Analysis Of Financial Time Series, Midterm Exam 2023/05/02

Problem A: Answer the following questions with detailed derivation.

1. Assume $(1-0.5B)X_t = (1+0.6B)(1-0.3B)a_t$, where $\{a_t\}\sim WN(0,1)$.
   (a) Express X_t as an $MA(\infty)$ process.
   (b) Find the autocovariance generating function of X_t based on the result of (a).
   (c) Find a recursive formula for the autocorrelation function of X_t and then derive the general formula for the acf, for $k \geq 3$.

2. Assume that the quarterly log return r_t of an asset follows the model:
   $(1-0.4B)(1-B)(1-B^4)r_t = (1-0.2B^4)a_t, where\ \{a_t\}\sim WN(0,1)$.
   (a) What is the variance of the series $w_t = (1-B)(1-B^4)r_t$ with regular and seasonal differencing ?
   (b) Suppose that $r_{799} = 0.9, r_{798} = 0.5, r_{797} = 0.7, r_{796} = 0.6, r_{795} = 0.4, r_{794} = 0.3, a_{799} = 0.1, a_{798} = 0.3, a_{797} = -0.4, a_{796} = -0.5\ and\ a_{795} = 0.2$. Find the 1-step ahead forecast of r_{800} at the forecast origin $t = 799$?
   (c) What is the forecast error of the above 1-step ahead prediction ? What is the 95% 1-step ahead prediction interval of r_{800} ?

3. Consider the following AR(3) model, where $\{a_t\}\sim WN(0,1)$:
   $$(405 - 639B + 497B^2 - 196B^3)Z_t = a_t.$$
   Do the above AR(3) model imply existence of a business cycle ? If yes, derive the average period of the cycle.

4. Let $Z_t = Usin(9t+\theta) + \cos(9t+\theta)$, where U and θ are independent random variables, with $E(U) = 0$, $var(U) = 1$ and $\theta\sim Unif(-\pi,\pi)$.
   Is $\{Z_t\}$ covariance stationary? State your reason.

Problem B: Using R or python to answer the following questions.

1. Consider the data "data_1" from 2022/1/3 to 2023/3/31. The columns are (date, AAPL price, INTC price, MSFT price), where AAPL price, INTC price and MSFT price are the adjusted prices of Apple, Intel and Microsoft, respectively. A portfolio in constructed form 2022/1/3 to 2022/12/30 with the initial capital \$100 and the following allocation: $w_{AAPL} = 30\%, w_{INTC} = 50\% \; and \; w_{MSFT} = 20\%$. The invertor changes the portfolio capital allocation from 2023/1/3 to 2023/3/31, with $w_{AAPL} = 20\%, w_{INTC} = 20\% \; and \; w_{MSFT} = 60\%$.

   (a) What is the 4-period log-return of the portfolio from 2022-04-25 to 2022-04-29 ?
   (b) What is the average daily simple return of the portfolio from 2022-04-01 to 2022-04-29 ?
   (c) What is the cumulative simple return of the portfolio from 2022/1/3 to 2022/12/30 ?
   (d) What is the cumulative log return of the portfolio from 2022/1/3 to 2023/3/31 ?

2. Consider the data "data_2". Use 5% significance level to answer the following questions for both time series:

   (a) Build a regression model using ten_years_rate as the dependent variable and three_years_rate as independent variable. Perform the goodness of fit test on the residuals of the fitted model.
   (b) Build a regression model using the first difference of ten_years_rate as the dependent variable and the first difference of three_years_rate as independent variable. Perform the goodness of fit test on the residuals of the fitted regression model.
   (c) Build a regression with time series error model using the first difference of ten_years_rate as the dependent variable and the first difference of three_years_rate as independent variable. Perform the goodness of fit test on the residuals of the fitted regression model.
   (d) Based on the results of (a), (b) and (c), which model would you suggest to use ? Justify your answer.
   (e) Set $X_t = \text{ten\_years\_rate}_t - \text{three\_years\_rate}_t$. Build a time series model for X_t and check it goodness of fit test on the residuals.