## 1. Problem 25.2-2

$$t = 0.002$$

$$P'_{A} = 4 \cdot 10^{-8}$$

$$\alpha^{*} = 10$$

$$L_{f} = 0.002$$

$$x_{f} = 0.413$$

$$P_{h} = 80$$

$$P_{l} = 20$$

$$x_{0} = 0.3$$

Finding  $y_p$ 

$$r = \frac{P_l}{P_h}$$

$$r = \frac{20}{80} = 0.25$$

$$a = 1 - \alpha^*$$

$$a = 1 - 10 = -9$$

$$b = -1 + \alpha^* + \frac{1}{r} + \frac{x_0}{r} (\alpha^* - 1)$$

$$b = -1 + 10 \frac{1}{0.25} + \frac{0.3}{0.25} (10 - 1) = 23.8$$

$$c = \frac{-\alpha^* x_0}{r}$$

$$c = \frac{-10 \cdot 0.3}{0.25} = -12$$

$$y_p = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$y_p = \frac{-23.8 + \sqrt{23.8^2 - 4 \cdot -9 \cdot -12}}{2 \cdot -9}$$

$$y_p = 0.678$$

Finding fraction permeated

$$x_0 = \frac{x_f - \theta y_p}{1 - \theta}$$
$$0.3 = \frac{0.413 - \theta \cdot 0.678}{1 - \theta}$$

Solve for  $\theta$ 

$$\begin{aligned} & \boxed{\theta = 0.299} \\ A_m &= \frac{\theta L_f y_p}{\frac{P_A'}{t} \left( P_h x_0 - P_l y_h \right)} \\ A_m &= \frac{0.299 \cdot 0.002 \cdot 0.678}{\frac{4 \cdot 10^{-8}}{0.002} \left( 80 \cdot 0.3 - 20 \cdot 0.678 \right)} \\ & \boxed{A_m = 1.94 \text{ cm}^3} \end{aligned}$$

## 2. Problem 25.2-3

(a)

$$P'_{A} = 4 \cdot 10^{-9}$$
  
 $\alpha^* = 10$   
 $L_f = 2000$   
 $\theta = 0.529$   
 $P_h = 40$   
 $P_l = 10$   
 $x_f = 0.5$   
 $t = 0.0015$ 

Finding  $y_p$ 

$$a_{1} = \theta + \frac{P_{l}}{P_{h}} - \frac{P_{l}}{P_{h}} \theta - \alpha^{*}\theta - \alpha^{*}\frac{P_{l}}{P_{h}} + \alpha^{*}\frac{P_{l}}{P_{h}}\theta$$

$$a_{1} = 0.529 + \frac{10}{40} - \frac{10}{40} \cdot 0.529 - 10 \cdot 0.529 - 10 \cdot \frac{10}{40} + 10 \cdot \frac{10}{40} \cdot 0.529$$

$$a_{1} = -5.82$$

$$b_{1} = 1 - \theta - x_{f} - \frac{P_{l}}{P_{h}} + \frac{P_{l}}{P_{h}}\theta + \alpha^{*}\theta + \alpha^{*}\frac{P_{l}}{P_{h}} - \alpha^{*}\frac{P_{l}}{P_{h}}\theta + \alpha^{*}x_{f}$$

$$b_{1} = 1 - .529 - 0.5 - \frac{10}{40} + \frac{10}{40} \cdot 0.529 + 10 \cdot 0.529 + 10 \cdot \frac{10}{40} - 10 \cdot \frac{10}{40} \cdot 0.529 + 10 \cdot 0.5$$

$$b_{1} = 11.3$$

$$c_{1} = -\alpha^{*}x_{f}$$

$$c_{1} = -10 \cdot 0.5 = -5$$

$$y_{p} = \frac{-b_{1} + \sqrt{b_{1}^{2} - 4a_{1}c_{1}}}{2a_{1}}$$

$$y_{p} = \frac{-11.3 + \sqrt{11.3^{2} - 4 \cdot -5.82 \cdot -5}}{2 \cdot -5.8}$$

$$y_{p} = 0.678$$

$$x_{0} = \frac{0.5 - 0.529 \cdot 0.678}{1 - 0.529}$$

$$x_{0} = 0.3$$

$$A_{m} = \frac{\theta L_{f}y_{p}}{\frac{P_{l}'}{t} (P_{h}x_{0} - P_{l}y_{h})}}{\theta - \frac{10}{0.0015} (40 \cdot 0.3 - 10 \cdot 0.678)}$$

$$A_{m} = \frac{0.529 \cdot 2000 \cdot 0.678}{\frac{4\cdot10^{-9}}{0.0015} (40 \cdot 0.3 - 10 \cdot 0.678)}$$

$$A_{m} = 5.15 \cdot 10^{7} \text{ cm}^{3}$$

(b)

$$x_{0M} = \frac{x_f \left[ 1 + (\alpha^* - 1) \frac{P_l}{P_h} (1 - x_f) \right]}{\alpha^* (1 - x_f) + x_f}$$
$$x_{0M} = \frac{0.5 \left[ 1 + (10 - 1) \frac{10}{40} (1 - 0.5) \right]}{10 (1 - 0.5) + 0.5}$$
$$x_{0M} = 0.193$$

(c)

Same as part b except that  $x_f = 0.6$ 

$$x_{0M} = \frac{0.6 \left[ 1 + (10 - 1) \frac{10}{40} (1 - 0.6) \right]}{10 (1 - 0.6) + 0.6}$$

$$\boxed{x_{0M} = 0.2478}$$

## 3. Problem 25.2-4

(a)

$$x_{f} = 0.413$$

$$\alpha^{*} = 10$$

$$P_{l} = 20$$

$$P_{h} = 80$$

$$x_{0M} = \frac{x_{f} \left[ 1 + (\alpha^{*} - 1) \frac{P_{l}}{P_{h}} (1 - x_{f}) \right]}{\alpha^{*} (1 - x_{f}) + x_{f}}$$

$$x_{0M} = \frac{0.413 \left[ 1 + (10 - 1) \frac{20}{80} (1 - 0.413) \right]}{10 (1 - 0.413) + 0.413}$$

$$x_{0M} = 0.153$$

(b)

$$\alpha^* = 5$$

$$x_{0M} = \frac{0.413 \left[ 1 + (5 - 1) \frac{20}{80} (1 - 0.413) \right]}{5 (1 - 0.413) + 0.413}$$

$$\boxed{x_{0M} = 0.196}$$

(c)

$$\alpha^* = 1$$

$$x_{0M} = \frac{0.413 \left[ 1 + (1 - 1) \frac{20}{80} (1 - 0.413) \right]}{1 (1 - 0.413) + 0.413}$$

$$x_{0M} = 0.413$$

