RESEARCH STATEMENT

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1. Previous work

I completed my PhD with Cristóbal Rojas and Sebastián Barbieri, and my PhD work was mainly about topological dynamical systems over groups, subshifts of finite type (SFT) on groups, and connections between dynamical and recursive properties of symbolic systems.

In order to explain my work it is convenient to mention a famous theorem of Kechris and Rosendal stating that in the space of continuous actions $\mathbb{Z} \curvearrowright \{0,1\}^{\mathbb{N}}$ there is a residual conjugacy class [15]. A countable group G is said to have the strong topological Rokhlin property (STRP) if the space of continuous actions $G \curvearrowright \{0,1\}^{\mathbb{N}}$ contains a residual conjugacy class. It is an open problem to classify those countable or finitely generated groups with the STRP.

In [8] I proved a sufficient condition for the failure of the STRP of a recursively presented group in terms of Medvedev degrees of SFTs. The criterion states that it is sufficient to construct a single SFT with nonzero Medvedev degree. Medvedev degrees are a complexity measure which shares some properties with entropy, and is defined using notions from recursion theory. Combining the result in [8] with known results about Medvedev degrees of SFTs, one obtains a number of new examples of groups without the STRP. One also recovers previous results (e.g. that $\mathbb{Z}^d$ does not have the STRP for $d > 2$, proved by Hochman in [14]). These results about the STRP are a motivation for the classification of Medvedev degrees of SFTs over different groups.

One of the central topics in my PhD thesis is the classification of Medvedev degrees of SFTs over different groups. In [4] I proved a classification of the Medvedev degrees of effective subshifts over groups with decidable word problem. In [1] we used these results to classify the Medvedev degrees of SFTs of groups of polynomial growth, groups of the form $G \times H$, with G, H finitely generated infinite and with decidable word problem, some hyperbolic groups, and others. We continue to work on this [9].

These problems about Medvedev degrees took me to problems in recursive combinatorics (see [4]). This derived in the works [11, 5] on effective graph theory.

I also worked is understanding the class of topological factors of symbolic systems and SFTs. In [3] we exhibited new classes of factors of SFTs using recursive techniques. We also developed some tools relating computable analysis and subshifts on groups, which were important for my work on Medvedev degrees.

I also worked on automorphism groups of subshifts. In [2] we computed the center of the automorphism group of a strongly irreducible subshift on an infinite group. This work generalizes existing results about automorphism groups of subshifts on $\mathbb{Z}$.

Furthermore, I also proved some general undecidability results for dynamical properties of SFTs on $\mathbb{Z}^2$ and other groups [6].

After my PhD I have kept working on directions related to the ones mentioned here, both alone and with other researchers [10].

2. Current work

After I finished my PhD I have been a postdoctoral researcher with Adam Kanigowski. I am currently working in entropy-type invariants such as slow entropy and sequence entropy, both for topological systems and measure-preserving systems.

I have been specifically interested in dynamical systems which are skew products driven by a cocycle, and in understanding how entropy-type invariants of these systems are related to the growth of the Birkhoff sums of the cocycle. This relation is explored in a precise manner in my recent work [7] about topological slow entropy. The main research direction for my remaining time here in Poland is the measure-preserving version of the same problem. The preprint [7] also contains a generalization of a classic result of Goodman [13] about topological sequence entropy. I am also working on generalizing this result to actions of amenable groups (in preparation).

3. Future work

One research direction that currently interests me is the following. Consider a topological or measure-preserving dynamical system $T\colon X \to X$ and a map $\tau\colon X \to G$ taking values in an abelian group G. Define $\tau^n(x) = \sum_{i=0}^{n-1} \tau(T^i(x))$, $n \in \mathbb{N}$, $x \in X$, and consider the *range*

$$R_n(x) = \{\tau^i(x) : i = 0, \dots, n-1\}$$

One can interpret $(\tau^n(x))_{n \geq 0}$ as a walk over G driven by τ and the dynamics of T. Then $R_n(x)$ is the set of places in G that the walker visits in the first n steps. Consider the following aspects of the range of the walk:

- Growth: how does the cardinality of $R_n(x)$ evolve with time?
- Shape: how does the boundary of $R_n(x)$ evolve with time? (in the sense of [12])

I am interested in understanding the interplay between these properties of the walk driven by τ, and entropy-type invariants of skew products dynamical systems defined by τ. This interplay is studied in [7] for $G = \mathbb{Z}$ and thus these questions are a follow-up in a more general context.

References

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