Time Series HW5

30/40

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2.

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Using the daily log return of IBM (IBM), Intel (INTC), 3M (MMM), Microsoft (MSFT) and Citi-group (C) from 2020/1/1 to 2023/02/28 to calculate and plot the sample ACVF and ACF for lag 1 ~ 20.

```
# from 正常，to 少一天

getSymbols("IBM",from="2020-01-01",to='2023-03-01')   ### IBM
```

```
## [1] "IBM"
```

```
# head(IBM)

getSymbols("INTC",from="2020-01-01",to='2023-03-01')  ### Intel
```

```
## [1] "INTC"
```

```
# head(INTC)

getSymbols("MMM",from="2020-01-01",to='2023-03-01')   ### 3M
```

```
## [1] "MMM"
```

```
# head(MMM)

getSymbols("MSFT",from="2020-01-01",to='2023-03-01')  ### Microsoft(微軟)
```

```
## [1] "MSFT"
```

```
# head(MSFT)

getSymbols("C",from="2020-01-01",to='2023-03-01')     ### Citi-group(花旗)
```

```
## [1] "C"
```

```
# head(C)
```

取出 adjust price

```
IBM_adjust = IBM$IBM.Adjusted
INTC_adjust = INTC$INTC.Adjusted
MMM_adjust = MMM$MMM.Adjusted
MSFT_adjust = MSFT$MSFT.Adjusted
C_adjust = C$C.Adjusted
```

Log Return : $r_t = ln(P_t) - ln(P_{t-1})$

- $r_t = ln(R_t + 1)$
- $r_t = ln(P_t) - ln(P_{t-1})$

```
IBM_log_return = na.omit(diff(log(IBM_adjust)))
head(IBM_log_return)
```

```
##              IBM.Adjusted
## 2020-01-03 -0.0080071157
## 2020-01-06 -0.0017880171
## 2020-01-07  0.0006708205
## 2020-01-08  0.0083119169
## 2020-01-09  0.0105127111
## 2020-01-10 -0.0003657492
```

```
INTC_log_return = na.omit(diff(log(INTC_adjust)))
head(INTC_log_return)
```

```
##             INTC.Adjusted
## 2020-01-03 -0.0122377101
## 2020-01-06 -0.0028327243
## 2020-01-07 -0.0168266684
## 2020-01-08  0.0006782729
## 2020-01-09  0.0055806366
## 2020-01-10 -0.0060895234
```

```
MMM_log_return = na.omit(diff(log(MMM_adjust)))
head(MMM_log_return)
```

```
##              MMM.Adjusted
## 2020-01-03 -0.0086483776
## 2020-01-06  0.0009522201
## 2020-01-07 -0.0040391231
## 2020-01-08  0.0152291770
## 2020-01-09  0.0031505485
## 2020-01-10 -0.0040368314
```

```
MSFT_log_return = na.omit(diff(log(MSFT_adjust)))
head(MSFT_log_return)
```

```
##            MSFT.Adjusted
## 2020-01-03  -0.012529967
## 2020-01-06   0.002581557
## 2020-01-07  -0.009159705
## 2020-01-08   0.015803000
## 2020-01-09   0.012415426
## 2020-01-10  -0.004637861
```

```
C_log_return = na.omit(diff(log(C_adjust)))
head(C_log_return)
```

```
##              C.Adjusted
## 2020-01-03 -0.019015054
## 2020-01-06 -0.003141641
## 2020-01-07 -0.008722762
## 2020-01-08  0.007589167
## 2020-01-09  0.009031853
## 2020-01-10 -0.010418905
```

calculate and plot the sample ACVF and ACF for lag 1 ~ 20.

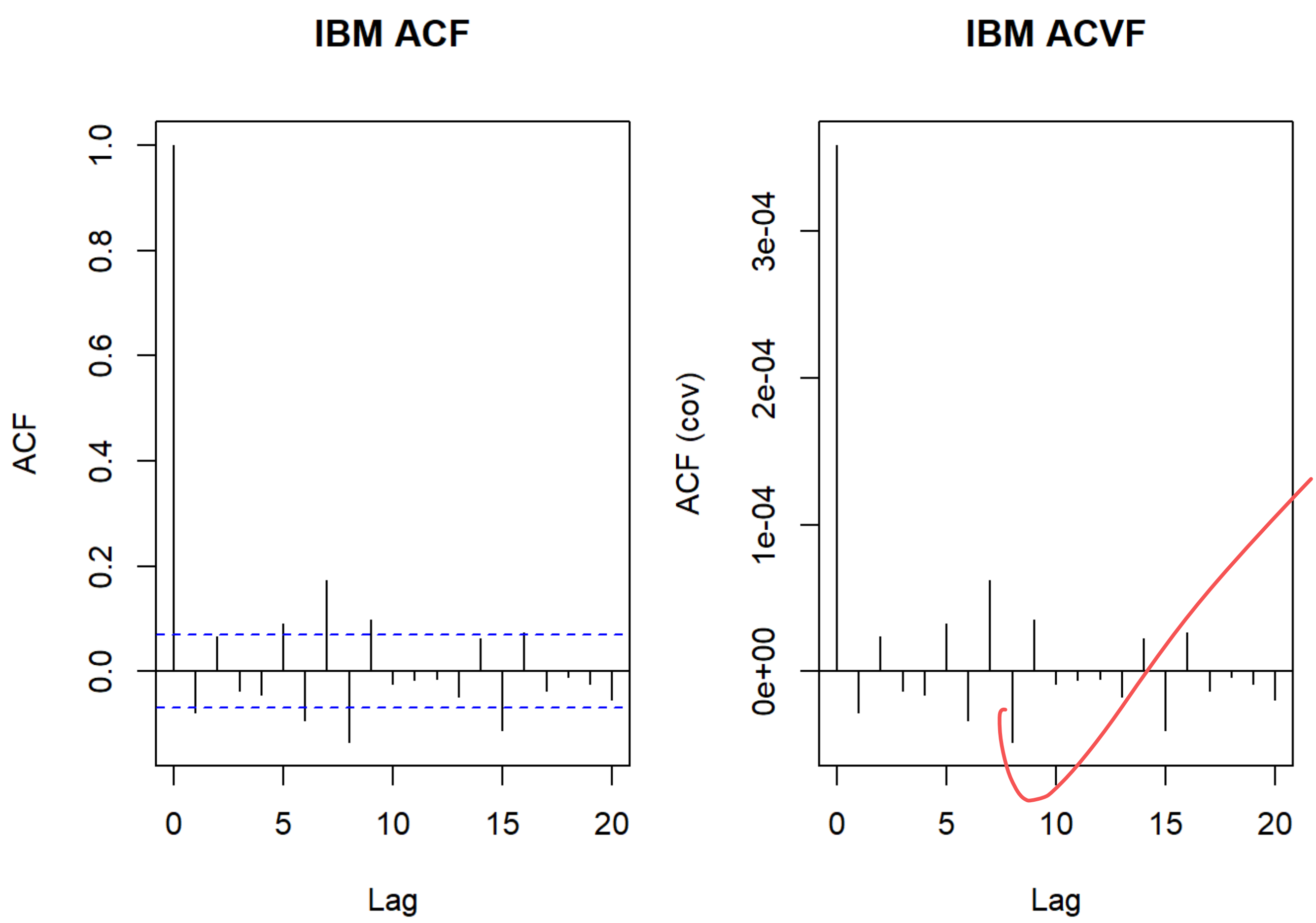
```
par(mfrow=c(1,2),mar=c(4,4,4,1)) # 邊:下左上右
acf(IBM_log_return, type = "correlation", lag.max = 20, plot = F)
```

```
##
## Autocorrelations of series 'IBM_log_return', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000 -0.078  0.067 -0.038 -0.045  0.092 -0.094  0.172 -0.134  0.099 -0.024
##     11     12     13     14     15     16     17     18     19     20
## -0.015 -0.015 -0.048  0.062 -0.112  0.073 -0.037 -0.011 -0.023 -0.053
```

```
acf(IBM_log_return, type = "correlation", lag.max = 20, plot = T, main="IBM ACF")
acf(IBM_log_return, type = "covariance", lag.max = 20, plot = F)
```

```
##
## Autocovariances of series 'IBM_log_return', by lag
##
##         0         1         2         3         4         5         6         7
##  3.58e-04 -2.79e-05  2.40e-05 -1.35e-05 -1.60e-05  3.28e-05 -3.37e-05  6.17e-05
##         8         9        10        11        12        13        14        15
## -4.80e-05  3.55e-05 -8.73e-06 -5.54e-06 -5.46e-06 -1.72e-05  2.23e-05 -4.00e-05
##        16        17        18        19        20
##  2.61e-05 -1.34e-05 -4.07e-06 -8.26e-06 -1.90e-05
```

```
acf(IBM_log_return, type = "covariance", lag.max = 20, plot = T, main="IBM ACVF")
```



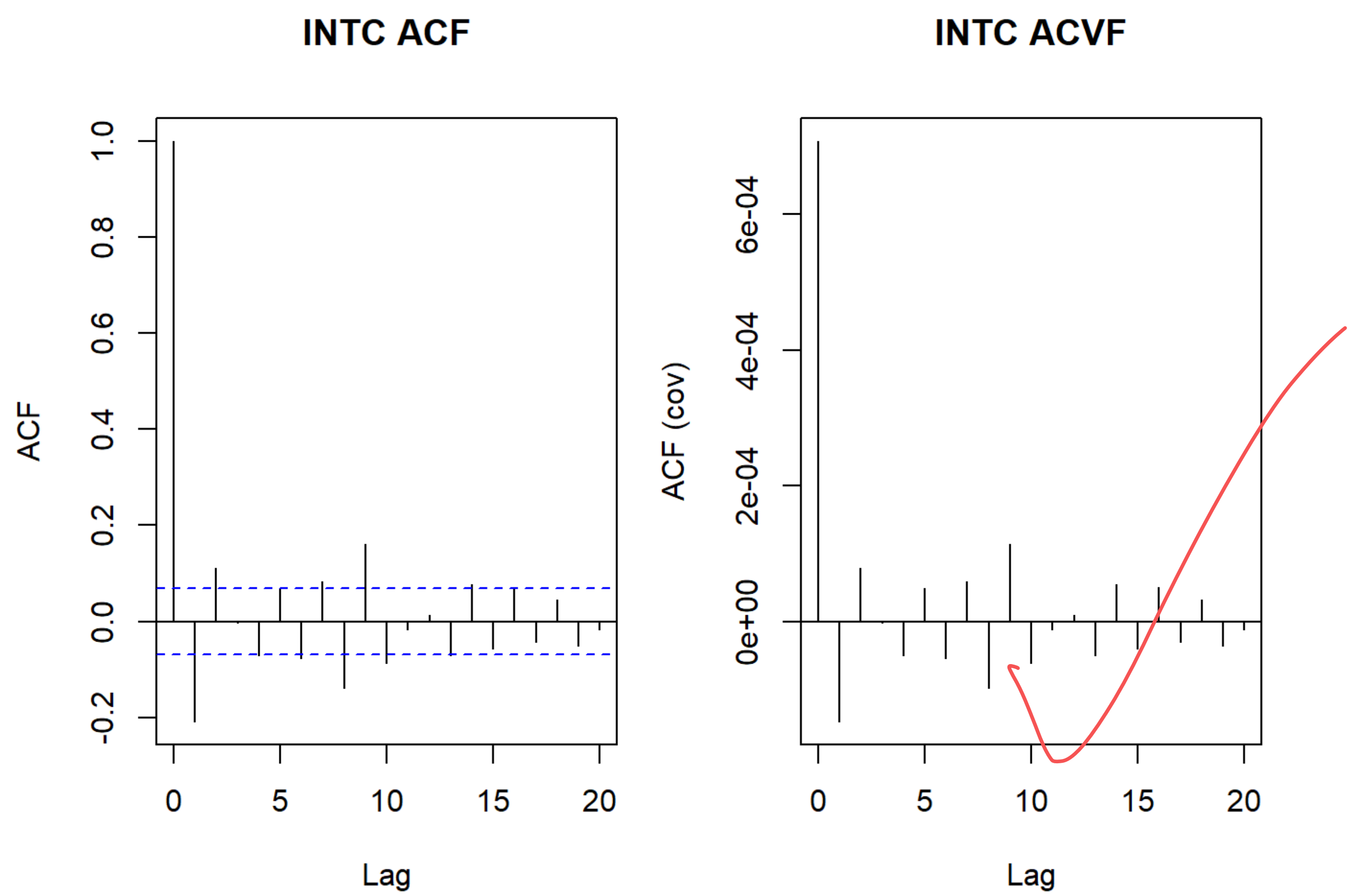

```
par(mfrow=c(1,2),mar=c(4,4,4,1)) # 邊:下左上右
acf(INTC_log_return, type = "correlation", lag.max = 20, plot = F)
```

```
##
## Autocorrelations of series 'INTC_log_return', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000 -0.208  0.110 -0.002 -0.071  0.070 -0.077  0.083 -0.138  0.162 -0.088
##     11     12     13     14     15     16     17     18     19     20
## -0.016  0.013 -0.071  0.078 -0.057  0.071 -0.042  0.045 -0.051 -0.018
```

```
acf(INTC_log_return, type = "correlation", lag.max = 20, plot = T, main="INTC ACF")
acf(INTC_log_return, type = "covariance", lag.max = 20, plot = F)
```

```
##
## Autocovariances of series 'INTC_log_return', by lag
##
##         0         1         2         3         4         5         6         7
##  7.07e-04 -1.47e-04  7.79e-05 -1.74e-06 -5.03e-05  4.94e-05 -5.47e-05  5.87e-05
##         8         9        10        11        12        13        14        15
## -9.76e-05  1.15e-04 -6.20e-05 -1.16e-05  8.92e-06 -5.01e-05  5.49e-05 -4.04e-05
##        16        17        18        19        20
##  5.00e-05 -2.99e-05  3.20e-05 -3.63e-05 -1.27e-05
```

```
acf(INTC_log_return, type = "covariance", lag.max = 20, plot = T, main="INTC ACVF")
```



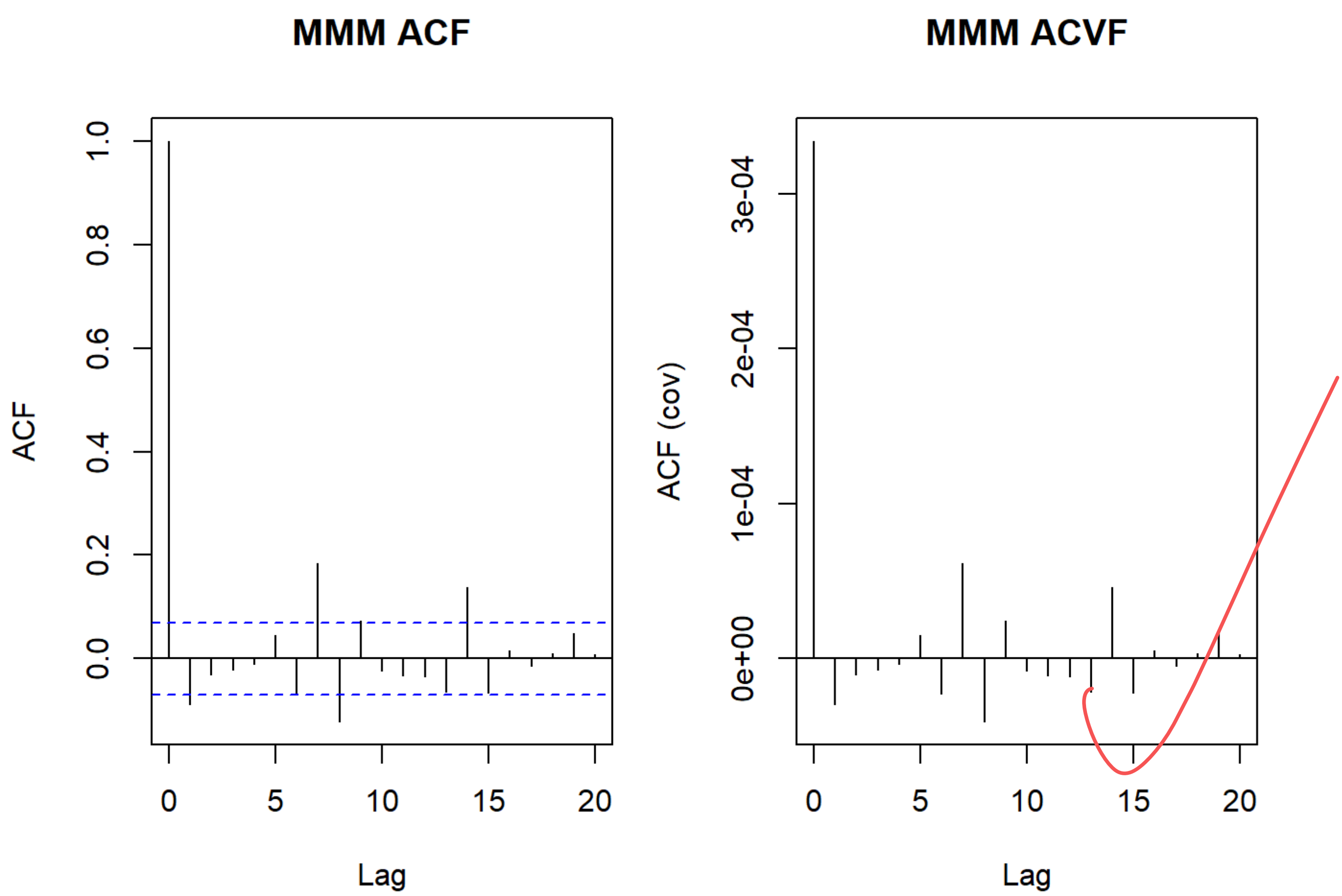

```
par(mfrow=c(1,2),mar=c(4,4,4,1)) # 邊:下左上右
acf(MMM_log_return, type = "correlation", lag.max = 20, plot = F)
```

```
##
## Autocorrelations of series 'MMM_log_return', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000 -0.089 -0.032 -0.023 -0.011  0.044 -0.068  0.184 -0.122  0.072 -0.025
##     11     12     13     14     15     16     17     18     19     20
## -0.033 -0.035 -0.065  0.138 -0.066  0.014 -0.015  0.009  0.049  0.007
```

```
acf(MMM_log_return, type = "correlation", lag.max = 20, plot = T, main="MMM ACF")
acf(MMM_log_return, type = "covariance", lag.max = 20, plot = F)
```

```
##
## Autocovariances of series 'MMM_log_return', by lag
##
##         0         1         2         3         4         5         6         7
##  3.34e-04 -2.96e-05 -1.07e-05 -7.62e-06 -3.75e-06  1.48e-05 -2.28e-05  6.13e-05
##         8         9        10        11        12        13        14        15
## -4.06e-05  2.41e-05 -8.18e-06 -1.12e-05 -1.18e-05 -2.16e-05  4.61e-05 -2.21e-05
##        16        17        18        19        20
##  4.83e-06 -4.97e-06  3.09e-06  1.63e-05  2.43e-06
```

```
acf(MMM_log_return, type = "covariance", lag.max = 20, plot = T, main="MMM ACVF")
```



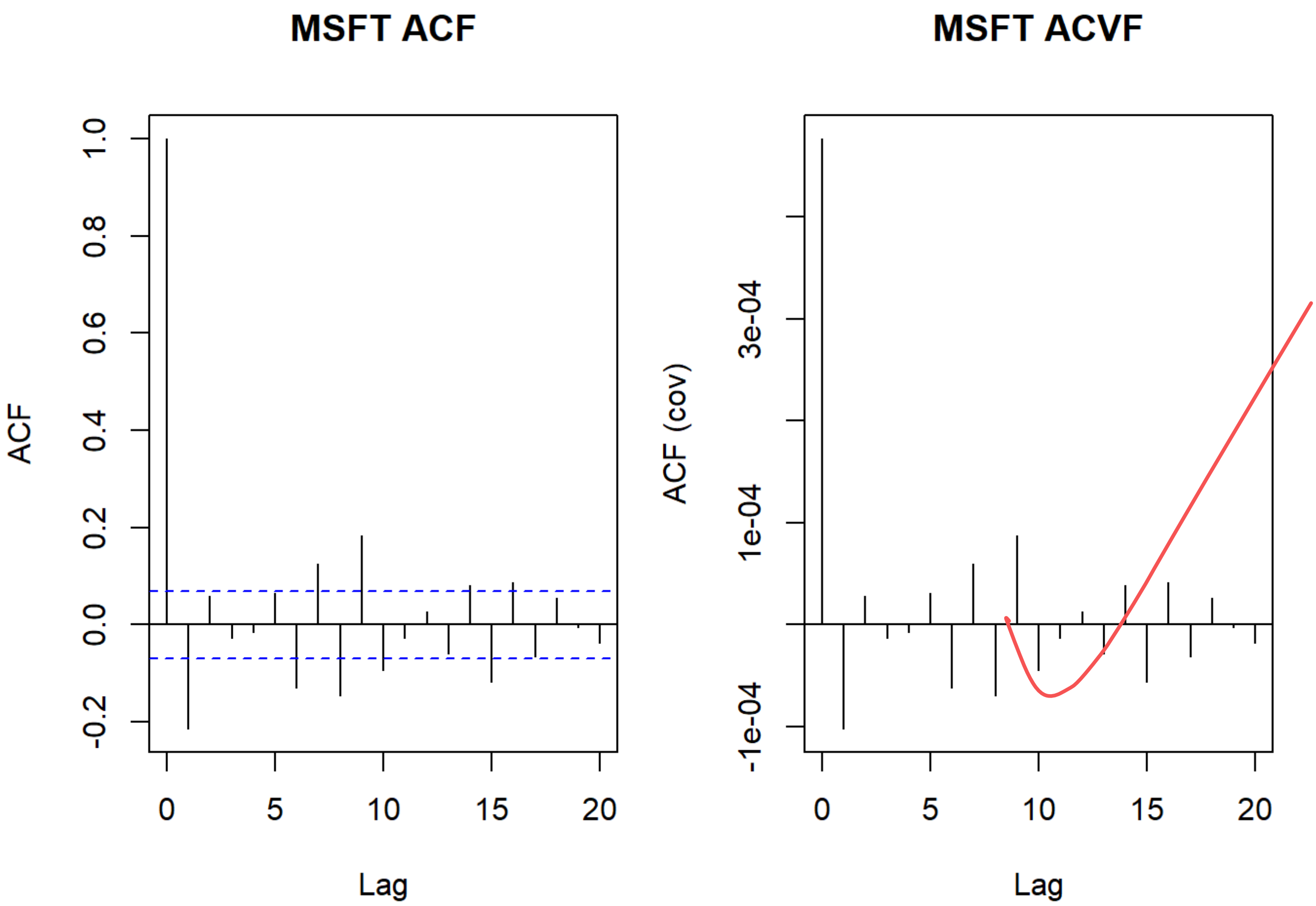

```
par(mfrow=c(1,2),mar=c(4,4,4,1)) # 邊:下左上右
acf(MSFT_log_return, type = "correlation", lag.max = 20, plot = F)
```

```
##
## Autocorrelations of series 'MSFT_log_return', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000 -0.214  0.058 -0.028 -0.015  0.064 -0.129  0.125 -0.145  0.183 -0.094
##     11     12     13     14     15     16     17     18     19     20
## -0.027  0.026 -0.060  0.082 -0.118  0.087 -0.066  0.055 -0.005 -0.038
```

```
acf(MSFT_log_return, type = "correlation", lag.max = 20, plot = T, main="MSFT ACF")
acf(MSFT_log_return, type = "covariance", lag.max = 20, plot = F)
```

```
##
## Autocovariances of series 'MSFT_log_return', by lag
##
##         0         1         2         3         4         5         6         7
##  4.77e-04 -1.02e-04  2.79e-05 -1.34e-05 -7.18e-06  3.05e-05 -6.16e-05  5.96e-05
##         8         9        10        11        12        13        14        15
## -6.92e-05  8.72e-05 -4.49e-05 -1.28e-05  1.25e-05 -2.87e-05  3.90e-05 -5.60e-05
##        16        17        18        19        20
##  4.14e-05 -3.12e-05  2.62e-05 -2.26e-06 -1.80e-05
```

```
acf(MSFT_log_return, type = "covariance", lag.max = 20, plot = T, main="MSFT ACVF")
```



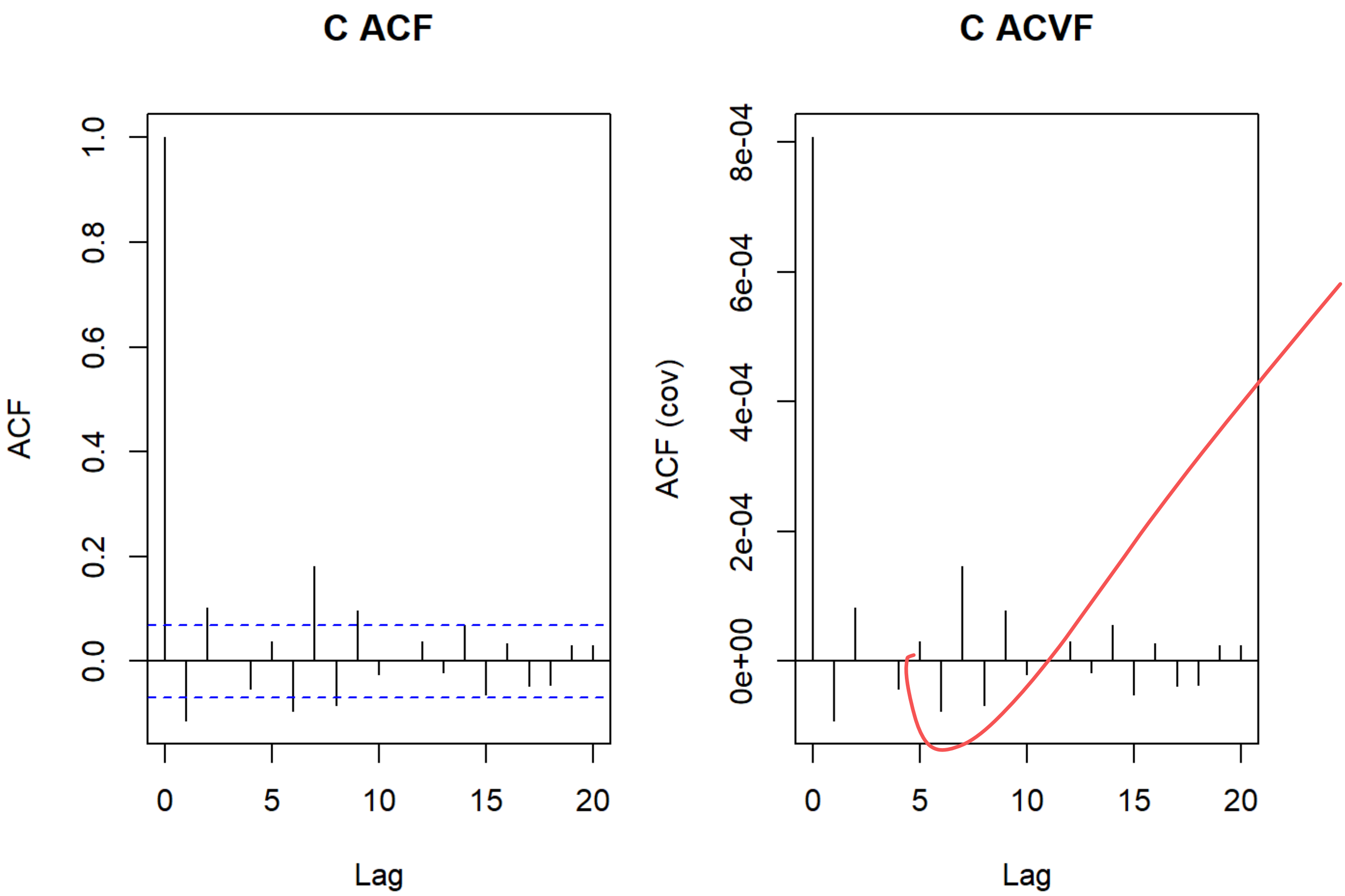

```
par(mfrow=c(1,2),mar=c(4,4,4,1)) # 邊:下左上右
acf(C_log_return, type = "correlation", lag.max = 20, plot = F)
```

```
##
## Autocorrelations of series 'C_log_return', by lag
##
##      0      1      2      3      4      5      6      7      8      9     10
##  1.000 -0.113  0.101  0.000 -0.053  0.038 -0.096  0.181 -0.084  0.096 -0.025
##     11     12     13     14     15     16     17     18     19     20
##  0.001  0.038 -0.021  0.068 -0.064  0.034 -0.046 -0.045  0.030  0.030
```

```
acf(C_log_return, type = "correlation", lag.max = 20, plot = T, main="C ACF")
acf(C_log_return, type = "covariance", lag.max = 20, plot = F)
```

```
##
## Autocovariances of series 'C_log_return', by lag
##
##         0         1         2         3         4         5         6         7
##  8.07e-04 -9.11e-05  8.19e-05  3.10e-08 -4.31e-05  3.09e-05 -7.71e-05  1.46e-04
##         8         9        10        11        12        13        14        15
## -6.81e-05  7.77e-05 -2.03e-05  6.13e-07  3.05e-05 -1.72e-05  5.52e-05 -5.19e-05
##        16        17        18        19        20
##  2.78e-05 -3.73e-05 -3.67e-05  2.42e-05  2.43e-05
```

```
acf(C_log_return, type = "covariance", lag.max = 20, plot = T, main="C ACVF")
```



- R 時間序列分析(一) (https://rpubs.com/ivan0628/TimeSeries01)

B08204005 高念慈

Subject: Time series, 2023.3.24. HW 5 No.: Date: ..../..../....

10

1. Is the following a 有效 valid autocorrelation function for a real-valued covariance stationary process? Why?

$$\rho_k : \begin{cases} 1, & \text{if } k = 0 \\ \phi, \ \frac{1}{2} < |\phi| < 1, & \text{if } |k| = 1 \\ 0, & \text{if } |k| \geq 2 \end{cases}$$

No ※ ρ_k is not non-negative definite ※

If $\phi > \frac{1}{2}$, $K = [\rho(i-j)]_{i,j=1}^{n}$, a is the n-component vector: $(1,-1,1\cdots)'$ 寫完整.

then

$$a'Ka = n - 2(n-1)\phi < 0, \text{ for } n > \frac{2\phi}{2\phi-1} \quad ※$$

$$K = \begin{pmatrix} \rho(0) & \rho(1) & & \rho(n-1) \\ \rho(1) & \rho(0) & & \vdots \\ \rho(2) & & \ddots & \vdots \\ \vdots & & & \rho(1) \\ \rho(n-1) & & & \rho(0) \end{pmatrix}_{n\times n} = \begin{pmatrix} 1 & \phi & 0 & \cdots & 0 \\ \phi & 1 & \phi & & \vdots \\ 0 & \phi & 1 & \ddots & \phi \quad 0 \\ \vdots & & \phi & & 1 \quad \phi \\ 0 & \cdots & & 0 \quad \phi & 1 \end{pmatrix}$$

$$a'Ka = (1-\phi, 2\phi-1, 1-2\phi, 2\phi-1, 1-2\phi \cdots) \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \\ \vdots \end{pmatrix} = (1-2\phi)(n-2) + (1-\phi)2$$

assume ✓

$$a'Ka < 0 \iff (1-\phi)2 < (2\phi-1)(n-2)$$

$$\iff \frac{2(1-\phi)}{2\phi-1} + 2 < n \iff \frac{2\phi}{2\phi-1} < n \quad ※$$

Similarly, if $\phi < -\frac{1}{2}$, $a = (1,1,1,1\cdots)'$

then

$$a'Ka = n + 2(n-1)\phi < 0, \text{ for } n < \frac{2\phi}{2\phi+1} \quad ※$$

Thus, ρ_k is not a covariance stationary process ※

再代數字舉個例子更好.

Subject : .... No. : Date : ..../........./.........

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3. Consider a stationary series with theoretical 理論上的 autocorrelation function

$$\rho_k = \phi^k, \quad |\phi| < 1, \quad k = 1, 2, 3 \dots$$

Find the variance of the sample ACF $\hat{\rho}_k$ using Bartlett's approximation.

(Hint: equation from lecture 2 PPT p.34 p.35)

$$Cov(\hat{\rho}_k, \hat{\rho}_{k+j}) \approx \frac{1}{n} \sum_{i=-\infty}^{\infty} (\rho_i \rho_{i+j} + \rho_{i+k+j}\rho_{i-k} - 2\rho_k \rho_i \rho_{i-k-j}$$

$k > 0,\ k+j > 0$

$$- 2\rho_{k+j}\rho_i \rho_{i-k} + 2\rho_k \rho_{k+j} \rho_i^2) \quad \#$$

For large n, $\hat{\rho}_k \xrightarrow{D} N(\rho_k, Var(\hat{\rho}_k))$

$$Var(\hat{\rho}_k) \approx \frac{1}{n} \sum_{i=-\infty}^{\infty} (\rho_i^2 + \rho_{i+k}\rho_{i-k} - 4\rho_k \rho_i \rho_{i-k} + 2\rho_k^2 \rho_i^2) \quad \#$$

題目給

If $\rho_k = 0$, for $k > m$, then

$$Var(\hat{\rho}_k) \approx \frac{1}{n}(1 + 2\rho_1^2 + \cdots 2\rho_m^2), \text{ for } k > m \quad \#$$

$$\Rightarrow Var(\hat{\rho}_k) \approx \frac{1}{n} \sum_{i=-\infty}^{\infty} (\phi^{2i} + \phi^{2i} - 4\phi^{2i} + 2\phi^{2k+2i})$$

$$= \frac{1}{n} \sum_{i=-\infty}^{\infty} (+2\phi^{2i}(\phi^{2k} - 1))$$

$$= \frac{2}{n}(\phi^{2k} - 1) \sum_{i=-\infty}^{\infty} \phi^{2i}$$

$$= \frac{2}{n}(\phi^{2k} - 1)(2 \underbrace{\sum_{i=0}^{\infty} \phi^{2i}}_{= \frac{1}{1-\phi^2}} - 1) \qquad \left(\frac{a_1(1-r^n)}{1-r}\right)$$

$$= \frac{2}{n}(\phi^{2k} - 1)\left(\frac{\phi^2}{1-\phi^2}\right) \quad \#$$

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4. Show $\hat{\gamma}_k$ is always positive semi-definite, but $\hat{\hat{\rho}}_k$ isn't necessarily so.

$$\hat{\Gamma}_n = \begin{pmatrix} \hat{\gamma}_0 & \hat{\gamma}_1 & \cdots & \hat{\gamma}_{n-1} \\ & \hat{\gamma}_0 & & \\ & & \ddots & \\ \hat{\gamma}_{n-1} & \cdots & \hat{\gamma}_1 & \hat{\gamma}_0 \end{pmatrix}_{n \times n} = \frac{1}{n} T T'$$

$$\forall \vec{\alpha} \in \mathbb{R}^n \quad \vec{\alpha}' \hat{\Gamma}_n \vec{\alpha} = n^{-1}(\alpha' T)(T' \alpha) = n^{-1} \| T' \vec{\alpha} \|^2 \geq 0, \ \forall n \quad \#$$

$$T = \begin{bmatrix} 0 & \cdots & 0 & z_1 - \bar{z}_n & \cdots & z_n - \bar{z}_n \\ & 0 & z_1 - \bar{z}_n & & z_n - \bar{z}_n & 0 \\ \vdots & & & & & \vdots \\ & & \text{(每次往左一格)} & & & \\ \vdots & & & & & \vdots \\ z_1 - \bar{z}_n & \cdots & z_{n-1} - \bar{z}_n, & z_n - \bar{z}_n & \cdots & 0 \end{bmatrix}_{n \times (2n-1)} \quad T' = \begin{bmatrix} 0 & 0 & \cdots & z_1 - \bar{z}_n \\ \vdots & & & \vdots \\ 0 & z_1 - \bar{z}_n & & z_{n-1} - \bar{z}_n \\ z_1 - \bar{z}_n & z_2 - \bar{z}_n & & z_n - \bar{z}_n \\ z_2 - \bar{z}_n & \vdots & & 0 \\ \vdots & \vdots & & \vdots \\ & z_n - \bar{z}_n & & \vdots \\ z_n - \bar{z}_n & 0 & & 0 \end{bmatrix}$$

$$\hat{\hat{\gamma}}_k = \frac{1}{n-k} \sum_{t=1}^{n-k} (z_t - \bar{z}_n)(z_{t+k} - \bar{z}_n)$$

Assume $z_1 = 1$, $z_2 = 5$, $z_3 = 2$, $z_4 = 5$, $z_5 = 2$

$\Rightarrow \bar{z}_5 = 3$

$$\hat{\hat{\gamma}}_0 = \frac{4+4+1+4+1}{5} = \frac{14}{5}$$

$$\hat{\hat{\gamma}}_1 = \frac{(-2)(2) + 2\cdot(-1) + (2)(-1) + 2(-1)}{4} = -\frac{5}{2}$$

$$\hat{\hat{\gamma}}_2 = \frac{2(-1) + (-1)(2) + 2(-1)}{3} = -2$$

$$\hat{\hat{\gamma}}_3 = \frac{(-1)(2) + 2\times(-1)}{2} = -2$$

$$\hat{\hat{\gamma}}_4 = \frac{2\times(-1)}{1} = -2$$

$$\Rightarrow \hat{\hat{\gamma}}_k : \begin{pmatrix} \frac{14}{5} & -\frac{5}{2} & -2 & -2 & -2 \\ -\frac{5}{2} & \frac{14}{5} & -\frac{5}{2} & -2 & -2 \\ -2 & -\frac{5}{2} & \frac{14}{5} & -\frac{5}{2} & -2 \\ -2 & -2 & -\frac{5}{2} & \frac{14}{5} & -\frac{5}{2} \\ -2 & -2 & -2 & -\frac{5}{2} & \frac{14}{5} \end{pmatrix}$$

$\exists\ a \in \mathbb{R}^5$ $a : ($[illegible]$)$, s.t.

$a \hat{\hat{\gamma}}_k a' = ($[illegible]$)\begin{pmatrix} \text{[illegible]} \end{pmatrix} = \underline{\quad -1 \quad} < 0$

看不到

Hence $\hat{\hat{\gamma}}$ isn't positive semi-define #