1.1 Problem Set

A small grocery store has only one checkout counter. Customers arrive at this checkout counter at random from 1 to 8 minutes apart. Each possible value of inter arrival time has the same probability of occurrence. The service times vary from 1 to 6 minutes with the probabilities shown in table 2. The problem is to analyze the system by simulating the arrival and service of 6 customers.

Table.1 Distribution of arrived time

Interarrival time (Min)	1	2	3	4	5	6	7	8
Probability	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125

Table-2 Distribution of service time

Service time (Min)	1	2	3	4	5	6
Probability	0.20	0.10	0.30	0.15	0.20	0.05

Calculate the following:

i. The average waiting time for a customer

ii. The probability that a customer has to wait in the queue

iii. The fraction of idle time of the server

iv. The average service time

v. The average time between arrivals

vi. The average waiting time of those who wait

vii. The average time a customer spends in the system.

Use the following sequence of random number,

Random digit for arrival are: 905, 727, 125, 848, 609.

Random digit for service time are: 14, 30, 94, 53, 37, 79.

1.2 Solution:

Calculate arrival time distribution and assign a random number,

Time between arrival	Probability	Cumulative probability	Random digit assignment
1	0.125	0.125	00 – 125
2	0.125	0.250	126 – 250
3	0.125	0.375	251 – 375
4	0.125	0.500	376 - 500
5	0.125	0.625	501 - 625
6	0.125	0.750	626 – 750
7	0.125	0.875	751 – 875
8	0.125	1.000	876 - 000

Calculate service time distribution and assign a random number,

Service time	Probability	Cumulative probability	Random digit assignment
1	0.20	0.20	00 - 20
2	0.10	0.30	21 – 30
3	0.30	0.60	31 – 60
4	0.15	0.75	61 – 75
5	0.20	0.95	76 - 95
6	0.05	1.00	96 - 00

Determining time between arrival,

Customer	Random Digit	IAT
1	-	-
2	905	8
3	727	6
4	125	1
5	848	7
6	609	5

Determining service time,

Customer	Random Digit	Service time
1	14	1
2	30	2
3	94	5
4	53	3
5	37	3
6	79	5

Simulation Table

Customer	IAT	Arrival Time	Service time	Time service begins	Waiting time	Time service ends	Time spent in system	Idle time of server
1	-	0	1	0	0	1	1	0
2	8	8	2	8	0	10	2	7
3	6	14	5	14	0	19	5	4
4	1	15	3	19	4	22	7	0
5	7	22	3	22	0	25	3	0
6	5	27	5	27	0	32	5	2

Here,

$Total\ number\ of\ customer = 6,$

$Cumulative\ IAT\ = 27,$

$Total\ service\ time = 19,$

$Number\ of\ customers\ waiting\ in\ queue\ = 1,$

$Total\ time\ customer\ wait\ in\ queue = 4,$

$Total\ idle\ time\ of\ server\ = 13,$

$Total\ run\ time\ of\ server\ =\ 32,$

$Total\ time\ spent\ in\ system = 23$

Therefore,

i. $The\ average\ waiting\ time\ for\ a\ customer = \frac{Total\ time\ customer\ wait\ in\ queue}{Total\ number\ of\ customer}$

$$= \frac{4}{6} = 0.67\ mins$$

ii. $The\ probability\ that\ a\ customer\ has\ to\ wait\ in\ the\ queue$

$$= \left(\frac{Number\ of\ customers\ waiting\ in\ queue}{Total\ number\ of\ customer}\right) \times 100 = \frac{1}{6} \times 100 = 16.67\%$$

iii. $The\ fraction\ of\ idle\ time\ of\ the\ server\ = \left(\frac{Total\ idle\ time\ of\ server}{Total\ run\ time\ of\ server}\right) \times 100$

$$= \frac{13}{32} \times 100 = 40.625\%$$

iv. $The\ average\ service\ time = \frac{Total\ service\ time}{Total\ number\ of\ customer} = \frac{19}{6} = 3.17\ mins$

v. $The\ average\ time\ between\ arrivals = \frac{Cumulative\ IAT}{Total\ number\ of\ customer - 1} = \frac{27}{5} = 5.4\ mins$

vi. $The\ average\ waiting\ time\ of\ those\ who\ wait\ = \frac{Total\ time\ customer\ wait\ in\ queue}{Number\ of\ customers\ waiting\ in\ queue}$

$$= \frac{4}{1} = 4\ mins$$

vii. $The\ average\ time\ a\ customer\ spends\ in\ the\ system = \frac{Total\ time\ spent\ in\ system}{Total\ number\ of\ customer}$

$$= \frac{23}{6} = 3.83\ mins$$

1.3 Simulation in Excel:

Simulation Table,

Serial	random_number_for_IAT	inter_arrivel_time	arrival_time	random_number_for_service_time	service_time	time_service_begins	waiting_time	time-service_end	time_spent_in_system	idle_tme_of_server
1	0	0	0	97	6	0	0	6	6	0
2	479	4	4	36	3	6	2	9	5	0
3	990	8	12	46	3	12	0	15	3	3
4	92	1	13	47	3	15	2	18	5	0
5	917	8	21	80	5	21	0	26	5	3
6	202	2	23	16	1	26	3	27	4	0
6					21		7		28	6

Value calculation,

		total number of customer	Cumulative IAT	total service time	number of customer waiting in queue	total time customer wait in queue	total idle time of server
		6	23	21	3	7	6
The average waiting time for a customer	1.17 mins						
The probability that a customer has to wait in the queue	50 %						
The fraction of idle time of the server	21 %						
The average service time	3.5 mins						
The average time between arrivals	4.6 mins						
The average waiting time of those who wait	2.33 mins						
The average time a customer spends in the system	4.67 mins						

1.4 Simulation using python:

Code:

```
from random import randrange
customer_in_server = []
ultimate_customer_info = []
customer_in_queue = []
rn_iat = 0
iat = 0
rn_st = 0
st = 0
stb = 0
wt = 0
its = 0

def inter_arrival_time(k):
  for i in range(k):
    if (i == 0):
      customer_in_queue = [i, 0, 0, 0]

    else:
      global stb, wt, its
      rn_iat = randrange(1000)

      if (rn_iat < 126):
        iat = 1

      elif (rn_iat < 251):
        iat = 2

      elif (rn_iat < 376):
        iat = 3

      elif (rn_iat < 501):
        iat = 4

      elif (rn_iat < 626):
        iat = 5

      elif (rn_iat < 751):
        iat = 6

      elif (rn_iat < 876):
        iat = 7

      elif (rn_iat < 1001):
        iat = 8
```

```
        customer_in_server = customer_in_queue.copy()
      customer_in_queue = [i, rn_iat, iat]

      at = customer_in_server[3] + customer_in_queue[2]
      customer_in_queue.append(at)
      stb = max(customer_in_server[8], customer_in_queue[3])
      wt = customer_in_server[8] - at
      if wt < 0 :
        wt = 0
      its = at - customer_in_server[8]
      if its < 0 :
        its = 0

    a, b = server_time()
    customer_in_queue.append(a)
    customer_in_queue.append(b)
    customer_in_queue.append(stb)
    customer_in_queue.append(wt)
    tse = customer_in_queue[5] + customer_in_queue[6]
    customer_in_queue.append(tse)
    tsis = customer_in_queue[8] - customer_in_queue[3]
    customer_in_queue.append(tsis)
    customer_in_queue.append(its)

    ultimate_customer_info.append(customer_in_queue)

def server_time():
  rn_st = randrange(100)

  if (rn_st < 21):
    st = 1

  elif (rn_st < 31):
    st = 2

  elif (rn_st < 61):
    st = 3

  elif (rn_st < 76):
    st = 4

  elif (rn_st < 96):
    st = 5

  elif (rn_st < 101):
    st = 6
```

```
    return rn_st, st

inter_arrival_time(6)

import pandas as pd
df = pd.DataFrame(ultimate_customer_info, columns = ['Customer no.', ' rn
                - iat ', ' iat ', ' at ', ' rn_st ', ' st ', ' stb ', ' wt ', ' tse ', ' tsis ', ' its '])

df = df.set_index('Customer no.')

print(df)
```

Output:

```
                rn-iat      iat      at      rn_st      st      stb      wt      tse      tsis      its
Customer no.
0                    0        0       0         90       5        0       0        5         5        0
1                  487        4       4         40       3        5       1        8         4        0
2                  946        8      12          7       1       12       0       13         1        4
3                  900        8      20         75       4       20       0       24         4        7
4                  869        7      27          5       1       27       0       28         1        3
5                  603        5      32         49       3       32       0       35         3        4
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