CHEN 461 HW12

April 29, 2023

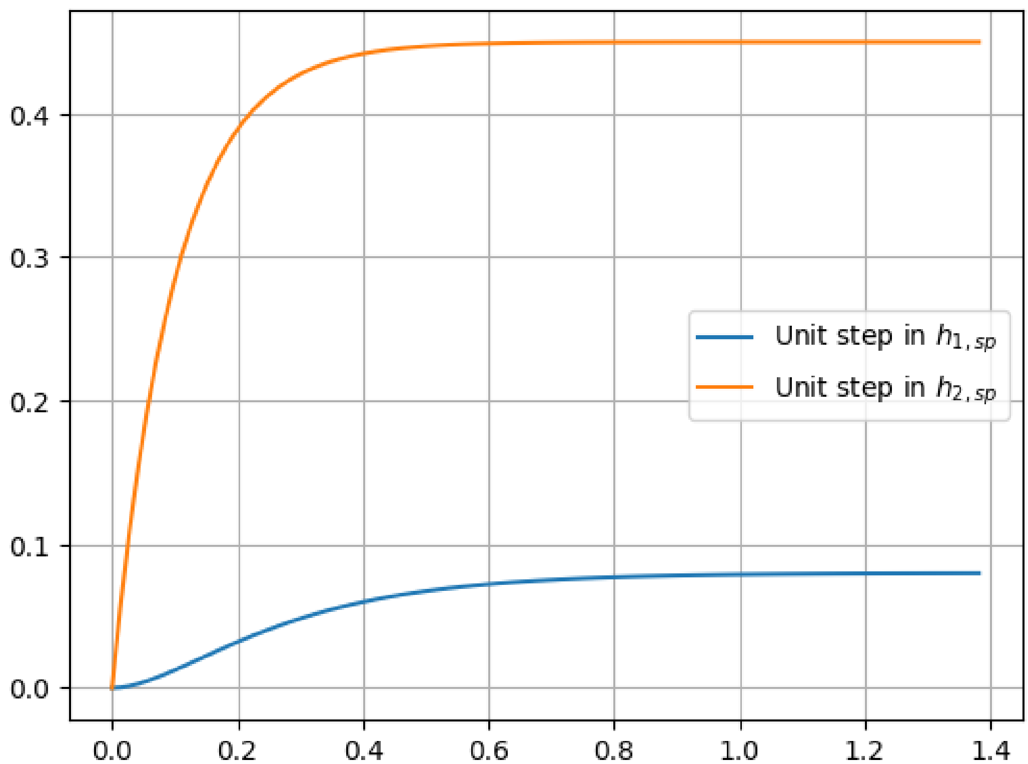
```
[ ]: from control import ss, tf, step_response, input_output_response
from matplotlib.pyplot import plot, xlabel, ylabel, title, legend, grid
from numpy import zeros, ones, linspace, array

from sympy import symbols, expand, simplify, exp
from sympy.abc import s, t, lamda, theta
from sympy.matrices import Matrix
```

1 Problem 18.3

```
[ ]: h_1, h_2, A_1, A_2, R_1, R_2, h_1sp, h_2sp, k_c1, k_c2 = symbols("h_1, h_2,␣
  ↪A_1, A_2, R_1, R_2, h_1sp, h_2sp, k_c1, k_c2")

F_in1 = k_c1 * (h_1sp - h_1)
F_in2 = k_c2 * (h_2sp - h_2)

f_1 = expand((F_in1 - h_1 / R_1) / A_1)
f_2 = expand((F_in2 + h_1 / R_1 - h_2 / R_2) / A_2)

A_sym = Matrix([
    [f_1.coeff(h_1), f_1.coeff(h_2)],
    [f_2.coeff(h_1), f_2.coeff(h_2)],
])

B_sym = Matrix([
    [f_1.coeff(h_1sp), f_1.coeff(h_2sp)],
    [f_2.coeff(h_1sp), f_2.coeff(h_2sp)],
])

c_sym = Matrix([[0, 1 / R_2]])

d = 0

sub_dict = {A_1: 1, A_2: 0.5, R_1: 1, R_2: 2, k_c1: 4, k_c2: 4.5}

A = array(A_sym.subs(sub_dict), dtype=float)
B = array(B_sym.subs(sub_dict), dtype=float)
```

```
c = array(c_sym.subs(sub_dict), dtype=float)

sys = ss(A, B, c, d)
t, y = step_response(sys)

plot(t, y[0, 0], label=r"Unit step in $h_{1,sp}$")
plot(t, y[0, 1], label=r"Unit step in $h_{2,sp}$")
grid()
legend(loc="right")
```

[ ]: <matplotlib.legend.Legend at 0x18b0794fad0>



2 Problem 19.1

2.1 Part A

[ ]:
```
k, tau_0, tau_1, tau_2, tau_3 = symbols("k, tau_0, tau_1, tau_2, tau_3")

Gpp = 1
Gpm = k * (1 + tau_0 * s) / (1 + tau_1 * s) / (1 + tau_2 * s) / (1 + tau_3 * s)
r = 2
```

```
G_c = simplify(1 / ((lamda * s + 1)**r - Gpp) / Gpm)

G_c
```

[ ]: $$\frac{(s\tau_1 + 1)(s\tau_2 + 1)(s\tau_3 + 1)}{k(s\tau_0 + 1)\left((\lambda s + 1)^2 - 1\right)}$$

2.2 Part B

[ ]:
```
Gpp = (1 - tau_0 * s) / (1 + tau_0 * s)

G_c = simplify(1 / ((lamda * s + 1)**r - Gpp) / Gpm)

G_c
```

[ ]: $$\frac{(s\tau_1 + 1)(s\tau_2 + 1)(s\tau_3 + 1)}{k\left(s\tau_0 + (\lambda s + 1)^2(s\tau_0 + 1) - 1\right)}$$

2.3 Part C

[ ]:
```
Gpp = exp(-theta * s)
Gpm = k / (1 + tau_1 * s) / (1 + tau_2 * s)
r = 2
G_c = simplify(1 / ((lamda * s + 1)**r - Gpp) / Gpm)

G_c
```

[ ]: $$\frac{(s\tau_1 + 1)(s\tau_2 + 1)e^{s\theta}}{k\left((\lambda s + 1)^2 e^{s\theta} - 1\right)}$$

2.3.1 Pade

[ ]:
```
Gpp = 1
pade_1 = (1 - theta * s / 2) / (1 + theta * s / 2)
Gpm = k / (1 + tau_1 * s) / (1 + tau_2 * s) * pade_1
r = 2
G_c = simplify(1 / ((lamda * s + 1)**r - Gpp) / Gpm)

G_c
```

[ ]: $$-\frac{(s\tau_1 + 1)(s\tau_2 + 1)(s\theta + 2)}{k(s\theta - 2)\left((\lambda s + 1)^2 - 1\right)}$$