CHEN 324 HW 2

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1. Problem 19.1-9

   (a) Derivation

$$
\begin{aligned}
N_B &= 0 \\
N_A &= -cD_{AB}\frac{dx_A}{dz} + \frac{C_A}{c}(N_A + 0) \\
N_A &= -cD_{AB}\frac{dx_A}{dz} + \frac{C_A}{c}N_A \\
c &= \frac{P}{RT} \\
P_A &= x_A P \\
dx_A &= \frac{dP_A}{P} \\
\frac{C_A}{c} &= \frac{P_A}{P} \\
N_A &= -\frac{P}{RT}D_{AB}\frac{dP_A}{Pdz} + \frac{P_A}{P}N_A \\
N_A &= -\frac{D_{AB}}{RT}\frac{dP_A}{dz} + \frac{P_A}{P}N_A \\
N_A\left(1 - \frac{P_A}{P}\right) &= -\frac{D_{AB}}{RT}\frac{dP_A}{dz} \\
N_A\int_{z_1}^{z_2} dz &= -\frac{D_{AB}}{RT}\int_{P_{A1}}^{P_{A2}} \frac{dP_A}{1 - \frac{P_A}{P}} \\
N_A(z_2 - z_1) &= \frac{D_{AB}P}{RT}\ln\left(\frac{P - P_{A2}}{P - P_{A1}}\right) \\
P_{BM} &= \frac{P_{A1} - P_{A2}}{\ln\left(\frac{P - P_{A2}}{P - P_{A1}}\right)} \\
N_A(z_2 - z_1) &= \frac{D_{AB}P(P_{A1} - P_{A2})}{RTP_{BM}} \\
N_A &= \frac{\rho_A}{M_A}\frac{dz}{dt} \\
r_1 &= z_2 - z_1 \\
\frac{\rho_A}{M_A}\int_{r_2}^{r_1} r_1 dr &= \frac{D_{AB}P(P_{A1} - P_{A2})}{RTP_{BM}}\int dt \\
\frac{\rho_A}{2M_A}(r_1^2 - r_2^2) &= \frac{D_{AB}P(P_{A1} - P_{A2})}{RTP_{BM}}t_F \\
r_2 &= 0 \\
\frac{\rho_A}{2M_A}r_1^2 &= \frac{D_{AB}P(P_{A1} - P_{A2})}{RTP_{BM}}t_F
\end{aligned}
$$

$$
\boxed{t_F = \frac{\rho_A RTP_{BM}r_1^2}{2M_A D_{AB}P(P_{A1} - P_{A2})}}
$$

(b) Calculation

$$
\begin{aligned}
t_F &= \frac{\rho_A R T P_{BM} r^2}{2 M_A D_{AB} P (P_{A1} - P_{A2})} \\
\rho_A &= 866 \\
R &= 8.314 \\
T &= 25.9^\circ\mathrm{C} = 299.05\mathrm{K} \\
P_{A1} &= 3.84 \\
P_{A2} &= 0 \\
P_{BM} &= \frac{3.82 - 0}{\ln\left(\frac{101.325-0}{101.325-3.84}\right)} = 99.39 \\
r_1 &= 0.002 \\
M_A &= 92.14 \\
D_{AB} &= 0.086 \cdot 10^{-4} \\
P &= 101.325 \\
t_F &= \frac{866 \cdot 8.314 \cdot 299.05 \cdot 99.39 \cdot 0.002^2}{2 \cdot 92.14 \cdot 0.086 \cdot 10^{-4} \cdot 101.325 (3.84 - 0)} \\
\boxed{t_F &= 1388.2\mathrm{s}}
\end{aligned}
$$

2. Problem 19.1-21

(a)

$$2\text{A} \rightarrow \text{B}$$

$$\frac{dC_A}{t} = -\nabla N_A + r_A$$

Steady state

$$\begin{aligned}
-\nabla N_A + r_A &= 0 \\
N_A &= -D_{AB}\frac{dC_A}{dz} + x_A N_T \\
\frac{1}{2}N_A &= -N_B \\
N_T &= N_A - \frac{1}{2}N_A \\
N_T &= \frac{1}{2}N_A \\
N_A &= -D_{AB}\frac{dC_A}{dz} + \frac{1}{2}N_A x_A \\
N_A &= \frac{-D_{AB}C dx_A}{(1-\frac{x_A}{2})dz} \\
N_A \int dz &= -D_{AB}C \int \frac{dx_A}{1-\frac{x_A}{2}} \\
C &= \frac{P}{RT} \\
\boxed{N_A &= \frac{2D_{AB}P}{\delta RT}\ln\left(\frac{x_{A2}-2}{x_{A1}-2}\right)}
\end{aligned}$$

(b) Calculation

$$\begin{aligned}
D_{AB} &= 0.2 \cdot 10^{-4} \\
P &= 101.32 \\
X_{A1} &= 0.97 \\
X_{A2} &= 0 \\
\delta &= 0.0013 \\
R &= 8.314 \\
T &= 298 \\
N_A &= \frac{2 \cdot 0.2 \cdot 10^{-4} \cdot 101.32}{0.0013 \cdot 8.314 \cdot 298}\ln\left(\frac{0-2}{0.97-2}\right) \\
\boxed{N_A &= 8.35 \cdot 10^{-4}\frac{\text{kgmol}}{\text{m}^2 \cdot \text{s}}}
\end{aligned}$$

(c) Finite reaction rate

$$
\begin{aligned}
N_A A_s &= k_1' C_{A2} V_L \\
C_{A2} &= \frac{x_{A2} P}{RT} \\
N_A A_s &= k_1' \frac{x_{A2} P}{RT} V_L \\
x_{A2} &= \frac{N_A A_s RT}{k_1' P V_L} \\
N_A &= \frac{2 D_{AB} P}{\delta RT} \ln\left(\frac{x_{A2} - 2}{x_{A1} - 2}\right) \\
\boxed{N_A &= \frac{2 D_{AB} P}{\delta RT} \ln\left(\frac{\frac{N_A A_s RT}{k_1' P V_L} - 2}{x_{A1} - 2}\right)}
\end{aligned}
$$

(d) Calculation

$$
\begin{aligned}
N_A &= \frac{2 D_{AB} P}{\delta RT} \ln\left(\frac{\frac{N_A A_s RT}{k_1' P V_L} - 2}{x_{A1} - 2}\right) \\
D_{AB} &= 0.2 \cdot 10^{-4} \\
P &= 101.32 \\
X_{A1} &= 0.97 \\
X_{A2} &= 0 \\
\delta &= 0.0013 \\
R &= 8.314 \\
T &= 298 \\
k_1' &= 0.53 \cdot 10^{-2}
\end{aligned}
$$

Solve the system

$$
\boxed{N_A = 1.766 \cdot 10^{-4} \frac{\text{kgmol}}{\text{m}^2 \cdot \text{s}}}
$$

3. Problem 19.3-1

Steady state no reaction

$$N_A = \frac{D_{AB}C}{1-x_A}\frac{dx_A}{dz}$$
$$N_A = \frac{D_{AB}C}{\delta x_{BM}}(x_{A1} - x_{A2})$$
$$N_A = \frac{D_{AB}}{\delta x_{BM}}(C_{A1} - C_{A2})$$
$$C_A = \frac{SP_A}{22.414}$$
$$x_{BM} = 1$$
$$N_A = \frac{D_{AB}S}{t_M 22.414}(P_{A1} - P_{A2})$$
$$D_{AB} = 0.11 \cdot 10^{-9}$$
$$S = 0.90$$
$$t_M = 0.030$$
$$P_{A1} = 2$$
$$P_{A2} = 1$$
$$A_c = 4 \cdot 10^{-4}$$
$$N_A = \frac{D_{AB}S}{t_M 22.414}(P_{A1} - P_{A2})$$
$$N_A = \frac{D_{AB}S}{t_M 22.414}(P_{A1} - P_{A2})A_c$$
$$N_A = \frac{0.11 \cdot 10^{-9} \cdot 0.90}{0.030 \cdot 22.414}(2-0) \cdot 4 \cdot 10^{-4}$$
$$\boxed{N_A = 1.1778 \cdot 10^{-13} \frac{\text{kgmol}(CO_2)}{\text{m}^2 \cdot \text{s}}}$$

4. Problem 20.10-4

Convection resistance is zero

$$\frac{k_c}{D_{AB}}\sqrt{D_{AB}t} = \infty$$

$$t = 3600$$

$$C_0 = 0.14$$

$$C_1 = 0.03$$

for $x = 0.005$m

$$\frac{x}{2\sqrt{D_{AB}t}} = \frac{0.005}{2\sqrt{1.29 \cdot 10^{-8} \cdot 3600}} = 0.367$$

From Figure 14.3-3

$$\frac{C - C_0}{C_1 - C_0} \approx 0.6$$

$$\frac{C - 0.14}{0.03 - 0.14} = 0.6$$

$$\boxed{C = 0.074}$$

for $x = 0.01$m

$$\frac{x}{2\sqrt{D_{AB}t}} = \frac{0.01}{2\sqrt{1.29 \cdot 10^{-8} \cdot 3600}} = 0.734$$

From Figure 14.3-3

$$\frac{C - C_0}{C_1 - C_0} \approx 0.28$$

$$\frac{C - 0.14}{0.03 - 0.14} = 0.28$$

$$\boxed{C = 0.109}$$

for $x = 0.02$m

$$\frac{x}{2\sqrt{D_{AB}t}} = \frac{0.02}{2\sqrt{1.29 \cdot 10^{-8} \cdot 3600}} = 1.467$$

From Figure 14.3-3

$$\frac{C - C_0}{C_1 - C_0} \approx 0.039$$

$$\frac{C - 0.14}{0.03 - 0.14} = 0.039$$

$$\boxed{C = 0.136}$$

Plot of depth v weight%: