

Basic physics - Introduction

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Scope of the course

Material coverage:

0. Physics (introduction to the course, computer tools, mathematics)
1. Mechanics (description of motion, causes of motion).
2. Study of motion: harmonic oscillator, projectile motion, mathematical pendulum, etc.
3. Waves (optics, sound waves, interference, Doppler effect).
4. Electromagnetism (Maxwell equations, Electromagnetic waves).
5. Electricity (Kirchoff Laws, RLC, diodes).
6. Measurements (speed of light, wavelength of laser, g from mathematical pendulum, static characteristics of semiconductor diodes and bipolar transistor).
7. Modern Physics I (Atoms, Quantum Mechanics, Particle Physics).
8. Modern Physics II (Special and General Relativity, Cosmology, and Astrophysics).

Exercises:

- Complete several MS Forms assignments in MS Teams.
- Practice and understand better lecture material... **How?**

Organizational Matters

All materials useful for the course will be collected in a GitHub repository.

<https://github.com/dchorazkiewicz/Physics-Lectures>

-- Bookmark this page!

Access will be open, so there's no need to log in. This will streamline work and ensure equal access for everyone.

Passing course

- **Exam:** you need positive grade from exercises! Exam using MS Forms.
- **Exercises:** we plan 8 subject/sections based on topics (Physics Intro+Mechanics+Study of motion, Waves, Electromagnetism, Electricity, Measurements, Modern Physics I, Modern Physics II).
- All will be ending with the **big MS Forms** questions/problems to complete. Your final grade will depend from it! (week or two for completing)

Weekly exercises: **small MS Forms** or/and **github list** with problems (engage you, make you familiar with the problems and learn course topics).

Idea standing behind the course

Course Vision:

- This course is designed as a journey through the vast and intriguing world of physics, exploring fundamental principles that govern the Universe.
- We aim to provide a comprehensive overview of the basic branches of physics, connecting important concepts with cutting-edge tools.

Utilizing Modern Tools and Techniques:

- Apply cutting-edge tools and software used in physics research and industry.
- Gain experience with simulations, data analysis, and computational models.
- Equip yourself with practical skills that bridge theory with real-world applications.

Tools you can use in all classes and at exams (!):

- WolframAlpha - <https://www.wolframalpha.com/> (free version is enough)
- Colab - <https://colab.research.google.com/> (free version is enough)
- ChatGPT - <https://chat.openai.com/> (free version is enough)
- Gemini Google - <https://gemini.google.com/app> (free version is enough)
- Geogebra - <https://www.geogebra.org/>

You have to be a driver of this tools!



Tools you can and should(!) use during classes and exams

FROM THE MAKERS OF WOLFRAM LANGUAGE AND MATHEMATICA



pendulum

NATURAL LANGUAGE MATH INPUT

EXTENDED KEYBOARD EXAMPLES UPLOAD RANDOM

Assuming "pendulum" refers to a formula | Use as referring to physical system or a music act or a word instead

Computational Inputs:

Assuming small-oscillation pendulum | Use full pendulum instead

Calculate: period

» length: 1 m
» initial angle: 5°
» gravitational acceleration: 1 g

Also include: mass | moment of inertia

Compute

Input information

small-oscillation pendulum

length 1 meter

initial angle 5° (degrees)

gravitational acceleration 1 g (standard acceleration due to gravity on the surface of the earth)

Results

More units

Step-by-step solution

period 2.006 seconds

frequency 0.4984 Hz (hertz)

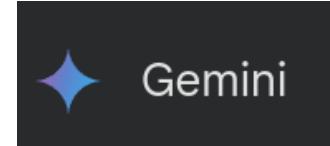
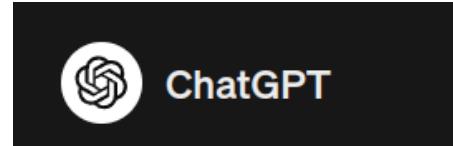
maximum speed 27.32 cm/s (centimeters per second)
= 0.6111 mph (miles per hour)
= 0.9835 km/h (kilometers per hour)

Equation

$$T = 2\pi \sqrt{\frac{l}{g}} \quad f = \frac{1}{T}$$

$$v_{\max} = \sqrt{2gl(1 - \cos(\theta_0))}$$

FROM THE MAKERS OF WOLFRAM LANGUAGE AND MATHEMATICA



- <https://www.wolframalpha.com/>
- <https://chat.openai.com/>
- <https://gemini.google.com/app>
- <https://colab.research.google.com/>
- <https://www.geogebra.org/>

ChatGPT



ChatGPT

A mathematical pendulum, also known as a simple pendulum, is an idealized model used in physics to describe the motion of a pendulum under the influence of gravity. It consists of a weightless, inextensible cord of fixed length l , with a mass m (referred to as a bob) attached to one end, while the other end is fixed at a pivot point. When displaced from its equilibrium position and released, the mass swings back and forth due to the gravitational force, undergoing periodic motion.

The motion of a mathematical pendulum is described by the differential equation:

$$\frac{d^2\theta}{dt^2} + \frac{g}{l} \sin(\theta) = 0$$

where:

- θ is the angular displacement from the vertical,
- g is the acceleration due to gravity,
- l is the length of the pendulum.

For small angles (θ), the $\sin(\theta)$ term can be approximated by θ (in radians), leading to a simpler form of the equation that describes simple harmonic motion:

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$$

The period T of the pendulum, which is the time it takes for the pendulum to complete one full swing back and forth, is given by:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

This formula shows that the period T is independent of the mass m and depends only on the length l of the pendulum and the gravitational acceleration g .

Gemini



A mathematical pendulum is an idealized model of a real pendulum. It aims to simplify the real-world behavior by making certain assumptions, making it easier to analyze and understand its motions. Here are the key characteristics of a mathematical pendulum:

Idealizations:

- **Point mass:** The bob (the swinging object) is treated as a point with all its mass concentrated at a single location.
- **Massless string/rod:** The string or rod suspending the bob has no mass, so it doesn't contribute to the movement.
- **Frictionless:** There is no friction or air resistance acting on the bob, so its swings wouldn't slow down over time.
- **Small angles:** The pendulum swings back and forth only through small angles compared to its length. This allows us to use sine and cosine functions for calculations, which simplifies the equations.
- **Planar motion:** The bob moves only in a vertical plane determined by its initial swing.

Motion:

- **Oscillation:** Due to gravity, the bob oscillates back and forth about its equilibrium position (hanging straight down).
- **Period:** The time it takes to complete one full swing (back and forth) is called the period and is independent of the bob's mass, depending only on the length of the pendulum and the acceleration due to gravity (g).
- **Frequency:** This is the number of swings per second and is inversely proportional to the period.



ChatGPT

A mathematical pendulum, or simple pendulum, is a model consisting of a mass (the "bob") attached to a string or rod of fixed length, which swings freely under the influence of gravity. The motion is described by a second-order differential equation derived from Newton's second law and assuming small angles (linear approximation):

$$\frac{d^2\theta}{dt^2} + \frac{g}{L} \sin(\theta) = 0$$

where:

- θ is the angular displacement from the vertical,
- g is the acceleration due to gravity (9.81 m/s^2 on the surface of the Earth),
- L is the length of the pendulum.

For small angles ($\theta \ll 1$ radian), $\sin(\theta) \approx \theta$, simplifying the equation to:

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$$

This simplification leads to simple harmonic motion, which can be solved analytically. However, to solve the non-linear equation for any angle, we typically use numerical methods. Below is a Python code snippet using `scipy.integrate.solve_ivp` to solve the differential equation for a mathematical pendulum with an initial angle:

```
python
Copy code

import numpy as np
from scipy.integrate import solve_ivp
import matplotlib.pyplot as plt

# Constants
g = 9.81 # acceleration due to gravity, in m/s^2
L = 1.0 # length of pendulum, in meters

# Differential equation for the pendulum (theta'' + (g/L)*sin(theta) = 0)
def pendulum(t, y):
    theta, theta_dot = y
    dtheta_dt = theta_dot
    dtheta_dot_dt = -(g/L) * np.sin(theta)
    return [dtheta_dt, dtheta_dot_dt]

# Initial conditions: theta = 0.2 radians, theta_dot = 0.0 radians/s
y0 = [0.2, 0.0]

# Time span for the solution
t_span = (0, 10) # 10 seconds
t_eval = np.linspace(t_span[0], t_span[1], 1000)

# Solve the differential equation
sol = solve_ivp(pendulum, t_span, y0, t_eval=t_eval, method='RK45')

# Plot the solution
plt.plot(sol.t, sol.y[0])
plt.xlabel('Time (s)')
plt.ylabel('Angular displacement (radians)')
plt.title('Pendulum Motion')
plt.grid(True)
plt.show()
```

This code defines the differential equation for the pendulum's motion, solves it over a time span of 10 seconds for a pendulum of length 1 meter with an initial displacement of 0.2 radians, and plots the angular displacement over time.

GPT + Colab



MP.ipynb

File Edit View Insert Runtime Tools Help

+ Code + Text

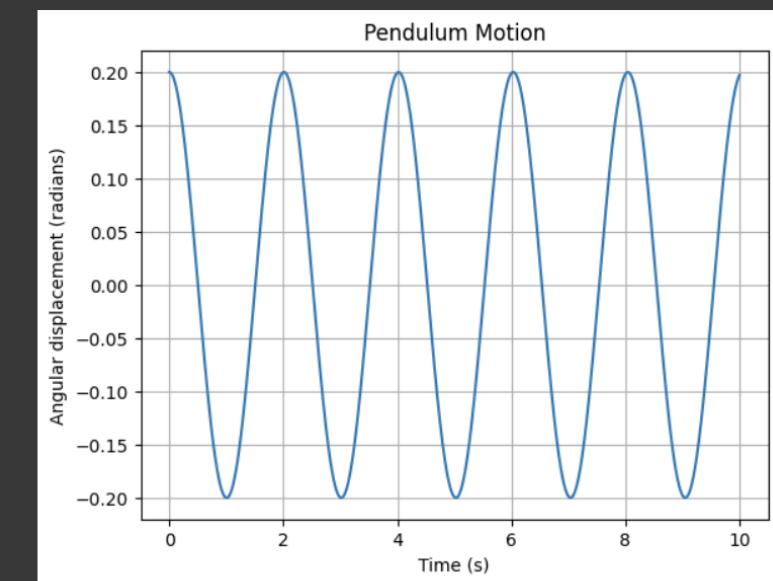


os

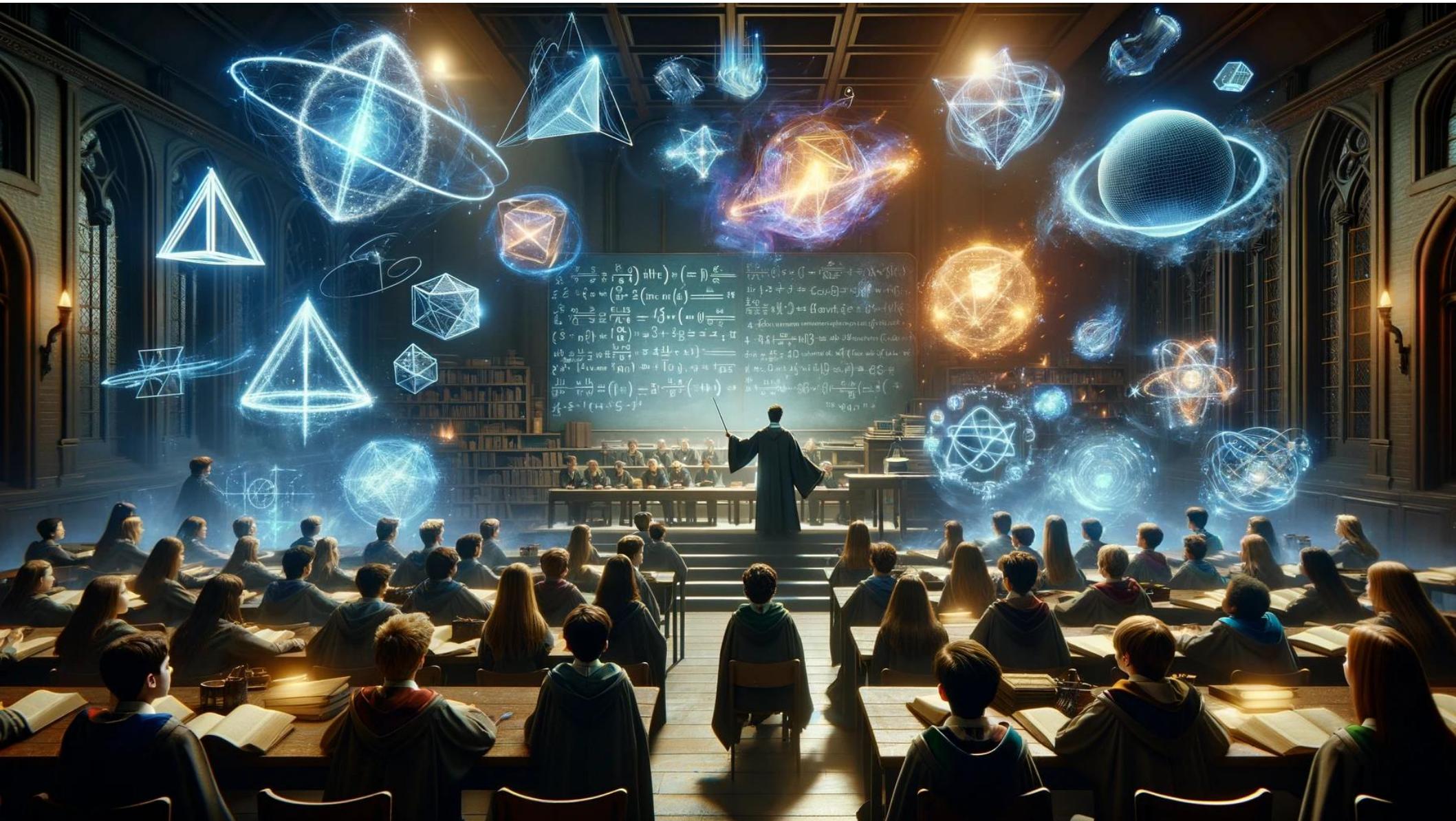
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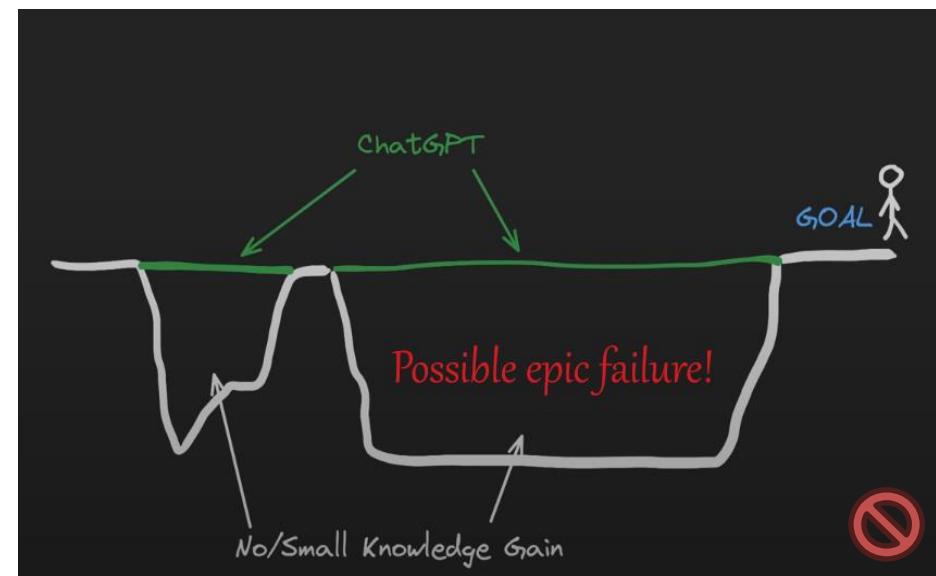
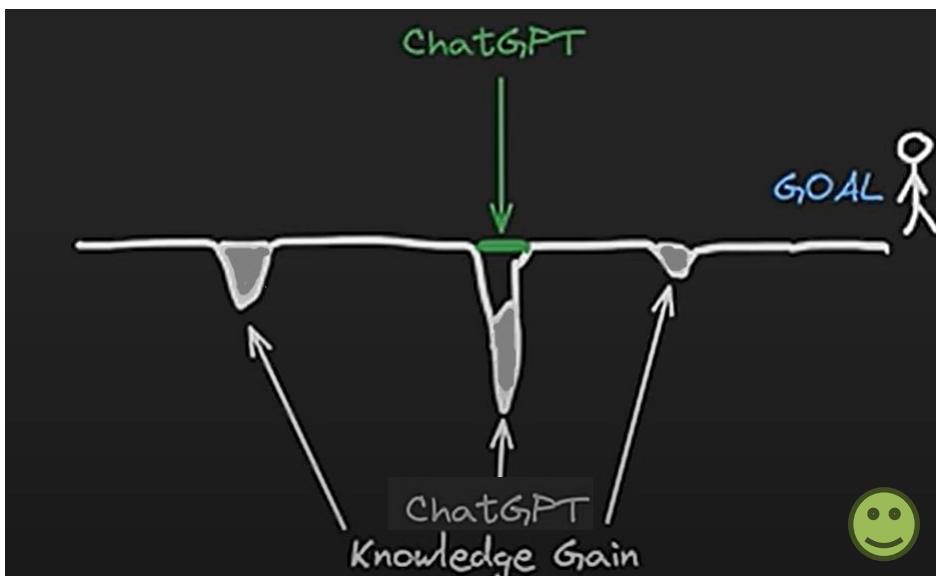
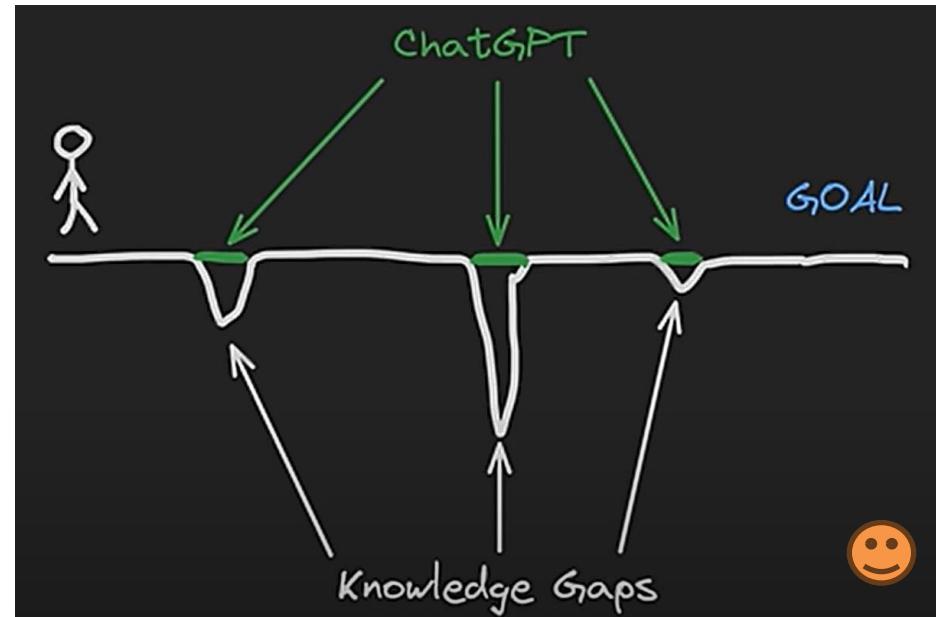
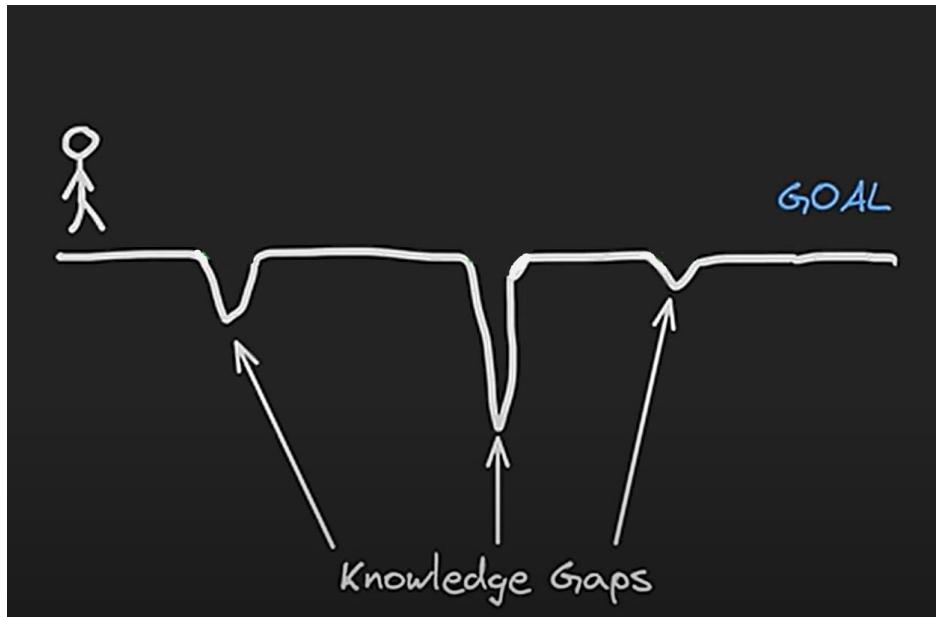
```
[1]  1 import numpy as np
2 from scipy.integrate import solve_ivp
3 import matplotlib.pyplot as plt
4
5 # Constants
6 g = 9.81 # acceleration due to gravity, in m/s^2
7 L = 1.0 # length of pendulum, in meters
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9 # Differential equation for the pendulum (theta'' + (g/L)*sin(theta) = 0)
10 def pendulum(t, y):
11     theta, theta_dot = y
12     dtheta_dt = theta_dot
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14     return [dtheta_dt, dtheta_dot_dt]
15
16 # Initial conditions: theta = 0.2 radians, theta_dot = 0.0 radians/s
17 y0 = [0.2, 0.0]
18
19 # Time span for the solution
20 t_span = (0, 10) # 10 seconds
21 t_eval = np.linspace(t_span[0], t_span[1], 1000)
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23 # Solve the differential equation
24 sol = solve_ivp(pendulum, t_span, y0, t_eval=t_eval, method='RK45')
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26 # Plot the solution
27 plt.plot(sol.t, sol.y[0])
28 plt.xlabel('Time (s)')
29 plt.ylabel('Angular displacement (radians)')
30 plt.title('Pendulum Motion')
31 plt.grid(True)
32 plt.show()
33
```



We literally casting spells!



How one should use ChatGPT



From https://www.youtube.com/watch?v=6CGtwF_5kzY

ChatGPT as bicycle for a mind!

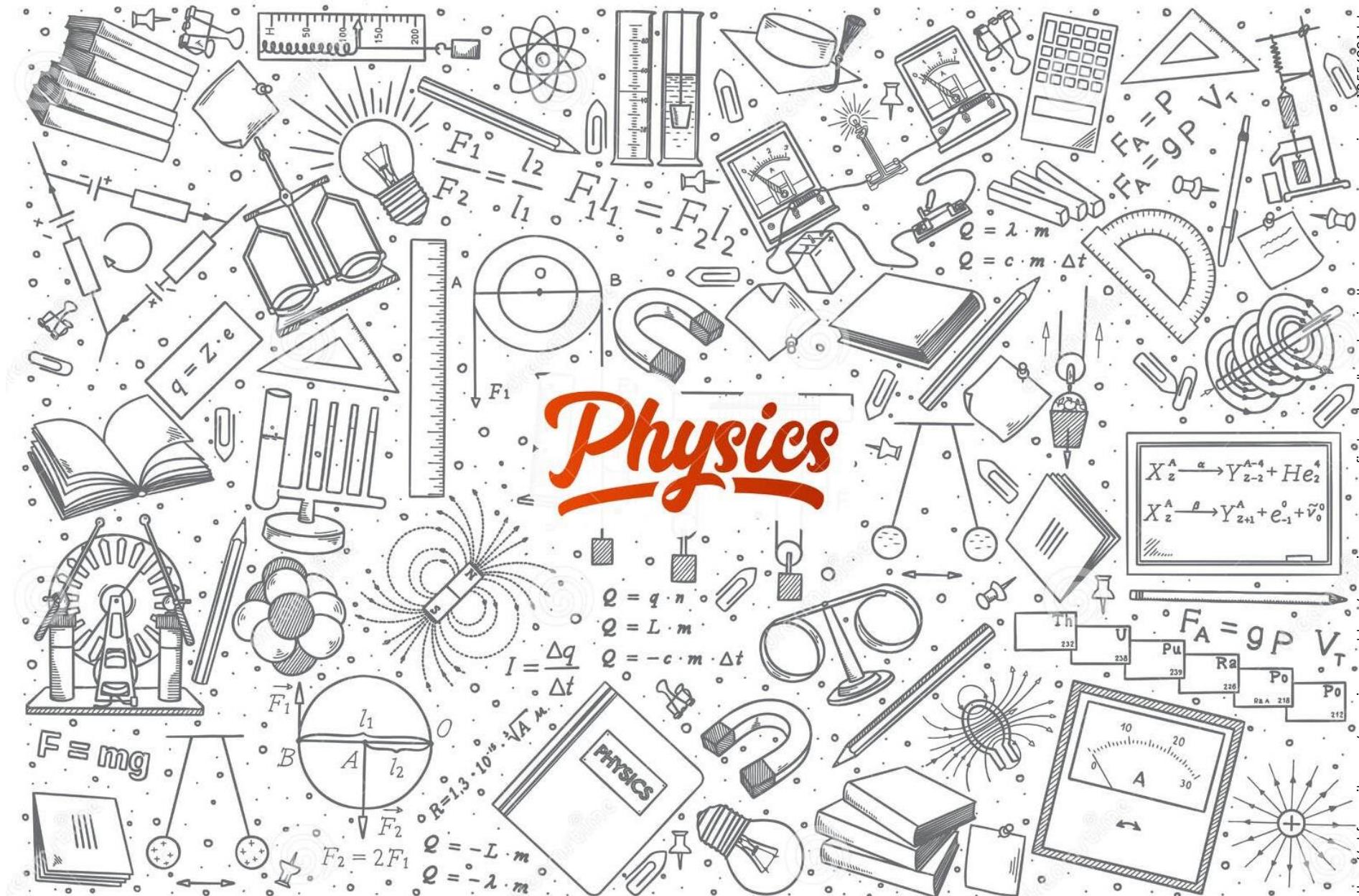
One person army to solve the problems



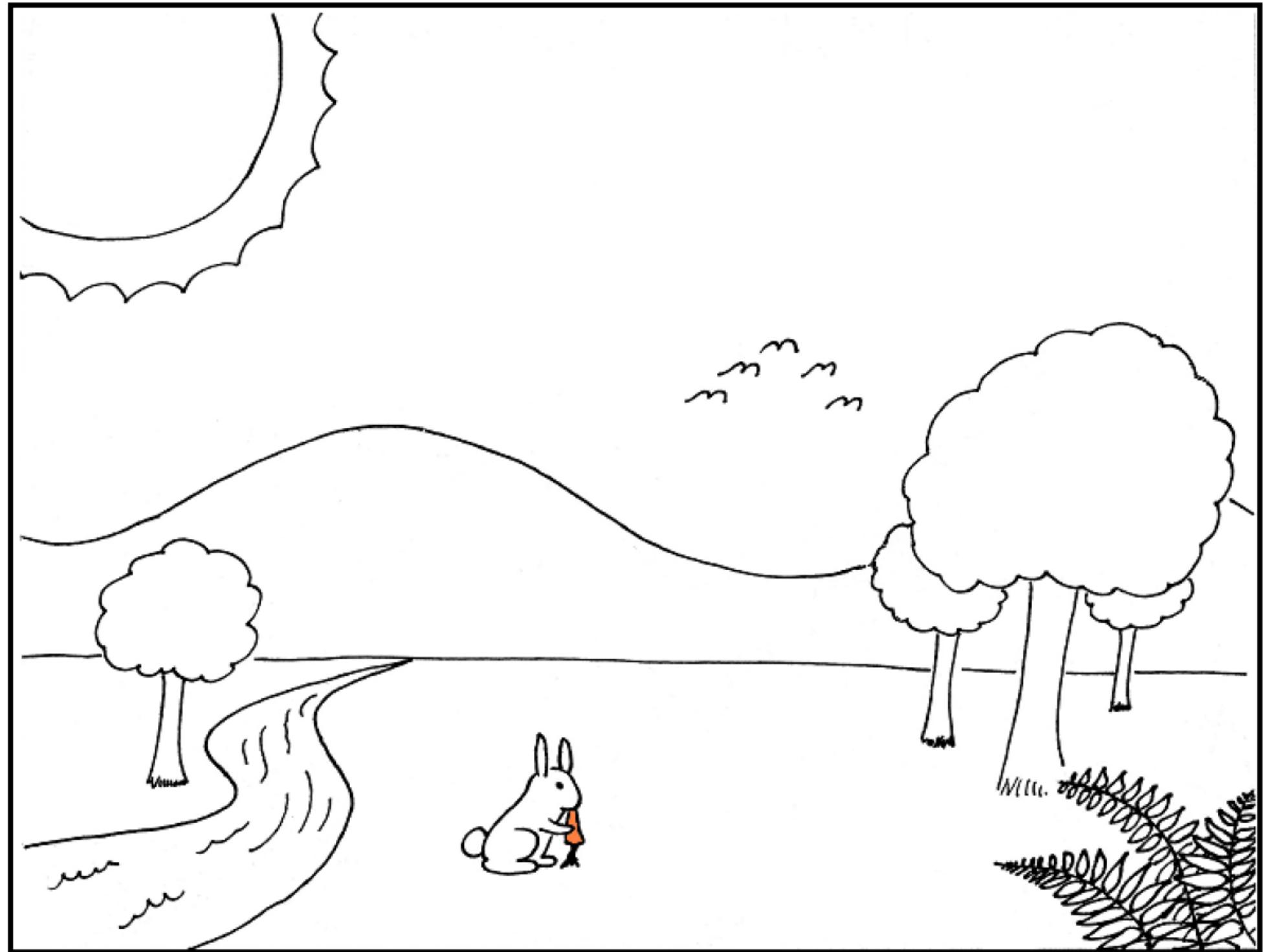
(Math)gyver

That doesn't mean that we discourage the teamwork!

What is Physics?

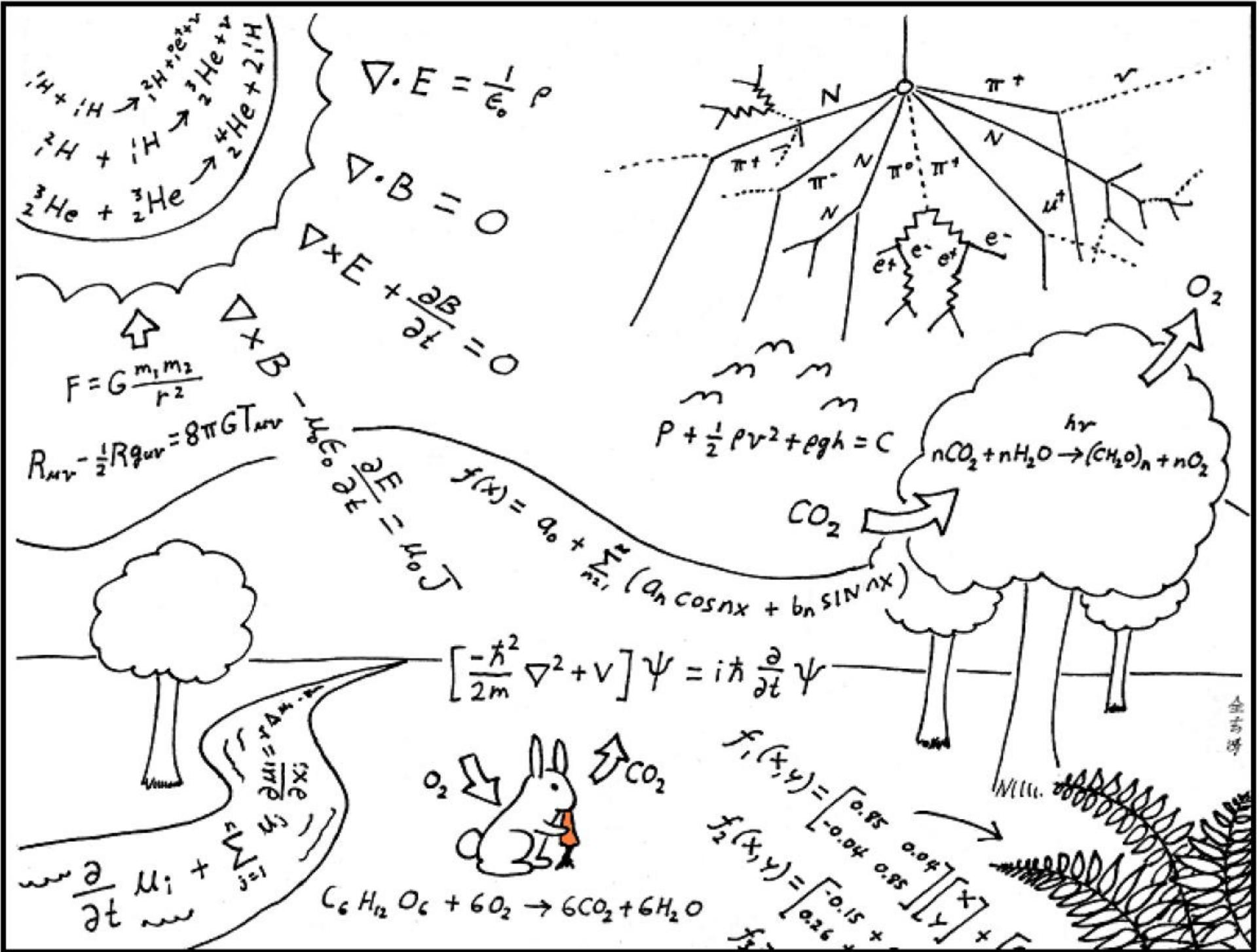


An exploration into **how things work** and **why!**



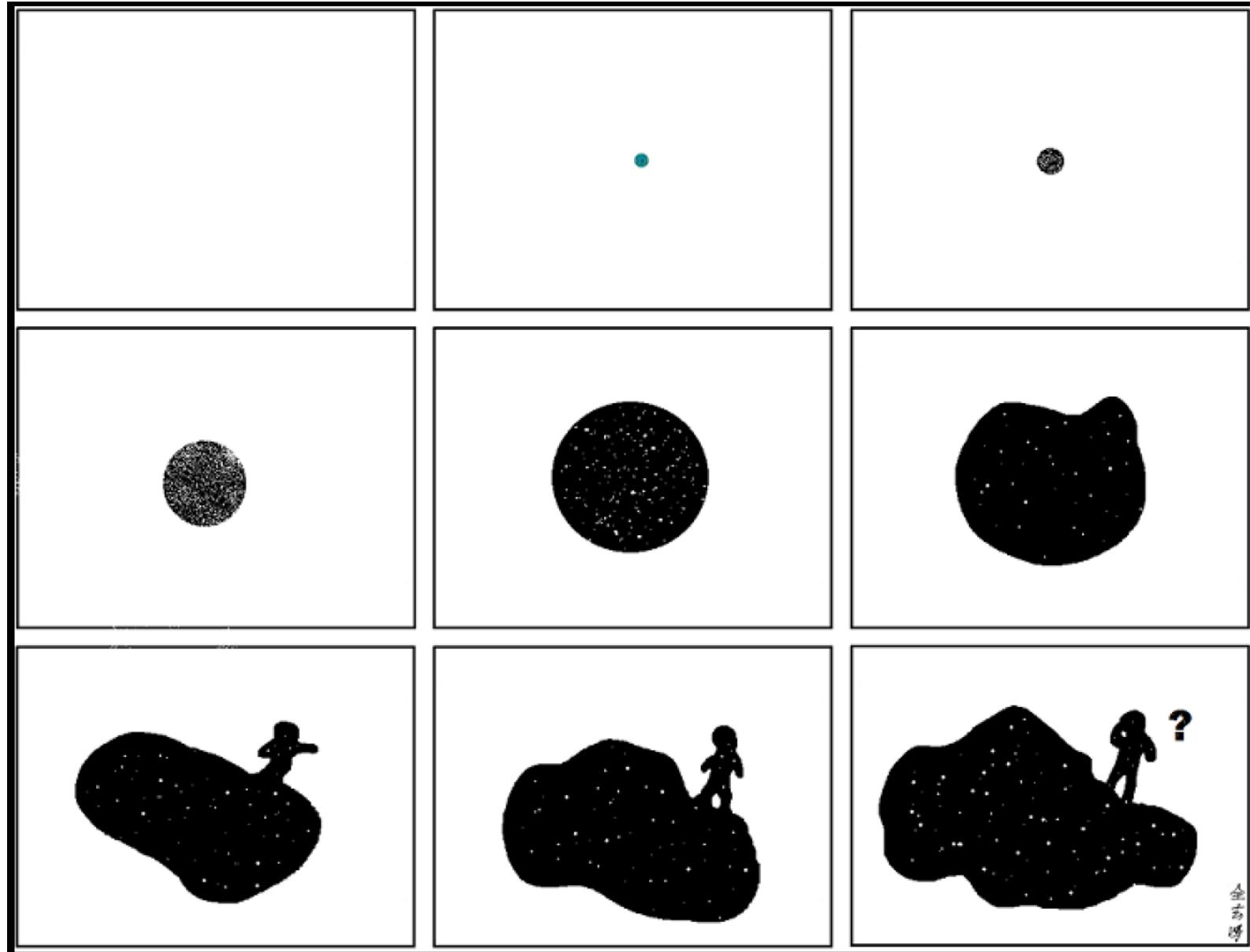
From: <https://abstrusegoose.com/>

Abstroose Goose



This is how scientists see the world.

Universe trying to understand itself



From <https://abstusegoose.com/>

„The real voyage of **discovery** consists not in seeking new landscapes,
but in having **new eyes.**” Marcel Proust

Physics has many subcategories:

- **Mechanics**- the study of motion and causes of motion
- **Thermodynamics**- behavior of heat
- **Waves**
- **Optics**- behavior of light
- **Acoustics**- behavior of sound
- **Electricity and Electromagnetism**- behavior of charged things
- **Modern Physics**- nuclear physics, particle physics, quantum physics, relativity, astrophysics, cosmology

Physics

Theoretical Physics

- **Focus:** Development of **mathematical models** and conceptual frameworks to understand physical phenomena.
- **Goal:** **Predict phenomena** that have not yet been observed and provide an abstract understanding of the physical world.
- **Examples:** String theory, Quantum mechanics, General relativity.

Experimental Physics

- **Focus:** Empirical **testing and measurement** of physical phenomena.
- **Goal:** **Verify theoretical predictions** through **experiments** and observations. Finding new phenomena signalizing new physics.
- **Examples:** Particle physics experiments at CERN, Astrophysical observations, Laboratory measurements of physical constants.

Physical Quantity = (amount) and [units]

Work = 10 [J]

Physical quantities are properties or characteristics of physical systems that can be quantified or measured.

Examples:

- potential energy $E = m g h = 100$ [J]
- elastic force $F_x = -k x = -1.2$ [N]
- Temperature $T = 273$ [K]
- Temperature $T = 0$ [deg Celcius]

The SI Units System (International System of Units)

Globally accepted system of universal & comfortable units.

Provides a consistent framework for scientific and technical measurements worldwide.

There are **seven base units** in the SI system, each corresponding to a fundamental physical quantity:

1. Length: meter (m)
2. Mass: kilogram (kg)
3. Time: second (s)
4. Electric Current: ampere (A)
5. Temperature: kelvin (K)
6. Amount of Substance: mole (mol)
7. Luminous Intensity: candela (cd)



The SI Units System (International System of Units)

Derived SI Units: from the base units through multiplication, division, or other mathematical operations.

Examples: area (square meters, m^2), volume (cubic meters, m^3), speed (meters per second, m/s), force (newton, $\text{N}=\text{kg m/s}^2$).

Name	Symbol	Quantity	In SI base units	In other SI units
hertz	Hz	frequency	s^{-1}	
newton	N	force, weight	$\text{kg}\cdot\text{m}\cdot\text{s}^{-2}$	
pascal	Pa	pressure, stress	$\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-2}$	$\text{N/m}^2 = \text{J/m}^3$
joule	J	energy, work, heat	$\text{kg}\cdot\text{m}^2\cdot\text{s}^{-2}$	$\text{N}\cdot\text{m} = \text{Pa}\cdot\text{m}^3$
watt	W	power, radiant flux	$\text{kg}\cdot\text{m}^2\cdot\text{s}^{-3}$	J/s
coulomb	C	electric charge	$\text{s}\cdot\text{A}$	
volt	V	electric potential, voltage, emf	$\text{kg}\cdot\text{m}^2\cdot\text{s}^{-3}\cdot\text{A}^{-1}$	$\text{W/A} = \text{J/C}$
farad	F	capacitance	$\text{kg}^{-1}\cdot\text{m}^{-2}\cdot\text{s}^4\cdot\text{A}^2$	$\text{C/V} = \text{C}^2/\text{J}$
ohm	Ω	resistance, impedance, reactance	$\text{kg}\cdot\text{m}^2\cdot\text{s}^{-3}\cdot\text{A}^{-2}$	$\text{V/A} = \text{J}\cdot\text{s}/\text{C}^2$
siemens	S	electrical conductance	$\text{kg}^{-1}\cdot\text{m}^{-2}\cdot\text{s}^3\cdot\text{A}^2$	Ω^{-1}
weber	Wb	magnetic flux	$\text{kg}\cdot\text{m}^2\cdot\text{s}^{-2}\cdot\text{A}^{-1}$	$\text{V}\cdot\text{s}$
tesla	T	magnetic flux density	$\text{kg}\cdot\text{s}^{-2}\cdot\text{A}^{-1}$	Wb/m^2
henry	H	inductance	$\text{kg}\cdot\text{m}^2\cdot\text{s}^{-2}\cdot\text{A}^{-2}$	Wb/A

SI Prefixes

Used to form decimal multiples and submultiples of SI units.

Facilitates the expression of very large or very small measurements in a compact form.

Examples:

kilo- (k, 10^3), km

milli- (m, 10^{-3}), mm

micro- (μ , 10^{-6}), μm

nano-(n, 10^{-9}), nm

Examples:

HectoPascal,

Megabyte,

GigaHertz,

MegaJoul

Prefix		Base 10	Decimal	Adoption [nb 1]
Name	Symbol			
quetta	Q	10^{30}	1 000 000 000 000 000 000 000 000 000 000	2022 ^[12]
ronna	R	10^{27}	1 000 000 000 000 000 000 000 000 000 000	
yotta	Y	10^{24}	1 000 000 000 000 000 000 000 000 000 000	1991
zetta	Z	10^{21}	1 000 000 000 000 000 000 000 000 000 000	
exa	E	10^{18}	1 000 000 000 000 000 000 000 000 000 000	1975 ^[13]
peta	P	10^{15}	1 000 000 000 000 000 000 000 000 000 000	
tera	T	10^{12}	1 000 000 000 000 000 000 000 000 000 000	1960
giga	G	10^9	1 000 000 000 000 000 000 000 000 000 000	
mega	M	10^6	1 000 000 000 000 000 000 000 000 000 000	1873
kilo	k	10^3	1 000 000 000 000 000 000 000 000 000 000	1795
hecto	h	10^2	1 000 000 000 000 000 000 000 000 000 000	
deca	da	10^1	1 000 000 000 000 000 000 000 000 000 000	
—	—	10^0	1 000 000 000 000 000 000 000 000 000 000	
deci	d	10^{-1}	0.1 000 000 000 000 000 000 000 000 000 000	1795
centi	c	10^{-2}	0.01 000 000 000 000 000 000 000 000 000 000	
milli	m	10^{-3}	0.001 000 000 000 000 000 000 000 000 000 000	
micro	μ	10^{-6}	0.000 001 000 000 000 000 000 000 000 000 000	1873
nano	n	10^{-9}	0.000 000 001 000 000 000 000 000 000 000 000	1960
pico	p	10^{-12}	0.000 000 000 001 000 000 000 000 000 000 000	
femto	f	10^{-15}	0.000 000 000 000 001 000 000 000 000 000 000	
atto	a	10^{-18}	0.000 000 000 000 000 001 000 000 000 000 000	
zepto	z	10^{-21}	0.000 000 000 000 000 000 001 000 000 000 000	1991
yocto	y	10^{-24}	0.000 000 000 000 000 000 000 001 000 000 000	
ronto	r	10^{-27}	0.000 000 000 000 000 000 000 000 001 000 000	
quecto	q	10^{-30}	0.000 000 000 000 000 000 000 000 000 001 000	2022 ^[12]

The SI System (International System of Units)

Conversion:

- In September of 1999, after almost 10 months of travel to Mars, the **Mars Climate orbiter** was lost.
- **327 million dollar** mission failed due to...
wrong conversion from miles to km.

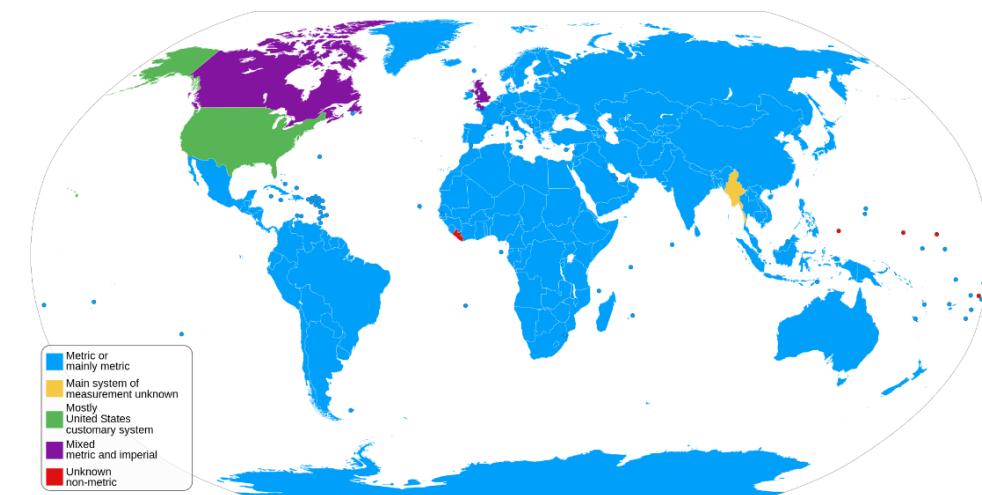
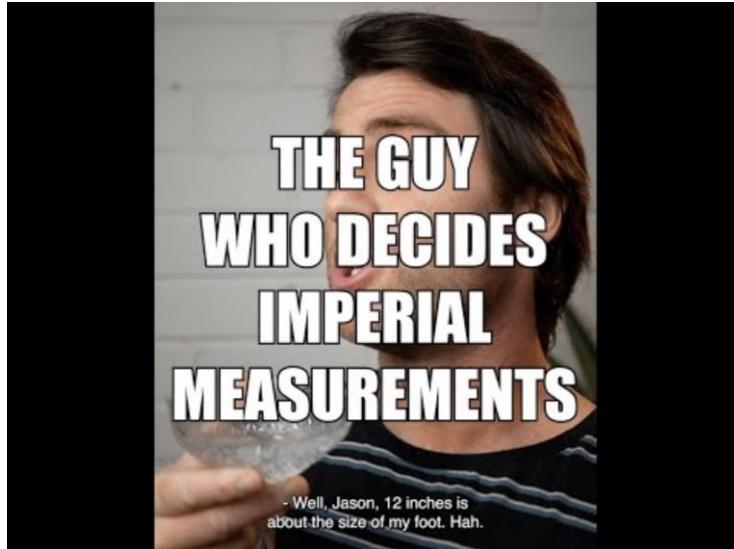
Value „1” meant different things in different modules:

- USA built (1 mile), Europe built (1 km)

SI Units vs Imperial

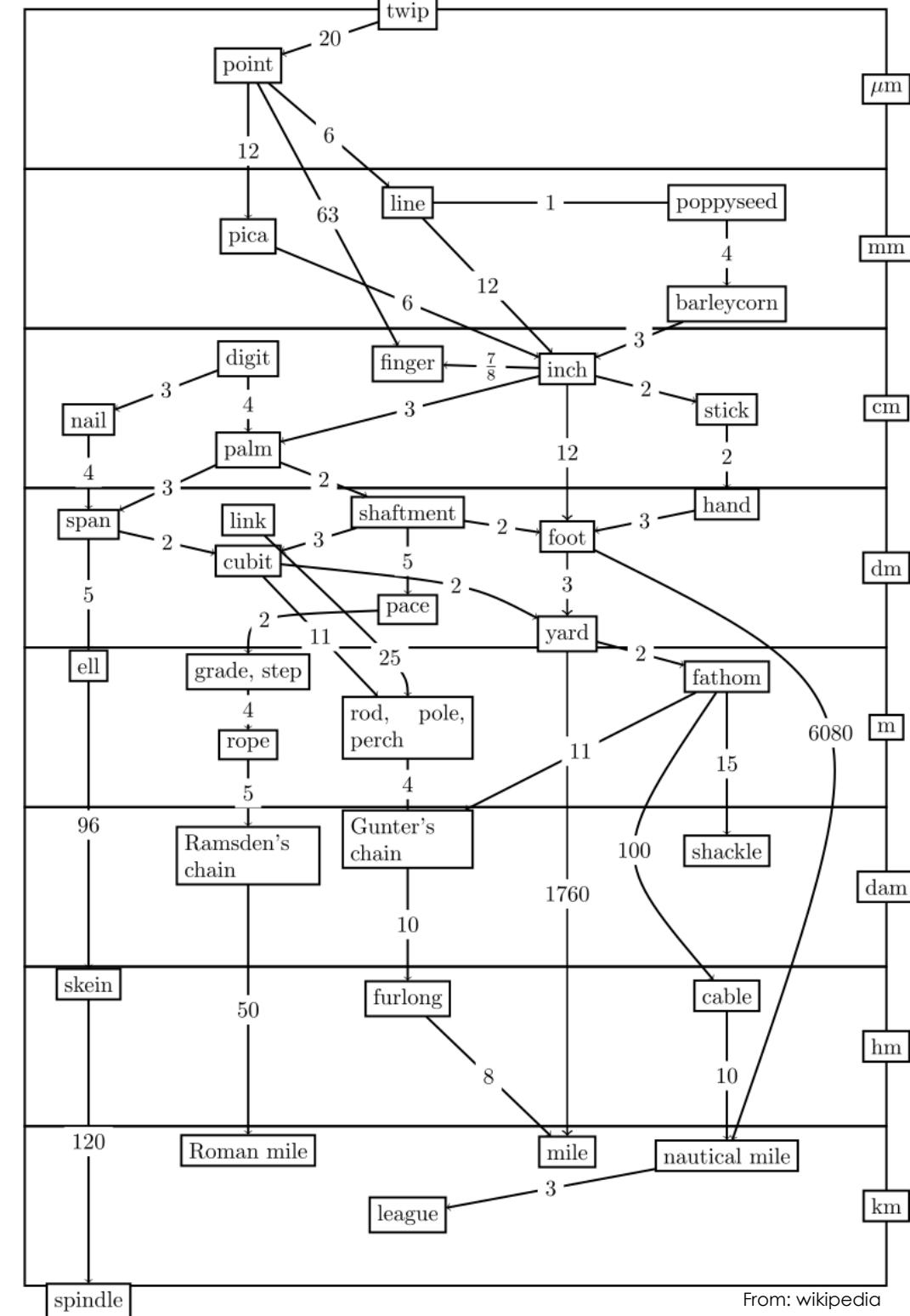
Worth to see what kind of mess it is!

<https://youtu.be/GDUt-Kbxqsg?si=ZnAfMPbDVrdRqvpi>



From: wikipedia

World adoption:

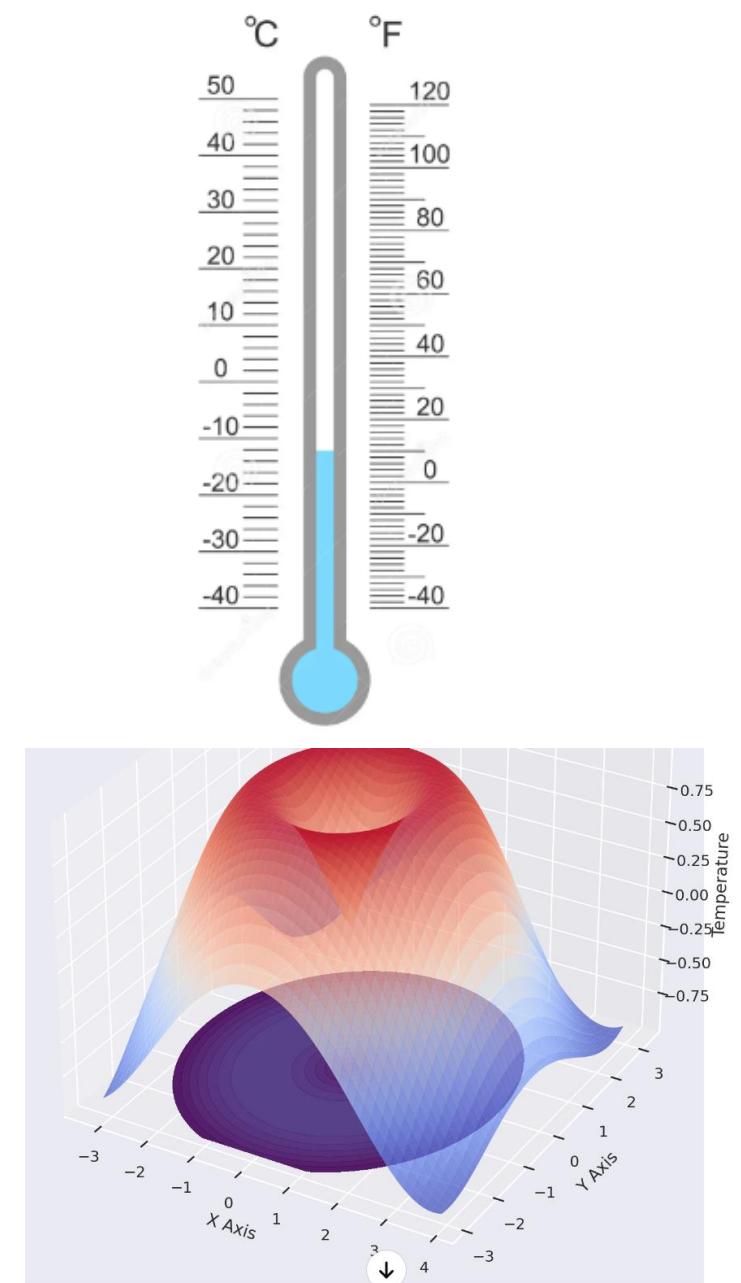
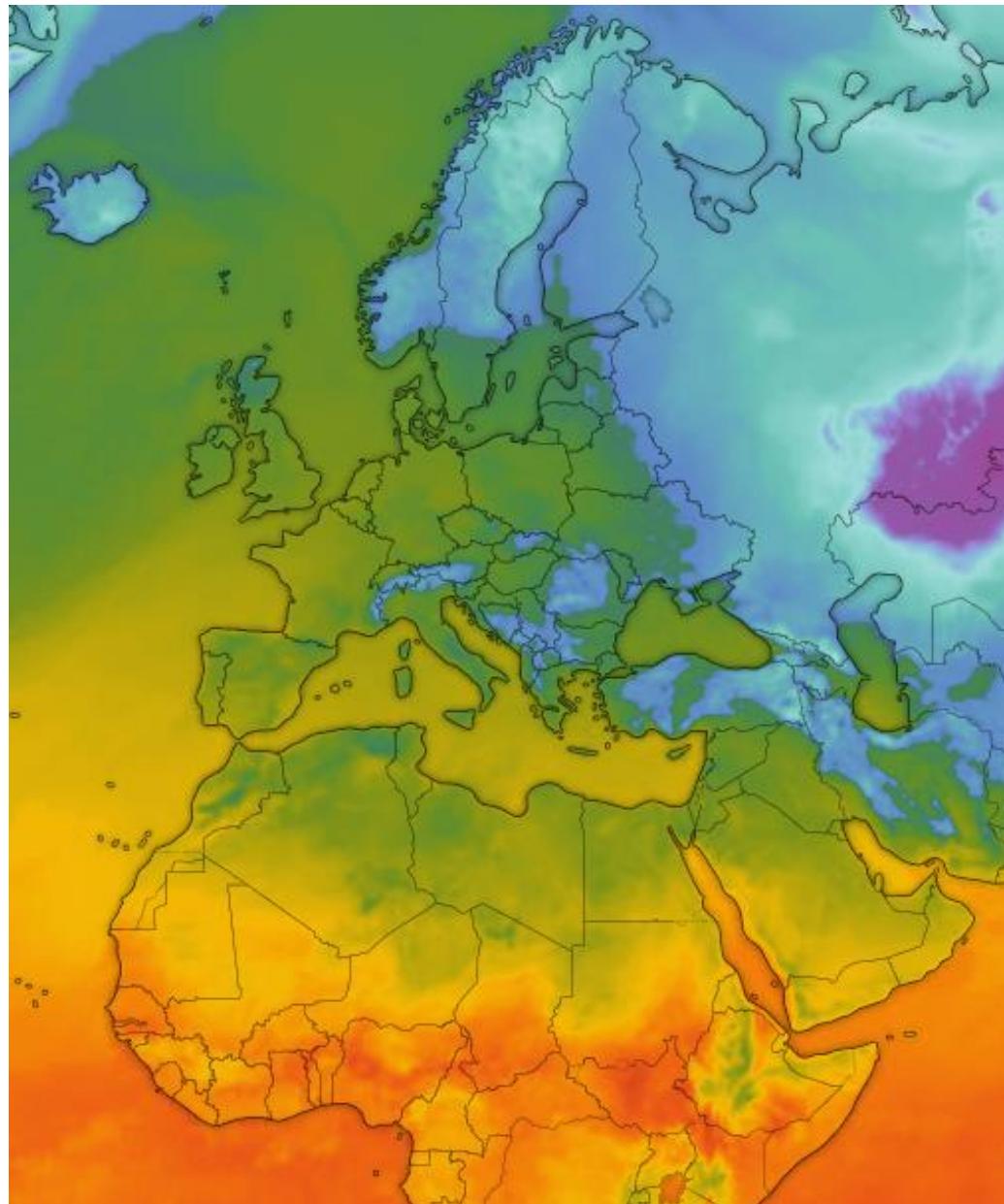


Introduction to Mathematical Tools in Physics

- **Physics** employs mathematics to **describe** and **analyze** the physical world.
- **Mathematics** is required for objective cross-validation and agreement of concepts, achieving conceptual precision, and repeatability of world description.
- **We will need** a few fundamental things in expressing physical quantities and their interactions:
 - Scalars, vectors, and matrices
 - Derivatives
 - Integrals
 - Differential equations

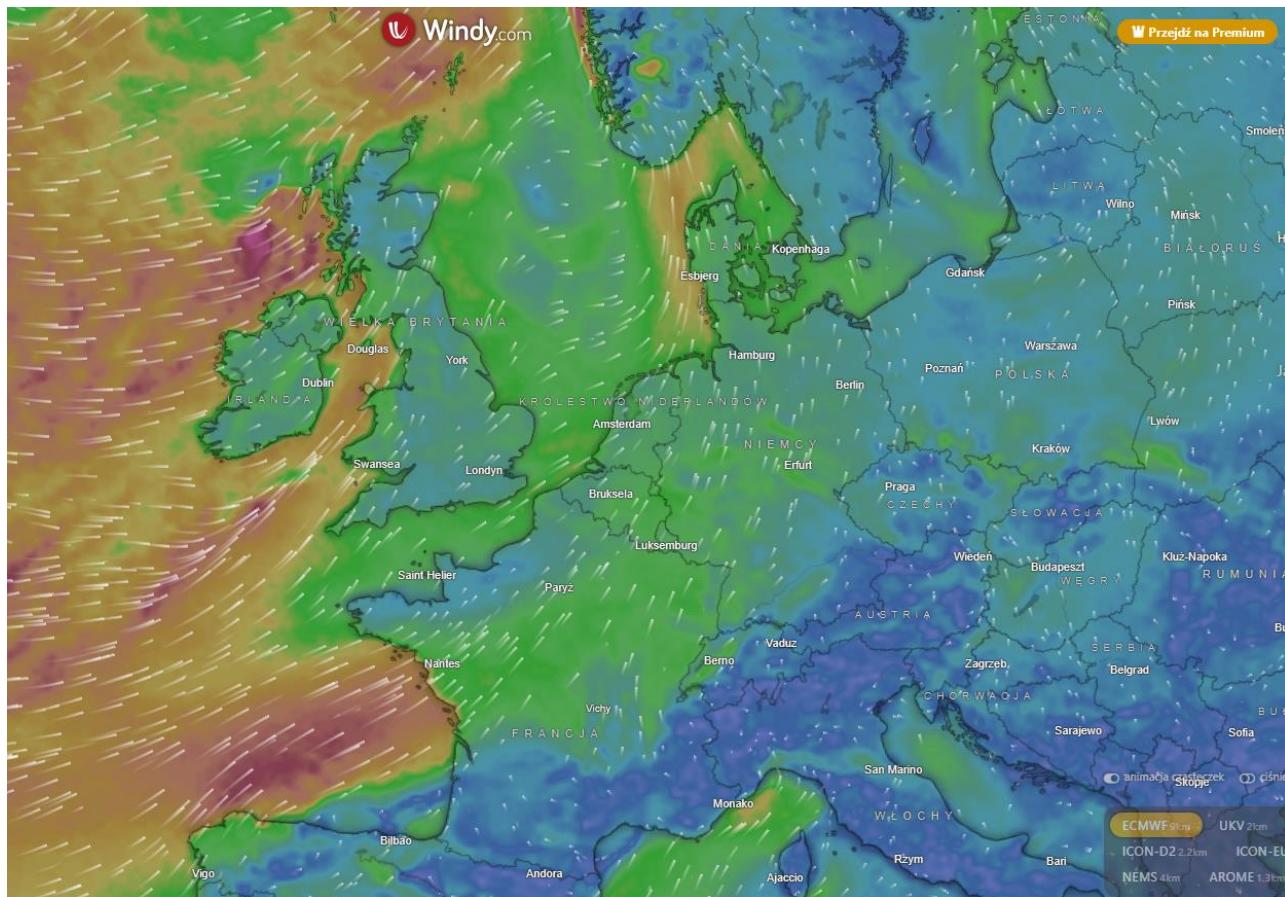
<https://www.windy.com/?50.670,17.918,5>

Scalar: temperature



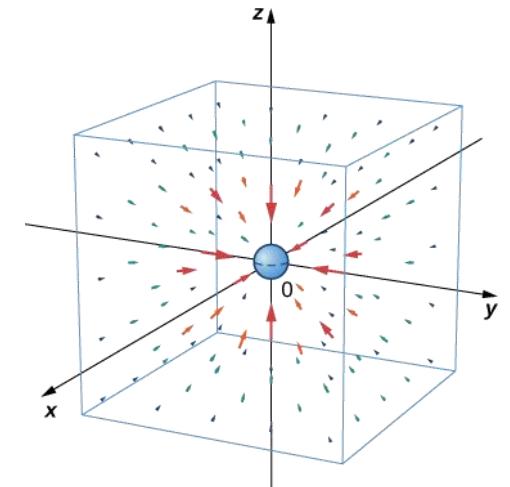
<https://www.windy.com/?50.670,17.918,5>

Vector: wind

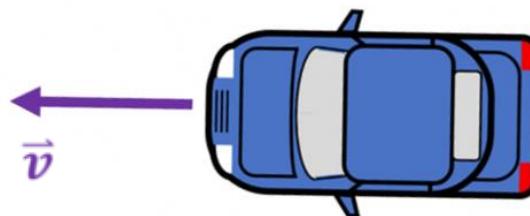


From: <https://www.windy.com>

Vector: electric field



Vector: car velocity

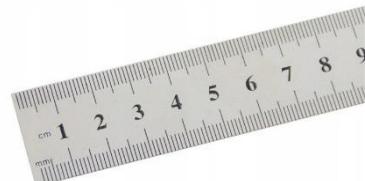


Scalars:

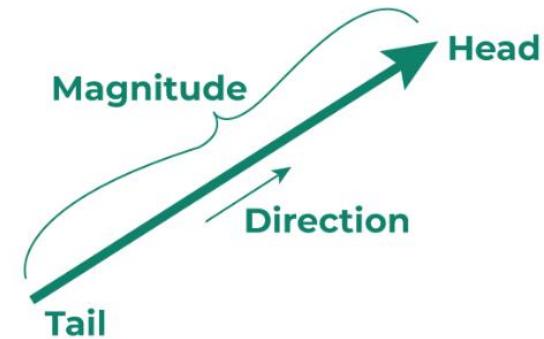
- **Definition:** A scalar is a quantity that is fully described by a magnitude (or numerical value) alone. „Something on a scale”
- **Characteristics:** Has no direction.
- **Examples in Physics:** temperature, mass, energy, work, potential, density, and time.
- **Usage:** Used to describe quantities that are not directional but are essential in physical calculations.



$$\phi = -G \frac{M}{r}$$



- **Vectors:**
- **Definition:** A vector is a quantity that has both magnitude and direction.



- **Characteristics:** Represented graphically by an arrow; the length represents the magnitude, and the direction represents the vector's direction.

$$\vec{v} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

- **Examples in Physics:** velocity, force, position in 3D, electric field, magnetic field

$$\vec{v} \times \vec{B} = \mu \epsilon \vec{j} + \frac{\partial \vec{E}}{\partial t}$$

$$\epsilon \vec{v} \cdot \vec{E} = \rho$$

$$\vec{v} \cdot \vec{B} = 0$$

- **Usage:** crucial for understanding motion, forces, fields.

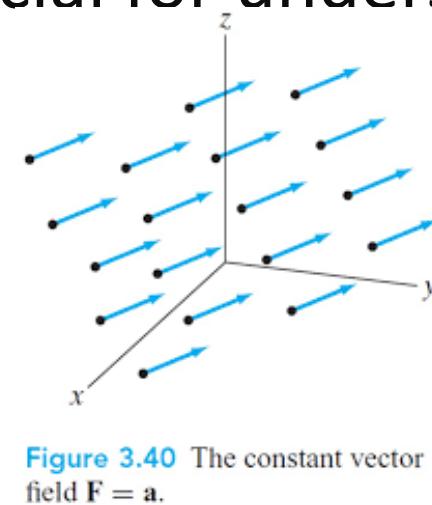
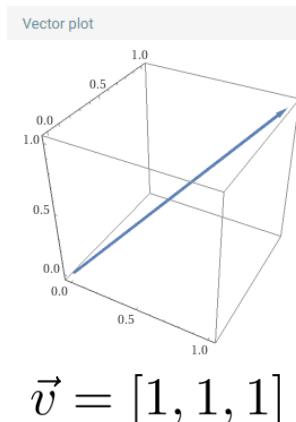
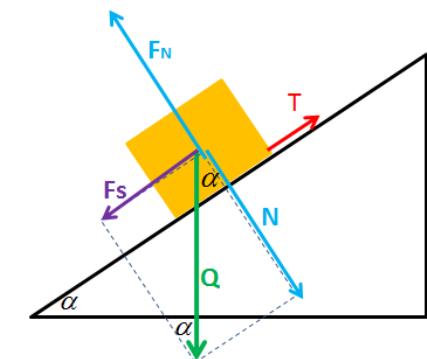
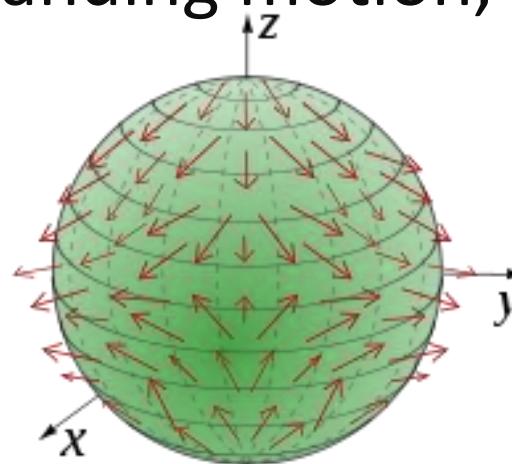


Figure 3.40 The constant vector field $\mathbf{F} = \mathbf{a}$.



• Vectors:

☰ GeoGebra Calculator Suite Graphing

● A = (0, 0) ⋮

● B = (0, 6) ⋮

● C = (-2, 8) ⋮

● D = (0, 6) ⋮

● $u = \text{Vector}(A, B)$ ⋮

● $= \begin{pmatrix} 0 \\ 6 \end{pmatrix}$

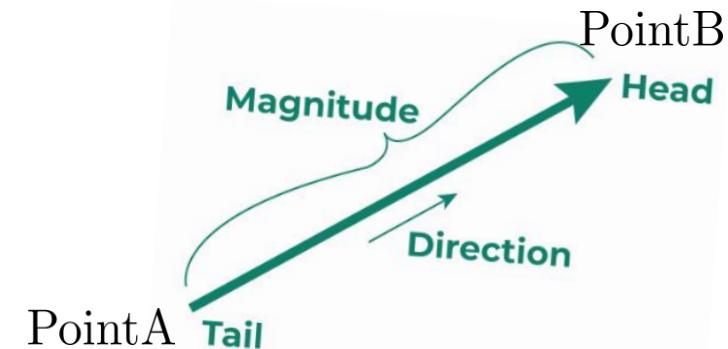
● $v = \text{Vector}(C, D)$ ⋮

● $= \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

● $w = u + v$ ⋮

● $= \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

+ Input...



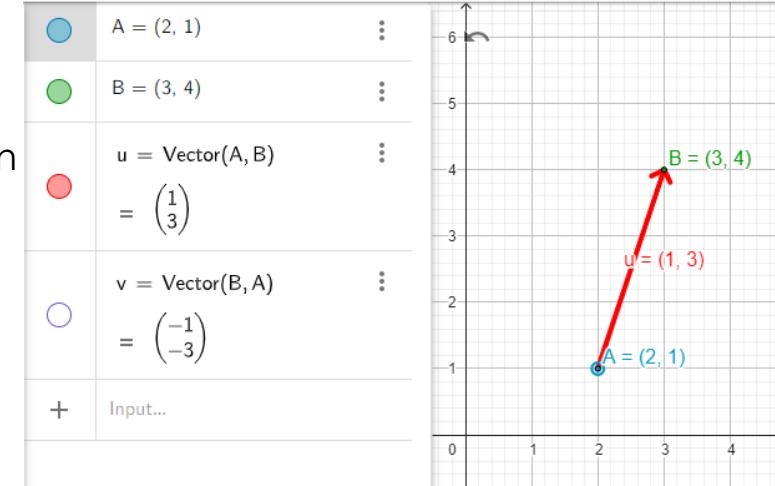
$$\text{Point } A = (A_x, A_y)$$

$$\text{Point } B = (B_x, B_y)$$

$$\vec{AB} = (B_x - A_x, B_y - A_y)$$

<https://www.geogebra.org/calculator/qavkaahz>

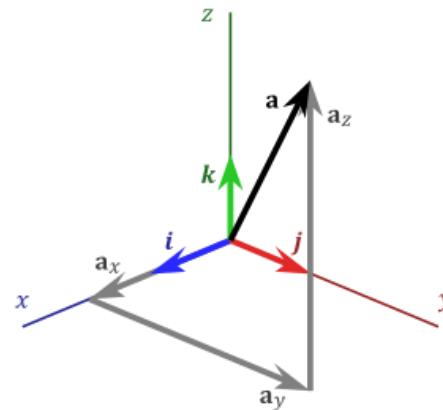
GeoGebra Calculator Suite Graphing



Vectors: 2D and 3D

$$\vec{a} \equiv (a_x, a_y)$$

$$\vec{a} \equiv (a_x, a_y, a_z)$$

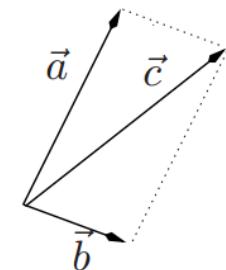


Vectors Manipulation

Addition: <https://www.geogebra.org/m/Cy8bxakS>

$$\vec{a} + \vec{b} = \vec{c}$$

$$(a_x + b_x, a_y + b_y, a_z + b_z) = (c_x, c_y, c_z)$$



Length:

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}.$$

Scalar product:

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z .$$

Angle between vectors:

$$\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

Ortogonal(perpendicular):

$$\vec{a} \cdot \vec{b} = 0 \iff \vec{a} \perp \vec{b}$$

- **Matrices:**
- **Definition:** A matrix is a rectangular array of numbers, symbols, or expressions arranged in rows and columns.
- **Characteristics:** Used to represent and solve systems of linear equations, transformation of coordinates, and more.
- **Examples in Physics:** Representation of quantum states, tensors in general relativity, rotation matrices.
- **Usage:** Essential in simulations, quantum mechanics, and other areas requiring complex calculations and transformations.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -2 & -5 \end{bmatrix}$$

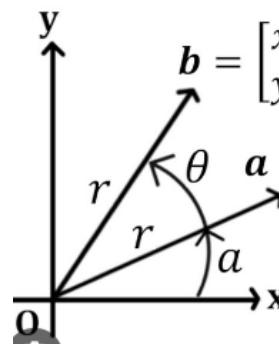
$$g_{\mu\nu} = \begin{pmatrix} -\left(1 - \frac{2GM}{rc^2}\right) & 0 & 0 & 0 \\ 0 & \left(1 - \frac{2GM}{r^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 2 \\ 0 & 4 & 7 \\ -3 & -4 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 5 \\ 7 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} i & 0 \\ 0 & 1-i \end{bmatrix}$$



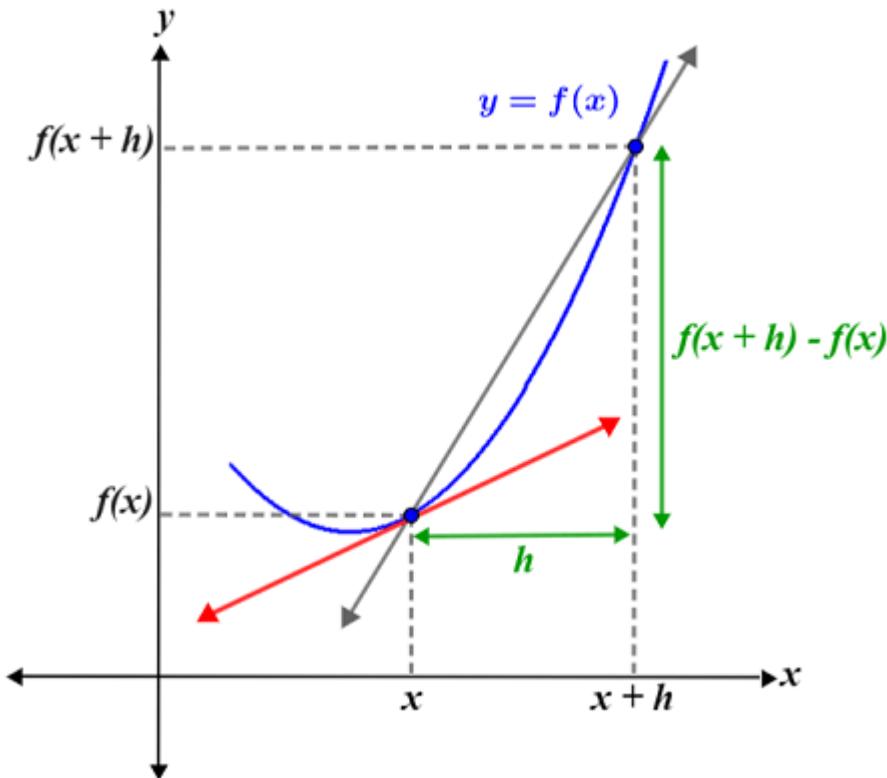
Rotation Matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = R(\theta) \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Derivative

Computationally: the limit of the average rate of change of the function over an interval as the interval approaches zero.



$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

Mathematically, it is the limit of the average rate of change of the function over a small interval of argument, as the interval approaches zero.

Velocity computed from distance over time

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

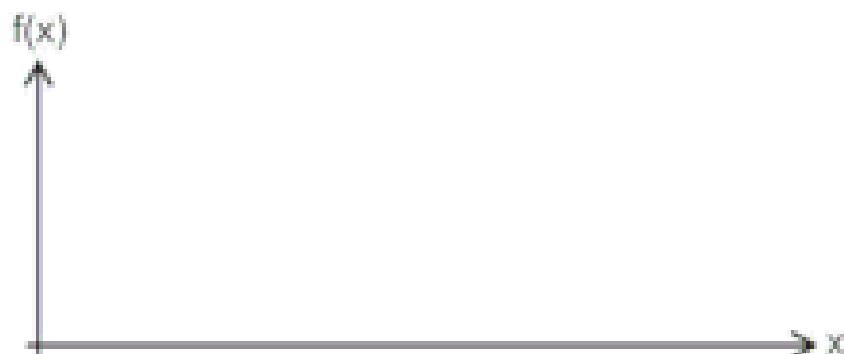
$$v(t) = x'(t) = \lim_{\Delta t \rightarrow 0} \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

Derivative

- **Definition:** The derivative of a function represents the rate of change at a certain point.
- **Geometrical Interpretation:** the derivative at a point corresponds to the slope of the tangent line to the function at point x .



Useful for
optimisations:



$$f'(x) = 0$$

function
extremum
(min or max)

Derivative

Physical Interpretation gives rate of change quantities that change over time or space, indicating how fast or slow the function value is changing at that specific point.

Example: velocity (derivative of position with respect to time), acceleration (derivative of velocity with respect to time).

Importance: These concepts are fundamental in understanding motion and dynamics.

Notation: The derivative of a function $f(x)$ can be denoted

$$f'(x), \frac{df}{dx}, \frac{d}{dx} f(x)$$

Rules of Differentiation: use programs! (if not, long list of rules)

$$(\sin(x))' = \cos(x)$$

$$(\cos(x))' = -\sin(x)$$

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$(x^n)' = nx^{n-1}$$

Derivative of x^4

NATURAL LANGUAGE

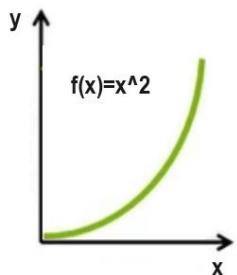
Derivative

$$\frac{d}{dx}(x^4) = 4x^3$$

Derivative

Derivative of function over argument.

Mathematics: argument x on plot, function $y(x)$ or $f(x)$

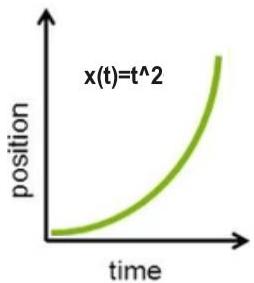


$$f'(x), \frac{df}{dx}$$

$$f'(x) = \frac{d}{dx} f(x), \quad f''(x) = \frac{d^2}{dx^2} f(x)$$

Higher derivatives

Physics: argument t , function $x(t)$ or $U(t)$



$$\dot{x}(t), \frac{dx}{dt}$$

$$\dot{x}(t) = \frac{d}{dt} x(t), \quad \ddot{x}(t) = \frac{d^2}{dt^2} x(t)$$

Higher derivatives

Integrals

Definition:

- **Opposite of differentiation**, restoring the function from its rate of change. Inverses, just like pairs: (log and exp), (sqrt and $\wedge 2$).
- Mathematical process of finding the integral of a function, representing the **area under the curve** of the function on a graph.
- **Adding up** tiny bits.

Types of Integrals:

- **Indefinite Integrals:** Represent the family of all antiderivative of a function, without specific limits. (**function**)
- **Definite Integrals:** Calculate the net area (number!) under a curve between two points, providing a specific numerical value. (**number**)

Integrals as Opposite of differentiation

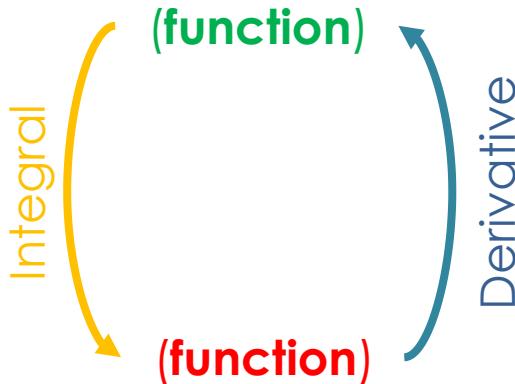
$$\int \underline{f(x) dx} = \underline{F(x) + C}$$

INPUT:
function

OUTPUT:
function

$$\int f(x) dx = F(x) + C$$

$$(F(x) + C)' = f(x)$$



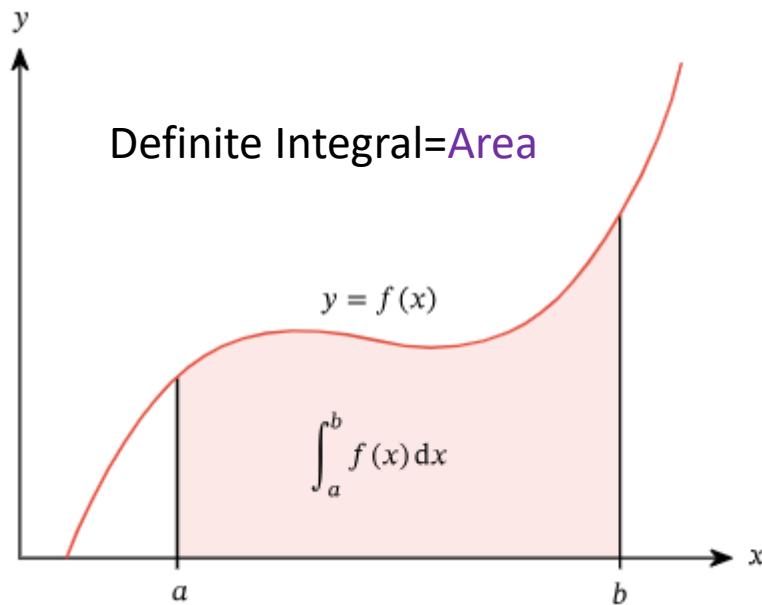
- $$\int adx = ax + c$$
- $$\int x^n dx = \frac{1}{n+1} x^{n+1} + c \quad \text{dla } n \neq -1$$
- $$\int \frac{1}{x} dx = \ln |x| + c$$
- $$\int a^x dx = \frac{a^x}{\ln a} \quad \text{gdzie } \ln a = \log_e a$$
- $$\int e^x dx = e^x + c$$
- $$\int \sin x dx = -\cos x + c$$
- $$\int \cos x dx = \sin x + c$$
- $$\int \frac{1}{\cos^2 x} dx = \operatorname{tg} x + c$$
- $$\int \frac{1}{\sin^2 x} dx = -\operatorname{ctg} x + c$$
- $$\int \frac{1}{1+x^2} x dx = \operatorname{arctg} x + c$$

Integral $x^2 dx$

Indefinite integral

$$\int x^2 dx = \frac{x^3}{3} + \text{constant}$$

Integrals as area under the curve



Wolfram Alpha

integral xsin(3x) from 0 to pi/4

NATURAL LANGUAGE

MATH INPUT

Definite integral

$$\int_0^{\frac{\pi}{4}} x \sin(3x) dx = \frac{4 + 3\pi}{36\sqrt{2}} \approx 0.26369$$

Explore the construction...

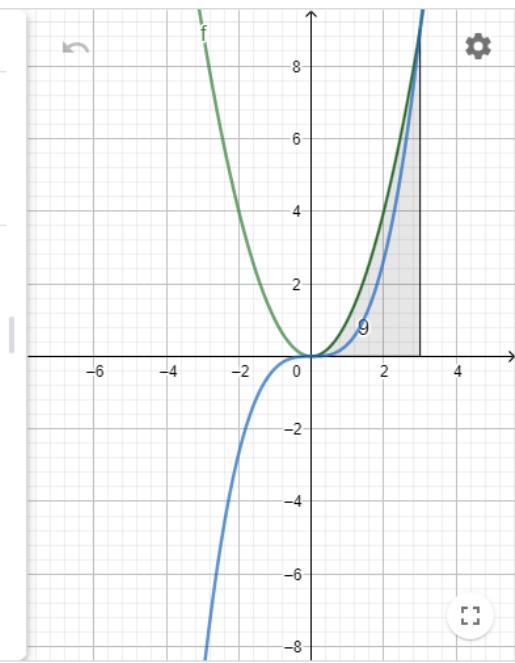
≡	<input type="radio"/>	$f(x) = x^2$
	<input type="radio"/>	$\int f dx$ = $\frac{1}{3} x^3$
	<input type="radio"/>	$\int_0^3 f dx$ = 9
	<input type="radio"/>	

$$\int f dx$$

= $\frac{1}{3} x^3$

$$\int_0^3 f dx$$

= 9

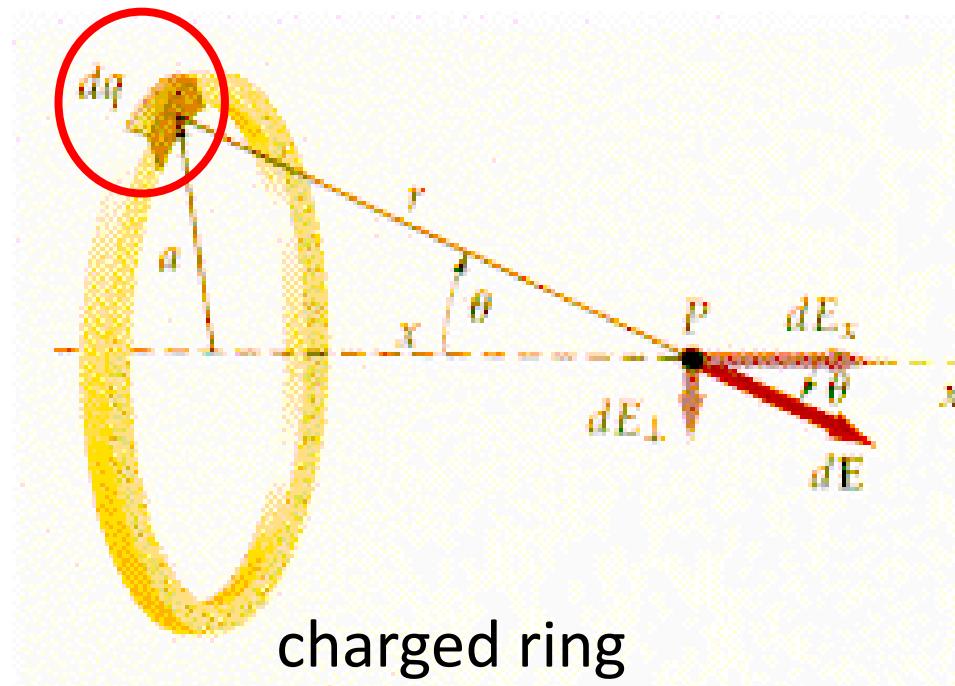


Geogebra

GeoGebra Calculator Suite

Integrals as summing up tiny things

charge line
element



charged ring

$$E_x = \int \frac{kx dq}{(x^2 + a^2)^{3/2}} = \frac{kx}{(x^2 + a^2)^{3/2}} \int dq = \frac{kxQ}{(x^2 + a^2)^{3/2}}$$