PHYS 2030
Homework 5 Solutions.

October 22, 2014

1. a. -

   b. Let us denote $v = \frac{dx}{dt}$, then the undriven Duffing oscillator can be rewritten as the following system of ODEs.

   $$\frac{dx}{dt} = v$$
   $$\frac{dv}{dt} = -\alpha x - \beta x^3 - \delta v$$

   This system cannot be represented in matrix form, because of its nonlinearity, due to $-\beta x^3$ term.

   c. To find the equilibrium points we need to find such $v_0 = v(t_0)$ and $x_0 = x(t_0)$ for which $\frac{dx}{dt}\big|_{t,x_0,v_0} = 0$ and $\frac{dv}{dt}\big|_{t,x_0,v_0} = 0$. These v_0 and x_0 represent solutions that are constant for all time t.
   For our system of ODEs we have

   $$\frac{dx}{dt} = v = 0$$
   $$\frac{dv}{dt} = -\alpha x - \beta x^3 - \delta v = 0$$

   Simplifying, we get

   $$v = 0$$
   $$x(\alpha + \beta x^2) = 0$$

   d. In the linear ODE there is one nonlinear term, $-\beta x^3$.