Time Series HW8

40/40

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1. 10

$$(a)Z_t = a_t$$

- i. Simulate 100 observations from the following process:
  Where a_t $WN(0,1)$
  Compute the sample ACF of $\{Z_t\}$,i.e,$\hat{\rho}_k$ for k=1,2,3

模擬

```
set.seed(20230414)
WN_sim <- arima.sim(list(order=c(0,0,0), sq=sqrt(1)), n=100)
acf(WN_sim, type="correlation", plot=FALSE, lag.max=3)
```

```
##
## Autocorrelations of series 'WN_sim', by lag
##
##      0      1      2      3
##  1.000 -0.033 -0.042  0.014
```

- ii. Repeating (i) for 1000 times to find the values of
  the mean, variance and covariance of$\hat{\rho}_1$,$\hat{\rho}_2$ and $\hat{\rho}_3$

mean, variance and covariance

```
set.seed(20230414)

M = 1000

rho_1 = rep(0,M);rho_2 = rep(0,M);rho_3 = rep(0,M)

for (i in 1:M){
  WN_sim = arima.sim(list(order=c(0,0,0), sq=sqrt(1)), n=100)
  ar100_acf = acf(WN_sim, type="correlation", plot=FALSE, lag.max=3)[[1]]  # ACF
  rho_1[i] = ar100_acf[2]
  rho_2[i] = ar100_acf[3]
  rho_3[i] = ar100_acf[4]
}

cbind(mean_rho = c(mean(rho_1),mean(rho_2),mean(rho_3)),
      var_rho = c(var(rho_1),var(rho_2),var(rho_3)))
```

```
##           mean_rho     var_rho
## [1,] -0.013203291 0.009791758
## [2,] -0.008934660 0.009809279
## [3,] -0.009377751 0.010092048
```

covariance

```
rbind(co_rho12 = cov(rho_1,rho_2),
      co_rho23 = cov(rho_2,rho_3),
      co_rho13 = cov(rho_1,rho_3))
```

```
##                   [,1]
## co_rho12 -8.861869e-05
## co_rho23 -9.272359e-06
## co_rho13 -8.363077e-04
```

- iii. Compare the result of (ii) with the Bartlett's formula.(p35,36)

$$\because \text{White noise } Var(\hat{\rho}_k) \approx \frac{1}{n}$$

$$\therefore Var(\hat{\rho}_1) = Var(\hat{\rho}_2) = Var(\hat{\rho}_3) \approx 0.01$$

$$\text{White noise process, each } \rho i, \rho j \text{ for } i \neq j \text{ are approximately uncorrelated}$$

$$\therefore Cov(\hat{\rho}_1, \hat{\rho}_2) = Cov(\hat{\rho}_1, \hat{\rho}_3) = Cov(\hat{\rho}_2, \hat{\rho}_3) \approx 0$$

- 結果都跟(ii)差不多

$$(b) Z_t = a_t - 1.5a_{t-1}$$

- i. Simulate 100 observations from the following process:
  Where a_t $WN(0, 1)$
  Compute the sample ACF of $\{Z_t\}$,i.e,$\hat{\rho}_k$ for k=1,2,3

模擬

```
set.seed(20230414)
WN_sim2 = arima.sim(model = list(order=c(0,0,1), ma=c(-1.5)),
                    sd = sqrt(1),
                    n = 100)
acf(WN_sim2, type="correlation", plot=FALSE, lag.max=3)
```

```
##
## Autocorrelations of series 'WN_sim2', by lag
##
##      0      1      2      3
##  1.000 -0.454 -0.029  0.019
```

- ii. Repeating (i) for 1000 times to find the values of
  the mean, variance and covariance of $\hat{\rho}_1$, $\hat{\rho}_2$ and $\hat{\rho}_3$

mean, variance and covariance

```
set.seed(20230414)

M = 1000

rho_1 = rep(0,M);rho_2 = rep(0,M);rho_3 = rep(0,M)

for (i in 1:M){
  WN_sim2 = arima.sim(model = list(order=c(0,0,1), ma=c(-1.5)),
                      sd = sqrt(1), n = 100)
  ar100_acf = acf(WN_sim2, type="correlation", plot=FALSE, lag.max=3)[[1]]  # ACF
  rho_1[i] = ar100_acf[2]
  rho_2[i] = ar100_acf[3]
  rho_3[i] = ar100_acf[4]
}

cbind(mean_rho = c(mean(rho_1),mean(rho_2),mean(rho_3)),
      var_rho = c(var(rho_1),var(rho_2),var(rho_3)))
```

```
##          mean_rho     var_rho
## [1,] -0.4541391934 0.005449826
## [2,] -0.0040762526 0.013105427
## [3,]  0.0004236458 0.013248544
```

covariance

```
rbind(co_rho12 = cov(rho_1,rho_2),
      co_rho23 = cov(rho_2,rho_3),
      co_rho13 = cov(rho_1,rho_3))
```

```
##                   [,1]
## co_rho12 -0.006766043
## co_rho23 -0.008239759
## co_rho13  0.001503095
```

- iii. Compare the result of (ii) with the Bartlett's formula.
- 紙上

3. 10

Consider the monthly U.S. unemployment rate from January 1948 to March 2009 in the file m-unrate.txt. The data are seasonally adjusted and obtained from the Federal Reserve Bank of St Louis. Build an AR time series model for the series and use the model to forecast the unemployment rate for the April, May, June, and July of 2009. In addition, does the fitted model imply the existence of business cycles? Why? (Note that there are more than one model fits the data well. You only need an adequate model.)

the monthly U.S. unemployment rate

```
unrate = read.table('https://faculty.chicagobooth.edu/-/media/faculty/ruey-s-tsay/teaching/ft
s3/m-unrate.txt',header = T)
head(unrate)
```

```
##   Year Mon Day Rate
## 1 1948   1   1  3.4
## 2 1948   2   1  3.8
## 3 1948   3   1  4.0
## 4 1948   4   1  3.9
## 5 1948   5   1  3.5
## 6 1948   6   1  3.6
```

Build an AR time series model (You only need an adequate model.)

1. 找**P**

```
m1 = ar(unrate$Rate, aic=TRUE, method='mle')
m1$order
```

```
## [1] 11
```

```
m11 = arima(unrate$Rate, order=c(11,0,0))
m11
```

```
##
## Call:
## arima(x = unrate$Rate, order = c(11, 0, 0))
##
## Coefficients:
##          ar1     ar2      ar3      ar4     ar5      ar6      ar7     ar8
##       0.9886  0.2375  -0.0741  -0.0630  0.0301  -0.1283  -0.0426  0.0539
## s.e.  0.0367  0.0516   0.0525   0.0525  0.0526   0.0524   0.0526  0.0527
##          ar9     ar10     ar11  intercept
##       -0.0146  -0.1293  0.1259     5.6554
## s.e.   0.0526   0.0518  0.0371     0.4422
##
## sigma^2 estimated as 0.03867:  log likelihood = 150.03,  aic = -274.07
```

2. 刪不重要的係數

- ar3 : -0.17896726,0.03084811
- ar4 : -0.16791970,0.04196004
- ar5 : -0.07516071,0.13542866
- ar7 : -0.14788610,0.06262882
- ar8 : -0.05139841,0.15921055
- ar9 : -0.11978259,0.09062277

係數可以一次一個刪，不要同時一起刪。

```
Box.test(m11$residuals, lag=20, type="Ljung-Box", fitdf=11) # reject no residual serial corre
lation
```

```
##
##  Box-Ljung test
##
## data:  m11$residuals
## X-squared = 24.827, df = 9, p-value = 0.003168
```

```
rbind(m11$coef-2*sqrt(diag(m11$var.coef)),m11$coef+2*sqrt(diag(m11$var.coef)))
```

```
##            ar1       ar2         ar3         ar4         ar5         ar6
## [1,] 0.9151829 0.1343485 -0.17896726 -0.16791970 -0.07516071 -0.23316059
## [2,] 1.0621163 0.3407452  0.03084811  0.04196004  0.13542866 -0.02349939
##              ar7         ar8         ar9        ar10       ar11 intercept
## [1,] -0.14788610 -0.05139841 -0.11978259 -0.23282452 0.05163457  4.770950
## [2,]  0.06262882  0.15921055  0.09062277 -0.02580673 0.20009208  6.539806
```

3. 新 **MODEL**

```
new_m11 = arima(unrate$Rate, order=c(11,0,0), fixed=c(NA,NA,0,0,0,NA,0,0,0,NA,NA,NA))
new_m11
```

```
##
## Call:
## arima(x = unrate$Rate, order = c(11, 0, 0), fixed = c(NA, NA, 0, 0, 0, NA, 0,
##     0, 0, NA, NA, NA))
##
## Coefficients:
##          ar1     ar2  ar3  ar4  ar5      ar6  ar7  ar8  ar9     ar10    ar11
##       0.9805  0.1695    0    0    0  -0.1728    0    0    0  -0.1216  0.1282
## s.e.  0.0361  0.0432    0    0    0   0.0270    0    0    0   0.0433  0.0364
##       intercept
##          5.6605
## s.e.     0.4337
##
## sigma^2 estimated as 0.03904:  log likelihood = 146.55,  aic = -279.11
```

```
Box.test(new_m11$residuals, lag=20, type="Ljung-Box", fitdf=11-6) # reject no residual serial
correlation，但 p-value 有上升
```

```
##
##  Box-Ljung test
##
## data:  new_m11$residuals
## X-squared = 32.716, df = 15, p-value = 0.005137
```

```
1-pchisq(32.716,20-5)  # p-value:0.005137207
```

```
## [1] 0.005137207
```

forecast the unemployment rate for the April, May, June, and July of 2009.

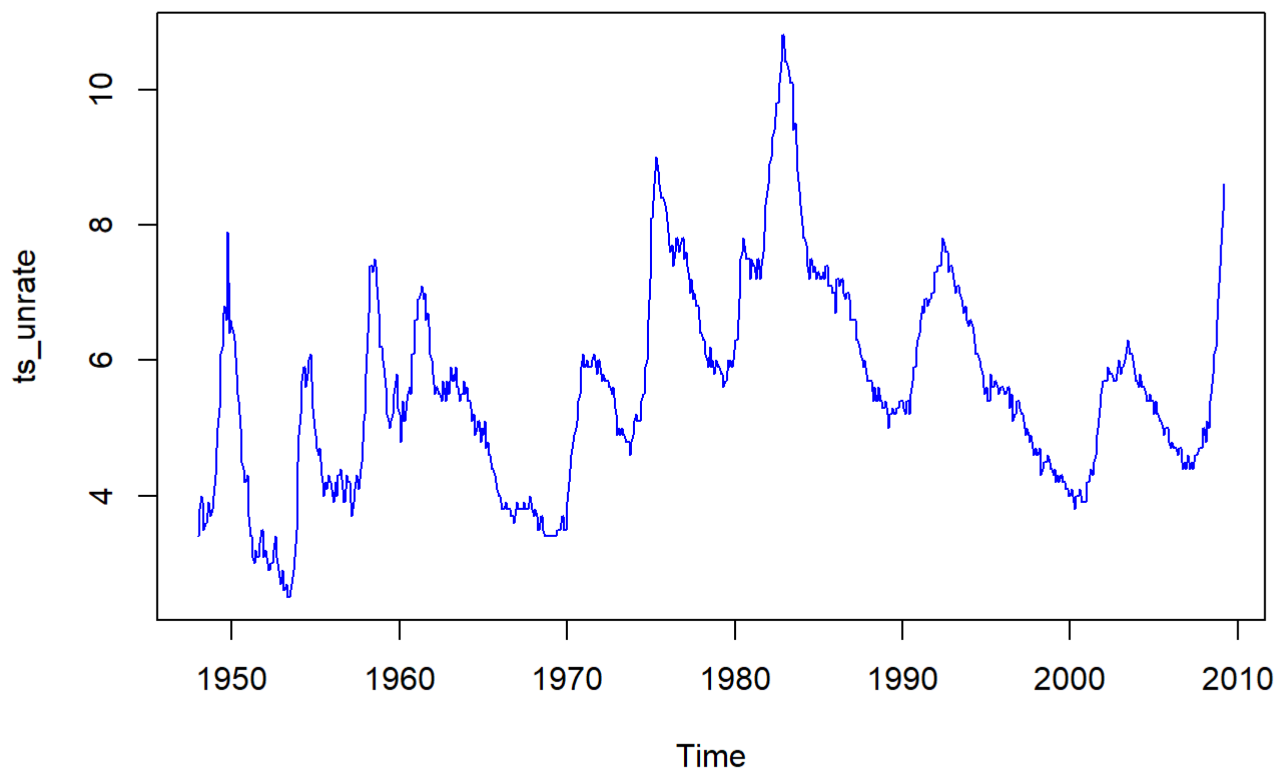
4. 預測

```
predict(new_m11, n.ahead = 4)$pred
```

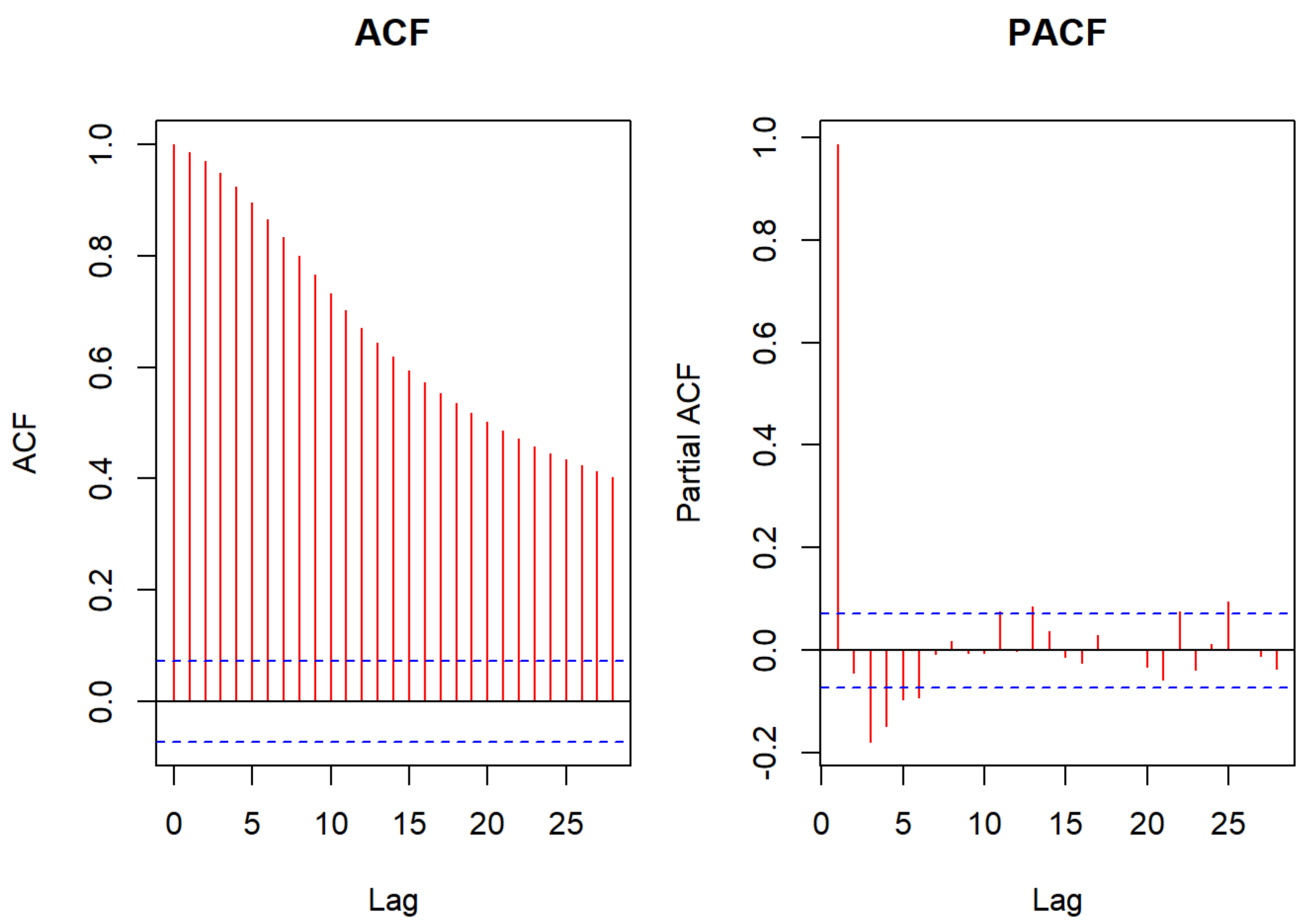
```
## Time Series:
## Start = 736
## End = 739
## Frequency = 1
## [1] 8.796765 8.981996 9.112555 9.246428
```

does the fitted model imply the existence of business cycles? Why?

```
ts_unrate = ts(unrate$Rate, frequency=12, start=c(1948,1))
plot(ts_unrate,type="l",col="blue")
```




```
par(mfrow=c(1,2),mar=c(4,4,4,1)) # 邊:下左上右
acf(unrate$Rate, type = "correlation", col="Red", main="ACF")
acf(unrate$Rate, type = "partial", col="Red", main="PACF")
```




```
pl = c(1,-new_m11$coef[1:11])              # 特徵方程式，mean為[12]
roots = polyroot(pl)                       # 找解
cbind(roots=roots, Mod_roots=Mod(roots))   # 每個根的長度
```

```
##                         roots   Mod_roots
##  [1,]  0.7213096+1.0870135i 1.304563+0i
##  [2,] -1.1427487+0.3989529i 1.210388+0i
##  [3,] -0.7723762-0.9739137i 1.243010+0i
##  [4,]  1.1354496+0.2017578i 1.153235+0i
##  [5,]  0.0144528+1.2121540i 1.212240+0i
##  [6,] -0.7723762+0.9739137i 1.243010+0i
##  [7,]  0.0144528-1.2121540i 1.212240+0i
##  [8,]  1.0357787+0.0000000i 1.035779+0i
##  [9,]  1.1354496-0.2017578i 1.153235+0i
## [10,] -1.1427487-0.3989529i 1.210388+0i
## [11,]  0.7213096-1.0870135i 1.304563+0i
```

差分.

AR(11) business cycles

```
knitr::include_graphics("C:/Users/user/Desktop/time_series/AR(11)_business_cycle.jpg")
```

在一般的AR(n)模型中，经济周期的平均长度可以通过计算周期成分的特征根来估计。具体来说，如果AR(n)模型的特征根为ρ_1, ρ_2, ..., ρ_n，则经济周期的平均长度为：

T = (2π) / (acos[(ρ_1+ρ_2+...+ρ_n)/n√(ρ_1ρ_2...ρ_n)])

其中，acos表示反余弦函数。

需要注意的是，这个公式只适用于AR(n)模型的情况，并且假设模型的噪声项是白噪声。此外，平均长度只是一个近似值，实际上经济周期的长度可能因多种因素而有所变化。

```
k = 2*pi/acos(Re(roots[1]/Mod(roots)[1]))
k # AR(2) : 6.379253
```

```
## [1] 6.379253
```

```
k = 2*pi/acos((sum(roots)/prod(roots)^(1/11)))
k # AR(11) : 9.438663
```

```
## [1] 9.438663-0i
```

- Yes,因為特徵方程式的根含有虛數
- And the Business cycles of the fitted model is about 9.5 months.

4. 10

Consider the weekly yields(產量) of Moody’s Aaa and Baa seasoned bonds from January 5, 1962, to April 10, 2009. The data are obtained from the Federal Reserve Bank of St Louis. Weekly yields are averages of daily yields. Obtain the summary statistics (sample mean, standard deviation, skewness, excess kurtosis, minimum, and maximum) of the two yield series. Are the bond yields skewed? Do they have heavy tails? Answer the questions using 5% significance level

weekly yields(產量) of Moody’s Aaa and Baa seasoned bonds

```
Aaa = read.table("C:/Users/user/Desktop/time_series/HW/w-Aaa.txt",header = F)
head(Aaa)
```

```
##     V1 V2 V3   V4
## 1 1962  1  5 4.43
## 2 1962  1 12 4.42
## 3 1962  1 19 4.42
## 4 1962  1 26 4.41
## 5 1962  2  2 4.42
## 6 1962  2  9 4.42
```

```
Baa = read.table("C:/Users/user/Desktop/time_series/HW/w-Baa.txt",header = F)
head(Baa)
```

```
##     V1 V2 V3   V4
## 1 1962  1  5 5.11
## 2 1962  1 12 5.09
## 3 1962  1 19 5.08
## 4 1962  1 26 5.08
## 5 1962  2  2 5.07
## 6 1962  2  9 5.08
```

the summary statistics

- sample mean
- standard deviation
- skewness
- excess kurtosis
- minimum
- maximum

```
basicStats(Aaa$V4)            # Aaa
```

```
##                 X..Aaa.V4
## nobs          2467.000000
## NAs              0.000000
## Minimum          4.190000
## Maximum         15.850000
## 1. Quartile      5.985000
## 3. Quartile      8.930000
## Mean             7.830109
## Median           7.540000
## Sum          19316.880000
## SE Mean          0.048697
## LCL Mean         7.734618
## UCL Mean         7.925601
## Variance         5.850323
## Stdev            2.418744
## Skewness         0.857092
## Kurtosis         0.578605
```

```
mean(Aaa$V4,na.rm=T)          # sample mean
```

```
## [1] 7.830109
```

```
sqrt(var(Aaa$V4,na.rm=T))     # standard deviation
```

```
## [1] 2.418744
```

```
skewness(Aaa$V4,na.rm=T)      # skewness
```

```
## [1] 0.857092
## attr(,"method")
## [1] "moment"
```

```
kurtosis(Aaa$V4,na.rm=T)       # excess kurtosis
```

```
## [1] 0.5786054
## attr(,"method")
## [1] "excess"
```

```
min(Aaa$V4,na.rm=T)            # minimum
```

```
## [1] 4.19
```

```
max(Aaa$V4,na.rm=T)            # maximum
```

```
## [1] 15.85
```

```
basicStats(Baa$V4)             # Baa
```

```
##                 X..Baa.V4
## nobs         2467.000000
## NAs             0.000000
## Minimum         4.780000
## Maximum        17.290000
## 1. Quartile     6.990000
## 3. Quartile    10.200000
## Mean            8.847122
## Median          8.350000
## Sum         21825.850000
## SE Mean         0.054704
## LCL Mean        8.739852
## UCL Mean        8.954392
## Variance        7.382486
## Stdev           2.717073
## Skewness        0.929779
## Kurtosis        0.760896
```

```
mean(Baa$V4,na.rm=T)           # sample mean
```

```
## [1] 8.847122
```

```
sqrt(var(Baa$V4,na.rm=T))      # standard deviation
```

```
## [1] 2.717073
```

```
skewness(Baa$V4,na.rm=T)       # skewness
```

```
## [1] 0.9297785
## attr(,"method")
## [1] "moment"
```

```
kurtosis(Baa$V4,na.rm=T)       # excess kurtosis
```

```
## [1] 0.760896
## attr(,"method")
## [1] "excess"
```

```
min(Baa$V4,na.rm=T)            # minimum
```

```
## [1] 4.78
```

```
max(Baa$V4,na.rm=T)            # maximum
```

```
## [1] 17.29
```

skewed?

skewness is zero test

```
skaaa = skewness(Aaa$V4, na.rm=TRUE)  # Aaa:0.857092
# Compute test statistic
t = nrow(Aaa)                         # 2467 # sum(is.na(Aaa$V4)) : 0
tl = skaaa/sqrt(6/t)                  # 17.37946
# Compute p-value
pv = 2*(1 - pnorm(tl,lower.tail = TRUE))
pv                                    # 0
```

```
## [1] 0
## attr(,"method")
## [1] "moment"
```

```
skbaa = skewness(Baa$V4, na.rm=TRUE)  # Baa:0.9297785
# Compute test statistic
t = nrow(Baa)                         # 2467 # sum(is.na(Baa$V4)) : 0
tl = skbaa/sqrt(6/t)                  # 18.85335
# Compute p-value
pv = 2*(1 - pnorm(tl,lower.tail = TRUE))
pv                                    # 0
```

```
## [1] 0
## attr(,"method")
## [1] "moment"
```

- p-value = 0 < 0.05， reject H0，skewness of the Aaa yield series is not zero
- p-value = 0 < 0.05， reject H0，skewness of the Baa yield series is not zero
- 都右偏(Aaa:0.857092/Baa:0.9297785 (Aaa:0.857092/Baa:0.9297785))

skewness.norm.test

```
skewness.norm.test(Aaa$V4)  # Aaa:0.857092 # p-value < 2.2e-16，reject H0
```

```
##
##  Skewness test for normality
##
## data:  Aaa$V4
## T = 0.85761, p-value < 2.2e-16
```

```
skewness.norm.test(Baa$V4)  # Baa:0.929779 # p-value < 2.2e-16，reject H0
```

```
##
##  Skewness test for normality
##
## data:  Baa$V4
## T = 0.93034, p-value < 2.2e-16
```

have heavy tails?

kurtosis is zero test

```
Aaa_ku = kurtosis(Aaa$V4, na.rm=TRUE)  # Aaa:0.5786054
# Compute test statistic
k = nrow(Aaa)                          # 2467 # sum(is.na(Aaa$V4)) : 0
kl = (Aaa_ku)/sqrt(24/k)               # 5.866262
# Compute p-value
pv = 2*(1-pnorm(kl,lower.tail = TRUE))
pv                                     # 4.457306e-09
```

```
## [1] 4.457306e-09
## attr(,"method")
## [1] "excess"
```

```
Baa_ku = kurtosis(Baa$V4, na.rm=TRUE)  # Baa:0.760896
# Compute test statistic
k = nrow(Baa)                          # 2467 # sum(is.na(Baa$V4)) : 0
kl = (Baa_ku)/sqrt(24/k)               # 7.714438
# Compute p-value
pv = 2*(1-pnorm(kl,lower.tail = TRUE))
pv                                     # 1.221245e-14
```

```
## [1] 1.221245e-14
## attr(,"method")
## [1] "excess"
```

- p-value = 4.457306e-09 < 0.05， reject H0，kurtosis of the Aaa yield series is not 3
- p-value = 1.221245e-14 < 0.05， reject H0，kurtosis of the Baa yield series is not 3
- 都厚尾(Aaa:0.5786054/# (Aaa:0.5786054/#) Baa:0.760896)

kurtosis.norm.test

```
kurtosis.norm.test(Aaa$V4)  # Aaa:0.578605,p-value < 2.2e-16,reject H0
```

```
##
##  Kurtosis test for normality
##
## data:  Aaa$V4
## T = 3.5815, p-value < 2.2e-16
```

```
kurtosis.norm.test(Baa$V4)  # Baa:0.760896,p-value < 2.2e-16,reject H0
```

```
##
##  Kurtosis test for normality
##
## data:  Baa$V4
## T = 3.7639, p-value < 2.2e-16
```

高念慈 B08204005

No.: HW8

Subject: Time Series

Date: 2023/4/14

1. (iii) Compare the results of (ii) with Bartlett's formula

(b) $Z_t = a_t - 1.5 a_{t-1}$, $\{a_t\} \sim WN(0, 1)$

P53 . P35

$\rho_1 = \dfrac{-\theta}{1+\theta^2} = \dfrac{6}{13}$, $\theta = -1.5$

0.005449826

Case 1 (lag = 1), $Var(\hat{\rho}_1) \approx \dfrac{1 - 3\rho_1^2 + 4\rho_1^4}{n} \approx 0.00542453$ #

Case 2 (lag > 1), $Var(\hat{\rho}_k) \approx \dfrac{1 + 2\rho_1^2}{n} \approx 0.01426036 \leftrightarrow 0.01310542$

$\therefore Var(\hat{\rho}_2) = Var(\hat{\rho}_3) = 0.01426036$ # $\leftrightarrow 0.01324854$

$Cov(\hat{\rho}_1, \hat{\rho}_2) \approx \dfrac{2(\rho_1 - \rho_1^3)}{n} \approx -0.007264452$ # $\leftrightarrow -0.006766043$

$Cov(\hat{\rho}_2, \hat{\rho}_3) \approx \dfrac{2\rho_1}{n} \approx -0.009230769$ # $\leftrightarrow -0.008239959$

$Cov(\hat{\rho}_1, \hat{\rho}_3) \approx \dfrac{\rho_1^2}{n} \approx 0.002130178$ # $\leftrightarrow 0.001503095$

10 結果都很接近(ii)

2. Suppose the daily log return of a security follows the model 証券

$r_t = 0.01 + 0.2 r_{t-2} + a_t$, $\{a_t\} \sim$ Gaussian $WN(0, 0.02)$

(i) What are the mean and variance of r_t ?

(ii) Compute the lag-1 and lag-2 autocorrelations of r_t.

(iii) Assume $r_{100} = -0.01$, $r_{99} = 0.02$, Compute 1 and 2-step-ahead forecasts of the return series at the forecast origin $t = 100$

(iiii) What are the associated sd of the forecast errors?

(i) mean : $E(r_t) = 0.01 + 0.2 E(r_{t-2}) + E(a_t)$

$0.8\mu = 0.01$, $\mu = \dfrac{0.01}{0.8} = 0.0125$ #

Variance : $Var(r_t) = Var(0.2 r_{t-2} + a_t)$

$= 0.04 Var(r_{t-2}) + Var(a_t) + 0.4 \underbrace{Cov(r_{t-2}, a_t)}_{0}$

$\Rightarrow Var(r_t) = 0.04 Var(r_{t-2}) + 0.02$

$\Rightarrow Var(r_t) = \dfrac{0.02}{1-0.04} = \dfrac{1}{48}$ #

Double A

No. :

Date : ..../........../..........

Subject : ....

(ii) ACF 1 & 2 $(\rho_1 . \rho_2)$

$$\dot{r}_t r'_{t-k} = 0.2 r'_{t-2} r'_{t-k} + a_t r'_{t-k} \quad (\times r'_{t-k})$$

$$E(r'_t r'_{t-k}) = 0.2 E(r'_{t-2} r'_{t-k}) + 0$$

$$\gamma_k = 0.2 \gamma_{k-2} , \quad k \in \mathbb{Z}$$

$$\Rightarrow \rho_k = 0.2 \rho_{k-2}$$

$k=1$, $\rho_1 = 0.2 \rho_{-1} \Rightarrow 0.8 \rho_1 = 0$, $\rho_1 = 0$ #

$k=2$, $\rho_2 = 0.2 \rho_0 \Rightarrow \rho_2 = 0.2$ #

(iii) 1-step : $\hat{r}_n(1) = r_{n+1}$

(iiii) 2-step : $\hat{r}_n(2) = r_{n+2}$

$$\Rightarrow \hat{r}_{100}(1) = E(r_{101} \mid r_{100} \cdots r_1)$$

$$= E(0.01 + 0.2 r_{99} + a_{101} \mid r_{100} \cdots r_1)$$

$$= 0.01 + 0.2 E(r_{99})$$

$$= 0.01 + 0.2 \times 0.02 = 0.014 \quad \#$$

$$e_{100}(1) = r_{101} - \hat{r}_{100}(1)$$

$$= 0.01 + 0.2 r_{99} + a_{101} - (0.01 + 0.2 \cdot E(r_{99}))$$

$$= a_{101}$$

$Var(a_{101}) = 0.02 \Rightarrow SD(a_{101}) = \sqrt{0.02} = 0.1414$ #

$$\Rightarrow \hat{r}_{100}(2) = E(0.01 + 0.2 r_{100} + a_{102} \mid r_{100} \cdots r_1)$$

$$= 0.01 + 0.2(-0.01) + 0$$

$$= 0.008 \quad \#$$

$$e_{100}(2) = r_{102} - \hat{r}_{100}(2)$$

$$= 0.01 + 0.2 r_{100} + a_{102} - (0.01 + 0.2(-0.01))$$

$$= a_{102}$$

$Var(a_{102}) = 0.02 \Rightarrow SD(a_{102}) = 0.1414$ #