Study Guide: Camera Models, Geometry, and Real-World Aspects

This guide covers fundamental concepts related to camera operation, image formation, and the mathematical models used to describe them.

1. *The Pinhole Camera Model (Sources 1-4)*

The pinhole camera model is a foundational concept for understanding how images are formed.

Image Formation Basics:

- An image is created when a **light source** illuminates an **object**, and the reflected light reaches a **sensor**.
- The model helps answer how a camera creates an image by "looking" at the real world.

The Pinhole Concept:

- A pinhole selectively allows light rays to reach the sensor.
- **Perfect definition** (sharp image) is achieved if only one ray per point reaches the sensor, which ideally happens if the hole is just a point. This theoretical point-sized hole gives rise to the **pinhole camera model**.
- This model has been used for centuries, exemplified by the "camera obscura".

Modeling a Camera with Pinhole Elements:

- The pinhole model simplifies a real camera, neglecting certain effects to provide a starting point for studying geometric projection.
- **Main elements**:
  - **Optical center (O)**: The location of the pinhole.
  - **Image plane (Π)**: The surface where the image is formed.
  - **Optical axis (Z)**: An imaginary line perpendicular to the image plane and passing through the optical center.
  - **Principal point**: The intersection of the image plane and the optical axis.
  - **Focal length (f)**: The distance between the optical center and the image plane.
- **Reference systems**:
  - A **camera reference system (XYZ)** describes the camera's position and orientation.
  - An **image plane reference system (xy)** describes positions on the image plane.
  - The **right-hand rule** is used for orientation.

2. *Projective Geometry (Sources 5-20)*

Projective geometry provides a quantitative description of the camera projection process.

Describing Projection Quantitatively:

- It defines the relationship between a **3D point in the world (P)** and its **2D projection on the image plane (p)**.
- For easier work, projection is often considered on a plane parallel to the image plane, located in front of the optical center at distance `f`, which avoids the upside-down effect while maintaining the same geometrical relation.

Field of View (FoV):

- The FoV defines the limits of the world framed by the camera.

- It is described by the angle 2φ, where φ is the angle under which a point P is seen.
- FoV depends on the **sensor size (d)** and the **focal length (f)**, mathematically expressed as $\varphi = \arctan(d / 2f)$. Horizontal and vertical FoVs are commonly provided.

Camera Projection Equations:
- Using the similar triangle rule, the 2D projected coordinates (xp, yp) relate to the 3D world coordinates (Xp, Yp, Zp) and focal length (f) as:
  - $xp = f * Xp / Zp$
  - $yp = f * Yp / Zp$
- **Important**: Projecting 3D points onto a 2D surface leads to the **loss of distance information (Zp)**.

Homogeneous Coordinates and Projection Matrix:
- **Homogeneous coordinates** are a "mathematical trick" to extend N-dimensional points into (N+1) coordinates, allowing geometric transformations to be represented as matrix multiplications.
- The projection equations can be rewritten in **matrix form** using homogeneous coordinates: $\tilde{m} \approx P \tilde{M}$.
- **P** is the **projection matrix**, which describes how the 3D world is mapped onto the image plane. For a basic pinhole model with focal length f, $P = [f\ 0\ 0\ 0;\ 0\ f\ 0\ 0;\ 0\ 0\ 1\ 0]$. When $f=1$, this represents the essential perspective projection.

Mapping to Image Coordinates (Pixels):
- The projected x, y coordinates (metric distances) need to be converted to pixel coordinates (u, v) for digital images.
- The origin of pixel coordinates is typically the **top-left corner**.
- Conversion factors $ku = 1/w$ (pixel width) and $kv = 1/h$ (pixel height) are used.
- The mapping includes the principal point coordinates ($u0, v0$):
  - $u = u0 + x_p / w = u0 + k_u * x_p$
  - $v = v0 + y_p / h = v0 + k_v * y_p$
- Combining these with the projection equations yields:
  - $u = u0 + f_u * Xp / Zp$
  - $v = v0 + f_v * Yp / Zp$
  - where $f_u = k_u * f$ and $f_v = k_v * f$ are the **focal lengths in pixels**.

Camera Matrix (Intrinsic Parameters):
- The combined projection can be expressed using the **camera matrix K** (also called the intrinsic matrix): $P = K\ [I \mid 0]$.
- $K = [fu\ 0\ u0;\ 0\ fv\ v0;\ 0\ 0\ 1]$.
- The elements of K ($fu, fv, u0, v0$, (and implicitly ku, kv), f) are called **intrinsic parameters**. They define the inherent projection characteristics of the camera.

Camera vs. Real World (Extrinsic Parameters):
- To relate the camera's reference frame to a separate world reference frame, a **rototranslation matrix T** is used.
- $T = [R\ t;\ 0\ 1]$, where R is a **rotation matrix** and t is a **translation vector**.
- The complete projection process then becomes: $\tilde{m} \approx P\ T\ \tilde{M}$.
- The parameters of T (3 for translations, 3 for rotations) are called **extrinsic parameters**. They define the relationship between the camera and the world.

Projection Recap:
- The full projection process involves three transformations:
  1) **3D to 3D**: From world coordinates to camera coordinates (rototranslation).

  2) **3D to 2D**: From camera coordinates to the sensor plane (projection).

  3) **2D to 2D**: From sensor plane to sensor coordinates (pixels) (scaling and translation).

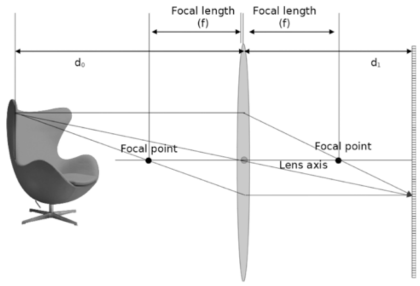
Inverse Projection:
- The inverse transformation (from 2D image back to 3D world) is generally **not invertible**.
- This non-invertibility is due to the **loss of the Z-dimension** during 3D-to-2D projection and **pixel quantization**.
- Inverse projection is possible if:
  - The result is accepted as the **direction of the object** rather than its exact 3D position.
  - **Additional constraints** (e.g., knowledge of a ground plane) provide location along the ray.
  - The **quantization effect is neglected** (assuming pixel location is the same as the projected point), which is acceptable for high-resolution sensors.

3. *Cameras and Lenses (Sources 21-29)*

Real cameras use lenses to overcome the limitations of the simple pinhole model, but lenses introduce their own complexities.

Pinhole Limitations & Adding a Lens:
- The pinhole model has a **trade-off**: a smaller hole gives sharper images but lets in less light (low intensity).
- A **lens** provides sharp images without needing a tiny pinhole, gathering more light.

Thin Lens Model:
- A lens is added, centered on the pinhole, with its main axis on the optical axis.
- A "thin" lens is modeled as a 2D plane where light deviation occurs, neglecting its width.
- **Effect of a lens**: Lenses deviate light rays to **focus** them.
  - Rays passing through the center of the lens are **not deviated**.
  - Rays parallel to the lens axis are deviated through the **focal point**.
- **Thin Lens Equation**: This equation relates the object distance (`d0`), image distance (`d1`), and lens focal length (`f`): `1/d0 + 1/d1 = 1/f`.



Focus and Depth of Field (DoF):
- o Points at a specific distance are **in focus**. Objects at other distances appear out of focus, creating a **circle of confusion**.
- o Changing the distance between the lens and the sensor alters the focus characteristics.
- o **Depth of Field** is the range of distances within which objects appear acceptably in focus, meaning the circle of confusion is negligible.
- o **Aperture controls DoF**: A **smaller aperture (larger f-number)** increases the depth of field, allowing more of the scene to be in focus.

Focal Length Comparison:
- o The term "focal length" has two meanings:
  - ▪ In a **thin lens**, it's the distance where parallel rays intersect.
  - ▪ In the **pinhole camera model**, it's the distance between the pinhole and the sensor.

Non-Ideal Lenses: Distortion and Aberrations:
- o Real lenses are not ideal and introduce effects like **distortion** and **chromatic aberrations**.
- o **Distortion** is a deviation from ideal projection behavior.
  - ▪ **Radial distortion**: The amount of distortion depends on the distance of a point from the image center. It causes straight lines to appear curved. It's often modeled by a polynomial function using coefficients `k1, k2, k3` (e.g., `x_corr = x * (1 + k1*r^2 + k2*r^4 + k3*r^6)`).
  - ▪ **Tangential distortion**: Caused by non-ideal alignment between the lens and sensor. It is usually negligible and modeled with `p1, p2` parameters.
- o **Chromatic aberration (dispersion)**: The refractive index of the lens depends on the light's wavelength, causing different colors to focus at different points. This results in **color fringes** near image edges.
- o Camera models can be coupled with distortion estimation to compensate for these effects.

4. *Camera Calibration (Sources 30-38)*

Camera calibration is the process of estimating the intrinsic and extrinsic parameters of a camera.

What is Camera Calibration?:
- o It's the process of **estimating camera parameters**, including **intrinsic parameters** (which define the camera's internal geometry and lens properties, like focal length, principal point, and distortion coefficients) and **extrinsic parameters** (which define the camera's position and orientation in the world).
- o Calibration is crucial for quantitative applications, such as measuring projection characteristics or relating image points to the real world (e.g., determining distances).
- o Camera orientation can be expressed using angles like **yaw, pitch, and roll** (or pan and tilt).

The Calibration Process Outline:
- o **General steps**:
  1. Acquire an **object of known shape and dimensions** (a **calibration pattern**, e.g., a checkerboard, where corners serve as easily recognizable points).
  2. Take **multiple pictures (N images)** of this pattern from different viewpoints.
  3. For each image, identify the **3D positions of the pattern's corners** in its own reference system (`P_i,j`) and their corresponding **2D pixel coordinates** in the image (`p_i,j`).

4. **Initialize intrinsic parameters** (`fu, fv, u0, v0`, and distortion coefficients `k1, k2, k3, p1, p2`) and **extrinsic parameters** (rototranslation `T`) to default values.
5. Solve a **non-linear least squares problem** to minimize the difference between the observed 2D image points and the projected 3D world points: `min || K [I|0] T_i P_i,j - p_i,j ||^2`.

Challenges and Improvements:

- The minimization process can be unstable due to the large number of parameters.
- **Good initial guesses** are highly desirable.
- **Homography** can provide a good initial guess: For a planar object (like a checkerboard) and neglecting distortion, the transformation from the object to the image plane can be represented by a homography. (*Se l'oggetto che stai inquadrando è piatto (come una scacchiera), e trascuri la distorsione della lente, allora il modo in cui l'oggetto si proietta sull'immagine può essere descritto bene da una omografia (homography)).*
  (Una omografia è una matrice 3×3 che descrive come un piano si trasforma da una vista all'altra).
- **Number of views needed**:
  - Each view of a planar object (e.g., checkerboard) provides **8 constraints** (from 4 "free" corner points). More points are useful for measuring distortion.
  - Neglecting distortion, a minimum of **2 views** are theoretically needed to determine the 4 or 5 intrinsic and 6 extrinsic parameters.
  - In practice, a **larger number of views (e.g., 10 or more)** is needed for a stable and accurate calibration due to the instability of the minimization process.

6. Image Mapping (Sources 39-46)

Image mapping deals with how pixels are transformed spatially in an image.

Geometric Transforms:

- A geometric transform modifies the **spatial relationship among pixels**.
- It involves two steps: **coordinate transformation** (e.g., `x', y' = T(x, y)`) and **image mapping/resampling**.

Forward Mapping:

- Directly applies the transformation from **source pixels** to **destination pixels**.
- A significant drawback is that it can **leave empty pixels** in the destination image if multiple source pixels map to the same destination, or if transformed coordinates fall between integer pixel locations.

Inverse/Backward Mapping:

- Inverts the process: For each pixel in the **destination image**, it calculates the corresponding location in the **source image**.
- **Advantages**:
  - Every pixel at the destination is visited once.
  - Every pixel at the destination is covered by the transformation, ensuring no empty pixels.
- **Interpolation** is used to determine the pixel value when the calculated source location falls between discrete pixel centers. Common interpolation methods include nearest neighbor, bilinear, and bicubic.

Look-Up Table (LUT):

- If the same mapping is applied to many images, the locations of the source pixels can be pre-calculated and saved into a **Look-Up Table (LUT)** (one for x-values, one for y-values). This avoids re-evaluating the transformation repeatedly.

OpenCV Functions:

- Libraries like OpenCV provide functions for image mapping, such as `cv::warpAffine()`, which handles affine transformations and manages the low-level details of backward mapping and interpolation automatically.
- These functions require a transformation matrix (e.g., a 2x3 matrix for affine transforms) as input.

7. Real Cameras (Sources 47-58)

This section discusses additional practical considerations and effects in real-world cameras beyond the idealized models.

Perspective Projection Consequences:

- Cameras fundamentally act as "dimensionality reduction machines," projecting the 3D world onto a 2D surface.
- This transformation results in the **loss of certain information**: angles, distances, and parallelism (though straight lines remain straight).

The Role of Lenses:

- Lenses allow cameras to **gather more light** compared to a pinhole.
- They also necessitate the concept of **focus**.

Perspective vs. Viewpoint & Focal Length:

- While focal length changes the subject size in the image, the **viewpoint (camera distance to subject)** also significantly affects perspective.
- One can compensate for a change in focal length by moving the viewpoint to keep the subject size constant, but this will alter the background and potentially introduce different distortions (e.g., the "Vertigo effect").
- A **large FoV (small focal length)** means the camera is close to the subject, leading to more distortion.
- A **small FoV (large focal length)** means the camera is further from the subject, leading to less distortion.

Image Sensors:

- Modern digital cameras use **sensors (CCD or CMOS)** to convert photons into electrons. These are essentially grayscale sensors.
- **Sensing colors**:
  - **3-chip color sensors**: Use a trichroic prism to split incident light, sending R, G, and B components to separate dedicated imagers.
  - **Single-chip color sensors**: Use a **Bayer Color Filter Array (CFA)** or mosaic pattern on a single imager, requiring **interpolation (demosaicing)** to provide complete color information for each pixel.
  - **Direct image sensors (e.g., Foveon)**: Do not require interpolation, minimizing information loss.

Exposure Control: Aperture and Shutter Speed:

- **Exposure** (the total amount of light reaching the sensor) is controlled by two main parameters: **aperture** and **shutter speed**.
- **Aperture**:
  - The diameter of the lens opening.
  - Expressed as an **f-number (f/N)**, where $A = f/N$ (A is aperture diameter, f is focal length, N is f-number).
  - A **smaller f-number (e.g., f/2.0)** means a **larger aperture opening**.
  - **F-numbers are used** because if the f-number (f/N) is constant, the amount of light gathered is constant regardless of the lens's focal length.
  - Typical f-number progression (e.g., f/2.0, f/2.8, f/4) ensures that each stop roughly **halves or doubles the amount of light**.
- **Shutter Speed**:
  - The **exposure time**: the duration for which the sensor is exposed to light.
  - Typical values include fractions of a second (e.g., 1/60s, 1/1000s).
  - **Electronic shutters** are used in video cameras, controlling exposure electronically.
  - **Global shutter**: All pixels are acquired simultaneously.
  - **Rolling shutter**: The image is acquired row by row, which can lead to distortion with fast motion.
- **Reciprocity**: Various combinations of shutter speed and aperture can yield the **same overall exposure**.
  - **Choice of shutter speed** affects motion: **shorter exposure** (faster shutter speed) **freezes motion**, while **longer exposure** (slower shutter speed) results in **motion blur**.
  - **Choice of aperture** impacts **depth of field** and diffraction effects.
    Large aperture = Small DoF
    Small aperture = Large DoF