es. 1) Dato il problema

$$\begin{aligned} \min \quad & 2x_1 - x_2 \\ \text{s.t.} \quad & x_1 + 2x_2 \geq 7 \\ & 2x_1 - x_2 \geq -6 \\ & -3x_1 + 2x_2 \geq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$$

applicare le condizioni di complementarietà primale-duale per verificare se la soluzione $[x_1, x_2] = [0, 6]$ è ottima.

sol:

1. $\bar{x} \in X$:

$x_1 + 2x_2 \geq 7 \Rightarrow 0 + 2 \cdot 6 \geq 7 \rightarrow 12 \geq 7$ ok

$2x_1 - x_2 \geq -6 \Rightarrow 2 \cdot 0 - 6 \geq -6 \rightarrow -6 \geq -6$ ok

$-3x_1 + 2x_2 \geq 8 \Rightarrow -3 \cdot 0 + 2 \cdot 6 \geq 8 \Rightarrow 12 \geq 8$ ok

$x_1 \geq 0, x_2 \geq 0 \Rightarrow 0 \geq 0 ; 6 \geq 0$ ok

D:
$$\begin{aligned} \max \quad & 7y_1 - 6y_2 + 8y_3 \\ \text{s.t.} \quad & y_1 + 2y_2 - y_3 \leq 2 \\ & 2y_1 - y_2 + 2y_3 \leq -1 \\ & y_1, y_2, y_3 \geq 0 \end{aligned}$$

2) $\bar{y}_1, \bar{y}_2, \bar{y}_3$ t.c.
$$\begin{aligned} & \bar{y}_1 + 2\bar{y}_2 - \bar{y}_3 \leq 2 \\ & 2\bar{y}_1 - \bar{y}_2 + 2\bar{y}_3 \leq -1 \\ & \bar{y}_1, \bar{y}_2, \bar{y}_3 \geq 0 \end{aligned}$$

3) $\bar{x}^T \cdot S_d = 0$

$S_{d_1} = 2 - \bar{y}_1 - 2\bar{y}_2 + \bar{y}_3$

$S_{d_2} = -1 - 2\bar{y}_1 + \bar{y}_2 - 2\bar{y}_3$

$$\Rightarrow [0 \ 6] \begin{bmatrix} 2 - \bar{y}_1 - 2\bar{y}_2 + \bar{y}_3 \\ -1 - 2\bar{y}_1 + \bar{y}_2 - 2\bar{y}_3 \end{bmatrix} = -6 - 12\bar{y}_1 + 6\bar{y}_2 - 12\bar{y}_3 = 0$$

4) $y^T \cdot S_p = 0$

$x_1 + 2x_2 \geq 7 \Rightarrow 12 \geq 7$ 5

$2x_1 - x_2 \geq -6 \Rightarrow -6 \geq -6$ 0

$-3x_1 + 2x_2 \geq 8 \Rightarrow 12 \geq 8$ 4

$\Rightarrow$ $S_{p_1} = 5$, $S_{p_2} = 0$, $S_{p_3} = 4$

$$\begin{bmatrix}\bar{y}_1 & \bar{y}_2 & \bar{y}_3\end{bmatrix}\begin{bmatrix}5\\0\\4\end{bmatrix} = 5\bar{y}_1 + 4\bar{y}_3 = 0$$

5)

$$\begin{cases}-6 - 12\bar{y}_1 + 6\bar{y}_2 - 12\bar{y}_3 = 0\\ 5\bar{y}_1 + 4\bar{y}_3 = 0\end{cases} \Rightarrow \begin{cases}-6 - 12\bar{y}_1 + 6\bar{y}_2 - 12\bar{y}_3 = 0\\ 5\bar{y}_1 = 0\\ 4\bar{y}_3 = 0\end{cases} \Rightarrow \begin{cases}\bar{y}_2 = 1\\ \bar{y}_1 = 0\\ \bar{y}_3 = 0\end{cases}$$

$$\begin{aligned}&y_1 + 2y_2 - y_3 \leq 2 & & 2 \leq 2 \text{ (ok)}\\ &2y_1 - y_2 + 2y_3 \leq -1 & \Rightarrow\ & -1 \leq -1 \text{ (ok)}\\ &y_1, y_2, y_3 \geq 0 & & 0, 1, 0 \geq 0 \text{ (ok)}\end{aligned}$$

es.2) $\max\ x_2+x_3$

s.t. $-x_1-x_2+2x_3\le 1$

$-2x_1+x_2\le 2$

$2x_2\ge -3$

$2x_1+x_3=2$

x_1 libera, $x_2\ge 0$, $x_3\le 0$

$[x_1,x_2,x_3]=[1,4,0]$

sol:

1) $-x_1-x_2+2x_3\le 1 \Rightarrow -1-4\le 1 \Rightarrow -5\le 1$ (ok)

$-2x_1+x_2\le 2 \Rightarrow -2+4\le 2 \Rightarrow 2\le 2$ (ok)

$2x_2\ge -3 \Rightarrow 2\cdot 4\ge -3$ (ok)

$2x_1+x_3=2 \Rightarrow 2=2$ (ok)

$x_2\ge 0 \Rightarrow 4\ge 0$ (ok) e $0\ge 0$ (ok)

D: $\min\ y_1+2y_2-3y_3+2y_4$

$-y_1-2y_2+2y_4=0$

$-y_1+y_2+2y_3\ge 1$

$2y_1+y_4\le 1$

$y_1,y_2\ge 0$ $y_3\le 0$ y_4 libera

2) $\bar{y}_1\ \bar{y}_2\ \bar{y}_3\ \bar{y}_4$ t.c. $-\bar{y}_1-2\bar{y}_2+2\bar{y}_4=0$

$-\bar{y}_1+\bar{y}_2+2\bar{y}_3\ge 1$

$2\bar{y}_1+\bar{y}_4\le 1$

$\bar{y}_1,\bar{y}_2\ge 0$ $\bar{y}_3\le 0$ $\bar{y}_4$ libera

3) $x^T\cdot s_d=0$

$s_{d1}=0$

$s_{d2}=-\bar{y}_1+\bar{y}_2+2\bar{y}_3-1 \Rightarrow [1\ 4\ 0]\begin{bmatrix}0\\ -\bar{y}_1+\bar{y}_2+2\bar{y}_3-1\\ 1-2\bar{y}_1-\bar{y}_4\end{bmatrix}=-\bar{y}_1+\bar{y}_2+2\bar{y}_3-1=0$

$s_{d3}=1-2\bar{y}_1-\bar{y}_4$

4) $y^T \cdot Sp = 0$

$Sp_1 = 6 \quad Sp_2 = 0 \quad Sp_3 = 11 \quad Sp_4 = 0$

$$[\bar{y}_1\ \bar{y}_2\ \bar{y}_3\ \bar{y}_4] \begin{bmatrix} 6 \\ 0 \\ 11 \\ 0 \end{bmatrix} = 6\bar{y}_1 + 11\bar{y}_3 = 0$$

$$\begin{cases} -\bar{y}_1 + \bar{y}_2 + 2\bar{y}_3 - 1 = 0 \\ 6\bar{y}_1 + 11\bar{y}_3 = 0 \end{cases} \Rightarrow \begin{cases} \bar{y}_2 = 1 \\ \bar{y}_3 = 0 \\ \bar{y}_1 = 0 \end{cases}$$

$$-y_1 - 2y_2 + 2y_4 = 0 \Rightarrow -2 + 2y_4 = 0 \Rightarrow \bar{y}_4 = 1$$

$$\begin{aligned} &-y_1 - 2y_2 + 2y_4 = 0 \\ &-y_1 + y_2 + 2y_3 \geq 1 \\ &2y_1 + y_4 \leq 1 \\ &y_1, y_2 \geq 0 \quad y_3 \leq 0 \quad y_4 \text{ libera} \end{aligned} \Rightarrow \begin{cases} 0 = 0 \quad (ok) \\ 1 \geq 1 \quad (ok) \\ 1 \leq 1 \quad (ok) \\ (ok). \end{cases}$$

3)

$$\min \; 2x_1 + x_2 + x_3$$
$$-x_1 - x_2 + 2x_3 + x_4 \geq 1$$
$$-x_1 + x_2 - 2x_4 \leq 2$$
$$2x_2 + x_3 = -3$$
$$x_1, x_4 \geq 0 \quad x_2 \leq 0 \quad x_3 \in \mathbb{R}$$

$(0, 0, -3, 7)$

1) $1 \geq 1$ (ok)

$-14 \leq 2$ (ok)

$-3 = -3$ (ok)

D: $\max \; y_1 + y_2 - 3y_3$
$$-y_1 - y_2 \leq 2$$
$$-y_1 + y_2 + 2y_3 \geq 1$$
$$2y_1 + y_3 = 1$$
$$y_1 - 2y_2 \leq 0$$
$$y_1 \geq 0, \; y_2 \leq 0, \; y_3 \in \mathbb{R}$$

2) $-\bar{y}_1 - \bar{y}_2 \leq 2$
$$-\bar{y}_1 + \bar{y}_2 + 2\bar{y}_3 \geq 1$$
$$2\bar{y}_1 + \bar{y}_3 = 1$$
$$\bar{y}_1 - 2\bar{y}_2 \leq 0$$
$$\bar{y}_1 \geq 0, \; \bar{y}_2 \leq 0 \quad \bar{y}_3 \in \mathbb{R}$$

3) $x^T s_d = 0$

$$s_{d_1} = 2 + \bar{y}_1 + \bar{y}_2$$
$$s_{d_2} = -\bar{y}_1 + \bar{y}_2 + 2\bar{y}_3 - 1 \Rightarrow [0 \; 0 \; -3 \; 7] \begin{bmatrix} 2 + \bar{y}_1 + \bar{y}_2 \\ -\bar{y}_1 + \bar{y}_2 + 2\bar{y}_3 - 1 \\ 0 \\ 2\bar{y}_2 - \bar{y}_1 \end{bmatrix} = 14\bar{y}_2 - 7\bar{y}_1 = 0$$
$$s_{d_3} = 0$$
$$s_{d_4} = 2\bar{y}_2 - \bar{y}_1$$

4) $y^T s_p = 0$

s_p:

$s_{p_1} = 0 \qquad s_{p_2} = 16 \qquad s_{p_3} = 0$

$$\begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} 0 \\ 16 \\ 0 \end{bmatrix} = 16 y_2 = 0$$

$$\begin{cases} 16 y_2 = 0 \\ 14 y_2 - 7 y_2 = 0 \end{cases} \Rightarrow \begin{cases} y_2 = 0 \\ y_1 = 0 \end{cases}$$

$2y_1 + y_3 = 1 \Rightarrow y_3 = 1$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$-y_1 - y_2 \leq 2$ — $0 \leq 2$ ok
$-y_1 + y_2 + 2y_3 \geq 1$ — $2 \geq 1$ ok
$2y_1 + y_3 = 1$ — $1 = 1$ ok
$y_1 - 2y_2 \leq 0$ — $0 \leq 0$ ok
$y_1 \geq 0, \; y_2 \leq 0, \; y_3 \in \mathbb{R}$ — ok

4) $\max\ x_1 - x_2$

$x_2 \le 1$

$2x_1 + x_2 \le 5$

$-x_1 - 3x_2 = 10$

$x_1 + x_2 \ge -2$

$x_1 \ge 0$

$x_2 \in \mathbb{R}$

$[2, -4]$

1)

$-4 \le 1$ (ok)

$0 \le 5$ ok

$10 = 10$ (ok)

$-2 \ge -2$ (ok)

D: $\min\ y_1 + 5y_2 + 10y_3 - 2y_4$

$2y_2 - y_3 + y_4 \ge 1$

$y_1 + y_2 - 3y_3 + y_4 = -1$

$y_1, y_2 \ge 0 \quad y_4 \le 0 \qquad y_3 \in \mathbb{R}$

3 $x^T \cdot s_d = 0$

s_d:

$s_{d1} = 2\bar{y}_2 - \bar{y}_3 + \bar{y}_4 - 1$

$s_{d2} = 0$

$$[2, -4] \begin{bmatrix} 2y_2 - y_3 + y_4 - 1 \\ 0 \\ \end{bmatrix} =$$

$= 4y_2 - 2y_3 + 2y_4 - 2$

$y^T \cdot s_p$

$s_{p1} = 5 \quad s_{p2} = 5 \quad s_{p3} = 0 \quad s_{p4} = 0$

$$[y_1\ y_2\ y_3\ y_4] \begin{bmatrix} 5 \\ 5 \\ 0 \\ 0 \end{bmatrix} = \quad 5\bar{y}_1 + 5\bar{y}_2 = 0$$

$\cancel{y_1} + \cancel{y_2} - 3y_3 + y_4 = -1$

$-3y_3 + 1 + y_3 = -1 \Rightarrow y_3 = 1$

$y_4 = 2$

$$\begin{cases} 4y_2 - 2y_3 + 2y_4 - 2 = 0 \\ 5y_1 = 0 \\ 5y_2 = 0 \end{cases} \Rightarrow \begin{cases} -y_3 + y_4 - 1 = 0 \\ y_1 = y_2 = 0 \end{cases} \Leftrightarrow \begin{cases} y_4 = 1 + y_3 \\ y_1 = y_2 = 0 \end{cases}$$

5) $\max\ 2x_1 + x_2$

$$
\begin{aligned}
x_1 + x_2 &\leq 4 \\
2x_1 - x_2 + x_3 &\geq 8 \\
x_1 + 2x_2 - x_3 &= -1 \\
x_1 + x_2 + 2x_3 &= 4 \\
x_1, x_3 &\in \mathbb{R} \\
x_2 &\leq 0
\end{aligned}
$$

D: $\min\ 4y_1 + 8y_2 - y_3 + 4y_4$

$$
\begin{aligned}
y_1 + 2y_2 + y_3 + y_4 &= 2 \\
y_1 - y_2 + 2y_3 + y_4 &\leq 1 \\
y_2 - y_3 + 2y_4 &= 0 \\
y_1 \geq 0 \quad y_2 &\leq 0 \\
y_3, y_4 &\in \mathbb{R}
\end{aligned}
$$