Session 3
13.11.2025

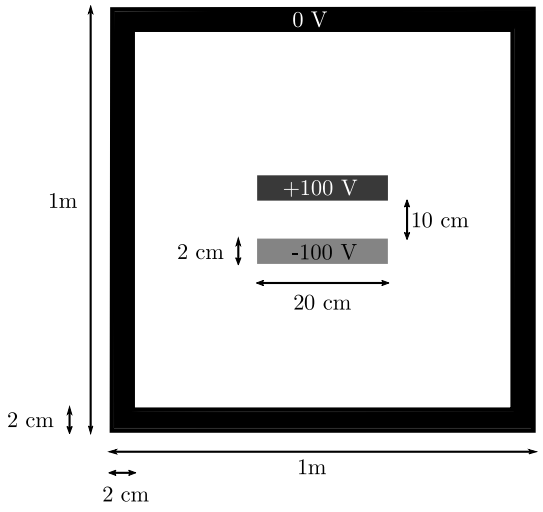
Exercise 3.1 - Geometry and boundary conditions

We work in two dimensions and simulate a square area of size $L \times L$, e.g. $L = 1m$. A regular grid with N points in each direction is introduced. The grid points are $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ with $x_i/h \in \{0, 1, \ldots, N-1\}$. The lattice spacing is $h = \frac{L}{N-1}$.

- Set up a double precision matrix `phi` and a logical matrix `bndry` of size $N \times N$. The value `phi(ix,iy)` is the electric potential at position $\begin{pmatrix} (\texttt{ix}-1)h \\ (\texttt{iy}-1)h \end{pmatrix}$, while the corresponding entry in `bndry` is `true` if this point is a boundary point and `false` otherwise. Make these two matrices and L global variables (hint: `global`).

- Write a function
  `rect(minx, maxx, miny, maxy, val)`
  It should set all entries in `phi` that have coordinates $\vec{x} \in [\texttt{minx}, \texttt{maxx}] \times [\texttt{miny}, \texttt{maxy}]$ to the value `val`. And the corresponding points in `bndry` to `true`.

- Use the function `rect` to initialize the following geometry (a condensator in a grounded box).



Work with $N = 101, 201$ or 401.

Methods to determine the field values on the internal points will be developed next week.

Exercise 3.2 - Differential Operators
Consider the function $\phi : \mathbb{R}^2 \to \mathbb{R}$,

$$\phi(x, y) = \sin^2(x)\ \cos^3(y) + \cos^2(y)$$

- Create a matrix of size $N \times N$ with e.g. $N = 20$ or $N = 30$, that contains the function values $\phi(x, y)$ on a regular grid with $x \in [0, \pi]$ and $y \in [0, 2\pi]$.
- Plot this function (hint: `surf`)
- Compute $\vec{E} = \mathrm{grad}(\phi)$ and $\rho = \Delta\phi$ using finite differences with $O(h^2)$ accuracy. Store the two components of the gradient in two matrices `Ex` and `Ey`. (hint: compute the derivatives only for non-boundary points, i.e. the result will be an $(N-2) \times (N-2)$ matrix.
- Make a contour plot of ϕ and a quiver plot of $\vec{E}$ together. (hint `contour`, `quiver`)