高念慈 B08204005

25/30.

10 Subject : 時間序列

No. : HW9

Date : 112 / 4 / 24

1. Suppose the simple return of a monthly bond index follows the MA(1) Model : $R_t = a_t + 0.2\, a_{t-1}$, $\sigma_a = 0.025$, Assume $a_{100} = 0.01$ Compute 1 step and 2 step ahead forecasts of return at origin $t = 100$. What are the sd of the associated forecast errors? Compute lag 1 . lag 2 autocorrelation of return series.

1 step : $\hat{R}_{100}(1) = E(R_{101} \mid R_1 \cdots R_{100})$

$= E(a_{101} + 0.2\, a_{100} \mid R_1 \cdots R_{100})$

$= 0.2\, E(a_{100})$

$= 0.2 \times 0.01 = 0.002$ #

error : $R_{101} - \hat{R}_{100}(1) = a_{101}$, sd : $\sqrt{Var(a_{101})} = 0.025$ #

2 step : $\hat{R}_{100}(2) = E(R_{102} \mid R_1 \cdots R_{100})$

$= E(a_{102} + 0.2\, a_{101} \mid R_1 \cdots R_{100})$

$= 0.2 \cdot E(a_{101}) = 0$ #

$e_{100}(2) : R_{102} - \hat{R}_{100}(2) = a_{102} + 0.2\, a_{101}$

$Var(a_{102} + 0.2\, a_{101}) = \sigma_a^2 + 0.04\, \sigma_a^2 = (1.04) \cdot 0.025^2$

$\Rightarrow sd(e_{100}(2)) = \sqrt{1.04} \cdot 0.025 \doteq 0.0254951$ #

ACF $\rho_1 . \rho_2$, $\gamma(B) = \sigma_a^2 (1 - \theta_1 B)(1 - \theta_1 B^{-1})$

$= \sigma_a^2 (-\theta_1 B^{-1} + (1+\theta_1^2) - \theta_1 B)$

$$\Rightarrow \gamma_k = \begin{cases} (1+\theta_1^2)\sigma_a^2 , & k=0 \\ -\theta_1 \sigma_a^2 , & k=1 \\ 0 , & k>1 \end{cases} \Rightarrow \rho_k = \begin{cases} \dfrac{-\theta_1}{1+\theta_1^2} , & k=1 \\ 0 , & k>1 \end{cases}$$

$$\Rightarrow \rho_1 = \frac{-(-0.2)}{1+(-0.2)^2} = 0.1923077 \ \#$$

$\rho_2 = 0$ #

Double A

Time Series HW9

B082040005 高念慈

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7

2.

Consider the monthly simple returns of the Decile 1, Decile 2, Decile 9, and Decile 10 of NYSE/AMEX/NASDAQ based on market capitalization. The data span is from January 1970 to December 2008, and the data are obtained from CRSP.

```
data2 = read.table("C:/Users/user/Desktop/time_series/HW/m-deciles08.txt", header=T)
head(data2)
```

```
##       date   CAP1RET   CAP2RET   CAP9RET  CAP10RET
## 1 19700130  0.054383 -0.004338 -0.073082 -0.076874
## 2 19700227  0.020264  0.020155  0.064185  0.059512
## 3 19700331 -0.031790 -0.028090 -0.004034 -0.001327
## 4 19700430 -0.184775 -0.193004 -0.115825 -0.091112
## 5 19700529 -0.088189 -0.085342 -0.085565 -0.053193
## 6 19700630 -0.059476 -0.085212 -0.046605 -0.048133
```

(a)

For the return series of Decile 2 and Decile 10, test the null hypothesis that the first 12 lags of autocorrelations are zero at the 5% level. Draw your conclusion.

```
Decile_2 = data2[,3]
Box.test(Decile_2,lag=12,type="Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  Decile_2
## X-squared = 55.736, df = 12, p-value = 1.335e-07
```

According to the p-value < 0.05 ,
we reject the H0 is such that the first 12 lags of Decile_2 autocorrelations have some nonzero terms .(Thus, the time series of Decile_2 is not a white noise.)

```
Decile_10 = data2[,5]
Box.test(Decile_10,lag = 12,type = "Ljung-Box")
```

```
##
##  Box-Ljung test
##
## data:  Decile_10
## X-squared = 10.687, df = 12, p-value = 0.5559
```

According to the p-value > 0.05 , we accept the null hypothesis the first 12 lags of Decile_10 autocorrelations are zero.
(Considered as white noise process)

(b)

Build an ARMA model for the return series of Decile 2. Perform model checking and write down the fitted model.

```
ts_Decile_2 = ts(Decile_2,frequency=12,start=c(1970,1))
model1 = ar(ts_Decile_2,method = "mle")
model1$order
```

```
## [1] 12
```

```
m1 = arima(ts_Decile_2,order = c(12,0,0))
phi_0 = (1-sum(m1$coef[1:12]))*mean(Decile_2)
phi_0
```

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```
## [1] 0.008135085
```

residual 沒檢定 -2.

```
round(m1$coef,5)
```

```
##       ar1       ar2       ar3       ar4       ar5       ar6       ar7       ar8
##   0.21028  -0.07803  -0.03588   0.00285  -0.04375  -0.00239   0.00262  -0.07878
##       ar9      ar10      ar11      ar12 intercept
##  -0.01953   0.06895  -0.04536   0.24471   0.01023
```

```
m1$sigma2
```

```
## [1] 0.003770542
```

$Rt = 0.00814 + 0.21028Rt - 1 - 0.07803Rt - 2 - 0.03588Rt - 3 + 0.00285Rt - 4 - 0.04375Rt - 5 - 0.00239Rt - 6 + 0.00262Rt - 7 -$

(c)

Use the fitted ARMA model to produce 1-to 12-step-ahead forecasts of the series and the associated standard errors of forecasts.

```
forecast2 = predict(m1,n.ahead = 12)
forecast2
```

```
## $pred
##               Jan           Feb           Mar           Apr           May
## 2009  1.101801e-02  2.168344e-02  1.827985e-02  1.625908e-02  2.740593e-02
##               Jun           Jul           Aug           Sep           Oct
## 2009  1.615496e-02 -1.313376e-03  2.565188e-03 -2.745673e-02 -2.918551e-02
##               Nov           Dec
## 2009 -2.501148e-02 -5.641547e-05
##
## $se
##             Jan        Feb        Mar        Apr        May        Jun
## 2009 0.06140474 0.06274765 0.06278199 0.06288786 0.06289420 0.06294301
##             Jul        Aug        Sep        Oct        Nov        Dec
## 2009 0.06295157 0.06295195 0.06310736 0.06318306 0.06330522 0.06330589
```

8

3.

Consider the monthly log returns of CRSP equal-weighted index from January 1962 to December 1999 for 456 observations. You may obtain the data from CRSP directly or from the file m- ew6299.txt on the Web.

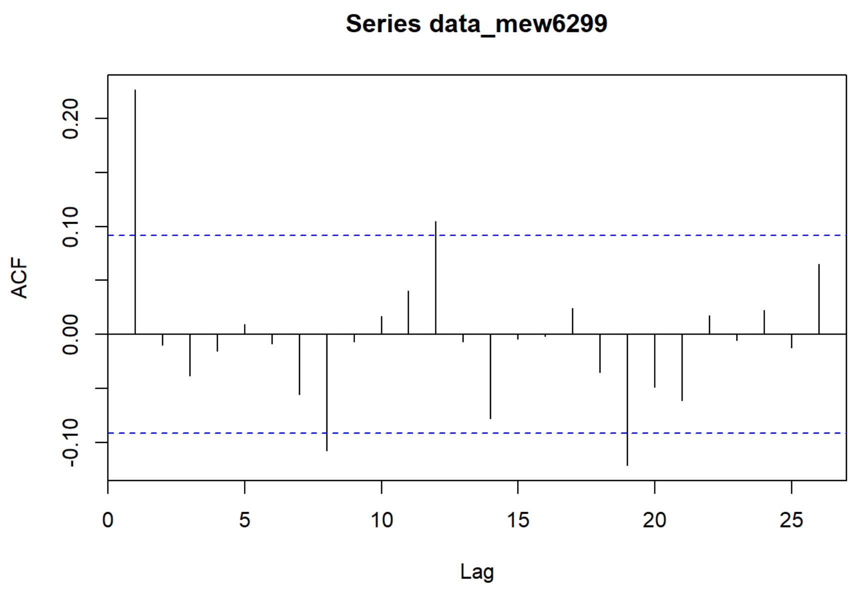
```
data_mew6299 = read.table('https://faculty.chicagobooth.edu/-/media/faculty/ruey-s-tsay/teaching/fts3/m-ew6299.txt')
head(data_mew6299)
```

```
##       V1
## 1  -0.792
## 2   1.532
## 3  -0.596
## 4  -7.049
## 5 -10.319
## 6  -8.880
```

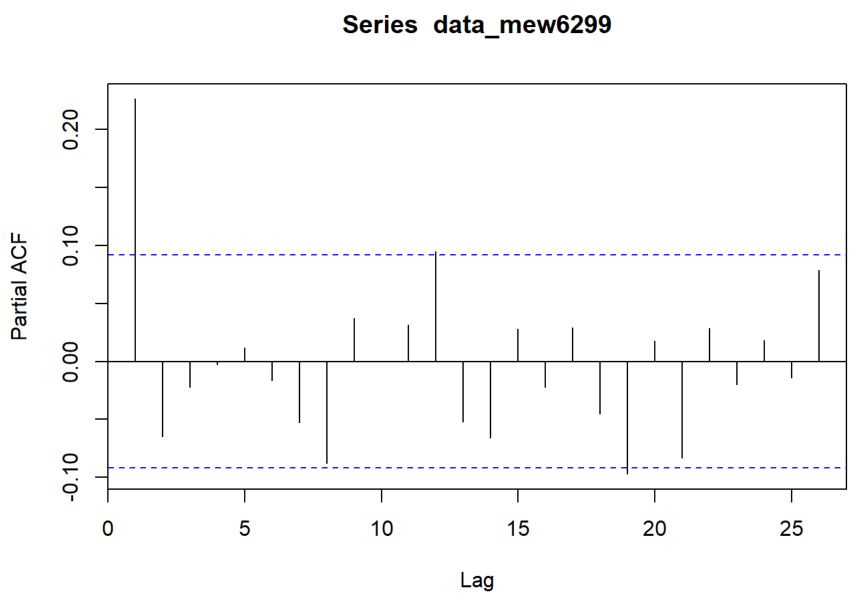
```
library(TSA)
eacf(data_mew6299[,1])
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x o o o o o o x o o o  x  o  o
## 1 x o o o o o o x o o o  x  o  o
## 2 x o o o o o o x o o o  o  o  o
## 3 x x o o o o o o o o o  o  o  o
## 4 x o o o o o o o o o o  o  o  o
## 5 x o x o o o o o o o o  o  o  o
## 6 x x x o o o o o o o o  o  o  o
## 7 x x o x o x x o o o o  o  o  o
```

```
acf(data_mew6299)
```




```
pacf(data_mew6299)
```



(a)

Build an AR model for the series and check the fitted model.

```
m4=ar(data_mew6299[,1],method = "mle")
m4$order
```

```
## [1] 1
```

```
model3 = arima(data_mew6299[,1],order=c(1,0,0))
model3
```

residual 沒檢定一

```
##
## Call:
## arima(x = data_mew6299[, 1], order = c(1, 0, 0))
##
## Coefficients:
##          ar1  intercept
##       0.2267     1.0626
## s.e.  0.0456     0.3297
##
## sigma^2 estimated as 29.68:  log likelihood = -1420.11,  aic = 2844.22
```

```
phi_0 = (1-sum(model3$coef[1]))*1.0626
rbind(phi_0=phi_0) # phi_0 as the intercept of AR(1) model
```

```
##            [,1]
## phi_0 0.8217582
```

```
rbind(sigma_a = sqrt(model3$sigma2))
```

```
##             [,1]
## sigma_a 5.448175
```

- $AR(1)mo\ del : rt = 0.8218 + 0.2267rt - 1 + at, \sigma a = 5.448$

(b)

Build an MA model for the series and check the fitted model.

```
model4 = arima(data_mew6299[,1],order=c(0,0,1))
model4
```

residual 沒檢定. -1

```
##
## Call:
## arima(x = data_mew6299[, 1], order = c(0, 0, 1))
##
## Coefficients:
##          ma1  intercept
##       0.2385     1.0605
## s.e.  0.0449     0.3153
##
## sigma^2 estimated as 29.59:  log likelihood = -1419.37,  aic = 2842.73
```

```
rbind(sigma_a=sqrt(model4$sigma2))
```

```
##             [,1]
## sigma_a 5.439245
```

- $MA(1)mo\ del : rt = 1.0605 + at + 0.2385at - 1, \sigma a = 5.439$

(c)

Compute 1- and 2-step-ahead forecasts of the AR and MA models built in the previous two questions.

- $AR(1)$

```
forecast1=predict(model3,n.ahead = 2)
forecast1
```

```
## $pred
## Time Series:
## Start = 457
## End = 458
## Frequency = 1
## [1] 2.601682 1.411453
##
## $se
## Time Series:
## Start = 457
## End = 458
## Frequency = 1
## [1] 5.448175 5.586364
```

- $MA(1)$

```
forecast2=predict(model4,n.ahead = 2)
forecast2
```

```
## $pred
## Time Series:
## Start = 457
## End = 458
## Frequency = 1
## [1] 2.250303 1.060512
##
## $se
## Time Series:
## Start = 457
## End = 458
## Frequency = 1
## [1] 5.439245 5.591797
```

(d)

Compare the fitted AR and MA models.

$$AR(1)mo\ del : (1 - 0.2267B)rt = 0.8218 + at$$

long division

$$\because \frac{1}{1-\phi_1 B} = 1 + \phi_1 B + \phi_1^2 B^2 + \phi_1^3 B^3 \ldots$$

$$\therefore MArepresentationo\ \ f\ AR(1)model = \frac{0.8218}{1-0.2267} + a_t(1 + 0.2267B + 0.2267^2 B^2 + \ldots)$$

$$= 1.063 + a_t + 0.2267a_{t-1} + 0.0514a_{t-2} + 0.0117a_{t-3} + \ldots$$

$$(MA(1)mo\ del : rt = 1.0605 + at + 0.2385a_{t-1}, \sigma a = 5.439)$$

- Thus, AR(1) model and MR(1) model are basically the same
- and equally adequate to the time series of log return of m-ew6299 data.