1. Problem 7.7

(a)

$$\frac{dC_R}{dt} = -\frac{F}{V}C_R - k_1 C_R - k_3 C_R^2 + \frac{F}{V}C_{R,in}$$
$$\frac{dC_P}{dt} = -\frac{F}{V}C_P - k_2 C_P + k_1 C_R$$

At steady state

$$0 = -\frac{F_s}{V}C_R - k_1 C_R - k_3 C_R^2 + \frac{F_s}{V}C_{R,in,s}$$
$$0 = -\frac{F_s}{V}C_P - k_2 C_P + k_1 C_R$$
$$F_s = 21$$
$$C_{R,in,s} = 5.5$$
$$V = 1$$
$$k_1 = 50$$
$$k_2 = 54$$
$$k_3 = 4$$

Solve the system of equations

$$\boxed{C_{R,s} = 1.5}$$
$$\boxed{C_{P,s} = 1}$$

(b)

Linearize in deviation form

$$\frac{dC_R}{dt} = f_1(x,u)$$
$$\frac{dC_P}{dt} = f_2(x,u)$$
$$A = \begin{bmatrix} \frac{\partial f_1}{\partial C_R}(x_s,u_s) & \frac{\partial f_1}{\partial C_P}(x_s,u_s) \\ \frac{\partial f_2}{\partial C_R}(x_s,u_s) & \frac{\partial f_2}{\partial C_P}(x_s,u_s) \end{bmatrix}$$
$$A = \begin{bmatrix} -\left(\frac{F_s}{V} + k_1 + 2k_3 C_{R,s}\right) & 0 \\ k_1 & -\left(\frac{F_s}{V} + k_2\right) \end{bmatrix}$$
$$B = \begin{bmatrix} \frac{\partial f_1}{\partial C_{R,in}}(x_s,u_s) & \frac{\partial f_1}{\partial F}(x_s,u_s) \\ \frac{\partial f_2}{\partial C_{R,in}}(x_s,u_s) & \frac{\partial f_2}{\partial F}(x_s,u_s) \end{bmatrix}$$
$$B = \begin{bmatrix} F_s & C_{R,in,s} - C_{R,s} \\ 0 & -C_{P,s} \end{bmatrix}$$
$$A = \begin{bmatrix} -\left(\frac{21}{1} + 50 + 2 \cdot 4 \cdot 1.5\right) & 0 \\ 50 & -\left(\frac{21}{1} + 54\right) \end{bmatrix}$$
$$A = \begin{bmatrix} -83 & 0 \\ 50 & -75 \end{bmatrix}$$
$$B = \begin{bmatrix} 21 & 5.5 - 1.5 \\ 0 & -1 \end{bmatrix}$$
$$B = \begin{bmatrix} 21 & 4 \\ 0 & -1 \end{bmatrix}$$
$$c = \begin{bmatrix} 0 & 1 \end{bmatrix}$$
$$d = 0$$

Linearized system

$$\frac{d}{dt}\begin{bmatrix} \overline{C}_R \\ \overline{C}_P \end{bmatrix} = \begin{bmatrix} -83 & 0 \\ 50 & -75 \end{bmatrix}\begin{bmatrix} \overline{C}_R \\ \overline{C}_P \end{bmatrix} + \begin{bmatrix} 21 & 4 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} \overline{C}_{R,in} \\ \overline{F} \end{bmatrix}$$
$$y = \overline{C}_P$$

Check asymptotic stability

$$0 = \begin{bmatrix} \lambda - (-83) & 0 \\ 50 & \lambda - (-75) \end{bmatrix}$$
$$\lambda = -75, -83$$

Eigenvalues are negative, and so the system is asymptotically stable.

(c)

Non-linear system

$$\frac{dC_R}{dt} = -\frac{F}{V}C_R - k_1 C_R - k_3 C_R^2 + \frac{F}{V}C_{R,in}$$
$$\frac{dC_P}{dt} = -\frac{F}{V}C_P - k_2 C_P + k_1 C_R$$

Solve the system of ODEs with the following parameters

$$F = 28$$
$$C_{R,in} = 5.5$$
$$V = 1$$
$$k_1 = 50$$
$$k_2 = 54$$
$$k_3 = 4$$

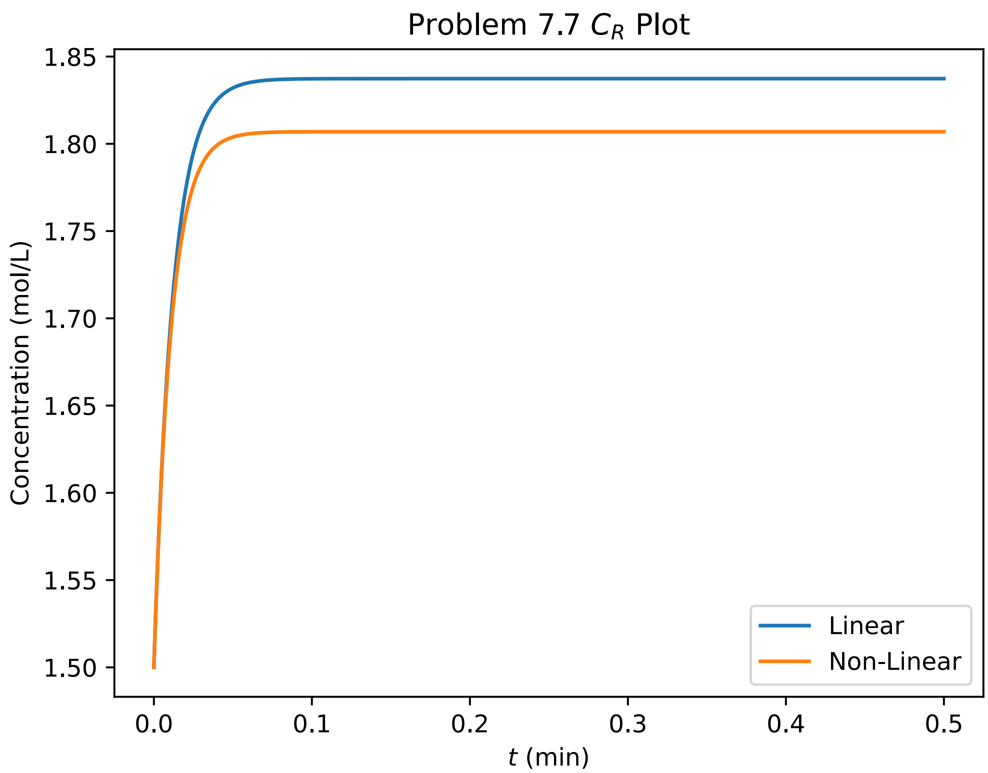
Linear system

$$\frac{d\overline{C}_R}{dt} = -83\overline{C}_R + 21\overline{C}_{R,in} + 4\overline{F}$$
$$\frac{d\overline{C}_P}{dt} = 50\overline{C}_R - 75\overline{C}_P - \overline{F}$$

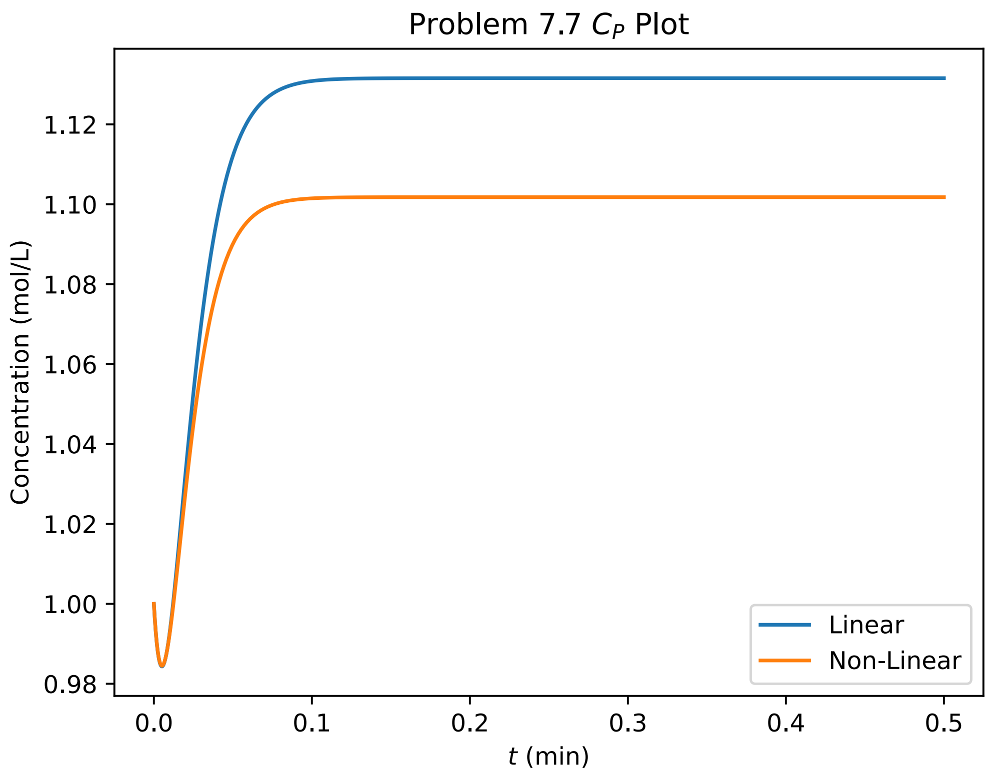
Solve the system of ODEs with the following parameters

$$\overline{F} = 7$$
$$\overline{C}_{R,in} = 0$$
$$k_1 = 50$$
$$k_2 = 54$$
$$k_3 = 4$$

C_R Plot:



C_P Plot:

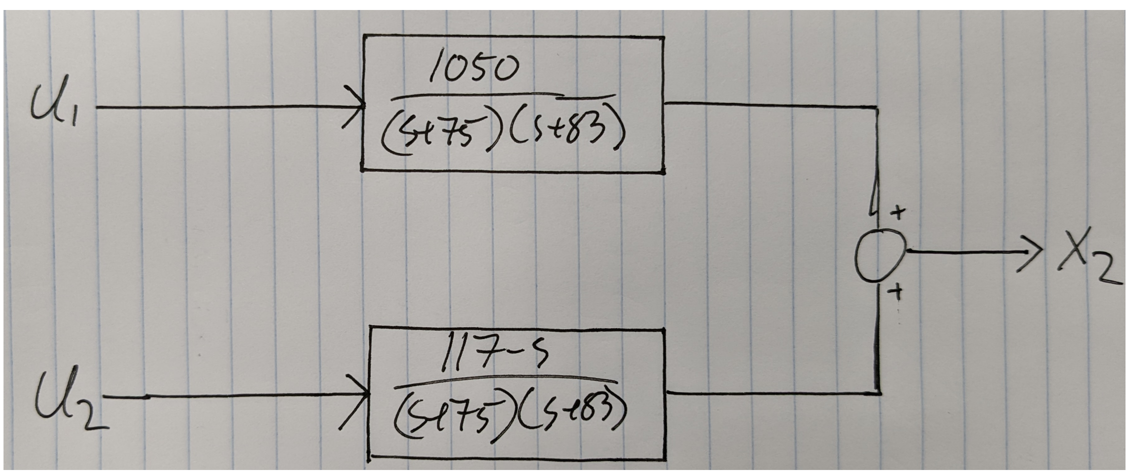


Problem 7.7 C_P Plot
Concentration (mol/L)
1.12
1.10
1.08
1.06
1.04
1.02
1.00
0.98
0.0
0.1
0.2
0.3
0.4
0.5
t (min)
Linear
Non-Linear

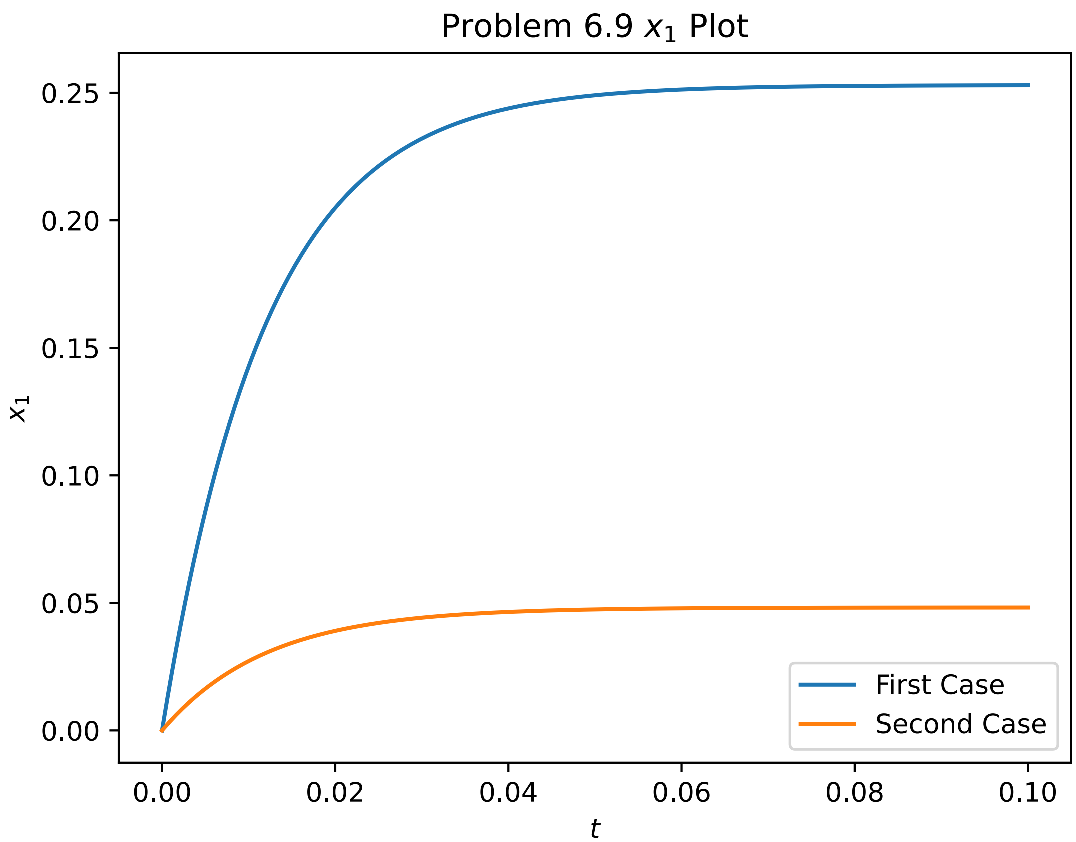
2. Problem 6.9

(a)

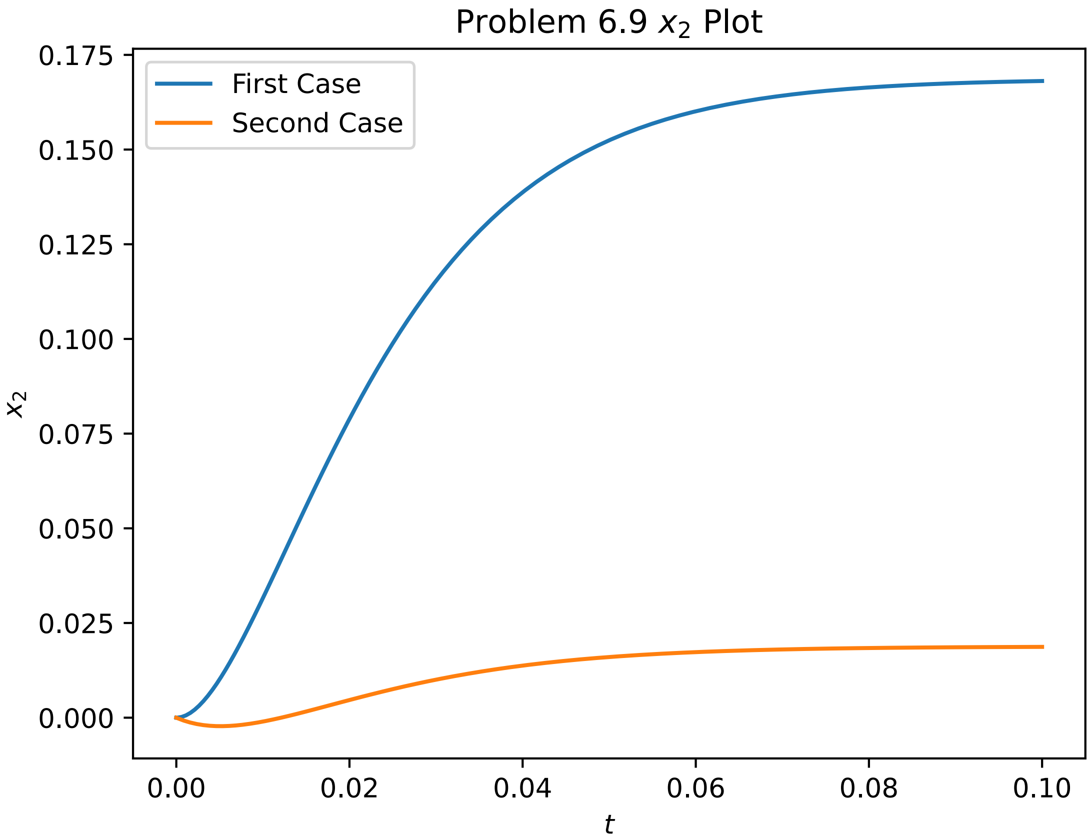
$$\frac{dx_1}{dt} = -83x_1 + 21u_1 + 4u_2$$
$$\frac{dx_2}{dt} = 50x_1 - 75x_2 - u_2$$
$$sX_1 = -83X_1 + 21U_1 + 4U_2$$
$$X_1 = \frac{21}{s+83}U_1 + \frac{4}{s+83}U_2$$
$$sX_2 = 50X_1 - 75X_2 - U_2$$
$$X_2(s+75) = 50X_1 - U_2$$
$$X_2(s+75) = 50\left(\frac{21}{s+83}U_1 + \frac{4}{s+83}U_2\right) - U_2$$
$$X_2(s+75) = \frac{1050}{s+83}U_1 + \frac{117-s}{s+83}U_2$$
$$X_2 = \frac{1050}{(s+83)(s+75)}U_1 + \frac{117-s}{(s+83)(s+75)}U_2$$
$$\boxed{G_1(s) = \frac{1050}{(s+83)(s+75)}}$$
$$\boxed{G_2(s) = \frac{117-s}{(s+83)(s+75)}}$$



x_1 Plot:


Problem 6.9 x_1 Plot
0.25
0.20
0.15
0.10
0.05
0.00
x_1
0.00
0.02
0.04
0.06
0.08
0.10
t
First Case
Second Case

x_2 Plot:


Problem 6.9 x_2 Plot
First Case
Second Case
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
x_2
0.00
0.02
0.04
0.06
0.08
0.10
t