Assignment 7

If not specifically specified, use 5% significance level to draw conclusions in the exercises.

1. Find the autocovariance generating function of the following models.
   (a) $Z_t = \mu + a_t - 0.5a_{t-1} + 2a_{t-2} + 3a_{t-3},\ a_t \sim WN(0,1)$.
   (b) $Z_t = 0.5Z_{t-1} + a_t,\ a_t \sim WN(0,\sigma_a^2)$.

2. Consider the following $AR(2)$ models:
   (i) $Z_t - 0.6Z_{t-1} - 0.3Z_{t-2} = a_t$.
   (ii) $Z_t - 0.8Z_{t-1} + 0.5Z_{t-2} = a_t$.

   (a) Find the general expression for ρ_k.
   (b) Plot the ρ_k, for k = 0,1,2,...,10.
   (c) Calculate σ_Z^2 by assuming $\sigma_a^2 = 1$.

3. Given the $AR(2)$ process : $Z_t = Z_{t-1} - 0.25Z_{t-2} + a_t$
   (a) Calculate ρ_1.
   (b) Use ρ_0, ρ_1 as starting values and the difference equation to obtain the general expression for ρ_k.
   (c) Calculate the value ρ_k for k = 1,2,...,10.

4. Derive the stationary region of ϕ_1 and ϕ_2 for $AR(2)$ model:
   $$(1 - \phi_1 B - \phi_2 B^2)\dot{Z}_t = a_t.$$