Assignment 6

1. (i) Show that the ACF ρ_k for the $AR(1)$ process satisfies the difference equation

$$\rho_k - \phi_1 \rho_{k-1} = 0, \qquad for\ k \geq 1.$$

(ii) Find the general expression for ρ_k.

2. (i) Simulate 100 observations from the following $AR(1)$ process:

$$Z_t = 0.5 - 0.5 Z_{t-1} + \epsilon_t,$$

Where $Z_0 = 0$ and $\epsilon_t \overset{i.i.d.}{\sim} N(0, 0.3)$. Compute the sample ACF $\hat{\rho}_k$ for k = 0, 1, 2, …, 10 and PACF $\hat{\phi}_{kk}$ for k = 0, 1, 2, …, 10.

(ii) Repeating the procedure of (i) for 1000 times, find the values of the mean, variance and covariance of $\hat{\rho}_1$, $\hat{\rho}_2$ and $\hat{\rho}_3$.

(iii) Compare the result of (ii) with the Bartlett's approximation.

3. Simulate a series of 100 observations from each of the following models where the a_t is a Gaussian white noise process with $E(a_t) = 0$ and $Var(a_t) = 1$:
   (a) $Z_t - 0.5 Z_{t-1} = a_t$
   (b) $Z_t + 0.98 Z_{t-1} = a_t$
   (c) $Z_t - 1.3 Z_{t-1} + 0.4 Z_{t-2} = a_t$
   (d) $Z_t - 1.2 Z_{t-1} + 0.8 Z_{t-2} = a_t$

   For each case, plot the simulated series, and calculate and study its sample ACF $\hat{\rho}_k$ for k = 0, 1, 2, …, 20 and PACF $\hat{\phi}_{kk}$ for k = 0, 1, 2, …, 20.