Assignment 5

1. Is the following a valid autocorrelation function for a real-valued covariance stationary process ? Why ?

$$\rho_k = \begin{cases} 1, & \text{if } k = 0, \\ \phi, & \frac{1}{2} < |\phi| < 1, \text{ if } |k| = 1, \\ 0, & \text{if } |k| \geq 2. \end{cases}$$

2. Using the daily log return of IBM (IBM), Intel (INTC), 3M (MMM), Microsoft (MSFT) and Citi-group (C) from 2020/1/1 to 2023/02/28 to calculate and plot the sample ACVF and ACF for lag 1 ~ 20.

3. Consider a stationary series with theoretical autocorrelation function

$$\rho_k = \phi^k, \ |\phi| < 1, \ k = 1, 2, 3, \ldots$$

   Find the variance of the sample autocorrelation function $\widehat{\rho_k}$ using Bartlett's approximation. (Hint: Please use the equation of "lecture 2 PPT" on page 34 or page 35)

4. Show that the estimator $\widehat{\gamma_k}$ is always positive semi-definite, but $\widehat{\widehat{\gamma_k}}$ is not necessarily so.