# Es. PROGRAMMAZIONE LINEARE

es. 1.1 Porre in forma canonica i seguenti programmi lineari.

a) $\min\ 3x_1+4x_2-2x_3$

$x_1+2x_2-x_3\geq 5$

$2x_1+4x_3=12$

$x_1+x_2+x_3\leq 15$

$x_1,x_2\geq 0$, $x_3$ libera

sol.: $x_3=x_4-x_5$ con $x_4,x_5\geq 0$ ; $2x_1+4x_3\leq 12$, $2x_1+4x_3\geq 0$ $\Rightarrow$ $2x_1+4x_4-4x_5\leq 12$, $-2x_1-4x_4+4x_5\leq -12$

$\max\ -3x_1-4x_2+2x_4-2x_5$

$-x_1-2x_2+x_3\leq -5$

$2x_1+4x_4-4x_5\leq 12$

$-2x_1-4x_4+4x_5\leq -12$

$x_1+x_2+x_4-x_5\leq 15$

$x_1,x_2,x_4,x_5\geq 0$

b) $\max\ 4x_1-x_2$

$x_1+x_2-x_3=8$

$3x_1+x_3\leq 7$

$x_1\geq 0$, $x_2$ libera, $x_3\leq 0$

sol.:

$x_2=x_4-x_5$ con $x_4,x_5\geq 0$ ; $x_3=-x_3'$ ; $x_1+x_2-x_3\leq 8$, $x_1+x_2-x_3\geq 8$ $\Rightarrow$ $x_1+x_4-x_5+x_3'\leq 8$, $-x_1-x_4+x_5-x_3'\leq -8$

$\max\ 4x_1-x_4+x_5$

$x_1+x_4-x_5+x_3'\leq 8$

$-x_1-x_4+x_5-x_3'\leq -8$

$3x_1-x_3'\leq 7$

$x_1,x_3',x_4,x_5\geq 0$

c) $\min\ 8x_1 - x_2 + x_3$

$x_1 + x_3 \geq 4$

$x_2 - x_3 \leq 7$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0,\ x_3 \leq 0$

sol.: $x_3 = -x_3'$

$\max\ -8x_1 + x_2 + x_3'$

$-x_1 + x_3' \leq -4$

$x_2 + x_3' \leq 7$

$x_1 - x_2 \leq 2$

$x_1, x_2, x_3' \geq 0$

d) $\max\ 4x_1 - x_2$

$x_1 + 2x_2 \leq 2$

$2x_1 + 7x_2 = 8$

$x_1 \geq 0,\ x_2 \leq 0$

sol.: $x_2 = -x_2'$ ; $\begin{array}{l} 2x_1 + 7x_2 \leq 8 \\ 2x_1 + 7x_2 \geq 8 \end{array} \Rightarrow \begin{array}{l} 2x_1 - 7x_2' \leq 8 \\ -2x_1 + 7x_2' \leq -8 \end{array}$

$\max\ 4x_1 + x_2'$

$x_1 - 2x_2' \leq 2$

$2x_1 - 7x_2' \leq 8$

$-2x_1 + 7x_2' \leq -8$

$x_1, x_2' \geq 0$

e)

$\min\ 4x_1 + 5x_2 - x_3 + 2x_4$

$x_1 + x_2 \geq 4$

$x_2 + x_3 \leq 7$

$x_3 - x_4 \leq 2$

$x_1 - x_4 = 12$

$x_1, x_2, x_3 \geq 0,\ x_4$ libero

sol.: $x_4 = x_5 - x_6$ com $x_5, x_6 \geq 0$ ; $\begin{array}{l} x_1 - x_4 \leq 12 \\ x_1 - x_4 \geq 12 \end{array} \Rightarrow \begin{array}{l} x_1 - x_5 + x_6 \leq 12 \\ -x_1 + x_5 - x_6 \leq -12 \end{array}$

$$\max \ -4x_1 - 5x_2 + x_3 - 2x_5 + 2x_6$$
$$-x_1 - x_2 \leq -4$$
$$x_2 + x_3 \leq 7$$
$$x_3 - x_5 + x_6 \leq 2$$
$$x_1 - x_5 + x_6 \leq 12$$
$$-x_1 + x_5 - x_6 \leq -12$$
$$x_1, x_2, x_3, x_5, x_6 \geq 0$$

f) $\max \ 2x_1 + 4x_3$
$$x_1 + x_2 + x_3 \leq 12$$
$$x_1 - x_2 \geq 2$$
$$x_2 + x_3 \leq 4$$
$$x_1 \geq 0, \ x_2 \text{ libera}, \ x_3 \leq 0$$

sol.: $x_2 = x_4 - x_5$ con $x_4, x_5 \geq 0$ ; $x_3 = -x_3'$

$$\max \ 2x_1 - 4x_3'$$
$$x_1 + x_4 - x_5 - x_3' \leq 12$$
$$-x_1 + x_4 - x_5 \leq -2$$
$$x_4 - x_5 - x_3' \leq 4$$
$$x_1, x_3', x_4, x_5 \geq 0$$

es. 1.2 Porre in forma standard i programmi lineari dell'esercizio 1.1

a) $\min \ 3x_1 + 4x_2 - 2x_3$
$$x_1 + 2x_2 - x_3 \geq 5$$
$$2x_1 + 4x_3 = 12$$
$$x_1 + x_2 + x_3 \leq 15$$
$$x_1, x_2 \geq 0, \ x_3 \text{ libera.}$$

sol.:

$$\max \ -3x_1 - 4x_2 + 2x_4 - 2x_5$$
$$x_1 + 2x_2 - x_4 + x_5 - x_6 = 5$$
$$2x_1 + 4x_4 - 4x_5 = 12$$
$$x_1 + x_2 + x_4 - x_5 + x_7 = 15$$
$$x_1, x_2, x_4, x_5, x_6, x_7 \geq 0$$

b) $\max\ 4x_1 - x_2$

$x_1 + x_2 - x_3 = 8$

$3x_1 + x_3 \leq 7$

$x_1 \geq 0,\ x_2$ libera $,\ x_3 \leq 0$

sol.:

$\max\ 4x_1 - x_4 + x_5$

$x_1 + x_4 - x_5 + x_3' = 8$

$3x_1 - x_3' + x_6 = 7$

$x_1, x_3', x_4, x_5, x_6 \geq 0$

c) $\min\ 8x_1 - x_2 + x_3$

$x_1 + x_3 \geq 4$

$x_2 - x_3 \leq 7$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0,\ x_3 \leq 0$

sol:

$\max\ -8x_1 + x_2 + x_3'$

$x_1 - x_3' - x_4 = 4$

$x_2 + x_3' + x_5 = 7$

$x_1 - x_2 + x_6 = 2$

$x_1, x_2, x_3', x_4, x_5, x_6 \geq 0$

d) $\max\ 4x_1 - x_2$

$x_1 + 2x_2 \leq 2$

$2x_1 + 7x_2 = 8$

$x_1 \geq 0,\ x_2 \leq 0$

sol.:

$\max\ 4x_1 + x_2'$

$x_1 - 2x_2' + x_3 = 2$

$2x_1 - 7x_2' = 8$

$x_1, x_2', x_3 \geq 0$

e) $\min\ 4x_1+5x_2-x_3+2x_4$

$$x_1+x_2\geq 4$$
$$x_2+x_3\leq 7$$
$$x_3-x_4\leq 2$$
$$x_1-x_4=12$$
$$x_1,x_2,x_3\geq 0,\ x_4 \text{ libero}$$

sol.:

$\max\ -4x_1-5x_2+x_3-2x_5+2x_6$

$$x_1+x_2-x_7=4$$
$$x_2+x_3+x_8=7$$
$$x_3-x_5+x_6+x_9=2$$
$$x_1-x_5+x_6=12$$
$$x_1,x_2,\dots,x_9\geq 0$$

f) $\max\ 2x_1+4x_3$

$$x_1+x_2+x_3\leq 12$$
$$x_1-x_2\geq 2$$
$$x_2+x_3\leq 4$$
$$x_1\geq 0,\ x_2 \text{ libero},\ x_3\leq 0$$

sol.:

$\max\ 2x_1-4x_3'$

$$x_1+x_4-x_5-x_3'+x_6=12$$
$$x_1+x_4-x_5-x_7=2$$
$$x_4-x_5-x_3'+x_8=4$$
$$x_1,x_3',x_4,x_5,x_6,x_7,x_8\geq 0$$

es. 1.3 Risolvere i seguenti programmi lineari utilizzando il metodo del simplesso

a)

$$\max\ 3x_1+2x_2-5x_3 \qquad\qquad \max\ 3x_1+2x_2-5x_3$$

$$\begin{cases} 4x_1-2x_2+2x_3\leq 4 \\ 2x_1+x_2+x_3\leq 1 \\ x_1,x_2,x_3\geq 0 \end{cases} \xrightarrow{f.s.} \begin{cases} 4x_1-2x_2+2x_3+s_1=4 \\ 2x_1+x_2+x_3+s_2=1 \\ x_1,x_2,x_3,s_1,s_2\geq 0 \end{cases}$$

sol.

| | | $x_1$ | | | | |
|---|---|---|---|---|---|---|
| | | 3 | 2 | -5 | 0 | 0 |
| $s_1$ | 4 | 4 | -2 | 2 | 1 | 0 |
| $s_2$ | 1 | 2 | 1 | 1 | 0 | 1 |

$\Rightarrow$

| | | | $x_2$ | | | |
|---|---|---|---|---|---|---|
| | $-3/2$ | 0 | $1/2$ | $-13/2$ | 0 | $-3/2$ |
| $s_1$ | 2 | 0 | -4 | -2 | 1 | -2 |
| $x_1$ | $1/2$ | 1 | $1/2$ | $1/2$ | 0 | $1/2$ |

| | | | | | | |
|---|---|---|---|---|---|---|
| | -2 | -1 | 0 | -7 | 0 | -2 |
| $s_1$ | 6 | 8 | 0 | 4 | 1 | 2 |
| $x_2$ | 1 | 2 | 1 | 1 | 0 | 1 |

$$\Rightarrow z^* = 2 \quad e \quad x^* = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 6 \\ 0 \end{pmatrix}$$

b) $\max\ x_1 - 2x_2 + 3x_3$

$x_1 - 2x_2 + x_3 \leq 2$

$3x_1 - x_2 - 2x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$

$\xrightarrow{f.s.}$

$\max\ x_1 - 2x_2 + 3x_3$

$x_1 - 2x_2 + x_3 + S_1 = 2$

$3x_1 - x_2 - 2x_3 + S_2 = 6$

$x_1, x_2, x_3, S_1, S_2 \geq 0$

sol:

| | | $x_1$ | $x_2$ | $x_3$ | | |
|---|---|---|---|---|---|---|
| | | 1 | -2 | 3 | 0 | 0 |
| $S_1$ | 2 | 1 | -2 | 1 | 1 | 0 |
| $S_2$ | 6 | 3 | -1 | -2 | 0 | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | -6 | -2 | 4 | 0 | -3 | 0 |
| $x_3$ | 2 | 1 | -2 | 1 | 1 | 0 |
| $S_2$ | 10 | 5 | -5 | 0 | 2 | 1 |

$\Rightarrow z^* = +\infty$

c) $\max\ 2x_1 + x_2 + 3x_3$

$x_1 + x_2 + x_3 \leq 2$

$2x_1 + 3x_2 + 8x_3 \leq 12$

$x_1, x_2, x_3 \geq 0$

$\xrightarrow{f.s.}$

$\max\ 2x_1 + x_2 + 3x_3$

$x_1 + x_2 + x_3 + S_1 = 2$

$2x_1 + 3x_2 + 8x_3 + S_2 = 12$

$x_1, x_2, x_3, S_1, S_2 \geq 0$

sol:

| | | | | | | |
|---|---|---|---|---|---|---|
| | | 2 | 1 | 3 | 0 | 0 |
| $S_1$ | 2 | 1 | 1 | 1 | 1 | 0 |
| $S_2$ | 12 | 2 | 3 | 8 | 0 | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | -4 | 0 | -1 | 1 | -2 | 0 |
| $x_1$ | 2 | 1 | 1 | 1 | 1 | 0 |
| $S_2$ | 8 | 0 | 1 | 6 | -2 | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | $-16/3$ | 0 | $-7/6$ | 0 | $-5/6$ | $-1/6$ |
| $x_1$ | $2/3$ | 1 | $5/6$ | 0 | $4/3$ | $-1/6$ |
| $x_3$ | $4/3$ | 0 | $1/6$ | 1 | $-1/3$ | $1/6$ |

$$\Rightarrow z^* = 16/3 \quad e \quad x^* = \begin{pmatrix} 2/3 \\ 0 \\ 4/3 \\ 0 \\ 0 \end{pmatrix}$$

d) $\min\ 3x_1+x_2-2x_3-x_4$

$$\begin{cases} 2x_1+x_2-x_3+3x_4 \leq 8 \\ -x_1+2x_2-2x_3+2x_4 \leq 4 \\ x_1+x_3 \leq 10 \\ x_1,\dots,x_4 \geq 0 \end{cases} \Rightarrow \quad \min\ 3x_1+x_2-2x_3-x_4 \quad \begin{cases} 2x_1+x_2-x_3+3x_4+x_5=8 \\ -x_1+2x_2-2x_3+2x_4+x_6=4 \\ x_1+x_3+x_7=10 \\ x_1,\dots,x_7 \geq 0 \end{cases}$$

sol :

| | | | | $x_3$ | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 3 | 1 | -2 | -1 | 0 | 0 | 0 |
| $x_5$ | 8 | 2 | 1 | -1 | 3 | 1 | 0 | 0 |
| $x_6$ | 4 | -1 | 2 | -2 | 2 | 0 | 1 | 0 |
| $x_7$ | 10 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

$\Downarrow$

| | | | | | $x_4$ | | | |
|---|---|---|---|---|---|---|---|---|
| | 20 | 5 | 1 | 0 | -1 | 0 | 0 | 2 |
| $x_5$ | 18 | 3 | 1 | 0 | 3 | 1 | 0 | 1 |
| $x_6$ | 24 | 1 | 2 | 0 | 2 | 0 | 1 | 2 |
| $x_3$ | 10 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 26 | 6 | 4/3 | 0 | 0 | 1/3 | 0 | 7/3 |
| $x_4$ | 6 | 1 | 1/3 | 0 | 1 | 1/3 | 0 | 1/3 |
| $x_6$ | +12 | -1 | 4/3 | 0 | 0 | -2/3 | 1 | 4/3 |
| $x_3$ | 10 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

$$\Rightarrow z^* = -26 \quad x^* = \begin{pmatrix} 0 \\ 0 \\ 10 \\ 6 \\ 0 \\ +12 \\ 0 \end{pmatrix}$$

e) $\max\ x_1 + 3x_2 - x_3$

$2x_1 + x_2 \leq 3$

$x_1 + x_2 + 3x_3 \leq 6$

$2x_1 + x_2 + 3x_3 \leq 8$

$x_1, x_2, x_3 \geq 0$

$\longrightarrow$

$\max\ x_1 + 3x_2 - x_3$

$2x_1 + x_2 + x_4 = 3$

$x_1 + x_2 + 3x_3 + x_5 = 6$

$2x_1 + x_2 + 3x_3 + x_6 = 8$

$x_1, \dots, x_6 \geq 0$

sol:

| | | | $x_2$ | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 3 | -1 | 0 | 0 | 0 |
| $x_4$ | 3 | 2 | 1 | 0 | 1 | 0 | 0 |
| | 6 | 1 | 1 | 3 | 0 | 1 | 0 |
| | 8 | 2 | 1 | 3 | 0 | 0 | 1 |
| | -9 | -5 | 0 | -1 | -3 | 0 | 0 |
| $x_2$ | 3 | 2 | 1 | 0 | 1 | 0 | 0 |
| $x_5$ | 3 | -1 | 0 | 3 | -1 | 1 | 0 |
| $x_6$ | 5 | 0 | 0 | 3 | -1 | 0 | 1 |

$\Rightarrow z^* = 9$ e $x^* = \begin{pmatrix} 0 \\ 3 \\ 0 \\ 0 \\ 3 \\ 5 \end{pmatrix}$

f) max $4x_1 + x_2 + 5x_3$

$$\begin{aligned} -x_1 + x_2 &\leq 1 \\ 2x_2 - x_3 &\leq 2 \\ x_1 + x_3 &\leq 1 \\ x_1, x_2, x_3 &\geq 0 \end{aligned} \longrightarrow \max\ 4x_1 + x_2 + 5x_3 \quad \begin{aligned} -x_1 + x_2 + x_4 &= 1 \\ 2x_2 - x_3 + x_5 &= 2 \\ x_1 + x_3 + x_6 &= 1 \\ x_1, \dots, x_6 &\geq 0 \end{aligned}$$

sol :

| | | $x_2$ ↓ | | | | |
|---|---|---|---|---|---|---|
| | 4 | 1 | 5 | 0 | 0 | 0 |
| $x_4$ 1 ← | -1 | 1 | 0 | 1 | 0 | 0 |
| $x_5$ 2 | 0 | 2 | -1 | 0 | 1 | 0 |
| $x_6$ 1 | 1 | 0 | 1 | 0 | 0 | 1 |

⇒

| | | | $x_3$ ↓ | | | |
|---|---|---|---|---|---|---|
| -1 | 5 | 0 | 5 | -1 | 0 | 0 |
| $x_2$ 1 | -1 | 1 | 0 | 1 | 0 | 0 |
| $x_5$ 0 | 2 | 0 | -1 | -2 | 1 | 0 |
| $x_6$ 1 ← | 1 | 0 | 1 | 0 | 0 | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| -6 | 0 | 0 | 0 | -1 | 0 | -5 |
| $x_2$ 1 | -1 | 1 | 0 | 1 | 0 | 0 |
| $x_5$ 1 | 3 | 0 | 0 | -2 | 1 | 1 |
| $x_3$ 1 | 1 | 0 | 1 | 0 | 0 | 1 |

$$\Rightarrow z^* = 6 \quad x^* = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

**es. 1.4** Risolvere i seguenti programmi lineari utilizzando il metodo del simplesso.

**a)** $\min\ 6x_1 + x_2 + 3x_3$

$$10x_1 - 2x_2 + 5x_3 \geq 15$$
$$x_1 - x_2 + 3x_3 \geq 6$$
$$x_1, x_2, x_3 \geq 0$$

**sol.:**

$$\begin{array}{l} \min\ 6x_1 + x_2 + 3x_3 \\ 10x_1 - 2x_2 + 5x_3 - x_4 = 15 \\ x_1 - x_2 + 3x_3 - x_5 = 6 \\ x_1, \dots, x_5 \geq 0 \end{array} \longrightarrow \begin{array}{l} \min\ a_1 + a_2 \\ 10x_1 - 2x_2 + 5x_3 - x_4 + a_1 = 15 \\ x_1 - x_2 + 3x_3 - x_5 + a_2 = 6 \\ x_1, \dots, x_5, a_1, a_2 \geq 0 \end{array}$$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| $a_1$ | 15 | 10 | -2 | 5 | -1 | 0 | 1 | 0 |
| $a_2$ | 6 | 1 | -1 | 3 | 0 | -1 | 0 | 1 |

⇓

| | | | | $x_3$ | | | | |
|---|---|---|---|---|---|---|---|---|
| | -21 | -11 | 3 | -8 | 1 | 1 | 0 | 0 |
| | 15 | 10 | -2 | 5 | -1 | 0 | 1 | 0 |
| $a_2$ | 6 | 1 | -1 | 3 | 0 | -1 | 0 | 1 |

⇓

| | | | | | | $x_5$ | | |
|---|---|---|---|---|---|---|---|---|
| | -5 | -25/3 | 1/3 | 0 | 1 | -5/3 | 0 | 8/3 |
| $a_1$ | 5 | 25/3 | -1/3 | 0 | -1 | 5/3 | 1 | -5/3 |
| $x_3$ | 2 | 1/3 | -1/3 | 1 | 0 | -1/3 | 0 | 1/3 |

⇓

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| $x_5$ 3 | 5 | -5 | 0 | $-3/5$ | 1 | $3/5$ | -1 |
| $x_3$ 3 | 2 | -2 | 1 | $-1/5$ | 0 | $1/5$ | 0 |

$\Downarrow$ II° FASE

| | | | | | |
|---|---|---|---|---|---|
| | 6 | 1 | 3 | 0 | 0 |
| $x_5$ 3 | 5 | -5 | 0 | $-3/5$ | 1 |
| $x_3$ 3 | 2 | -2 | 1 | $-1/5$ | 0 |

$\longrightarrow$

| | | | | | |
|---|---|---|---|---|---|
| -9 | 0 | 7 | 0 | $3/5$ | 0 |
| 3 | 5 | -5 | 0 | $-3/5$ | 1 |
| 3 | 2 | -2 | 1 | $-1/5$ | 0 |

$\Rightarrow z^* = 9 \qquad x^* = \begin{pmatrix} 0 \\ 0 \\ 3 \\ 0 \\ 3 \end{pmatrix}$

b) $\min \; 7x_1 + 2x_2 - 5x_3 - x_4$

$$4x_1 + 3x_2 + 2x_4 \geq 2$$
$$-5x_1 - 3x_2 + x_3 - x_4 \leq 1$$
$$x_1, \dots, x_4 \geq 0$$

$\longrightarrow$ $\min \; 7x_1 + 2x_2 - 5x_3 - x_4$

$$4x_1 + 3x_2 + 2x_4 - S_1 = 2$$
$$-5x_1 - 3x_2 + x_3 - x_4 + S_2 = 1$$
$$x_1, \dots, S_1, S_2 \geq 0$$

$\longrightarrow$ $\min \; a_1$

$$4x_1 + 3x_2 + 2x_4 - S_1 + a_1 = 2$$
$$-5x_1 - 3x_2 + x_3 - x_4 + S_2 = 1$$
$$x_1, \dots, S_1, S_2, a_1 \geq 0$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| $a_1$ 2 | 4 | 3 | 0 | 2 | -1 | 0 | 1 |
| $S_2$ 1 | -5 | -3 | 1 | -4 | 0 | 1 | 0 |

| | $x_1$ | | | | | | |
|---|---|---|---|---|---|---|---|
| -2 | -4 | -3 | 0 | -2 | 1 | 0 | 0 |
| $a_1$ 2 | 4 | 3 | 0 | 2 | -1 | 0 | 1 |
| $S_2$ 1 | -5 | -3 | 1 | -4 | 0 | 1 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| $x_1$ $1/2$ | 1 | $3/4$ | 0 | $1/2$ | $-1/4$ | 0 | $1/4$ |
| $S_2$ $7/2$ | 0 | $3/4$ | 1 | $3/2$ | $-5/4$ | 1 | $5/4$ |

$\downarrow$ II° FASE

| | | | | | | |
|---|---|---|---|---|---|---|
| | 7 | 2 | -5 | -1 | 0 | 0 |
| $x_1$ $1/2$ | 1 | $3/4$ | 0 | $1/2$ | $-1/4$ | 0 |
| $S_2$ $7/2$ | 0 | $3/4$ | 1 | $3/2$ | $-5/4$ | 1 |

$\downarrow$

| | | | $x_3$ | | | |
|---|---|---|---|---|---|---|
| $-7/2$ | 0 | $-13/4$ | $-5$ | $-9/2$ | $7/4$ | 0 |
| $x_1$ $1/2$ | 1 | $3/4$ | 0 | $1/2$ | $-1/4$ | 0 |
| $s_2$ $7/2$ | 0 | $3/4$ | 1 | $3/2$ | $-5/4$ | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 14 | 0 | $1/2$ | 0 | 3 | $-9/2$ | 5 |
| $x_1$ $1/2$ | 1 | $3/4$ | 0 | $1/2$ | $-1/4$ | 0 |
| $x_3$ $7/2$ | 0 | $3/4$ | 1 | $3/2$ | $-5/4$ | 1 |

$\Rightarrow z^* \to -\infty$

c) $\min\ 2x_1+x_2+4x_3$

$$x_1+x_2+2x_3=3$$
$$2x_1+x_2+3x_3=5$$
$$x_1,x_2,x_3\geq 0$$

$\longrightarrow$

$\min\ a_1+a_2$

$$x_1+x_2+2x_3+a_1=3$$
$$2x_1+x_2+3x_3+a_2=5$$
$$x_1,\dots,x_3,a_1,a_2\geq 0$$

| | | | | | |
|---|---|---|---|---|---|
| | 0 | 0 | 0 | 1 | 1 |
| 3 | 1 | 1 | 2 | 1 | 0 |
| 5 | 2 | 1 | 3 | 0 | 1 |

$\rightarrow$

| | | | | | |
|---|---|---|---|---|---|
| | | $x_2$ | | | |
| -8 | -3 | -2 | -5 | 0 | 0 |
| $a_1$: 3 | 1 | 1 | 2 | 1 | 0 |
| $a_2$: 5 | 2 | 1 | 3 | 0 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| | $x_1$ | | | | |
| -2 | -1 | 0 | -1 | 2 | 0 |
| $x_2$: 3 | 1 | 1 | 2 | 1 | 0 |
| $a_2$: 2 | 1 | 0 | 1 | -1 | 1 |

$\rightarrow$

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 |
| $x_2$: 1 | 0 | 1 | 1 | 2 | -1 |
| $x_1$: 2 | 1 | 0 | 1 | -1 | 1 |

II FASE:

| | | | |
|---|---|---|---|
| | 2 | 1 | 4 |
| $x_2$: 1 | 0 | 1 | 1 |
| $x_1$: 2 | 1 | 0 | 1 |

$\rightarrow$

| | | | |
|---|---|---|---|
| -1 | 2 | 0 | 3 |
| 1 | 0 | 1 | 1 |
| 2 | 1 | 0 | 1 |

$\rightarrow$

| | | | |
|---|---|---|---|
| -5 | 0 | 0 | 1 |
| $x_2$: 1 | 0 | 1 | 1 |
| $x_1$: 2 | 1 | 0 | 1 |

$\Rightarrow z^* = 5 \qquad x^* = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

d) $\max\ x_1+x_2+x_3$

$$-x_1-2x_2+x_3+x_4=5$$
$$4x_2+x_3+2x_4=2$$
$$3x_2-2x_4\leq 6$$
$$x_1,\dots,x_4\geq 0$$

$\longrightarrow$

$\min\ a_1+a_2$

$$-x_1-2x_2+x_3+x_4+a_1=2$$
$$4x_2+x_3+2x_4+a_2=2$$
$$3x_2-2x_4+x_5=6$$
$$x_1,\dots,x_5,a_1,a_2\geq 0$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| $a_1$ 5 | -1 | -2 | 1 | 1 | 0 | 1 | 0 |
| $a_2$ 2 | 0 | 4 | 1 | 2 | 0 | 0 | 1 |
| $x_5$ 6 | 0 | 3 | 0 | -2 | 1 | 0 | 0 |

$x_3$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| -7 | 1 | -2 | -2 | -3 | 0 | 0 | 0 |
| $a_1$ 5 | -1 | -2 | 1 | 1 | 0 | 1 | 0 |
| $a_2$ 2 | 0 | 4 | 1 | 2 | 0 | 0 | 1 |
| $x_5$ 6 | 0 | 3 | 0 | -2 | 1 | 0 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| -3 | 1 | 6 | 0 | 1 | 0 | 0 | 2 |
| $a_1$ 3 | -1 | -6 | 0 | -1 | 0 | 1 | -1 |
| $x_3$ 2 | 0 | 4 | 1 | 2 | 0 | 0 | 1 |
| $x_5$ 6 | 0 | 3 | 0 | -2 | 1 | 0 | 0 |

$\rightarrow$ non ammette sol. am.

e) $\max\ 4x_1+3x_2-x_3+2x_4$ $\longrightarrow$ $\min\ a_1+a_2$

$$\begin{aligned}-x_1+2x_2+x_3-x_4&\geq 2\\ 2x_1-x_2+x_3-5x_4&\leq -3\\ x_1,\dots,x_4&\geq 0\end{aligned}
\qquad\longrightarrow\qquad
\begin{aligned}-x_1+2x_2+x_3-x_4-x_5+a_1&=2\\ -2x_1+x_2-x_3+5x_4-x_6+a_2&=3\end{aligned}$$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| $a_1$ 2 | -1 | 2 | 1 | -1 | -1 | 0 | 1 | 0 |
| $a_2$ 3 | -2 | 1 | -1 | 5 | 0 | -1 | 0 | 1 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| -5 | 3 | -3 | 0 | -4 $x_4$ | 1 | 1 | 0 | 0 |
| $a_1$ 2 | -1 | 2 | 1 | -1 | -1 | 0 | 1 | 0 |
| $a_2$ 3 | -2 | 1 | -1 | 5 | 0 | -1 | 0 | 1 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| -13/5 | 7/5 | -11/5 | -4/5 $x_3$ | 0 | 1 | 1/5 | 0 | 4/5 |
| $a_1$ 13/5 | -7/5 | 11/5 | 4/5 | 0 | -1 | -1/5 | 1 | 1/5 |
| $x_4$ 3/5 | -2/5 | 1/5 | -1/5 | 1 | 0 | -1/5 | 0 | 1/5 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| $x_3$ 13/4 | -7/4 | 11/4 | 1 | 0 | -5/4 | -1/4 | 5/4 | 1/4 |
| $x_4$ 5/4 | -3/4 | 11/4 | 0 | 1 | -1/4 | -1/4 | 1/4 | 1/4 |

II FASE

| | | | | | | |
|---|---|---|---|---|---|---|
| | $4$ | $3$ | $-1$ | $2$ | $0$ | $0$ |
| $x_3$ $13/4$ | $-7/4$ | $11/4$ | $1$ | $0$ | $-5/4$ | $-1/4$ |
| $x_4$ $5/4$ | $-3/4$ | $3/4$ | $0$ | $1$ | $-5/20$ | $-1/4$ |

| | | | | | | |
|---|---|---|---|---|---|---|
| $3/4$ | $15/4$ | $17/4$ | $0$ | $0$ | $-3/4$ | $-3/4$ |
| $13/4$ | $-7/4$ | $11/4$ | $1$ | $0$ | $-5/4$ | $-1/4$ |
| $5/4$ | $-3/4$ | $3/4$ | $0$ | $1$ | $-5/20$ | $-1/4$ |

| | | | | | | |
|---|---|---|---|---|---|---|
| | $8$ | $0$ | $0$ | $-17/3$ | $2/3$ | $2/3$ |
| $-4/3$ $x_3$ | $1$ | $0$ | $1$ | $-11/3$ | $-1/3$ | $7/12$ |
| $5/3$ $x_2$ | $-1$ | $1$ | $0$ | $4/3$ | $-1/3$ | $-1/3$ |

| | | | | | | |
|---|---|---|---|---|---|---|
| $32/3$ | $0$ | $0$ | $-8$ | $71/3$ | $10/3$ | $-4$ |
| $-4/3$ | $1$ | $0$ | $1$ | $-11/3$ | $-1/3$ | $7/12$ |
| $1/3$ | $0$ | $1$ | $1$ | $-7/3$ | $-2/3$ | $1/4$ |

$\Rightarrow z^* = +\infty$

f) $$\min \; x_1+x_2-2x_3$$
$$2x_1+x_2+x_3=2$$
$$3x_1+x_2+2x_3=5$$
$$x_1+2x_2+x_3=4$$
$$x_1,x_2,x_3\geq 0$$

$\longrightarrow$

$$\min \; a_1+a_2+a_3$$
$$2x_1+x_2+x_3+a_1=2$$
$$3x_1+x_2+2x_3+a_2=5$$
$$x_1+2x_2+x_3+a_3=4$$
$$x_1,x_2,x_3,a_1,a_2,a_3\geq 0$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | 0 | 0 | 0 | 1 | 1 | 1 |
| $a_1$ | 2 | 2 | 1 | 1 | 1 | 0 | 0 |
| $a_2$ | 5 | 3 | 1 | 2 | 0 | 1 | 0 |
| $a_3$ | 4 | 1 | 2 | 1 | 0 | 0 | 1 |

$\Downarrow$

| | | $x_1$ | | | | | |
|---|---|---|---|---|---|---|---|
| | -11 | -6 | -4 | -4 | 0 | 0 | 0 |
| $a_1$ | 2 | 2 | 1 | 1 | 1 | 0 | 0 |
| | 5 | 3 | 1 | 2 | 0 | 1 | 0 |
| | 4 | 1 | 2 | 1 | 0 | 0 | 1 |

$\Downarrow$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | -5 | 0 | -1 | -1 | 3 | 0 | 0 |
| $x_1$ | 1 | 1 | 1/2 | 1/2 | 1/2 | 0 | 0 |
| $a_2$ | 2 | 0 | -1/2 | 1/2 | -3/2 | 1 | 0 |
| $a_3$ | 3 | 0 | 3/2 | 1/2 | -1/2 | 0 | 1 |

$\Rightarrow$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | -3 | 2 | 0 | 0 | 4 | 0 | 0 |
| $x_2$ | 2 | 2 | 1 | 1 | 1 | 0 | 0 |
| $a_2$ | 3 | 1 | 0 | 1 | -1 | 1 | 0 |
| $a_3$ | 0 | -3 | 0 | -1 | -2 | 0 | 1 |

$\leftarrow$ non esiste sol. ammissibile

**es. 1.5** Per i programmi lineari dell'es. 1.3, dire se le basi finali risultano ottime cambiando l'obiettivo come segue

a) $\max 7x_1 + x_2$

| | | | | | |
|---|---|---|---|---|---|
| | 7 | 1 | 0 | 0 | 0 |
| $s_1$ 6 | 8 | 0 | 4 | 1 | 2 |
| $x_2$ 1 | 2 | 1 | 1 | 0 | 1 |

$\longrightarrow$

| | | | | | |
|---|---|---|---|---|---|
| -1 | 5 | 0 | -1 | 0 | -1 |
| $s_1$ 6 | 8 | 0 | 4 | 1 | 2 |
| $x_2$ 1 | 2 | 1 | 1 | 0 | 1 |

$\Rightarrow x^*$ non ottima

b) $\min 4x_1 + 5x_2 - x_3$

| | | | | | |
|---|---|---|---|---|---|
| | 4 | 5 | -1 | 0 | 0 |
| $x_3$ 2 | 1 | -2 | 1 | 1 | 0 |
| $s_2$ 10 | 5 | -5 | 0 | 2 | 1 |

$\longrightarrow$

| | | | | | |
|---|---|---|---|---|---|
| 2 | 5 | 3 | 0 | 1 | 0 |
| 2 | 1 | -2 | 1 | 1 | 0 |
| 10 | 5 | -5 | 0 | 2 | 1 |

$\Rightarrow x^*$ è ottima

c) $\min x_1 + x_2 + x_3$

| | | | | | |
|---|---|---|---|---|---|
| | 1 | 1 | 1 | 0 | 0 |
| $x_1$ $2/3$ | 1 | $5/6$ | 0 | $4/3$ | $-1/6$ |
| $x_3$ $4/3$ | 0 | $1/6$ | 1 | $-1/3$ | $1/6$ |

$\longrightarrow$

| | | | | | |
|---|---|---|---|---|---|
| -2 | 0 | 0 | 0 | -1 | 0 |
| $2/3$ | 1 | $5/6$ | 0 | $4/3$ | $-1/6$ |
| $4/3$ | 0 | $1/6$ | 1 | $-1/3$ | $1/6$ |

$\Rightarrow x^*$ non è ottima

d) $\min\ 5x_1+3x_2-2x_3-x_4$

| | | 5 | 3 | -2 | -1 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|
| $x_4$ | 6 | 1 | 1/3 | 0 | 1 | 1/3 | 0 | 1/3 |
| $x_6$ | +12 | -1 | 4/3 | 0 | 0 | -2/3 | 1 | 4/3 |
| $x_3$ | 10 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

$\Downarrow$

| | 6 | 6 | 10/3 | -2 | 0 | 1/3 | 0 | 1/3 |
|---|---|---|---|---|---|---|---|---|
| $x_4$ | 6 | 1 | 1/3 | 0 | 1 | 1/3 | 0 | 1/3 |
| $x_6$ | 12 | -1 | 4/3 | 0 | 0 | -2/3 | 1 | 4/3 |
| $x_3$ | 10 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

$\Downarrow$

| 26 | 8 | 10/3 | 0 | 0 | 1/3 | 0 | 7/3 |
|---|---|---|---|---|---|---|---|
| 6 | 1 | 1/3 | 0 | 1 | 1/3 | 0 | 1/3 |
| 12 | -1 | 4/3 | 0 | 0 | -2/3 | 1 | 4/3 |
| 10 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

$\Rightarrow x^*$ è ottimo

e) max $5x_1 + 3x_2$

| | | 5 | 3 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $x_2$ | 3 | 2 | 1 | 0 | 1 | 0 | 0 |
| $x_5$ | 3 | -1 | 0 | 3 | -1 | 1 | 0 |
| $x_6$ | 5 | 0 | 0 | 3 | -1 | 0 | 1 |

| | -9 | -1 | 0 | 0 | -3 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| | 3 | 2 | 1 | 0 | 1 | 0 | 0 |
| | 3 | -1 | 0 | 3 | -1 | 1 | 0 |
| | 5 | 0 | 0 | 3 | -1 | 0 | 1 |

$\Rightarrow \bar{x}$ e' ottimo

f) min $x_1 + 2x_2 - x_3$

| | | 1 | 2 | -1 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $x_2$ | 1 | -1 | 1 | 0 | 1 | 0 | 0 |
| $x_5$ | 1 | 3 | 0 | 0 | -2 | 1 | 1 |
| $x_3$ | 1 | 1 | 0 | 1 | 0 | 0 | 1 |

| | -1 | 4 | 0 | 0 | -2 | 0 | 1 |
|---|---|---|---|---|---|---|---|
| | 1 | -1 | 1 | 0 | 1 | 0 | 0 |
| | 1 | 3 | 0 | 0 | -2 | 1 | 1 |
| | 1 | 1 | 0 | 1 | 0 | 0 | 1 |

$\Rightarrow \bar{x}$ non e' ottima

es 1.6 Per ognuno dei programmi lineari dell'es. 1.3, identificare la matrice $A_B^{-1}$.

a) $A_B^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

b) $A_B^{-1} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

c) $A_B^{-1} = \begin{pmatrix} 4/3 & -1/6 \\ -1/3 & 1/6 \end{pmatrix}$

d) $A_B^{-1} = \begin{pmatrix} 1/3 & 0 & 1/3 \\ -2/3 & 1 & 4/3 \\ 0 & 0 & 1 \end{pmatrix}$

e) $A_B^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

f) $A_B^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

es. 1.7

Risolvere i seguenti programmi lineari utilizzando il metodo grafico

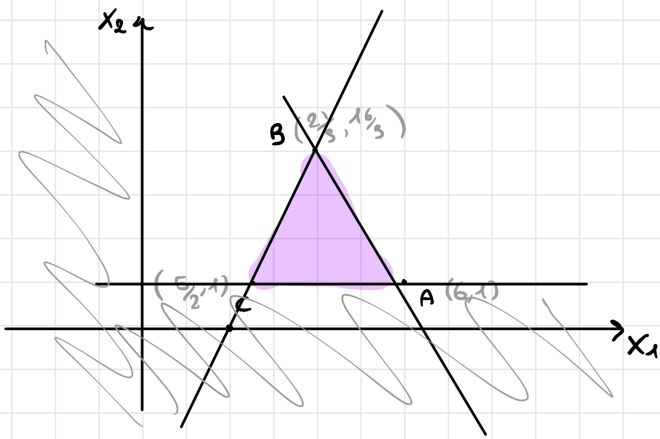
a) $\max\ x_1 + x_2$

$2x_1 - x_2 \geq 4$

$x_1 + 4x_2 \leq 10$

$x_2 \geq 1$

$x_1, x_2 \geq 0$




• $2x_1 - x_2 = 4$

$\Rightarrow -x_2 = +4 - 2x_1 \Rightarrow x_2 = -4 + 2x_1$ $\left(\frac{5}{2}, 1\right)$

• $x_1 + 4x_2 \leq 10 \Rightarrow$

$x_1 \leq 10 - 4x_2$ $(6,1)$

• $\begin{cases} 2x_1 - x_2 \geq 4 \\ x_1 + 4x_2 \leq 10 \end{cases} \Rightarrow \begin{cases} x_2 = 2x_1 - 4 \\ x_1 = 10 - 4x_2 \end{cases} \Rightarrow$

$\begin{cases} x_2 = 20 - 8x_2 - 4 \\ x_1 = 10 - 4x_2 \end{cases} \Rightarrow \begin{cases} x_2 = 16/9 \\ x_1 = 26/9 \end{cases}$

$f(A): \frac{5}{2} + 1 = \frac{5+2}{2} = 7/2$

$f(B) = \frac{26}{9} + \frac{16}{9} = \frac{42}{9} = 14/3$

$f(C) = 6 + 1 = 7 \Rightarrow x_1^* = 6$

$x_2^* = 1$

b)

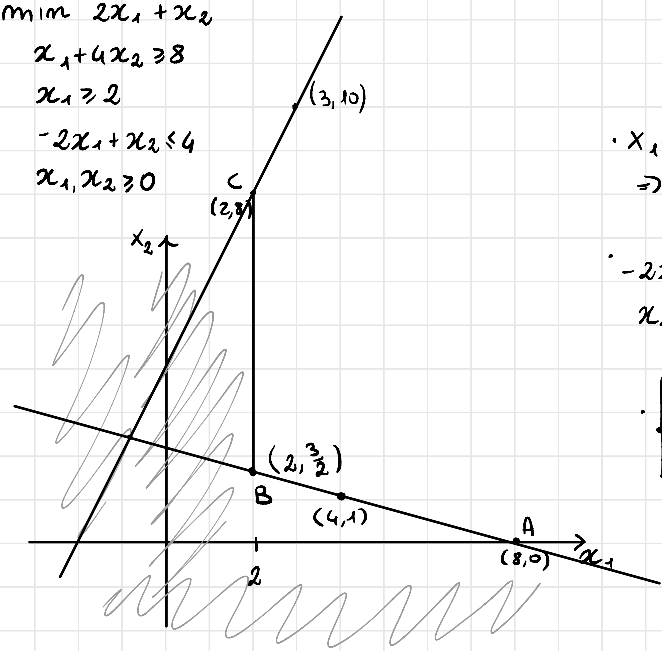
$\min\ 2x_1 + x_2$

$x_1 + 4x_2 \geq 8$

$x_1 \geq 2$

$-2x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$




- $x_1 + 4x_2 \geq 8$
  $\Rightarrow x_1 = 8 - 4x_2$ (8,0); (4,1)

- $-2x_1 + x_2 \leq 4$
  $x_2 \leq 4 + 2x_1$ (2,8) (3,10)

- $\begin{cases} x_1 + 4x_2 \geq 8 \\ -2x_1 + x_2 \leq 4 \end{cases} \Rightarrow \begin{cases} x_1 = 8 - 4x_2 \\ x_2 = 4 + 2x_1 \end{cases}$

$\Rightarrow \begin{cases} x_1 = 8 - 4(4 + 2x_1) \\ x_2 = 4 + 2x_1 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x_1 = 8 - 16 - 8x_1 \\ x_2 = 4 + 2x_1 \end{cases} \Rightarrow \begin{cases} 9x_1 = -8 \\ x_2 = 4 + 2x_1 \end{cases} \Rightarrow \begin{cases} x_1 = -8/9 \\ x_2 = 20/9 \end{cases}$

- $\begin{cases} x_1 \geq 2 \\ x_1 + 4x_2 \geq 8 \end{cases} \Rightarrow \begin{cases} x_1 = 2 \\ x_2 = 3/2 \end{cases}$

$x_1^* = 2$; $x_2^* = 3/2$

$f(A) = 16$ ; $f(B) = 4 + \frac{3}{2} = \frac{8+3}{2} = 11/2$ ; $f(C) = 10$

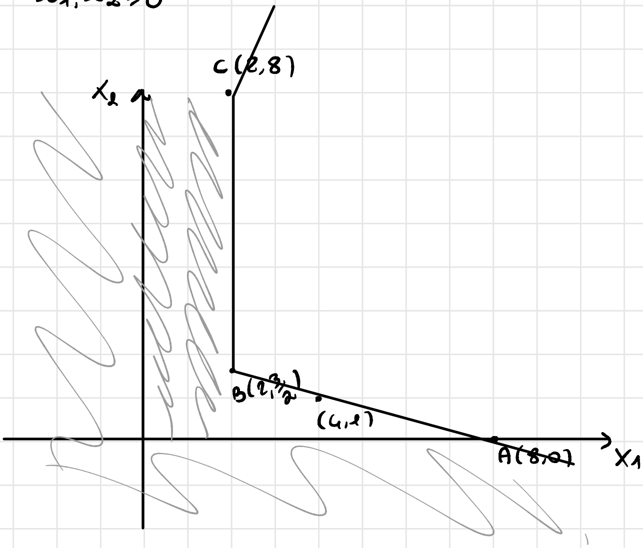
c) $\max\ 2x_1 - x_2$

$x_1 + 4x_2 \geq 8$

$x_1 \geq 2$

$-2x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$




• $x_1 + 4x_2 = 8$

$x_1 = 8 - 4x_2$ (8,0)

(4,1)

• $-2x_1 + x_2 = 4$

$x_2 = 4 + 2x_1$

(2,8)

$\begin{cases} x_1 = 2 \\ x_1 + 4x_2 = 8 \end{cases} \Rightarrow \begin{cases} x_1 = 2 \\ x_2 = 3/2 \end{cases}$

$f(B) = 4 - \frac{3}{2} = \frac{8-3}{2} = 5/2$

$f(A) = 16$

$f(C) = 4 - 8 = -4$ illimitato

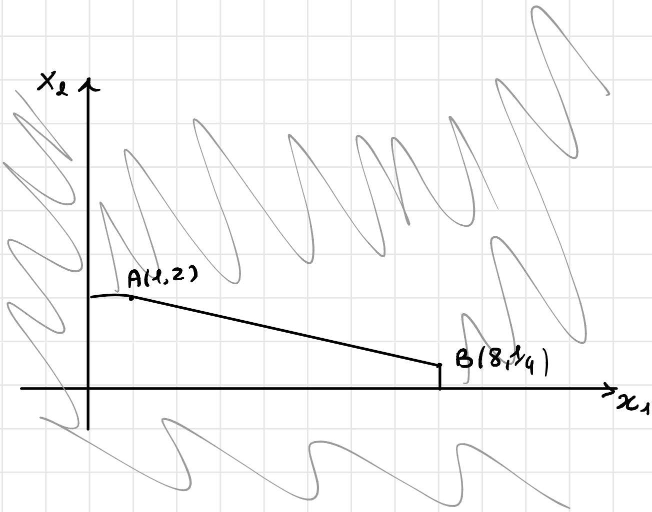
d) $\max \ \frac{2}{3}x_1 + \frac{8}{3}x_2$

$x_1 + 4x_2 \le 9$

$x_1 \le 8$

$x_2 \le 2$

$x_1, x_2 \ge 0$




• $x_1 + 4x_2 = 9$

$x_1 = 9 - 4x_2$ (1,2)

$\begin{cases} x_1 + 4x_2 = 9 \\ x_1 = 8 \end{cases} \Rightarrow \begin{cases} x_2 = 1/4 \\ x_1 = 8 \end{cases}$

$f(A) = 6$

$\Rightarrow x_1^* = 1$

$f(B) = 6$ $\quad x_2^* = 2$

# DUALITA'

**es 2.1** Partendo dalla forma standard, determinare i duali dei programmi lineari dell'es. 1.3

a) $\max\ 3x_1 + 2x_2 - 5x_3$

$4x_1 - 2x_2 + 2x_3 \leq 4$

$2x_1 + x_2 + x_3 \leq 1$

$x_1, x_2, x_3 \geq 0$

$\xrightarrow{f.s.}$

$\max\ 3x_1 + 2x_2 - 5x_3$

$4x_1 - 2x_2 + 2x_3 + x_4 = 4$

$2x_1 + x_2 + x_3 + x_5 = 1$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

D: $\min\ 4y_1 + y_2$

$4y_1 + 2y_2 \geq 3$

$-2y_1 + y_2 \geq 2$

$2y_1 + y_2 \geq -5$

$y_1 \geq 0\ ;\ y_2 \geq 0$

b) $\max\ x_1 - 2x_2 + 3x_3$

$x_1 - 2x_2 + x_3 \leq 2$

$3x_1 - x_2 - 2x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$

$\xrightarrow{f.s.}$

$\max\ x_1 - 2x_2 + 3x_3$

$x_1 - 2x_2 + x_3 + x_4 = 2$

$3x_1 - x_2 - 2x_3 + x_5 = 6$

$x_1, x_2, \dots, x_5 \geq 0$

D: $\min\ 2y_1 + 6y_2$

$y_1 + 3y_2 \geq 1$

$-2y_1 - y_2 \geq -2$

$y_1 - 2y_2 \geq 3$

$y_1, y_2 \geq 0$

c) $\max\ 2x_1 + x_2 + 3x_3$

$x_1 + x_2 + x_3 \leq 2$

$2x_1 + 3x_2 + 8x_3 \leq 12$

$x_1, x_2, x_3 \geq 0$

$\xrightarrow{f.s.}$

$\max\ 2x_1 + x_2 + 3x_3$

$x_1 + x_2 + x_3 + x_4 = 2$

$2x_1 + 3x_2 + 8x_3 + x_5 = 12$

$x_1, \dots, x_5 \geq 0$

D: $\min\ 2y_1 + 12y_2$

$y_1 + 2y_2 \geq 2$

$y_1 + 3y_2 \geq 1$

$y_1 + 8y_2 \geq 3$

$y_1, y_2 \geq 0$

d) $\min\ 3x_1+x_2-2x_3-x_4$
$2x_1+x_2-x_3+3x_4 \leq 8$
$-x_1+2x_2-2x_3+2x_4 \leq 4$
$x_1+x_3 \leq 10$
$x_1,\dots,x_4 \geq 0$

$\xrightarrow{f.s.}$

$\min\ 3x_1+x_2-2x_3-x_4$
$2x_1+x_2-x_3+3x_4+x_5=8$
$-x_1+2x_2-2x_3+2x_4+x_6=4$
$x_1+x_3+x_7=10$
$x_1,\dots,x_7 \geq 0$

D: $\max\ 8y_1+4y_2+10y_3$
$2y_1-y_2+y_3 \leq 3$
$y_1+2y_2 \leq 1$
$-y_1-2y_2+y_3 \leq -2$
$3y_1+2y_2 \leq -1$
$y_1,y_2,y_3 \leq 0$

e) $\max\ x_1+3x_2-x_3$
$2x_1+x_2 \leq 3$
$x_1+x_2+3x_3 \leq 6$
$2x_1+x_2+3x_3 \leq 8$
$x_1,x_2,x_3 \geq 0$

$\xrightarrow{f.s.}$

$\max\ x_1+3x_2-x_3$
$2x_1+x_2+x_4=3$
$x_1+x_2+3x_3+x_5=6$
$2x_1+x_2+3x_3+x_6=8$
$x_1,\dots,x_6 \geq 0$

D: $\min\ 3y_1+6y_2+8y_3$
$2y_1+y_2+2y_3 \geq 1$
$y_1+y_2+y_3 \geq 3$
$3y_2+3y_3 \geq -1$
$y_1,y_2,y_3 \geq 0$

f) $\max\ 4x_1+x_2+5x_3$
$-x_1+x_2 \leq 1$
$2x_2-x_3 \leq 2$
$x_1+x_3 \leq 1$
$x_1,x_2,x_3 \geq 0$

$\xrightarrow{f.s.}$

$\max\ 4x_1+x_2+5x_3$
$-x_1+x_2+x_4=1$
$2x_2-x_3+x_5=2$
$x_1+x_3+x_6=1$
$x_1,\dots,x_6 \geq 0$

D: $\min\ y_1+2y_2+y_3$
$-y_1+y_3 \geq 4$
$y_1+2y_2 \geq 1$
$-y_2+y_3 \geq 5$ $\quad y_1,y_2,y_3 \geq 0$

es. 2.2 Partendo dalla forma standard, determinare i duali dei prog. lineari dell. es. 1.4.

a) $\min\ 6x_1 + x_2 + 3x_3$

$$10x_1 - 2x_2 + 5x_3 \geq 15$$
$$x_1 - x_2 + 3x_3 \geq 6$$
$$x_1, x_2, x_3 \geq 0$$

$\xrightarrow{f.s.}$

$\min\ 6x_1 + x_2 + 3x_3$

$$10x_1 - 2x_2 + 5x_3 - x_4 = 15$$
$$x_1 - x_2 + 3x_3 - x_5 = 6$$
$$x_1, \ldots, x_5 \geq 0$$

D: $\max\ 15y_1 + 6y_2$

$$10y_1 + y_2 \leq 6$$
$$-2y_1 - y_2 \leq 1$$
$$5y_1 + 3y_2 \leq 3$$
$$-y_1, -y_2 \leq 0$$

?

b) $\min\ 7x_1 + 2x_2 - 5x_3 - x_4$

$$4x_1 + 3x_2 + 2x_4 \geq 2$$
$$-5x_1 - 3x_2 + x_3 - x_4 \leq 1$$
$$x_1, \ldots, x_4 \geq 0$$

$\xrightarrow{f.s.}$

$\min\ 7x_1 + 2x_2 - 5x_3 - x_4$

$$4x_1 + 3x_2 + 2x_4 - x_5 = 2$$
$$-5x_1 - 3x_2 + x_3 - x_4 + x_6 = 1$$
$$x_1, \ldots, x_6 \geq 0$$

D: $\max\ 2y_1 + y_2$

$$4y_1 - 5y_2 \leq 7$$
$$3y_1 - 3y_2 \leq 2$$
$$y_2 \leq -5$$
$$2y_1 - y_2 \leq -1$$
$$-y_1 \leq 0 \quad y_2 \leq 0$$

?

c) $\min\ 2x_1 + x_2 + 4x_3$

$$x_1 + x_2 + 2x_3 = 3$$
$$2x_1 + x_2 + 3x_3 = 5$$
$$x_1, x_2, x_3 \geq 0$$

D: $\max\ 3y_1 + 5y_2$

$$y_1 + 2y_2 \leq 2$$
$$y_1 + y_2 \leq 1$$
$$2y_1 + 3y_2 \leq 4$$

d) $\max\ x_1+x_2+x_3$

$$\begin{aligned}-x_1-2x_2+x_3+x_4&=5\\4x_2+x_3+2x_4&=2\\3x_2-2x_4&\leq 6\\x_1,\dots,x_4&\geq 0\end{aligned} \longrightarrow \begin{aligned}&\max\ x_1+x_2+x_3\\-x_1-2x_2+x_3+x_4&=5\\4x_2+x_3+2x_4&=2\\3x_2-2x_4+x_5&=6\\x_1,\dots,x_5&\geq 0\end{aligned}$$

D: $\min\ 5y_1+2y_2+6y_3$

$$\begin{aligned}-y_1&\geq 1\\-2y_1+4y_2+3y_2&\geq 1\\y_1+y_2&\geq 1\\y_1+2y_2-2y_3&\geq 0\\y_3&\geq 0\end{aligned}$$

e) $\max\ 4x_1+3x_2-x_3+2x_4$

$$\begin{aligned}-x_1+2x_2+x_3-x_4&\geq 2\\2x_1-x_2+x_3-5x_4&\leq -3\\x_1,\dots,x_4&\geq 0\end{aligned} \longrightarrow \begin{aligned}&\max\ 4x_1+3x_2-x_3+2x_4\\-x_1+2x_2+x_3-x_4-x_5&=2\\2x_1-x_2+x_3-5x_4+x_6&=-3\\x_1,\dots,x_6&\geq 0\end{aligned}$$

D: $\min\ 2y_1-3y_2$

$$\begin{aligned}-y_1+2y_2&\geq 4\\2y_1-y_2&\geq 3\\y_1+y_2&\geq -1\\-y_1-5y_2&\geq 2\\-y_1&\geq 0\\y_2&\geq 0\end{aligned}$$

f) $\min\ x_1+x_2-2x_3$

$$\begin{aligned}2x_1+x_2+x_3&=2\\3x_1+x_2+2x_3&=5\\x_1+2x_2+x_3&=4\\x_1,\dots,x_3&\geq 0\end{aligned} \longrightarrow$$

D: $\max\ 2y_1+5y_2+4y_3$

$$\begin{aligned}2y_1+3y_2+y_3&\leq 1\\y_1+y_2+2y_3&\leq 1\\y_1+2y_2+y_3&\leq -2\end{aligned}$$