Euler Method

$$\frac{dy(t)}{dt} = f(t, y(t))$$

$$y(t + \Delta t) = y(t) + f(t, y(t))\Delta \text{t} + O(\Delta t^2)$$

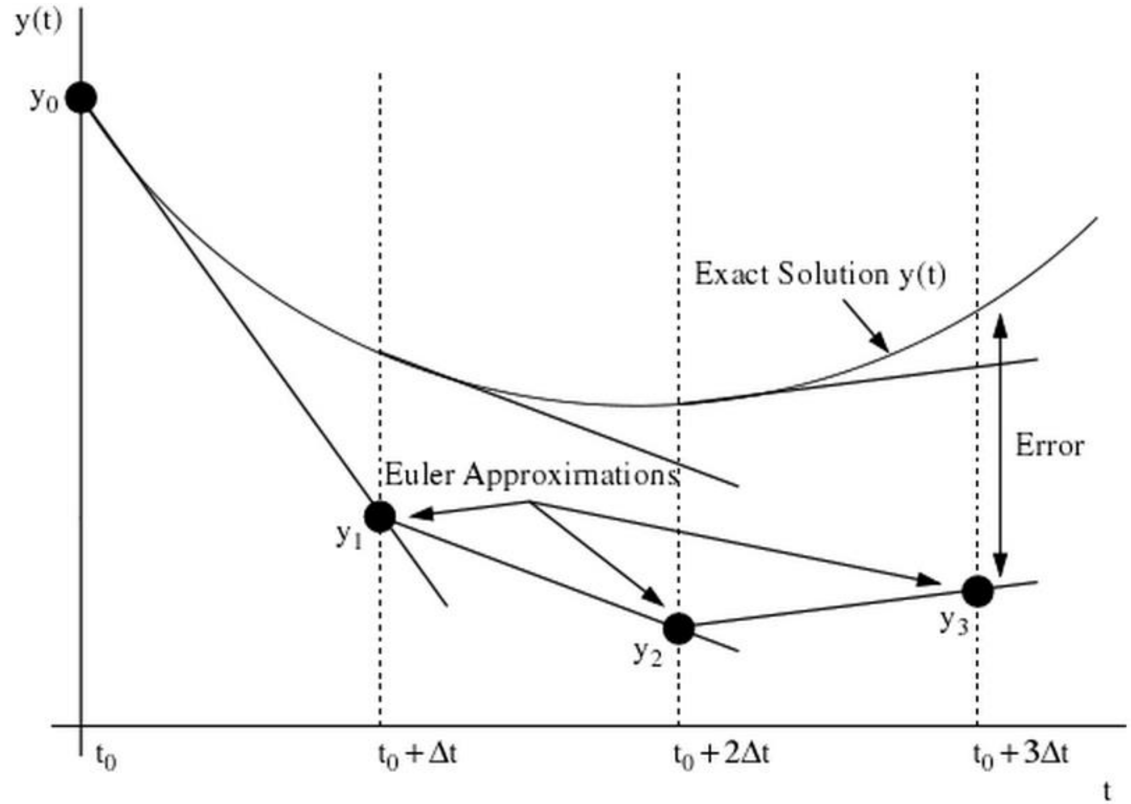

Figure 1. Graphical description of the Euler method (Kutz, 2013)

Implement the Euler method as a function that can be called from MATLAB environment in the following format

$$\text{function } [\text{tVal, yVal}]=\text{EulerMethod}(\text{f,deltaT,t0,tf,y0})$$

f = function, $f(t, y(t))$
deltaT = step size, Δt
t0 = initial value of the argument
tf = final time of the argument
y0 = initial value, $y(t_0)$

tVal = equally spaced array of arguments
yVal = array of computed values

- Test the function by numerically solving the ODE governing radioactive decay,

$$\frac{dN(t)}{dt} = \frac{1}{\tau} N(t)$$

where $N(t)$ is the number of atoms at time t, τ is decay constant for a given kind of atom.

- Solve this equation analytically and compute the error of the Euler method.

- Plot in one figure two plots, where on the first one both numerical and analytical results are shown. On the second plot show the computed error as a function of time.

- Label the axes and give titles to the plots. Play with options for plotting to make the graphs clearly distinct and readable.

- Vary the time step size, decay constant and initial parameters and observe the effect on the error.

- Export the figure in .pdf format.

- More elaborate expression for the Euler method:

$$y(t+\Delta t) = y(t) + \Delta t\left[A\,f\big(t, y(t)\big) + B\,f\Big(t + P\Delta t, y(t) + Q\Delta t f\big(t, y(t)\big)\Big)\right] + O(\Delta t^3),$$

where A, B, P, Q are some constants. One can prove by using Taylor series that

$$\begin{aligned} A + B &= 1 \\ PB &= 1/2 \\ BQ &= 1/2 \end{aligned}$$

Derive y_{n+1} for $A = 0$. This is the 2nd-order Runge-Kutta method.

- Modify the Euler Method function to have an option of whether to use the Euler method or the 2nd-order Runge-Kutta method.

- Use the radioactive decay ODE again to observe the difference in errors between the two methods. Which method is better?