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# Fast and accurate single image super-resolution via an energy-aware improved deep residual network



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#### ABSTRACT

Recently, convolutional neural network (CNN) based single image super-resolution (SISR) solutions have demonstrated significant progress on restoring accurate high-resolution image based on its corresponding low-resolution version. However, most state-of-the-art SISR approaches attempt to achieve higher accuracy by pursuing deeper or more complicated models, which adversely increases computational cost. To achieve a good balance between restoration accuracy and computational speed, we make simple but effective modifications to the structure of residual blocks and skip-connections between stacked layers, and then propose a novel energy-aware training loss to adaptively adjust the restoration of high-frequency and low-frequency image regions. Extensive qualitative and quantitative evaluation results on benchmark datasets verify the effectiveness of the proposed techniques that they significantly improve SISR accuracy while causing no/ignorable extra computational loads.

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#### 1. Introduction

Recently, single image super-resolution (SISR) technique, which aims to recover the high-resolution (HR) image from its corresponding low-resolution (LR) version, has attracted substantial attention from both the academic and industrial communities, facilitating a broad range of applications such as security surveillance, autonomous driving, and medical analysis [1]. Over the past decades, many machine learning algorithms have been developed for SISR, such as sparse coding [2,3], local linear regression [4,5] and random forest [6]. The underlying principle is that the HR image can be reconstructed by learning the nonlinear LR-to-HR mapping relationship via numerous training pairs.

Due to its powerful learning capacity, deep learning has become a prevalent tool for computer vision tasks [7–10]. Nowadays, the SISR problem has achieved significant progress by adopting deep convolutional neural networks (CNNs) architectures. SRCNN [11] proposed a three-layer CNN model to learn the nonlinear LR-to-HR mapping function. It is the first time that deep learning technique is applied to SISR problem. Although SRCNN

is a lightweight CNN model, it achieved significantly better image restoration performances than many previous machine-learning-based SR methods such as sparse coding (SC) [2,3], neighbor embedding (NE) [12] and anchored neighborhood regression (ANR) [4].

Following this pioneering work, many researchers attempted to achieve more accurate SISR results by either increasing the depth of the network or deploying more complex architectures. For instance, VDSR [13] is a 20-layer deep super-resolution convolutional network (VDSR), and more recent DRRN [14], SRDenseNet [15], and MemNet [16] SISR models contain 52, 68, and 80 layers, respectively. However, the above mentioned SISR methods typically contain a large number of network parameters and require heavy computational loads. Even with graphics processing unit (GPU) acceleration, their running time is still far from real-time, which adversely decrease their applicability for image pre-processing tasks. For instance, the running times of DRRN [14] and MemNet [16] processing a 640 × 480 image on a PC equipped with NVIDIA GTX 1080Ti GPU (11GB memory) are 10.9856 s and 15.0263 s, respectively. Recently, a number of fast-speed SISR methods [17,18] have been proposed to overcome the slow running time limit. However, performances of these shallow CNN models, evaluated using several widely-used metrics (peak signal-to-noise ratio - PSNR, structural similarity index - SSIM, and information fidelity criterion - IFC), are not comparable with the ones adopting deeper networks.

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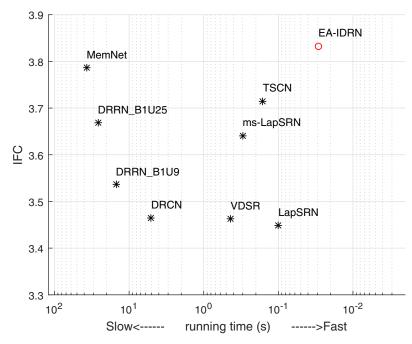


Fig. 1. Compared with the state-of-the-art SISR solutions [13,14,16,21,24–26], our proposed EA-IDRN model achieves the highest IFC value on Urban100 dataset with the scale factor × 4 and runs real-time. All SISR methods are performed on a PC equipped with NVIDIA GTX 1080Ti GPU (11GB memory), Cuda 8.0 and Cudnn 5.1.

To achieve fast and accurate SISR, we present a number of effective techniques which can improve SISR accuracy while causing no/ignorable extra computational loads. The first improvement is to optimize the network architecture. The recently introduced residual blocks [14,19] and skip-connections [15,20-22] are proven helpful techniques to facilitate better training of deep CNNs for higher-level computer vision problems such as image classification and detection. We investigate residual blocks and skip-connections with different structures in an attempt to identify the optimal way to build up a residual network for the low-level SISR task. The second improvement is the design of a novel energy-aware training loss. Although L2 loss is the most widely used one in the stateof-the-art SISR approaches, it correlates poorly with human perception of image quality [23] and suffers from losing image details. Motivated by the fact that high-frequency edges or textures tend to disappear during image downsampling, we propose a novel energy-aware training loss to adaptively adjust the restoration of image regions with different characteristics.

Based on the above improvements (**a.** an optimized residual network architecture and **b.** an energy-aware training loss), we present a compact but powerful Energy-Aware Improved Deep Residual Network (EA-IDRN) SISR model which is very accurate and runs real-time. Our proposed 23-layer EA-IDRN model outperforms other state-of-the-art SISR methods, even including the 52-layer DRRN [14] and 80-layer MemNet [16] on Urban100 dataset with the scale factor  $\times 4$  and has a real-time running speed ( $\times 30$  fps), as illustrated in Fig. 1. Overall, the contributions of this paper are mainly summarized as follows:

- We investigate a number of design options for building up deep residual networks and make modifications to the structure of residual blocks for feature extraction and skipconnections between stacked residual blocks. Experimental results show these simple but effective modifications increase SISR performance substantially.
- We propose a novel loss function, incorporating the pixelwise gradient responses with the  $L_1$  loss, to achieve more accurate SISR results. The energy-aware loss adaptively assigns high-valued back-propagated gradients for

- textured/edge image regions, thus can better recover highfrequency image details which tend to lose in low-resolution images.
- Based on the above improvements, we present an Energy-Aware Improved Deep Residual Network (EA-IDRN) SISR model, which is trained end-to-end, for fast and accurate restoration of low-resolution images. EA-IDRN achieves higher accuracy and significantly faster running time compared with state-of-the-art deep-learning-based SISR approaches [11,13,14,16,21,24-27].

The remainder of this paper is organized as follows. We first review a number of learning-based SISR methods and different choices of loss functions in Section 2. Then Section 3 provides details of important components in our proposed Energy-Aware Improved Deep Residual Network (EA-IDRN). Qualitative and quantitative evaluation results are provided in Section 4 to show the effectiveness of our method. Finally, Section 5 concludes this paper.

## 2. Related work

Over the past decades, substantial approaches [2,4,5,28–35] have been proposed to solve the single image super-resolution (SISR) problem. Although the interpolation-based [28,29,36,37] and reconstruction-based methods [38,39] are extremely simple and fast, they cannot achieve satisfactory restoration results (difficult to recover high-frequency signals such as edges or textures). In this paper, we focus on learning-based SISR methods which aim to infer the complex LR-to-HR mapping function based on a large number of training samples.

#### 2.1. Machine-learning-based approaches

Freeman et al. firstly utilized learning-based methods for low-level SISR problems [40,41]. However, the solution space is too vast to explore, therefore many subsequent methods embedded some prior information to improve the accuracy and speed of SISR. Sparse coding super-resolution (SCSR) methods proposed by Yang et al. [2,3] assumed that LR and HR images share the same sparse

representation. In [12,42], neighbor embedding (NE) algorithms generated super-resolution results based on the assumption that low-dimensional non-linear manifolds in LR and HR feature space have a similar local geometry. To mitigate the computational load, Timofte et al. [4,5] proposed to apply a number of linear regressors to anchor the neighbors locally. Timofte et al. also presented seven ways to improve the super-resolution performance [32]. In addition, several methods [43,44] exploited the self-similarity property in natural images and constructed the internal training pairs (without external datasets) based on the scale-space pyramid.

#### 2.2. Deep-learning-based approaches

Recently, deep learning techniques have been employed to achieve breakthrough results of SISR. Dong et al. [11,27] proposed the first deep learning based SISR method. The Super-Resolution Convolutional Neural Network (SRCNN) is a light-weight model (three layers) but significantly surpasses the state-of-theart methods [2,4] at that time. Following this pioneering work, Kim et al. presented deeper networks (VDSR [13] and DRCN [21]) to achieve better generalization capacity for more accuracy image restoration. However, the gradient vanishing problem caused by deeper networks will make the training process unstable. Global residual learning [13] and recursive layers [21] are employed to ease the problem. To achieve higher reconstruction accuracy, Tai et al. developed the 52-layer DRRN model [14] which utilizes local and global residual learning and recursive layers and the 80layer MemNet model [16] which contains persistent memory units and multiple supervisions. It is experimentally shown that training/deploying a deconvolution layer [17] (or a sub-pixel layer [18]) to directly construct the HR images at the end of the network provides a feasible way to reduce the computational cost and achieve higher restoration accuracy. By adopting this post-upsampling strategy, improved SISR results are achieved in EDSR [45] (using enhanced residual blocks) [19], SRDenseNet [15](using dense blocks) [46], and RDN [47] (using residual dense blocks). More recently, Zhang et al. [48] presented a very deep residual channel attention networks model (RCAN), which reaches the state-of-theart SISR performance. It is noted that EDSR, SRDenseNet, RDN and RCAN are trained using the high-resolution DIV2K [49,50] dataset (containing 800 training images of 2K resolution) or ImageNet [51] subset. Their training processes take a long time to complete as well as the predicting processes.

It is noted that most previous researchers attempted to achieve more accurate SISR results by either increasing depth of the network or deploying more complex architectures. However, these very deep CNN models contain a large number of parameters and cannot deliver real-time speed. It is critically important to increase the efficiency of SISR models to facilitate image pre-processing tasks. Hui et al. [25] proposed a compact two-stage convolutional network (TSCN) to achieve fast inference time and state-of-theart SR results on four benchmark datasets simultaneously. Lai et al. presented LapSRN and ms-LapSRN models [24,26] in which progressive reconstruction strategy is employed to improve both restoration accuracy and testing speed. Towards fast and accurate SISR, we present a novel Energy-Aware Improved Deep Residual Network (EA-IDRN) SISR model, which performs favorably compared to state-of-the-art SISR methods in terms of both accuracy and efficiency.

# 2.3. Loss function

The loss function provides critical information to guide the tuning of weights and biases of DNN models. Despite its importance, the design of a loss function suitable for the changeling SISR task

has not received too much attention yet. By far, mean square error (MSE) loss or  $L_2$  loss is the most widely used loss function for SISR [11,13,14,16,17,21,27]. The reason behind its popularity is that its calculation is similar to the one used in PSNR which is a major SISR evaluation indicator. However, SISR models based on  $L_2$  loss function are difficult to handle the uncertainty inherent in recovering lost high-frequency details such as textures or edges and tend to generate over-smoothed outputs [52,53]. Recently, Lim et al. [45] experimentally reported that  $L_1$  loss is a better option than  $L_2$  loss to achieve improved SISR performance. Furthermore, Lai et al. [24] proposed a robust Charbonnier loss function which essentially is a differentiable variant of L1. Researchers from Twitter presented a perceptual loss function which contains an adversarial loss and a content loss. However, the artificially generated high-frequency details may be "fake" texture patterns, which are not suitable for some applications demands accurate restoration. In this paper, we present a novel energy-aware training loss to adaptively adjust the restoration of image regions with different characteristics and achieve high-fidelity SISR results.

# 3. Our approach

In this section, we present an end-to-end Energy-Aware Improved Deep Residual Network (EA-IDRN) model for fast and accurate SISR. The architecture of proposed EA-IDRN model is illustrated in Fig. 2. We first present a baseline deep residual network which consists of many stacked residual blocks. Then we make improvements to this baseline architecture by modifying the core residual blocks and the skip-connections between them. Finally, we propose a novel energy-aware loss function to achieve high-quality image restoration.

#### 3.1. Baseline architecture

Recently, deep residual networks (RseNet) have been successfully employed to solve the ill-posed SISR problem [14,20,45,54]. In this paper, we investigate different design options in an attempt to construct better ResNet structures. For this purpose, we firstly present a baseline architecture of RseNet and then make modifications to it. As illustrated in Fig. 3(a), two convolutional layers are embedded to extract the low-level features  $F_{-1}$  and  $F_0$  on the original LR input  $I_{LR}$  as

$$F_{-1} = w_{-1} * I_{LR} + b_{-1}, (1)$$

$$F_0 = w_0 * F_{-1} + b_0, (2)$$

where  $w_{-1:0}$  and  $b_{-1:0}$  represent the filtering weights and biases of the first and second convolutional layers, \* denotes the convolutional operation.  $F_0$  is then fed into a number of stacked residual blocks to extract high-level features for HR image reconstruction. Without loss of generality, we employ the residual block used in EDSR [45]. After adding N residual blocks, our baseline model also embeds a deconvolution layer at the end of the network to reconstruct the final HR output  $I_{SR}$  as

$$I_{SR} = Deconv(F_N \uparrow s), \tag{3}$$

where  $F_N$  is the output of the  $N_{th}$  residual block, and  $Deconv(\cdot)$  represents the deconvolution operation which generates the input signal by a sum over convolutions of the feature maps (as opposed to the input) with learned filters [55].  $\uparrow$  denotes the up-sampling operation, and s denotes the up-sampling factor. The advantage of deploying a deconvolution layer is two-fold. First, it avoids artifacts induced by hand-crafted image pre-processing techniques (e.g., bicubic interpolation). Moreover, it accelerates SISR reconstruction process by conducting computationally expensive convolutional operations on LR images [17,24].

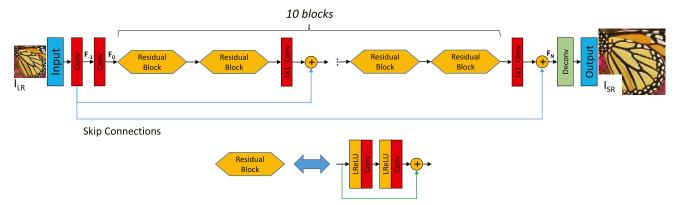
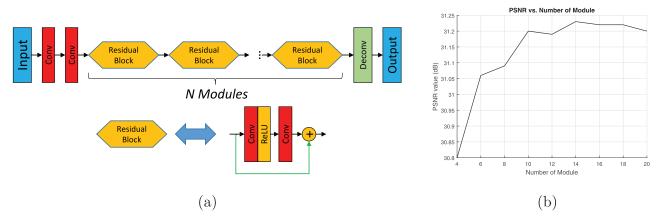


Fig. 2. The architecture of our proposed EA-IDRN model. A number of enhanced residual blocks are stacked, and skip connections and  $1 \times 1$  convolutional layers are added between them to boost the overall SISR performance. The final output is directly reconstructed from low-resolution features by a deconvolution layer.



**Fig. 3.** (a) The architecture of the baseline deep residual network in which *N* residual blocks [45] are stacked. (b) The curve of PSNR vs. Number of residual blocks. All of the quantitative results are obtained on the Urban100 dataset with the scale factor  $\times$  2.

The performance of the baseline models containing different number of residual blocks N are evaluated on the Urban100 dataset with the scale factor  $\times 2$  and the comparative results (PSNR vs. Number of residual blocks) are shown in Fig. 3(b). It is noted that using more residual blocks can generally achieve a higher PSNR value when the number of residual blocks N is less than 10. However, the performance is marginally improved or even drops when N is larger than 10. The underlying reason is that stacking too many layers/blocks will incur the gradient vanishing/exploring problem and cause network training difficulty. Moreover, adding more residual blocks significantly increases model parameters, and thus decreases the running-time. In our baseline, we set N = 10 to achieve a good balance between model complexity and good performance, and then present a number of effective techniques to improve accuracy of SISR while incurring negligible computational overhead.

#### 3.2. Network optimization

Residual blocks [14,19] and skip-connections [21,56] are two important building blocks of deep residual networks. In this section, we investigate different design options of residual blocks and skip-connections for fast and accurate SISR.

**Residual block:** Fig. 4 shows residual blocks with different structure designs including (a) the one used in SRResNet [20], (b) the one used in EDSR [45], (c) our modified version A, and (d) our modified version B. Following the research work of Lim et al. [45], we remove the batch normalization (BN) layers in our modified versions. The advantages are two-fold. First, it can reduce the

GPU memory usage, therefore save the running time. Second, it avoids the feature normalization operation and increases the range flexibility of network. It is noticed that both SRResNet and EDSR adopt the ReLU activation function in their residual blocks. If the activation value of a certain neuron is below zero (dead neuron [57]), it will not be activated using the ReLU function and the derivatives for all preceding neurons linked to it will become zeros according to the chain rule of derivation. Too many inactive neuron chains will impede the back-propagation of derivatives and reduce the learning ability of network. In our modified version A, we replace the ReLU function (i.e., f(x) = max(0, x)) with the Leaky ReLU (i.e., f(x) = max(0.2x, x)) which assigns a nonzero slope for zero or negative activation signals. This modification ensures that there is always a non-zero gradient flowing backwards during the training process. In our modified version B, we add an extra Leaky ReLU layer before convolution operation to embed more nonlinear terms into network following the research work presented by He et al. [56]. Since the Leaky ReLU layer does not involve additional parameters and its mathematical calculation is simple, only a small amount of computational cost is added in our modified version B. The above mentioned four residual blocks have different structure designs and will lead to different SISR performances. Their comparative evaluation is provided in Section 4.3.

**Skip-connections:** Adding skip connections between multiple-stacked layers enables gradient signal to back-propagate directly from the higher-level features to lower-level ones, alleviating the gradient vanishing/exploring problem [22]. In our implementation, element-wise addition and channel-wise concatenation achieve very similar SISR performances, while the concatenation fusion

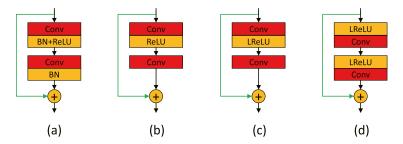
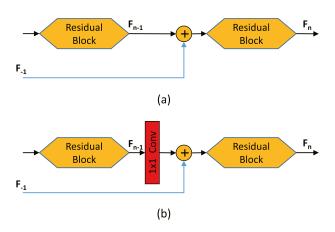


Fig. 4. Residual blocks with different structure designs. (a) Residual block used in SRResNet [20], (b) residual block used in EDSR [45] (removing the BN operations), (c) modified residual block version A (replacing the ReLU activation layer with Leaky ReLU), and (d) modified residual block version B (adding an extra Leaky ReLU layer before convolutional layers). The performance of these four different residual blocks are systematically evaluated in Section 4.3.



**Fig. 5.** Various types of skip connections. (a) Skip connections are directly added between different layers [15,20-22], (b) Skip connections and  $1 \times 1$  convolutional layers are added between different layers. The performance of these two different designs is systematically evaluated in Section 4.3.

will increase the channel number of features, adding extra computational time for subsequent convolutional operations. Thus, we adopt the addition function for feature fusion. In Fig. 5, we consider two different design options. The first one directly adds skip connections between different layers, which is adopted in most CNN-based SISR approaches [15,20–22]. In this case, the output of the  $n_{th}$  residual block  $F_n$  is calculated as

$$F_n = Res(F_{-1} + F_{n-1}), (4)$$

where  $Res(\cdot)$  denotes the residual block. The second option utilizes a  $1 \times 1$  convolutional layer and skip connections to connect different layers and the output of the  $n_{th}$  residual block is

$$F_n = Res(F_{-1} + w_n * F_{n-1} + b_n), \tag{5}$$

where  $w_n$  and  $b_n$  represent the filtering weights and biases of the nth  $1 \times 1$  convolutional layer respectively. We will experimentally evaluate these two different designs and discuss the best way to deploy skip connections to achieve good accuracy and fast speed in Section 4.3.

## 3.3. Loss function

The loss function calculates the pixel-wise difference between the ground truth HR image and the restored SISR output, which is critical to update the weights and biases of DNN models. Despite its importance, the design of a loss function suitable for the changeling SISR task has not attracted much research attention. Most deep learning based SISR methods [11,13–17,21,27] adopt  $L_2$  loss (i.e., mean square error loss or Euclidean loss) as the training

loss. The formulation of  $L_2$  loss and the back-propagated derivative for a pixel p are formulated as

$$\mathcal{L}_{L_2}(P) = \frac{1}{2} \sum_{p \in P} ||I^{SR}(p) - I^{GT}(p)||_2^2, \tag{6}$$

$$\partial \mathcal{L}_{L_2}(P)/\partial I^{SR}(p) = I^{SR}(p) - I^{GT}(p), \tag{7}$$

where p indicates the index of a pixel in the image patch P,  $I^{GT}$  and  $I^{SR}$  represent the ground truth and restored images, and  $||\cdot||_2$  denotes the  $L_2$  norm. Note that, although  $\mathcal{L}_{L_2}(P)$  is a function of the patch as a whole, the derivatives are back-propagated for each pixel in this patch. However, as pointed out by Zhao et al. in [58], the widely used  $L_2$  loss function suffers from some well-known limitations including  $L_2$  correlates poorly with human perception [23] and  $L_2$  loss is based on the assumption of a Gaussian noise model which is not valid in general.

More recently, some researchers experimentally discover that CNN models trained with  $L_1$  loss can achieve higher restoration accuracy [45,47,58]. The  $L_1$  loss function and its derivative for a pixel p are formulated as

$$\mathcal{L}_{L_1}(P) = \sum_{p \in P} ||I^{SR}(p) - I^{GT}(p)||_1, \tag{8}$$

$$\partial \mathcal{L}_{L_1}(P)/\partial I^{SR}(p) = sign[I^{SR}(p) - I^{GT}(p)], \tag{9}$$

where  $||\cdot||_1$  denotes the  $L_1$  norm. However, the back-propagated derivative of  $L_1$  loss is fixed to 1 or -1 for image regions with different characteristics (e.g., textured or smooth) which is suboptimal. Motivated by the fact that high-frequency edges or textures tend to disappear during image degradation while low-frequency smooth image regions remain almost unchanged, we propose a novel gradient energy-aware (EA) training loss to adaptively adjust the strategy how high-frequency and low-frequency image regions are restored. We formulate the EA loss as the weighted sum of a  $L_1$  loss and a gradient energy component as

$$\mathcal{L}_{EA}(P) = \alpha \sum_{p \in P} ||I^{SR}(p) - I^{GT}(p)||_{1} + \beta \sum_{p \in P} E(p)||I^{SR}(p) - I^{GT}(p)||_{1}$$

$$= \sum_{p \in P} [\alpha + \beta E(p)]||I^{SR}(p) - I^{GT}(p)||_{1}, \qquad (10)$$

$$\partial \mathcal{L}_{FA}(P)/\partial I^{SR}(p) = [\alpha + \beta E(p)] \cdot sign[I^{SR}(p) - I^{GT}(p)], \tag{11}$$

where  $\alpha$  and  $\beta$  denote the weights for  $L_1$  loss and the energy-aware item, respectively. In our implementation, we simply set  $\alpha=\beta=0.5$ . E(p) represents the pixel-wise gradient energy which is calculated as

$$E(p) = \frac{1}{2}(|G_x(p)| + |G_y(p)|), \tag{12}$$

where  $G_X(p)$  and  $G_Y(p)$  denote the horizontal and vertical gradient values of pixel p which are calculated using two  $3 \times 3$  Sobel kernels. It is noted that he energy-aware loss  $\mathcal{L}_{EA}$  adaptively assigns

**Table 1** The settings of deconvolution layers with the scale factors  $\times 2$ ,  $\times 3$  and  $\times 4$ .

scales	× 2	× 3	× 4
Kernel size	4 × 4	5 × 5	6 × 6
padding	1	1	1

high-valued back-propagated gradients for high-frequency image regions, emphasizing the recovery of edges and textures which are mostly lost in low-resolution images. Comparative results of different lose functions are provided in Section 4.3.

#### 4. Experimental results

#### 4.1. Dataset and evaluation metric

**Training datasets:** Following [13,21], we train our networks on RGB91 dataset from Yang et al. [3] and another 200 images from Berkeley Segmentation Dataset (BSD) [59]. To expand our training dataset, three data augmentation techniques are utilized including: 1. Rotation: rotate image by  $90^\circ$ ,  $180^\circ$  and  $270^\circ$ . 2. Flipping: horizontally flip image. 3. Scaling: downscale image with the scale factors of 0.9, 0.8, 0.7, 0.6 and 0.5. After data augmentation, we randomly crop these images into  $48 \times 48$  sub-images to generate our HR images. The LR images are obtained by down-sampling corresponding HR images through bicubic interpolation.

**Testing datasets:** Five commonly used public benchmark datasets are utilized for evaluating the performance of our EA-IDRN method. Set5 [42] and Set14 [60] datasets are widely used in SISR, B100 [59] contains 100 natural images collected from BSD, and Urban100 [43] consists of 100 real-world urban scene images which are rich of structures. In addition, Manga109 [61], a collection of 109 Japanese comic images, is also employed.

**Evaluation metrics:** We adopt peak signal-to-noise-ratio (PSNR) and structural similarity index (SSIM) [62] to evaluate the SISR performance. In addition, information fidelity criterion (IFC) metric, which correlates well with human perception [63], is also utilized to assesses the image quality. Since training is performed on the luminance channel [5,27] (Y channel of YCbCr color space), all three metrics are calculated on the Y channel accordingly. For fair comparison, we crop off boundary image pixels according to [27].

# 4.2. Implementation details

Our EA-IDRN model consists of N=10 residual blocks. All convolutional layers have 64 channels of features and the kernel sizes are fixed to  $3\times3$  except the  $1\times1$  convolutional layers before skip connections. Zero-padding is employed to avoid the sizes of feature maps from shirking. In addition, the settings of deconvolution layers for scale factors  $\times 2$ ,  $\times 3$  and  $\times 4$  are summarized in Table 1.

We implement our EA-IDRN with Caffe [64] framework and train this model on NVIDIA GTX 1080Ti with Cuda 8.0 and Cudnn 5.1 for 50 epochs (when fine-tunning the  $\times$ 3 and  $\times$ 4 models,

10 epochs are enough). Adam [65] solver is utilized to optimize the weights by setting  $\beta_1=0.9$ ,  $\beta_2=0.999$  and  $\epsilon=1e-8$ . The batch size is set to 64 and the learning rate is fixed to 1e-4. For weights initialization, we adopt the method described in [66] and the biases are initialized to zeros. It takes roughly 16 hours to train our proposed EA-IDRN for scale factor  $\times$  2. We train our EA-IDRN for scale factor  $\times$  3 and  $\times$  4 by initializing the weights with pretrained  $\times$  2 model. This fine-tuning strategy accelerates the training process and enhances the performance [45]. The source code will be made publicly available in the future.

#### 4.3. Performance analysis

In this section, we set up ablation experiments to evaluate the performance (accuracy and running time) of different designs of (1) residual blocks, (2) skip connections, and (3) loss function. According to Section 3.1, our baseline model contains 10 stacked residual blocks to achieve a good balance between model complexity and good performance.

Residual blocks: We evaluate four residual blocks with different designs as illustrated in Fig. 4. For fair comparison, these four different residual blocks are evaluated based on the same baseline model trained using  $L_1$  loss without skip connections and the comparative results (PSNR, SSIM, IFC, and running time) are illustrated in Table 2. We made three important observations. First of all, it is demonstrated that removing BN layers in the residual block to increase the range flexibility of network is an effective technique to achieve higher SISR accuracy with faster running time. For instance, the PSNR index increases from 31.11 dB to 31.20 dB while the running time reduces from 0.098s to 0.063s. Our experimental results are consistent with previous research of Lim et al. [45]. Second, using the Leak ReLU layer to replace the original ReLU in a residual block is another effective technique to improve SISR accuracy without adding extra computational costs. In our experiments, the PSNR index increases from 31.20 to 31.25 while the running time remains at 0.063 s. The underlying reason is that the Leak ReLU function will assign a nonzero slope for negative signals to avoid dead neuron and ensure a non-zero gradient always flowing backwards during the training process [57]. Finally, adding a Leaky ReLU layer before convolution operation can embed more nonlinear terms into network and lead to further improvement of SISR accuracy. The PSNR index increases from 31.25 dB to 31.28 dB. Since the mathematical calculation of Leaky ReLU function is simple, only a small amount of computational overhead is added in our modified pre-activation residual module. Our experiments show that adding 10 Leaky ReLU layers only increase the running time by 0.002 s.

**Skip-connections:** We evaluate two different options to incorporate skip connections into deep residual networks as illustrated in Fig. 5. The experiments are conducted based on the baseline model which employs the best performing modified residual block B and is trained using  $L_1$  loss. Table 3. illustrates the experimental results (PSNR, SSIM, IFC, and running time) on Urban100 dataset with the scale factor  $\times$  2. Although directly adding skip connections between layers at different depths is a proven effective

**Table 2** We calculate the PSNR SSIM IFC values and running times of the baseline model using residual blocks (RB) with different designs on Urban100 dataset with the scale factor  $\times$  2.

	RB in SRResNet [20]	RB in EDSR [45]	Modification A	Modification B
PSNR (dB)	31.11	31.20	31.25	31.28
SSIM	0.9180	0.9192	0.9196	0.9200
IFC	9.3893	9.4065	9.4384	9.5303
Time (s)	0.098	0.063	0.063	0.065

**Table 3**We calculate the PSNR SSIM IFC values and running times of the baseline model using skip connection (SC) with different designs on Urban100 dataset with the scale factor  $\times 2$ .

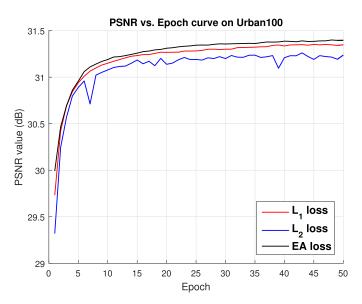
	No SC	Direct SC 10 times	1 × 1 Conv. SC 10 times	1 × 1 Conv. No SC 10 times	1 × 1 Conv. SC 5 times
PSNR (dB)	31.28	31.29	31.36	31.24	31.34
SSIM IFC	0.9200 9.5303	0.9202 9.5153	0.9206 9.5638	0.9190 9.5037	0.9205 9.5622
Time (s)	0.065	0.066	0.078	0.077	0.071

**Table 4** We calculate the PSNR SSIM IFC values and running times of the baseline model using different loss functions on Urban100 dataset with the scale factor  $\times 2$ .

Loss function	L <sub>2</sub> loss	$L_1$ loss	EA loss
PSNR (dB)	31.24	31.34	31.39
SSIM	0.9194	0.9205	0.9209
IFC	9.4695	9.5622	9.7056
Time (s)	0.071	0.071	0.071

technique in many higher-level computer vision problems such as image classification and detection [56], our experimental results show that it only achieves marginal improvement for the low-level SISR task. The PSNR index slightly increases from 31.28 dB to 31.29 dB while the IFC index even drops from 9.5303 to 9.5153. In comparison, a noticeable improvement of restoration accuracy is achieved by adding a  $1 \times 1$  convolutional layer before the skip connection. The feature maps extracted in a residual block is adaptively adjusted through a  $1 \times 1$  convolutional layer before combined with output of a shallower layer, facilitating the training of more distinct features for high-quality image restoration. As the result, the PSNR index increases from 31.28 dB to 31.36 dB, SSIM index increases from 0.9200 to 0.9206, and IFC index increases from 9.5303 to 9.5638. To verify the improvement is not caused by the increase of the number of network layers (adding 10  $1 \times 1$ convolutional layers), we also employ  $1 \times 1$  convolutional layers without adding skip connections ( $1 \times 1$  Conv. No SC 10 times). As shown in Table 3, it is observed that stacking more layers into the network without adding skip connections will even lead to the decrease of SISR accuracy (PSNR decreases from 31.28 dB to 31.24 dB, SSIM decreases from 0.9200 to 0.9190, and IFC decreases from 9.5303 to 9.5037). The underlying reason for this degradation is the gradient vanishing problem triggered by deeper layers. Our experiments show that skip connections provide a useful technique to alleviate the gradient vanishing problem [15] and adding a  $1 \times 1$  convolutional layer before each skip connection achieves the optimal SISR performance. Furthermore, we investigate different ways to deploy skip connections. Instead of densely connect the first convolutional layer with every single residual block (10 times), we deploy the  $1 \times 1$  convolution and skip connection every two residual blocks (5 times). It is noted that the SISR accuracy marginally drops (e.g., PSNR index decreases from 31.36 dB to 31.34 dB, SSIM index decreases from 0.9206 to 0.9205, and IFC index decreases from 9.5638 to 9.5622) while the computational time substantially improves from 0.078 s to 0.071 s. Therefore, we implement  $1 \times 1$  convolution and skip connection every two residual blocks to achieve a good balance between restoration accuracy and computational speed.

**Loss function:** We evaluate three different loss functions ( $L_1$  loss,  $L_2$  loss, and our proposed EA loss) using the baseline model which employs the modified residual block B as the basic module and contains 5 1 × 1 Conv. skip connections. Table 4 lists the PSNR, SSIM and IFC indexes and running times of three different loss functions on Urban100 dataset with the scale factor × 2. We notice that training with our proposed EA loss reaches the highest PSNR



**Fig. 6.** The PSNR vs. Epoch curves driven by three different loss functions: L1, L2 and EA. The experiments are conducted on Urban100 dataset with the scale factor  $\times$  2.

value of 31.39 dB, which surpasses the  $L_1$  loss by 0.05 dB and the  $L_2$  loss by 0.15 dB. Although  $L_2$  loss is the most widely used one, it tends to over-smooth the image details and causes perceptually poor quality. Our experimental results show that  $L_1$  loss is a better option of loss function for image restoration tasks, which are consistent with previous research works [45,47,48,67]. However,  $L_1$  loss function cannot differentiate image regions with different characteristics (e.g., high-frequency edge region and low-frequency smooth region) and restore them equally. In comparison, our proposed gradient EA training loss allows our model to adaptively adjust how high-frequency and low-frequency image regions are restored. Gradient energy information is utilized to emphasize the information recovery for regions with textures or edges to achieve higher SISR accuracy. It is worth mentioning that EA loss does not incur any extra computational overhead during the testing phase. We also visualize the convergence process of these three different training losses in Fig. 6. It is observed that our proposed EA loss achieves higher PSNR values while using less training epochs. Fig. 7 visualize the computed gradient energy and the comparative SISR results using  $L_1$  loss and our proposed EA loss. It is observed that the proposed energy-aware loss adaptively assigns high-valued back-propagated gradients for high-frequency image regions, leading to better SISR restoration results particularly for regions with textures or edges.

# 4.4. Comparisons with state-of-the-arts

In this section, we compare our proposed EA-IDRN with a number of state-of-the-art SISR methods qualitatively and quantitatively. Two machine-learning-based approaches (Aplus [5] and

Table 5 Benchmark results of several state-of-the-art SISR methods. We compare the average PSNR(dB)/SSIM values with the scale factors  $\times 2$ ,  $\times 3$  and  $\times 4$  on Set5, Set14, B100, Urban100 and Manga109. **Red** and **blue** indicate the best and the second best performance, respectively. It is noted that the metrics are calculated on Y channel

App Sel SRI	cubic olus [5] IfExSR [43] CNN [27] DSR [13] RCN [21] pSRN [24] RRN [14] emNet [16] CN [25] s-LapSRN [26] L-IDRN (ours) cubic olus [5] IfExSR [43]	33.66 / 0.9299 36.54 / 0.9544 36.50 / 0.9536 36.66 / 0.9542 37.53 / 0.9587 37.63 / 0.9588 37.44 / 0.9581 37.74 / 0.9591 37.78 / 0.9597 37.88 / 0.9602 37.70 / 0.9590 37.91 / 0.9603 30.39 / 0.8682 32.58 / 0.9088	30.24 / 0.8688 32.28 / 0.9056 32.22 / 0.9034 32.45 / 0.9067 33.03 / 0.9124 33.04 / 0.9118 32.96 / 0.9117 33.23 / 0.9136 33.28 / 0.9142 33.28 / 0.9147 33.25 / 0.9138 33.32 / 0.9154 27.55 / 0.7742	29.56   0.8431 31.21   0.8863 31.17   0.8853 31.36   0.8879 31.90   0.8960 31.85   0.8942 31.78   0.8941 32.05   0.8978 32.08   0.8978 32.09   0.8985 32.02   0.8970 32.12   0.8990	26.88 / 0.8403 29.20 / 0.8938 29.52 / 0.8965 29.51 / 0.8946 30.76 / 0.9140 30.75 / 0.9133 30.39 / 0.9093 31.23 / 0.9188 31.31 / 0.9195 31.29 / 0.9198 31.13 / 0.9180	30.81 / 0.9341 35.37 / 0.9680 35.12 / 0.9660 35.60 / 0.9663 37.15 / 0.9738 37.63 / 0.9740 37.21 / 0.9731 38.03 / 0.9753 38.07 / 0.9755 37.71 / 0.9747
Sel SRI VD DR Lap DR Me TSO ms EA 3 Bio Ap Sel SRI	IfExSR [43] CCNN [27] DSR [13] RCN [21] PSRN [24] RRN [14] emNet [16] CN [25] S-LapSRN [26] L-IDRN (ours) cubic olus [5] IfExSR [43]	36.50 / 0.9536 36.66 / 0.9542 37.53 / 0.9587 37.63 / 0.9588 37.44 / 0.9581 37.74 / 0.9591 37.78 / 0.9597 37.88 / 0.9602 37.70 / 0.9590 37.91 / 0.9603 30.39 / 0.8682	32.22 / 0.9034 32.45 / 0.9067 33.03 / 0.9124 33.04 / 0.9118 32.96 / 0.9117 33.23 / 0.9136 33.28 / 0.9142 33.28 / 0.9147 33.25 / 0.9138 33.32 / 0.9154	31.17 / 0.8853 31.36 / 0.8879 31.90 / 0.8960 31.85 / 0.8942 31.78 / 0.8941 32.05 / 0.8978 32.08 / 0.8978 32.09 / 0.8985 32.02 / 0.8970 32.12 / 0.8990	29.52 / 0.8965 29.51 / 0.8946 30.76 / 0.9140 30.75 / 0.9133 30.39 / 0.9093 31.23 / 0.9188 31.31 / 0.9195 31.29 / <b>0.9198</b> 31.13 / 0.9180	35.12 / 0.9660 35.60 / 0.9663 37.15 / 0.9738 37.63 / 0.9740 37.21 / 0.9731 37.88 / 0.9749 38.03 / <b>0.975</b> 2 <b>38.07</b> / 0.9750
SRI VD DR Laj DR M6 TS0 ms EA 3 Bic Ap Sel	CNN [27]  DSR [13]  CN [21]  pSRN [24]  RRN [14]  emNet [16]  CN [25]  s-LapSRN [26]  a-IDRN (ours)  cubic  cubic  lfExSR [43]	36.66 / 0.9542 37.53 / 0.9587 37.63 / 0.9588 37.44 / 0.9581 37.74 / 0.9591 37.78 / 0.9597 37.88 / 0.9602 37.70 / 0.9590 37.91 / 0.9603 30.39 / 0.8682	32.45 / 0.9067 33.03 / 0.9124 33.04 / 0.9118 32.96 / 0.9117 33.23 / 0.9136 33.28 / 0.9142 33.28 / 0.9147 33.25 / 0.9138 33.32 / 0.9154	31.36 / 0.8879 31.90 / 0.8960 31.85 / 0.8942 31.78 / 0.8941 32.05 / 0.8973 32.08 / 0.8978 <b>32.09 / 0.8985</b> 32.02 / 0.8970 <b>32.12 / 0.8990</b>	29.51 / 0.8946 30.76 / 0.9140 30.75 / 0.9133 30.39 / 0.9093 31.23 / 0.9188 <b>31.31</b> / 0.9195 31.29 / <b>0.9198</b> 31.13 / 0.9180	35.60 / 0.9663 37.15 / 0.9738 37.63 / 0.9740 37.21 / 0.9731 37.88 / 0.9749 38.03 / <b>0.975</b> 3 <b>38.07</b> / 0.9750
VD DR Laj DR Me TSO ms EA 3 Bio Ap Sel	OSR [13] RCN [21] pSRN [24] RRN [14] emNet [16] CN [25] s-LapSRN [26] L-IDRN (ours) cubic olus [5] IfExSR [43]	37.53 / 0.9587 37.63 / 0.9588 37.44 / 0.9581 37.74 / 0.9591 37.78 / 0.9597 <b>37.88</b> / <b>0.9602</b> 37.70 / 0.9590 <b>37.91</b> / <b>0.9603</b> 30.39 / 0.8682	33.03 / 0.9124 33.04 / 0.9118 32.96 / 0.9117 33.23 / 0.9136 33.28 / 0.9147 33.28 / 0.9147 33.25 / 0.9138 33.32 /0.9154	31.90 / 0.8960 31.85 / 0.8942 31.78 / 0.8941 32.05 / 0.8973 32.08 / 0.8978 32.09 / 0.8985 32.02 / 0.8970 32.12 /0.8990	30.76 / 0.9140 30.75 / 0.9133 30.39 / 0.9093 31.23 / 0.9188 <b>31.31</b> / 0.9195 31.29 / <b>0.9198</b> 31.13 / 0.9180	37.15   0.9738 37.63   0.9740 37.21   0.9731 37.88   0.9749 38.03   <b>0.975</b> 5 <b>38.07</b>   0.9750
DR Laj DR Me TS0 ms EA 3 Bio Ap Sel	RCN [21] pSRN [24] RRN [14] emNet [16] CN [25] s-LapSRN [26] I-IDRN (ours) cubic olus [5] IfExSR [43]	37.63 / 0.9588 37.44 / 0.9581 37.74 / 0.9591 37.78 / 0.9597 <b>37.88</b> / <b>0.9602</b> 37.70 / 0.9590 <b>37.91</b> / <b>0.9603</b> 30.39 / 0.8682	33.04 / 0.9118 32.96 / 0.9117 33.23 / 0.9136 33.28 / 0.9142 33.28 / 0.9147 33.25 / 0.9138 33.32 /0.9154	31.85 / 0.8942 31.78 / 0.8941 32.05 / 0.8973 32.08 / 0.8978 <b>32.09</b> / <b>0.8985</b> 32.02 / 0.8970 <b>32.12</b> / <b>0.8990</b>	30.75 / 0.9133 30.39 / 0.9093 31.23 / 0.9188 <b>31.31</b> / 0.9195 31.29 / <b>0.9198</b> 31.13 / 0.9180	37.63 / 0.9740 37.21 / 0.9731 37.88 / 0.9749 38.03 / <b>0.975</b> 5 <b>38.07</b> / 0.9750
Laj DR Me TSO ms EA 3 Bio Ap Sel	pSRN [24] RRN [14] emNet [16] CN [25] s-LapSRN [26] I-IDRN (ours) cubic blus [5] IfExSR [43]	37.44 / 0.9581 37.74 / 0.9591 37.78 / 0.9597 <b>37.88 / 0.9602</b> 37.70 / 0.9590 <b>37.91 / 0.9603</b> 30.39 / 0.8682	32.96 / 0.9117 33.23 / 0.9136 33.28 / 0.9142 33.28 / 0.9147 33.25 / 0.9138 <b>33.32</b> / <b>0.9154</b>	31.78 / 0.8941 32.05 / 0.8973 32.08 / 0.8978 <b>32.09</b> / <b>0.8985</b> 32.02 / 0.8970 <b>32.12</b> / <b>0.8990</b>	30.39 / 0.9093 31.23 / 0.9188 <b>31.31</b> / 0.9195 31.29 / <b>0.9198</b> 31.13 / 0.9180	37.21 / 0.9731 37.88 / 0.9749 38.03 / <b>0.975</b> 3 <b>38.07</b> / 0.9750
DR Me TSI ms EA 3 Bic Ap Sel	RRN [14] emNet [16] CN [25] s-LapSRN [26] t-IDRN (ours) cubic olus [5] IfExSR [43]	37.74 / 0.9591 37.78 / 0.9597 37.88 / 0.9602 37.70 / 0.9590 37.91 / 0.9603 30.39 / 0.8682	33.23 / 0.9136 33.28 / 0.9142 33.28 / 0.9147 33.25 / 0.9138 <b>33.32</b> / <b>0.9154</b>	32.05 / 0.8973 32.08 / 0.8978 <b>32.09</b> / <b>0.8985</b> 32.02 / 0.8970 <b>32.12</b> / <b>0.8990</b>	31.23 / 0.9188 31.31 / 0.9195 31.29 / <b>0.9198</b> 31.13 / 0.9180	37.88 / 0.9749 38.03 / <b>0.975</b> 5 <b>38.07</b> / 0.9750
Me TSO ms EA 3 Bic Ap Sel SRO	emNet [16] CN [25] s-LapSRN [26] L-IDRN (ours) cubic olus [5] IfExSR [43]	37.78   0.9597 37.88   0.9602 37.70   0.9590 37.91   0.9603 30.39   0.8682	33.28 / 0.9142 33.28 / 0.9147 33.25 / 0.9138 <b>33.32</b> / <b>0.9154</b>	32.08 / 0.8978 32.09 / 0.8985 32.02 / 0.8970 32.12 /0.8990	<b>31.31</b> / 0.9195 31.29 / <b>0.9198</b> 31.13 / 0.9180	38.03 / <b>0.975</b> 5 <b>38.07</b> / 0.9750
TSO ms EA 3 Bic Ap Sel SRO	CN [25] s-LapSRN [26] A-IDRN (ours) cubic blus [5] IfExSR [43]	<b>37.88</b> / <b>0.9602</b> 37.70 / 0.9590 <b>37.91</b> / <b>0.9603</b> 30.39 / 0.8682	33.28 / 0.9147 33.25 / 0.9138 <b>33.32</b> / <b>0.9154</b>	<b>32.09</b> / <b>0.8985</b> 32.02 / 0.8970 <b>32.12</b> / <b>0.8990</b>	31.29 / <b>0.9198</b> 31.13 / 0.9180	<b>38.07</b> / 0.9750
ms EA 3 Bio Ap Sel SR	s-LapSRN [26] A-IDRN (ours) cubic olus [5] IfExSR [43]	37.70 / 0.9590 <b>37.91</b> / <b>0.9603</b> 30.39 / 0.8682	33.25 / 0.9138 <b>33.32</b> / <b>0.9154</b>	32.02 / 0.8970 <b>32.12</b> / <b>0.8990</b>	31.13 / 0.9180	
3 Bic Ap Sel SR	a-IDRN (ours) cubic olus [5] lfExSR [43]	<b>37.91</b> / <b>0.9603</b> 30.39 / 0.8682	33.32 /0.9154	32.12 /0.8990		37.71 / 0.9747
3 Bio Ap Sel SR	cubic blus [5] lfExSR [43]	30.39 / 0.8682	'	'	21 20 / 0 0200	
Ap Sel SR	olus [5] lfExSR [43]		27.55   0.7742		31.39 / 0.9209	<b>38.23</b> / <b>0.975</b> 1
Sel SR	lfExSR [43]	32.58 / 0.9088		27.21 / 0.7385	24.46 / 0.7349	26.96 / 0.8546
SR			29.13 / 0.8188	28.29 / 0.7835	26.03 / 0.7973	29.93 / 0.8120
		32.64 / 0.9097	29.15 / 0.8196	28.29 / 0.7840	26.46 / 0.8090	29.61 / 0.9050
VD	CNN [27]	32.75 / 0.9090	29.29 / 0.8215	28.41 / 0.7863	26.24 / 0.7991	30.48 / 0.9117
	OSR [13]	33.66 / 0.9213	29.77 / 0.8314	28.82 / 0.7976	27.14 / 0.8279	32.00 / 0.9329
DR	RCN [21]	33.82 / 0.9226	29.76 / 0.8311	28.80 / 0.7963	27.15 / 0.8276	32.31 / 0.9360
	pSRN [24]	- / -	- / -	- / -	- / -	- / -
	RRN_B1U25 [14]	34.03 / 0.9244	29.96 / 0.8349	<b>28.95</b> / 0.8004	27.53 / <b>0.8378</b>	<b>32.71</b> / 0.9379
	emNet [16]	34.09 / 0.9248	<b>30.00</b> / 0.8350	<b>28.96</b> / 0.8001	<b>27.56</b> / <b>0.8376</b>	32.79 / 0.938
	CN [25]	<b>34.18</b> / <b>0.9256</b>	<b>29.99</b> / 0.8351	<b>28.95</b> / 0.8012	27.46 / 0.8362	32.68 / <b>0.938</b>
	s-LapSRN [26]	- / -	- / -	- / -	- / -	- / -
	-IDRN (ours)	34.19 / 0.9260	30.00 / 0.8358	28.96   0.8018	27.43 / 0.8366	32.69 / <b>0.938</b>
	cubic	28.42 / 0.8104	26.00 / 0.7027	25.96 / 0.6675	23.14 / 0.6577	24.91 / 0.7846
	olus [5]	30.28 / 0.8603	27.32 / 0.7491	26.82 / 0.7087	24.32 / 0.7183	27.03 / 0.8510
	IfExSR [43]	30.30 / 0.8620	27.38 / 0.7516	26.84 / 0.7106	24.80 / 0.7377	26.80 / 0.8410
	CNN [27]	30.48 / 0.8628	27.50 / 0.7513	26.90 / 0.7103	24.52   0.7226	27.58 / 0.8555
	OSR [13]	31.35 / 0.8838	28.02 / 0.7678	27.29   0.7252	25.18 / 0.7525	28.88 / 0.885
	RCN [21]	31.53 / 0.8854	28.03 / 0.7673	27.24 / 0.7233	25.14 / 0.7511	28.98 / 0.8870
	pSRN [24]	31.52 / 0.8854	28.08 / 0.7687	27.31 / 0.7255	25.21 / 0.7545	29.08 / 0.8883
	RRN_B1U25 [14]	31.68 / 0.8888	28.21 / 0.7721	27.38 / 0.7284	<b>25.44</b> / 0.7638	29.44 / 0.894
	emNet [16]	31.74 / 0.8893	<b>28.26</b> / 0.7723	27.40 / 0.7281	<b>25.50</b> / 0.7630	29.64 / 0.896
	CN [25]	31.82 / 0.8907	28.28 / 0.7734	27.42   0.7301	25.44 / 0.7644	29.48 / 0.895
	s-LapSRN [26] A-IDRN (ours)	31.72 / 0.8891 <b>31.76</b> / <b>0.8903</b>	28.25   0.7730 <b>28.26</b>   <b>0.7735</b>	<b>27.42</b>   0.7296 <b>27.41</b>   <b>0.7300</b>	<b>25.50</b> / <b>0.7661</b> 25.42 / 0.7635	<b>29.53</b> / <b>0.895</b> 29.44 / 0.894
EA	A-IDKIV (OUIS)	31.70 / 0.8903	28.20   0.7733	27.41 / 0.7300	23.42   0.7033	25.44   0.054

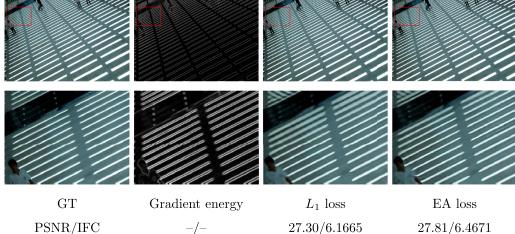


Fig. 7. Visualization of the computed gradient energy and the comparative SISR results using  $L_1$  loss and our proposed EA loss.

SelfExSR [43]) and eight deep-learning-based methods (SRCNN [11,27], VDSR [13], DRCN [21], LapSRN [24], DRRN [14], MemNet [16], TSCN [25], and ms-LapSRN [26]) are considered. Source codes or pre-trained models of these methods are publicly available. For fair comparison, SISR models trained using very large datasets<sup>1</sup>

(e.g., [15,20]) or high-resolution datasets<sup>2</sup> (e.g., [45,47,48]) are not considered in this part.

We show quantitative evaluation results on five testing datasets (Set5 [42], Set14 [60], B100 [59], Urban100 [43], and Manga109 [61]) with the scale factors  $\times 2$ ,  $\times 3$ ,  $\times 4$  in Table 5 (PSNR and

<sup>&</sup>lt;sup>1</sup> ImageNet dataset or its subset [51].

<sup>&</sup>lt;sup>2</sup> DIV2K dataset [49,50].

**Table 6** Average IFC values with the scale factors  $\times$  2,  $\times$  3 and  $\times$  4 on Set5, Set14, B100, Urban100 and Manga109 datasets. **Red** and **blue** indicate the best and the second best performance, respectively. It is noted that the IFC value is calculated on Y channel.

Dataset	Scale	Bicubic	VDSR [13]	LapSRN [24]	DRRN [14]	MemNet [16]	TSCN [25]	ms-LapSRN [26]	EA-IDRN
	× 2	6.083	8.580	8.401	8.670	8.850	9.175	8.628	9.314
Set5	$\times 3$	3.580	5.203	_	5.394	5.503	5.544	_	5.679
	$\times 4$	2.329	3.542	3.515	3.700	3.787	3.766	3.697	3.899
	$\times 2$	6.105	8.159	8.042	8.280	8.469	8.729	8.236	8.902
Set14	$\times 3$	3.473	4.691	_	4.870	4.958	4.970	_	5.090
	$\times 4$	2.237	3.106	3.089	3.249	3.309	3.286	3.202	3.382
	$\times 2$	5.619	7.494	7.295	7.513	7.665	7.871	7.475	7.982
B100	$\times 3$	3.138	4.151	_	4.235	4.300	4.350	_	4.423
	$\times 4$	1.978	2.679	2.618	2.746	2.778	2.792	2.692	2.868
	$\times 2$	6.245	8.629	8.441	8.889	9.122	9.442	8.881	9.706
Urban100	$\times 3$	3.620	5.159	_	5.440	5.560	5.559	_	5.703
	$\times 4$	2.361	3.462	3.448	3.669	3.786	3.715	3.641	3.833
	$\times 2$	6.230	8.886	8.912	9.212	9.470	9.976	9.115	10.220
Manga109	$\times 3$	3.522	5.310	_	5.681	5.798	5.846	_	6.004
	$\times 4$	2.283	3.633	3.661	3.903	4.038	3.967	3.854	4.096

**Table 7** Average running time for scale factors  $\times 4$  on three common resolution settings including  $480 \times 360$ ,  $640 \times 480$  and  $1280 \times 720$ . **Red** and **blue** indicate the best and the second best performance, respectively.

Dataset	Resolution	VDSR [13]	LapSRN [24]	DRRN [14]	MemNet [16]	TSCN [25]	ms-LapSRN [26]	EA-IDRN
Time (s)		0.0413	0.0278	5.0254	8.2985	0.0349	0.0538	0.0080
	$640 \times 480$	0.0859	0.0443	10.9856	15.0263	0.0668	0.0827	0.0118
	$1280 \times 720$	0.2789	0.1188	25.0669	35.2654	0.1528	0.2270	0.0307

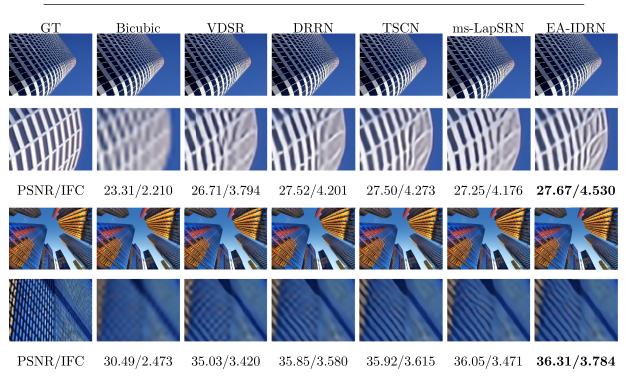


Fig. 8. Qualitative comparisons. Top: image "img005" from Urban100 dataset with scale factor  $\times$  4. Bottom: image "img012" from Urban100 dataset with scale factor  $\times$  4. Please zoom in on screen for better visualization.

SSIM indexes) and Table 6 (IFC index). In many cases, our proposed light-weight EA-IDRN model (23 layers in total) achieves even higher PSNR and SSIM values than some very deep networks (depth > 50), such as the 52-layer DRRN [14] and 80-layer MemNet [16]. It is worth mentioning that our EA-IDRN model achieves the new state-of-the-art results based on the IFC index, which correlates better with human perception [63], for all five testing datasets with 3 different scale factors ( $\times$ 2,  $\times$ 3, and  $\times$ 4).

In Table 7, we show the averaged running time of different SISR methods to process 100 input images of three different resolutions including  $480 \times 360,\ 640 \times 480$  and  $1280 \times 720$ . The testing are conducted on a PC which is equipped with NVIDIA GTX 1080Ti

GPU (11 GB memory). It is noted that our proposed EA-IDRN runs much faster than other state-of-the-art SISR methods<sup>3</sup> and can still achieve real-time speed (>30 fps) on processing  $1280 \times 720$  images

Some comparative results with state-of-the-art deep-learning-based SISR methods are shown in Fig. 8. Overall our EA-IDRN model can achieve better image restoration results. It is observed

<sup>&</sup>lt;sup>3</sup> Caffe-implemented DRRN [14] and MemNet [16] are quite time-consuming. The main reason behind this phenomenon is due to the limitation of GPU memory, these two methods need to divide the testing image into small patches and test on these patches, then collect all the time together.

**Table 8** Detailed Comparisons with EDSR and RDN. Depth indicates the number of convolution and deconvolution layers (only layers with kernel size larger than  $1\times1$  are counted). The running times are tested on  $1280\times720$  resolution images with scale  $\times4$  by a PC equipped with NVIDIA GTX 1080Ti (11GB memory), Cuda 8.0 and Cudnn 5.1.

Models	EA-IDRN		EDSR	RDN
Training Data	RGB91+B200	DIV2K	DIV2K	DIV2K
Depth	23	23	69	133
Set5 × 2	37.91 dB	37.98 dB	38.11 dB	38.24 dB
Set $14 \times 2$	33.32 dB	33.52 dB	33.92 dB	34.01 dB
$B100 \times 2$	32.12 dB	32.17 dB	32.32 dB	32.34 dB
Time (s)	0.0307	0.0307	10.0579	6.3548

that parallel lines and lattice texture pattern in red highlighted region processed by our EA-IDRN method are much sharper and clearer than the results of other SISR methods. Moreover, our method can effectively suppress undesired artifacts or distortions which are helpful for other high-level image processing tasks such as target detection and medical analysis.

It is possible to make use of the high-resolution DIV2K dataset (containing 800 training images of 2K resolution) to train our EA-IDRN model. The comparative evaluation results (in terms of accuracy and computational speed) with EDSR [45] and RDN [47] are shown in Table 8. It is noted that using a larger or higher resolution training dataset (e.g., DIV2K or ImageNet) can generally lead to higher SISR accuracy. For instance, our EA-IDRN model trained using the high-resolution DIV2K dataset achieves higher PSNR than the one trained using low-resolution images from RGB91 and BSD200 (37.98 dB vs. 37.91 dB on Set5 with scale  $\times$  2). The results indicate that the performance of EA-IDRN model can be further boosted using a higher quality training set (e.g., DIV2K or ImageNet). However, the improvement is not significant. The underlying principle is that our EA-IDRN is a 23-layer model and using a very large training dataset could not significantly boost the performance of this light-weight network. Our experimental results are consistent with the ones reported in [27]. We could stack more modules, increase the filter number of convolution layer (more parameters) and use a larger training dataset to further boost the performance, although it is beyond the scope of this paper. It is worth mentioning that our proposed light-weight EA-IDRN model run significantly faster than EDSR and RDN (EDSR vs. RDN vs. EA-IDRN: 10.0579s vs. 6.3548s vs. 0.0307s), which is more suitable to facilitate image pre-processing tasks.

## 5. Conclusion

In this paper, a compact but powerful Energy-Aware Improved Deep Residual Network (EA-IDRN) model is proposed for fast and accuracy SISR. We present a number of simple but effective structure modifications to two basic building blocks (residual blocks and skip connections) in deep residual networks. In addition, we propose a novel energy-aware training loss to adaptively adjust how high-frequency and low-frequency image regions are restored. Gradient energy information is utilized to emphasize the restoration of regions with textures or edges to achieve higher SISR accuracy. It is worth mentioning that these improvements can significantly increase SISR accuracy while causing no/ignorable extra computational loads. Extensive experimental results on multiple benchmark datasets demonstrate that our EA-IDRN method achieves more accurate results with faster speed.

## **Conflict of interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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