	$\$1.10471$
Weekly	52	$\frac{0.1}{52}$	$\$1.10506$
Daily	365	$\frac{0.1}{365}$	$\$1.10516$
Continuously	∞		$\$1.10517$

- 對任一個給定的離散複利皆可以找到一個連續複利 r 與之對應，使兩者在到期時有相同的本利和。E.g. 在上列中 semi-Annual 離散複利 (0.05) 所對應的連續複利的解為

$$1.1025 = \exp(r) \Rightarrow r = 0.0975803.$$

- Present value (現值):

$$C = A \cdot \exp[-r \times n]$$

- Continuously compounded return (log return):
當 $n = 1$ 且利率隨時間改變 $(r = r_t)$，

$$\begin{aligned} P_t &= P_{t-1} e^{r_t} \\ \Rightarrow r_t &= \ln(P_t) - \ln(P_{t-1}) \\ &= \ln\left(\frac{P_t}{P_{t-1}}\right) \\ &= \ln(1 + R_t) \end{aligned}$$

因此連續複利 r_t 又稱為對數報酬率 (log return)。

- Multiperiod log return (多期對數報酬率) $r_t(k)$:

$$\begin{aligned} P_t &= P_{t-k} e^{r_t(k)} \\ \Rightarrow r_t(k) &= \ln\left(\frac{P_t}{P_{t-k}}\right) \\ &= \ln\left(1 + \frac{P_t - P_{t-k}}{P_{t-k}}\right) \\ &= \ln(1 + R_t(k)). \end{aligned}$$

$$\begin{aligned} r_t(k) &= \ln[1 + R_t(k)] \\ &= \ln[(1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1})] \\ &= \ln(1 + R_t) + \ln(1 + R_{t-1}) + \cdots + \ln(1 + R_{t-k+1}) \\ &= r_t + r_{t-1} + \cdots + r_{t-k+1}. \end{aligned}$$

(R: simple return, r: log return, P: price.)

Example of computing log returns

Use the previous daily prices.

Day	1	2	3	4	5
Price	37.84	38.49	37.12	37.60	36.30

1. What is the log return from day 1 to day 2?

Ans: $r_2 = \ln(38.49) - \ln(37.84) = 0.017.$

2. What is the log return from day 1 to day 5?

Ans: $r_5(4) = \ln(36.3) - \ln(37.84) = -0.042.$

3. It is easy to verify $r_5(4) = r_2 + \cdots + r_5$.

Portfolio's simple return

- Consider a portfolio of N assets.
- Let w_i = the capital allocation (%) of the i-th asset

$$\sum_{i=1}^{N} w_i = 1$$

- Let V_t= the portfolio value at time t
  (個股購買股數乘以當時股價)

$$V_t = \sum_{i=1}^{N} \frac{w_i V_{t-1}}{P_{it-1}} P_{it} = \sum_{i=1}^{N} w_i V_{t-1}(1 + R_{it})$$

$$\frac{V_t}{V_{t-1}} = \sum_{i=1}^{N} w_i(1 + R_{it}) = 1 + \sum_{i=1}^{N} w_i R_{it}$$

- Portfolio (one-period) simple return: N assets

$$R_{p,t} = \frac{V_t}{V_{t-1}} - 1 = \sum_{i=1}^{N} w_i R_{it}$$

Portfolio return example

- Portfolio (one-period) simple return: N assets

$$R_{p,t} = \sum_{i=1}^{N} w_i R_{it}$$

- Portfolio (one-period) log return: N assets

$$r_{p,t} = \ln(1 + R_{p,t}) = \ln(1 + \sum_{i=1}^{N} w_i R_{it})$$

- Example:
  An investor holds stocks of IBM, Microsoft and Citi-Group. Assume that her capital allocation is 30%, 30% and 40%. Given the following monthly simple returns (see Table 1.2):IBM $= 1.42\%$, Microsoft $= 3.37\%$ and Citi-Group $= 2.20\%$. What is the mean simple return of her stock portfolio?

Ans: $R_{p,t} = 0.3 \times 1.42 + 0.3 \times 3.37 + 0.4 \times 2.20 = 2.32.$

Dividend payment

- 除權
  是指公司將「股票股利」發給投資人。舉例來說，若公司決定發放「1 元股票股利」，你手中原來有 1000 股（也就是 1 張股票），將會變成 1100 股。換句話說，你手中多了 100 股的股票。為什麼「1 元股票股利 = 100 股的股票」呢？那是因為台灣規定股票的「票面價」是「1 股 10 元」，所以股票股利必須除以 10 元。
  股票股利 =1000 股 *(1/10)=100 股
- 除息
  是指公司讓股利直接以「發現金」的方式，發給投資人。假設你手中有 1000 股（也就是 1 張）台積電，公司今年將配發 7 元現金，那麼你就會拿到 7000 元的現金。
  現金股利 =1000 股 *7=7000 元

Dividend payment

- 除權息後股價會跌
  以台積電為例，台積電今年沒有配發股票股利，只有現金股利 7 元。那麼在 6 月 26 日除息那天，股價就會較前一個收盤日「少 7 元」，因為這 7 元已經配發給股東了。
  不過，投資人不能以為，前一個交易日收盤價是 217 元，到了 6 月 26 日，台積電股價就是直接以 210 元開盤。因為還有許多盤前就掛好要買進或賣出台積電的單子，所以當股市一開盤開始交易，台積電的股價不太可能直接停在 210 元。
  一般來說，除權息後，都會造成股價短暫（或永久）的下跌，如果市場看好這家公司，就會積極參與「填權息」行情。例如台積電，配息前 217 元，配息後 210 元，趁著 210 元買進的投資人，就是看好大家會積極地買進買進，讓台積電股價再次回到除權息前的 217 元。這樣，在除權息時買進的人，就賺到了 7 元的「價差」，也就是賺到了「填權息行情」。然而，也有公司除權息後，股價就再也沒回到除權息前的價格。

Dividend payment

- 當一個資產在時間 t 時，會支付 D_t 的股利時，則其在時間 $t-1$ 報酬率的計算公式應修正如下:

$$R_t = \frac{P_t + D_t}{P_{t-1}} - 1$$

$$r_t = \ln(P_t + D_t) - \ln(P_{t-1}).$$

- Excess return (超額報酬率): (adjusting for risk)
  - $Z_t = R_t - R_{0t}$, Z_t 為簡單超額報酬，R_{0t} 為簡單無風險利率。
  - $z_t = r_t - r_{0t}$, z_t 為為對數超額報酬，r_{0t} 為對數無風險利率。
- 在財務中，超額報酬 (excess return) 可視為一種 " 賣空 **reference asset** 買進 **risky asset**" 的套利 (arbitrage) 投資組合的收益 (payoff)。由於此種操作不需要初始投資額，可視為一種套利 (arbitrage) 的投資組合。

- A long financial position = owing the asset.
- A short position = selling asset one does not own.
- selling short (放空):
  - 賣方出售的證券是借來的，交割給買方後，希望將來能以較低的價格買回，好從預期中的價格跌勢獲利。
  - 擁有空頭部位 (short position) 的人，負有交割所賣證券的法律責任，直到軋平部位為止。如果投資人認為股市將下跌，向證券商借入股票賣出，於行情確實下跌之後，再從市場中買進股票還給證券商。
  - 譬如，於七月一日借入 $1,000$ 股甲公司股票，以每股 8 元賣出；於八月一日，以每股 7 元的價格買進 $1,000$ 股甲公司股票；於是你這次放空獲利 $1,000$ 元，但需扣除買賣佣金和借券費用。
  - 在 short position maintained 的期間，若該資產發放現金股利 (cash dividends)，則這些現金股利將發放給 short sale 的買方。且這個 short seller 也必須補償等值的現金股利給借貸者 (lender)。

- Relationship between log return and simple return:

$$r_t = \ln(1 + R_t), \quad R_t = e^{r_t} - 1.$$

If the return are in percentage, then

$$r_t = 100 \times \ln\left(1 + \frac{R_t}{100}\right), \quad R_t = [\exp(r_t/100) - 1] \times 100.$$

- Temporal aggregation of the return produces

$$1 + R_t(k) = (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1}),$$
$$r_t(k) = r_t + r_{t-1} + \cdots + r_{t-k+1}.$$

- Example:
  If the monthly log returns of an asset are 4.46%, -7.34% and 10.77%, then what is the corresponding quarterly log return?

Ans: $4.46 - 7.34 + 10.77 = 7.89\%$.

- Example:
  If the monthly simple returns of an asset are 4.46%, -7.34% and 10.77%, then what is the corresponding quarterly simple return?

Ans:

$$\begin{aligned} R &= (1 + 0.0446)(1 - 0.0734)(1 + 0.1077) - 1 \\ &= 1.0721 - 1 = 0.0721 = 7.21\%. \end{aligned}$$

Distributional properties of returns

- What is the distribution of $\{r_{it}; i = 1, \ldots, N; t = 1, \ldots, T\}$?
- Moments of a random variable X with density $f(x)$: l-th moment

$$m_l' = E(X^l) = \int_{-\infty}^{\infty} x^l f(x) dx$$

  - First moment: mean or expectation of X.
  - l-th central moment

$$m_l = E[(X - \mu_x)^l] = \int_{-\infty}^{\infty} (x - \mu_x)^l f(x) dx,$$

  - 2nd central moment $=$ Variance of X.
  - Skewness (symmetry) and kurtosis (fat-tails)

$$S(x) = E\left[\frac{(X - \mu_x)^3}{\sigma_x^3}\right], \quad K(x) = E\left[\frac{(X - \mu_x)^4}{\sigma_x^4}\right].$$

  - $K(x) - 3$: Excess kurtosis.

- Why are mean and variance of returns important?
  They are concerned with long-term return and risk, respectively.
- Why is symmetry of interest in financial study?
  Symmetry has important implications in holding short or long financial positions and in risk management.
- Why is kurtosis important?
  1. Related to volatility forecasting, efficiency in estimation and tests, etc.
  2. High kurtosis implies heavy (or long) tails in distribution.

Estimation

Data: $\{x_1, \ldots, x_T\}$

- sample mean:

$$\hat{\mu}_x = \frac{1}{T}\sum_{t=1}^{T} x_t,$$

- sample variance:

$$\hat{\sigma}_x^2 = \frac{1}{T-1}\sum_{t=1}^{T}(x_t - \hat{\mu}_x)^2$$

- sample skewness:

$$\hat{S}(x) = \frac{1}{(T-1)\hat{\sigma}_x^3}\sum_{t=1}^{T}(x_t - \hat{\mu}_x)^3$$

Data: $\{x_1, \ldots, x_T\}$

- sample kurtosis:

$$\hat{K}(x) = \frac{1}{(T-1)\hat{\sigma}_x^4} \sum_{t=1}^{T} (x_t - \hat{\mu}_x)^4$$

Under normality assumption,

$$\hat{S}(x) \sim N\left(0, \frac{6}{T}\right), \quad \hat{K}(x) - 3 \sim N\left(0, \frac{24}{T}\right).$$

Some simple tests for normality (for large T).

1. Test for symmetry:

$$S^* = \frac{\hat{S}(x)}{\sqrt{6/T}} \sim N(0,1)$$

   if normality holds.
   Decision rule: Reject H_0 of a symmetric distribution if $|S^*| > Z_{\alpha/2}$ or p-value is less than α.

2. Test for tail thickness:

$$K^* = \frac{\hat{K}(x) - 3}{\sqrt{24/T}} \sim N(0,1)$$

   if normality holds.
   Decision rule: Reject H_0 of kurtosis $= 3$ if $|K^*| > Z_{\alpha/2}$ or p-value is less than α.

Some simple tests for normality (for large T).

3. A joint test(Jarque-Bera test):

$$JB = (K^*)^2 + (S^*)^2 \sim \chi_2^2$$

   if normality holds, where χ_2^2 denote a chi-square distribution with 2 degrees of freedom.
   Decision rule: Reject H_0 of a symmetric distribution if $JB > \chi_2^2(\alpha)$ or p-value is less than α.
4. Pearson's chi-square test.
5. Kolmogorov-Smirnov (K-S) test.
6. Shapiro-Wilk Test (檢定資料是否為常態分佈).
7. Anderson Darling Test (檢定資料是否為某種特殊分佈)

The Shapiro-Wilk Test (常態檢定)

- The Shapiro-Wilk test (1965) 檢定樣本 $x_1, x_2, \ldots, x_n$ 是否為常態分佈.
- This test has done very well in comparison studies with other goodness of fit tests.
- Test statistic W:

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\left(\sum_{i=1}^{n}(x_i - \overline{x})^2\right)^2}$$

- $x_{(j)}$：樣本順序統計量。
- a_j：由樣本順序統計量的 mean, variance 及 covariances 所求得的常數。
- 當檢定統計量 W 的值小的時候 $\rightarrow$ 拒絕常態假設。
- 利用 Monte Carlo 模擬法求檢定統計量 W 的百分位數 (Pearson and Hartley (1972), Table 16)。

The Anderson-Darling Test (特定分佈檢定)

- The Anderson-Darling test (Stephens, 1974) 檢定樣本 x_1, x_2, …, x_n 是否來自某一個特定分佈分佈。
- 為 Kolmogorov-Smirnov (K-S) test 的一種修正檢定，將更多的權重分配給分佈的 tail part。
- The K-S test is distribution free in the sense that the critical values do not depend on the specific distribution being tested.
- H_0：資料來自某一特定分佈，H_a：資料不是來自某一特定分佈
- Test statistic: $A^2 = -n - S$, where

$$S = \sum_{i=1}^{n} \frac{2i-1}{n}[\ln F(x_{(i)}) + \ln(1 - F(x_{(n+1-i)}))]$$

F 為該特定分佈的 cdf. n 為樣本大小. $x_{(i)}$ 為順序統計量.

The Anderson-Darling Test (特定分佈檢定)

- 優點：是一個更 sensitive 的檢定。
- 缺點：必須對每一個分佈分別計算其檢定統計量 A 的 critical values。
- Tabulated values and formulas (Stephens, 1974, 1976, 1977, 1979) for a few specific distributions (normal, lognormal, exponential, Weibull, logistic, extreme value type 1).
- 為單邊檢定。
- 當檢定統計量 A 的值大於 critical value 時，拒絕 H_0。

Data source

CRSP: Center for research in security prices, 為隸屬於 Univ. of Chicago 的一個資料中心，出售各種財務金融方面的 database e.g. stock price 及 index 的資料庫。
CRSP 中關於 index 的資料庫，包含 NYSE, AMEX, NASDAQ (但不包含 ADRs)。

- ADRs (American Depositary Receipts) 稱為美國信託憑證是一種憑證，使美國人可以在美國境內買賣外國的股票。
- Market-value weighted index:
  A stock index in which each stock affect the index in **proportion** to its market value.
  E.g. Nasdaq Composite Index, S&P 500, Wilshire 5000 Equity Index.

- 各財經資料庫網站:
  Wharton WRDS
  http://wrds.wharton.upenn.edu/(隸屬於 Univ. of Pennsylvania 的資料中心).
- Federal Reserve Bank at St. Louis
  http://research.stlouisfed.org/fred2/
- Federal Reserve Bank 聯邦準備銀行構成美國聯邦準備制度 (Federal Reserve System) 的十二家中央銀行,依 1913 年聯邦準備法 (Federal Reserve Act) 而設立,目的是管理美國的貨幣、銀行和信用。十二家聯邦準備銀行設於波士頓、紐約、費城、克利夫蘭、里契蒙、亞特蘭大、芝加哥、聖路易、明尼亞波利斯、堪薩斯市、達拉斯、舊金山。
- Data sets of the textbook:
  http://faculty.chicagobooth.edu/ruey.tsay/teaching/fts3/

R PACKAGES

- The main package used is R, which is a free software available from `http://www.r-project.org`.
- One can click *CRAN* on its Web page to select a nearby *CRAN Mirror* to download and install the software and selected packages.
- The following packages are needed in R: (fBasics, fGarch, quantmod, fUtilities, fUnitRoots, MTS, nnet, evir, urca)
- You may want to install the complete package Rmetrics.
- This can be done in R using the following two commands:
source("http://www.rmetrics.org/Rmetrics.R")
install.Rmetrics()

- R and S-Plus are objective-oriented software. They enable users to create many objects. For instance, one can use the command ts to create a time series object.
- Treating time series data as a time series object in R has some advantages, but it requires some learning to get used to it.
- It is, however, not necessary to create a time series object in R to perform the analyses discussed in this book.

- 關於現行目錄的相關指令：
  - getwd()：可得知目前現行目錄的位置
  - setwd(" ")：變更目前現行目錄的位置至 " "

  例如：欲將現行目錄設定成 D:/time，則輸入

  setwd("D:/time")

  並執行即可。
- 關於變更目前現行目錄的位置，亦可透過下列方式：
  1. 點選「檔案」
  2. 點選「變更現行目錄」
  3. 選擇你想要指定的目錄
  4. 最後，點選「確定」

- 讀取資料：
  先將資料放到指定的「現行目錄」裡，輸入

  $$\text{read.table("test.txt")}$$

  並執行即可讀取「文字檔 test」。
- 存取資料：
  輸入

  $$\text{write.table(x ,file="test.txt")}$$

  其中 x 代表欲存取的數值，file="test.txt" 代表欲存取的檔名，執行後，會發現指定的「現行目錄」裡會有「文字檔 test」。

Example

- Consider the monthly simple returns of the General Motors stock from January 1975 to December 2008; see Exercise 1.2. The data have 408 observations.
- The following R commands are used to illustrate the points: (In the following program code > is the prompt character and % denotes explanation:)

```
> da=read.table("m-gm3dx7508.txt",header=T)   % Load data
> gm=da[,2]   % Column 2 contains GM stock returns
> gm1=ts(gm,frequency=12,start=c(1975,1))
% Creates a ts object.
> par(mfcol=c(2,1))   % Put two plots on a page.
> plot(gm,type='l')
> plot(gm1,type='l')
> acf(gm,lag=24)
> acf(gm1,lag=24)
```

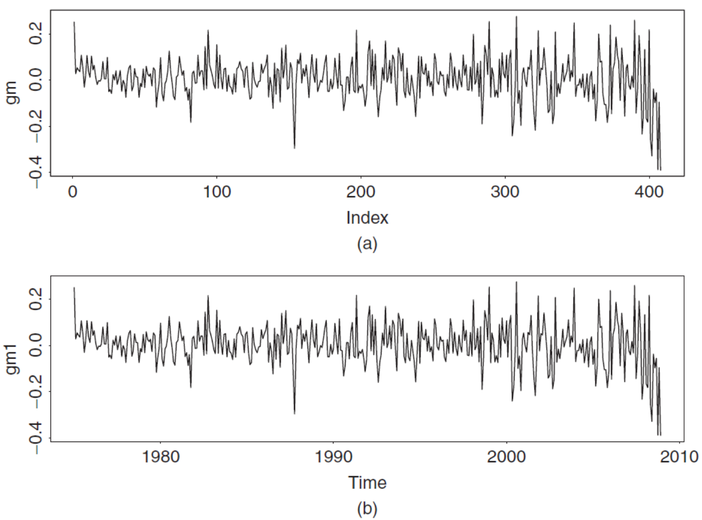


Figure: Time plots of monthly simple returns to General Motors stock from January 1975 to December 2008: (a) and (b) are without and with time series object, respectively.

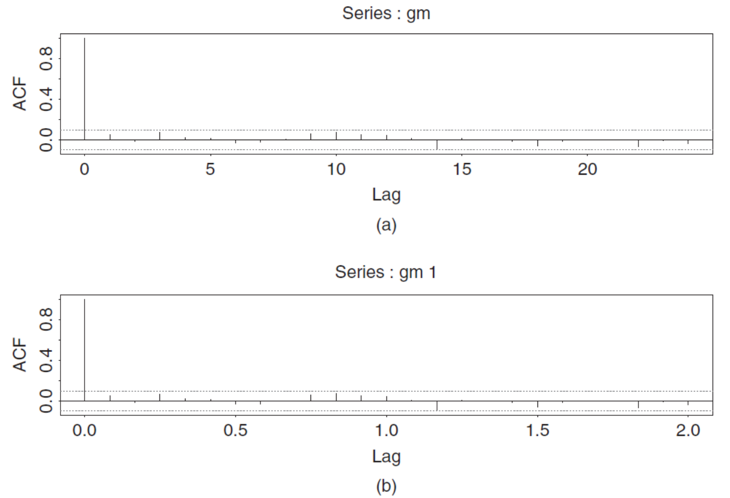


Figure: Sample ACFs of the monthly simple returns to General Motors stock from January 1975 to December 2008: (a) and (b) are without and with time series object, respectively.

- In the ts command
  - $\mathrm{frequency} = 12$ says that the time unit is year and there are 12 equally spaced observations in each time unit.
  - start $=$ c$(1975, 1)$ means the starting time is January 1975.
- Frequency and start are the two basic arguments needed in R to create a time series object. For further details, please use $\mathrm{help}(\mathrm{ts})$ in R to obtain details of the command.
- Here $\mathrm{gm1}$ is a time series object in R, but gm is not.

- Figures 1.7 and 1.8 show, respectively, the time plot and autocorrelation function (ACF) of the returns of GM stock.
- In each figure, the upper plot is produced without using time series object, whereas the lower plot is produced by a time series object.
- The upper and lower plots are identical except for the horizontal label.
    - For the time plot, the time series object uses calendar time to label the x axis, which is preferred.
    - For the ACF plot, the time series object uses fractions of time unit in the label, not the commonly used time lags.

Example 1.2: R Demonstration
利用 R 分析 IBM 報酬率

```
> library(fBasics)   % Load the package fBasics.
> da=read.table("d-ibm3dx7008.txt",header=T)   % Load the data.
% header=T means 1st row of the data file contains
% variable names. The default is header=F, i.e., no names.

> dim(da)   % Find size of the data: 9845 rows and 5 columns.
[1] 9845   5
> da[1,]   % See the first row of the data
      Date        rtn     vwretd    ewretd     sprtrn   % column names
1  19700102  0.000686  0.012137  0.03345  0.010211

> ibm=da[,2]   % Obtain IBM simple returns
> sibm=ibm*100   % Percentage simple returns
```

```
> basicStats(sibm)   % Compute the summary statistics
                     sibm
nobs          9845.000000   % Sample size
NAs              0.000000   % Number of missing values
Minimum        -22.963000
Maximum         13.163600
1. Quartile     -0.857100   % 25th percentile
3. Quartile      0.883300   % 75th percentile
Mean             0.040161   % Sample mean
Median           0.000000   % Sample median
Sum            395.387600   % Sum of the percentage simple returns
SE Mean          0.017058   % Standard error of the sample mean
LCL Mean         0.006724   % Lower bound of 95% conf.
                            % interval for mean
UCL Mean         0.073599   % Upper bound of 95% conf.
                            % interval for mean
Variance         2.864705   % Sample variance
Stdev            1.692544   % Sample standard error
Skewness         0.061399   % Sample skewness
Kurtosis         9.916359   % Sample excess kurtosis.
```

% **Alternatively, one can use individual commands as follows:**

```
> mean(sibm)
[1]  0.04016126
> var(sibm)
[1]  2.864705
> sqrt(var(sibm))    % Standard deviation
[1]  1.692544
> skewness(sibm)
[1]  0.06139878
attr(,"method")
[1]  "moment"
> kurtosis(sibm)
[1]  9.91636
attr(,"method")
[1]  "excess"
```

% **Simple tests**

```
> s1=skewness(sibm)
> t1=s1/sqrt(6/9845)    % Compute test statistic
> t1
[1]  2.487093
> pv=2*(1-pnorm(t1))    % Compute p-value.
> pv
[1]  0.01287919
```

% **Turn to log returns in percentages**

```
> libm=log(ibm+1)*100
> t.test(libm)    % Test mean being zero.
        One Sample t-test
data: libm
t = 1.5126, df = 9844, p-value = 0.1304
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
-0.007641473   0.059290531
```

% **The result shows that the hypothesis of zero expected return**
% **cannot be rejected at the 5% or 10% level.**

```
> normalTest(libm,method='jb')
```
% **Normality test**

```
Title:
 Jarque - Bera Normality Test
Test Results:
  STATISTIC:
    X-squared: 60921.9343
  P VALUE:
    Asymptotic p Value: < 2.2e-16
```

% **The result shows the normality for log-return is rejected.**

IBM 的月報酬率 1/1926~12/2008

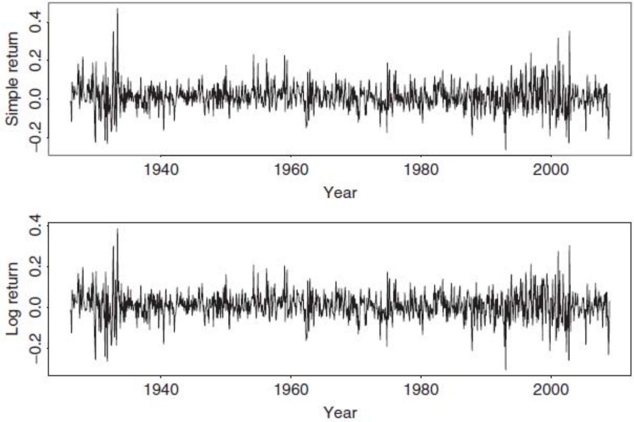


Figure 1.2 Time plots of monthly returns of IBM stock from January 1926 to December 2008. Upper panel is for simple returns, and lower panel is for log returns.

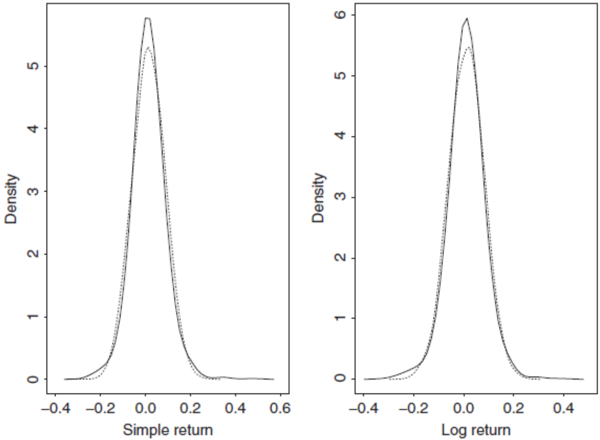


Figure: Comparison of empirical and normal densities for the month return of IBM stock. The normal density, shown by the dashed.

Empirical dist of asset returns tends to be **skewed** to the **left** with **heavy tails** and has a higher peak than normal dist.

加權指數 (VW) 報酬，1/1926~2008

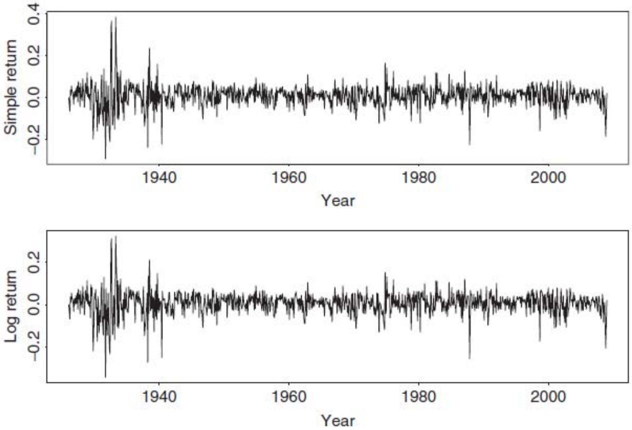


Figure 1.3 Time plots of monthly returns of value-weighted index from January 1926 to December 2008. Upper panel is for simple returns, and lower panel is for log returns.

TABLE 1.2 Descriptive Statistics for Daily and Monthly Simple and Log Returns of Selected Indexes and Stocks[a]

Security	Start	Size	Mean	Standard Deviation	Skewness	Excess Kurtosis	Minimum	Maximum
				Daily Simple Returns (%)				
SP	70/01/02	9845	0.029	1.056	−0.73	22.81	−20.47	11.58
VW	70/01/02	9845	0.040	1.004	−0.62	18.02	−17.13	11.52
EW	70/01/02	9845	0.076	0.814	−0.77	17.08	−10.39	10.74
IBM	70/01/02	9845	0.040	1.693	0.06	9.92	−22.96	13.16
Intel	72/12/15	9096	0.108	2.891	−0.15	6.13	−29.57	26.38
3M	670/01/02	9845	0.045	1.482	−0.36	13.34	−25.98	11.54
Microsoft	86/03/14	5752	0.123	2.359	−0.13	9.92	−30.12	19.57
Citi-Grp	86/10/30	5592	0.067	2.602	1.80	55.25	−26.41	57.82
				Daily Log Returns (%)				
SP	70/01/02	9845	0.023	1.062	−1.17	30.20	−22.90	10.96
VW	70/01/02	9845	0.035	1.008	−0.94	21.56	−18.80	10.90
EW	70/01/02	9845	0.072	0.816	−1.00	17.76	−10.97	10.20
IBM	70/01/02	9845	0.026	1.694	−0.27	12.17	−26.09	12.37
Intel	72/12/15	9096	0.066	2.905	−0.54	7.81	−35.06	23.41
3M	70/01/02	9845	0.034	1.488	−0.78	20.57	−30.08	10.92
Microsoft	86/03/14	5752	0.095	2.369	−0.63	14.23	−35.83	17.87
Citi-Grp	86/10/30	5592	0.033	2.575	0.22	33.19	−30.66	45.63
				Monthly Simple Returns (%)				
SP	26/01	996	0.58	5.53	0.32	9.47	−29.94	42.22
VW	26/01	996	0.89	5.43	0.15	7.69	−29.01	38.37
EW	26/01	996	1.22	7.40	1.52	14.94	−31.28	66.59
IBM	26/01	996	1.35	7.15	0.44	3.43	−26.19	47.06
Intel	73/01	432	2.21	12.85	0.32	2.70	−44.87	62.50
3M	46/02	755	1.24	6.45	0.22	0.98	−27.83	25.80
Microsoft	86/04	273	2.62	11.08	0.66	1.96	−34.35	51.55
Citi-Grp	86/11	266	1.17	9.75	−0.47	1.77	−39.27	26.08
				Monthly Log Returns (%)				
SP	26/01	996	0.43	5.54	−0.52	7.93	−35.58	35.22
VW	26/01	996	0.74	5.43	−0.58	6.85	−34.22	32.47
EW	26/01	996	0.96	7.14	0.25	8.55	−37.51	51.04
IBM	26/01	996	1.09	7.03	−0.07	2.62	−30.37	38.57
Intel	73/01	432	1.39	12.80	−0.55	3.06	−59.54	48.55
3M	46/02	755	1.03	6.37	−0.08	1.25	−32.61	22.95
Microsoft	86/04	273	2.01	10.66	0.10	1.59	−42.09	41.58
Citi-Grp	86/11	266	0.68	10.09	−1.09	3.76	−49.87	23.18

[a]Returns are in percentages and the sample period ends on December 31, 2008. The statistics are defined in eqs. (1.10)–(1.13), and VW, EW and SP denote value-weighted, equal-weighted, and S&P composite index.

1. 平均報酬率：日 (接近 0) < 月。
   標準差：指數 < 個股；日 < 月。
2. skewness 存在，但不是十分偏斜。
   Excess kurtosis 十分顯著：個股 < 指數；月 < 日。
3. 簡單報酬率與對數報酬率皆為厚尾分佈，且有相似的分佈性質。

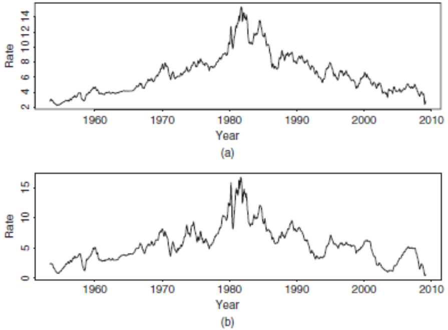


Figure 1.5 Time plots of monthly U.S. interest rates from April 1953 to February 2009: (a) 10-year Treasury constant maturity rate and (b) 1-year maturity rate.

Figure: 10 年期及 1 年期的利率：約 12×55 筆月資料.

- 10 年期 & 1 年期的國庫券利率 (Treasury bill)：美國財政部發行的短期證券。發行期間一年或一年以下。通常發行期間為 52 天、91 天或 182 天。
- 美國政府的短期債務工具，每周標售，只採登錄形式，並以折價銷售，到期則按面額償還。最低面值為 10,000 美元，此後以 1,000 美元的倍數遞增。

- Treasury bill 的字面翻譯，是「寶貴地帳單」。它是國家向人民借款，並於還款時提供利息的證券，而這種證券的優點是人民不再像過去一樣，只要政府徵稅，就得無條件給錢，現在買政府的債券，還能在到期日取回「Bill」，又賺利息錢。
- Treasury note 財政部中期公債: 美國財政部發行的中期債券發行期間在二年以上、十年以下。只採登錄形式。發行期限為二年或三年的中期公債，最低面值為 5,000 美元，其他中期公債的最低面值則為 1,000 美元；所有的中期公債都在最低面值之上以 1,000 美元的倍數遞增。
- Treasury bond 財政部長期公債: 美國財政部發行的長期固定收益證券。發行期間為十年期以上的美國政府債券。1986 年 4 月以後，新發行的財政部長期公債不可贖回，期限為二十年或三十年，只採登錄形式，最低面值為 1,000 美元。

各種國庫券報酬率

TABLE 1.3 Descriptive Statistics of Selected U.S. Financial Time Series[a]

Maturity	Mean	Standard Deviation	Skewness	Excess Kurtosis	Minimum	Maximum
Monthly Bond Returns: Jan. 1952 to Dec. 2008, $T = 684$						
1–12 months	0.45	0.35	2.47	13.14	−0.40	3.52
12–24 months	0.49	0.67	1.88	15.44	−2.94	6.85
24–36 months	0.52	0.98	1.37	12.92	−4.90	9.33
48–60 months	0.53	1.40	0.60	4.83	−5.78	10.06
61–120 months	0.55	1.69	0.65	4.79	−7.35	10.92
Monthly Treasury Rates: April 1953 to February 2009, $T = 671$						
1 year	5.59	2.98	1.02	1.32	0.44	16.72
3 years	5.98	2.85	0.95	0.95	1.07	16.22
5 years	6.19	2.77	0.97	0.82	1.52	15.93
10 years	6.40	2.69	0.95	0.61	2.29	15.32
Weekly Treasury Bill Rates: End on March 27, 2009.						
3 months	5.07	2.82	1.08	1.80	0.02	16.76
6 months	5.52	2.73	0.99	1.53	0.20	15.76

[a]The data are in percentages. The weekly 3-month Treasury bill rate started from January 8, 1954, and the 6-month rate started from December 12, 1958. The sample sizes for Treasury bill rates are 2882 and 2625, respectively. Data sources are given in the text.

1. Treasury bond 的月報酬率 (1/1952∼12/2008)：不同到期日的 mean return 大致相同；stdev：隨到期時間增加而增加。
2. Treasury rate 的月報酬率 (4/1953∼2/2009) 與 Treasury bill 的週報酬率 (∼ 3/27/2009)：mean：隨到期時間增加而增加；stdev：隨到期時間增加而減少。
3. 大部分的 excess kurtosis > 0。

Normal and lognormal dists

- Y is lognormal if $X = \ln(Y)$ is normal.
- If $X \sim N(\mu, \sigma^2)$, then $Y = \exp(X)$ is lognormal with

$$E(Y) = \exp\left(\mu + \frac{\sigma^2}{2}\right), \quad Var(Y) = \exp(2\mu+\sigma^2)[\exp^{\sigma^2} - 1].$$

- Conversely, if Y is lognormal with mean μ_y and variance σ_y^2, then $X = \ln(Y)$ is normal with mean and variance

$$E(X) = \ln\left[\frac{\mu_y}{\sqrt{1 + \frac{\sigma_y^2}{\mu_y^2}}}\right], \quad Var(X) = \ln\left[1 + \frac{\sigma_y^2}{\mu_y^2}\right].$$

Application

- If the monthly log return (r_t) of an asset (IBM stock) is normally distributed with mean $= 0.0109$ and standard deviation $= 0.0703$, then what is the mean and stdev of its simple return?
- If r_t is normally distributed, then $1 + R_t = \exp(r_t)$ has a lognormal distribution.

Step 1: The mean and variance of $Y_t = \exp(r_t)$ are

$$E(Y) = \exp\left[0.0109 + \frac{0.0703^2}{2}\right] = 1.01346$$

$$Var(Y) = \exp(2\times0.0109+0.0703^2)[\exp^{0.0703^2} - 1] = 0.00508.$$

Step 2: Simple return is $R_t = \exp(r_t) - 1 = Y_t - 1$. Therefore,

$$E(R) = E(Y) - 1 = 0.01346$$

$Var(R_t) = Var(Y_t) = 0.00508$, stdev$=$ $\sqrt{Var(R)} = 0.0713.$

(Remark: See the monthly IBM stock simple returns in Table 1.2. sample mean $= 0.0135$, sample stdev $= 0.0715$)

Likelihood function of returns

- 考慮資產，T 期的對數報酬率 $\{r_1, \ldots, r_T\}$
- 目的：利用過去報酬率的資料來估計分佈的參數並推論報酬率的性質
- Basic concept:
  Joint dist = Conditional dist × Marginal dist,

$$f(x, y) = f(x|y)f(y)$$

For two consecutive returns r_1 and r_2, we have

$$f(r_2, r_1) = f(r_2|r_1)f(r_1).$$

For three returns r_1, r_2 and r_3, by repeated application,

$$\begin{aligned} f(r_3, r_2, r_1) &= f(r_3|r_2, r_1)f(r_2, r_1) \\ &= f(r_3|r_2, r_1)f(r_2|r_1)f(r_1) \end{aligned}$$

In general, we have

$$
\begin{aligned}
& f(r_T, r_{T-1}, \ldots, r_2, r_1) \\
=& f(r_T | r_{T-1}, \ldots, r_2, r_1) f(r_{T-1}, \ldots, r_1) \\
= & \qquad \vdots \\
= & \left[\prod_{t=2}^{T} f(r_t | r_{t-1}, \ldots, r_1) \right] f(r_1),
\end{aligned}
$$

If $r_t | r_{t-1}, \ldots, r_2, r_1$ is normal with mean μ_t and variance σ_t^2, then likelihood function becomes

$$f(r_T, r_{T-1}, \ldots, r_2, r_1) = \prod_{t=2}^{T} \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left[\frac{-(r_t - \mu_t)^2}{2\sigma_t^2}\right] f(r_1).$$

For simplicity, if $f(r_1)$ is ignored, then the likelihood function becomes

$$f(r_T, r_{T-1}, \ldots, r_2, r_1) = \prod_{t=2}^{T} \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left[\frac{-(r_t - \mu_t)^2}{2\sigma_t^2}\right].$$

This is the *conditional* likelihood function of the returns under normality.
Other dists, e.g. Student-t, can be used to handle heavy tails.

Likelihood function of multivariate returns

- Multivariate returns: joint pdf of log returns of N assets at time $t = 1, 2, \cdots, T$
- 考慮 N 個資產，T 期的對數報酬率 $\{r_{i,t};\ i = 1,\ 2,\ \cdots,\ N, t = 1,\ 2,\ \cdots,\ T\}$ 的聯合分佈：

$$F_{\vec{Y}}(r_{11}, \cdots, r_{N1}; r_{12},\ \cdots,\ r_{N2}; \cdots; r_{1T},\ \cdots,\ r_{NT}; \vec{Y}, \vec{\vartheta})$$

  $\vec{Y}$：given environmental variables (環境變數向量)，通常視為給定的變數
  $\vec{\vartheta}$：distribution parameter vector (分佈的參數向量)
- 目的：利用過去報酬率的資料來估計 $\vec{\vartheta}$ 並推論 $\{r_{i,t}\}$

Process considered

- return series (e.g. ch. 1,2,5)
- volatility processes (e.g. ch. 3,4,9) :conditional standard deviation of the return over time (variability of returns vary over time and appear in cluster) 對選擇權的定價扮演重要角色
- continuous-time processes (ch. 6)
- extreme events (ch. 7):
  - negative extreme returns：對風險管理是一個重要的考量
  - positive returns：對擁有 short position 者是一個重要的考量
  - 探討極值 (extreme) 發生的頻率、大小，以及經濟變數對極值的影響
- multivariate series (ch8,9)

Normal distribution assumption for simple return R_t ?

- Assume the simple returns $\{R_{it}; t = 1, \cdots, T\}$ are iid normal rv's.
- $R_t = \frac{P_t - P_{t-1}}{P_{t-1}} > -1$. Note that the lower bound of the simple return is -1, yet normal distribution is defined on $(-\infty, \infty)$.
- If R_t is normal distributed, then the k-period simple return

$$R_t(k) = (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1}) - 1$$

  is not normal distributed (since it is a product of one-period returns.)
- Furthermore, empirical simple returns have positive excess kurtosis.

Normal distribution for log return r_t ?

- Assuming log return r_t has a normal distribution implies $1 + R_t$ has a lognormal distribution.
- pdf of log normal distribution :
  $f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{-(\ln x - \mu)^2}{2\sigma^2}}, x > 0$
- All moments of log normal distribution exist and are given by $E(X^k) = \mu_k = e^{k\mu + k^2\sigma^2/2}$; yet the moment generating function for the log-normal distribution does not exist!
- If we assume the log returns $\{r_t; t = 1, \cdots, T\}$ are iid normal rv's, then the k-period log return

$$r_t(k) = r_t + r_{t-1} + \cdots + r_{t-k+1}$$

  is also normal distributed.
- Since the simple return $R_t = \exp(r_t) - 1$, thus the lower bound for $R_t(\geq -1)$ is satisfied.
- Yet empirical log returns have positive excess kurtosis !

Stable distribution

- stable under addition:
  stable rv + stable rv 仍為 stable 分佈
  meets the need of continuously compounded return r_t
- capable of capturing excess kurtosis
- non-normal stable distributions do not have a finite variance
  e.g. Cauchy distribution which is symmetric wrt its median but has infinite variance
  conflict with most finance theories!
- statistical modeling using non-normal stable distributions is difficult!

Scale mixture of normal distributions

- pdf:
$$f(x) = (1-\alpha)\phi(\frac{x-\mu}{\sigma_1}) + \alpha\phi(\frac{x-\mu}{\sigma_2}),$$
$\phi(\cdot)$: the pdf of standard normal distribution.
- $$r_t \sim (1-X)N(\mu, \sigma_1^2) + XN(\mu, \sigma_2^2)$$
$X \sim$ Bernoulli with $P(X=1) = \alpha$
σ_1^2 is small, and σ_2^2 is relatively large.
- The large value of σ_2^2 is to enable the mixture to put more mass at the tail of its distribution.
- α is small says that the majority of the returns follow a simple normal distribution.
- advantages: maintain the tractability of normal, have finite higher moments, and can capture the excess kurtosis
- Yet it is hard to estimate the mixture parameter α.

- Figure 1.1 shows the pdfs of normal, cauchy and mixture of normal.

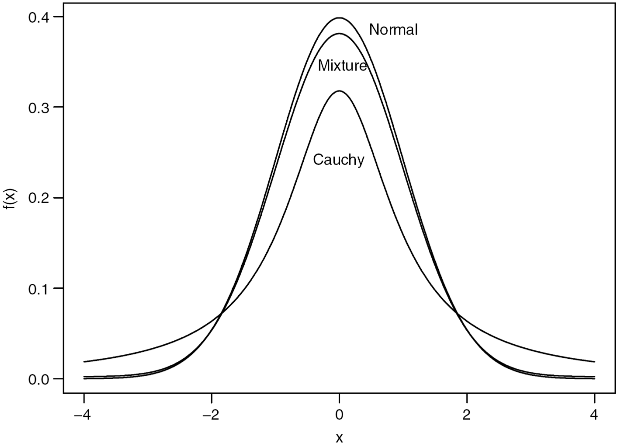


Figure 1.1. Comparison of finite mixture, stable, and standard normal density functions.

Capital Asset Pricing Model

- Capital Asset Pricing Model (CAPM)(資本資產定價模式)
- 由 Sharpe, Linter, Treynor 及 Mossin 等人提出的模型。
- 假設在一個有效率的市場，資本資產 (個別證券或投資組合) 的報酬率僅與系統風險 (即市場的報酬) 相關，是一種單期的模型。
- CAPM 模型著重於 N 個資產，同期對數報酬率的聯合分佈，即

$$\{r_{1t}, \cdots, r_{Nt}\}$$

的聯合分佈。

Capital Asset Pricing Model (CAPM)

- CAPM Model:
  $E(R_i) = R_f + \beta_i[E(R_m) - R_f]$
  $E(R_i)$：資產 i 的期望報酬
  R_f：無風險利率
  $E(R_m)$：市場的期望報酬
  $E(R_m) - R_f$：市場的風險溢酬
  $E(R_i) - R_f$：資產 i 的風險溢酬

- CAPM 的 Market Model
  - Market Model $R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + \epsilon_i$
  - The least squares estimate of β_i is

$$\hat{\beta}_i = \frac{Cov(R_i, R_m)}{Var(R_m)}$$

CAPM 的 Market Model

- β_i 是用來衡量資產 i 的系統風險
  $\beta_i = 1$：平均風險證券 (或稱市場投資組合風險證券，中性證券)
  $\beta_i > 1$：高風險證券 (或進取性證券)
  $\beta_i < 1$：低風險證券 (或防衛性證券)
- 不同類型證券由小到大的風險順序：
  短期國庫券、商業本票、浮動利率債券、第一抵押債券、第二抵押債券、附屬信用債券、可轉換債券、收益債券、特別股、可轉換特別股、普通股、認股權證、選擇權、金融期貨。

CAPM 的涵意與限制

- CAPM 的涵意
  1. 報酬率均等原則 (單一價格法則)：
     系統風險相同的證券所提供的報酬率也要相同，否則就會出現套利的機會；而投資人爭相套利的結果，會使系統風險相同的證券所提供的報酬率又趨於一致。通常金融市場效率越高，套利機會消失的越快。
- CAPM 運用的限制
  1. 投資者對證券報酬率的機率分佈有同質預期假設不容易成立
  2. 投資市場是完全市場的假設不易成立
  3. 借貸利率不相等
  4. 各種證券無法完全細分，使投資者可以進行多角化投資
  5. 只以系統風險為唯一解釋變數，無考慮通貨膨脹等因素
  6. 貝他係數的估計不易

市場效率性

- CAPM 模型在效率的市場的假設下，假設資本資產 (個別證券或投資組合) 的報酬率僅與系統風險 (即市場的報酬) 相關。由於報酬率為不可預測的，若在實證中發現報酬率具有相關性具有預測性，則對於效率市場的假設，將會有爭議性.
- 金融市場的效率性可分為強勢，半強勢及弱勢效率
  - 強勢效率：市場中的公開與非公開的資訊均可即時的反映市場的價格
  - 半強勢效率：僅有公開資訊可即時的反映在市場的價格
- CAPM v.s. Time Series Model
  - 時間序列模型：著重於單個資產，不同期對數報酬率的聯合分佈，如第 i 個資產 $\{r_{it}, \cdots, r_{iT}\}$ 的聯合分佈

低頻與高頻的資料

- 低頻資料 (e.g. daily log returns)：treated as continuous random variables
- 高頻資料 (e.g. tick-by-tick return)：discreteness becomes an issue
  紐約證券交易所 (NYSE,New York Stock Exchange)：
  - 在 1997 年 7 月以前：a tick $= 1/8$ dollar
  - 1997 年 7 月 ∼ 2001 年 1 月：a tick $= 1/16$ dollar
  - 2000 年 8 月 28 日，NYSE 將 7 檔股票的 tick 設為 a tick $= 1/100$ dollar
    American Stock Exchange (AMEX) 將 6 檔股票及兩類選擇權的 tick 設為 tick $= 1/100$ dollar
  - 2000 年 9 月 25 日及 12 月 4 日，NYSE 再各加入 57 檔及 94 檔股票，將其 tick 設為 tick $= 1/100$ dollar
  - 2001 年 1 月 29 日之後，NYSE 及 AMEX 全面採用 a tick $= 1/100$ dollar

OTC (over-the-counter)：店頭市場；櫃臺買賣中心

- 證券市場的一種
- 有價證券不在集中交易市場上以競價方式買賣，而在證券商的營業櫃檯以議價方式進行的交易行為，稱為櫃檯買賣。
- 由櫃檯買賣所形成的市場稱為櫃檯買賣市場，又稱店頭交易，英文為 Over-the-Counter，簡稱 OTC。
- 公開發行公司申請將其所發行的證券 (包括股票與公司債) 在證券商營業處所買賣者稱為上櫃申請，經核准可以在證券商營業處所為櫃檯買賣的股票稱為上櫃股票，也就是說，可以在櫃檯買賣市場發行與流通的股票叫做上櫃股票。
- 在美國, 店頭市場透過電話成交, 而且在店頭市場是可以議價的, 而不是像在紐約證券交易所 (NYSE)，是採用全自動化運作。
- 店頭市場的交易人可以自行操作，也可以擔任顧客的仲介人。
- 店頭市場是各類債券的主要交易市場。