PHYS 2030
Lab 4

October 6, 2014

1. Implement 2nd-order Runge-Kutta method for $\frac{d\mathbf{y(t)}}{dt} = f(t, \mathbf{y}(t))$, which is the following

$$\mathbf{y}(t + \Delta t) = \mathbf{y}(t) + \Delta t \cdot f\left(t + \frac{\Delta t}{2}, \mathbf{y}(t) + \frac{\Delta t}{2} \cdot f(t, \mathbf{y}(t))\right)$$

2. Modify the 2nd-order Runge-Kutta method function to solve one body 1D mechanics problems, i.e., of the form

$$\frac{d^2x(t)}{dt^2} = \frac{(F_{net})_x}{m}$$

   This function must have two inputs for initial conditions: $x(t_0)$ and $\frac{dx(t_0)}{dt} = v(t_0)$. The input function is $\frac{(F_{net})_x}{m}$.

3. - Test your new function by solving the problem of simple harmonic motion (SHO) modeled by a body of mass m on a spring with spring constant k. The equation of motion is

$$\frac{d^2x(t)}{dt^2} = -\frac{k}{m}x$$

   - Compare with analytical solution given by

$$x(t) = A\sin(\omega t + \phi_0),$$

     where A is the oscillation amplitude, ω is the angular frequency and ϕ_0 is the initial phase. Both the amplitude and the initial phase can be derived from the initial conditions. The initial phase is easy to find and for the amplitude use energy conservation.

   - Pick your own initial conditions and perform the numerical computation for a set of time steps.

- On one plot show the position-vs-time graphs for all time steps. Make sure it is easy to distinguish different computed data sets.
- Show on one plot both the analytic solution and the data corresponding to the smallest element of your set of time steps.

4. - Go to MatLab *Help* file and read about syntax for `ode45`.
   - Solve the SHO problem with `ode45` with the same initial conditions as before.
   - Show on separate graphs the computed data and associated phase space. Comment on the shape of the phase space.