Numerical Simulation of Inferior and Superior Atmospheric Mirages

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Abstract

Atmospheric mirages are optical phenomena caused by the refraction of light rays passing through air layers of varying temperatures and densities. This paper presents a numerical study of two distinct types of mirages: the inferior mirage, commonly observed in deserts, and the superior mirage (Fata Morgana), often seen over cold bodies of water. We employ a ray-tracing technique based on Fermat's principle, solving the optical path differential equations using a fourth-order Runge-Kutta (RK4) integrator. We define mathematical models for the refractive index profiles characteristic of each environment: an exponential decay model for the heated desert surface and a dual exponential inversion model for the cold ocean surface. The simulation results successfully demonstrate the upward curvature of rays in the inferior mirage and the complex ducting and downward curvature associated with the superior mirage.

Abstract

What is happening? Light bends when it travels through air that changes with height. **Why does air change?** Temperature changes $\Rightarrow$ density changes $\Rightarrow$ refractive index changes. **What we do in this report:**

- We invent two realistic formulas $n(y)$: one for hot desert air, one for cold-ocean inversion.
- We treat light as rays and compute their paths using RK4 (a numerical method).
- We plot the rays to see if they behave like real mirages.

Big idea: Mirages come from real curved rays, not from imagination.

Contents

Team Contributions **2**

1 Introduction **3**
1.1 Objectives . . . 3
1.2 Notation (Symbols) . . . 4

2 Physical Phenomenon and Theoretical Background **5**
2.1 The Physics of Atmospheric Refraction . . . 5
2.1.1 Temperature, Density, and Refractive Index . . . 5
2.2 Schematic of the Phenomenon . . . 6

2.3 Snell's Law 6
2.4 Mechanism of the Inferior Mirage 7
2.5 Mechanism of the Superior Mirage (Fata Morgana) 7
2.6 Mathematical Approximation 8
2.7 Assumptions and Validity 8

3 Physical Models and Methodology 9
3.1 Ray Equation 9
3.2 Desert Atmosphere Model (Inferior Mirage) 9
3.3 Ocean Atmosphere Model (Superior Mirage) 10

4 Implementation 10

5 Results and Discussion 11
5.1 Inferior Mirage Simulation 11
5.2 Superior Mirage Simulation 12

6 Conclusion 12

Team Contributions

This project was realized through the collaborative effort of the following team members:

Member Name	Contribution
Hammadi Nibel	Python/RK4 ray-tracing simulation, figure/CSV generation, and final PDF compilation.
Endahmane Reham	Physical model definition, key assumptions, and explanation of refractive-index gradients and ray bending.
RAKI Meriem	Physics + Snell's law study; simplified refractive-index modeling ideas; contributed to layered-to-continuous ray-marching approach.
MESSAHLI Hiba Manel	Explanation of mirage formation and image interpretation using Snell's law; helped simplify into a usable mathematical description.
Amani Takelait	Numerical ray-propagation algorithm and parameter-effect analysis on ray curvature.
Lounnaci Yasmine	Theoretical background research: atmosphere optics, temperature-gradient mirage mechanisms, mirage types (incl. Fata Morgana), and examples/applications.

Table 1: Distribution of tasks among team members.

Note: *The colored "Friendly Explanation" boxes included throughout this document are intended to provide simplified, intuitive summaries of the technical content to assist in understanding.*

1 Introduction

A mirage is a naturally occurring optical phenomenon in which light rays are bent to produce a displaced image of distant objects or the sky. The fundamental physical principle behind mirages is the variation of the refractive index of air, n, which depends on air density and, consequently, temperature.

Mirages are broadly classified into two categories:

- **Inferior Mirage:** Occurs when the ground is significantly hotter than the air above it (e.g., a desert or asphalt road). The air near the ground is less dense and has a lower refractive index than the cooler air above. This gradient causes light rays traveling downwards to bend upwards, creating an inverted image below the object derived from sky light.
- **Superior Mirage:** Occurs when the air near the surface is colder than the air above it (a temperature inversion). This is common over cold oceans. The denser air near the surface has a higher refractive index. This causes light rays to bend downwards. Under specific conditions, rays can become trapped in an atmospheric duct, leading to complex distortions known as the Fata Morgana.

This work implements a computational framework to simulate these phenomena by integrating the ray equations through defined atmospheric models.

Introduction

If you remember only one thing: your eyes assume light travels in straight lines.
But in the atmosphere:

- Near hot ground: air is hotter, less dense, and usually has a slightly smaller refractive index.
- Higher up: air is cooler, denser, with slightly larger refractive index.

What this causes: rays curve. When a curved ray reaches your eye, your brain continues it backward in a straight line and places the image in the wrong place.
That wrong placement is the "mirage image."

1.1 Objectives

The primary objectives of this study are:

1. To develop physically motivated refractive index models for desert and ocean atmospheric conditions that capture the essential thermal stratification responsible for mirage formation.
2. To numerically solve the ray propagation equations derived from Fermat's principle using a robust fourth-order Runge-Kutta integration scheme.
3. To visualize and analyze the resulting ray trajectories, demonstrating the characteristic upward bending in inferior mirages and the ducting behavior in superior mirages.
4. To validate the simulation framework against known optical phenomena and assess the computational accuracy and limitations of the approach.

Objectives (what we will actually build)

Goal 1: Build the atmosphere. We choose formulas for $n(y)$ (refractive index vs height).
Goal 2: Build the "ray engine." We convert physics into differential equations, then integrate them with RK4 step-by-step.
Goal 3: Make it visible. We plot $n(y)$ and many rays to see the mirage effect.
Goal 4: Check if it makes sense. Do desert rays curve upward? Do ocean rays duct? If yes, our model is consistent with real mirage behavior.

1.2 Notation (Symbols)

Symbol	Meaning	Units
x	Horizontal distance	m
y	Height above surface	m
s	Ray path parameter (step variable)	m
$n(y)$	Refractive index profile	(dimensionless)
θ	Ray angle (relative to normal)	rad or deg
v_x, v_y	Tangent components of ray direction	(dimensionless here)
H, h_1, h_2	Scale heights for profiles	m
$\Delta n, A, B$	Profile amplitudes	(dimensionless)

Table 2: Main symbols used in the report.

Symbols

When you read equations, always ask:

- "Is this a position?" (usually x, y)
- "Is this a direction?" (usually v_x, v_y or an angle θ)
- "Is this describing the air?" (usually $n(y)$ and dn/dy)

If you keep those three categories in mind, the math becomes much easier.

2 Physical Phenomenon and Theoretical Background

2.1 The Physics of Atmospheric Refraction

A mirage is not an optical illusion in the sense of a hallucination; it is a real physical phenomenon caused by the bending of light rays as they pass through air layers of varying density. This bending is governed by the refractive index of air, n, which depends on its density ρ.

Atmospheric refraction

Simple story:

1. Air is not the same everywhere. Near the ground it can be hotter/colder than higher up.
2. Hot air has lower density; cold air has higher density.
3. The refractive index n depends on density (more dense air usually means slightly larger n).
4. Light bends toward the region with larger n).

Result: even in "empty-looking" air, rays can curve because n changes with height.

2.1.1 Temperature, Density, and Refractive Index

The fundamental cause of atmospheric refraction is the variation of air temperature.

- **Heating:** Near a hot surface (e.g., asphalt or desert sand), the air heats up and expands.
- **Density Decrease:** Hotter air is less dense (ρ decreases).
- **Refractive Index:** The refractive index is approximately proportional to density ($n-1 \propto \rho$). Therefore, hot air has a lower refractive index than cooler air.

This creates a vertical gradient in the refractive index, dn/dy. Light rays traveling through this gradient are bent toward the region of higher refractive index (cooler, denser air).

Temperature $\rightarrow n(y)$

Step 1: Temperature changes with height. Example: near hot ground, T is high at $y = 0$, lower above.
Step 2: Density changes. Hotter air expands and becomes less dense, so $\rho(y)$ is smaller near the ground.
Step 3: Refractive index changes. Since $n - 1$ roughly follows density, $n(y)$ becomes smaller near the ground and larger above.
Step 4: Rays bend. Light bends toward higher n, so it tends to curve upward in the desert case (because higher n is above).

2.2 Schematic of the Phenomenon

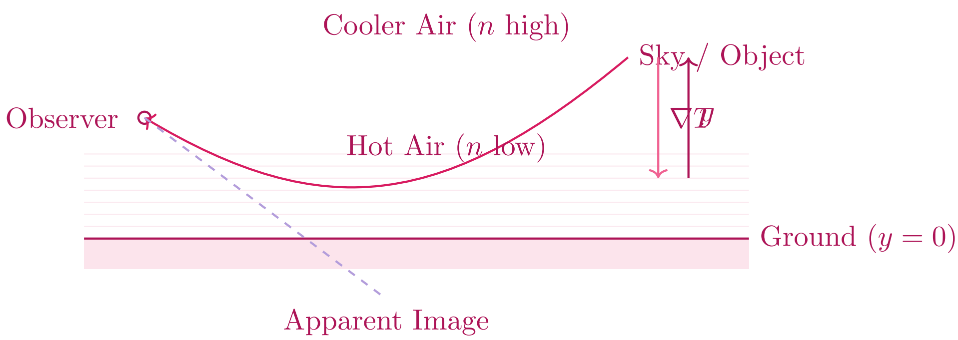


Figure 1: Inferior mirage schematic: curved rays can reach the observer even if they dip close to the hot surface, and the observer extrapolates them backward as a straight line.

How to read this schematic

Look at the solid red line: That is the REAL path of light. It curves because the air density changes. **Look at the dashed line:** That is the APPARENT path. Your eye/brain assumes light travels straight, so it projects the ray backwards. **Result:** You see the sky/object coming from the ground (the "mirage").

2.3 Snell's Law

The bending of light is governed by Snell's law at each atmospheric interface.

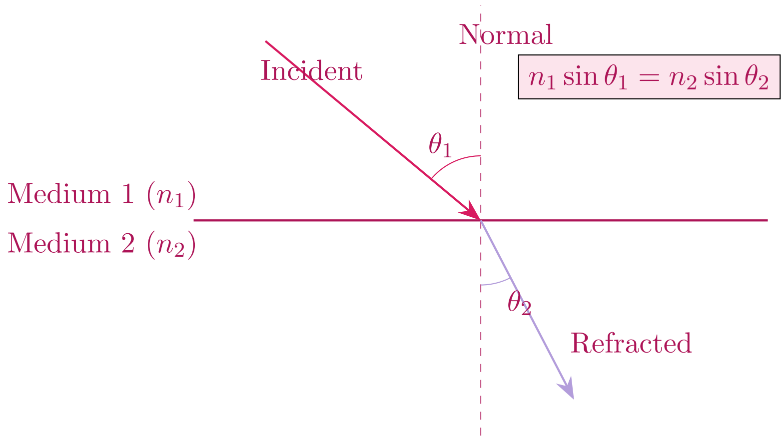


Figure 2: Snell's law at an interface. In the atmosphere, many thin layers produce continuous bending.

Snell's law

Meaning of Snell's law: it tells how the direction changes when the refractive index changes.
Two key facts:

- If n becomes bigger, the ray bends *toward the normal.*
- If n becomes smaller, the ray bends *away from the normal.*

In the atmosphere: we do not have one big interface. We have many tiny layers. So Snell's law is applied "little by little," and the ray becomes a **smooth curve**.

Layered invariant idea (important): in a layered medium, one often writes

$$n \sin \psi = \text{constant},$$

with ψ measured toward the normal of the layers. This is the same spirit as Snell's law, but used continuously across many layers.

2.4 Mechanism of the Inferior Mirage

The "inferior" mirage is the most common type, seen on hot days.

1. **Conditions:** The ground is significantly hotter than the air above it ($T_{ground} > T_{air}$).
2. **Gradient:** Temperature decreases with height $\rightarrow$ Density increases with height $\rightarrow$ Refractive index increases with height ($dn/dy > 0$).
3. **Ray Bending:** Light rays coming from the sky or distant objects travel downwards into hotter, lower-index air.
4. **Curvature:** The gradient causes these rays to curve *upward*.
5. **Observation:** The brain assumes rays are straight, so it places the image below the horizon (reflection-like look).

Inferior mirage

What you see on a hot road: a "wet" shiny area far away.
What is really happening:

- Some rays from the sky travel downward.
- Near the hot ground, n is smaller.
- The gradient bends the ray back upward before it hits the ground.
- That upward-bent ray reaches your eye.

Why it looks like water: your brain extends the ray backward as a straight line and thinks it came from below the horizon, like a reflection.

2.5 Mechanism of the Superior Mirage (Fata Morgana)

The "superior" mirage occurs under opposite conditions (temperature inversion).

1. **Conditions:** The surface is colder than the air above it.
2. **Gradient:** Refractive index decreases with height near the surface ($dn/dy < 0$).
3. **Curvature:** Rays bend *downward*.
4. **Ducting:** Rays can become trapped in a layer and travel long distances.

Superior mirage

Cold surface + warmer air above can create an "inversion layer."
Effect on rays:

- Rays bend downward, sometimes staying close to the surface.
- If the bending is strong enough, rays can be "guided" inside a layer (ducting).

What you see: far objects can look lifted, stretched, or duplicated (Fata Morgana).

2.6 Mathematical Approximation

While the real atmospheric profile is complex, for small vertical distances, the variation can be approximated and connected to Snell's law.

- **Local linear approximation:**

$$n(y) \approx n_0 + a(y - y_0), \qquad a = \left.\frac{dn}{dy}\right|_{y_0}.$$

- **Continuous layered invariant:**

$$n(y)\,\sin\theta(y) = \text{constant}.$$

- **Why we integrate ODEs:** Near the surface gradients can be strong and non-linear, so numerical integration is more reliable than a simple approximation.

Mathematical approximation

1) Linear approximation: what does it mean? If a curve is smooth, a tiny part of it looks like a line. So we approximate $n(y)$ near y_0 by a line with slope $a = \frac{dn}{dy}$.
2) Why do we care about the sign of a?

- If $a > 0$: n increases with height. Rays tend to curve upward (typical inferior mirage).
- If $a < 0$: n decreases with height. Rays tend to curve downward (possible superior mirage / ducting).

3) The invariant $n \sin\theta =$ const: what does it help with? It is a "shortcut rule" in a layered medium: it tells you the angle must change if n changes. So you can predict bending direction without solving everything.
4) Why not stop there? Because in real mirages, the strongest changes happen very close to the surface, and a simple straight-line approximation may be too rough. That's why we solve the full ODEs numerically.

2.7 Assumptions and Validity

To simulate these phenomena numerically, we assume:

- **2D:** horizontal uniformity.
- **Geometric optics:** rays, not waves.
- **Static medium:** no turbulence/wind.

Assumptions

Why 2D? Because we focus on "height changes" which are the main cause of bending in many mirage situations.
Why rays not waves? Because atmospheric layers are meters thick, while light wavelength is about 10^{-6} m; on this scale, ray optics usually works well.
Why static? Turbulence makes mirages dance and flicker. We remove it to see the clean physics first.

3 Physical Models and Methodology

3.1 Ray Equation

Light travels through a medium with a spatially varying refractive index $n(x, y)$. According to Fermat's principle, light takes the path that minimizes the optical path length

$$S = \int n\, ds.$$

This leads to the ray equation:

$$\frac{d}{ds}\left(n\frac{d\mathbf{r}}{ds}\right) = \nabla n \tag{1}$$

For a stratified atmosphere where n varies only with height y (i.e., $n = n(y)$), the 2D equations of motion for a ray $[x(s), y(s)]$ with tangent vector $[v_x, v_y]$ become:

$$\frac{dx}{ds} = v_x, \qquad \frac{dy}{ds} = v_y \tag{2}$$

$$\frac{dv_x}{ds} = -\frac{v_y v_x}{n}\frac{dn}{dy} \tag{3}$$

$$\frac{dv_y}{ds} = \frac{v_x^2}{n}\frac{dn}{dy} \tag{4}$$

Ray equations

We track 4 quantities:

$$(x, y, v_x, v_y).$$

Eqs. (2): update position from direction: if v_x is positive, x increases; if v_y is negative, the ray is going downward.
Eqs. (3)-(4): update direction from the air gradient:

- Everything depends on $\frac{dn}{dy}$.
- If $\frac{dn}{dy} > 0$, Eq. (4) gives $dv_y/ds > 0$ (because $v_x^2 \geq 0$), so v_y increases: the ray turns upward.
- If $\frac{dn}{dy} < 0$, then $dv_y/ds < 0$: the ray turns downward.

So the sign of dn/dy basically decides the mirage type.

3.2 Desert Atmosphere Model (Inferior Mirage)

For the desert scenario, we model the intense heating near the ground using a perturbed exponential profile. The base index n_{base} is modified by a term that decreases rapidly with height:

$$n(y) = n_{\text{base}} - \Delta n \cdot \exp(-y/H) \tag{5}$$

where Δn determines the strength of the gradient at the surface and H is the scale height of the boundary layer. The gradient is positive everywhere ($\frac{dn}{dy} > 0$), which forces rays to curve upward (away from the ground).

Desert model (what the formula means)

At $y = 0$: $\exp(0) = 1$, so $n(0) = n_{\text{base}} - \Delta n$. That makes n smaller right at the hot surface. As y increases: $\exp(-y/H)$ becomes smaller quickly, so the "hot effect" disappears and $n(y)$ returns toward n_{base}.
Why exponential? It is a simple way to model "strong near the surface, weak a few meters above," which is realistic for a thin heated boundary layer.

3.3 Ocean Atmosphere Model (Superior Mirage)

For the ocean scenario, we model a temperature inversion where a layer of cold air sits below warmer air. We use a dual-exponential profile to capture both the inversion and the normal atmospheric lapse rate above it:

$$n(y) = n_{\text{base}} + A \cdot \exp(-y/h_1) - B \cdot \exp(-y/h_2) \tag{6}$$

Here, the term with A represents the cold, high-density layer near the surface (increasing n), while the term with B represents the standard decrease of density with altitude. The interplay between these terms creates a region where $\frac{dn}{dy} < 0$, causing rays to bend downward. If the curvature of the ray matches the curvature of the Earth (or in this flat-earth approximation, if the ray bends down sufficiently to stay within the layer), "ducting" occurs.

Ocean model (what the two exponentials do)

We combine two effects:

- $+Ae^{-y/h_1}$: adds a near-surface "extra high n" layer (cold dense air).
- $-Be^{-y/h_2}$: subtracts a slower-decaying part (representing normal decrease with height).

Depending on parameters, this can create a region where $dn/dy < 0$. That is exactly the condition that can bend rays downward and create ducting.

4 Implementation

The simulation is implemented in Python. The core solver uses the explicit fourth order Runge-Kutta (RK4) method to integrate the system of first-order differential equations derived in Section 2.1.

- **Integration Step:** We use a fixed step size ds (0.5m for desert, 10m for ocean) to ensure numerical stability.
- **Parameters:**
  - Desert: $\Delta n = 2.4 \times 10^{-4}$, $H = 3.0$ m.
  - Ocean: $A = 1.2 \times 10^{-4}$, $B = 4.0 \times 10^{-5}$, $h_1 = 12$ m, $h_2 = 40$ m.

RK4 ray tracing

What RK4 means: it is a method that takes a small step in s and estimates the new state accurately by sampling the slope multiple times inside the step.
What is the "state"?

$$\text{state} = (x, y, v_x, v_y).$$

Algorithm outline:

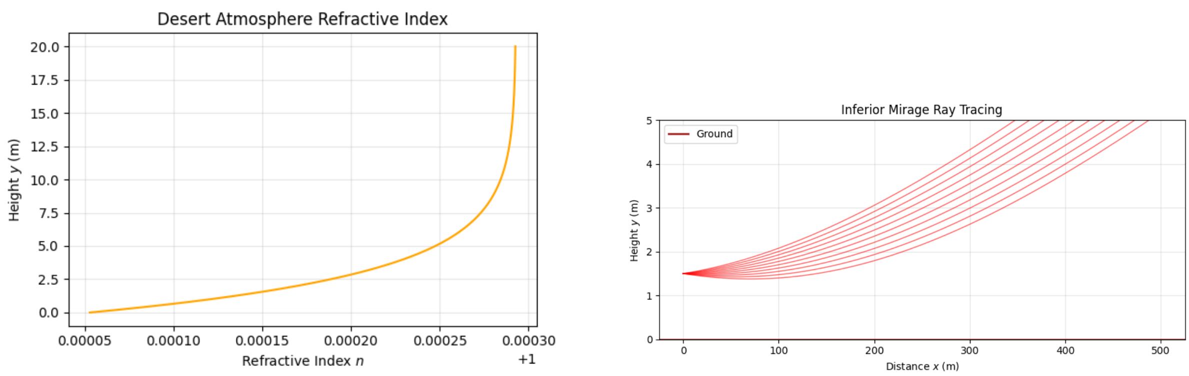
1. Choose an initial point (x_0, y_0) and a launch angle (or directly v_x, v_y).
2. At each step:
   - Compute $n(y)$ and dn/dy.
   - Use Eqs. (2)–(4) to compute the derivatives.
   - Apply RK4 to update (x, y, v_x, v_y).
3. Stop when:
   - $y \leq 0$ (ray hits the surface), or
   - x goes beyond the plot range, or
   - number of steps reaches a limit.
4. Save the ray as a CSV file with columns x, y.

Why CSV? LaTeX can plot the exact computed points, so your graphs are "perfect" (not hand-drawn curves).

5 Results and Discussion

5.1 Inferior Mirage Simulation

Figure 1a shows the refractive index profile for the desert model. The sharp decrease in n approaches the ground ($y = 0$). Figure 1b illustrates the trajectories of light rays. Rays emitted downwards (negative initial angle) are refracted upwards by the positive gradient. To an observer, these rays appear to originate from below the ground, creating the illusion of a reflection (often mistaken for water).



(a) Refractive Index Profile $n(y)$ **(b)** Ray Trajectories

Figure 3: Simulation of the Inferior (Desert) Mirage.

How to understand Figure

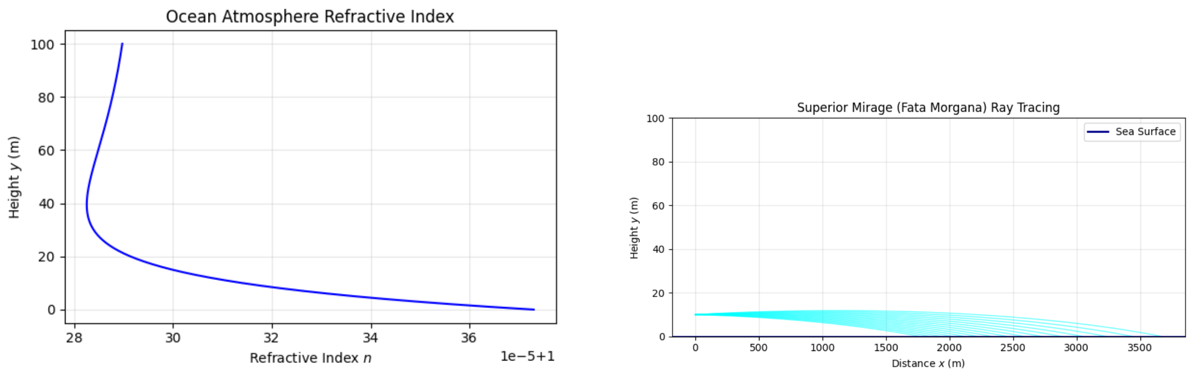
Look at panel (a): near $y = 0$, $n(y)$ changes fast. That is the "bending zone."
Look at panel (b): some rays go down, then curve up.

- If a ray curves up and reaches the observer, the observer receives sky light from below the horizon line.
- The brain thinks it is a reflection: that is the inferior mirage.

So: **strong gradient near ground** + **upward-curving rays** = "water on the road" effect.

5.2 Superior Mirage Simulation

Figure 2a displays the refractive index profile for the ocean model, showing a characteristic "kink" or inversion where density is highest near the surface. Figure 2b shows the ray traces. Rays launched horizontally or with slight elevation can be trapped in the duct created by the inversion layer ($y < 20$m). These rays travel long distances, allowing objects (like ships) to be seen well beyond the geometric horizon or to appear vertically stretched (looming).



(a) Refractive Index Profile $n(y)$ **(b)** Ray Trajectories

Figure 4: Simulation of the Superior (Ocean) Mirage.

How to understand Figure

Panel (a) can create a layer where rays are "pulled" downward.
Panel (b): if you see rays staying close to the surface for a long distance, that is **ducting**.

- Ducting means light can travel farther than usual near the surface.
- That allows you to see distant objects that normally would be hidden by Earth's curvature (in real life).
- It also causes strange vertical stretching and distortions (Fata Morgana).

6 Conclusion

This report combined physical intuition, refractive-index modeling, and RK4 ray tracing to reproduce key features of inferior and superior atmospheric mirages.

Conclusion

Desert: hot ground $\Rightarrow$ $dn/dy > 0$ $\Rightarrow$ rays curve upward $\Rightarrow$ "water-like reflection."
Ocean inversion: $dn/dy < 0$ in a layer $\Rightarrow$ rays curve downward and may duct $\Rightarrow$ distant objects appear lifted/distorted.
Numerical part: RK4 just follows the rules step-by-step and gives the ray curves you plot from CSV.