

# Continuous Control With Deep Reinforcement Learning

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# Motivation

## Problem

1. While DQN solves problems with high-dimensional observation spaces, it can only handle discrete and low-dimensional action spaces
2. Large, non-linear function approximators for  $Q$  function has often been avoided since theoretical performance guarantees are impossible

→ *Using Deep Deterministic Policy Gradient method*

# Method

## Notation

$E$ : An Environment in discrete timesteps

$x_i$ : An observation received at each timestep  $t$

$R_t = \sum_{k=t}^T \gamma^{k-t} r(s_k, a_k)$ : Total Discount Return

$V^\pi(s) = E[R_t | S_1 = s; \pi]$ : Value function, Expected total discounted reward

$Q^\pi(s, a) = E[R_t | S_1 = s, A_1 = a; \pi]$ : Action Value Function

$\mu_\theta(s)$ : Deterministic target policy

$\mu'$ : Different behavior policy (in off-policy setting)

# Method

## Review & Background

\*Bellman Equation

$$Q^\pi(s_t, a_t) = \mathbb{E}_{r_t, s_{t+1} \sim \mathcal{E}}[r(s_t, a_t) + \gamma \mathbb{E}_{a_{t+1} \sim \pi}[Q^\pi(s_{t+1}, a_{t+1})]]$$

\*Bellman Equation with Deterministic policy

$$Q^\pi(s_t, a_t) = \mathbb{E}_{r_t, s_{t+1} \sim \mathcal{E}}[r(s_t, a_t) + \gamma \mathbb{E}_{a_{t+1} \sim \pi}[Q^\pi(s_{t+1}, \mu(s_{t+1}))]]$$

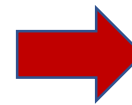
\*Critic Loss Function

$$L(\theta^Q) = \mathbb{E}_{\mu'}[(Q_\theta(s_t, a_t) - y_t)^2], \quad \text{s.t. } y_t = r(s_t, a_t) + \gamma Q_\theta(s_{t+1}, \mu(s_{t+1}))$$

Directly implement Q learning with  $L(\theta^Q)$  to be unstable. Q is also updated by target value  $y_t$  is also function of parameter  $\theta$ , but it is typically ignored

\*Off-Policy Deterministic Actor-Critic (OPDAC)

$$J_\beta(\mu_\theta) = \int_S \rho^\beta V^\mu(s) ds = \int_S \rho^\beta(s) Q^\mu(s, \mu_\theta(s)) ds$$
$$\nabla_\theta J_\beta(\mu_\theta) \approx \int_S \rho^\beta(s) \nabla_\theta \mu_\theta(a|s) Q^\mu(s, a) ds = \mathbb{E}_{s \sim \rho^\beta}[\nabla_\theta \mu_\theta(s) \nabla_a Q^\mu(s, a)|_{a=\mu_\theta(s)}]$$



$$\delta_t = r_t + \gamma Q^w(s_{t+1}, a_{t+1}) - Q^w(s_t, a_t)$$
$$w_{t+1} = w_t + \alpha_w \delta_t \nabla_w Q^w(s_t, a_t)$$
$$\theta_{t+1} = \theta_t + \alpha_\theta \nabla_\theta \mu_\theta(s_t) \nabla_a Q^w(s_t, a_t)|_{a=\mu_\theta(s)}$$

# Method

## Algorithm

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**Algorithm 1** DDPG algorithm

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Randomly initialize critic network  $Q(s, a|\theta^Q)$  and actor  $\mu(s|\theta^\mu)$  with weights  $\theta^Q$  and  $\theta^\mu$ .  
Initialize target network  $Q'$  and  $\mu'$  with weights  $\theta^{Q'} \leftarrow \theta^Q$ ,  $\theta^{\mu'} \leftarrow \theta^\mu$ .  
Initialize replay buffer  $R$   
**for** episode = 1, M **do**  
  Initialize a random process  $\mathcal{N}$  for action exploration  
  Receive initial observation state  $s_1$   
  **for** t = 1, T **do**  
    Select action  $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$  according to the current policy and exploration noise  
    Execute action  $a_t$  and observe reward  $r_t$  and observe new state  $s_{t+1}$   
    Store transition  $(s_t, a_t, r_t, s_{t+1})$  in  $R$   
    Sample a random minibatch of  $N$  transitions  $(s_i, a_i, r_i, s_{i+1})$  from  $R$   
    Set  $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$   
    Update critic by minimizing the loss:  $L = \frac{1}{N} \sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$   
    Update the actor policy using the sampled gradient:

$$\nabla_{\theta^\mu} \mu|_{s_i} \approx \frac{1}{N} \sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu} \mu(s|\theta^\mu)|_{s_i}$$

Update the target networks:

$$\begin{aligned}\theta^{Q'} &\leftarrow \tau \theta^Q + (1 - \tau) \theta^{Q'} \\ \theta^{\mu'} &\leftarrow \tau \theta^\mu + (1 - \tau) \theta^{\mu'}\end{aligned}$$

**end for**  
**end for**

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# Method

## Algorithm

### Update Actor

$$\nabla_{\theta} \mu_{\theta} \approx \mathbb{E}_{s \sim \mu'} [\nabla_{\theta} Q(s, a | \theta^Q) |_{s=s_t, a=\mu_{\theta}(s_t)}] = \mathbb{E}_{s \sim \mu'} [\nabla_{\theta} Q(s, a | \theta^Q) |_{s=s_t, a=\mu_{\theta}(s_t)}]$$

### Review – Deterministic Policy Gradient Theorem

$$J(\mu_{\theta}) = \int_s \rho^{\mu}(s) r(s, \mu_{\theta}(s)) ds = \mathbb{E}_{s \sim \rho^{\mu}} [r(s, \mu_{\theta}(s))]$$
$$\nabla_{\theta} J(\mu_{\theta}) = \int_s \rho^{\mu}(s) \nabla_{\theta}(s) \nabla_a Q^{\mu}(s, a) \Big|_{a=\mu_{\theta}(s)} ds = \mathbb{E}_{s \sim \rho^{\mu}} [\nabla_{\theta} \mu_{\theta}(s) \nabla_a Q^{\mu}(s, a) \Big|_{a=\mu_{\theta}(s)}]$$

### Behavior policy

$$\mu'(s_t) = \mu_{\theta_t}^{\mu}(s_t) + N, N \text{ is a noise process}$$

*Noise makes exploration on continuous action spaces*

### ‘Soft’ target update

$$\theta' = \tau \theta + (1 - \tau) \theta'$$

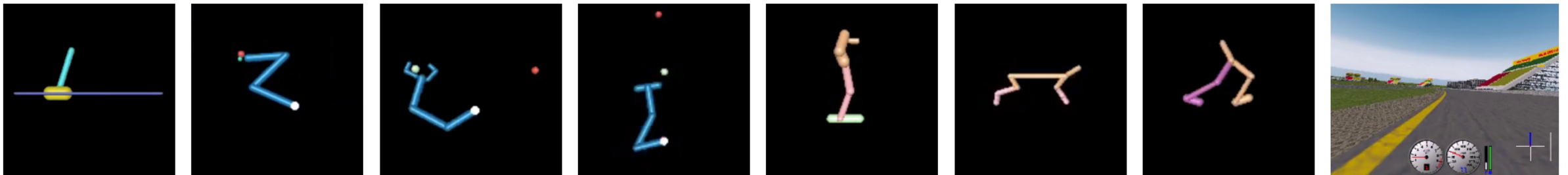
*Target values are constrained to change slowly, greatly improving the stability of learning*

# Experiments

## Method

1. Low-dimensional state and high-dimensional state
2. For each timestep of agent, repeating the agent's action and simulate 3 timesteps  
– 3\*RGB  $\rightarrow$  9 feature maps, Normalize RGB value into  $[0, 1]$
3. Evaluate the policy periodically (In inference, remove the noise  $\mathcal{N}$ )
4. Ablation study for target value and batch normalization
5. DPG vs DDPG
6. Q-Learning vs DDPG

## Results



# Experiments

## Results

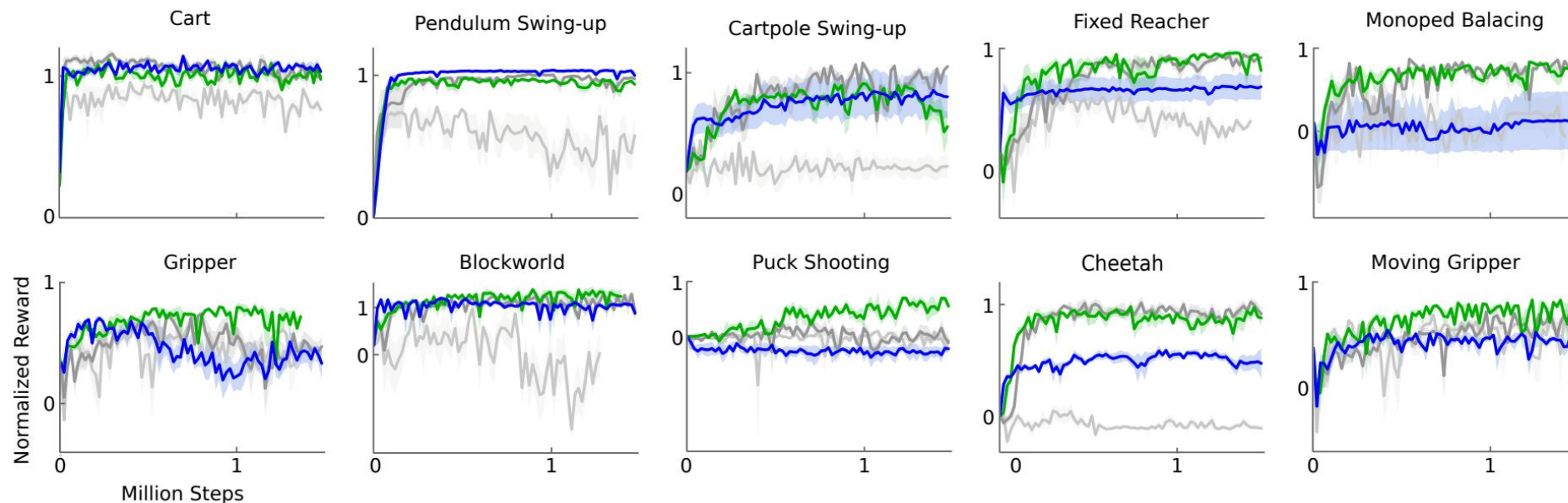


Figure 2: Performance curves for a selection of domains using variants of DPG: original DPG algorithm (minibatch NFQCA) with batch normalization (light grey), with target network (dark grey), with target networks and batch normalization (green), with target networks from pixel-only inputs (blue). Target networks are crucial.



# Experiments

## Results

| ddpg                   |               |                 |              | dpg with replay buffer and batchnorm |                |                  |
|------------------------|---------------|-----------------|--------------|--------------------------------------|----------------|------------------|
| environment            | $R_{av,lowd}$ | $R_{best,lowd}$ | $R_{av,pix}$ | $R_{best,pix}$                       | $R_{av,cntrl}$ | $R_{best,cntrl}$ |
| blockworld1            | 1.156         | 1.511           | 0.466        | 1.299                                | -0.080         | 1.260            |
| blockworld3da          | 0.340         | 0.705           | 0.889        | 2.225                                | -0.139         | 0.658            |
| canada                 | 0.303         | 1.735           | 0.176        | 0.688                                | 0.125          | 1.157            |
| canada2d               | 0.400         | 0.978           | -0.285       | 0.119                                | -0.045         | 0.701            |
| cart                   | 0.938         | 1.336           | 1.096        | 1.258                                | 0.343          | 1.216            |
| cartpole               | 0.844         | 1.115           | 0.482        | 1.138                                | 0.244          | 0.755            |
| cartpoleBalance        | 0.951         | 1.000           | 0.335        | 0.996                                | -0.468         | 0.528            |
| cartpoleParallelDouble | 0.549         | 0.900           | 0.188        | 0.323                                | 0.197          | 0.572            |
| cartpoleSerialDouble   | 0.272         | 0.719           | 0.195        | 0.642                                | 0.143          | 0.701            |
| cartpoleSerialTriple   | 0.736         | 0.946           | 0.412        | 0.427                                | 0.583          | 0.942            |
| cheetah                | 0.903         | 1.206           | 0.457        | 0.792                                | -0.008         | 0.425            |
| fixedReacher           | 0.849         | 1.021           | 0.693        | 0.981                                | 0.259          | 0.927            |
| fixedReacherDouble     | 0.924         | 0.996           | 0.872        | 0.943                                | 0.290          | 0.995            |
| fixedReacherSingle     | 0.954         | 1.000           | 0.827        | 0.995                                | 0.620          | 0.999            |
| gripper                | 0.655         | 0.972           | 0.406        | 0.790                                | 0.461          | 0.816            |
| gripperRandom          | 0.618         | 0.937           | 0.082        | 0.791                                | 0.557          | 0.808            |
| hardCheetah            | 1.311         | 1.990           | 1.204        | 1.431                                | -0.031         | 1.411            |
| hopper                 | 0.676         | 0.936           | 0.112        | 0.924                                | 0.078          | 0.917            |
| hyq                    | 0.416         | 0.722           | 0.234        | 0.672                                | 0.198          | 0.618            |
| movingGripper          | 0.474         | 0.936           | 0.480        | 0.644                                | 0.416          | 0.805            |
| pendulum               | 0.946         | 1.021           | 0.663        | 1.055                                | 0.099          | 0.951            |
| reacher                | 0.720         | 0.987           | 0.194        | 0.878                                | 0.231          | 0.953            |
| reacher3daFixedTarget  | 0.585         | 0.943           | 0.453        | 0.922                                | 0.204          | 0.631            |
| reacher3daRandomTarget | 0.467         | 0.739           | 0.374        | 0.735                                | -0.046         | 0.158            |
| reacherSingle          | 0.981         | 1.102           | 1.000        | 1.083                                | 1.010          | 1.083            |
| walker2d               | 0.705         | 1.573           | 0.944        | 1.476                                | 0.393          | 1.397            |
| torcs                  | -393.385      | 1840.036        | -401.911     | 1876.284                             | -911.034       | 1961.600         |

### Normalize the score using two baselines

-Naïve policy (sample action from a uniform distribution)

→ Mean: 0

-iLQG(Todrov & Li, 2005) ~ planning based solver

→ Mean:1

# Discussion

1. DDPG algorithm robustly solve the challenging problems across a variety of domains, even when using raw pixels for observations
2. Experimental results demonstrate that stable learning without the need for any modifications between environments
3. DDPG needs fewer steps of experience than was used by DQN learning to find solutions in the Atari domain
4. DDPG requires a large number training episodes to find solutions