Minimum distance faculty

Marco D'Amico - Niccoló Didoni

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1 Solution

The problem required to add a constraint to express the fact that a customer has to be assigned to the closest active centre.

Let us consider the following constraint

$$x_{ij} d_{ij} \leq d_{ih} y_h + D(1 - y_h) \quad \forall i \in C \ \forall j, h \neq j \in S \tag{1}$$

where D is a large positive constant (ideally larger than the biggest distance between any two customer and centre).

Let us check that constraint 1 works for every combination of x and y.

- If $x_{ij} = 0$ and $y_h = 0$ (i.e. customer i isn't assigned to centre j and centre $h \neq j$ isn't active) then distance d_{ij} can be whatever with respect to d_{ih} and the constraint is trivially satisfied (because we defined $D \geq 0$).

$$\begin{aligned} 0 \cdot d_{ij} &\leq d_{ih} \cdot 0 + D(1 - 0) \\ 0 &\leq D \end{aligned}$$

- If $x_{ij} = 0$ and $y_h = 1$ (i.e. customer i isn't assigned to centre j and centre $h \neq j$ is active) then distance d_{ij} can be whatever with respect to d_{ih}. Distance d_{ij} can be smaller than d_{ih} because there could exist an active centre k so that $d_{ik} \leq d_{ij} \leq d_{ih}$. The constraint is always true because the distance d parameter is non negative by construction.

$$\begin{aligned} 0 \cdot d_{ij} &\leq d_{ih} \cdot 1 + D(1 - 1) \\ 0 &\leq d_{ih} \end{aligned}$$

- If $x_{ij} = 1$ and $y_h = 0$ (i.e. customer i is assigned to centre j and centre $h \neq j$ is not active) then the distances d_{ih} and d_{ij} can be whatever because h is not active.

$$\begin{aligned} 1 \cdot d_{ij} &\leq d_{ih} \cdot 0 + D(1 - 0) \\ d_{ij} &\leq D \end{aligned}$$

  The constraint is true because we defined D as a big constant bigger than the maximum distance d_{max} between a customer and a centre.

- If $x_{ij} = 1$ and $y_h = 1$ (i.e. customer i is assigned to centre j and centre $h \neq j$ is active) then distance d_{ij} must be smaller than distance d_{ih}.

$$\begin{aligned} 1 \cdot d_{ij} &\leq d_{ih} \cdot 1 + D(1 - 1) \\ d_{ij} &\leq d_{ih} \end{aligned}$$

Since the constraint holds for every combination of x and y, it correctly models the fact that a customer has to be assigned to the closest active centre.