

1. Problem 7.7

(a)

$$\begin{aligned}\frac{dC_R}{dt} &= -\frac{F}{V}C_R - k_1C_R - k_3C_R^2 + \frac{F}{V}C_{R,in} \\ \frac{dC_P}{dt} &= -\frac{F}{V}C_P - k_2C_P + k_1C_R\end{aligned}$$

At steady state

$$\begin{aligned}0 &= -\frac{F_s}{V}C_R - k_1C_R - k_3C_R^2 + \frac{F_s}{V}C_{R,in,s} \\ 0 &= -\frac{F_s}{V}C_P - k_2C_P + k_1C_R \\ F_s &= 21 \\ C_{R,in,s} &= 5.5 \\ V &= 1 \\ k_1 &= 50 \\ k_2 &= 54 \\ k_3 &= 4\end{aligned}$$

Solve the system of equations

$$\boxed{C_{R,s} = 1.5}$$

$$\boxed{C_{P,s} = 1}$$

(b)

Linearize in deviation form

$$\begin{aligned}A &= \begin{bmatrix} -\left(\frac{F_s}{V} + k_1 + 2k_3C_{R,s}\right) & 0 \\ k_1 & -\left(\frac{F_s}{V} + k_2\right) \end{bmatrix} \\ B &= \begin{bmatrix} F_s & C_{R,in,s} - C_{R,s} \\ 0 & -C_{P,s} \end{bmatrix} \\ A &= \begin{bmatrix} -\left(\frac{21}{1} + 50 + 2 \cdot 4 \cdot 1.5\right) & 0 \\ 50 & -\left(\frac{21}{1} + 54\right) \end{bmatrix} \\ A &= \begin{bmatrix} -83 & 0 \\ 50 & -75 \end{bmatrix} \\ B &= \begin{bmatrix} 21 & 5.5 - 1.5 \\ 0 & -1 \end{bmatrix} \\ B &= \begin{bmatrix} 21 & 4 \\ 0 & -1 \end{bmatrix} \\ c &= [0 \quad 1] \\ d &= 0\end{aligned}$$

Linearized system

$$\begin{aligned}\frac{d}{dt} \begin{bmatrix} \overline{C}_R \\ \overline{C}_P \end{bmatrix} &= \begin{bmatrix} -83 & 0 \\ 50 & -75 \end{bmatrix} \begin{bmatrix} \overline{C}_R \\ \overline{C}_P \end{bmatrix} + \begin{bmatrix} 21 & 4 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \overline{C}_{R,in} \\ \overline{F} \end{bmatrix} \\ y &= \overline{C}_P\end{aligned}$$

Check asymptotic stability

$$0 = \begin{bmatrix} \lambda - (-83) & 0 \\ 50 & \lambda - (-75) \end{bmatrix}$$
$$\lambda = -75, -83$$

Eigenvalues are negative, and so the system is asymptotically stable.

(c)

Non-linear system

$$\frac{dC_R}{dt} = -\frac{F}{V}C_R - k_1C_R - k_3C_R^2 + \frac{F}{V}C_{R,in}$$
$$\frac{dC_P}{dt} = -\frac{F}{V}C_P - k_2C_P + k_1C_R$$

Solve the system of ODEs with the following parameters

$$F = 28$$
$$C_{R,in} = 5.5$$
$$V = 1$$
$$k_1 = 50$$
$$k_2 = 54$$
$$k_3 = 4$$

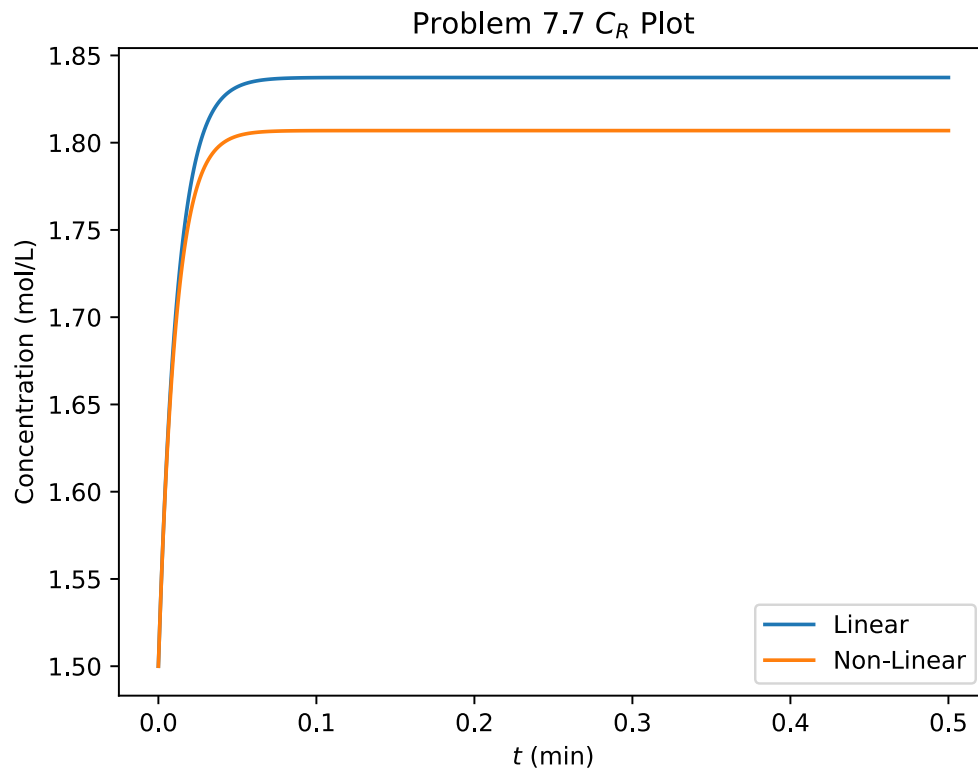
Linear system

$$\frac{d\bar{C}_R}{dt} = -83\bar{C}_R + 21\bar{C}_{R,in} + 4\bar{F}$$
$$\frac{d\bar{C}_P}{dt} = 50\bar{C}_R - 75\bar{C}_P - \bar{F}$$

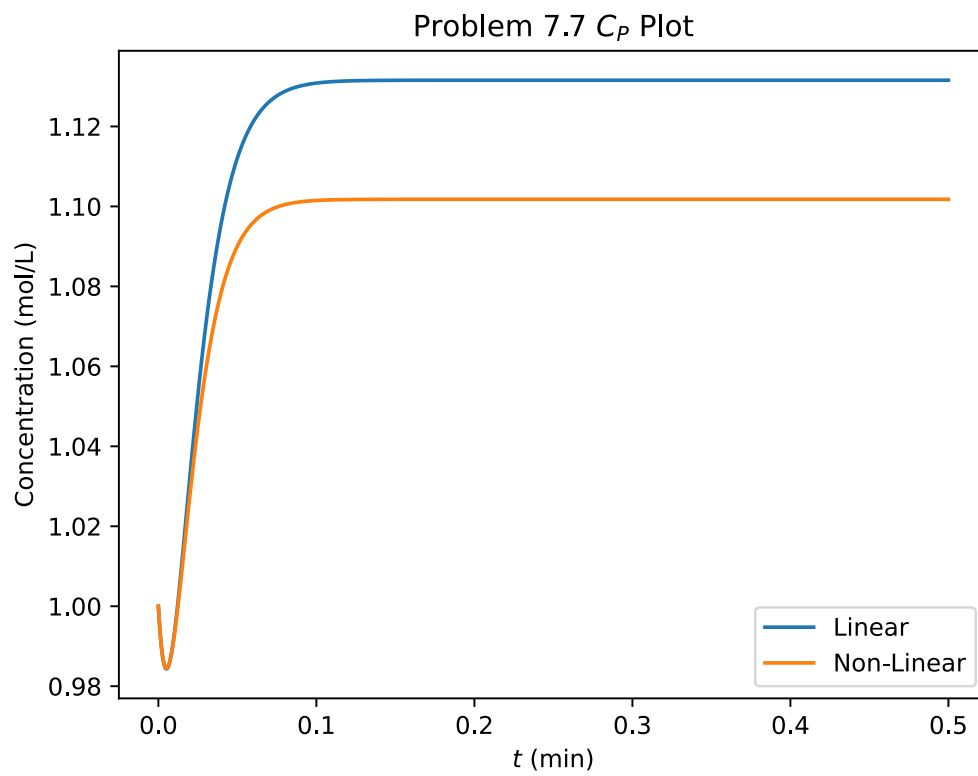
Solve the system of ODEs with the following parameters

$$\bar{F} = 7$$
$$\bar{C}_{R,in} = 0$$
$$k_1 = 50$$
$$k_2 = 54$$
$$k_3 = 4$$

C_R Plot:



C_P Plot:



2. Problem 6.9

(a)

$$\frac{dx_1}{dt} = -83x_1 + 21u_1 + 4u_2$$

$$\frac{dx_2}{dt} = 50x_1 - 75x_2 - u_2$$

$$sX_1 = -83X_1 + 21U_1 + 4U_2$$

$$X_1 = \frac{21}{s+83}U_1 + \frac{4}{s+83}U_2$$

$$sX_2 = 50X_1 - 75X_2 - U_2$$

$$X_2(s+75) = 50X_1 - U_2$$

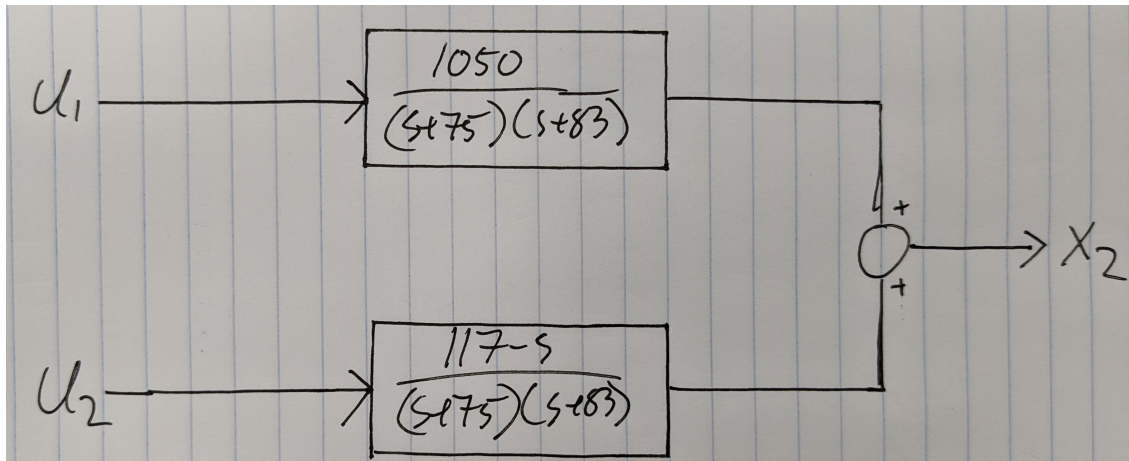
$$X_2(s+75) = 50 \left(\frac{21}{s+83}U_1 + \frac{4}{s+83}U_2 \right) - U_2$$

$$X_2(s+75) = \frac{1050}{s+83}U_1 + \frac{117-s}{s+83}U_2$$

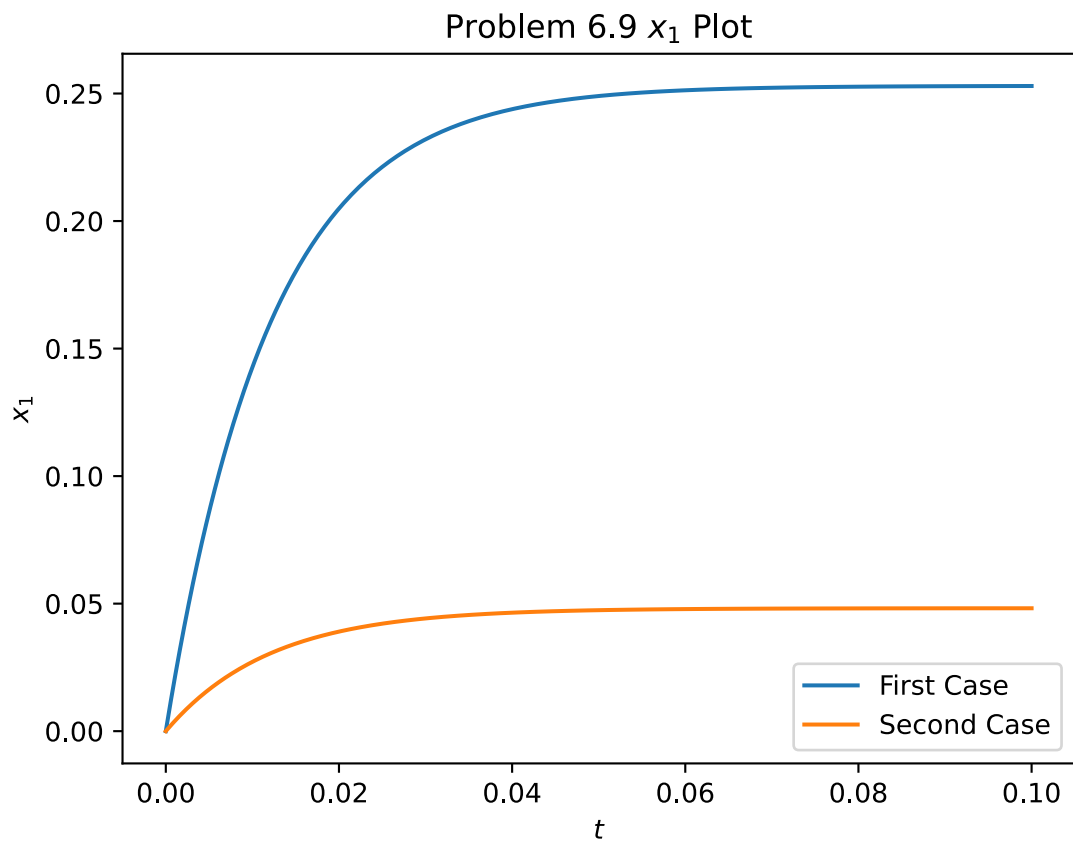
$$X_2 = \frac{1050}{(s+83)(s+75)}U_1 + \frac{117-s}{(s+83)(s+75)}U_2$$

$$G_1(s) = \frac{1050}{(s+83)(s+75)}$$

$$G_2(s) = \frac{117-s}{(s+83)(s+75)}$$



x_1 Plot:



x_2 Plot:

