1. Problem A.3

   (a)

$$
\begin{aligned}
F(s) &= \frac{4}{s^2(s^2+4s+4)} \\
F(s) &= \frac{4}{s^2(s+2)^2} \\
\frac{4}{s^2(s+2)^2} &= \frac{A_1}{s} + \frac{A_2}{s^2} + \frac{B_1}{s+2} + \frac{B_2}{(s+2)^2} \\
4 &= A_1(s^3+4s^2+4s) + A_2(s^2+4s+4) + B_1(s^3+2s^2) + B_2 s^2 \\
4 &= s^3(A_1+B_1) + s^2(4A_1+A_2+2B_1+B_2) + s(4A_1+4A_2) + 4(A_2) \\
A_2 &= 1 \\
A_1 &= -1 \\
B_1 &= 1 \\
B_2 &= 1 \\
F(s) &= \frac{-1}{s} + \frac{1}{s^2} + \frac{1}{s+2} + \frac{1}{(s+2)^2} \\
\mathcal{L}^{-1}\{F(s)\} &= \mathcal{L}^{-1}\left\{\frac{-1}{s} + \frac{1}{s^2} + \frac{1}{s+2} + \frac{1}{(s+2)^2}\right\} \\
&\boxed{f(t) = -1 + t + e^{-2t} + te^{-2t}}
\end{aligned}
$$

2. A.9

$$
\begin{aligned}
\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y &= 2 \\
y(0) = \frac{dy}{dt}(0) = \frac{d^2y}{dt^2}(0) &= 0 \\
\mathcal{L}\left\{\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y\right\} &= \mathcal{L}\{2\} \\
s^3Y(s) + 3s^2Y(s) + 4sY(s) + 2Y(s) &= \frac{2}{s} \\
Y(s)\left(s^3 + 3s^2 + 4s + 2\right) &= \frac{2}{s} \\
\frac{2}{s(s+1)(s^2+2s+2)} &= Y(s) \\
\frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{s^2+2s+2} &= \frac{2}{s(s+1)(s^2+2s+2)} \\
A(s^3+3s^2+4s+2) + B(s^3+2s^2+2s) + (Cs^3+Cs^2+Ds^2+Ds) &= 2 \\
s^3(A+B+C) + s^2(3A+2B+C+D) + s(4A+2B+D) + 2(A) &= 2 \\
A &= 1 \\
C &= 1 \\
B &= -2 \\
D &= 0 \\
\frac{1}{s} - \frac{2}{s+1} + \frac{s}{s^2+2s+2} &= Y(s) \\
\frac{1}{s} - \frac{2}{s+1} + \frac{s}{(s+1)^2+1} &= Y(s) \\
\mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{2}{s+1} + \frac{s+1}{(s+1)^2+1} - \frac{1}{(s+1)^2+1}\right\} &= \mathcal{L}^{-1}\{Y(s)\} \\
\boxed{1 - 2e^{-t} + e^{-t}\cos t - e^{-t}\sin t = y(t)}
\end{aligned}
$$

3. A.12

$$
\begin{aligned}
\frac{x_1}{dt} &= -2x_1 + 2x_2 + f(t) \\
\frac{x_2}{dt} &= x_1 - 3x_2 \\
x_1(0) &= x_2(0) = 0 \\
\mathcal{L}\left\{\frac{x_1}{dt}\right\} &= \mathcal{L}\{\ -2x_1 + 2x_2 + f(t)\} \\
\mathcal{L}\left\{\frac{x_2}{dt}\right\} &= \mathcal{L}\{\ x_1 - 3x_2\} \\
sX_1(s) &= -2X_1(s) + 2X_2(s) + F(s) \\
sX_2(s) &= X_1(s) - 3X_2(s)
\end{aligned}
$$

Rearrange:

$$
\begin{aligned}
X_1(s)(s+2) &= 2X_2(s) + F(s) \\
X_1(s) &= X_2(2)(s+3)
\end{aligned}
$$

Substitute:

$$
\begin{aligned}
X_1(s) &= \frac{F(s)(s+3)}{(s+1)(s+4)} \\
X_2(s) &= \frac{F(s)}{(s+1)(s+4)} \\
X_1(s) &= \frac{F(s)}{3}\left(\frac{2}{s+1} + \frac{1}{s+4}\right) \\
\mathcal{L}^{-1}\{X_1(s)\} &= \mathcal{L}^{-1}\left\{\frac{F(s)}{3}\left(\frac{2}{s+1} + \frac{1}{s+4}\right)\right\} \\
x_1(t) &= \frac{f(t)}{3} * \left(2e^{-t} + e^{-4t}\right) \\
X_2(s) &= \frac{F(s)}{3}\left(\frac{1}{s+1} - \frac{1}{s+4}\right) \\
\mathcal{L}^{-1}\{X_2(s)\} &= \mathcal{L}^{-1}\left\{\frac{F(s)}{3}\left(\frac{1}{s+1} - \frac{1}{s+4}\right)\right\} \\
x_2(t) &= \frac{f(t)}{3} * \left(e^{-t} - e^{-4t}\right)
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
x_1(t) &= \frac{f(t)}{3} * \left(2e^{-t} + e^{-4t}\right) \\
x_2(t) &= \frac{f(t)}{3} * \left(e^{-t} - e^{-4t}\right)
\end{aligned}
}
$$

4. A.14

$$
\begin{aligned}
f(t) &= \frac{t}{\epsilon^2}\mathcal{H}\left(t\right) - \frac{t}{\epsilon^2}\mathcal{H}\left(t-\epsilon\right) + \left(\frac{2}{\epsilon} - \frac{t}{\epsilon^2}\right)\mathcal{H}\left(t-\epsilon\right) - \left(\frac{2}{\epsilon} - \frac{t}{\epsilon^2}\right)\mathcal{H}\left(t-2\epsilon\right) \\
f(t) &= \frac{t}{\epsilon^2}\mathcal{H}(t) + \left(\frac{2}{\epsilon} - \frac{2t}{\epsilon^2}\right)\mathcal{H}(t-\epsilon) - \left(\frac{2}{\epsilon} - \frac{t}{\epsilon^2}\right)\mathcal{H}(t-2\epsilon) \\
f(t) &= \frac{t}{\epsilon^2}\mathcal{H}(t) - \frac{2}{\epsilon^2}(t-\epsilon)\mathcal{H}(t-\epsilon) + \frac{1}{\epsilon^2}(t-2\epsilon)\mathcal{H}(t-2\epsilon) \\
\mathcal{L}\{f(t)\} &= \mathcal{L}\left\{\ \frac{t}{\epsilon^2}\mathcal{H}(t) - \frac{2}{\epsilon^2}(t-\epsilon)\mathcal{H}(t-\epsilon) + \frac{1}{\epsilon^2}(t-2\epsilon)\mathcal{H}(t-2\epsilon)\right\} \\
F(s) &= \frac{1}{\epsilon^2 s^2} - \frac{2}{\epsilon^2 s^2}e^{-\epsilon s} + \frac{1}{\epsilon^2 s^2}e^{-2\epsilon s} \\
&\boxed{F(s) = \frac{1 - 2e^{-\epsilon s} + e^{-2\epsilon s}}{\epsilon^2 s^2}}
\end{aligned}
$$

As ϵ approaches 0, $F(s)$ becomes the line $F(s) = 1$. The peak of the triangle in $f(t)$ moves closer to the y-axis, and it eventually becomes a straight line that runs along the y-axis with infinite height.