Assignment 4

1. Let Z_t be a sequence of independent random variables defined as
   Z_t = +1 or -1 with equal probability of 1/2 if t is even, and
   Z_t = Z_{t-1} if t is odd.

   (a) Is the process first order stationary in distribution?
   (b) Is it second order stationary in distribution?

2. Let $Zt = U\, sin(2\pi\omega t) + V\, cos(2\pi\omega t)$, where U and V are independent random variables, each with mean 0 and variance 1.

   (a) Is Z_t strictly stationary?
   (b) Is Z_t covariance stationary?

3. Verify the following properties for the autocorrelation function of a stationary process:

   (a) $\rho_0 = 1$
   (b) $|\rho_k| \leq 1$
   (c) $\rho_k = \rho_{-k}$

4. Prove or disprove the following processes are covariance stationary:

   (a) $Z_t = Asin(2\pi\omega t + \theta)$ where A is a constant, and θ is a random variable which has a uniformly distribution on $[0, 2\pi]$.
   (b) $Z_t = Asin(2\pi\omega t + \theta)$ where A is a random variable with zero mean and unit variance, and θ is a constant.
   (c) $Z_t = (-1)^t A$ where A is a random variable with zero mean and unit variance.