# Practical Work - Bayesian image analysis - IMA203
# NAME :

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| Bayesian analysis<br>for image classification |

**Objective of the session :**

In this PW we will perform the binary classification of a grayscale image "Iobservee.png" (image of the observations, realization $y$ of the field $Y$) using a Markovian model.
In this ideal case, we are given the ideal solution $x$ (binary image "IoriginalBW.png"), realization of the field of classes $X$, which will be used to evaluate the quality of the solution $\hat{x}$ that we will obtain. (NB : In practice usually, we don't have access to $x$).
You have to fill by hand-writing the printed version of the practical work (this document) and upload the filled jupyter notebook on e-campus.

This report should be given on the 9th of december in the IMA204 course. You must do it in pair (2 students), put both names on the document. A filled notebook for each student should be also uplodaded on e-campus for the 8th of december.

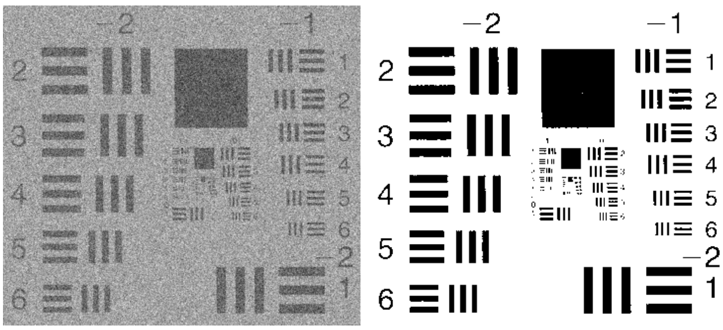

FIGURE 1 – Observed image $y$ on the left (gray levels) and "ideal" binary image $x$ (on the right) that we are trying to recover.

The objective is to estimate $x$ from $y$ using a prior on $P(X)$ in the form of a Markovian model. We note $x_s$ the class of the pixel $s$ (that we are looking for), and $y_s$ the observed gray level. The objective is to use a global model on the random field $X$ to classify the image. As we have seen in class, this amounts to minimizing the following energy :

$$U(x|y) = \sum_s -\ln(P(Y_s = y_s|X_s = x_s)) + \sum_c U_c(x_s, s \in c)$$

## 1 Analysis of the gray level distributions

In this part, we learn the probabilities $P(Y_s = y_s|X_s)$, that is to say $P(Y_s = y_s|X_s = 0)$ and $P(Y_s = y_s|X_s = 1)$. This is equivalent to studying the histogram of gray levels of pixels that are in class 0 and pixels that are in class 1.

To perform this training, we need to select pixels belonging to class 0 on the one hand (dark area of the observed image), and pixels belonging to class 1 on the other hand (light area of the observed image).

— Q1 What are the distributions followed by the grey levels in these two classes ? Give the means and variances of the two classes that you have estimated.

From the notebook it's evident that the gray levels follow normal Gaussian distributions for each class (look at the next page).

In particular for class 0 we have that P(Y_s = y_s I X_s = 0) ~ N(96.96484848484849, 520.5781583103765)

For class 1 we have P(Y_s = y_s I X_s = 1) ~ N(164.04833333333335, 489.64433055555554)

In the following, we assume that the variances are equal in order to simplify the energy expressions.

Suppose that we do not use a Markov model on $X$ and that we classify a pixel only according to its grey level by comparing $P(Y_s = y_s|X_s = 0)$ and $P(Y_s = y_s|X_s = 1)$.

— Q2 Show that this amounts to thresholding the image and give the value of the optimal threshold as a function of the parameters found previously (we say that we are doing a classification by punctual (=in each pixel) maximum likelihood).

— Q2bis Show graphically the threshold by drawing the histograms corresponding to the two conditional distributions.

$$P(Y_s = y_s \mid X_s = i) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp - \left( \frac{(y_s - \mu_i)^2}{2\sigma_i^2} \right)$$

— From the results found for $P(Y_s = y_s|X_s)$, write the likelihood energy (data attachment term) :

$$U_{attdo} = \sum_s -\ln(P(Y_s = y_s|X_s = x_s))$$

look at next page

## 2 Ising model for regularization

To improve the thresholding results, it is necessary to introduce a regularisation (global prior model).

Consider the function $\Delta(x_s, x_t) = 0$ if $x_s = x_t$, and $\Delta(x_s, x_t) = 1$ otherwise.

— Q4a Write the second-order clique potential for this Ising model as a function of $\Delta(x_s, x_t)$ where $x_s$ and $x_t$ are the classes of neighbouring pixels $s$ and $t$ in 4-connexity and the regularisation parameter $\beta$. This model will be 0 when the two neighbouring pixels are equal and $+\beta$ otherwise.

Write the *global* energy of the whole field and the local conditional energy for a site $s$ using the results previously established for the data attachment energy and the regularization energy defined previously.

Reminder : the global energy contains all the clique potentials in the image, the local conditional energy at a site $s$ contains only the clique potentials that contain $s$.

Tip : the energy is defined to within one additive constant and one multiplicative constant (the minimum of K+K'U is equivalent to the minimum of U). It is better to simplify the writing of the energy as much as possible in order to do the programming afterwards.

— Q4b Global energy :

— Q4c Local conditional energy :

$$U_{\mathrm{attdo}} = \sum_{s} \left( \frac{(y_s - \mu_{x_s})^2}{2\sigma^2} + \ln(\sqrt{2\pi\sigma^2}) \right)$$

— Q5 Write the local conditional energies for classes 0 and 1 of the central pixel, using the following local neighbourhood configuration : neighbours in states 0, 1, 1, 1, and assuming that the grey level of the pixel is $y_s = 105$, and using the mean and variance values found previously.

class 0:

U(0 I 105, 0, 1, 1, 1) = (105 - 96.97)^2 + beta*(0+1+1+1) = 64,4809 + 3beta

class 1:

U(0 I 105, 0, 1, 1, 1) = (105 - 164.05)^2 + beta*(1+0+0+0) = 3.486,9025 + beta

— Q6 In which class will this pixel be put if it is assigned the class that locally minimises energy ?

— Q7 Considering the *global* energy of the field, what is the solution $x$ when $\beta$ is 0 ?

When β=0, the global energy of the field simplifies because the regularization term is completely removed.
We could interpret that considering that when β=0 pixels are classified solely based on their individual intensities relative to the class distributions.
The result in this case may be noisy, especially in regions with high intensity variation, because neighboring pixels can be assigned different labels without penalty.

— Q8 Considering the *global* energy of the field, what is the solution $x$ when $\beta$ is $+\infty$ ?

— Q9 How will the solution vary when $\beta$ increases? Comment on the interest of this Markovian model.

In this case the influence of the regularization term becomes stronger, so the solution favors regions where neighboring pixels share the same label, creating larger homogeneous segments. This reduces noise and avoids overly pixelated or noisy segmentations, especially in areas where the data term alone might lead to isolated classifications.
If β becomes too large we could have over-smoothing: distinct objects or regions with different intensities would be merged into a single label, losing important details.
The Markovian model introduces spatial context into the segmentation process, making it more robust compared to pixel-wise classification. It is highly effective for real-world segmentation tasks because it balances data fidelity with spatial regularity, but as β increases, boundaries between regions may blur. and small objects or details can be lost.

# 3 Optimization by ICM algorithm

We will optimise the global energy defined above, using the ICM (Iterated Conditional Modes) algorithm which consists of minimising the local conditional energy of the pixels one after the other, starting from a good initialisation of the classes. This algorithm converges to a local minimum but is very fast.

Complete the function to program the ICM, taking into account the data attachment term you have learned.

— Q10 How can we choose a good initialization of the solution? Justify your answer.

— Q11 With what value of $\beta$ do you get a good solution (i.e. the closest to the given "ideal" image "IoriginaleBW.png")? Compare this result with the result of the optimal thresholding.

Considering the file .ipynb of Angelo Minieri we get a good result with β=1800 and after ICM the noise has been reduced significantly compared to the results of the optimal thresholding. Moreover the regions of black and white pixels are much smoother and better defined.

— Q12 Try with other initialisations (with a constant image, with a random image). Comment on their influence.

# 4 Optimization by simulated annealing

Program the function of the simulated annealing which allows to update an image by sampling with the Gibbs distribution a posteriori with a fixed temperature T.

— Q13 Compare the results obtained by the Iterated Conditional Modes algorithm and by simulated annealing. Do you observe the expected results of the course ?

The answer depends on the parameters chosen and, since we chose different parameters, we'll write the answer on the file .ipynb.

Considering the file .ipynb of Angelo Minieri the simulated annealing result show a clear improvement over the ICM algorithm. In this case in fact the initial high temperature allows the algorithm to explore the solution space more thoroughly, avoiding poor local minima. This leades to a segmentation that is smoother, with far fewer misclassified pixels and improved preservation of thin structures and fine details, particularly in noisy regions.
ICM in contrast converge deterministically and this casuses it to get stuck in local minima. This results in noisier segmentation with isolated errors and poorer boundary preservation.
The compariosn confirms the theoretical expectations: Simulated Annealing, especially with an appropriately high starting temperature and gradual cooling, produces a segmentation closer to the global optimum, while ICM trades accuracy for speed and simplicity.