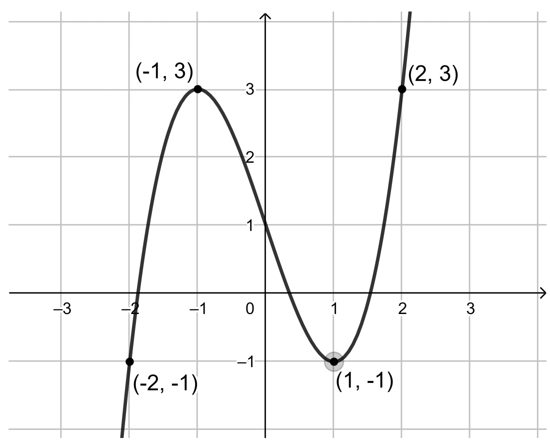
Directions:

- Do the problems only for the Learning Targets that you need to take, and feel ready to take. If you have already earned Level 2 on a Learning Target, do not attempt the problems for that Target. You can skip a Target if you need more time to practice with it, and attempt it on the next Checkpoint. Remember, most Learning Targets appear on 4 consecutive Checkpoints.
- Scan your work for each Learning Target to its own PDF file, and submit it to the appropriate "dropbox" on Brightspace. **Please do not submit more than one Learning Target in the same PDF, and make sure you are submitting it to the right Brightspace area.**
- If you are handwriting, submit your work by **scanning your work** using a scanning app or scanning device; **do not just take a picture** but scan your work to a clear, legible, black and white PDF file. **Brightspace will permit only PDF file uploads.**
- Please consult the grading criteria found in the Information on Learning Targets and Checkpoints document (also available in Brightspace) prior to submitting your work, to make sure your submission has met all of the requirements.
- Please use only the approved resources to double-check your work against errors prior to submitting your work.

Learning Target 1: *I can find the average rate of change of a function and the average velocity of an object on an interval.*

1. Let $f(x) = x + \sqrt{x}$. Find the average rate of change in f on the intervals $[1,3]$ and $[1,1.01]$. *If you round, round your answer to four decimal places.*
2. Let $g(x)$ be the graph shown at right. Find the average rate of change in g on the intervals $[-1,1]$ and $[-1,2]$.



3. A car is moving down a straight racetrack, and its distance s (in feet) from an observation booth on the track at time t seconds is given by the following table:

Time	0	15	30	45	60
Distance	10	100	220	450	540

Find the car's average velocity from $t=0$ to $t=15$ seconds and from $t=45$ to $t=60$ seconds.

Learning Target 2 (Core): *I can find one- and two-sided limits of a function at a point and at infinity using numerical, graphical, and algebraic methods.*

1. Complete the table of values below using the function

$$f(x) = \frac{\sqrt{x} - 3}{x - 9}$$

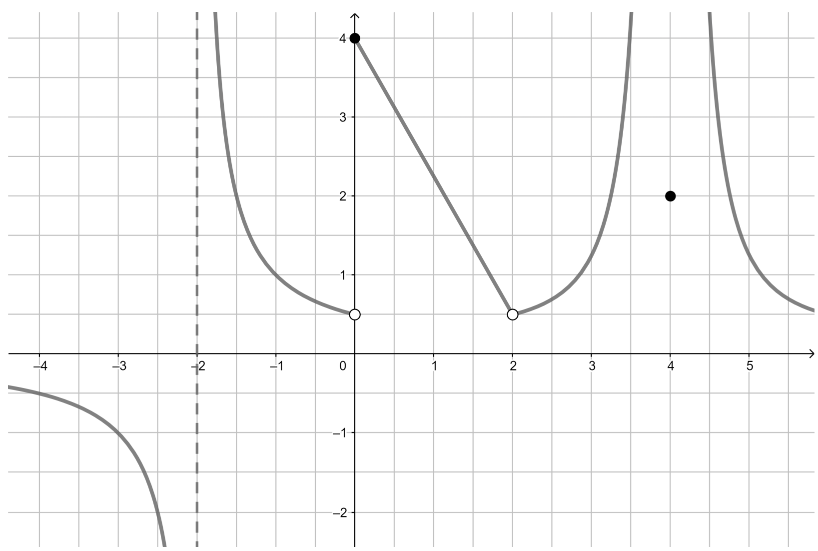
Then state the value of $\lim_{x \to 9} f(x)$ and explain your reasoning based on the evidence in the table. You do not need to show your work on computing the table values, but they must all be correct.

x	8.9	8.99	8.999	9.001	9.01	9.1
$f(x)$						

2. Using only algebra (no graphs or tables), evaluate

$$\lim_{x \to 1} \frac{4x^2 - x - 3}{x - 1}$$

3. The function $h(x)$ is shown below. State the value of each limit shown below the graph. If the limit doesn't exist, write "DNE" and then explain why.



(a) $\lim_{x \to -2^+} h(x)$

(b) $\lim_{x \to 0^-} h(x)$

(c) $\lim_{x \to 0^+} h(x)$

(d) $\lim_{x \to 2} h(x)$

Learning Target 3: *I can find the derivative of a function (both at a point and as a function) and the instantaneous velocity of an object using the definition of the derivative.*

Consider the function $f(x) = 2x^2 - x + 5$.

1. Write out the correct limit expression that would compute $f'(1)$.
2. Find the exact value of $f'(1)$ by computing the limit indicated in part (1), using algebraic techniques (not a number table or a graph).

Note: Your solution *must* begin with a correct statement of the limit. Your solution *can only* be found by evaluating the limit algebraically. No "shortcut" methods from later parts of this course (or that you may have learned if you took Calculus 12) are allowed (except in your notes to check your answer). *All significant algebra steps* must be shown and done correctly.

Learning Target 4: *I can use graphical and analytic methods to do the following: determine whether a function is continuous at a given point, determine whether a given function differentiable at a given point, find the discontinuities of a given function, find points at which a function is not differentiable, state whether a discontinuity is removable or non-removable.*

Consider the function

$$f(x) = \begin{cases} 2x + \tan(x), & x \geq 0 \\ x^2, & x < 0 \end{cases}$$

1. Show, using the definition of continuity, that f is continuous at $x = 0$. Your demonstration must make use of the definition of continuity in terms of a limit or it will miss the learning target.
2. Identify one place where f is not continuous. What kind of discontinuity is it? Removable or non-removable? No justification required.
3. Determine whether f is differentiable at $x = 0$. Your argument must use the limit definition of the derivative. You might find it easier to use the definition given in this form:

$$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$$

   Hint: Consider the two one-sided limits in the definition of the derivative.

Learning Target 5 (Core): *I can use derivative notation correctly, state the units of a derivative, estimate the value of a derivative using difference quotients, and correctly interpret the meaning of a derivative in context.*

A wind turbine generates electricity which is then sold on the open market. The faster the wind blows, the more electricity is generated and therefore the more money is earned. The revenue earned (R, in dollars) is therefore a function of the wind speed (v, in kilometres per hour). Denote this relationship $R = f(v)$.

1. Suppose $f'(12) = 6$. State the units of measurement for the numbers 12 and 6. (Clearly indicate which is which.)
2. Still assuming $f'(12) = 6$, explain the meaning of this statement in ordinary terms and without using any technical math jargon.

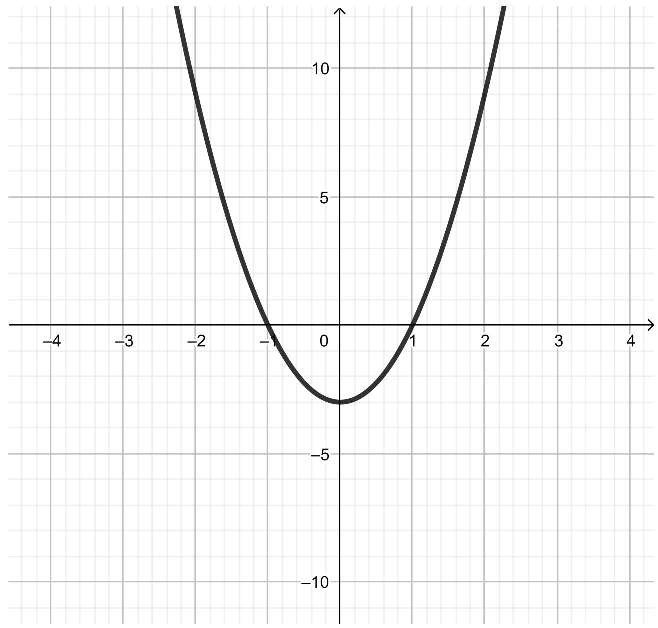
Learning Target 6: *Given information about a function and its first two derivatives, I can correctly give information about the function and its first two derivatives and the increasing/decreasing behaviour and concavity of the function (and vice versa).*

Below right is the graph of the first derivative f' of a function f. To repeat, this is **not** the graph of f, it is the graph of f'.

1. On what interval or intervals is the original function increasing? State your answer and give a clear, single sentence explaining your reasoning using the graph of f'.
2. On what interval or intervals is the original function f concave up? State your answer and give a clear, single sentence explaining your reasoning using the graph of f'.

Notes:

- In your explanations, you may refer only what has been taught so far in MAT 181 (not topics not yet covered, which you may have covered if you took Calculus 12).
- Your explanations must refer to all functions explicitly by name. refer to f, f', or f''; or "the original", or "the first derivative", or "the second derivative". **Do not** refer to "it", "the graph", "the function", "the line", etc. without also making it explicit to which function you are referring.



Learning Target 7 (Core): *I can compute basic-level derivatives using algebraic shortcut methods and solve simple application problems. (Functions involved will include constant, power, polynomial, exponential, and sine/cosine functions; applications include rates of change and slopes/equations of tangent lines/estimation using local linearization.)*

In each of the items below:

- use only the derivative computation rules found in sections 2.1 and 2.2 of the text. **Do not use the limit definition or any rules not found in Sections 2.1 and 2.2**. Use of these will result in the work being not assessable.
- **If algebra is needed to simplify the function before finding the derivative, show all your algebra work.**
- Yon must work using the variables given in the problem statement.
- Make sure your answer is clearly indicated (for example, by circling it).
- Your work will be assessed based on your processes as well as your final answers, so show all of your work.

1. Compute the derivatives of the following functions:
   (a) $y = x^5 + x^{1/5} + 5x + 5^x - 5$
   (b) $g(x) = 5\cos(x) + 15\sin(x)$
   (c) $h(x) = \dfrac{x^5 + x^2}{x^3}$ (Remember: Use *only* the rules from Sections 2.1 and 2.2. The Product, Quotient, and Chain Rules are off limits.)
2. Find an equation for the tangent line to the graph of $y = x^2 - 10x + 20$ at $x = 10$. Show all of your work for this part.
3. The height (h, in feet) of a ball that is fired straight up is a function of time (t, in seconds) given by $h(t) = 64t - 32t^2$. Find the instantaneous velocity of the ball at $t = 1$ seconds. Show all of your work.

Learning Target 8 (Core): *I can compute derivatives correctly using the Product, Quotient, and Chain rules. (Functions involved will be those under "basic".)*

Find the derivatives of each of the indicated functions, keeping in mind the following instructions:

- **In each, state the rules you are using, in the correct order of use. You need only list when you use the Product, Quotient, or Chain Rules. In the case of the Chain Rule, clearly indicate the decomposition of the function by stating the "inner" and "outer" functions first. Correct answers given without this information will not meet the criteria for acceptability.**
- Show your work in a clear and logical order and circle/box your answer. DO NOT simplify your answers.

1. $y = \dfrac{e^x}{\sin(x)}$
2. $y = \left(x^2 + x + 1\right)\sin(x)$
3. $y = e^{\cos x}$

Learning Target 9: *I can compute the derivatives of inverse functions, including logarithmic and inverse trigonometric functions.*

Find the derivative of $y = \log_2(x)$ by first solving for x as in terms of y and then working to find dy/dx expressed as a function of x. Show all of your work. The point $(1,0)$ is on the graph of $y = \log_2(x)$. Demonstrate the reciprocal slope property at this point.

Learning Target 10: *I can compute the derivative of an implicitly-defined function and find the slope of the tangent line to an implicit curve.*

Consider the plane curve described by $y^4 + 9x^2 = x^4 + 4y^2$.

1. Verify that the point $(3,2)$ is on the curve. Show your work.
2. Find $\dfrac{dy}{dx}$ using implicit differentiation.
3. Find the slope of the tangent line to the curve at $(3,2)$. Give your answer exactly, not using decimals. Be sure to express the slope in as simple a form as possible.

Learning Target 11: *I can compute advanced-level derivatives using algebraic short-cut methods. (Functions involved will include basic ones plus logarithmic, trig, and inverse trigonometric functions along with Product, Quotient, and Chain Rules and multiple rules in combination.)*

Find the derivatives of each of the functions given below, keeping in mind the following standards:

- In each, state the rules you are using, in the correct order of use. You need only list when you are using the *product*, *quotient*, or *chain rules*.
- In the case of the chain rule, clearly indicate the decomposition of the function by stating the "inner" and "outer" functions first, or clearly identify the intermediate variables you use if you use Leibniz notation. Correct answers given without this information will not meet the criteria for acceptability.
- Show your work in a clear and logical order and circle/box your answer.
- If you use logarithmic differentiation, you must rewrite your answer using a common denominator. Otherwise, do not simplify your answers (although obvious simplifications are always welcome).

1. $y = \ln\left(\dfrac{x+1}{\sqrt{x-1}}\right)$
2. $y = \sqrt{\ln(x) \cdot e^x}$
3. $y = e^{\sqrt{x}\cdot\cos(x)}$

Learning Target 12 (Core): *I can find the critical values of a function, determine where the function is increasing and decreasing, and apply the first and second derivative tests to classify the critical points as local extrema.*

Consider the function $f(x) = 6x^5 - 15x^4 - 30x^3 + 120x^2 - 120x$.

1. Find all the critical values of the function. Make your reasoning clear by giving all calculus and algebra steps clearly and neatly.
2. Make a first derivative sign chart for the function and use it to determine the intervals on which the function is increasing and decreasing. Your sign chart must be legible and formatted using the rules we discussed in class, and you must state which points you are using as test points.
3. Using either the First or Second Derivative Test (your choice), classify each critical value as a local maximum, local minimum, or neither and explain your reasoning.

Acceptable work requires ALL calculus steps used to arrive at your answer; however, you may use technology to perform any non-calculus task, like finding output values of functions, without showing the steps.

Learning Target 13: *I can determine the intervals of concavity of a function and find all of its points of inflection.*

Consider the function $f(x) = x^4 - 3x^3$. Use calculus (not visual estimation from a graph) to find the intervals on which f is concave up and the intervals on which f is concave down, and state its inflection points.

If you construct a sign chart, make sure the chart has all the required properties that we have discussed. **Acceptable work requires showing ALL calculus steps used** to arrive at your answer; however, you may use technology to perform any non-calculus task, like finding output values of functions, without showing the steps.

Note that this is **not** the same function you worked with in Learning Target 12, so there is no work that you can recycle this time.

Learning Target 14: *I can use the Extreme Value Theorem to find the absolute maximum and minimum values of a continuous function on a closed interval.*

For each of the following, use calculus (not visual estimation from a graph) to determine the absolute extreme values of the given function on the specified interval. You may assume that each function is continuous on the interval. **Acceptable work requires showing ALL calculus steps used** to arrive at your answer; however, you may use technology to perform any non-calculus task, like finding output values of functions, without showing the steps.

- $f(x) = 6x^5 - 15x^4 - 30x^3 + 120x^2 - 120x$ on $[0,3]$
- $g(x) = e^{x^2-4x}$ on $[0,3]$

The first function above is the same function you worked with in Learning Target 12. If you attempted that Learning Target, then you may state the **relevant** results from that Learning Target here without redoing the work to obtain them.

Learning Target 15 (Core): *I can set up and use derivatives to solve applied optimisation problems.*

Set up and solve the optimization problem found below the bullet list. **In order for your work to meet quality standards, it needs to contain ALL of the following. Check carefully before turning in your work to make sure you've included each one.**

- A clear indication of what each variable in the solution represents (including units)
- A clear statement of what quantity your are optimizing
- A formula for the quantity you are optimizing and a clear indication of how you obtained it
- If you use a constraint in the problem, a clear statement of what that constraint is and how you used it
- The use of a derivative to find the input that optimises your quantity, and state the optimal value of the quantity.
- Reasoning that explains why your solution is correct – *explain how you know that value optimizes the target quantity.*

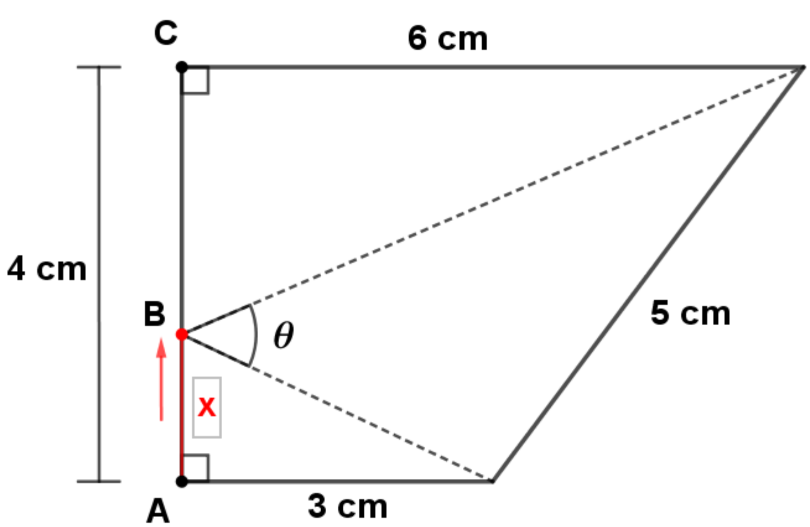
Statement of the problem: A factory is making a container with an open top and a square base. It has to be built so that its volume is 300 cubic feet. What are the dimensions of the container that uses the least amount of materials? (Your answer here must include all three dimensions – length, width, and height – and needs to contain all the information listed above.)

Learning Target 16: *I can set up related rates problems and use derivatives to solve them.*

Use the DREDS method (Diagram, Rates, Equation, Differentiate, Substitute) illustrated in class to solve the following problem. Each part of the method must be clearly presented. Your diagram should be drawn neatly, and large enough to be easily read. Be sure to write a clear concluding statement, including appropriate units. **If you do not follow these instructions, your work will miss the target.**

Statement of the problem:

Point B moves from point A to point C at 2 cm/s in the accompanying diagram. At what rate is θ changing when $x = 4$ cm?



Learning Target 17: *I can calculate the area under a curve, net change, and displacement using geometric formulas and Riemann sums.*

Consider the function $f(x) = 10 - e^x$. Estimate the area under the curve, above the x-axis, and between $x = 0$ and $x = 2$, using the following Riemann sums. **On each one:** Clearly state the value of Δx, clearly state which points you are using to construct the rectangles, and show the setup of your calculation. **Keep all approximations to four (4) decimal places.**

1. R_8
2. M_3

Learning Target 18: *I can explain the meaning of each part of the definition of the definite integral in terms of a graph, and interpret the definite integral in terms of areas, net change, and displacement.*

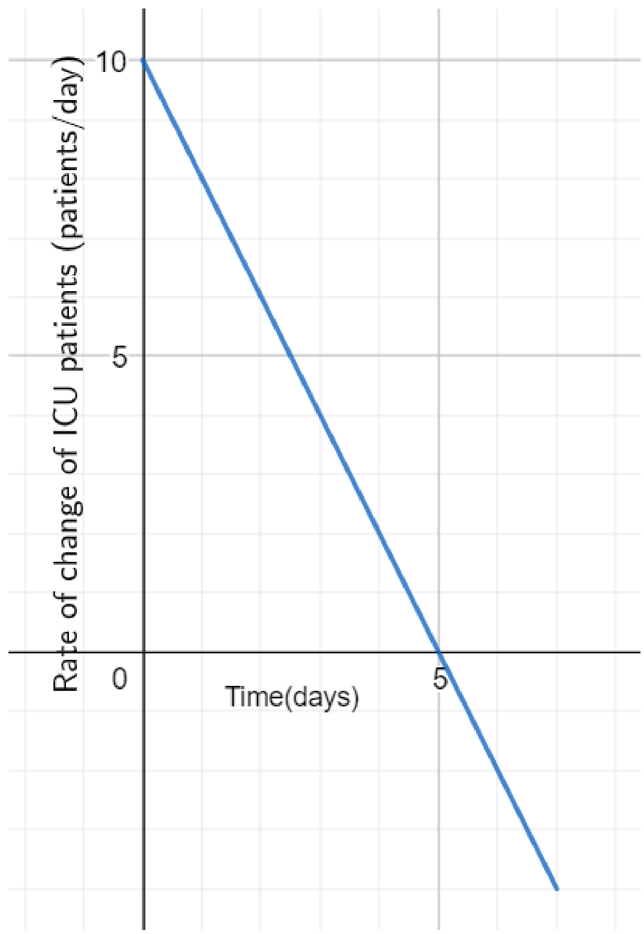
The graph below shows the rate at which the number of patients in an intensive care unit (ICU) at a hospital was changing, measured in "patients per day," during a one-week period (7 days). A negative rate indicates that more patients were leaving the ICU than were entering.

1. Let the function whose graph is shown be called $r(t)$. State the units of the definite integral

$$\int_0^7 r(t)\,dt$$

and briefly explain how you know.

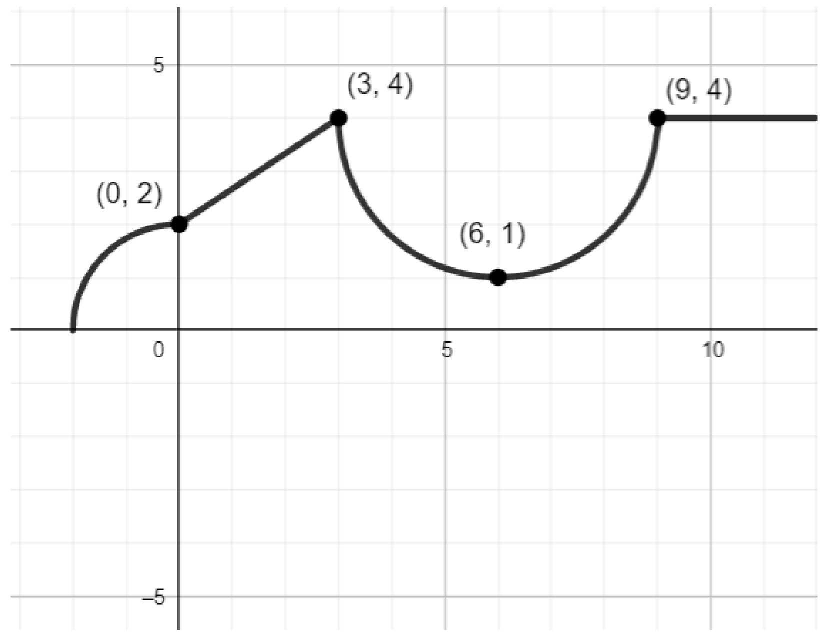
2. At the **end** of the week shown here, were there *more* patients in the ICU than at the beginning of the week, were there *fewer* patients, or was the number of patients the same? State your answer clearly and give a clear, well-reasoned explanation.
3. If there were 30 patients in the ICU at the beginning of the week, how many are there at the end of the week? State your answer clearly and make your reasoning clear.



Learning Target 19: *I can evaluate a definite integral using geometric formulas and the properties of the definite integral.*

The graph of $y = f(x)$ is shown below. All curved portions are arcs of circles. Everything else is made up of line segments. Four of the points are labelled. **Using only the graph**, evaluate the **exact** value – no decimal approximations – of each of the integrals shown below. **Note: antidifferentiation is not allowed in solutions to this problem, nor are decimal approximations.** Also, **show all of your work ==** answers with insufficient work or no work will not meet the grading criteria.

1. $\int_{-2}^{0} f(x)\,dx$
2. $\int_{3}^{9} f(x)\,dx$
3. $\int_{-2}^{12} f(x)\,dx$



Learning Target 20 (Core): *I can find antiderivatives of a function and evaluate a definite integral using the Fundamental Theorem of Calculus*

Find the exact value of each of the following definite integrals by using the Fundamental Theorem of Calculus (not geometry or Riemann sums).

A correct solution must do the following:

1. Clearly show the antiderivative of each integrand, with the "evaluation line" indicating where the antiderivative is to be evaluated.
2. Give the **exact value** of the answer with *no decimal approximations* and that is **fully simplified**
3. Give a decimal approximation of the exact answer that agrees with the exact form to **four decimal places**.

Here is an example from your textbook that exhibits the required elements. If any of your solutions are missing any of these three requirements, they will not meet the target.

$$\int_0^4 (4x - x^2)\,dx = \left(2x^2 - \frac{1}{3}x^3\right)\Big|_0^4$$
$$= \left(2(4)^2 - \frac{1}{3}4^3\right) - (0 - 0) = 32/3. \approx 10.6667$$

1. $\int_1^\pi \left(\frac{1}{\sqrt[3]{x}} + \sin(x)\right) dx$
2. $\int_0^2 \left(10 - e^x\right) dx$