

[A change in the chosen weight of a memristor directly influences the functional form of $g_\infty(\Delta)$. In turn, this functional form guides the evolution toward a steady state that aligns with the desired one. It is important to note that the entire functional form is significant, not just the steady-state value of the potential drop and the corresponding conductance, as explained in Appendix A.] -> add in paper

Dynamical evolution to the steady state

In this section, we explain more in depth the relaxation toward a steady-state of a memristor network. Specifically, we want to emphasise that in a network where the applied voltage is constant at the input nodes, the dynamical dependence on the steady-state conductance is fundamental to reach desired values of potential drops.

For simplicity, we consider a voltage divider, as shown in Fig. 1, where we set $\Delta P = 0$ and $\Delta \rho = 0$.

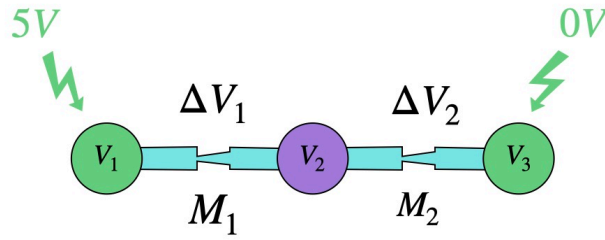


Figure 1

Therefore, the dynamical evolution of the conductance of each memristor discretized in time-steps Δt , shown in its general form in Eq. (2), reduces to

$$g_m(t + \Delta t) = g_m(t) + \frac{g_{m,\infty}(\Delta V_m(t)) - g_m(t)}{\tau} \Delta t, \quad (1)$$

with initial condition $g_m(t = 0) = g_0$. At each time-step, the potential drop $\Delta V_m(t)$ across each memristor is calculated using Kirchoff's law, that reduces to the expressions

$$\Delta V_1(t) = \frac{5g_2(t)}{g_1(t) + g_2(t)}, \quad \Delta V_2(t) = \frac{5g_1(t)}{g_1(t) + g_2(t)}, \quad (2)$$

found by minimizing the power dissipated by the resistances with respect to the free potential $\partial P / \partial V_2 = 0$.

The algorithm works as follows:

- At time $t = 0$, the potentials $\Delta V_1(t = 0)$ and $\Delta V_2(t = 0)$ are computed from Eq. (2) using the initial condition $g_1(t = 0) = g_2(t = 0) = g_0$.
- In the following time-step $t = \Delta t$, the potentials $\Delta V_1(t = 0)$ and $\Delta V_2(t = 0)$ are used to compute conductances $g_m(\Delta t)$ using Eq. (1).
- The previous step is iterated, and the system reaches the steady-state when $g_m(t) = g_{m,\infty}(t)$.

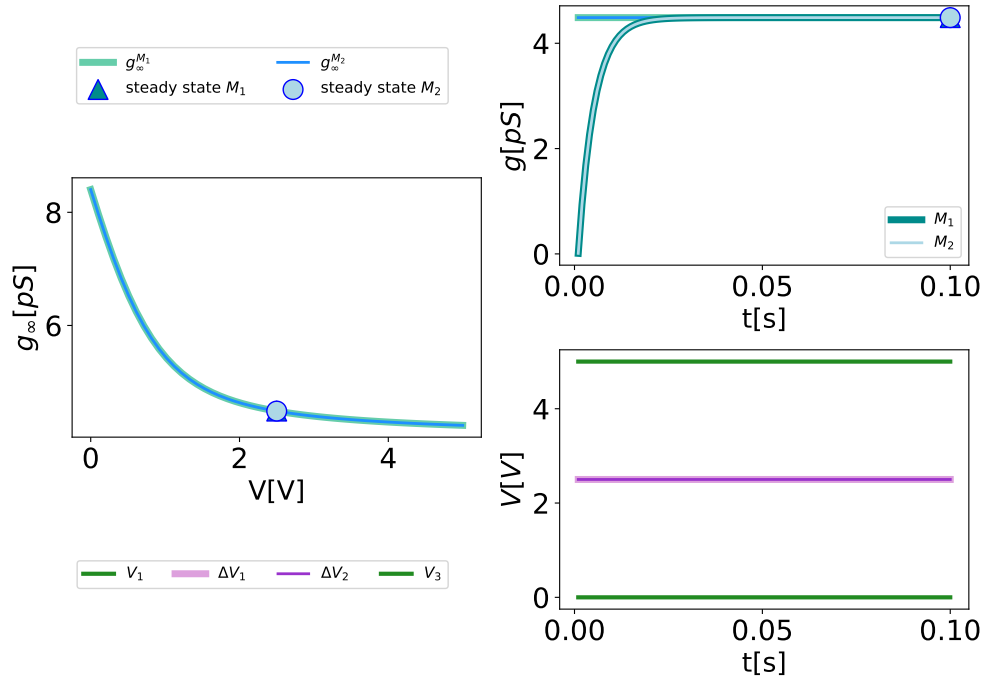


Figure 2: Relaxation toward the steady state when $g_{1,\infty}(V) = g_{2,\infty}(V)$.

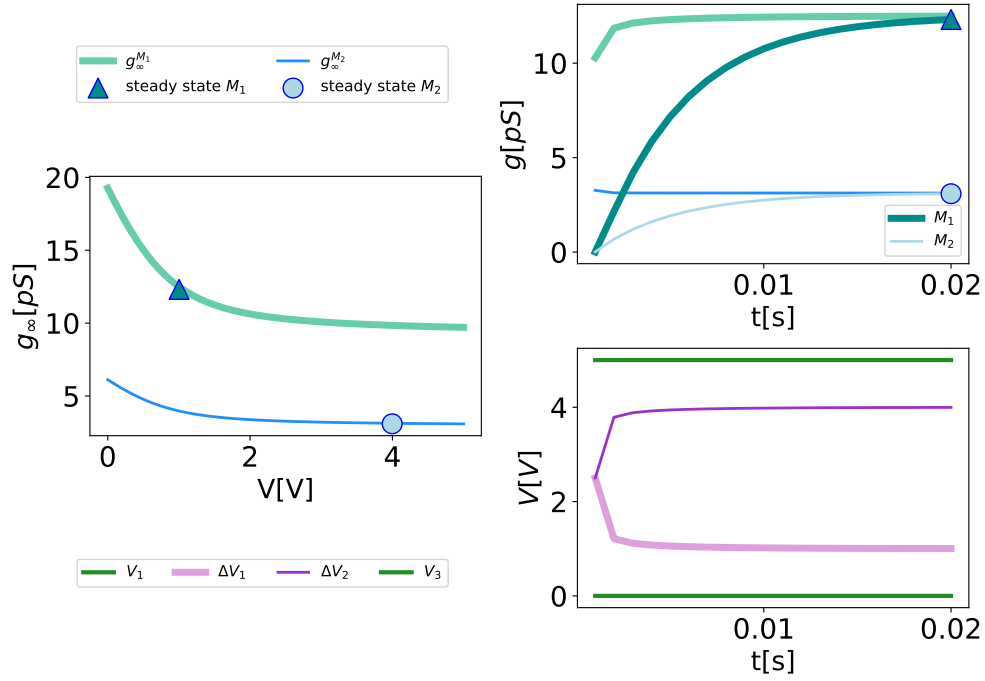


Figure 3: Relaxation toward the steady state when the system has been trained to give $V_2 = 4V$. The training sets the two different functional forms of g_∞ seen in the right figure.

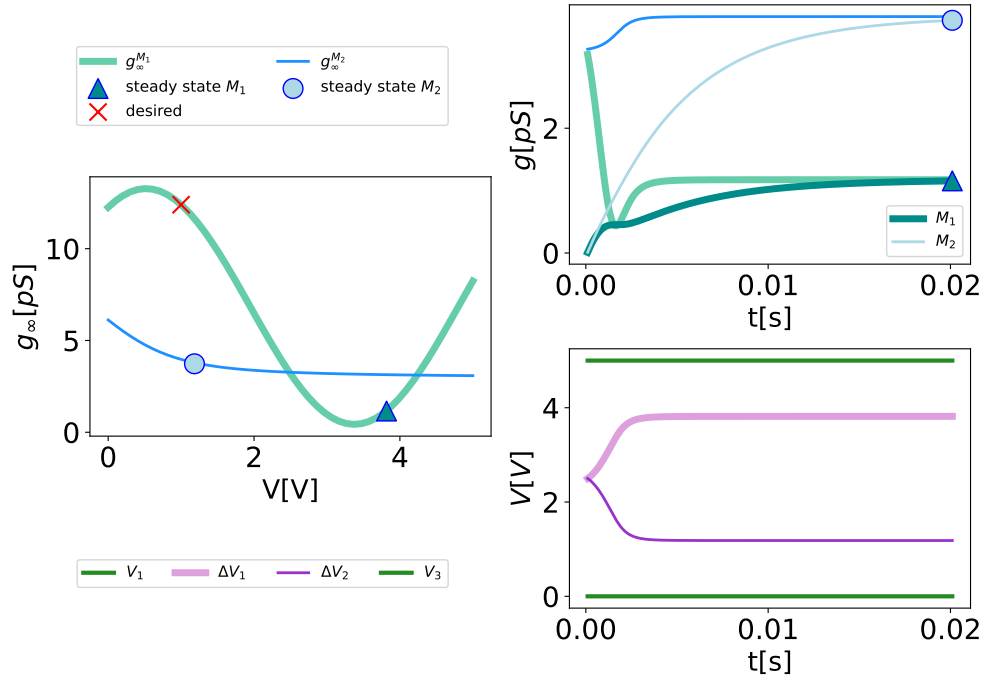


Figure 4: We consider the trained system in Figure 3, but we define the function $g_{1,\infty}$ such that it passes through the steady state point (triangle in Figure 3), now indicated with a red cross, but with a different functional form. Due to the different functional form, the system reaches a steady state that doesn't coincide with the desired in Figure 3.