



# Using high performance libraries

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# 7 Motifs in HPC..

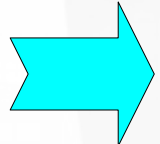
Phil Colella (LBL) identified **7 kernels** of which most simulation and data analysis program are composed:

- Dense Linear Algebra
  - Ex: solve  $Ax=B$  or  $Ax=\lambda x$  where  $A$  is a dense matrix
- Sparse Linear Algebra
  - Ex: solve  $Ax=B$  or  $Ax=\lambda x$  where  $A$  is a sparse matrix (mostly zero)
- Operation on structured Grids:
  - Ex:  $ANEW_j() = 4 * (A(i,j) - A(i-1,j) - A(i+1,j) - A(i,j-1) - A(i,j+1))$
- Operation on unstructured Grids:
  - Ex; similar but list of neighbours varies from entry to entry
- Spectral Methods
  - Ex: Fast Fourier Transform (FFT)
- Particle Methods
  - Ex: Compute electrostatic forces on  $n$ -particles
- Monte Carlo
  - Ex: many independent simulation using different inputs

**Where should you start optimizing your application ?**

# Optimization Techniques

- There are basically three different categories:
  - Improve memory performance (the most important)
  - Improve CPU performance
  - Use already highly optimized libraries/subroutines
    - The easiest and more efficient way.. 



# What are Performance libraries ?

- Routines for common (math) functions such as vector and matrix operations, fast Fourier transform etc. written in a specific way to take advantage of capabilities of the CPU.
- Each CPU type normally has its own version of the library specifically written or compiled to maximally exploit that architecture

# What are Performance libraries ?

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# Why use performance libraries ?



- Compilers can optimize code only to a certain point. Effective programming needs deep knowledge of the platform
- Performance libraries are designed to use the CPU in the most efficient way, which is not necessarily the most straightforward way.
- It is normally best to use the libraries supplied by or recommended by the CPU vendor
- On modern hardware they are hugely important, as they most efficiently exploit caches, special instructions and parallelism

# Any other reason apart from optimization ?

- Usage of libraries
  - Make coding easier. Complicated modules can be used from existing routines
  - Increase portability of code as standard libraries exist for ALL computing platforms (well optimized)
- Lego approach: build your own code using already available bricks..

we dont want just to optimize the code, we want to make the code easy, save time dont reinvent the wheel. Increase portability, by changing library knowing library allows to use lego approach



# What is available ?

- Linear Algebra: BLAS/LAPACK/SCALAPACK
  - FFT:
    - FFTW
  - ODE/PDE
    - PETSC
  - Machine Learning
    - Tensorflow
- etc.

FFT of the West

Ordinary  
Differential  
Equations/  
partial  
differential  
equations  
Integrate heavy  
PDE  
  
DL2

thousands of  
frameworks a lot  
of stuff:  
tensorflow  
caffe, torch  
pitorch  
framework  
where to build a  
machine  
learning library  
Orwood  
parallelizing  
tensorflow

a lot of libraries  
available

scalable  
pack(paralllel  
approach)

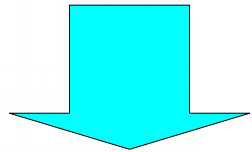
linear algebra  
package: almost  
linear algrbra  
partition

# Should I write my own algorithm for L. A. ?

99.99% of time NO

- Tons of libraries out there
- Well tested
- Extremely efficient in 99.99% of the case
- With some “de facto” standard implemented

Allows you to be  
portable, adapt  
the code



PORTABILITY IS COMING (almost) FOR FREE

# Why Linear algebra ?

## 3 Basic Linear Algebra Problems in

1. Linear Equations: Solve  $Ax=b$  for  $x$
2. Least Squares: Find  $x$  that minimizes  $\|r\|_2 \equiv \sqrt{\sum r_i^2}$  where  $r=Ax-b$ 
  - Statistics: Fitting data with simple functions
- 3a. Eigenvalues: Find  $\lambda$  and  $x$  where  $Ax = \lambda x$ 
  - Vibration analysis, e.g., earthquakes, circuits
- 3b. Singular Value Decomposition:  $A^T Ax = \sigma^2 x$ 
  - Data fitting, Information retrieval

Lots of variations depending on structure of  $A$

- $A$  symmetric, positive definite, banded, ...

# Why dense Linear Algebra ?

majority of  
algorithms are  
well defined in  
dense linear  
algebra

- Many large matrices are sparse, but ...
  - Dense algorithms easier to understand
  - Some applications yields large dense matrices
  - LINPACK Benchmark ([www.top500.org](http://www.top500.org))
    - “How fast is your computer?” =  
“How fast can you solve dense  $Ax=b$ ?”
- Large sparse matrix algorithms often yield smaller (but still large) dense problems

large sparse  
matrix,  
operations  
sparpack sparse

dense problem  
problem solved  
by dense  
matrices

# BLAS: **B**asic **L**inear **A**lgebra **S**ubprograms

# BLAS history (1/3)

- In the beginning it was libraries like **EISPACK** (for eigenvalue problems)
- Then the **BLAS-1** were invented (1973-1977)
  - Create a standard library of 15 operations (mostly) on vectors need a standard library to create a
  - “AXPY” ( $y = \alpha \cdot x + y$ ), dot product, scale ( $x = \alpha \cdot x$ ), etc
  - Up to 4 versions of each (S/D/C/Z), 46 routines, 3300 LOC
  - **Language: FORTRAN**
    - single,  
double, complex, z
- Goals
  - Common “pattern” to ease portability
    - no dynamic allocation
    - no pointers
  - Robustness, via careful coding (avoiding over/underflow) --> Accuracy
  - Portability (common interface)
  - Efficiency via machine specific implementations
  - Maintainability
- Why **BLAS-1**? They do  $O(n)$  ops on  $O(n)$  data
  - Used in libraries like LINPACK (for linear systems)
  - Source of the name “LINPACK Benchmark” (not the code!)
    - not important

# BLAS history (2/3)

- But the BLAS-1 weren't enough
  - Consider AXPY (  $y = \alpha \cdot x + y$  ):  $2n$  flops on  $3n$  read/writes
  - Computational intensity =  $(2n)/(3n) = 2/3$  poor intensity
  - Too low to run near peak speed (read/write dominates)
- So the BLAS-2 were developed (1984-1986) benchmark to read/write for memory
  - Standard library of 25 operations (mostly) on matrix/vector pairs
  - “general matrix vector operations” =  $\alpha \cdot A \cdot x + \beta \cdot x$ , “GER”:  $A = A + \alpha \cdot x \cdot y^T$ ,  $x = T^{-1} \cdot x$
  - Up to 4 versions of each (S/D/C/Z), 66 routines, 18K LOC
- Why **BLAS-2** ?
  - They do  $O(n^2)$  ops on  $O(n^2)$  data
  - So computational intensity still just  $\sim (2n^2)/(n^2) = 2$
  - OK for vector machines, but not for machine with caches computational intensity is the number of operations im

# BLAS history (3/3)

- The next step: BLAS-3 (1987-1988)
  - Standard library of 9 operations (mostly) on matrix/matrix pairs
    - “GEMM”:  $C = \alpha \cdot A \cdot B + \beta \cdot C$ ,  $C = \alpha \cdot A \cdot A^T + \beta \cdot C$ ,  $C = T^{-1} \cdot B$   
general matrix matrix operation
  - Up to 4 versions of each (S/D/C/Z), 30 routines, 10K LOC
  - Why BLAS 3? They do  $O(n^3)$  ops on  $O(n^2)$  data
  - So computational intensity  $(2n^3)/(4n^2) = n/2$  – big at last!
    - Good for machines with caches, other mem. hi
  - Performing implementations left to others..

here i am  
bounded by the  
cpu activity,  
doing cpu  
operations and  
not only memory  
access

Vendor has to  
choose the  
specific routine



# Where are BLAS ?

<http://www.netlib.org/blas>

- Source: 142 routines, 31K LOC,
- Testing: 28K LOC
- Reference (unoptimized) implementation only !
  - [http://www.netlib.org/blas/#\\_reference\\_blas\\_version\\_3\\_5\\_0](http://www.netlib.org/blas/#_reference_blas_version_3_5_0)
  - Ex: 3 nested loops for GEMM

reference  
implementations  
, not so much  
performance

three nested  
loops you can  
write

# BLAS list

quick look

## Level 1 BLAS

	dim	scalar	vector	vector	scalars	5-element array
SUBROUTINE xROTG (					A, B, C, S)	
SUBROUTINE xROTMG(					D1, D2, A, B,	PARAM )
SUBROUTINE xROT ( N,			I, INCX, Y, INCY,		C, S)	
SUBROUTINE xROTH ( N,			I, INCX, Y, INCY,			PARAM )
SUBROUTINE xSWAP ( N,			I, INCX, Y, INCY )			
SUBROUTINE xSCAL ( N,		ALPHA,	I, INCX )			
SUBROUTINE xCOPY ( N,			I, INCX, Y, INCY )			
SUBROUTINE xAXPY ( N,		ALPHA,	I, INCX, Y, INCY )			
FUNCTION xDOT ( N,			I, INCX, Y, INCY )			
FUNCTION xDOTU ( N,			I, INCX, Y, INCY )			
FUNCTION xDOTC ( N,			I, INCX, Y, INCY )			
FUNCTION xxDOT ( N,			I, INCX, Y, INCY )			
FUNCTION xRNRM2 ( N,			I, INCX )			
FUNCTION xASUM ( N,			I, INCX )			
FUNCTION IxAMAX( N,			I, INCX )			

Generate plane rotation  
 Generate modified plane rotation  
 Apply plane rotation  
 Apply modified plane rotation  
 $x \leftrightarrow y$   
 $x \leftarrow \alpha x$   
 $y \leftarrow x$   
 $y \leftarrow \alpha x + y$   
 $dot \leftarrow x^T y$   
 $dot \leftarrow x^T y$   
 $dot \leftarrow x^H y$   
 $dot \leftarrow \alpha + x^T y$   
 $norm2 \leftarrow \|x\|_2$   
 $asum \leftarrow \|re(x)\|_1 + \|im(x)\|_1$   
 $amax \leftarrow 1^{*}k \ni |re(x_k)| + |im(x_k)|$   
 $\quad = \max\{|re(x_k)| + |im(x_k)|\}$

prefixes  
 S, D  
 S, D  
 S, D  
 S, D  
 S, D, C, Z  
 S, D, C, Z, CS, ZD  
 S, D, C, Z  
 S, D, C, Z  
 S, D, DS  
 C, Z  
 C, Z  
 SDS  
 S, D, SC, DZ  
 S, D, SC, DZ  
 S, D, C, Z

## Level 2 BLAS

options	dim	b-width	scalar	matrix	vector	scalar	vector
xGEMV ( TRANS,	M, N,		ALPHA, A, LDA,	X, INCX,	BETA, Y, INCY )		
xGEMV ( TRANS,	M, N, KL, KU,		ALPHA, A, LDA,	X, INCX,	BETA, Y, INCY )		
xHEMV ( UPLO,	N,		ALPHA, A, LDA,	X, INCX,	BETA, Y, INCY )		
xSBMV ( UPLO,	N, K,		ALPHA, A, LDA,	X, INCX,	BETA, Y, INCY )		
xSPMV ( UPLO,	N,		ALPHA, AP,	X, INCX,	BETA, Y, INCY )		
xSYMV ( UPLO,	N,		ALPHA, A, LDA,	X, INCX,	BETA, Y, INCY )		
xSBMV ( UPLO,	N, K,		ALPHA, A, LDA,	X, INCX,	BETA, Y, INCY )		
xSPMV ( UPLO,	N,		ALPHA, AP,	X, INCX,	BETA, Y, INCY )		
xTRMV ( UPLO, TRANS, DIAG,	N,		A, LDA, X, INCX )				
xTRMV ( UPLO, TRANS, DIAG,	N, K,		A, LDA, X, INCX )				
xTPMV ( UPLO, TRANS, DIAG,	N,		AP, X, INCX )				
xTRSV ( UPLO, TRANS, DIAG,	N,		A, LDA, X, INCX )				
xTRSV ( UPLO, TRANS, DIAG,	N, K,		A, LDA, X, INCX )				
xTPSV ( UPLO, TRANS, DIAG,	N,		AP, X, INCX )				
options	dim	scalar	vector	matrix	matrix	scalar	matrix
xGER (	M, N,	ALPHA, X, INCX,	Y, INCY,	A, LDA )			
xGERU (	M, N,	ALPHA, X, INCX,	Y, INCY,	A, LDA )			
xGERC (	M, N,	ALPHA, X, INCX,	Y, INCY,	A, LDA )			
xHER ( UPLO,	N,	ALPHA, X, INCX,		A, LDA )			
xHPR ( UPLO,	N,	ALPHA, X, INCX,		AP )			
xHER2 ( UPLO,	N,	ALPHA, X, INCX,	Y, INCY,	A, LDA )			
xHPR2 ( UPLO,	N,	ALPHA, X, INCX,	Y, INCY,	AP )			
xSYR ( UPLO,	N,	ALPHA, X, INCX,		A, LDA )			
xSPR ( UPLO,	N,	ALPHA, X, INCX,		AP )			
xSYR2 ( UPLO,	N,	ALPHA, X, INCX,	Y, INCY,	A, LDA )			
xSPR2 ( UPLO,	N,	ALPHA, X, INCX,	Y, INCY,	AP )			

$y \leftarrow \alpha Ax + \beta y, y \leftarrow \alpha A^T x + \beta y, y \leftarrow \alpha A^H x + \beta y, A - m \times n$   
 $y \leftarrow \alpha Ax + \beta y, y \leftarrow \alpha A^T x + \beta y, y \leftarrow \alpha A^H x + \beta y, A - m \times n$   
 $y \leftarrow \alpha Ax + \beta y$   
 $y \leftarrow \alpha Ax + \beta y$   
 $y \leftarrow \alpha Ax + \beta y$   
 $y \leftarrow \alpha Ax + \beta y$   
 $y \leftarrow \alpha Ax + \beta y$   
 $y \leftarrow \alpha Ax + \beta y$   
 $x \leftarrow Ax, x \leftarrow A^T x, x \leftarrow A^H x$   
 $x \leftarrow Ax, x \leftarrow A^T x, x \leftarrow A^H x$   
 $x \leftarrow Ax, x \leftarrow A^T x, x \leftarrow A^H x$   
 $x \leftarrow A^{-1} x, x \leftarrow A^{-T} x, x \leftarrow A^{-H} x$   
 $x \leftarrow A^{-1} x, x \leftarrow A^{-T} x, x \leftarrow A^{-H} x$   
 $x \leftarrow A^{-1} x, x \leftarrow A^{-T} x, x \leftarrow A^{-H} x$   
 $A \leftarrow \alpha xy^T + A, A - m \times n$   
 $A \leftarrow \alpha xy^T + A, A - m \times n$   
 $A \leftarrow \alpha xy^H + A, A - m \times n$   
 $A \leftarrow \alpha xx^H + A$   
 $A \leftarrow \alpha xx^H + A$   
 $A \leftarrow \alpha xy^H + y(\alpha x)^H + A$   
 $A \leftarrow \alpha xy^H + y(\alpha x)^H + A$   
 $A \leftarrow \alpha xx^T + A$   
 $A \leftarrow \alpha xx^T + A$   
 $A \leftarrow \alpha xy^T + \alpha yx^T + A$   
 $A \leftarrow \alpha xy^T + \alpha yx^T + A$

S, D, C, Z  
 S, D, C, Z  
 C, Z  
 C, Z  
 C, Z  
 S, D  
 S, D  
 S, D  
 S, D, C, Z  
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 S, D, C, Z  
 S, D, C, Z  
 S, D, C, Z  
 S, D, C, Z  
 S, D  
 C, Z  
 C, Z  
 C, Z  
 C, Z  
 C, Z  
 S, D  
 S, D  
 S, D  
 S, D

## Level 3 BLAS

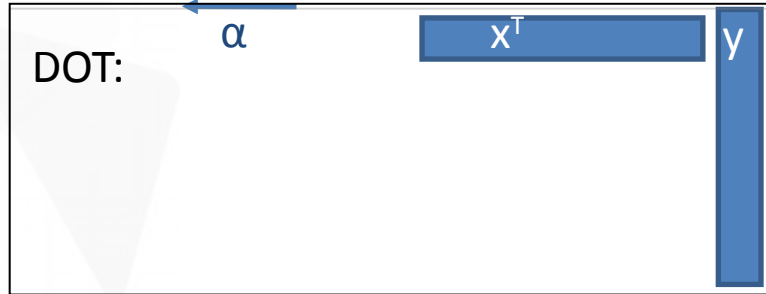
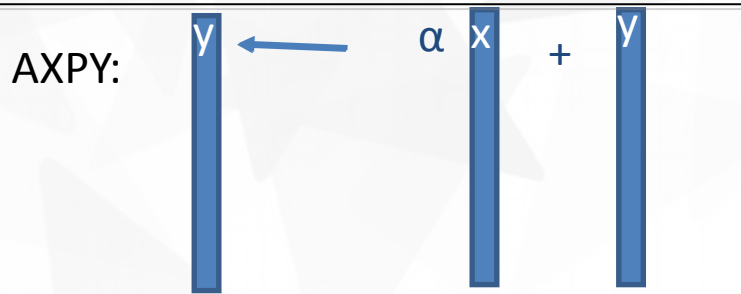
options	dim	scalar	matrix	matrix	scalar	matrix
xGEMM ( TRANSA, TRANSB,	M, N, K,	ALPHA, A, LDA,	B, LDB,	BETA, C, LDC )		
xSYMM ( SIDE, UPLO,	M, N,	ALPHA, A, LDA,	B, LDB,	BETA, C, LDC )		
xHEMM ( SIDE, UPLO,	M, N,	ALPHA, A, LDA,	B, LDB,	BETA, C, LDC )		
xSYRK ( UPLO, TRANS,	N, K,	ALPHA, A, LDA,		BETA, C, LDC )		
xHERK ( UPLO, TRANS,	N, K,	ALPHA, A, LDA,		BETA, C, LDC )		
xSYR2X( UPLO, TRANS,	N, K,	ALPHA, A, LDA,	B, LDB,	BETA, C, LDC )		
xHER2X( UPLO, TRANS,	N, K,	ALPHA, A, LDA,	B, LDB,	BETA, C, LDC )		
xTRMM ( SIDE, UPLO, TRANSA,	DIAG, M, N,	ALPHA, A, LDA,	B, LDB )			
xTRSM ( SIDE, UPLO, TRANSA,	DIAG, M, N,	ALPHA, A, LDA,	B, LDB )			

$C \leftarrow \alpha op(A)op(B) + \beta C, op(X) = X, X^T, X^H, C - m \times n$   
 $C \leftarrow \alpha AB + \beta C, C \leftarrow \alpha BA + \beta C, C - m \times n, A = A^T$   
 $C \leftarrow \alpha AB + \beta C, C \leftarrow \alpha BA + \beta C, C - m \times n, A = A^H$   
 $C \leftarrow \alpha AA^T + \beta C, C \leftarrow \alpha A^T A + \beta C, C - n \times n$   
 $C \leftarrow \alpha AA^H + \beta C, C \leftarrow \alpha A^H A + \beta C, C - n \times n$   
 $C \leftarrow \alpha AB^T + \alpha BA^T + \beta C, C \leftarrow \alpha A^T B + \alpha B^T A + \beta C, C - n \times n$   
 $C \leftarrow \alpha AB^H + \alpha BA^H + \beta C, C \leftarrow \alpha A^H B + \alpha B^H A + \beta C, C - n \times n$   
 $B \leftarrow \alpha op(A)B, B \leftarrow \alpha Bop(A), op(A) = A, A^T, A^H, B - m \times n$   
 $B \leftarrow \alpha op(A^{-1})B, B \leftarrow \alpha Bop(A^{-1}), op(A) = A, A^T, A^H, B - m \times n$

S, D, C, Z  
 S, D, C, Z  
 C, Z  
 S, D, C, Z  
 C, Z  
 S, D, C, Z  
 S, D, C, Z  
 S, D, C, Z  
 S, D, C, Z  
 S, D, C, Z

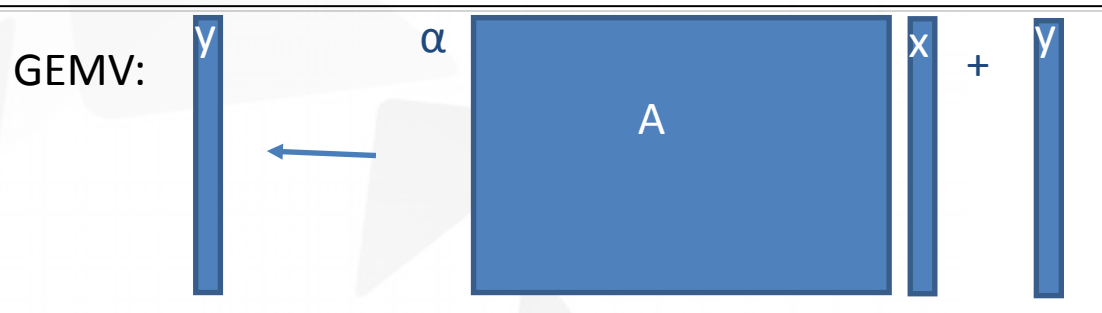
# Level 1, 2 and 3 BLAS

## Level 1 BLAS Vector-Vector operations



**2n FLOP**  
**2n memory reference**  
**RATIO: 1**

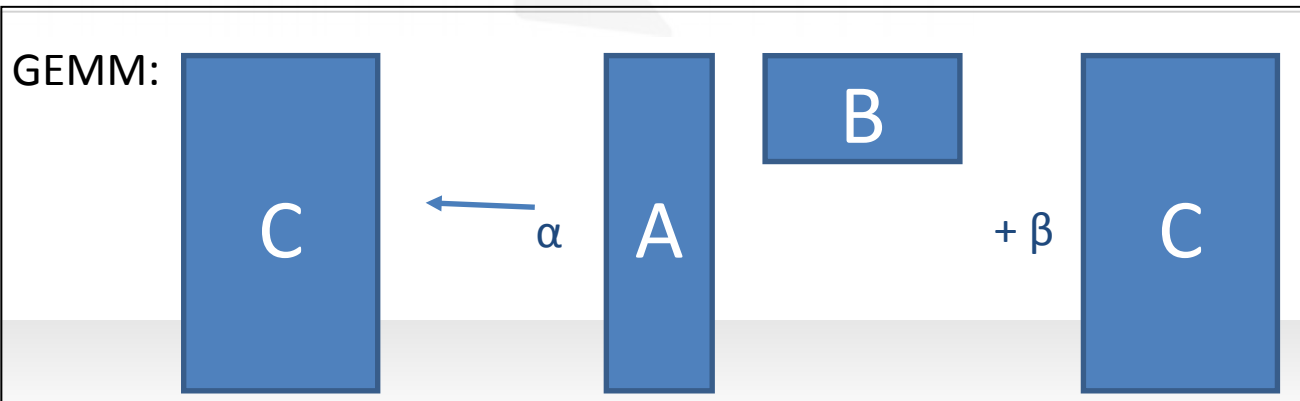
## Level 2 BLAS Matrix-Vector operations



quick look

**2n<sup>2</sup> FLOP**  
**n<sup>2</sup> memory references**  
**RATIO: 2**

## Level 3 BLAS Matrix-Matrix operations



**2n<sup>3</sup> FLOP**  
**4n<sup>2</sup> memory references**  
**RATIO: 2 n**

# Why BLAS so important?

- Because the BLAS are **efficient**, **parallel**, and **widely available**, they are common in the development of high quality linear algebra software.
- Performance of lot of applications depends a lot on the performance of the underlying BLAS.

numpy library  
for numerical  
operations on  
array, used for  
satellite  
computation,  
time-constraint  
problem, on  
their cluster

600 seconds  
numpy  
implementation  
was the  
reference  
implementation  
of blas  
reinstalled  
numpy.  
at the end it was  
250 seconds  
used OpenBLAS  
install OpenBlas  
most efficeint  
library for this  
architecture

# Standardization (BLAS example)

- Each BLAS Subroutines have a standardized layout
- BLAS is documented in the source code
- Man pages exist
- Vendor supplied docs
- Different BLAS implementations have the same calling sequence

```
SUBROUTINE DGEAM ( TRANSX, TRANSY, M, N, K, ALPHA, A, LDA, B, LDB,
#      BETA, C, LDC )
* .. SCALAR ARGUMENTS ..
* CHARACTER*1      TRANSX, TRANSY
* INTEGER          M, N, K, LDA, LDB, LDC
* DOUBLE PRECISION ALPHA, BETA
* .. ARRAY ARGUMENTS ..
* DOUBLE PRECISION A( LDA, * ), B( LDB, * ), C( LDC, * )
*
* ..
*
* PURPOSE
* =====
*
* DGEAM PERFORMS ONE OF THE MATRIX-MATRIX OPERATIONS
*
*   C := ALPHA*OP( A )*OP( B ) + BETA*C,
*
* WHERE OP( X ) IS ONE OF
*
*   OP( X ) = X   OR   OP( X ) = X',
*
* ALPHA AND BETA ARE SCALARS, AND A, B AND C ARE MATRICES, WITH OP( A )
* AN M BY K MATRIX, OP( B ) A K BY N MATRIX AND C AN M BY N MATRIX.
*
* PARAMETERS
* =====
*
* TRANSX - CHARACTER*1.
* ON ENTRY, TRANSX SPECIFIES THE FORM OF OP( A ) TO BE USED IN
* THE MATRIX MULTIPLICATION AS FOLLOWS:
*
*   TRANSX = 'N' OR 'N', OP( A ) = A,
*   TRANSX = 'T' OR 'T', OP( A ) = A',
*   TRANSX = 'C' OR 'C', OP( A ) = A'.
*
* UNCHANGED ON EXIT.
*
* TRANSY - CHARACTER*1.
* ON ENTRY, TRANSY SPECIFIES THE FORM OF OP( B ) TO BE USED IN
* THE MATRIX MULTIPLICATION AS FOLLOWS:
*
*   TRANSY = 'N' OR 'N', OP( B ) = B,
*   TRANSY = 'T' OR 'T', OP( B ) = B',
```

man pages  
blas given by  
netlib use  
openBLAS

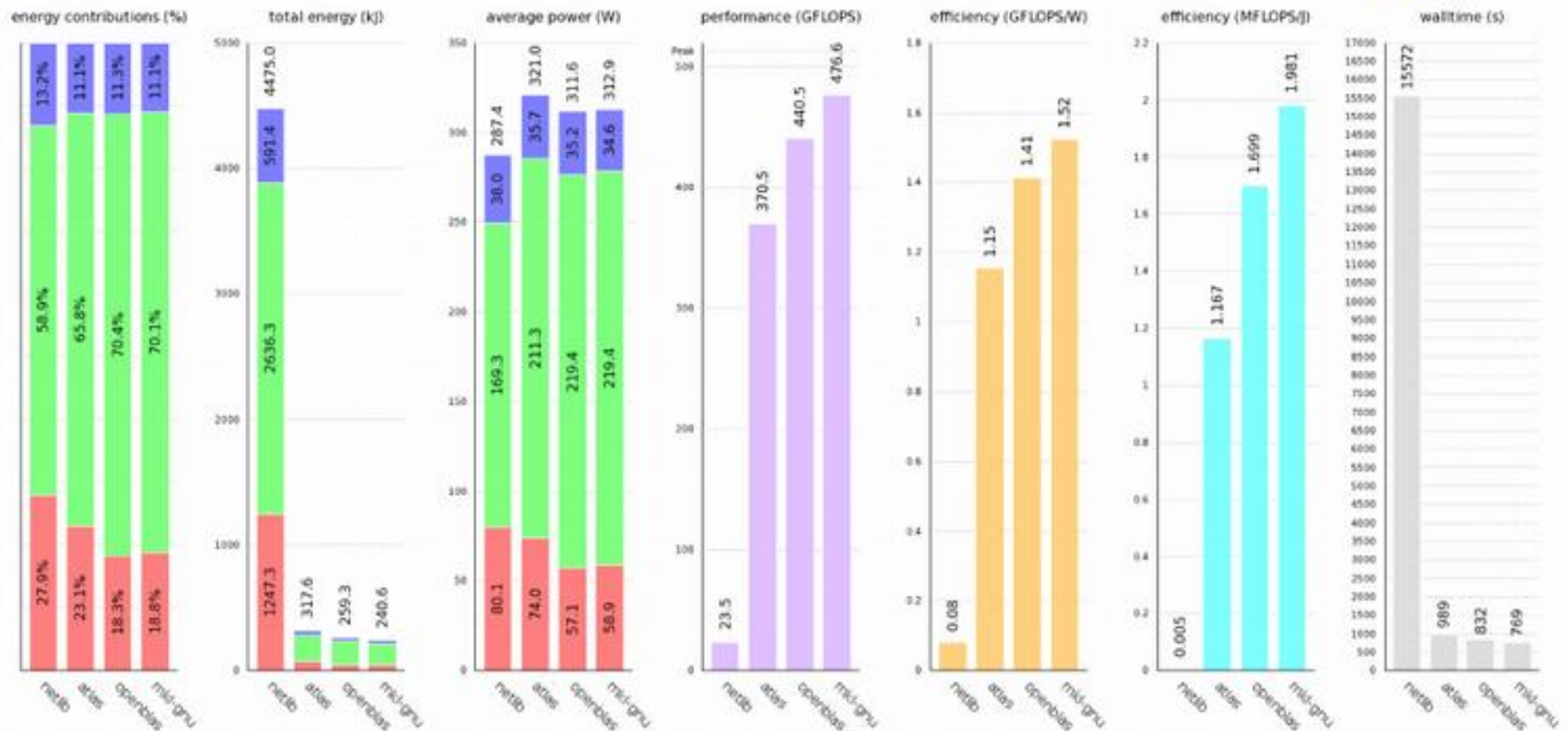
# Vendor/Optimized BLAS libraries

- ACML
  - The AMD Core Math Library, supporting the AMD processors
- **ATLAS**
  - Automatic Tuning of Linear Algebra Software: the start of the century: it tests the hardware and auto tunes itself performs well
- **Intel MKL**
  - The Intel Math Kernel Library, supporting x86 32-bits and 64-bits, includes optimizations for Intel Pentium, Core and Intel Xeon CPUs and Intel Xeon Phi; support for Linux, Windows and Mac OS X
- **cuBLAS**
  - Optimized for a large matrix matrix multiplication on the GPU
- cIBLAS
  - An OpenCL implementation of BLAS
- ESSL
  - IBM's power 9 machines with 4 g and Scientific Subroutines Library, supporting the PowerPC architecture under AIX and Linux
- GotoBLAS
  - Kazushige Goto's BSD-licensed implementation of BLAS, tuned in particular for Intel Itanium/Atom, VIA Nanoprocessor and AMD Opteron
- BLIS
  - BLAS-like framework for instantiation Software Highly optimized version of blas routines
- **OpenBLAS**
  - Optimized find GOTO based on Goto BLAS hosted a BLAS faster than openBLAS platform and other



# Blas efficiency: (from Moreno B. MHPC's thesis)

HPL power consumption on an Aurora node @COSINT using different BLAS libraries



standard  
implementations  
the higher the  
best  
mkl is the best

# What about my C++/C program ??

- BLAS routines are Fortran-style, when calling them from C-language programs, follow the Fortran-style calling conventions:
  - Pass variables by address, not by value.
  - Store your data in Fortran style, that is, column-major rather than row-major order.
- be aware that because the Fortran language is case-insensitive, the routine names can be both upper-case or lower-case, with or without the trailing underscore. For example, the following names are equivalent:  
    dgemm, DGEMM, dgemm\_, and DGEMM\_



# Use CBLAS

- C-style interface to the BLAS routines (<http://www.netlib.org/blas/blast-forum/cblas.tgz>)
- You can call CBLAS routines using regular C-style calls.
- The header file specifies enumerated values and prototypes of all the functions.
- For details and examples:

<https://software.intel.com/en-us/mkl-tutorial-c-multiplying-matrices-using-dgemm>

# Efficiency: q parameter (aka computational efficiency)

Table 2: Basic Linear Algebra Subroutines (BLAS)

Operation	Definition	Floating point operations	Memory references	$q$
<b>saxpy</b>	$y_i = \alpha x_i + y_i, i = 1, \dots, n$	$2n$	$3n + 1$	$2/3$
<b>Matrix-vector mult</b>	$y_i = \sum_{j=1}^n A_{ij}x_j + y_i$	$2n^2$	$n^2 + 3n$	$2$
<b>Matrix-matrix mult</b>	$C_{ij} = \sum_{k=1}^n A_{ik}B_{kj} + C_{ij}$	$2n^3$	$4n^2$	$n/2$

The parameter  $q$  is the ratio of flops to memory references.  
Generally:

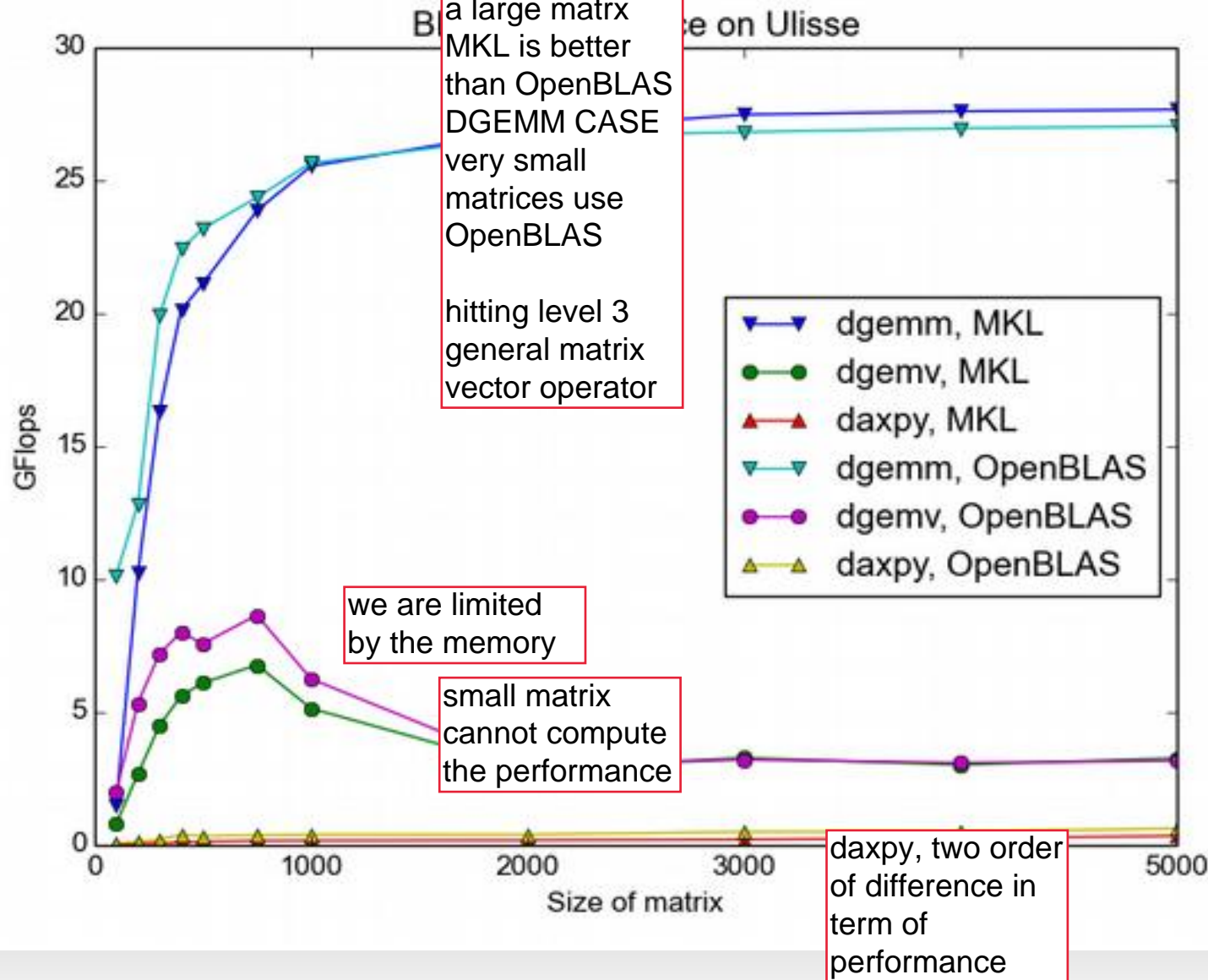
1. Larger values of  $q$  maximize useful work to time spent moving data.
2. The higher the level of the BLAS, the larger  $q$ .

# It follows..

- BLAS1 are memory bounded ! (for each computation a memory transfer is required)
- BLAS2 are not so memory bounded (can have good performance on super-scalar architecture)
- BLAS3 can be very efficient on super-scalar computers because **not memory bounded**

im bounded to  
memory too  
many memory  
access

# Blas Performance on Ulysse



# Proposed Exercise

- Create the previous graph for the HW/software we are using:
  - Using MKL
  - Using OpenBLAS
  - On a Ulisse old/new nodes
- Steps:
  - Install, if missing needed libraries in your directory
  - Write a small program to call the three routines
  - Write a script to collect all sizes of interest
  - Make nice plots

# Basic Linear Algebra Subroutines

Name	Description	Examples
Level-1 BLAS	Vector Operations	$C = \sum X_i Y_i$
Level-2 BLAS	Matrix-Vector Operations	$B_i = \sum_k A_{ik} X_k$
Level-3 BLAS	Matrix-Matrix Operations	$C_{ij} = \sum_k A_{ik} B_{kj}$

# How to link optimized libraries?

- Reference implementation: order matters !
  - LAPACK uses BLAS
  - => `-L/usr/local/lib -llapack -lblas`
- OpenBLAS:
  - Automatically includes lapack reference implementation so no need to specify anything else. Please check !
- ATLAS is written C with f77 wrappers:
  - `-L/opt/atlas/lib -lf77blas -latlas`
- MKL:
  - Generally complex and highly dependent on version and/or HW/SW implementation

<https://software.intel.com/en-us/articles/intel-mkl-link-line-advisor>

## Exercise 2: Running HPL on our clusters

- Check the README on github account



# A few notes:

- Standard input file should be present
- Beware of threads
  - How to control them ?

need an input file  
if you use mkl library it is  
multithreaded  
if parallel application uses  
multithreaded library  
check number of each thread

```
mpirun -np 24 hpl_command
```

hpl has been linked against hpl  
we want to use HPL in a  
multithreaded approach check how  
many threads i have, i have 24  
threads for each processor 24 mpi  
processes 24 threads spawn by  
each

control by exporting the numbers of  
threads that i want fix OMP\_NUM  
THREAD

it is best to have 24 MPI and 1  
thread

or 1 mpi process and 24 thread

TWO COMBINATIONS  
on HPL or a standard DGEM  
routine

# What about N ?

- N should be large enough to take ~75% of RAM..
  - $N = \sqrt{0.75 * \text{Number of Nodes} * \text{Minimum memory of any node} / 8}$
- You can compute it via:
  - <http://www.advancedclustering.com/act-kb/tune-hpl-dat-file/>

# HPL benchmark input file HPL.dat

```

HPLinpack benchmark input file
Innovative Computing Laboratory, University of Tennessee
HPL.out      output file name (if any)
6            device out (6=stdout,7=stderr,file)
1            # of problems sizes (N)
50000 Ns
1            # of NBS
768 block size
0            PMAP process mapping (0=Row-,1=Column-major)
1            # of process grids (P x Q)
4 1 2 1      Ps number of processors we
4 2 2 4      Qs have
16.0         threshold
1            # of panel fact
0 1 2        PFACTs (0=left, 1=Crout, 2=Right)
1            # of recursive stopping criteria
2 8          NBMINs (>= 1)
1            # of panels in recursion
2            NDIVs
1            # of recursive panel fact.
0 1 2        RFACTs (0=left, 1=Crout, 2=Right)
1            # of broadcast
0 2          BCASTs (0=1rg,1=1rM,2=2rg,3=2rM,4=3rg,5=3rM)
1            # of lookahead depth
1 0          DEPTHs (>=0)
1            SWAP (0=bin-exch,1=long,2=mix)
192          swapping threshold
1            L1 in (0=transposed,1=no-transposed) form
1            U  in (0=transposed,1=no-transposed) form
1            Equilibration (0=no,1=yes)
8            memory alignment in double (> 0)

```

number of  
problem sizes

50000 Ns

1 # of NBS

768 block size

0 PMAP process mapping (0=Row-,1=Column-major)

1 # of process grids (P x Q)

4 1 2 1 Ps number of processors we

4 2 2 4 Qs have

16.0 threshold

1 # of panel fact

0 1 2 PFACTs (0=left, 1=Crout, 2=Right)

1 # of recursive stopping criteria

2 8 NBMINs (>= 1)

1 # of panels in recursion

2 NDIVs

1 # of recursive panel fact.

0 1 2 RFACTs (0=left, 1=Crout, 2=Right)

1 # of broadcast

0 2 BCASTs (0=1rg,1=1rM,2=2rg,3=2rM,4=3rg,5=3rM)

1 # of lookahead depth

1 0 DEPTHs (>=0)

1 SWAP (0=bin-exch,1=long,2=mix)

192 swapping threshold

1 L1 in (0=transposed,1=no-transposed) form

1 U in (0=transposed,1=no-transposed) form

1 Equilibration (0=no,1=yes)

8 memory alignment in double (> 0)

this the input

files 4

blocksize 2

processor grid

8 cpu 8 cores

processes

run 32

experiments

just 1 instance

of everything

mpirun

# Parameters for HPL.dat input file

very large matrix  
is spawned in

6 times 4 and 4  
times 6

<b>N</b>	Problem size	<b>Pmap</b>	Process mapping
<b>NB</b>	Blocking factor	<b>threshold</b>	for matrix validity test
<b>P</b>	Rows in process grid	<b>Ndiv</b>	Panels in recursion
<b>Q</b>	Columns in process grid	<b>Nbmin</b>	Recursion stopping criteria
<b>Depth</b>	Lookahead depth	<b>Swap</b>	Swap algorithm
<b>Bcasts</b>	Panel broadcasting method	<b>L1, U</b>	to store triangle of panel
<b>Pfacts</b>	Panel factorization method	<b>Align</b>	Memory alignment
<b>Rfacts</b>	Recursive factorization method	<b>Equilibration</b>	

# Tips to get performance..

- Figure out a good **block size (NB)** for the matrix multiply routine. The best method is to try a few out. If you happen to know the block size used by the matrix-matrix multiply routine, a small multiple of that block size will do fine. This particular topic is discussed in the FAQs section.
- The process mapping should not matter if the nodes of your platform are single processor computers. If these nodes are **multi-processors**, a row-major mapping is recommended.
- **HPL likes "square" or slightly flat process grids.** Unless you are using a very small process grid, stay away from the 1-by-Q and P-by-1 process grids.

6 times 4 instead of 4 times 6  
THE REcommendations

# What are you supposed to do ?

- Let us read together the readme file.



**Thank you ...**



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