[A change in the chosen weight of a memristor directly influences the functional form of $g_\infty(\mathbf{\Delta})$. In turn, this functional form guides the evolution toward a steady state that aligns with the desired one. It is important to note that the entire functional form is significant, not just the steady-state value of the potential drop and the corresponding conductance, as explained in Appendix A.] -> add in paper

Dynamical evolution to the steady state

In this section, we explain more in depth relaxation towards a steady-state of a memristor network. Specifically, we want to emphasise that in a network where the applied voltage is constant at the input nodes, the dynamical dependence on the steady-state conductance is fundamental to reach desired values of potential drops.

For simplicity, we consider a voltage divider, as shown in Fig. 1, where we set $\Delta P = 0$ and $\Delta\rho = 0$.

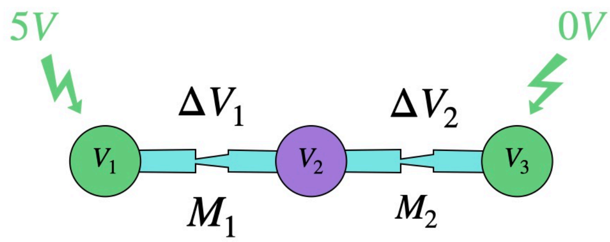

Figure 1

Therefore, the dynamical evolution of the conductance of each memristor discretized in timesteps Δt, shown in its general form in Eq. (2), reduces to

$$g_{\mathrm{m}}\left(t+\Delta t\right) = g_{\mathrm{m}}\left(t\right) + \frac{g_{\mathrm{m},\infty}\left(\Delta V_{\mathrm{m}}(t)\right) - g_{\mathrm{m}}\left(t\right)}{\tau}\Delta t \tag{1}$$

with initial condition $g_{\mathrm{m}}(t=0) = g_0$. At each time-step, the potential drop $\Delta V_m(t)$ across each memristor is calculated using Kirchoff law, that reduces to the expressions

$$\Delta V_1(t) = \frac{5g_2(t)}{g_1(t)+g_2(t)}, \qquad \Delta V_2(t) = \frac{5g_1(t)}{g_1(t)+g_2(t)} \tag{2}$$

found by minimizing the power dissipated by the resistances with respect to the free potential $\partial P/\partial V_2 = 0$.

The algorithm works as follows:

- At time $t = 0$, potentials $\Delta V_1(t=0)$ and $\Delta V_2(t=0)$ are computed from Eq. (2) using the initial condition $g_1(t=0) = g_2(t=0) = g_0$.
- In the following time-step $t = \Delta t$, potentials $\Delta V_1(t=0)$ and $\Delta V_2(t=0)$ are used to compute conductances $g_{\mathrm{m}}(\Delta t)$ with Eq. (1)
- The previous step is iterated, and the system reaches the steady-state when $g_{\mathrm{m}}(t) = g_{\mathrm{m},\infty}(t)$.

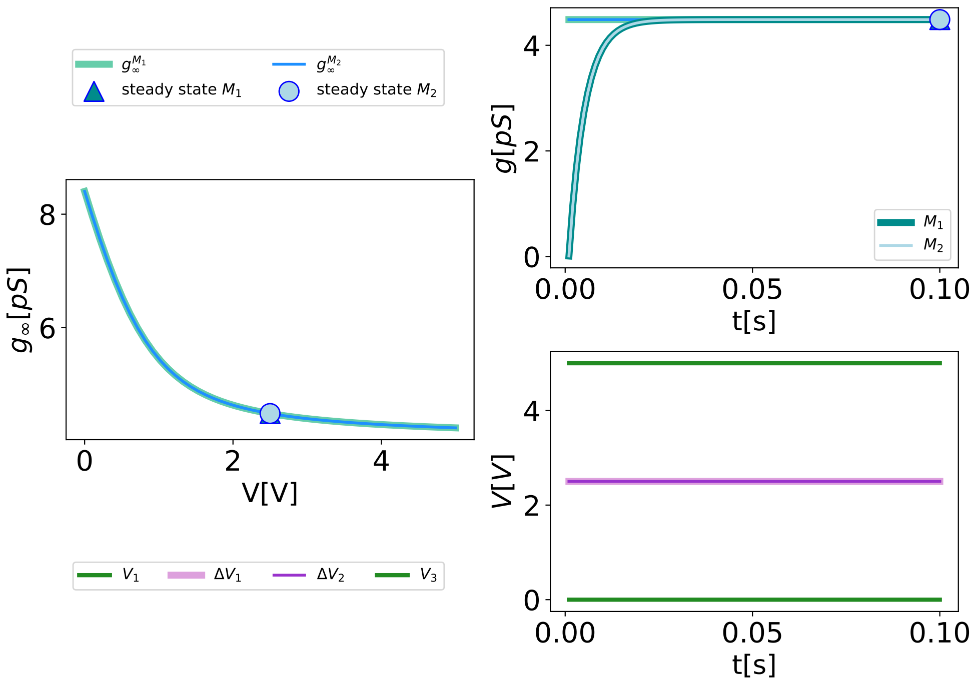

Figure 2: Relaxation towards steady-state when $g_{1,\infty}(V) = g_{2,\infty}(V)$.

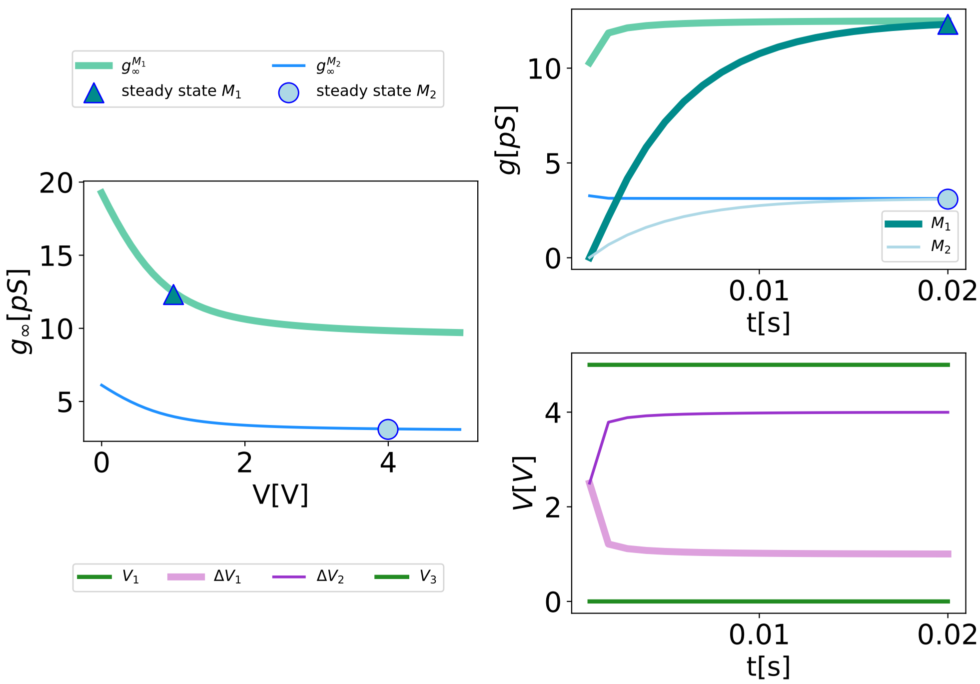

Figure 3: Relaxation towards steady-state when the system has been trained to give $V_2 = 4V$. The training sets the two different functional forms of g_∞ seen in the right figure.

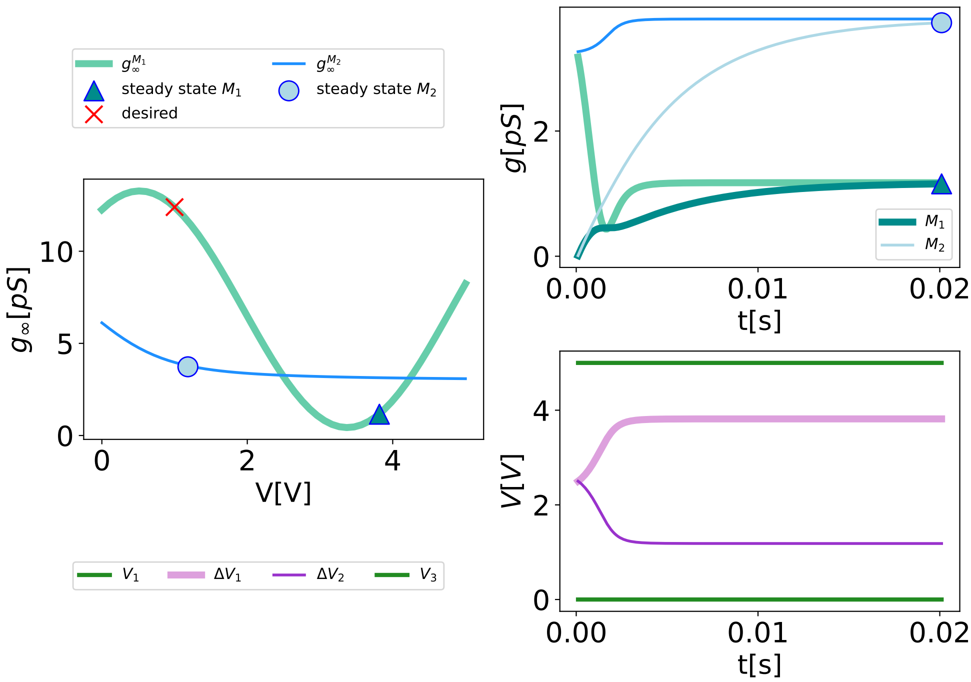

Figure 4: We consider the trained system in Figure 3, but we define the funtion $g_{1,\infty}$ such that it passes througth the steady-state point (triangle in Figure 3), not indicated with a red cross, but with a different functional form. Due to the different functinal form, the system reaches a steady-state that doesn't coincide with the desired in Figure 3.