Analysis Of Financial Time Series, Midterm Exam 2022/05/09

Problem A: Answer the following questions with detailed derivation.

1. Assume $X_t = (1-0.8B-0.7B^3+0.56B^{10})a_t$, where $\{a_t\}$ is a WN(0,1) process. Use the autocovariance generating function to compute the **autocorrelation** function(ACF) of $\{X_t\}$.

2. Consider the following AR(3) model, where $\{a_t\} \sim WN(0,1)$:

$$\left(1-\frac{3}{5}B\right)\left(1-\frac{9}{10}B+\frac{81}{100}B^2\right)Z_t = a_t.$$

   Do the above AR(3) model imply existence of a business cycle? If yes, derive the average period of the cycle.

3. Find the ACF of the following ARMA(1,1) process: $r_t = \phi_1 r_{t-1} + a_t - \theta_1 a_{t-1}$, where $a_t \sim WN(0,1)$. And derive the ACF of r_t when $\phi_1 = \theta_1$.

4. Let $Z_t = U\sin(5t+\theta) + V\cos(5t+\theta)$, where U, V and θ are independent random variables, with $\mathrm{E}(U) = \mathrm{E}(V) = 0$, $\mathrm{Var}(U) = \mathrm{Var}(V) = 1$ and $\theta \sim \mathrm{Unif}(-\pi,\pi)$. Is Z_t covariance stationary? State your reason.

5. Consider the following ARCH(2) model:

$$\begin{aligned} r_t &= 0.3 + 0.9r_{t-1} - 0.81r_{t-2} + a_t \\ a_t &= \sigma_t \epsilon_t \\ \sigma_t^2 &= 1 + 2\ a_{t-1}^2 + 4\ a_{t-2}^2 \end{aligned}$$

   where ϵ_t's are iid random variables with zero mean and unit variance. Show that $\{a_t^2\}$ is an AR(2) process.

Problem B: Using R or python to answer the following questions.

1. Consider the data "data1" from 2022-04-01 to 2022-04-29. The columns are (date, GOOG_price, TSLA_price, FB_price), where GOOG_price, TSLA_price and FB_price are adjusted prices of Google, Tesla and Meta, respectively. A portfolio of Stock Google, Tesla and Meta are considered, with $w_{GOOG} = 30\%$ (capital allocation for Stock Google), $w_{TSLA} = 50\%$ (capital allocation for Stock Tesla) and $w_{FB} = 20\%$ (capital allocation for Stock Meta).

   (a) What is the 4-period log-return of the portfolio from 2022-04-25 to 2022-04-29 ?

   (b) What is the average daily simple return of the portfolio from 2022-04-01 to 2022-04-29 ?

   (c) If the stock GOOG pays dividend \$0.25 on 2022-04-27, stock TSLA pays dividend \$1.05 on 2022-04-04 and stock FB pays dividend \$0.80 on 2022-04-11, find the average daily log return of the portfolio for the period 2022-04-01 to 2022-04-29.

2. Consider the data file "data2" from November 2010 to April 2022. The columns are (date, ts1), where ts1 is a time series. Use 5% significance level to answer the following questions for ts1:

   (a) Compute the log return of the time series ts1.

   (b) Are there serial correlations and ARCH effects in the log return of the time series ts1? Use Ljung-Box test Q(25).

   (c) Build ARCH models for the log return of time series ts1. Write down the fitted ARCH model and output the AIC.

   (d) Are there serial correlations and ARCH effect in the standardize residuals of the fitted model? Use Ljung-Box test Q(25).

   (e) Compute 1-step- to 5-step-ahead forecasts of the volatility based on the fitted model.

3. Consider the data file "data3". The columns are (date, year1_rate, year3_rate), where year1_rate and year3_rate are constant maturity rated of 1-year and 3-year, respectively. Use 5% significance level to answer the following questions for both time series:

   (a) Build a regression model using year3_rate as the dependent variable and year1_rate as independent variable. Perform the unit root test on the residuals of the fitted regression model. Use lag=5 to perform the test.

   (b) Build a regression model using first difference of year3_rate as the dependent variable and first difference of year1_rate as independent variable. Perform the unit root test on the residuals of the fitted regression model. Use lag=5 to perform the test.

   (c) Based on the results of (a) and (b), which models would you suggest to use respectively for the two time series? Justify your answer.

(d) Build a regression model with time series errors using first difference of year3_rate as the dependent variable and first difference of year1_rate as independent variable. Is there serial correlations in the residuals of the fitted model? Use Ljung-Box test Q(25).