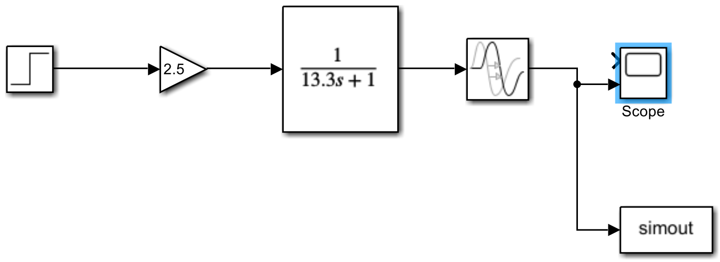
EE 407
HW#3

Q1. b) $t_1 = 23.7$ secs
$t_2 = 37$ secs } from the figure.

$\tau_p \cong \frac{1}{0{,}7}(37 - 23{,}7) = 13{,}3$ secs

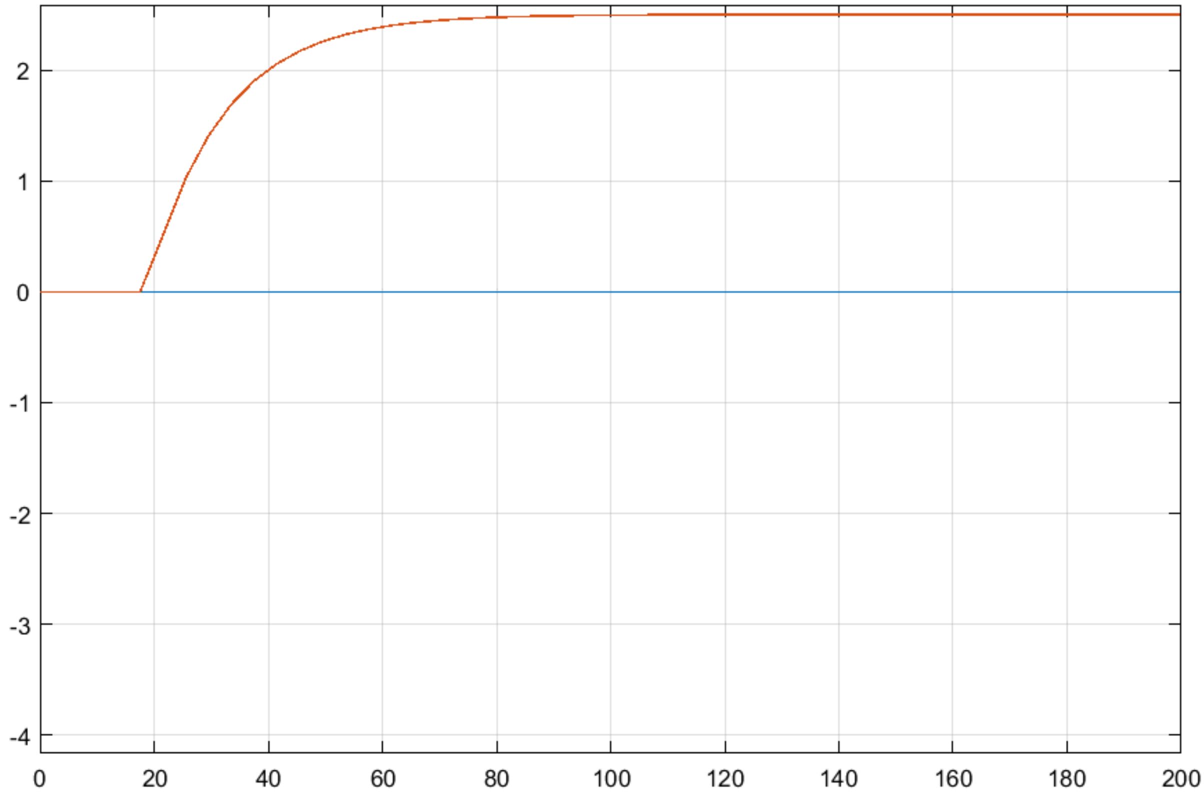
$\theta_p \cong 23{,}7 - 0{,}4 \cdot 13.3 = 18.38$ secs

$G(s) = \frac{e^{-18.38s}}{1 + 13.3s}$ $\quad K = \frac{\Delta y}{\Delta u} = 1$


2.5
1
13.3s + 1
Scope
simout

d.

e.


2
1
0
-1
-2
-3
-4
0
20
40
60
80
100
120
140
160
180
200

407

HW#

Q2. a) $A_1 \dot{h}_1 = q_i - q_1 = -\frac{1}{R_1} h_1 + q_i$

$A_2 \dot{h}_2 = -\frac{1}{R_2} h_2 + q_1 = -\frac{1}{R_2} h_2 + \frac{1}{R_1} h_1$

$A_3 \dot{h}_3 = -\frac{1}{R_3} h_3 + \frac{1}{R_2} h_2$

$\vdots$

$A_{n-1} \dot{h}_{n-1} = -\frac{1}{R_{n-1}} h_{n-1} + \frac{1}{R_{n-2}} h_{n-2}$

$A_n \dot{h}_n = -\frac{1}{R_n} h_n + \frac{1}{R_{n-1}} h_{n-1}$

$$\left\{\begin{aligned}
\dot{h}_1 &= -\frac{1}{A_1 R_1} h_1 + \frac{1}{A_1} q_i \\
\dot{h}_2 &= -\frac{1}{A_2 R_2} h_2 + \frac{1}{A_2 R_1} h_1 \\
\dot{h}_3 &= -\frac{1}{A_3 R_3} h_3 + \frac{1}{A_3 R_2} h_2 \\
&\vdots \\
\dot{h}_{n-1} &= -\frac{1}{A_{n-1} R_{n-1}} h_{n-1} + \frac{1}{A_{n-1} R_{n-2}} h_{n-2} \\
\dot{h}_n &= -\frac{1}{A_n R_n} h_n + \frac{1}{A_n R_{n-1}} h_{n-1}
\end{aligned}\right.$$

$$\dot{h} = \begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \\ \vdots \\ \dot{h}_n \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{1}{A_1R_1} & 0 & 0 & & & \\ \frac{1}{A_2R_1} & -\frac{1}{A_2R_2} & 0 & & 0 & \\ 0 & \frac{1}{A_3R_2} & -\frac{1}{A_3R_3} & & & \\ \vdots & 0 & & \ddots & & \\ 0 & 0 & 0 & \cdots & \frac{1}{A_nR_{n-1}} & \frac{1}{A_nR_n} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ \vdots \\ h_n \end{bmatrix}}_{h} + \underbrace{\begin{bmatrix} \frac{1}{A_1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{B} q_i$$

b) $\dot{h} = Ah + Bq_i \Rightarrow sH(s) = AH(s) + BQ_i(s) \Rightarrow H(s) = (sI-A)^{-1} BQ_i(s)$

$h_n = Ch + 0q_i \Rightarrow C = [0\ 0 \ldots 1]\, h \qquad H_n(s) = CH(s)$

$H_n(s) = C(sI-A)^{-1} B Q_i(s) \Rightarrow G(s) = \frac{H_n(s)}{Q_i(s)} = C(sI-A)^{-1}B$

$G(s) = \frac{H_n(s)}{Q_i(s)} = \frac{H_n(s)}{H_{n-1}(s)} \cdot \frac{H_{n-1}(s)}{H_{n-2}(s)} \cdots \frac{H_2(s)}{H_1(s)} \cdot \frac{H_1(s)}{Q_i(s)}$

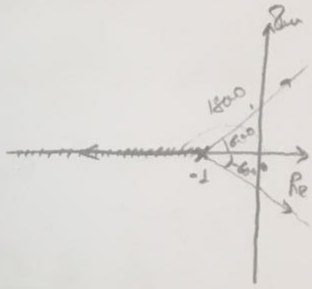
1/3)

Therefore we have a positive phase margin, which can never be zero or negative.

e) $n = 3$: $G(s) = \dfrac{1}{R_1 R_2 A_1 A_2 A_3 \left(s+\frac{1}{A_1 R_1}\right)\left(s+\frac{1}{A_2 R_2}\right)\left(s+\frac{1}{A_3 R_3}\right)}$

$\tau_i = R_i A_i = 1\,\text{min}$; $R_i = 1 \frac{\text{min}}{m^2} \Rightarrow A_i = 1\, m^2$

$\Rightarrow G(s) = \dfrac{1}{(s+1)(s+1)(s+1)}$ we have 3 poles: _



# of assymp.: $|n-m| = 3$

$\measuredangle$ b/w assymp.: $\pm\dfrac{(2k+1)180°}{3} = \pm 60;\ 180°$

$\sigma_0 = \dfrac{\sum p_i - \sum z_i}{n-m} = \dfrac{-3}{3} = -1$

$(s+1)(s+1)(s+1) + K\cdot 1 = s^3 + 3s^2 + 3s + 1 + K = 0$

s^3	1	3
Aux s^2	3	$(1+K)$
s	$\frac{8-K}{3}$	
1	$(1+K)$	

$K = 8$ Auxiliary polynomial:

$3s^2 + 1 + K = 0$

$3s^2 + 9 = 0 \Rightarrow s = \pm j\sqrt{3}$

$\omega = \pm\sqrt{3}$

$K_c = 8$; $\omega_{cu} = \pm\sqrt{3}$ rad/min

f

g

h

3. a: m_1: $m_1 c_1 \frac{dT_1(t)}{dt} = q_i - q_o - h_i A_i (T_1 - T_2)$

m_2: $m_2 c_2 \frac{dT_2(t)}{dt} = \cancel{q_i - q_o} - h_i A_1 (T_2 - T_1)$

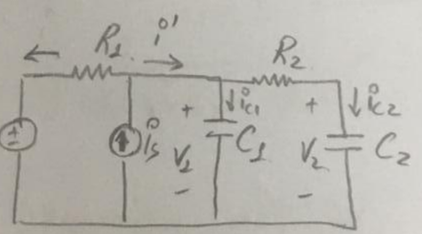
$= h_i A_i (T_1 - T_2)$

b) Their behaviour is similar to capacitors & inductors behaviours in the circuit. So:

m_1: $C_1 \frac{dV_1(t)}{dt} = i_{c1} = i' - \frac{V_1 - V_2}{R_2} = i_s - i - \frac{V_1 - V_2}{R_2}$

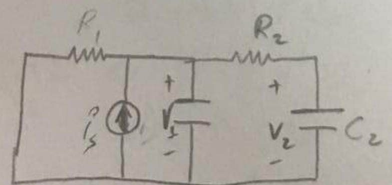
here. V-acts as T, i acts as energy (power)

m_2: $C_2 \frac{dV_2(t)}{dt} = \frac{V_1 - V_2}{R_2}$ $\Big\}$ => $V = T$; $q = i$; $C_1 = m_1 c_1$; $C_2 = m_2 c_2$; $R_1 = \frac{1}{h_o A_o}$; $R_2 = \frac{1}{h_i A_i}$



Equivalent circuit

c)



$G(s) = \frac{T_1(s)}{Q(s)}$:

$i_s = \frac{V_1}{R_1} + C_1 \frac{dV_1}{dt} + \frac{C_2 dV_2}{dt}$

in s domain: $i_s = V_1 \left(\frac{1}{R_1} + C_1 s + \frac{1}{R_2 + C_2 s} \right)$

$$\frac{V_1}{i_s} = \frac{1}{\frac{1}{R_1} + \frac{1}{1/sC_1} + \frac{1}{R_2 + 1/sC_2}} = \frac{R_2 + \frac{1}{sC_2}}{\frac{R_2}{R_1} + \frac{1}{R_1 s C_2} + sC_1 R_2 + \frac{C_1}{C_2} + 1} = \dots$$

... behaviours in the circuit is similar to capacitors in the circuit. So:

3C. ooo

$$= \frac{R_1 R_2 C_2 s + R_1 C_2}{R_2 C_2 s + 1 + R_1 C_1 C_2 s^2 + R_1 C_1 s + R_1 C_2 s} =$$

$$= \frac{R_1 C_2 + R_1 R_2 C_2 s}{1 + (R_1 C_1 + R_2 C_2 + R_1 C_2) s + R_1 C_1 C_2 s^2}$$

d) $G(s) = \dfrac{0{,}5\,(10s+1)}{2000s^2 + 410s + 1} =$

$t_{1/3} = t_0 + \theta_p + T_p \ln(1,5)$ =>

$\theta_p = t_{1/3} - t_0 - T_p \ln(1,5)$, when $t_0 = 0$ =>

$\theta_p = t_{1/3} - T_p \cdot 0,4$ ✓

b) i) Without dead band we wouldn't even have ON-OFF controller. It's the range of the input where controlle doesn't operate.
Without it, the controller's lifetime would be much lesser and it would consume much more energy. Temperature controllers, oven, cooling controller

ii) A self-regulating process: as this type of controller gives the output always proportional to error it will never reach zero-error steady state

iii) In the first case the speed is changing so the system must be follow set point. It should have aggressive mode

For the second case the set point is const therefore, moderate response should be used

iv)

EE402

HW#

Q 4. $y(t) = (1 - e^{-\frac{t-t_0-\theta_p}{T_p}}) u'(t - t_0 - \theta_p) \Delta y + y_0$

$$y(t) - y_0 = (1 - e^{-\frac{t-t_0-\theta_p}{T_p}}) \Delta y \quad , \; t \geq t_0 + \theta_p$$

$$1 - e^{-\frac{t-t_0-\theta_p}{T_p}} = \frac{y(t) - y_0}{\Delta y}$$

$$e^{-\frac{t-t_0-\theta_p}{T_p}} = 1 - \frac{y(t) - y_0}{\Delta y} \qquad -\left(\frac{t - t_0 - \theta_p}{T_p}\right) = \ln\left(\frac{\Delta y - y(t) + y_0}{\Delta y}\right)$$

$$t = -T_p \ln\left(\frac{\Delta y - y(t) + y_0}{\Delta y}\right) + t_0 + \theta_p$$

$$t_{1/3} = T_p \ln\left(\frac{\Delta y}{\Delta y - y_{1/3} + y_0}\right) + t_0 + \theta_p = T_p \ln\left(\frac{\Delta y}{\Delta y - \cancel{y_0} - \frac{\Delta y}{3} + \cancel{y_0}}\right) + t_0 + \theta_p$$

① $t_{1/3} = t_0 + \theta_p + T_p \ln(1,5)$

② $t_{2/3} = t_0 + \theta_p + T_p \ln(3)$

Subtract: ① - ②

$$t_{1/3} - t_{2/3} = T_p(\ln(1,5) - \ln(3))$$

$$t_{1/3} - t_{2/3} = T_p \ln\frac{1}{2}$$

$$t_{2/3} - t_{1/3} = \ln 2 \, T_p \Rightarrow T_p = \frac{1}{\ln 2}(t_{2/3} - t_{1/3}) = \frac{1}{0.7}(t_{2/3} - t_{1/3})$$

$$t_{1/3} = t_0 + \theta_p + T_p \ln(1,5) \Rightarrow$$

$$\theta_p = t_{1/3} - t_0 - T_p \ln(1,5) \quad , \text{ when } t_0 = 0 \Rightarrow$$

$$\theta_p = t_{1/3} - T_p \cdot 0,4 \quad ✓$$

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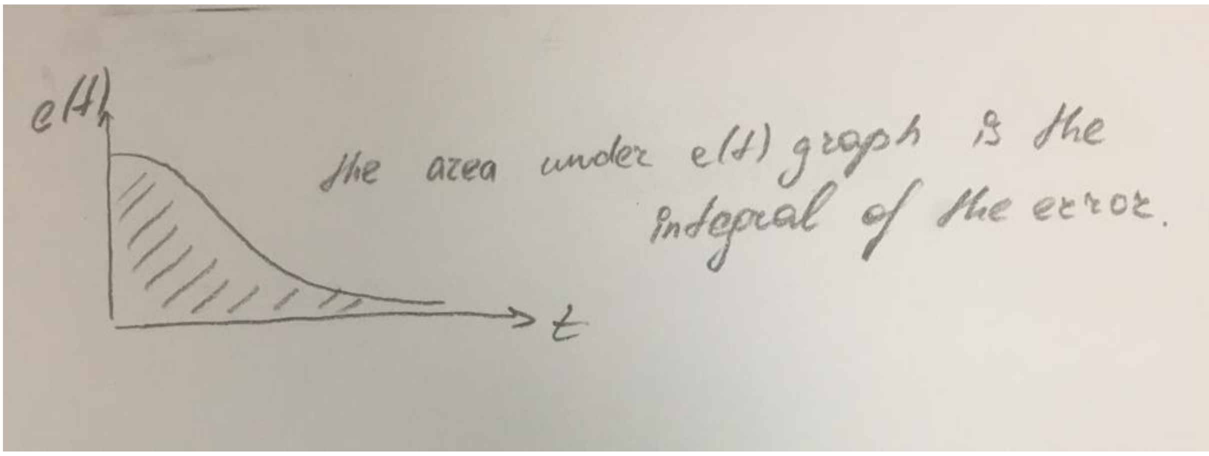
V. The controller Kp can be positive and negative. When Kp is positive the controller's mode is reverse acting mode. It means that when positive error value decreases it will apply more pressure on valve in order to decrease inflow rate of water and vice versa for increasing error value. When Kp is positive the controller is direct acting mode. It acts in an oppiste way to reverse acting mode

VI.



VII. Decay ratio and settling time, and the rise time and the peak time are positively correlated

IX. Derivative kick is the high output on a derivative controller for a small time. When we change when the set point step wise, a large error occurs which increases immediately the value of the derivative controller. It should be eliminated by taking the derivative of output because it can damage the controller over time . As derivative of the output doesn't change suddenly the derivvative value will not also change.

X. Ideal derivative action is detrimental to controller performance when there is a noise.