RESEARCH STATEMENT

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1. Previous work

I completed my PhD with Cristóbal Rojas and Sebastián Barbieri, and my thesis [4] was about topological dynamics and symbolic dynamics on groups. I was particularly interested in symbolic systems, subshifts of finite type (SFT), and their recursive properties.

A central topic of interest during my PhD was a complexity measure for symbolic systems called Medvedev degrees. These degrees share some properties with amenable topological entropy, and have been useful to provide information on the dynamics of symbolic systems over $\mathbb{Z}^2$ and other groups. This connection was studied or exploited by other authors in [15, 13, 12, 14, 10]. Works of mine on this subject include [5, 1].

Another topic in which I worked is understanding the class of topological factors of symbolic systems and SFTs. In [3] we exhibited new classes of factors of SFTs using recursive techniques.

Furthermore, I also proved some general undecidability results for dynamical properties of SFTs on $\mathbb{Z}^2$ and other groups [6].

I also worked in automorphism groups of symbolic systems. In [2] we generalized and strengthened classic result about automorphism groups of subshifts over $\mathbb{Z}$ to other groups.

After my PhD I have kept working on directions related to the ones mentioned here with other researchers [9, 8].

2. Current work

After I finished my PhD I have been a postdoctoral researcher with Adam Kanigowski. I am currently working in entropy-type invariants such as slow entropy and sequence entropy, both for topological systems and measure-preserving systems.

I have been specifically interested in dynamical systems which are skew products driven by a cocycle, and in understanding how entropy-type invariants of these systems are related to the growth of the Birkhoff sums of the cocycle. This relation is explored in a precise manner in my recent work [7] about topological slow entropy. The main research direction for my remaining time here in Poland is the measure-preserving version of the same problem. The preprint [7] also contains a generalization of a classic result of Goodman [11] about topological sequence entropy. I am also working on generalizing this result to actions of amenable groups.

3. Future work

I am interested in continuing to work with topological and measure-preserving dynamical systems. I am very curious and interested in learning new things.

A specific direction that interests me is the following. Consider a topological or measure preserving dynamical system $T\colon X \to X$ and a cocycle $\tau\colon X \to G$ taking values in a group G. Define

$\tau^n(x) = \sum_{i=0}^{n-1} \tau(T^i(x))$, $n \in \mathbb{N}$, $x \in X$, and consider the *range*

$$R_n(x) = \{\tau^i(x) : i = 0, \dots, n-1\}$$

One can interpret $(\tau^n(x))_{n\geq 0}$ as a walk over G driven by τ and the dynamics of T. Then $R_n(x)$ is the set of places in G that the walker visits in the first $n-1$ steps. Consider the following aspects of the range of the walk:

- Growth: how does the cardinality of $R_n(x)$ evolve with time?
- Shape: how does the boundary of $R_n(x)$ evolve with time?

I am specifically interested in understanding the interplay between these properties of the walk driven by τ, and entropy-type invariants of skew products dynamical systems defined by τ.

References

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