

# Assignment 12

1. This problem is concerned with the dynamic relationship between the spot and futures prices of the S&P 500 index. The data file `sp5may.dat` has three columns:  $\log(\text{futures price})$ ,  $\log(\text{spot price})$ , and cost-of-carry ( $\times 100$ ). The data were obtained from the Chicago Mercantile Exchange for the S&P 500 stock index in May 1993 and its June futures contract. The time interval is 1 minute (intraday). Several authors used the data to study index futures arbitrage. Here we focus on the first two columns. Let  $f_t$  and  $s_t$  be the log prices of futures and spot, respectively. Consider  $y_t = f_t - f_{t-1}$  and  $x_t = s_t - s_{t-1}$ . Build a regression model with time series errors between  $\{y_t\}$  and  $\{x_t\}$ , with  $y_t$  being the dependent variable.
2. The file `m-mrk4608.txt` contains monthly simple returns of Merck stock from June 1946 to December 2008. The file has two columns denoting date and simple return. Transform the simple returns to log returns.
  - (a) Is there any evidence of serial correlations in the log returns? Use autocorrelations and 5% significance level to answer the question. If yes, remove the serial correlations.
  - (b) Is there any evidence of ARCH effects in the log returns? Use the residual series if there are serial correlations in part (a). Use Ljung–Box statistics for the squared returns (or residuals) with 6 and 12 lags of autocorrelations and 5% significance level to answer the question.
  - (c) Identify an ARCH model for the data and fit the identified model. Write down the fitted model.
3. The file `m-3m4608.txt` contains two columns. They are date and the monthly simple return for 3M stock. Transform the returns to log returns.
  - (a) Is there any evidence of ARCH effects in the log returns? Use Ljung–Box statistics with 6 and 12 lags of autocorrelations and 5% significance level to answer the question.
  - (b) Use the PACF of the squared returns to identify an ARCH model. What is the fitted model?
  - (c) There are 755 data points. Refit the model using the first 750 observations and

use the fitted model to predict the volatilities for  $t$  from 751 to 755 (the forecast origin is 750).