CHEN 324 HW 3

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1. Problem 21.1-2

(a)

$$N_A = \frac{D^*_{AB}}{z_2 - z_1}(C_{A2} - C_{A1})$$
$$N_A = \frac{D^*_{AB}}{(z_2 - z_1)(1 - y_A)_M}(C_{A2} - C_{A1})$$

Lump scalars into k

$$k'_c = \frac{D^*_{AB}}{z_2 - z_1}$$
$$k_c = \frac{D^*_{AB}}{(z_2 - z_1)(1 - y_A)_M}$$
$$k_c = \frac{k'_c}{(1 - y_A)_M}$$
$$N_A = k_c(C_{A2} - C_{A1})$$
$$N_A = k_c(y_{A2} - y_{A1})C$$
$$N_A = k_y(y_{A2} - y_{A1})$$
$$k_y(y_{A2} - y_{A1}) = k_c(y_{A2} - y_{A1})C$$
$$k_y = k_c C$$
$$\boxed{k_y = \frac{k'_c C}{(1 - y_A)_M}}$$
$$C = \frac{P}{RT}$$
$$N_A = k_c(y_{A2} - y_{A1})\frac{P}{RT}$$
$$N_A = k_c(P_{A2} - P_{A1})\frac{1}{RT}$$
$$N_A = k_G(P_{A2} - P_{A1})$$
$$k_G(P_{A2} - P_{A1}) = k_c(P_{A2} - P_{A1})\frac{1}{RT}$$
$$k_G = \frac{k_c}{RT}$$
$$\boxed{k_G = \frac{k'_c}{RT(1 - y_A)_M}}$$

(b)

$$\begin{aligned}
N_A &= k_c(x_{A2} - x_{A1})C \\
k_c &= k_L \\
N_A &= k_L(x_{A2} - x_{A1})C \\
N_A &= k_x(x_{A2} - x_{A1}) \\
k_x(x_{A2} - x_{A1}) &= k_L(x_{A2} - x_{A1})C \\
&\boxed{k_x = k_L C} \\
k_L &= \frac{k'_L}{(1 - x_A)_M} \\
k'_x &= k'_L C \\
k_L &= \frac{k'_x}{C(1 - x_A)_M} \\
&\boxed{k'_x = k_L C(1 - x_A)_M}
\end{aligned}$$

(c)

$$\begin{aligned}
N_A &= k_G(P_{A1} - P_{A2}) \\
N_A &= k_G(y_{A1} - y_{A2})P \\
N_A &= k_y(y_{A1} - y_{A2}) \\
k_y(y_{A1} - y_{A2}) &= k_G(y_{A1} - y_{A2})P \\
&\boxed{k_y = k_G P} \\
C &= \frac{P}{RT} \\
P &= CRT \\
N_A &= k_G(y_{A1} - y_{A2})CRT \\
N_A &= k_G(C_{A1} - C_{A2})RT \\
N_A &= k_c(C_{A1} - C_{A2}) \\
k_c(C_{A1} - C_{A2}) &= k_G(C_{A1} - C_{A2})RT \\
&\boxed{k_c = k_G RT}
\end{aligned}$$

2. Problem 21.1-3

$$
\begin{aligned}
x_{A1} &= 2 \cdot 10^{-5} \\
P_{A1} &= 609 \cdot 2 \cdot 10^{-5} \\
P_{A1} &= 0.01218 \\
P_{A2} &= 0.05 \\
k'_c &= 9.567 \cdot 10^{-4} \\
k'_G &= \frac{k'_c}{RT} \\
k'_G &= \frac{9.567 \cdot 10^{-4}}{0.08306 \cdot 303.15} \\
k'_G &= 3.85 \cdot 10^{-5} \\
N_A &= k'_G(P_{A1} - P_{A2}) \\
N_A &= 9.567 \cdot 10^{-4} \cdot (0.01218 - 0.05) \\
&\boxed{N_A = -1.45 \cdot 10^{-6} \frac{\text{kgmol}}{\text{s} \cdot \text{m}^2}}
\end{aligned}
$$

3. Problem 21.3-3

$$
\begin{aligned}
T &= 338.6 \\
P &= 101320 \\
v &= 3.66 \\
\rho &= 1.043 \\
\mu &= 2.03 \cdot 10^{-5} \\
D_{AB} &= 0.288 \cdot 10^{-4}
\end{aligned}
$$

Adjust D_{AB} for elevated temperature:

$$
\begin{aligned}
D_{AB} &= 0.288 \cdot 10^{-4} \cdot \frac{338.6^{1.75}}{315^{1.75}} \\
D_{AB} &= 0.327 \cdot 10^{-4}
\end{aligned}
$$

(a) Single sphere

$$
\begin{aligned}
D &= 0.0254 \\
Re &= \frac{Dv\rho}{\mu} \\
Re &= \frac{0.0254 \cdot 1.043 \cdot 3.66}{2.03 \cdot 10^{-5}} \\
Re &= 4776 \\
Sc &= \frac{\mu}{\rho D_{AB}} \\
Sc &= \frac{2.03 \cdot 10^{-5}}{1.043 \cdot 0.327 \cdot 10^{-4}} \\
Sc &= 0.596
\end{aligned}
$$

Gas over sphere Sc = 0.6 - 2.7 and Re 1 - 48,000:

$$\begin{aligned}
Sh &= 2 + 0.552 \cdot Re^{0.53} Sc^{\frac{1}{3}} \\
Sh &= 2 + 0.552 \cdot 4776^{0.53} 0.596^{\frac{1}{3}} \\
Sh &= 43.4 \\
Sh &= k_c' \frac{D}{D_{AB}} \\
k_c' &= \frac{Sh D_{AB}}{D} \\
k_c' &= \frac{43.4 \cdot 0.327 \cdot 10^{-4}}{0.0254} \\
k_c' &= 0.0558 \\
k_c' \frac{P}{RT} &= k_G' P \\
k_G' &= \frac{k_c'}{RT} \\
k_G' &= \frac{0.0558}{8.314 \cdot 101320} \cdot \frac{\text{kgmol}}{1000\text{mol}} \\
&\boxed{k_G' = 1.98 \cdot 10^{-8} \frac{\text{kgmol}}{\text{s} \cdot \text{m}^2 \cdot \text{Pa}}}
\end{aligned}$$

(b) Packed bed of spheres

Gas over a packed bed Re = 10 - 10,000:

$$\begin{aligned}
J_D &= \frac{0.4548}{\epsilon} \cdot Re^{-0.4069} \\
\epsilon &= 0.35 \\
J_D &= \frac{0.4548}{0.35} \cdot 4776^{-0.4069} \\
J_D &= 0.0413 \\
J_D &= \frac{k_c'}{v} \cdot Sc^{\frac{2}{3}} \\
k_c' &= \frac{J_D v}{Sc^{\frac{2}{3}}} \\
k_c' &= \frac{0.0413 \cdot 3.66}{0.596^{\frac{2}{3}}} \\
k_c' &= 0.214 \\
k_G' &= \frac{0.214}{8.314 \cdot 101320} \cdot \frac{\text{kgmol}}{1000\text{mol}} \\
&\boxed{k_G' = 7.6 \cdot 10^{-8} \frac{\text{kgmol}}{\text{s} \cdot \text{m}^2 \cdot \text{Pa}}}
\end{aligned}$$

4. Problem 21.4-2

$$N_{Sc} = \frac{\mu_c}{\rho_c D_{AB}}$$
$$\mu_c = 6.947 \cdot 10^{-4}$$
$$\rho_c = 944$$
$$D_{AB} = 3.25 \cdot 10^{-9}$$
$$N_{Sc} = \frac{6.947 \cdot 10^{-4}}{944 \cdot 3.25 \cdot 10^{-9}}$$
$$N_{Sc} = 215$$
$$k_L' = \frac{2D_{AB}}{D_p} + 0.31 N_{Sc}^{-\frac{2}{3}} \left(\frac{\Delta\rho\mu_c g}{\rho_c^3} \right)$$
$$\rho_p = 1100$$
$$D_p = 0.667 \cdot 10^{-6}$$
$$k_L' = \frac{2 \cdot 3.25 \cdot 10^{-9}}{0.667 \cdot 10^{-6}} + 0.31 \cdot 215^{-\frac{2}{3}} \left(\frac{(944 - 1100) \cdot 6.947 \cdot 10^{-4} \cdot 9.81}{944^3} \right)$$
$$k_L' = 9.73 \cdot 10^{-3}$$
$$k_L = k_L'$$
$$N = k_L(C_{A1} - C_{A2})$$
$$C_{A2} = 0$$
$$C_{A1} = 2.29 \cdot 10^{-4}$$
$$N_A = 9.73 \cdot 10^{-3} \cdot (2.29 \cdot 10^{-4} - 0)$$
$$N_A = 2.23 \cdot 10^{-6} \frac{\text{kgmol}}{\text{m}^2\text{s}}$$
$$A = \pi D^2$$
$$V = \frac{\pi}{6} D^3$$
$$V_T = \frac{m}{\rho}$$
$$V_T = \frac{5}{1000 \cdot 1100} = 4.5 \cdot 10^{-6}$$
$$V = \frac{\pi}{6} \cdot (0.667 \cdot 10^{-6})^3 = 1.55 \cdot 10^{-19}$$
$$N = \frac{V_T}{V} = \frac{4.5 \cdot 10^{-6}}{1.55 \cdot 10^{-19}} = 2.93 \cdot 10^{13}$$
$$A = \pi \cdot (0.667 \cdot 10^{6})^2 = 1.39 \cdot 10^{-12}$$
$$A_T = NA = 2.93 \cdot 10^{13} \cdot 1.39 \cdot 10^{-12} = 40.9$$
$$N = A_T N_A = 2.23 \cdot 10^{-6} \cdot 40.5$$
$$\boxed{N = 9.12 \cdot 10^{-5} \frac{\text{kgmol}}{\text{s}}}$$

5. Problem 21.2-1

$$f\left(k_c', D, \rho, \mu, v, D_{AB}, g, \Delta\rho, L\right) = 0$$

Π groups:

$$\begin{aligned}
\Pi_1 &= D^{a_1}\rho^{b_1}\mu^{c_1}k_c' \\
\Pi_2 &= D^{a_2}\rho^{b_2}\mu^{c_2}v \\
\Pi_3 &= D^{a_3}\rho^{b_3}\mu^{c_3}D_{AB}^* \\
\Pi_4 &= D^{a_4}\rho^{b_4}\mu^{c_4}g \\
\Pi_5 &= D^{a_5}\rho^{b_5}\mu^{c_5}\Delta\rho \\
\Pi_6 &= D^{a_6}\rho^{b_6}\mu^{c_6}L
\end{aligned}$$

Π_1:

$$\begin{aligned}
\Pi_1 &= (L)^{a_1}\left(\frac{M}{L^3}\right)^{b_1}\left(\frac{M}{LT}\right)^{c_1}\left(\frac{L}{T}\right) \\
a_1 - 3b_1 - c_1 + 1 &= 0 \\
b_1 + c_1 &= 0 \\
-c_1 - 1 &= 0 \\
c_1 &= -1 \\
b_1 &= 1 \\
a_1 &= 1 \\
\Pi_1 &= \frac{D\rho k_c'}{\mu}
\end{aligned}$$

Π_2:

$$\begin{aligned}
\Pi_2 &= (L)^{a_1}\left(\frac{M}{L^3}\right)^{b_1}\left(\frac{M}{LT}\right)^{c_1}\left(\frac{L}{T}\right) \\
a_2 - 3b_2 - c_2 + 1 &= 0 \\
b_2 + c_2 &= 0 \\
-c_2 - 1 &= 0 \\
c_2 &= -1 \\
b_2 &= 1 \\
a_2 &= 1 \\
\Pi_2 &= \frac{D\rho v}{\mu} = N_{Re}
\end{aligned}$$

Π_3:

$$\Pi_3 = (L)^{a_1} \left(\frac{M}{L^3}\right)^{b_1} \left(\frac{M}{LT}\right)^{c_1} \left(\frac{L^2}{T}\right)$$

$$\begin{aligned}
a_3 - 3b_3 - c_3 + 2 &= 0 \\
b_3 + c_3 &= 0 \\
-c_3 - 1 &= 0 \\
c_3 &= -1 \\
b_3 &= 1 \\
a_3 &= 0 \\
\Pi_3 &= \frac{\rho D^*_{AB}}{\mu} = \frac{1}{N_{Sc}}
\end{aligned}$$

Π_4:

$$\Pi_4 = (L)^{a_1} \left(\frac{M}{L^3}\right)^{b_1} \left(\frac{M}{LT}\right)^{c_1} \left(\frac{L}{T^2}\right)$$

$$\begin{aligned}
a_4 - 3b_4 - c_4 + 1 &= 0 \\
b_4 + c_4 &= 0 \\
-c_4 - 2 &= 0 \\
c_4 &= -2 \\
b_4 &= 2 \\
\Pi_4 &= \frac{D^3 \rho^2 g}{\mu^2}
\end{aligned}$$

Π_5:

$$\Pi_5 = (L)^{a_1} \left(\frac{M}{L^3}\right)^{b_1} \left(\frac{M}{LT}\right)^{c_1} \left(\frac{M}{L^3}\right)$$

$$\begin{aligned}
a_5 - 3b_5 - c_5 - 3 &= 0 \\
b_5 + c_5 + 1 &= 0 \\
-c_5 &= 0 \\
c_5 &= 0 \\
b_5 &= -1 \\
a_5 &= 0 \\
\Pi_5 &= \frac{\Delta\rho}{\rho}
\end{aligned}$$

Π_6:

$$\Pi_6 = (L)^{a_1}\left(\frac{M}{L^3}\right)^{b_1}\left(\frac{M}{LT}\right)^{c_1}(L)$$

$$\begin{aligned}
a_6 - 3b_6 - c_6 + 1 &= 0 \\
b_6 + c_6 &= 0 \\
-c_6 &= 0 \\
c_6 &= 0 \\
b_6 &= 0 \\
a_6 &= -1 \\
\Pi_6 &= \frac{L}{D}
\end{aligned}$$

Final dimensionless groups:

$$\Pi_1 = \frac{D\rho k_c'}{\mu}, \Pi_2 = \frac{D\rho v}{\mu}, \Pi_3 = \frac{\rho D_{AB}^*}{\mu}, \Pi_4 = \frac{D^3\rho^2 g}{\mu^2}, \Pi_5 = \frac{\Delta\rho}{\rho}, \Pi_6 = \frac{L}{D}$$

Rearrangements:

$$\begin{aligned}
\frac{\Pi_1}{\Pi_3} &= \frac{\frac{D\rho k_c'}{\mu}}{\frac{\rho D_{AB}^*}{\mu}} = \frac{k_c' D}{D_{AB}^*} = N_{Sh} \\
\Pi_6^3 \cdot \Pi_4 \cdot \Pi_5 &= \frac{L^3}{D^3} \cdot \frac{D^3\rho^2 g}{\mu^2} \cdot \frac{\Delta\rho}{\rho} = \frac{gL^3\rho\Delta\rho}{\mu^2} = N_{Gr} \\
\Pi_2 &= \frac{Dv\rho}{\mu} = N_{Re} \\
\frac{1}{\Pi_3} &= \frac{\mu}{\rho D_{AB}^*} = N_{Sc}
\end{aligned}$$

Final dimensionless relationship:

$$\begin{aligned}
\frac{\Pi_1}{\Pi_3} &= f\left(\Pi_6^3 \cdot \Pi_4 \cdot \Pi_5, \Pi_2, \frac{1}{\Pi_3}\right) \\
&\boxed{\frac{k_c' D}{D_{AB}^*} = f\left(\frac{gL^3\rho\Delta\rho}{\mu^2}, \frac{Dv\rho}{\mu}, \frac{\mu}{\rho D_{AB}^*}\right)} \\
N_{Sh} &= f\left(N_{Gr}, N_{Re}, N_{Sc}\right)
\end{aligned}$$