Say no to ∞!!

Goal: Atkey & McBride paper

↳ shallow embedding of ~~CTT~~ nondep CGTT

What types do we want?

~~$1, \times, +, \to, \mu, \forall, \triangleright^{\kappa}$~~

- $1, \times, +, \to, \mu, \forall, \triangleright^{\kappa}$ ☺

Blue part: standard

Poly → Set → Set

$\Gamma \vdash x : A$

1	constant presheaf
$+, \times$	pointwise
$\to$	Kripke ← some work
μ	← some room ☺

- 'Easy' ⟷ 'Difficult'

~~$X^A \to X$~~

1. Polynomials via grammar $\underbrace{A, +, \times, \triangleright^{\kappa}, \times}$

2. also $X^A \to X$

3. W-types

4. AMB

↳ something with clock context!

POLY DEPENDS ON CLOCK CONTEXT!!!

$\forall$: combinatorics, insert on x'th place and quantification

$$\mathrm{Size} \times \dots \times \mathrm{Size} \to \mathrm{Set}$$

0:	1	$\to \mathrm{Set}$	A
1:	Size	$\to \mathrm{Set}$	$i \mapsto \mathrm{Set}$
	$\vdots$		
n	Size^n	$\to \mathrm{Set}$	$(i_1, \dots, i_n) \mapsto \mathrm{Set}$

$$(i_1, \dots \cancel{i_k}, i_n, i_{n+1}) \mapsto \mathrm{Set}$$

Δ^x

record ΔD $(A : \mathrm{Size}^{n+1} \to \mathrm{Set})$ $(x \in [0, \dots, n])$
~~$\mathrm{Size}^{n+1} \to \mathrm{Set}$~~ $(i_0, \dots, i_n) : \mathrm{Set}$

force : $(j : \mathrm{Size}< i_x) \to A(i_0, \dots j, \dots i_n)$

$$\mathrm{Tick} = \Sigma\, x \in [0, \dots, n] : \mathrm{Size}<(i_x)$$

$$\mathrm{Tick}\ x\ i = \mathrm{Size}<(i_x)$$

$$\mathrm{TickCon}\ x\ i = \mathrm{Tick}\ (i_x)$$

ALIAS FOR FIN

Context:
$x : A$
~~A type~~

Judgements
- ~~$\Gamma \vdash A$~~ $\Gamma \vdash \mathrm{ctx}$
- $\vdash A$ type
- $\Gamma \vdash x : A$
- $\Gamma \vdash x \equiv y : A$

not for types because no help

Term Theory

Not much, at most annoying.

How to do types?
1. Presheaves
2. Families

Equalities

Section Various

Type isos: ✓

Applicative functor $\triangleright$:
- bisimilarity $\Rightarrow$ equality
- funext
- UIP

Code: literate Programming

basics.lagda
funext, UIP } as postulates } 'unrelated' but necessary

$x_1 B_1, \ldots x_n B_n$

$\Gamma \vdash x : A$

$[\![A]\!] : Psh$

$[\![\Gamma]\!] : Psh^{\Gamma}$ A

$[\![x]\!] : Tm\,\Gamma\,A$

~~clock context~~ → Basic defs foundations of the TT NO TYPE FORMERS

presheaves

types contexts

types

term

Judgement

Built from this and introduce new things

$D\Gamma \vdash x : A$

Real Part

$+$ $\cdot$ const 1

$\to$ M $\forall$ ΔK

Polynomial

$$\frac{\begin{array}{l} A\ B : \text{Type} \\ C : \text{Type} \\ x : A \vdash y : C \\ x : B \vdash z : C \end{array}}{x : A+B \vdash (y, z) : C}
\qquad
\frac{\vdots}{\vdash A+B \to C}$$

Const
$$\frac{\Delta, \Theta \vdash A \text{ type}}{\Delta, x ; \Theta \vdash A \text{ type}}$$

$$\frac{\Delta, \Theta \vdash x : A}{\Delta, x ; \Theta \vdash x : A}$$

$$\frac{\Delta, \Theta \vdash x \equiv y}{\Delta, x ; \Theta \vdash x \equiv y}$$

$+$:
- $+$-type
- inl
- inr
- case ...
- 2 computation, η

$\times$:
- $\times$-type
- π_1
- π_2
- pair
- 2-computation rule, η

$\to$:
- $\to$-type
- λ
- app
- β, η

1:
- 1 type
- tt
- 1-elim
- 1 Computation, η

polynomial has focy's and eval

μ:
uses polynomial def of the type

$\circ$ sup: $F(\mu F) \to \mu F$

Primrec: $\dfrac{F\,C \to C}{\mu F \to C}$

1 computation rule

$\forall$

Constant presheaf!

$$\boxed{\dfrac{\Delta \vdash \Gamma\ ctx}{\Delta, X \vdash \Gamma\ ctx}}$$

$$\dfrac{\cancel{\Delta; \Gamma \vdash e : A}}{\Delta; \Gamma \vdash \Lambda X.}$$

$$\dfrac{\Delta \vdash \Gamma\ ctx \quad \Delta, X; \cancel{\emptyset}; \Gamma \vdash A\ type \quad \Delta, X; \Gamma \vdash e : A}{\Delta; \Gamma \vdash \Lambda X. e : \forall X. A}$$

$\cancel{\Delta \vdash \Gamma\ ctx}$

$$\boxed{\dfrac{\Delta \vdash A : type}{\Delta, X' \vdash A : type}}$$

$X_1, \ldots, X_n \mapsto A$
$[X_i \mapsto X_j]$
ith
$X_1, \ldots, X_j, \ldots X_n \vdash A$

$$\dfrac{\Delta, X' \vdash \Gamma\ ctx \quad \Delta; \emptyset \vdash A : type \quad \Delta; \Gamma \vdash e : \forall X. A}{\Delta; \Gamma \vdash e[X'] : \underline{A[X \mapsto X']}}$$

subst

What's a Clock context?

$\boxed{\text{1. Vector}}$ ← do it → reuse!

2. Fin n → Clock ← encode

$\begin{cases} 1 \mapsto a_1 \\ \vdots \\ n \mapsto a_n \end{cases}$

$[\cancel{a_0}, \ldots, a_n]$

$[a_1, \ldots, a_n]$

if $i \leq j$ then a_i

else a_{i+1}

$\Sigma n, \mathrm{Vec}\ \mathrm{Clock}\ n$

$\mathrm{ClockCtx}\ n = \mathrm{Vec}\ \mathrm{Clock}\ n$

$$\frac{\Delta ; \Theta \vdash A \text{ type} \qquad \kappa \in \Delta}{\Delta ; \Theta \vdash \triangleright^{\kappa} A : \text{type}}$$

$n : \mathbb{N}$

$\Delta : \mathrm{ClockCtx}\ n$

$\Gamma : \mathrm{Context}\ \Delta$

$A : \mathrm{Type}\ \Delta$

$i\ \cancel{:} < n \quad i : \mathrm{Fin}\ n$

$\kappa : \mathrm{Clock}$

$p : \kappa \in \Delta$

$\mathrm{Vec}\ A\ n \to \mathrm{Fin}\ n \to A$

$\kappa \in \Delta \qquad \cancel{i \in \Delta} \quad i : \mathrm{Fin}$

$\mathrm{ClockCtx}\ (n+1)\ \cancel{\&}\ i : \mathrm{Fin}(\ldots) \to \mathrm{ClockCtx}\ n$

remove
Size<
Tick at X (Size< (δ/K $\wedge$ X))
Size of clock

$$\frac{\Delta \qquad X \in \Delta}{\text{Tick } X}$$

$\text{Size}^n \to \text{Set}$

$(i_1, \ldots, i_n)$
$\wedge \qquad \wedge$
$(j_1, \ldots, j_n)$

√remove
√ClockCtx
√Size≤
√ClockCtx<
√ClockCtx≤
√Tick

$0, \ldots, n$

~~Size~~

$X_0, \ldots, X_m$

$\text{ClockCtx}^n \to i : \text{Fin } n \to \text{Size}$

$(i_1, \ldots, i_n)$
$\geq \qquad \geq$
$(j_1, \ldots, j_n)$

$(i_1, \ldots i_k \ldots i_n)$
$\vee$
$(i_1, \ldots j \ldots i_n)$

$\text{Set}^{\ldots} \to \text{Set}^{\ldots}$

$\Delta = (i_1, i_2, \ldots, i_x, \ldots, i_n)$

$$\begin{array}{c} i_x \\ \vee \\ j \end{array}$$

$\alpha = (i_1, i_2, \ldots, j, \ldots, i_n)$

$\Delta [i_x \mapsto j]$

$\Delta \, x = \text{if } x == X$ ~~$i == j$~~
then j
else ΔX

~~$\Delta [$~~

case $i = i$

case $i \neq i$

$$\frac{\begin{array}{c}\Delta : \text{ClockContext } n \\ i : \text{Fin } n \\ \alpha : \text{Size} \leq (\Delta \, i)\end{array}}{\text{ClockContext} \leq \Delta}$$

=

case $i = i$

with

$(\Delta [i \mapsto \alpha]) [i \mapsto \alpha']$

$\equiv \Delta [i \mapsto \alpha']$