## Membranes

$$\theta = \frac{V_p}{L_f}$$

$$\alpha^* = \frac{P_A'}{P_B'}$$

$$\frac{y_p}{1 - y_p} = \frac{\alpha^* \left[ x_0 - \left( \frac{P_l}{P_h} \right) y_p \right]}{(1 - x_0) - \left( \frac{P_l}{P_h} \right) (1 - y_p)}$$

## Case 1:

$$r = \frac{P_l}{P_h}$$

$$a = 1 - \alpha^*$$

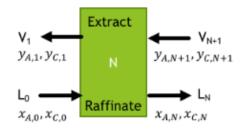
$$b = -1 + \alpha^* + \frac{1}{r} + \frac{x_0}{r} (\alpha^* - 1)$$

$$c = \frac{-\alpha^* x_0}{r}$$

$$y_p = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{split} y_p &= \frac{-b_1 + \sqrt{b_1^2 - 4a_1c_1}}{2a_1} \\ a_1 &= \theta + \frac{P_l}{P_h} - \frac{P_l}{P_h}\theta - \alpha^*\theta - \alpha^*\frac{P_l}{P_h} + \alpha^*\frac{P_l}{P_h}\theta \\ b_1 &= 1 - \theta - x_f - \frac{P_l}{P_h} + \frac{P_l}{P_h}\theta + \alpha^*\theta + \alpha^*\frac{P_l}{P_h} - \alpha^*\frac{P_l}{P_h}\theta + \alpha^*x_f \\ c_1 &= -\alpha^*x_f \\ x_0 &= \frac{x_f - \theta y_p}{1 - \theta} \\ y_p &= \frac{x_f - x_0(1 - \theta)}{\theta} \\ A_m &= \frac{\theta L_f y_p}{\frac{P_A'}{t}(P_h x_0 - P_l y_h)} \\ x_{0M} &= \frac{x_f \left[1 + (\alpha^* - 1)\frac{P_l}{P_h}\left(1 - x_f\right)\right]}{\alpha^*\left(1 - x_f\right) + x_f} \end{split}$$

## Liquid-Liquid Extraction



V: organic layer; L: aqueous layer;

Stripping: removing from aqueous layer

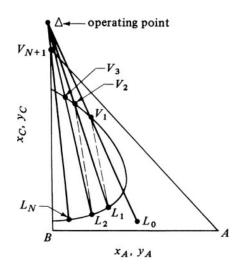
$$N = \frac{\ln\left[\frac{x_0 - y_{N+1}/m}{x_N - y_{N+1}/m}(1 - A) + A\right]}{\ln(1/A)}$$

Absorption: removing from organic layer 
$$N = \frac{\ln\left[\frac{y_{N+1} - mx_0}{y_1 - mx_0}\left(1 - \frac{1}{A}\right) + \frac{1}{A}\right]}{\ln(A)}$$

$$A = \frac{L}{mV}$$

$$A = \sqrt{A_N A_1}$$

$$\Delta x_{\Delta} = \frac{L_0 x_0 - V_1 y_1}{L_0 - V_1} = \frac{L_N x_N - V_{N+1} y_{N+1}}{L_N - V_{N+1}}$$



Adsorption

Mass Balance:  $q_0M + C_0S = q_fM + C_fS$ H from geometry:  $H_T = \frac{M}{\rho A_c \phi}$ ;  $\phi$ : void fraction

$$t_{t} = \int_{0}^{\infty} \left(1 - \frac{c}{c_{0}}\right) dt$$

$$t_{u} = \int_{0}^{t_{b}} \left(1 - \frac{c}{c_{0}}\right) dt$$

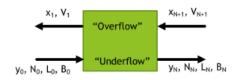
$$\eta = \frac{t_{u}}{t_{t}}$$

$$H_{B} = \frac{t_{u}}{t_{t}} H_{T}$$

$$H_{\text{UNB}} = \left(1 - \frac{t_{u}}{t_{b}}\right) H_{T}$$

$$\frac{Q_{1}}{D_{1}^{2}} = \frac{Q_{2}}{D_{2}^{2}}$$

$$Q = v \frac{\pi}{4} D^{2}$$
Leaching



 $\mathbf{V}$ : overflow layer;  $\mathbf{L}$ : liquid = A + C;  $\mathbf{x}$ : overflow liquid; y: liquid in slurry;

$$N = \frac{B}{L}$$

$$x_{\Delta} = \frac{L_{0}y_{0} - V_{1}x_{1}}{L_{0} - V_{1}} = \frac{L_{N}y_{N} - V_{N+1}x_{N+1}}{L_{N} - V_{N+1}}$$

$$N_{\Delta} = \frac{B}{L_{0} - V_{1}} = \frac{N_{0}L_{0}}{L_{0} - V_{1}}$$

