CHEN 461 HW 1

Mark Levchenko

January 2023

1. Problem 2.5

$$\frac{dM}{dt} = \rho F_{in} - \rho F_{out}$$
$$\frac{d(\rho V)}{dt} = \rho F_{in} - \rho F_{out}$$

Assume constant density.

$$\rho\frac{dV}{dt} = \rho F_{in} - \rho F_{out}$$
$$\frac{dV}{dt} = F_{in} - F_{out}$$
$$\frac{d(Ah)}{dt} = F_{in} - F_{out}$$
$$A\frac{dh}{dt} = F_{in} - F_{out}$$

Tank 1:

$$A_1\frac{dh_1}{dt} = F_{in} - F_{out,1}$$
$$F_{out} \approx c\sqrt{h}$$
$$F_{out,1} = c_1\sqrt{h_1}$$
$$A_1\frac{dh_1}{dt} = F_{in} - c_1\sqrt{h_1}$$
$$A_1 = \pi r_1^2$$
$$\pi r_1^2\frac{dh_1}{dt} = F_{in} - c_1\sqrt{h_1}$$
$$\frac{dh_1}{dt} = \frac{F_{in} - c_1\sqrt{h_1}}{\pi r_1^2}$$

Tank 2 is similar except that the outlet of Tank 1 is the inlet of Tank 2:

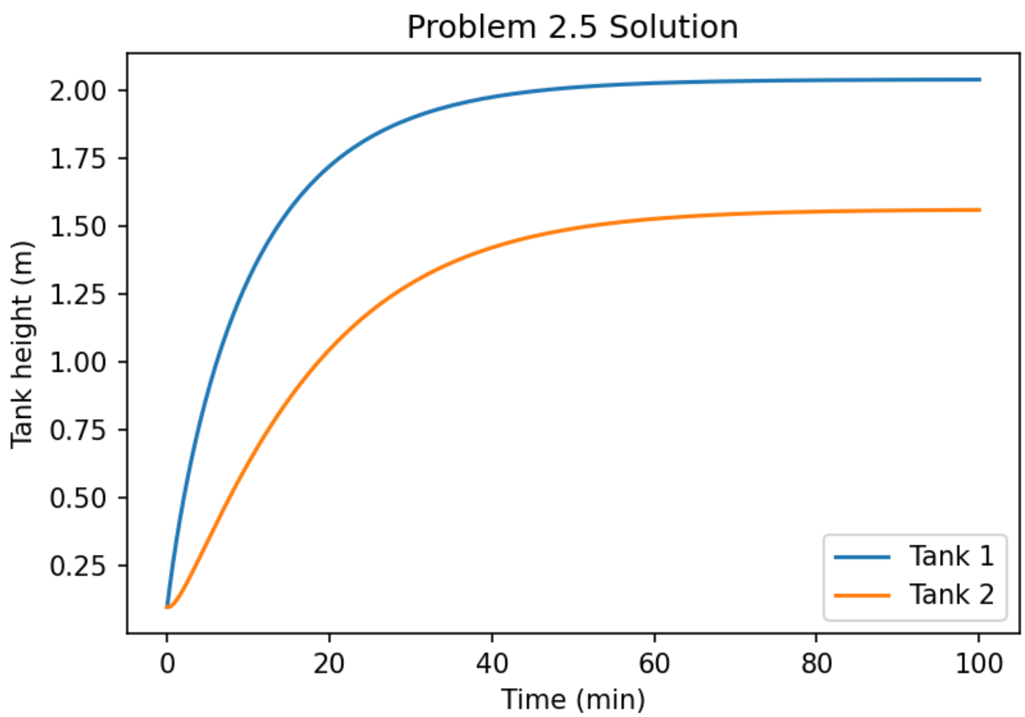
$$\frac{dh_2}{dt} = \frac{F_{in,2} - c_2\sqrt{h_2}}{\pi r_2^2}$$
$$F_{in,2} = F_{out,1} = c_1\sqrt{h_1}$$
$$\frac{dh_2}{dt} = \frac{c_1\sqrt{h_1} - c_2\sqrt{h_2}}{\pi r_2^2}$$

This problem becomes a system of coupled ODEs:

$$\boxed{\begin{aligned}\frac{dh_1}{dt} &= \frac{F_{in} - c_1\sqrt{h_1}}{\pi r_1^2} \\ \frac{dh_2}{dt} &= \frac{c_1\sqrt{h_1} - c_2\sqrt{h_2}}{\pi r_2^2}\end{aligned}}$$

$$\begin{aligned} r_1 = r_2 &= 1 \\ c_1 &= 0.7 \\ c_2 &= 0.8 \\ F_{in} &= 1 \end{aligned}$$

The system is solved in a computer. The output graph:



2. Problem 2.6

   Component R balance at constant volume:

$$
\begin{aligned}
V\frac{dC_R}{dt} &= FC_{in,R} - FC_R + V\sum_{j=1}^{m} r_{j,R} \\
V\frac{dC_R}{dt} &= FC_{in,R} - FC_R - Vr_1 + Vr_{-1} \\
r_1 &= k_1 C_R \\
r_{-1} &= k_{-1} C_P \\
V\frac{dC_R}{dt} &= FC_{in,R} - FC_R - Vk_1C_R + Vk_{-1}C_P \\
\frac{dC_R}{dt} &= \frac{F}{V}\left(C_{in,R} - C_R\right) - k_1C_R + k_{-1}C_P
\end{aligned}
$$

   Component P balance:

$$
\begin{aligned}
V\frac{dC_P}{dt} &= FC_{in,P} - FC_P + V\sum_{j=1}^{m} r_{j,P} \\
C_{in,P} &= 0 \\
V\frac{dC_P}{dt} &= -FC_P + Vr_1 - Vr_{-1} - Vr_2 \\
r_1 &= k_1 C_R \\
r_{-1} &= k_{-1} C_P \\
r_2 &= k_2 C_P^2 \\
V\frac{dC_P}{dt} &= -FC_P + Vk_1C_R - Vk_{-1}C_P - Vk_2C_P^2 \\
\frac{dC_P}{dt} &= -\frac{F}{V}C_P + k_1C_R - k_{-1}C_P - k_2C_P^2
\end{aligned}
$$

   Set of coupled ODEs that define the system:

$$
\boxed{
\begin{aligned}
\frac{dC_R}{dt} &= \frac{F}{V}\left(C_{in,R} - C_R\right) - k_1C_R + k_{-1}C_P \\
\frac{dC_P}{dt} &= -\frac{F}{V}C_P + k_1C_R - k_{-1}C_P - k_2C_P^2
\end{aligned}
}
$$

   Input variables: $C_{in,R}$

   State variables: C_R, C_P

   Output variables: C_P

   Parameters: k_1, k_{-1}, k_2, V, F

3. Problem 2.7

Overall mass balance:

$$\begin{aligned}
\frac{dM}{dt} &= M_{in,1} + M_{in,2} - M_{out} \\
\rho\frac{dV}{dt} &= \rho F_{in,1} + \rho F_{in,2} - \rho F_{out}
\end{aligned}$$

Assume a constant density and that the density of all streams is approximately the same.

$$\begin{aligned}
\frac{dV}{dt} &= F_{in,1} + F_{in,2} - F_{out} \\
V &= Ah \\
A\frac{dh}{dt} &= F_{in,1} + F_{in,2} - F_{out} \\
F_{out} &\approx ch \\
A\frac{dh}{dt} &= F_{in,1} + F_{in,2} - ch
\end{aligned}$$

Component mass balance:

$$\begin{aligned}
\frac{d(Vw)}{dt} &= w_{in,1}F_{in,1} + w_{in,2}F_{in,2} - wF_{out} \\
w_{in,1} &= 0 \\
\frac{d(Vw)}{dt} &= w_{in,2}F_{in,2} - wF_{out} \\
\frac{d(Vw)}{dt} &= w\frac{dV}{dt} + V\frac{dw}{dt} \\
w\frac{dV}{dt} + V\frac{dw}{dt} &= w_{in,2}F_{in,2} - wF_{out} \\
w(F_{in,1} + F_{in,2} - F_{out}) + V\frac{dw}{dt} &= w_{in,2}F_{in,2} - wF_{out} \\
wF_{in,1} + wF_{in,2} - wF_{out} + V\frac{dw}{dt} &= w_{in,2}F_{in,2} - wF_{out} \\
V\frac{dw}{dt} &= w_{in,2}F_{in,2} - w(F_{in,1} + F_{in,2}) \\
Ah\frac{dw}{dt} &= w_{in,2}F_{in,2} - w(F_{in,1} + F_{in,2})
\end{aligned}$$

Set of coupled ODEs for the mass balance:

$$\boxed{\begin{aligned}
\frac{dw}{dt} &= \frac{w_{in,2}F_{in,2} - w(F_{in,1} + F_{in,2})}{Ah} \\
\frac{dh}{dt} &= \frac{F_{in,1} + F_{in,2} - ch}{A}
\end{aligned}}$$

Energy balance, assuming constant C_p and that $C_v \approx C_p$:

$$\frac{d(V\rho C_v(T - T_{ref}))}{dt} = F_{in,1}\rho C_p(T_{in,1} - T_{ref}) + F_{in,2}\rho C_p(T_{in,2} - T_{ref}) - F_{out}\rho C_p(T - T_{ref}) + Q$$

Constant ρ and C_p

$$
\begin{aligned}
\frac{d(V(T-T_{ref}))}{dt} &= F_{in,1}(T_{in,1}-T_{ref}) + F_{in,2}(T_{in,2}-T_{ref}) - F_{out}(T-T_{ref}) + \frac{Q}{\rho C_p} \\
\frac{d(V(T-T_{ref}))}{dt} &= F_{in,1}T_{in,1} + F_{in,2}T_{in,2} - F_{out}T - T_{ref}(F_{in,1}+F_{in,2}-F_{out}) + \frac{Q}{\rho C_p} \\
\frac{d(V(T-T_{ref}))}{dt} &= (T-T_{ref})\frac{dV}{dt} + V\frac{d(T-T_{ref})}{dt} \\
\frac{dV}{dt} &= F_{in,1}+F_{in,2}-F_{out} \\
\frac{d(V(T-T_{ref}))}{dt} &= (T-T_{ref})(F_{in,1}+F_{in,2}-F_{out}) + V\frac{d(T-T_{ref})}{dt} \\
\frac{d(V(T-T_{ref}))}{dt} &= T(F_{in,1}+F_{in,2}-F_{out}) - T_{ref}(F_{in,1}+F_{in,2}-F_{out}) + V\frac{d(T-T_{ref})}{dt} \\
V\frac{d(T-T_{ref})}{dt} &= F_{in,1}T_{in,1} + F_{in,2}T_{in,2} - F_{out}T + \frac{Q}{\rho C_p} - T(F_{in,1}+F_{in,2}-F_{out}) \\
V\frac{d(T-T_{ref})}{dt} &= F_{in,1}(T_{in,1}-T) + F_{in,2}(T_{in,2}-T) + \frac{Q}{\rho C_p}
\end{aligned}
$$

Final differential equation for energy balance:

$$
\boxed{\frac{dT}{dt} = \frac{F_{in,1}(T_{in,1}-T) + F_{in,2}(T_{in,2}-T) + \frac{Q}{\rho C_p}}{Ah}}
$$

Final set of coupled ODEs with mass and energy:

$$
\boxed{
\begin{aligned}
\frac{dw}{dt} &= \frac{w_{in,2}F_{in,2} - w(F_{in,1}+F_{in,2})}{Ah} \\
\frac{dh}{dt} &= \frac{F_{in,1}+F_{in,2}-ch}{A} \\
\frac{dT}{dt} &= \frac{F_{in,1}(T_{in,1}-T) + F_{in,2}(T_{in,2}-T) + \frac{Q}{\rho C_p}}{Ah}
\end{aligned}
}
$$