
ACM 常用算法模板



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1 字符串处理

1.1 KMP

```

1  /*
2   * next[] 的含义: x[i-next[i]...i-1]=x[0...next[i]-1]
3   * next[i] 为满足 x[i-z...i-1]=x[0...z-1] 的最大 z 值 (就是 x 的自身匹配)
4   */
5  void kmp_pre(char x[],int m,int next[]){
6      int i,j;
7      j=next[0]=-1;
8      i=0;
9      while(i<m){
10         while(-1!=j && x[i]!=x[j])j=next[j];
11         next[++i]=++j;
12     }
13 }
14 /*
15  * kmpNext[i] 的意思:next'[i]=next[next[...[next[i]]]] (直到
16    next'[i]<0 或者 x[next'[i]]!=x[i])
17  * 这样的预处理可以快一些
18  */
19 void preKMP(char x[],int m,int kmpNext[]){
20     int i,j;
21     j=kmpNext[0]=-1;
22     i=0;
23     while(i<m){
24         while(-1!=j && x[i]!=x[j])j=kmpNext[j];
25         if(x[++i]==x[++j])kmpNext[i]=kmpNext[j];
26         else kmpNext[i]=j;
27     }
28 }
29 /*
30  * 返回 x 在 y 中出现的次数, 可以重叠
31  */
32 int next[10010];
33 int KMP_Count(char x[],int m,char y[],int n){//x 是模式串, y 是主串
34     int i,j;
35     int ans=0;
36     //preKMP(x,m,next);
37     kmp_pre(x,m,next);
38     i=j=0;
39     while(i<n){
40         while(-1!=j && y[i]!=x[j])j=next[j];
41         i++;j++;
42         if(j>=m){
43             ans++;
44             j=next[j];
45         }
46     }
47     return ans;

```

```

47 }
48 //经典题目: POJ 3167
49 /*
50  * POJ 3167 Cow Patterns
51  * 模式串可以浮动的模式匹配问题
52  * 给出模式串的相对大小, 需要找出模式串匹配次数和位置
53  * 比如说模式串: 1, 4, 4, 2, 3, 1 而主串: 5,6,2,10,10,7,3,2,9
54  * 那么 2,10,10,7,3,2 就是匹配的
55  *
56  * 统计比当前数小, 和于当前数相等的, 然后进行 kmp
57  */
58 const int MAXN=100010;
59 const int MAXM=25010;
60 int a[MAXN];
61 int b[MAXN];
62 int n,m,s;
63 int as[MAXN][30];
64 int bs[MAXM][30];
65 void init(){
66     for(int i=0;i<n;i++){
67         if(i==0){
68             for(int j=1;j<=25;j++)as[i][j]=0;
69         }
70         else{
71             for(int j=1;j<=25;j++)as[i][j]=as[i-1][j];
72         }
73         as[i][a[i]]++;
74     }
75     for(int i=0;i<m;i++){
76         if(i==0){
77             for(int j=1;j<=25;j++)bs[i][j]=0;
78         }
79         else{
80             for(int j=1;j<=25;j++)bs[i][j]=bs[i-1][j];
81         }
82         bs[i][b[i]]++;
83     }
84 }
85 int next[MAXM];
86 void kmp_pre(){
87     int i,j;
88     j=next[0]=-1;
89     i=0;
90     while(i<m){
91         int t11=0,t12=0,t21=0,t22=0;
92         for(int k=1;k<b[i];k++){
93             if(i-j>0)t11+=bs[i][k]-bs[i-j-1][k];
94             else t11+=bs[i][k];
95         }
96         if(i-j>0)t12=bs[i][b[i]]-bs[i-j-1][b[i]];
97         else t12=bs[i][b[i]];

```



```

98
99     for(int k=1;k<b[j];k++){
100         t21+=bs[j][k];
101     }
102     t22=bs[j][b[j]];
103     if(j==--1 || (t11==t21&& t12==t22)){
104         next[++i]=++j;
105     }
106     else j=next[j];
107 }
108 }
109 vector<int>ans;
110 void kmp(){
111     ans.clear();
112     int i,j;
113     kmp_pre();
114     i=j=0;
115     while(i<n){
116         int t11=0,t12=0,t21=0,t22=0;
117         for(int k=1;k<a[i];k++){
118             if(i-j>0)t11+=as[i][k]-as[i-j-1][k];
119             else t11+=as[i][k];
120         }
121         if(i-j>0)t12=as[i][a[i]]-as[i-j-1][a[i]];
122         else t12=as[i][a[i]];
123
124         for(int k=1;k<b[j];k++){
125             t21+=bs[j][k];
126         }
127         t22=bs[j][b[j]];
128         if(j==--1 || (t11==t21&& t12==t22)){
129             i++;j++;
130             if(j>=m){
131                 ans.push_back(i-m+1);
132                 j=next[j];
133             }
134         }
135         else j=next[j];
136     }
137 }
138 int main(){
139     while(scanf("%d%d%d",&n,&m,&s)==3){
140         for(int i=0;i<n;i++)scanf("%d",&a[i]);
141         for(int i=0;i<m;i++)scanf("%d",&b[i]);
142         init();
143         kmp();
144         printf("%d\n",ans.size());
145         for(int i=0;i<ans.size();i++)
146             printf("%d\n",ans[i]);
147     }
148     return 0;

```

149 | }

1.2 e-KMP

```

1  /*
2  * 扩展 KMP 算法
3  */
4  //next[i]:x[i...m-1] 与 x[0...m-1] 的最长公共前缀
5  //extend[i]:y[i...n-1] 与 x[0...m-1] 的最长公共前缀
6  void pre_EKMP(char x[],int m,int next[]){
7      next[0] = m;
8      int j = 0;
9      while( j+1 < m && x[j] == x[j+1] )j++;
10     next[1] = j;
11     int k = 1;
12     for(int i = 2; i < m; i++){
13         int p = next[k]+k-1;
14         int L = next[i-k];
15         if( i+L < p+1 )next[i] = L;
16         else{
17             j = max(0,p-i+1);
18             while( i+j < m && x[i+j] == x[j])j++;
19             next[i] = j;
20             k = i;
21         }
22     }
23 }
24 void EKMP(char x[],int m,char y[],int n,int next[],int extend[]){
25     pre_EKMP(x,m,next);
26     int j = 0;
27     while(j < n && j < m && x[j] == y[j])j++;
28     extend[0] = j;
29     int k = 0;
30     for(int i = 1;i < n;i++){
31         int p = extend[k]+k-1;
32         int L = next[i-k];
33         if(i+L < p+1)extend[i] = L;
34         else{
35             j = max(0,p-i+1);
36             while( i+j < n && j < m && y[i+j] == x[j] )j++;
37             extend[i] = j;
38             k = i;
39         }
40     }
41 }

```

1.3 Manacher

```

1  /*
2  * 求最长回文子串
3  */
4  const int MAXN=110010;

```

```

5 char Ma[MAXN*2];
6 int Mp[MAXN*2];
7 void Manacher(char s[],int len){
8     int l=0;
9     Ma[l++]='$';
10    Ma[l++]='#';
11    for(int i=0;i<len;i++){
12        Ma[l++]=s[i];
13        Ma[l++]='#';
14    }
15    Ma[l]=0;
16    int mx=0,id=0;
17    for(int i=0;i<l;i++){
18        Mp[i]=mx>i?min(Mp[2*id-i],mx-i):1;
19        while(Ma[i+Mp[i]]==Ma[i-Mp[i]])Mp[i]++;
20        if(i+Mp[i]>mx){
21            mx=i+Mp[i];
22            id=i;
23        }
24    }
25 }
26 /*
27  * abaaba
28  * i:      0 1 2 3 4 5 6 7 8 9 10 11 12 13
29  * Ma[i]: $ # a # b # a # a # b # a #
30  * Mp[i]: 1 1 2 1 4 1 2 7 2 1 4 1 2 1
31  */
32 char s[MAXN];
33 int main(){
34     while(scanf("%s",s)==1){
35         int len=strlen(s);
36         Manacher(s,len);
37         int ans=0;
38         for(int i=0;i<2*len+2;i++){
39             ans=max(ans,Mp[i]-1);
40         }
41         printf("%d\n",ans);
42     }
43     return 0;
44 }

```

1.4 AC 自动机

```

1 //=====
2 // HDU 2222
3 // 求目标串中出现了几个模式串
4 //=====
5 struct Trie{
6     int next[500010][26],fail[500010],end[500010];
7     int root,L;
8     int newnode(){
9         for(int i = 0;i < 26;i++)
10            next[L][i] = -1;

```

```

11         end[L++] = 0;
12         return L-1;
13     }
14     void init(){
15         L = 0;
16         root = newnode();
17     }
18     void insert(char buf[]){
19         int len = strlen(buf);
20         int now = root;
21         for(int i = 0; i < len; i++){
22             if(next[now][buf[i]-'a'] == -1)
23                 next[now][buf[i]-'a'] = newnode();
24             now = next[now][buf[i]-'a'];
25         }
26         end[now]++;
27     }
28     void build(){
29         queue<int>Q;
30         fail[root] = root;
31         for(int i = 0; i < 26; i++){
32             if(next[root][i] == -1)
33                 next[root][i] = root;
34             else{
35                 fail[next[root][i]] = root;
36                 Q.push(next[root][i]);
37             }
38         }
39         while( !Q.empty() ){
40             int now = Q.front();
41             Q.pop();
42             for(int i = 0; i < 26; i++){
43                 if(next[now][i] == -1)
44                     next[now][i] = next[fail[now]][i];
45                 else{
46                     fail[next[now][i]] = next[fail[now]][i];
47                     Q.push(next[now][i]);
48                 }
49             }
50         }
51     }
52     int query(char buf[]){
53         int len = strlen(buf);
54         int now = root;
55         int res = 0;
56         for(int i = 0; i < len; i++){
57             now = next[now][buf[i]-'a'];
58             int temp = now;
59             while( temp != root ){
60                 res += end[temp];
61                 end[temp] = 0;
62                 temp = fail[temp];

```

```

62         }
63     }
64     return res;
65 }
66 void debug(){
67     for(int i = 0;i < L;i++){
68         printf("id_=%3d,fail_=%3d,end_=%3d,chi_=",i,fail[i],end[i]);
69         for(int j = 0;j < 26;j++){
70             printf("%2d",next[i][j]);
71             printf("]\n");
72         }
73     }
74 };
75 char buf[1000010];
76 Trie ac;
77 int main(){
78     int T;
79     int n;
80     scanf("%d",&T);
81     while( T-- ){
82         scanf("%d",&n);
83         ac.init();
84         for(int i = 0;i < n;i++){
85             scanf("%s",buf);
86             ac.insert(buf);
87         }
88         ac.build();
89         scanf("%s",buf);
90         printf("%d\n",ac.query(buf));
91     }
92     return 0;
93 }

```

1.5 后缀数组

1.5.1 DA

```

1  /*
2  *suffix array
3  *倍增算法  $O(n \log n)$ 
4  *待排序数组长度为  $n$ ，放在  $0 \sim n-1$  中，在最后面补一个 0
5  *da(str ,sa,rank,height, n , );//注意是  $n$ ;
6  *例如:
7  *n = 8;
8  * num[] = { 1, 1, 2, 1, 1, 1, 1, 2, $ }; 注意 num 最后一位为 0，其他
   大于 0
9  *rank[] = 4, 6, 8, 1, 2, 3, 5, 7, 0 ;rank[0 ~ n-1] 为有效值，rank[n]
   必定为 0 无效值
10 *sa[] = 8, 3, 4, 5, 0, 6, 1, 7, 2 ;sa[1 ~ n] 为有效值，sa[0] 必定为  $n$  是
   无效值
11 *height[] = 0, 0, 3, 2, 3, 1, 2, 0, 1 ;height[2 ~ n] 为有效值

```

```

12 *
13 */
14 const int MAXN=20010;
15 int t1[MAXN],t2[MAXN],c[MAXN]; //求 SA 数组需要的中间变量, 不需要赋值
16 //待排序的字符串放在 s 数组中, 从 s[0] 到 s[n-1], 长度为 n, 且最大值小于 m,
17 //除 s[n-1] 外的所有 s[i] 都大于 0, r[n-1]=0
18 //函数结束以后结果放在 sa 数组中
19 bool cmp(int *r,int a,int b,int l){
20     return r[a] == r[b] && r[a+l] == r[b+l];
21 }
22 void da(int str[],int sa[],int rank[],int height[],int n,int m){
23     n++;
24     int i, j, p, *x = t1, *y = t2;
25     //第一轮基数排序, 如果 s 的最大值很大, 可改为快速排序
26     for(i = 0; i < m; i++) c[i] = 0;
27     for(i = 0; i < n; i++) c[x[i] = str[i]]++;
28     for(i = 1; i < m; i++) c[i] += c[i-1];
29     for(i = n-1; i >= 0; i--) sa[--c[x[i]]] = i;
30     for(j = 1; j <= n; j <= 1){
31         p = 0;
32         //直接利用 sa 数组排序第二关键字
33         for(i = n-j; i < n; i++) y[p++] = i; //后面的 j 个数第二关键字为
            空的最小
34         for(i = 0; i < n; i++) if(sa[i] >= j) y[p++] = sa[i] - j;
35         //这样数组 y 保存的就是按照第二关键字排序的结果
36         //基数排序第一关键字
37         for(i = 0; i < m; i++) c[i] = 0;
38         for(i = 0; i < n; i++) c[x[y[i]]]++;
39         for(i = 1; i < m; i++) c[i] += c[i-1];
40         for(i = n-1; i >= 0; i--) sa[--c[x[y[i]]]] = y[i];
41         //根据 sa 和 x 数组计算新的 x 数组
42         swap(x,y);
43         p = 1; x[sa[0]] = 0;
44         for(i = 1; i < n; i++)
45             x[sa[i]] = cmp(y,sa[i-1],sa[i],j)?p-1:p++;
46         if(p >= n) break;
47         m = p; //下次基数排序的最大值
48     }
49     int k = 0;
50     n--;
51     for(i = 0; i <= n; i++) rank[sa[i]] = i;
52     for(i = 0; i < n; i++){
53         if(k) k--;
54         j = sa[rank[i]-1];
55         while(str[i+k] == str[j+k]) k++;
56         height[rank[i]] = k;
57     }
58 }
59 int rank[MAXN],height[MAXN];
60 int RMQ[MAXN];
61 int mm[MAXN];

```

```

62 int best[20][MAXN];
63 void initRMQ(int n){
64     mm[0]=-1;
65     for(int i=1;i<=n;i++)
66         mm[i]=((i&(i-1))==0)?mm[i-1]+1:mm[i-1];
67     for(int i=1;i<=n;i++)best[0][i]=i;
68     for(int i=1;i<=mm[n];i++)
69         for(int j=1;j+(1<<i)-1<=n;j++){
70             int a=best[i-1][j];
71             int b=best[i-1][j+(1<<(i-1))];
72             if(RMQ[a]<RMQ[b])best[i][j]=a;
73             else best[i][j]=b;
74         }
75 }
76 int askRMQ(int a,int b){
77     int t;
78     t=mm[b-a+1];
79     b-=(1<<t)-1;
80     a=best[t][a];b=best[t][b];
81     return RMQ[a]<RMQ[b]?a:b;
82 }
83 int lcp(int a,int b){
84     a=rank[a];b=rank[b];
85     if(a>b)swap(a,b);
86     return height[askRMQ(a+1,b)];
87 }
88 char str[MAXN];
89 int r[MAXN];
90 int sa[MAXN];
91 int main()
92 {
93     while(scanf("%s",str) == 1){
94         int len = strlen(str);
95         int n = 2*len + 1;
96         for(int i = 0;i < len;i++)r[i] = str[i];
97         for(int i = 0;i < len;i++)r[len + 1 + i] = str[len - 1 - i]
98         ];
99         r[len] = 1;
100        r[n] = 0;
101        da(r,sa,rank,height,n,128);
102        for(int i=1;i<=n;i++)RMQ[i]=height[i];
103        initRMQ(n);
104        int ans=0,st;
105        int tmp;
106        for(int i=0;i<len;i++){
107            tmp=lcp(i,n-i);//偶对称
108            if(2*tmp>ans){
109                ans=2*tmp;
110                st=i-tmp;
111            }
112            tmp=lcp(i,n-i-1);//奇数对称

```

```

112         if(2*tmp-1>ans){
113             ans=2*tmp-1;
114             st=i-tmp+1;
115         }
116     }
117     str[st+ans]=0;
118     printf("%s\n",str+st);
119 }
120 return 0;
121 }

```

1.5.2 DC3

da[] 和 str[] 数组要开大三倍，相关数组也是三倍

```

1  /*
2   * 后缀数组
3   *  DC3  算法，复杂度 O(n)
4   *  所有的相关数组都要开三倍
5   */
6  const int MAXN = 2010;
7  #define F(x) ((x)/3+((x)%3==1?0:tb))
8  #define G(x) ((x)<tb?(x)*3+1:((x)-tb)*3+2)
9  int wa[MAXN*3],wb[MAXN*3],wv[MAXN*3],wss[MAXN*3];
10 int c0(int *r,int a,int b){
11     return r[a] == r[b] && r[a+1] == r[b+1] && r[a+2] == r[b+2];
12 }
13 int c12(int k,int *r,int a,int b){
14     if(k == 2)
15         return r[a] < r[b] || ( r[a] == r[b] && c12(1,r,a+1,b+1) );
16     else return r[a] < r[b] || ( r[a] == r[b] && wv[a+1] < wv[b+1] );
17 }
18 void sort(int *r,int *a,int *b,int n,int m){
19     int i;
20     for(i = 0;i < n;i++)wv[i] = r[a[i]];
21     for(i = 0;i < m;i++)wss[i] = 0;
22     for(i = 0;i < n;i++)wss[wv[i]]++;
23     for(i = 1;i < m;i++)wss[i] += wss[i-1];
24     for(i = n-1;i >= 0;i--)
25         b[--wss[wv[i]]] = a[i];
26 }
27 void dc3(int *r,int *sa,int n,int m){
28     int i, j, *rn = r + n;
29     int *san = sa + n, ta = 0, tb = (n+1)/3, tbc = 0, p;
30     r[n] = r[n+1] = 0;
31     for(i = 0;i < n;i++)if(i % 3 != 0)wa[tbc++] = i;
32     sort(r + 2, wa, wb, tbc, m);
33     sort(r + 1, wb, wa, tbc, m);
34     sort(r, wa, wb, tbc, m);
35     for(p = 1, rn[F(wb[0])] = 0, i = 1;i < tbc;i++)
36         rn[F(wb[i])] = c0(r, wb[i-1], wb[i]) ? p - 1 : p++;
37     if(p < tbc)dc3(rn,san,tbc,p);

```



```

38     else for(i = 0; i < tbc; i++) san[rn[i]] = i;
39     for(i = 0; i < tbc; i++) if(san[i] < tb) wb[ta++] = san[i] * 3;
40     if(n % 3 == 1) wb[ta++] = n - 1;
41     sort(r, wb, wa, ta, m);
42     for(i = 0; i < tbc; i++) wv[wb[i] = G(san[i])] = i;
43     for(i = 0, j = 0, p = 0; i < ta && j < tbc; p++)
44         sa[p] = c12(wb[j] % 3, r, wa[i], wb[j]) ? wa[i++] : wb[j
            ++];
45     for(; i < ta; p++) sa[p] = wa[i++];
46     for(; j < tbc; p++) sa[p] = wb[j++];
47 }
48 //str 和 sa 也要三倍
49 void da(int str[], int sa[], int rank[], int height[], int n, int m){
50     for(int i = n; i < n*3; i++)
51         str[i] = 0;
52     dc3(str, sa, n+1, m);
53     int i, j, k = 0;
54     for(i = 0; i <= n; i++) rank[sa[i]] = i;
55     for(i = 0; i < n; i++){
56         if(k) k--;
57         j = sa[rank[i]-1];
58         while(str[i+k] == str[j+k]) k++;
59         height[rank[i]] = k;
60     }
61 }

```

1.6 后缀自动机

1.6.1 基本函数

```

1  const int CHAR = 26;
2  const int MAXN = 250010;
3  struct SAM_Node{
4      SAM_Node *fa, *next[CHAR];
5      int len;
6      long long cnt;
7      void clear(){
8          fa = 0;
9          memset(next, 0, sizeof(next));
10         cnt = 0;
11     }
12 }pool[MAXN*2];
13 SAM_Node *root, *tail;
14 SAM_Node* newnode(int len){
15     SAM_Node* cur = tail++;
16     cur->clear();
17     cur->len = len;
18     return cur;
19 }
20 void SAM_init(){
21     tail = pool;

```

```

22     root = newnode(0);
23 }
24 SAM_Node* extend(SAM_Node* last,int x){
25     SAM_Node *p = last, *np = newnode(p->len+1);
26     while(p && !p->next[x])
27         p->next[x] = np, p = p->fa;
28     if(!p)np->fa = root;
29     else {
30         SAM_Node* q = p->next[x];
31         if(q->len == p->len+1)np->fa = q;
32         else {
33             SAM_Node* nq = newnode(p->len+1);
34             memcpy(nq->next,q->next,sizeof(q->next));
35             nq->fa = q->fa;
36             q->fa = np->fa = nq;
37             while(p && p->next[x] == q)
38                 p->next[x] = nq, p = p->fa;
39         }
40     }
41     return np;
42 }

```

1.6.2 例题

CC TSUBSTR

给了一个 Trie 树，求 Trie 子树上的第 k 大的子串。

```

1  /*
2   *  http://www.codechef.com/problems/TSUBSTR/
3   Input:
4   8 4
5   abcbBaca
6   1 2
7   2 3
8   1 4
9   4 5
10  4 6
11  4 7
12  1 8
13  abcdefghijklmnopqrstuvwxyz 5
14  abcdefghijklmnopqrstuvwxyz 1
15  bcdefghijklmnopqrstuvwxyz 5
16  abcdefghijklmnopqrstuvwxyz 100
17
18  Output:
19  12
20  aba
21
22  ba
23  -1
24  */
25  const int CHAR = 26;

```

```

26 const int MAXN = 250010;
27 struct SAM_Node{
28     SAM_Node *fa,*next[CHAR];
29     int len;
30     long long cnt;
31     void clear(){
32         fa = 0;
33         memset(next,0,sizeof(next));
34         cnt = 0;
35     }
36 }pool[MAXN*2];
37 SAM_Node *root,*tail;
38 SAM_Node* newnode(int len){
39     SAM_Node* cur = tail++;
40     cur->clear();
41     cur->len = len;
42     return cur;
43 }
44 void SAM_init(){
45     tail = pool;
46     root = newnode(0);
47 }
48 SAM_Node* extend(SAM_Node* last,int x){
49     SAM_Node *p = last, *np = newnode(p->len+1);
50     while(p && !p->next[x])
51         p->next[x] = np, p = p->fa;
52     if(!p)np->fa = root;
53     else {
54         SAM_Node* q = p->next[x];
55         if(q->len == p->len+1)np->fa = q;
56         else {
57             SAM_Node* nq = newnode(p->len+1);
58             memcpy(nq->next,q->next,sizeof(q->next));
59             nq->fa = q->fa;
60             q->fa = np->fa = nq;
61             while(p && p->next[x] == q)
62                 p->next[x] = nq, p = p->fa;
63         }
64     }
65     return np;
66 }
67 char str[MAXN];
68 struct Edge
69 {
70     int to,next;
71 }edge[MAXN*2];
72 int head[MAXN],tot;
73 void addedge(int u,int v){
74     edge[tot].to = v;
75     edge[tot].next = head[u];
76     head[u] = tot++;

```

```

77 }
78
79 SAM_Node *end[MAXN];
80 int topcnt[MAXN]; // 拓扑排序使用
81 SAM_Node *topsam[MAXN*2];
82 char s2[40];
83 int order[40];
84
85 int main()
86 {
87     int n,Q;
88     while(scanf("%d%d",&n,&Q) == 2){
89         scanf("%s",str+1);
90         memset(head,-1,sizeof(head));tot = 0;
91         int u,v;
92         for(int i = 1;i < n;i++){
93             scanf("%d%d",&u,&v);
94             addedge(u,v); addedge(v,u);
95         }
96         addedge(0,1);
97         SAM_init();
98         memset(end,0,sizeof(end));
99         end[0] = root;
100         queue<int>q;
101         q.push(0);
102         while(!q.empty()){
103             u = q.front();
104             q.pop();
105             for(int i = head[u];i != -1;i = edge[i].next){
106                 v = edge[i].to;
107                 if(end[v] != 0)continue;
108                 end[v] = extend(end[u],str[v]-'a');
109                 q.push(v);
110             }
111         }
112         memset(topcnt,0,sizeof(topcnt));
113         int num = tail - pool;
114         for(int i = 0;i < num;i++)topcnt[pool[i].len]++;
115         for(int i = 1;i <= n;i++)topcnt[i] += topcnt[i-1];
116         for(int i = 0;i < num;i++)topsam[--topcnt[pool[i].len]] = &
            pool[i];
117
118         for(int i = num-1;i >= 0;i--){
119             SAM_Node *p = topsam[i];
120             p->cnt = 1;
121             for(int i = 0;i < 26;i++)
122                 if(p->next[i])
123                     p->cnt += p->next[i]->cnt;
124         }
125         printf("%lld\n",root->cnt);
126         long long k;

```

```

127     while(Q--){
128         scanf("%s",s2);
129         for(int i = 0;i < 26;i++)order[i] = s2[i]-'a';
130         scanf("%lld",&k);
131         if(k > root->cnt){
132             printf("-1\n");
133             continue;
134         }
135         SAM_Node *p = root;
136         //这里的第 k 个子串是从空串算起的
137         while( (--k) > 0 ){
138             for(int i = 0;i < 26;i++)
139                 if(p->next[order[i]]){
140                     if(k <= p->next[order[i]]->cnt){
141                         printf("%c",'a'+order[i]);
142                         p = p->next[order[i]];
143                         break; //这个不要忘记
144                     }
145                     else k -= p->next[order[i]]->cnt;
146                 }
147             }
148         printf("\n");
149     }
150 }
151 return 0;
152 }

```

CF129 E

给了 n 个字符串，求每个字符串有多少个至少出现在 k 个字符串中的子串
fail 树，两遍 dfs, 经典题。

```

1  /* http://codeforces.com/contest/204/problem/E
2  input
3  3 1
4  abc
5  a
6  ab
7  output
8  6 1 3
9  input
10 7 4
11 rubik
12 furik
13 abab
14 baba
15 aaabbbababa
16 abababababa
17 zero
18 output
19 1 0 9 9 21 30 0
20 */
21 const int CHAR = 26;

```

```

22 const int MAXN = 100010;
23 //*****SAM*****
24 struct SAM_Node{
25     SAM_Node *fa,*next[CHAR];
26     int len;
27     void clear(){
28         fa = 0;
29         memset(next,0,sizeof(next));
30     }
31 }pool[MAXN*2];
32 SAM_Node *root,*tail;
33 SAM_Node* newnode(int len){
34     SAM_Node* cur = tail++;
35     cur->clear();
36     cur->len = len;
37     return cur;
38 }
39 void SAM_init(){
40     tail = pool;
41     root = newnode(0);
42 }
43 SAM_Node* extend(SAM_Node* last,int x){
44     SAM_Node *p = last, *np = newnode(p->len+1);
45     while(p && !p->next[x])
46         p->next[x] = np, p = p->fa;
47     if(!p)np->fa = root;
48     else {
49         SAM_Node* q = p->next[x];
50         if(q->len == p->len+1)np->fa = q;
51         else {
52             SAM_Node* nq = newnode(p->len+1);
53             memcpy(nq->next,q->next,sizeof(q->next));
54             nq->fa = q->fa;
55             q->fa = np->fa = nq;
56             while(p && p->next[x] == q)
57                 p->next[x] = nq, p = p->fa;
58         }
59     }
60     return np;
61 }
62 //*****Trie*****
63 struct Trie_Node{
64     int next[CHAR];
65     vector<int>belongs;
66 }trie[MAXN];
67 int trie_root,trie_tot;
68 int trie_newnode(){
69     memset(trie[trie_tot].next,-1,sizeof(trie[trie_tot].next));
70     trie[trie_tot].belongs.clear();
71     return trie_tot++;
72 }

```

```

73 void Trie_init(){
74     trie_tot = 0;
75     trie_root = trie_newnode();
76 }
77 void insert(char buf[],int id){
78     int now = trie_root;
79     int len = strlen(buf);
80     for(int i = 0;i < len;i++){
81         if(trie[now].next[buf[i]-'a'] == -1)
82             trie[now].next[buf[i]-'a'] = trie_newnode();
83         now = trie[now].next[buf[i]-'a'];
84         trie[now].belongs.push_back(id);
85     }
86 }
87 //***** fail 树*****
88 struct Edge{
89     int to,next;
90 }edge[MAXN*2];
91 int head[MAXN*2],tot;
92 void addedge(int u,int v){
93     edge[tot].to = v; edge[tot].next = head[u]; head[u] = tot++;
94 }
95 int MtoT[MAXN*2]; //SAM 结点映射到 Trie 结点
96 int cnt[MAXN*2];
97 int F[MAXN*2];
98 int find(int x){
99     if(F[x] == -1)return x;
100     return F[x] = find(F[x]);
101 }
102 void bing(int u,int v) //注意方向性
103 {
104     int t1 = find(u);
105     int t2 = find(v);
106     if(t1 != t2)F[t1] = t2;
107 }
108 int L[MAXN];
109 void Tarjan(int u){
110     for(int i = head[u];i != -1;i = edge[i].next){
111         Tarjan(edge[i].to);
112         bing(edge[i].to,u);
113     }
114     if(MtoT[u]){
115         int tt = MtoT[u];
116         int sz = trie[tt].belongs.size();
117         for(int i = 0;i < sz;i++){
118             int v = trie[tt].belongs[i];
119             cnt[find(L[v])]--;
120             cnt[u]++;
121             L[v] = u;
122         }
123     }

```

```

124 }
125 void dfs1(int u){
126     for(int i = head[u]; i != -1; i = edge[i].next){
127         dfs1(edge[i].to);
128         cnt[u] += cnt[edge[i].to];
129     }
130 }
131 long long ans[MAXN];
132 void dfs2(int u){
133     for(int i = head[u]; i != -1; i = edge[i].next){
134         int v = edge[i].to;
135         cnt[v] += cnt[u];
136         dfs2(v);
137     }
138     if(MtoT[u]){
139         int tt = MtoT[u];
140         int sz = trie[tt].belongs.size();
141         for(int i = 0; i < sz; i++){
142             int v = trie[tt].belongs[i];
143             ans[v] += cnt[u];
144         }
145     }
146 }
147
148 char str[MAXN];
149 SAM_Node *end[MAXN];
150 int main()
151 {
152     int n,k;
153     while(scanf("%d%d",&n,&k) == 2){
154         Trie_init();
155         for(int i = 0; i < n; i++){
156             scanf("%s",str);
157             insert(str,i);
158         }
159         SAM_init();
160         //根据 Trie 建立 SAM
161         memset(end,0,sizeof(end));
162         end[0] = root;
163         memset(MtoT,0,sizeof(MtoT));
164         MtoT[root-pool] = 0;
165         queue<int>q;
166         q.push(trie_root);
167         while(!q.empty()){
168             int u = q.front();
169             q.pop();
170             for(int i = 0; i < 26; i++){
171                 if(trie[u].next[i] == -1)continue;
172                 int v = trie[u].next[i];
173                 end[v] = extend(end[u],i);
174                 MtoT[end[v]-pool] = v;

```



```

175         q.push(v);
176     }
177 }
178 //建立 fail 树
179 int num = tail - pool;
180 memset(head,-1,sizeof(head));
181 tot = 0;
182 for(SAM_Node *p = pool+1;p < tail;p++)
183     addedge(p->fa - pool,p - pool);
184 memset(cnt,0,sizeof(cnt));
185 memset(F,-1,sizeof(F));
186 memset(L,0,sizeof(L));
187 Tarjan(0);
188 dfs1(0);
189 for(int i = 0;i < num;i++){
190     if(cnt[i] >= k)cnt[i] = pool[i].len - pool[i].fa->len;
191     else cnt[i] = 0;
192 }
193 memset(ans,0,sizeof(ans));
194 dfs2(0);
195 for(int i = 0;i < n;i++){
196     printf("%I64d",ans[i]);
197     if(i < n-1)printf("_");
198     else printf("\n");
199 }
200 }
201 return 0;
202 }

```

1.7 字符串 hash

HDU4622 求区间不相同子串个数

```

1  const int HASH = 10007;
2  const int MAXN = 2010;
3  struct HASHMAP{
4      int head[HASH],next[MAXN],size;
5      unsigned long long state[MAXN];
6      int f[MAXN];
7      void init(){
8          size = 0;
9          memset(head,-1,sizeof(head));
10     }
11     int insert(unsigned long long val,int _id){
12         int h = val%HASH;
13         for(int i = head[h]; i != -1;i = next[i])
14             if(val == state[i]){
15                 int tmp = f[i];
16                 f[i] = _id;
17                 return tmp;
18             }
19         f[size] = _id;

```

```

20     state[size] = val;
21     next[size] = head[h];
22     head[h] = size++;
23     return 0;
24 }
25 }H;
26 const int SEED = 13331;
27 unsigned long long P[MAXN];
28 unsigned long long S[MAXN];
29 char str[MAXN];
30 int ans[MAXN][MAXN];
31 int main(){
32     P[0] = 1;
33     for(int i = 1; i < MAXN; i++)
34         P[i] = P[i-1] * SEED;
35     int T;
36     scanf("%d",&T);
37     while(T--){
38         scanf("%s",str);
39         int n = strlen(str);
40         S[0] = 0;
41         for(int i = 1; i <= n; i++)
42             S[i] = S[i-1]*SEED + str[i-1];
43         memset(ans,0,sizeof(ans));
44         for(int L = 1; L <= n; L++){
45             H.init();
46             for(int i = 1; i + L - 1 <= n; i++){
47                 int l = H.insert(S[i+L-1] - S[i-1]*P[L],i);
48                 ans[i][i+L-1] ++;
49                 ans[l][i+L-1]--;
50             }
51         }
52         for(int i = n; i >= 0; i--)
53             for(int j = i; j <= n; j++)
54                 ans[i][j] += ans[i+1][j] + ans[i][j-1] - ans[i+1][j-1];
55         int m,u,v;
56         scanf("%d",&m);
57         while(m--){
58             scanf("%d%d",&u,&v);
59             printf("%d\n",ans[u][v]);
60         }
61     }
62     return 0;
63 }

```

2 数学

2.1 素数

2.1.1 素数筛选（判断 $\leq \text{MAXN}$ 的数是否素数）

```

1  /*
2   * 素数筛选，判断小于 MAXN 的数是不是素数。
3   * notprime 是一张表，为 false 表示是素数，true 表示不是素数
4   */
5  const int MAXN=1000010;
6  bool notprime[MAXN]; //值为 false 表示素数，值为 true 表示非素数
7  void init(){
8      memset(notprime,false,sizeof(notprime));
9      notprime[0]=notprime[1]=true;
10     for(int i=2;i<MAXN;i++){
11         if(!notprime[i]){
12             if(i>MAXN/i) continue; //防止后面 i*i 溢出（或者 i,j 用 long
                                   //long）
13             //直接从 i*i 开始就可以，小于 i 倍的已经筛选过了，注意是 j+=i
14             for(int j=i*i;j<MAXN;j+=i)
15                 notprime[j]=true;
16         }
17     }

```

2.1.2 素数筛选（筛选出小于等于 MAXN 的素数）

```

1  /*
2   * 素数筛选，存在小于等于 MAXN 的素数
3   * prime[0] 存的是素数的个数
4   */
5  const int MAXN=10000;
6  int prime[MAXN+1];
7  void getPrime(){
8      memset(prime,0,sizeof(prime));
9      for(int i=2;i<=MAXN;i++){
10         if(!prime[i]) prime[++prime[0]]=i;
11         for(int j=1;j<=prime[0]&&prime[j]<=MAXN/i;j++){
12             prime[prime[j]*i]=1;
13             if(i%prime[j]==0) break;
14         }
15     }
16 }

```

2.1.3 大区间素数筛选（POJ 2689）

```

1  /*
2   * POJ 2689 Prime Distance
3   * 给出一个区间 [L,U]，找出区间内容、相邻的距离最近的两个素数和
4   * 距离最远的两个素数。
5   *  $1 \leq L < U \leq 2,147,483,647$  区间长度不超过 1,000,000
6   * 就是要筛选出 [L,U] 之间的素数
7   */

```

```

8  const int MAXN=100010;
9  int prime[MAXN+1];
10 void getPrime(){
11     memset(prime,0,sizeof(prime));
12     for(int i=2;i<=MAXN;i++){
13         if(!prime[i])prime[++prime[0]]=i;
14         for(int j=1;j<=prime[0]&&prime[j]<=MAXN/i;j++){
15             prime[prime[j]*i]=1;
16             if(i%prime[j]==0)break;
17         }
18     }
19 }
20 bool notprime[1000010];
21 int prime2[1000010];
22 void getPrime2(int L,int R){
23     memset(notprime,false,sizeof(notprime));
24     if(L<2)L=2;
25     for(int i=1;i<=prime[0]&&(long long)prime[i]*prime[i]<=R;i++){
26         int s=L/prime[i]+(L%prime[i]>0);
27         if(s==1)s=2;
28         for(int j=s;(long long)j*prime[i]<=R;j++){
29             if((long long)j*prime[i]>=L)
30                 notprime[j*prime[i]-L]=true;
31         }
32     }
33     prime2[0]=0;
34     for(int i=0;i<=R-L;i++){
35         if(!notprime[i])
36             prime2[++prime2[0]]=i+L;
37 }
38 int main(){
39     getPrime();
40     int L,U;
41     while(scanf("%d%d",&L,&U)==2){
42         getPrime2(L,U);
43         if(prime2[0]<2)printf("There are no adjacent primes.\n");
44         else{
45             int x1=0,x2=1000000000,y1=0,y2=0;
46             for(int i=1;i<prime2[0];i++){
47                 if(prime2[i+1]-prime2[i]<x2-x1){
48                     x1=prime2[i];
49                     x2=prime2[i+1];
50                 }
51                 if(prime2[i+1]-prime2[i]>y2-y1){
52                     y1=prime2[i];
53                     y2=prime2[i+1];
54                 }
55             }
56             printf("%d,%d are closest,%d,%d are most distant.\n",
57                 x1,x2,y1,y2);
58         }
59     }
60 }

```

58 | }

2.2 素数筛选和合数分解

```

1 //*****
2 //素数筛选和合数分解
3 const int MAXN=10000;
4 int prime[MAXN+1];
5 void getPrime(){
6     memset(prime,0,sizeof(prime));
7     for(int i=2;i<=MAXN;i++){
8         if(!prime[i])prime[++prime[0]]=i;
9         for(int j=1;j<=prime[0]&&prime[j]<=MAXN/i;j++){
10             prime[prime[j]*i]=1;
11             if(i%prime[j]==0) break;
12         }
13     }
14 }
15 long long factor[100][2];
16 int fatCnt;
17 int getFactors(long long x){
18     fatCnt=0;
19     long long tmp=x;
20     for(int i=1;prime[i]<=tmp/prime[i];i++){
21         factor[fatCnt][1]=0;
22         if(tmp%prime[i]==0){
23             factor[fatCnt][0]=prime[i];
24             while(tmp%prime[i]==0){
25                 factor[fatCnt][1]++;
26                 tmp/=prime[i];
27             }
28             fatCnt++;
29         }
30     }
31     if(tmp!=1){
32         factor[fatCnt][0]=tmp;
33         factor[fatCnt++][1]=1;
34     }
35     return fatCnt;
36 }
37 //*****

```

2.3 扩展欧几里得算法（求 $ax+by=gcd$ 的解以及逆元）

```

1 //*****
2 //返回 d=gcd(a,b); 和对应于等式  $ax+by=d$  中的  $x,y$ 
3 long long extend_gcd(long long a,long long b,long long &x,long long
    &y){
4     if(a==0&&b==0) return -1;//无最大公约数
5     if(b==0){x=1;y=0;return a;}
6     long long d=extend_gcd(b,a%b,y,x);
7     y-=a/b*x;

```

```

8     return d;
9 }
10 //***** 求逆元 *****
11 //ax = 1(mod n)
12 long long mod_reverse(long long a,long long n){
13     long long x,y;
14     long long d=extend_gcd(a,n,x,y);
15     if(d==1) return (x%n+n)%n;
16     else return -1;
17 }

```

2.4 求逆元

2.4.1 扩展欧几里德法

见上面的写法

2.4.2 简洁写法

注意：这个只能求 $a < m$ 的情况，而且必须保证 a 和 m 互质

```

1 //求 ax = 1( mod m) 的 x 值，就是逆元 (0<a<m)
2 long long inv(long long a,long long m){
3     if(a == 1)return 1;
4     return inv(m%a,m)*(m-m/a)%m;
5 }

```

2.4.3 利用欧拉函数

mod 为素数, 而且 a 和 m 互质

```

1 long long inv(long long a,long long mod)//为素数mod
2 {
3     return pow_m(a,mod-2,mod);
4 }

```

2.5 模线性方程组

```

1 long long extend_gcd(long long a,long long b,long long &x,long long
    &y){
2     if(a == 0 && b == 0)return -1;
3     if(b == 0 ){x = 1; y = 0;return a;}
4     long long d = extend_gcd(b,a%b,y,x);
5     y -= a/b*x;
6     return d;
7 }
8 int m[10],a[10];//模数为 m, 余数为 a,X % m = a
9 bool solve(int &m0,int &a0,int m,int a){
10     long long y,x;
11     int g = extend_gcd(m0,m,x,y);
12     if( abs(a - a0)%g )return false;
13     x *= (a - a0)/g;
14     x %= m/g;

```

```

15     a0 = (x*m0 + a0);
16     m0 *= m/g;
17     a0 %= m0;
18     if( a0 < 0 )a0 += m0;
19     return true;
20 }
21 /*
22  * 无解返回 false, 有解返回 true;
23  * 解的形式最后为 a0 + m0 * t (0<=a0<m0)
24  */
25 bool MLES(int &m0 ,int &a0,int n)//解为 X = a0 + m0 * k
26 {
27     bool flag = true;
28     m0 = 1;
29     a0 = 0;
30     for(int i = 0;i < n;i++)
31         if( !solve(m0,a0,m[i],a[i]) )
32         {
33             flag = false;
34             break;
35         }
36     return flag;
37 }

```

2.6 随机素数测试和大数分解 (POJ 1811)

```

1  /* *****
2  * Miller_Rabin 算法进行素数测试
3  * 速度快可以判断一个 < 2^63 的数是不是素数
4  *
5  * *****/
6  const int S = 8; //随机算法判定次数一般 8~10 就够了
7  // 计算 ret = (a*b)%c      a,b,c < 2^63
8  long long mult_mod(long long a,long long b,long long c){
9      a %= c;
10     b %= c;
11     long long ret = 0;
12     long long tmp = a;
13     while(b){
14         if(b & 1){
15             ret += tmp;
16             if(ret > c)ret -= c;//直接取模慢很多
17         }
18         tmp <<= 1;
19         if(tmp > c)tmp -= c;
20         b >>= 1;
21     }
22     return ret;
23 }
24 // 计算 ret = (a^n)%mod
25 long long pow_mod(long long a,long long n,long long mod){
26     long long ret = 1;

```

```

27     long long temp = a%mod;
28     while(n){
29         if(n & 1)ret = mult_mod(ret,temp,mod);
30         temp = mult_mod(temp,temp,mod);
31         n >>= 1;
32     }
33     return ret;
34 }
35 // 通过  $a^{(n-1)}=1 \pmod n$  来判断 n 是不是素数
36 //  $n-1 = x * 2^t$  中间使用二次判断
37 // 是合数返回 true, 不一定是合数返回 false
38 bool check(long long a,long long n,long long x,long long t){
39     long long ret = pow_mod(a,x,n);
40     long long last = ret;
41     for(int i = 1;i <= t;i++){
42         ret = mult_mod(ret,ret,n);
43         if(ret == 1 && last != 1 && last != n-1)return true;//合数
44         last = ret;
45     }
46     if(ret != 1)return true;
47     else return false;
48 }
49 //*****
50 // Miller_Rabin 算法
51 // 是素数返回 true,(可能是伪素数)
52 // 不是素数返回 false
53 //*****
54 bool Miller_Rabin(long long n){
55     if( n < 2)return false;
56     if( n == 2)return true;
57     if( (n&1) == 0)return false;//偶数
58     long long x = n - 1;
59     long long t = 0;
60     while( (x&1)==0 ){x >>= 1; t++;}
61
62     srand(time(NULL));/* ***** */
63
64     for(int i = 0;i < S;i++){
65         long long a = rand()%(n-1) + 1;
66         if( check(a,n,x,t) )
67             return false;
68     }
69     return true;
70 }
71
72 //*****
73 // pollard_rho 算法进行质因数分解
74 //*****
75 long long factor[100];//质因数分解结果（刚返回时时无序的）
76 int tol;//质因素的个数, 编号 0~tol-1
77

```



```

78 long long gcd(long long a, long long b){
79     long long t;
80     while(b){
81         t = a;
82         a = b;
83         b = t % b;
84     }
85     if(a >= 0) return a;
86     else return -a;
87 }
88
89 //找出一个因子
90 long long pollard_rho(long long x, long long c){
91     long long i = 1, k = 2;
92     srand(time(NULL));
93     long long x0 = rand() % (x - 1) + 1;
94     long long y = x0;
95     while(1){
96         i++;
97         x0 = (mult_mod(x0, x0, x) + c) % x;
98         long long d = gcd(y - x0, x);
99         if(d != 1 && d != x) return d;
100        if(y == x0) return x;
101        if(i == k){y = x0; k += k;}
102    }
103 }
104 //对 n 进行素因子分解, 存入 factor. k 设置为 107 左右即可
105 void findfac(long long n, int k){
106     if(n == 1) return;
107     if(Miller_Rabin(n))
108     {
109         factor[tol++] = n;
110         return;
111     }
112     long long p = n;
113     int c = k;
114     while(p >= n) p = pollard_rho(p, c--); //值变化, 防止死循环 k
115     findfac(p, k);
116     findfac(n/p, k);
117 }
118 //POJ 1811
119 //给出一个N( $2 \leq N < 2^{54}$ ), 如果是素数, 输出"Prime", 否则输出最小的素因子
120 int main(){
121     int T;
122     long long n;
123     scanf("%d", &T);
124     while(T--){
125         scanf("%I64d", &n);
126         if(Miller_Rabin(n)) printf("Prime\n");
127         else{
128             tol = 0;

```

```

129         findfac(n,107);
130         long long ans = factor[0];
131         for(int i = 1;i < tol;i++)
132             ans = min(ans,factor[i]);
133         printf("%I64d\n",ans);
134     }
135 }
136 return 0;
137 }

```

2.7 欧拉函数

2.7.1 分解质因素求欧拉函数

```

1 getFactors(n);
2 int ret = n;
3 for(int i = 0;i < fatCnt;i++){
4     ret = ret/factor[i][0]*(factor[i][0]-1);
5 }

```

2.7.2 筛法欧拉函数

```

1 int euler[3000001];
2 void getEuler(){
3     memset(euler,0,sizeof(euler));
4     euler[1] = 1;
5     for(int i = 2;i <= 3000000;i++)
6         if(!euler[i])
7             for(int j = i;j <= 3000000; j += i){
8                 if(!euler[j])
9                     euler[j] = j;
10                euler[j] = euler[j]/i*(i-1);
11            }
12 }

```

2.7.3 求单个数的欧拉函数

```

1 long long eular(long long n){
2     long long ans = n;
3     for(int i = 2;i*i <= n;i++){
4         if(n % i == 0){
5             ans -= ans/i;
6             while(n % i == 0)
7                 n /= i;
8         }
9     }
10    if(n > 1)ans -= ans/n;
11    return ans;
12 }

```

2.7.4 线性筛（同时得到欧拉函数和素数表）

```

1 const int MAXN = 10000000;
2 bool check[MAXN+10];

```

```

3  int phi[MAXN+10];
4  int prime[MAXN+10];
5  int tot;//素数的个数
6  void phi_and_prime_table(int N){
7      memset(check,false,sizeof(check));
8      phi[1] = 1;
9      tot = 0;
10     for(int i = 2; i <= N; i++){
11         if( !check[i] ){
12             prime[tot++] = i;
13             phi[i] = i-1;
14         }
15         for(int j = 0; j < tot; j++){
16             if(i * prime[j] > N)break;
17             check[i * prime[j]] = true;
18             if( i % prime[j] == 0){
19                 phi[i * prime[j]] = phi[i] * prime[j];
20                 break;
21             }
22             else{
23                 phi[i * prime[j]] = phi[i] * (prime[j] - 1);
24             }
25         }
26     }
27 }

```

2.8 高斯消元（浮点数）

```

1  #define eps 1e-9
2  const int MAXN=220;
3  double a[MAXN][MAXN],x[MAXN];//方程的左边的矩阵和等式右边的值，求解之后 x
   存的就是结果
4  int equ,var;//方程数和未知数个数
5  /*
6  * 返回 0 表示无解,1 表示有解
7  */
8  int Gauss(){
9      int i,j,k,col,max_r;
10     for(k=0,col=0;k<equ&&col<var;k++,col++){
11         max_r=k;
12         for(i=k+1;i<equ;i++){
13             if(fabs(a[i][col])>fabs(a[max_r][col]))
14                 max_r=i;
15             if(fabs(a[max_r][col])<eps)return 0;
16             if(k!=max_r){
17                 for(j=col;j<var;j++){
18                     swap(a[k][j],a[max_r][j]);
19                     swap(x[k],x[max_r]);
20                 }
21                 x[k]/=a[k][col];
22                 for(j=col+1;j<var;j++)a[k][j]/=a[k][col];
23                 a[k][col]=1;

```

```

24         for(i=0;i<=u;i++)
25             if(i!=k){
26                 x[i]-=x[k]*a[i][col];
27                 for(j=col+1;j<var;j++)a[i][j]-=a[k][j]*a[i][col];
28                 a[i][col]=0;
29             }
30     }
31     return 1;
32 }

```

2.9 FFT

```

1 //HDU 1402 求高精度乘法
2 const double PI = acos(-1.0);
3 //复数结构体
4 struct Complex{
5     double x,y;//实部和虚部 x+yi
6     Complex(double _x = 0.0,double _y = 0.0){
7         x = _x;
8         y = _y;
9     }
10    Complex operator -(const Complex &b)const{
11        return Complex(x-b.x,y-b.y);
12    }
13    Complex operator +(const Complex &b)const{
14        return Complex(x+b.x,y+b.y);
15    }
16    Complex operator *(const Complex &b)const{
17        return Complex(x*b.x-y*b.y,x*b.y+y*b.x);
18    }
19 };
20 /*
21  * 进行 FFT 和 IFFT 前的反转变换。
22  * 位置 i 和 (i 二进制反转后位置) 互换
23  * len 必须为 2 的幂
24  */
25 void change(Complex y[],int len){
26     int i,j,k;
27     for(i = 1, j = len/2;i < len-1;i++){
28         if(i < j)swap(y[i],y[j]);
29         //交换互为小标反转的元素, i<j 保证交换一次
30         //i 做正常的 +1, j 左反转类型的 +1, 始终保持 i 和 j 是反转的
31         k = len/2;
32         while(j >= k){
33             j -= k;
34             k /= 2;
35         }
36         if(j < k)j += k;
37     }
38 }
39 /*
40  * 做 FFT

```

```

41  * len 必须为2^k形式
42  * on==1 时是 DFT, on== -1 时是 IDFT
43  */
44  void fft(Complex y[],int len,int on){
45      change(y,len);
46      for(int h = 2; h <= len; h <= 1){
47          Complex wn(cos(-on*2*PI/h),sin(-on*2*PI/h));
48          for(int j = 0;j < len;j+=h){
49              Complex w(1,0);
50              for(int k = j;k < j+h/2;k++){
51                  Complex u = y[k];
52                  Complex t = w*y[k+h/2];
53                  y[k] = u+t;
54                  y[k+h/2] = u-t;
55                  w = w*wn;
56              }
57          }
58      }
59      if(on == -1)
60          for(int i = 0;i < len;i++)
61              y[i].x /= len;
62  }
63  const int MAXN = 200010;
64  Complex x1[MAXN],x2[MAXN];
65  char str1[MAXN/2],str2[MAXN/2];
66  int sum[MAXN];
67  int main(){
68      while(scanf("%s%s",str1,str2)==2){
69          int len1 = strlen(str1);
70          int len2 = strlen(str2);
71          int len = 1;
72          while(len < len1*2 || len < len2*2)len<=1;
73          for(int i = 0;i < len1;i++)
74              x1[i] = Complex(str1[len1-1-i]-'0',0);
75          for(int i = len1;i < len;i++)
76              x1[i] = Complex(0,0);
77          for(int i = 0;i < len2;i++)
78              x2[i] = Complex(str2[len2-1-i]-'0',0);
79          for(int i = len2;i < len;i++)
80              x2[i] = Complex(0,0);
81          //求 DFT
82          fft(x1,len,1);
83          fft(x2,len,1);
84          for(int i = 0;i < len;i++)
85              x1[i] = x1[i]*x2[i];
86          fft(x1,len,-1);
87          for(int i = 0;i < len;i++)
88              sum[i] = (int)(x1[i].x+0.5);
89          for(int i = 0;i < len;i++){
90              sum[i+1]+=sum[i]/10;
91              sum[i]%=10;

```

```

92         }
93         len = len1+len2-1;
94         while(sum[len] <= 0 && len > 0)len--;
95         for(int i = len;i >= 0;i--)
96             printf("%c",sum[i]+'0');
97         printf("\n");
98     }
99     return 0;
100 }
101
102 //HDU 4609
103 //给出 n 条线段长度, 问任取 3 根, 组成三角形的概率。
104 //n<=10^5 用 FFT 求可以组成三角形的取法有几种
105 const int MAXN = 400040;
106 Complex x1[MAXN];
107 int a[MAXN/4];
108 long long num[MAXN]; //100000*100000 会超 int
109 long long sum[MAXN];
110 int main(){
111     int T;
112     int n;
113     scanf("%d",&T);
114     while(T--){
115         scanf("%d",&n);
116         memset(num,0,sizeof(num));
117         for(int i = 0;i < n;i++){
118             scanf("%d",&a[i]);
119             num[a[i]]++;
120         }
121         sort(a,a+n);
122         int len1 = a[n-1]+1;
123         int len = 1;
124         while( len < 2*len1 )len <= 1;
125         for(int i = 0;i < len1;i++)
126             x1[i] = Complex(num[i],0);
127         for(int i = len1;i < len;i++)
128             x1[i] = Complex(0,0);
129         fft(x1,len,1);
130         for(int i = 0;i < len;i++)
131             x1[i] = x1[i]*x1[i];
132         fft(x1,len,-1);
133         for(int i = 0;i < len;i++)
134             num[i] = (long long)(x1[i].x+0.5);
135         len = 2*a[n-1];
136         //减掉取两个相同的组合
137         for(int i = 0;i < n;i++)
138             num[a[i]+a[i]]--;
139         for(int i = 1;i <= len;i++)num[i]/=2;
140         sum[0] = 0;
141         for(int i = 1;i <= len;i++)
142             sum[i] = sum[i-1]+num[i];

```

```

143     long long cnt = 0;
144     for(int i = 0; i < n; i++){
145         cnt += sum[len] - sum[a[i]];
146         //减掉一个取大, 一个取小的
147         cnt -= (long long)(n-1-i)*i;
148         //减掉一个取本身, 另外一个取其它
149         cnt -= (n-1);
150         cnt -= (long long)(n-1-i)*(n-i-2)/2;
151     }
152     long long tot = (long long)n*(n-1)*(n-2)/6;
153     printf("%.7lf\n", (double)cnt/tot);
154 }
155 return 0;
156 }

```

2.10 高斯消元法求方程组的解

2.10.1 一类开关问题, 对 2 取模的 01 方程组

POJ 1681 需要枚举自由变元, 找解中 1 个数最少的

```

1 //对 2 取模的 01 方程组
2 const int MAXN = 300;
3 //有 equ 个方程, var 个变元。增广矩阵行数为 equ, 列数为 var+1, 分别为 0 到
  var
4 int equ, var;
5 int a[MAXN][MAXN]; //增广矩阵
6 int x[MAXN]; //解集
7 int free_x[MAXN]; //用来存储自由变元 (多解枚举自由变元可以使用)
8 int free_num; //自由变元的个数
9
10 //返回值为 -1 表示无解, 为 0 是唯一解, 否则返回自由变元个数
11 int Gauss(){
12     int max_r, col, k;
13     free_num = 0;
14     for(k = 0, col = 0; k < equ && col < var; k++, col++){
15         max_r = k;
16         for(int i = k+1; i < equ; i++){
17             if(abs(a[i][col]) > abs(a[max_r][col]))
18                 max_r = i;
19         }
20         if(a[max_r][col] == 0){
21             k--;
22             free_x[free_num++] = col; //这个是自由变元
23             continue;
24         }
25         if(max_r != k){
26             for(int j = col; j < var+1; j++)
27                 swap(a[k][j], a[max_r][j]);
28         }
29         for(int i = k+1; i < equ; i++){
30             if(a[i][col] != 0){
31                 for(int j = col; j < var+1; j++)

```

```

32         a[i][j] ^= a[k][j];
33     }
34 }
35 }
36 for(int i = k; i < equ; i++)
37     if(a[i][col] != 0)
38         return -1; //无解
39 if(k < var) return var - k; //自由变元个数
40 //唯一解, 回代
41 for(int i = var - 1; i >= 0; i--){
42     x[i] = a[i][var];
43     for(int j = i + 1; j < var; j++)
44         x[i] ^= (a[i][j] && x[j]);
45 }
46 return 0;
47 }
48 int n;
49 void init(){
50     memset(a, 0, sizeof(a));
51     memset(x, 0, sizeof(x));
52     equ = n * n;
53     var = n * n;
54     for(int i = 0; i < n; i++)
55         for(int j = 0; j < n; j++){
56             int t = i * n + j;
57             a[t][t] = 1;
58             if(i > 0) a[(i - 1) * n + j][t] = 1;
59             if(i < n - 1) a[(i + 1) * n + j][t] = 1;
60             if(j > 0) a[i * n + j - 1][t] = 1;
61             if(j < n - 1) a[i * n + j + 1][t] = 1;
62         }
63 }
64 void solve(){
65     int t = Gauss();
66     if(t == -1){
67         printf("inf\n");
68         return;
69     }
70     else if(t == 0){
71         int ans = 0;
72         for(int i = 0; i < n * n; i++)
73             ans += x[i];
74         printf("%d\n", ans);
75         return;
76     }
77     else
78     {
79         //枚举自由变元
80         int ans = 0x3f3f3f3f;
81         int tot = (1 << t);
82         for(int i = 0; i < tot; i++){

```



```

83         int cnt = 0;
84         for(int j = 0;j < t;j++){
85             if(i&(1<<j)){
86                 x[free_x[j]] = 1;
87                 cnt++;
88             }
89             else x[free_x[j]] = 0;
90         }
91         for(int j = var-t-1;j >= 0;j--){
92             int idx;
93             for(idx = j;idx < var;idx++){
94                 if(a[j][idx])
95                     break;
96                 x[idx] = a[j][var];
97                 for(int l = idx+1;l < var;l++){
98                     if(a[j][l])
99                         x[idx] ^= x[l];
100                 cnt += x[idx];
101             }
102             ans = min(ans,cnt);
103         }
104         printf("%d\n",ans);
105     }
106 }
107 char str[30][30];
108 int main(){
109     int T;
110     scanf("%d",&T);
111     while(T--){
112         scanf("%d",&n);
113         init();
114         for(int i = 0;i < n;i++){
115             scanf("%s",str[i]);
116             for(int j = 0;j < n;j++){
117                 if(str[i][j] == 'y')
118                     a[i*n+j][n*n] = 0;
119                 else a[i*n+j][n*n] = 1;
120             }
121         }
122         solve();
123     }
124     return 0;
125 }

```

2.10.2 解同余方程组

POJ 2947 Widget Factory

```

1 //求解对 MOD 取模的方程组
2 const int MOD = 7;
3 const int MAXN = 400;
4 int a[MAXN][MAXN]; //增广矩阵

```

```

5  int x[MAXN]; //最后得到的解集
6  inline int gcd(int a,int b){
7      while(b != 0){
8          int t = b;
9          b = a%b;
10         a = t;
11     }
12     return a;
13 }
14 inline int lcm(int a,int b){
15     return a/gcd(a,b)*b;
16 }
17 long long inv(long long a,long long m){
18     if(a == 1)return 1;
19     return inv(m%a,m)*(m-m/a)%m;
20 }
21 int Gauss(int equ,int var){
22     int max_r,col,k;
23     for(k = 0, col = 0; k < equ && col < var; k++,col++){
24         max_r = k;
25         for(int i = k+1; i < equ;i++){
26             if(abs(a[i][col]) > abs(a[max_r][col]))
27                 max_r = i;
28             if(a[max_r][col] == 0){
29                 k--;
30                 continue;
31             }
32             if(max_r != k)
33                 for(int j = col; j < var+1;j++)
34                     swap(a[k][j],a[max_r][j]);
35             for(int i = k+1;i < equ;i++){
36                 if(a[i][col] != 0){
37                     int LCM = lcm(abs(a[i][col]),abs(a[k][col]));
38                     int ta = LCM/abs(a[i][col]);
39                     int tb = LCM/abs(a[k][col]);
40                     if(a[i][col]*a[k][col] < 0)tb = -tb;
41                     for(int j = col;j < var+1;j++){
42                         a[i][j] = ((a[i][j]*ta - a[k][j]*tb)%MOD + MOD)
43                             %MOD;
44                     }
45                 }
46             }
47             for(int i = k;i < equ;i++){
48                 if(a[i][col] != 0)
49                     return -1;//无解
50             }
51             if(k < var) return var-k;//多解
52             for(int i = var-1;i >= 0;i--){
53                 int temp = a[i][var];
54                 for(int j = i+1; j < var;j++){
55                     temp -= a[i][j]*x[j];
56                 }
57             }
58         }
59     }
60     return k;
61 }

```

```

55         temp = (temp%MOD + MOD)%MOD;
56     }
57 }
58     x[i] = (temp*inv(a[i][i],MOD))%MOD;
59 }
60     return 0;
61 }
62 int change(char s[]){
63     if(strcmp(s,"MON") == 0) return 1;
64     else if(strcmp(s,"TUE")==0) return 2;
65     else if(strcmp(s,"WED")==0) return 3;
66     else if(strcmp(s,"THU")==0) return 4;
67     else if(strcmp(s,"FRI")==0) return 5;
68     else if(strcmp(s,"SAT")==0) return 6;
69     else return 7;
70 }
71 int main(){
72     int n,m;
73     while(scanf("%d%d",&n,&m) == 2){
74         if(n == 0 && m == 0)break;
75         memset(a,0,sizeof(a));
76         char str1[10],str2[10];
77         int k;
78         for(int i = 0;i < m;i++){
79             scanf("%d%s",&k,str1,str2);
80             a[i][n] = ((change(str2) - change(str1) + 1)%MOD + MOD)
                        %MOD;
81             int t;
82             while(k--){
83                 scanf("%d",&t);
84                 t--;
85                 a[i][t] ++;
86                 a[i][t]%MOD;
87             }
88         }
89         int ans = Gauss(m,n);
90         if(ans == 0){
91             for(int i = 0;i < n;i++)
92                 if(x[i] <= 2)
93                     x[i] += 7;
94             for(int i = 0;i < n-1;i++)printf("%d_",x[i]);
95             printf("%d\n",x[n-1]);
96         }
97         else if(ans == -1)printf("Inconsistent_data.\n");
98         else printf("Multiple_solutions.\n");
99     }
100     return 0;
101 }

```

2.11 整数拆分

```

1 //HDU 4651
2 //把数 n 拆成几个数（小于等于 n）相加的形式，问有多少种拆法。
3 const int MOD = 1e9+7;
4 int dp[100010];
5 void init(){
6     memset(dp,0,sizeof(dp));
7     dp[0] = 1;
8     for(int i = 1;i <= 100000;i++){
9         for(int j = 1, r = 1; i - (3 * j * j - j) / 2 >= 0; j++, r
            *= -1){
10             dp[i] += dp[i - (3 * j * j - j) / 2] * r;
11             dp[i] %= MOD;
12             dp[i] = (dp[i]+MOD)%MOD;
13             if( i - (3 * j * j + j) / 2 >= 0 ){
14                 dp[i] += dp[i - (3 * j * j + j) / 2] * r;
15                 dp[i] %= MOD;
16                 dp[i] = (dp[i]+MOD)%MOD;
17             }
18         }
19     }
20 }
21 int main(){
22     int T;
23     int n;
24     init();
25     scanf("%d",&T);
26     while(T--){
27         scanf("%d",&n);
28         printf("%d\n",dp[n]);
29     }
30     return 0;
31 }
32
33 //HDU 4658
34 //数 n(<=10^5) 的划分,相同的数重复不能超过 k 个。
35 const int MOD = 1e9+7;
36 int dp[100010];
37 void init(){
38     memset(dp,0,sizeof(dp));
39     dp[0] = 1;
40     for(int i = 1;i <= 100000;i++){
41         for(int j = 1, r = 1; i - (3 * j * j - j) / 2 >= 0; j++, r
            *= -1){
42             dp[i] += dp[i - (3 * j * j - j) / 2] * r;
43             dp[i] %= MOD;
44             dp[i] = (dp[i]+MOD)%MOD;
45             if( i - (3 * j * j + j) / 2 >= 0 ){
46                 dp[i] += dp[i - (3 * j * j + j) / 2] * r;
47                 dp[i] %= MOD;
48                 dp[i] = (dp[i]+MOD)%MOD;
49             }

```

```

50     }
51 }
52 }
53 int solve(int n,int k){
54     int ans = dp[n];
55     for(int j = 1, r = -1; n - k*(3 * j * j - j) / 2 >= 0; j++, r
        *= -1){
56         ans += dp[n - k*(3 * j * j - j) / 2] * r;
57         ans %= MOD;
58         ans = (ans+MOD)%MOD;
59         if( n - k*(3 * j * j + j) / 2 >= 0 ){
60             ans += dp[n - k*(3 * j * j + j) / 2] * r;
61             ans %= MOD;
62             ans = (ans+MOD)%MOD;
63         }
64     }
65     return ans;
66 }
67 int main(){
68     init();
69     int T;
70     int n,k;
71     scanf("%d",&T);
72     while(T--){
73         scanf("%d%d",&n,&k);
74         printf("%d\n",solve(n,k));
75     }
76     return 0;
77 }

```

2.12 求 A^B 的约数之和对 MOD 取模

```

1 //参考 POJ 1845
2 //里面有一种求 $1+p+p^2+p^3+\dots+p^n$ 的方法。
3 //需要素数筛选和合数分解的程序，需要先调用 getPrime();
4 long long pow_m(long long a,long long n){
5     long long ret = 1;
6     long long tmp = a%MOD;
7     while(n){
8         if(n&1)ret = (ret*tmp)%MOD;
9         tmp = tmp*tmp%MOD;
10        n >>= 1;
11    }
12    return ret;
13 }
14 //计算 $1+p+p^2+\dots+p^n$ 
15 long long sum(long long p,long long n){
16     if(p == 0)return 0;
17     if(n == 0)return 1;
18     if(n & 1){
19         return ((1+pow_m(p,n/2+1))%MOD*sum(p,n/2)%MOD)%MOD;
20     }

```

```

21     else return ((1+pow_m(p,n/2+1))%MOD*sum(p,n/2-1)+pow_m(p,n/2)%
22         MOD)%MOD;
23 }
24 //返回A^B的约数之和 % MOD
25 long long solve(long long A,long long B){
26     getFactors(A);
27     long long ans = 1;
28     for(int i = 0;i < fatCnt;i++){
29         ans *= sum(factor[i][0],B*factor[i][1])%MOD;
30         ans %= MOD;
31     }
32     return ans;
33 }

```

2.13 莫比乌斯反演

2.13.1 莫比乌斯函数

```

1  const int MAXN = 1000000;
2  bool check[MAXN+10];
3  int prime[MAXN+10];
4  int mu[MAXN+10];
5  void Moblus(){
6      memset(check,false,sizeof(check));
7      mu[1] = 1;
8      int tot = 0;
9      for(int i = 2; i <= MAXN; i++){
10         if( !check[i] ){
11             prime[tot++] = i;
12             mu[i] = -1;
13         }
14         for(int j = 0; j < tot; j++){
15             if(i * prime[j] > MAXN) break;
16             check[i * prime[j]] = true;
17             if( i % prime[j] == 0){
18                 mu[i * prime[j]] = 0;
19                 break;
20             }
21             else{
22                 mu[i * prime[j]] = -mu[i];
23             }
24         }
25     }
26 }

```

2.13.2 例题：BZOJ2301

对于给出的 n 个询问，每次求有多少个数对 (x,y) ，满足 $a \leq x \leq b, c \leq y \leq d$ ，且 $\gcd(x,y) = k$ ， $\gcd(x,y)$ 函数为 x 和 y 的最大公约数。 $1 \leq n \leq 50000, 1 \leq a \leq b \leq 50000, 1 \leq c \leq d \leq 50000, 1 \leq k \leq 50000$

```

1  const int MAXN = 1000000;
2  bool check[MAXN+10];
3  int prime[MAXN+10];

```

```

4  int mu[MAXN+10];
5  void Moblus(){
6      memset(check,false,sizeof(check));
7      mu[1] = 1;
8      int tot = 0;
9      for(int i = 2; i <= MAXN; i++){
10         if( !check[i] ){
11             prime[tot++] = i;
12             mu[i] = -1;
13         }
14         for(int j = 0; j < tot; j ++){
15             if( i * prime[j] > MAXN) break;
16             check[i * prime[j]] = true;
17             if( i % prime[j] == 0){
18                 mu[i * prime[j]] = 0;
19                 break;
20             }
21             else{
22                 mu[i * prime[j]] = -mu[i];
23             }
24         }
25     }
26 }
27 int sum[MAXN+10];
28 //找 [1,n],[1,m] 内互质的数的对数
29 long long solve(int n,int m){
30     long long ans = 0;
31     if(n > m)swap(n,m);
32     for(int i = 1, la = 0; i <= n; i = la+1){
33         la = min(n/(n/i),m/(m/i));
34         ans += (long long)(sum[la] - sum[i-1])*(n/i)*(m/i);
35     }
36     return ans;
37 }
38 int main(){
39     Moblus();
40     sum[0] = 0;
41     for(int i = 1;i <= MAXN;i++)
42         sum[i] = sum[i-1] + mu[i];
43     int a,b,c,d,k;
44     int T;
45     scanf("%d",&T);
46     while(T--){
47         scanf("%d%d%d%d",&a,&b,&c,&d,&k);
48         long long ans = solve(b/k,d/k) - solve((a-1)/k,d/k) - solve
            (b/k,(c-1)/k) + solve((a-1)/k,(c-1)/k);
49         printf("%lld\n",ans);
50     }
51     return 0;
52 }

```

2.14 Baby-Step Giant-Step

```

1 // (POJ 2417, 3243)
2 // baby_step giant_step
3 //  $a^x = b \pmod n$   $n$  是素数和不是素数都可以
4 // 求解上式  $0 \leq x < n$  的解
5 #define MOD 76543
6 int hs[MOD], head[MOD], next[MOD], id[MOD], top;
7 void insert(int x, int y) {
8     int k = x % MOD;
9     hs[top] = x, id[top] = y, next[top] = head[k], head[k] = top++;
10 }
11 int find(int x) {
12     int k = x % MOD;
13     for(int i = head[k]; i != -1; i = next[i])
14         if(hs[i] == x)
15             return id[i];
16     return -1;
17 }
18 int BSGS(int a, int b, int n) {
19     memset(head, -1, sizeof(head));
20     top = 1;
21     if(b == 1) return 0;
22     int m = sqrt(n * 1.0), j;
23     long long x = 1, p = 1;
24     for(int i = 0; i < m; ++i, p = p * a % n) insert(p * b % n, i);
25     for(long long i = m; ; i += m) {
26         if((j = find(x = x * p % n)) != -1) return i - j;
27         if(i > n) break;
28     }
29     return -1;
30 }

```

2.15 自适应 simpson 积分

```

1 double simpson(double a, double b) {
2     double c = a + (b - a) / 2;
3     return (F(a) + 4 * F(c) + F(b)) * (b - a) / 6;
4 }
5 double asr(double a, double b, double eps, double A) {
6     double c = a + (b - a) / 2;
7     double L = simpson(a, c), R = simpson(c, b);
8     if(fabs(L + R - A) <= 15 * eps) return L + R + (L + R - A) / 15.0;
9     return asr(a, c, eps / 2, L) + asr(c, b, eps / 2, R);
10 }
11 double asr(double a, double b, double eps) {
12     return asr(a, b, eps, simpson(a, b));
13 }

```

2.16 斐波那契数列取模循环节

必要时要上 unsigned long long
 HDU3977


```

1 long long gcd(long long a,long long b){
2     if(b == 0)return a;
3     return gcd(b,a%b);
4 }
5 long long lcm(long long a,long long b){
6     return a/gcd(a,b)*b;
7 }
8 struct Matrix{
9     long long mat[2][2];
10 };
11 Matrix mul_M(Matrix a,Matrix b,long long mod){
12     Matrix ret;
13     for(int i = 0;i < 2;i++){
14         for(int j = 0;j < 2;j++){
15             ret.mat[i][j] = 0;
16             for(int k = 0;k < 2;k++){
17                 ret.mat[i][j] += a.mat[i][k]*b.mat[k][j]%mod;
18                 if(ret.mat[i][j] >= mod)ret.mat[i][j] -= mod;
19             }
20         }
21     }
22     return ret;
23 }
24 Matrix pow_M(Matrix a,long long n,long long mod){
25     Matrix ret;
26     memset(ret.mat,0,sizeof(ret.mat));
27     for(int i = 0;i < 2;i++)ret.mat[i][i] = 1;
28     Matrix tmp = a;
29     while(n){
30         if(n&1)ret = mul_M(ret,tmp,mod);
31         tmp = mul_M(tmp,tmp,mod);
32         n >>= 1;
33     }
34     return ret;
35 }
36 long long pow_m(long long a,long long n,long long mod)//a^b % mod{
37     long long ret = 1;
38     long long tmp = a%mod;
39     while(n){
40         if(n&1)ret = ret*tmp%mod;
41         tmp = tmp*tmp%mod;
42         n >>= 1;
43     }
44     return ret;
45 }
46 //素数筛选和合数分解
47 const int MAXN = 1000000;
48 int prime[MAXN+1];
49 void getPrime(){
50     memset(prime,0,sizeof(prime));
51     for(int i = 2;i <= MAXN;i++){
52         if(!prime[i])prime[++prime[0]] = i;

```

```

52     for(int j = 1;j <= prime[0] && prime[j] <= MAXN/i;j++){
53         prime[prime[j]*i] = 1;
54         if(i%prime[j] == 0)break;
55     }
56 }
57 }
58 long long factor[100][2];
59 int fatCnt;
60 int getFactors(long long x){
61     fatCnt = 0;
62     long long tmp = x;
63     for(int i = 1;prime[i] <= tmp/prime[i];i++){
64         factor[fatCnt][1] = 0;
65         if(tmp%prime[i] == 0){
66             factor[fatCnt][0] = prime[i];
67             while(tmp%prime[i] == 0){
68                 factor[fatCnt][1]++;
69                 tmp /= prime[i];
70             }
71             fatCnt++;
72         }
73     }
74     if(tmp != 1){
75         factor[fatCnt][0] = tmp;
76         factor[fatCnt++][1] = 1;
77     }
78     return fatCnt;
79 }
80 //勒让德符号
81 int legendre(long long a,long long p){
82     if(pow_m(a,(p-1)>>1,p) == 1)return 1;
83     else return -1;
84 }
85 int f0 = 1;
86 int f1 = 1;
87 long long getFib(long long n,long long mod){
88     if(mod == 1)return 0;
89     Matrix A;
90     A.mat[0][0] = 0;
91     A.mat[1][0] = 1;
92     A.mat[0][1] = 1;
93     A.mat[1][1] = 1;
94     Matrix B = pow_M(A,n,mod);
95     long long ret = f0*B.mat[0][0] + f1*B.mat[1][0];
96     return ret%mod;
97 }
98 long long fac[10000000];
99 long long G(long long p){
100     long long num;
101     if(legendre(5,p) == 1)num = p-1;
102     else num = 2*(p+1);

```

```

103 //找出 num 的所有约数
104 int cnt = 0;
105 for(long long i = 1;i*i <= num;i++)
106     if(num%i == 0){
107         fac[cnt++] = i;
108         if(i*i != num)
109             fac[cnt++] = num/i;
110     }
111 sort(fac,fac+cnt);
112 long long ans;
113 for(int i = 0;i < cnt;i++){
114     if(getFib(fac[i],p) == f0 && getFib(fac[i]+1,p) == f1){
115         ans = fac[i];
116         break;
117     }
118 }
119 return ans;
120 }
121 long long find_loop(long long n){
122     getFactors(n);
123     long long ans = 1;
124     for(int i = 0;i < fatCnt;i++){
125         long long record = 1;
126         if(factor[i][0] == 2)record = 3;
127         else if(factor[i][0] == 3)record = 8;
128         else if(factor[i][0] == 5)record = 20;
129         else record = G(factor[i][0]);
130         for(int j = 1;j < factor[i][1];j++)
131             record *= factor[i][0];
132         ans = lcm(ans,record);
133     }
134     return ans;
135 }
136 void init(){
137     getPrime();
138 }
139 int main(){
140     init();
141     int T;
142     int iCase = 0;
143     int n;
144     scanf("%d",&T);
145     while(T--){
146         iCase++;
147         scanf("%d",&n);
148         printf("Case_#%d:_%I64d\n",iCase,find_loop(n));
149     }
150     return 0;
151 }

```

2.17 原根

定义：设 $m > 1, \gcd(a, m) = 1$, 使得 $a^d \equiv 1 \pmod{m}$ 成立的最小的正整数 d 为 a 对模 m 的阶，记为 $\delta_m(a)$.

如果 $\delta_m(a) = \varphi(m)$, 则称 a 是模 m 的原根.

定理：若 $m > 1, \gcd(a, m) = 1$, 正整数 d 满足 $a^d \equiv 1 \pmod{m}$, 则 $\delta_m(a)$ 整除 d .

定理：模 m 有原根的充要条件是 $m = 2, 4, p^n, 2p^n$, 其中 p 是奇质数, n 是任意正整数.

定理：如果模 m 有原根，那么它一定有 $\varphi(\varphi(m))$ 个原根.

定理：如果 p 是素数，那么素数 p 一定有原根，并且模 p 的原根的个数为 $\varphi(p-1)$.

求模素数 p 原根的方法：对 $p-1$ 素因子分解，即 $p-1 = p_1^{a_1} p_2^{a_2} \dots p_k^{a_k}$ 的标准分解式，若恒有

$$g^{\frac{p-1}{p_i}} \not\equiv 1 \pmod{p}$$

成立，则 g 就是 p 的原根。（对于合数求原根，只需要把 $p-1$ 换成 $\varphi(p)$ ）即可。

求素数的最小原根程序

```

1 //*****
2 //素数筛选和合数分解
3 const int MAXN=100000;
4 int prime[MAXN+1];
5 void getPrime(){
6     memset(prime,0,sizeof(prime));
7     for(int i=2;i<=MAXN;i++){
8         if(!prime[i])prime[++prime[0]]=i;
9         for(int j=1;j<=prime[0]&&prime[j]<=MAXN/i;j++){
10             prime[prime[j]*i]=1;
11             if(i%prime[j]==0) break;
12         }
13     }
14 }
15 long long factor[100][2];
16 int fatCnt;
17 int getFactors(long long x){
18     fatCnt=0;
19     long long tmp=x;
20     for(int i=1;prime[i]<=tmp/prime[i];i++){
21         factor[fatCnt][1]=0;
22         if(tmp%prime[i]==0){
23             factor[fatCnt][0]=prime[i];
24             while(tmp%prime[i]==0){
25                 factor[fatCnt][1]++;
26                 tmp/=prime[i];
27             }
28             fatCnt++;
29         }
30     }
31     if(tmp!=1){
32         factor[fatCnt][0]=tmp;
33         factor[fatCnt++][1]=1;
34     }
35     return fatCnt;
36 }

```

```

37 //*****
38 long long pow_m(long long a,long long n,long long mod){
39     long long ret = 1;
40     long long tmp = a%mod;
41     while(n){
42         if(n&1)ret = ret*tmp%mod;
43         tmp = tmp*tmp%mod;
44         n >>= 1;
45     }
46     return ret;
47 }
48 //求素数 P 的最小的原根
49 void solve(int P){
50     if(P == 2){
51         printf("1\n");
52         return;
53     }
54     getFactors(P-1);
55     for(int g = 2; g < P;g++){
56         bool flag = true;
57         for(int i = 0;i < fatCnt;i++){
58             int t = (P-1)/factor[i][0];
59             if(pow_m(g,t,P) == 1){
60                 flag = false;
61                 break;
62             }
63         }
64         if(flag){
65             printf("%d\n",g);
66             return;
67         }
68     }
69 }
70 int main(){
71     getPrime();
72     int T;
73     int P;
74     scanf("%d",&T);
75     while(T--){
76         scanf("%d",&P);
77         solve(P);
78     }
79     return 0;
80 }

```

2.18 快速数论变换

2.18.1 HDU4656 卷积取模

HDU4656

$x_k = b * c^{(2^k)} + d$, $F(x) = a_0x_0 + a_1x_1 + a_2x_2 + \dots + a_{n-1}x_{n-1}$ Given $n, b, c, d, a_0, \dots, a_{n-1}$,

calculate $F(x_0), \dots, F(x_{n-1})$.

$$\begin{aligned}
 F_{x_k} &= \sum_{i=0}^{n-1} a_i (bc^{2k} + d)^i \\
 &= \sum_{i=0}^{n-1} a_i \sum_{j=0}^i C_i^j (bc^{2k})^j d^{i-j} \\
 &= \sum_{j=0}^{n-1} (bc^{2k})^j j!^{-1} \sum_{i=j}^{n-1} a_i d^{i-j} i! (i-j)!^{-1} \\
 &= \sum_{j=0}^{n-1} (bc^{2k})^j j!^{-1} \sum_{i=0}^{n-1-j} a_{n-1-i} (n-1-i)! d^{n-1-i-j} (n-1-i-j)!^{-1} \\
 &= \sum_{j=0}^{n-1} (bc^{2k})^j j!^{-1} p_j \\
 &= \sum_{j=0}^{n-1} b^j j!^{-1} p_j c^{2jk} \\
 &= c^{k^2} \sum_{j=0}^{n-1} b^j j!^{-1} p_j c^{j^2} c^{-(k-j)^2} \\
 &= c^{k^2} q_k
 \end{aligned}$$

其中 p_j 和 q_k 都是卷积，可以使用 NTT 进行快速计算。

```

1 //*****
2 //快速数论变换 (NTT)
3 //求 A 和 B 的卷积，结果对 P 取模
4 //做长度为 N1 的变换，选取两个质数 P1 和 P2
5 //P1-1 和 P2-1 必须是 N1 的倍数
6 //E1 和 E2 分别是 P1,P2 的原根
7 //F1 是 E1 模 P1 的逆元,F2 是 E2 模 P2 的逆元
8 //I1 是 N1 对模 P1 的逆元,I2 是 N1 对模 P2 的逆元
9 //
10 //然后使用中国剩余定理，保证了结果是小于 MM=P1*P2 的
11 //M1 = (P2 对 P1 的逆元)*P2
12 //M2 = (P1 对 P2 的逆元)*P1
13
14 const int P = 1000003;//结果对 P 取模
15 const int N1 = 262144;// 2^18
16 const int N2 = N1+1;//数组大小
17 const int P1 = 998244353;//P1 = 2^23 * 7 * 17 + 1
18 const int P2 = 995622913;//P2 = 2^19 * 3 * 3 * 211 + 1
19 const int E1 = 996173970;
20 const int E2 = 88560779;
21 const int F1 = 121392023;//E1*F1 = 1(mod P1)
22 const int F2 = 840835547;//E2*F2 = 1(mod P2)
23 const int I1 = 998240545;//I1*N1 = 1(mod P1)
24 const int I2 = 995619115;//I2*N1 = 1(mod P2)
25 const long long M1 = 397550359381069386LL;

```

```

26 const long long M2 = 596324591238590904LL;
27 const long long MM = 993874950619660289LL;//MM = P1*P2
28 //计算 x*y 对 z 取模
29 long long mul(long long x,long long y,long long z){
30     return (x*y - (long long)(x/(long double)z*y+1e-3)*z+z)%z;
31 }
32 int trf(int x1,int x2){
33     return (mul(M1,x1,MM)+mul(M2,x2,MM))%MM%P;
34 }
35 int A[N2],B[N2],C[N2];
36 int A1[N2],B1[N2],C1[N2];
37 void fft(int *A,int PM,int PW){
38     for(int m = N1,h;h = m/2, m >= 2;PW = (long long)PW*PW%PM,m=h)
39         for(int i = 0,w=1;i < h;i++, w = (long long)w*PW%PM)
40             for(int j = i;j < N1;j += m){
41                 int k = j+h, x = (A[j]-A[k]+PM)%PM;
42                 (A[j]+=A[k])%=PM;
43                 A[k] = (long long)w*x%PM;
44             }
45     for(int i = 0,j = 1;j < N1-1;j++){
46         for(int k = N1/2; k > (i^=k);k /= 2);
47         if(j < i)swap(A[i],A[j]);
48     }
49 }
50 //计算 A 和 B 的卷积, 结果保存在 C 中, 结果对 P 取模
51 void mul(){
52     memset(C,0,sizeof(C));
53     memcpy(A1,A,sizeof(A));
54     memcpy(B1,B,sizeof(B));
55     fft(A1,P1,E1); fft(B1,P1,E1);
56     for(int i = 0;i < N1;i++)C1[i] = (long long)A1[i]*B1[i]%P1;
57     fft(C1,P1,F1);
58     for(int i = 0;i < N1;i++)C1[i] = (long long)C1[i]*I1%P1;
59     fft(A,P2,E2); fft(B,P2,E2);
60     for(int i = 0;i < N1;i++)C[i] = (long long)A[i]*B[i]%P2;
61     fft(C,P2,F2);
62     for(int i = 0;i < N1;i++)C[i] = (long long)C[i]*I2%P2;
63     for(int i = 0;i < N1;i++)C[i] = trf(C1[i],C[i]);
64 }
65 int INV[P];//逆元
66 const int MAXN = 100010;
67 int F[MAXN];//阶乘
68 int a[MAXN];
69 int pd[MAXN];
70 int pb[MAXN];
71 int pc2[MAXN];
72 int p[MAXN];
73 int main()
74 {
75     //预处理逆元
76     INV[1] = 1;

```

```

77     for(int i = 2; i < P; i++)
78         INV[i] = (long long)P/i*(P-INV[P%i])%P;
79     F[0] = 1;
80     for(int i = 1; i < MAXN; i++)
81         F[i] = (long long)F[i-1]*i%P;
82     int n,b,c,d;
83     while(scanf("%d%d%d%d",&n,&b,&c,&d) == 4){
84         for(int i = 0; i < n; i++)scanf("%d",&a[i]);
85         pd[0] = 1;
86         for(int i = 1; i < n; i++)
87             pd[i] = (long long)pd[i-1]*d%P;
88         memset(A,0,sizeof(A));
89         memset(B,0,sizeof(B));
90         for(int i = 0; i < n; i++)
91             A[i] = (long long)a[n-1-i]*F[n-1-i]%P;
92         for(int i = 0; i < n; i++)
93             B[i] = (long long)pd[i]*INV[F[i]]%P;
94         mul();
95         for(int i = 0; i < n; i++)p[i] = C[i];
96         reverse(p,p+n);
97         memset(A,0,sizeof(A));
98         pb[0] = 1;
99         for(int i = 1; i < n; i++)
100             pb[i] = (long long)pb[i-1]*b%P;
101         pc2[0] = 1;
102         int c2 = (long long)c*c%P;
103         for(int i = 1, s = c; i < n; i++){
104             pc2[i] = (long long)pc2[i-1]*s%P;
105             s = (long long)s*c2%P;
106         }
107         for(int i = 0; i < n; i++)
108             A[i] = (long long)pb[i]*INV[F[i]]%P*p[i]%P*pc2[i]%P;
109         memset(B,0,sizeof(B));
110         B[0] = 1;
111         for(int i = 1; i < n; i++)
112             B[i] = B[N1-i] = INV[pc2[i]];
113         mul();
114         for(int i = 0; i < n; i++)C[i] = (long long)C[i]*pc2[i]%P;
115         for(int i = 0; i < n; i++)
116             printf("%d\n",C[i]);
117     }
118     return 0;
119 }

```

2.19 其它公式

2.19.1 Polya

设 G 是 p 个对象的一个置换群，用 k 种颜色去染这 p 个对象，若一种染色方案在群 G 的作用下变为另一种方案，则这两个方案当作是同一种方案，这样的不同染色方案数为：

$$L = \frac{1}{|G|} \times \sum (k^{C(f)}), f \in G$$

$C(f)$ 为循环节, $|G|$ 表示群的置换方法数

对于有 n 个位置的手镯, 有 n 种旋转置换和 n 种翻转置换

对于旋转置换:

$C(f_i) = \gcd(n, i)$, i 表示一次转过 i 颗宝石, $i = 0$ 时 $c = n$;

对于翻转置换:

如果 n 为偶数: 则有 $\frac{n}{2}$ 个置换 $C(f) = \frac{n}{2}$, 有 $\frac{n}{2}$ 个置换 $C(f) = \frac{n}{2} + 1$

如果 n 为奇数: $C(f) = \frac{n}{2} + 1$

3 数据结构

3.1 划分树

```

1  /*
2   * 划分树（查询区间第 k 大）
3   */
4  const int MAXN = 100010;
5  int tree[20][MAXN]; //表示每层每个位置的值
6  int sorted[MAXN]; //已经排序好的数
7  int toleft[20][MAXN]; //toleft[p][i] 表示第 i 层从 1 到 i 有数分入左边
8
9  void build(int l, int r, int dep){
10     if(l == r) return;
11     int mid = (l+r)>>1;
12     int same = mid - l + 1; //表示等于中间值而且被分入左边的个数
13     for(int i = l; i <= r; i++) //注意是 l, 不是 one
14         if(tree[dep][i] < sorted[mid])
15             same--;
16     int lpos = l;
17     int rpos = mid+1;
18     for(int i = l; i <= r; i++){
19         if(tree[dep][i] < sorted[mid])
20             tree[dep+1][lpos++] = tree[dep][i];
21         else if(tree[dep][i] == sorted[mid] && same > 0){
22             tree[dep+1][lpos++] = tree[dep][i];
23             same--;
24         }
25         else
26             tree[dep+1][rpos++] = tree[dep][i];
27         toleft[dep][i] = toleft[dep][l-1] + lpos - l;
28     }
29     build(l, mid, dep+1);
30     build(mid+1, r, dep+1);
31 }
32
33 //查询区间第 k 大的数, [L,R] 是大区间, [l,r] 是要查询的小区间
34 int query(int L, int R, int l, int r, int dep, int k){
35     if(l == r) return tree[dep][l];
36     int mid = (L+R)>>1;
37     int cnt = toleft[dep][r] - toleft[dep][l-1];
38     if(cnt >= k){
39         int newl = L + toleft[dep][l-1] - toleft[dep][L-1];
40         int newr = newl + cnt - 1;
41         return query(L, mid, newl, newr, dep+1, k);
42     }
43     else{
44         int newr = r + toleft[dep][R] - toleft[dep][r];
45         int newl = newr - (r-l-cnt);
46         return query(mid+1, R, newl, newr, dep+1, k-cnt);
47     }
}

```

```

48 }
49 int main(){
50     int n,m;
51     while (scanf("%d%d",&n,&m)==2){
52         memset(tree,0,sizeof(tree));
53         for(int i = 1;i <= n;i++){
54             scanf("%d",&tree[0][i]);
55             sorted[i] = tree[0][i];
56         }
57         sort(sorted+1,sorted+n+1);
58         build(1,n,0);
59         int s,t,k;
60         while(m--){
61             scanf("%d%d%d",&s,&t,&k);
62             printf("%d\n",query(1,n,s,t,0,k));
63         }
64     }
65     return 0;
66 }

```

3.2 RMQ

3.2.1 一维

求最大值，数组下标从 1 开始。

求最小值，或者最大最小值下标，或者数组从 0 开始对应修改即可。

```

1  const int MAXN = 50010;
2  int dp[MAXN][20];
3  int mm[MAXN];
4  //初始化 RMQ, b 数组下标从 1 开始, 从 0 开始简单修改
5  void initRMQ(int n,int b[]){
6      mm[0] = -1;
7      for(int i = 1; i <= n;i++){
8          mm[i] = ((i&(i-1)) == 0)?mm[i-1]+1:mm[i-1];
9          dp[i][0] = b[i];
10     }
11     for(int j = 1; j <= mm[n];j++){
12         for(int i = 1;i + (1<<j) -1 <= n;i++){
13             dp[i][j] = max(dp[i][j-1],dp[i+(1<<(j-1))][j-1]);
14         }
15     }
16     //查询最大值
17     int rmq(int x,int y){
18         int k = mm[y-x+1];
19         return max(dp[x][k],dp[y-(1<<k)+1][k]);
20     }

```

3.2.2 二维

```

1  /*
2   * 二维 RMQ, 预处理复杂度 n*m*log*(n)*log(m)
3   * 数组下标从 1 开始

```

```

4  */
5  int val[310][310];
6  int dp[310][310][9][9]; //最大值
7  int mm[310]; //二进制位数减一，使用前初始化
8  void initRMQ(int n, int m){
9      for(int i = 1; i <= n; i++)
10         for(int j = 1; j <= m; j++)
11             dp[i][j][0][0] = val[i][j];
12     for(int ii = 0; ii <= mm[n]; ii++)
13         for(int jj = 0; jj <= mm[m]; jj++)
14             if(ii+jj)
15                 for(int i = 1; i + (1<<ii) - 1 <= n; i++)
16                     for(int j = 1; j + (1<<jj) - 1 <= m; j++){
17                         if(ii) dp[i][j][ii][jj] = max(dp[i][j][ii-1][jj], dp[i+(1<<(ii-1))][j][ii-1][jj]);
18                         else dp[i][j][ii][jj] = max(dp[i][j][ii][jj-1], dp[i][j+(1<<(jj-1))][ii][jj-1]);
19                     }
20 }
21 //查询矩形内的最大值 (x1<=x2,y1<=y2)
22 int rmq(int x1, int y1, int x2, int y2){
23     int k1 = mm[x2-x1+1];
24     int k2 = mm[y2-y1+1];
25     x2 = x2 - (1<<k1) + 1;
26     y2 = y2 - (1<<k2) + 1;
27     return max(max(dp[x1][y1][k1][k2], dp[x1][y2][k1][k2]), max(dp[x2][y1][k1][k2], dp[x2][y2][k1][k2]));
28 }
29 int main(){
30     //在外面对 mm 数组进行初始化
31     mm[0] = -1;
32     for(int i = 1; i <= 305; i++)
33         mm[i] = ((i&(i-1))==0)?mm[i-1]+1:mm[i-1];
34     int n, m;
35     int Q;
36     int r1, c1, r2, c2;
37     while(scanf("%d%d", &n, &m) == 2){
38         for(int i = 1; i <= n; i++)
39             for(int j = 1; j <= m; j++)
40                 scanf("%d", &val[i][j]);
41         initRMQ(n, m);
42         scanf("%d", &Q);
43         while(Q--){
44             scanf("%d%d%d%d", &r1, &c1, &r2, &c2);
45             if(r1 > r2) swap(r1, r2);
46             if(c1 > c2) swap(c1, c2);
47             int tmp = rmq(r1, c1, r2, c2);
48             printf("%d_", tmp);
49             if(tmp == val[r1][c1] || tmp == val[r1][c2] || tmp == val[r2][c1] || tmp == val[r2][c2])
50                 printf("yes\n");

```

```

51         else printf("no\n");
52     }
53 }
54 return 0;
55 }

```

3.3 树链剖分

3.3.1 点权

基于点权，查询单点值，修改路径的上的点权（HDU 3966 树链剖分 + 树状数组）

```

1  const int MAXN = 50010;
2  struct Edge{
3      int to,next;
4  }edge[MAXN*2];
5  int head[MAXN],tot;
6  int top[MAXN]; //top[v] 表示 v 所在的重链的顶端节点
7  int fa[MAXN]; //父亲节点
8  int deep[MAXN]; //深度
9  int num[MAXN]; //num[v] 表示以 v 为根的子树的节点数
10 int p[MAXN]; //p[v] 表示 v 对应的位置
11 int fp[MAXN]; //fp 和 p 数组相反
12 int son[MAXN]; //重儿子
13 int pos;
14 void init(){
15     tot = 0;
16     memset(head,-1,sizeof(head));
17     pos = 1; //使用树状数组，编号从头 1 开始
18     memset(son,-1,sizeof(son));
19 }
20 void addedge(int u,int v){
21     edge[tot].to = v; edge[tot].next = head[u]; head[u] = tot++;
22 }
23 void dfs1(int u,int pre,int d){
24     deep[u] = d;
25     fa[u] = pre;
26     num[u] = 1;
27     for(int i = head[u]; i != -1; i = edge[i].next){
28         int v = edge[i].to;
29         if(v != pre){
30             dfs1(v,u,d+1);
31             num[u] += num[v];
32             if(son[u] == -1 || num[v] > num[son[u]])
33                 son[u] = v;
34         }
35     }
36 }
37 void getpos(int u,int sp){
38     top[u] = sp;
39     p[u] = pos++;
40     fp[p[u]] = u;

```

```

41     if(son[u] == -1) return;
42     getpos(son[u],sp);
43     for(int i = head[u];i != -1;i = edge[i].next){
44         int v = edge[i].to;
45         if( v != son[u] && v != fa[u])
46             getpos(v,v);
47     }
48 }
49
50 //树状数组
51 int lowbit(int x){
52     return x&(-x);
53 }
54 int c[MAXN];
55 int n;
56 int sum(int i){
57     int s = 0;
58     while(i > 0)
59     {
60         s += c[i];
61         i -= lowbit(i);
62     }
63     return s;
64 }
65 void add(int i,int val){
66     while(i <= n){
67         c[i] += val;
68         i += lowbit(i);
69     }
70 }
71 //u->v 的路径上点的值改变 val
72 void Change(int u,int v,int val){
73     int f1 = top[u], f2 = top[v];
74     int tmp = 0;
75     while(f1 != f2){
76         if(deep[f1] < deep[f2]){
77             swap(f1,f2);
78             swap(u,v);
79         }
80         add(p[f1],val);
81         add(p[u]+1,-val);
82         u = fa[f1];
83         f1 = top[u];
84     }
85     if(deep[u] > deep[v]) swap(u,v);
86     add(p[u],val);
87     add(p[v]+1,-val);
88 }
89 int a[MAXN];
90 int main(){
91     int M,P;

```

```

92     while (scanf("%d%d%d",&n,&M,&P) == 3){
93         int u,v;
94         int C1,C2,K;
95         char op[10];
96         init();
97         for(int i = 1;i <= n;i++){
98             scanf("%d",&a[i]);
99         }
100        while(M--){
101            scanf("%d%d",&u,&v);
102            addedge(u,v);
103            addedge(v,u);
104        }
105        dfs1(1,0,0);
106        getpos(1,1);
107        memset(c,0,sizeof(c));
108        for(int i = 1;i <= n;i++){
109            add(p[i],a[i]);
110            add(p[i]+1,-a[i]);
111        }
112        while(P--){
113            scanf("%s",op);
114            if(op[0] == 'Q'){
115                scanf("%d",&u);
116                printf("%d\n",sum(p[u]));
117            }
118            else{
119                scanf("%d%d%d",&C1,&C2,&K);
120                if(op[0] == 'D')
121                    K = -K;
122                Change(C1,C2,K);
123            }
124        }
125    }
126    return 0;
127 }

```

3.3.2 边权

基于边权，修改单条边权，查询路径边权最大值（SPOJ QTREE 树链剖分 + 线段树）

```

1  const int MAXN = 10010;
2  struct Edge{
3      int to,next;
4  }edge[MAXN*2];
5  int head[MAXN],tot;
6  int top[MAXN]; //top[v] 表示 v 所在的重链的顶端节点
7  int fa[MAXN]; //父亲节点
8  int deep[MAXN]; //深度
9  int num[MAXN]; //num[v] 表示以 v 为根的子树的节点数
10 int p[MAXN]; //p[v] 表示 v 与其父亲节点的连边在线段树中的位置

```

```

11 int fp[MAXN]; //和 p 数组相反
12 int son[MAXN]; //重儿子
13 int pos;
14 void init(){
15     tot = 0;
16     memset(head,-1,sizeof(head));
17     pos = 0;
18     memset(son,-1,sizeof(son));
19 }
20 void addedge(int u,int v){
21     edge[tot].to = v; edge[tot].next = head[u]; head[u] = tot++;
22 }
23 //第一遍 dfs 求出 fa,deep,num,son
24 void dfs1(int u,int pre,int d){
25     deep[u] = d;
26     fa[u] = pre;
27     num[u] = 1;
28     for(int i = head[u]; i != -1; i = edge[i].next){
29         int v = edge[i].to;
30         if(v != pre){
31             dfs1(v,u,d+1);
32             num[u] += num[v];
33             if(son[u] == -1 || num[v] > num[son[u]])
34                 son[u] = v;
35         }
36     }
37 }
38 //第二遍 dfs 求出 top 和 p
39 void getpos(int u,int sp){
40     top[u] = sp;
41     p[u] = pos++;
42     fp[p[u]] = u;
43     if(son[u] == -1) return;
44     getpos(son[u],sp);
45     for(int i = head[u] ; i != -1; i = edge[i].next){
46         int v = edge[i].to;
47         if(v != son[u] && v != fa[u])
48             getpos(v,v);
49     }
50 }
51
52 //线段树
53 struct Node{
54     int l,r;
55     int Max;
56 }segTree[MAXN*3];
57 void build(int i,int l,int r){
58     segTree[i].l = l;
59     segTree[i].r = r;
60     segTree[i].Max = 0;
61     if(l == r)return;

```



```

62     int mid = (l+r)/2;
63     build(i<<1,l,mid);
64     build((i<<1)|1,mid+1,r);
65 }
66 void push_up(int i){
67     segTree[i].Max = max(segTree[i<<1].Max,segTree[(i<<1)|1].Max);
68 }
69 // 更新线段树的第 k 个值为 val
70 void update(int i,int k,int val){
71     if(segTree[i].l == k && segTree[i].r == k){
72         segTree[i].Max = val;
73         return;
74     }
75     int mid = (segTree[i].l + segTree[i].r)/2;
76     if(k <= mid)update(i<<1,k,val);
77     else update((i<<1)|1,k,val);
78     push_up(i);
79 }
80 //查询线段树中 [l,r] 的最大值
81 int query(int i,int l,int r){
82     if(segTree[i].l == l && segTree[i].r == r)
83         return segTree[i].Max;
84     int mid = (segTree[i].l + segTree[i].r)/2;
85     if(r <= mid)return query(i<<1,l,r);
86     else if(l > mid)return query((i<<1)|1,l,r);
87     else return max(query(i<<1,l,mid),query((i<<1)|1,mid+1,r));
88 }
89 //查询 u->v 边的最大值
90 int find(int u,int v){
91     int f1 = top[u], f2 = top[v];
92     int tmp = 0;
93     while(f1 != f2){
94         if(deep[f1] < deep[f2]){
95             swap(f1,f2);
96             swap(u,v);
97         }
98         tmp = max(tmp,query(1,p[f1],p[u]));
99         u = fa[f1]; f1 = top[u];
100     }
101     if(u == v)return tmp;
102     if(deep[u] > deep[v]) swap(u,v);
103     return max(tmp,query(1,p[son[u]],p[v]));
104 }
105 int e[MAXN][3];
106 int main(){
107     int T;
108     int n;
109     scanf("%d",&T);
110     while(T--){
111         init();
112         scanf("%d",&n);

```

```

113     for(int i = 0; i < n-1; i++){
114         scanf("%d%d%d", &e[i][0], &e[i][1], &e[i][2]);
115         addedge(e[i][0], e[i][1]);
116         addedge(e[i][1], e[i][0]);
117     }
118     dfs1(1, 0, 0);
119     getpos(1, 1);
120     build(1, 0, pos-1);
121     for(int i = 0; i < n-1; i++){
122         if(deep[e[i][0]] > deep[e[i][1]])
123             swap(e[i][0], e[i][1]);
124         update(1, p[e[i][1]], e[i][2]);
125     }
126     char op[10];
127     int u, v;
128     while(scanf("%s", op) == 1){
129         if(op[0] == 'D') break;
130         scanf("%d%d", &u, &v);
131         if(op[0] == 'Q')
132             printf("%d\n", find(u, v)); // 查询 u->v 路径上边权的最大值
133         else update(1, p[e[u-1][1]], v); // 修改第 u 条边的长度为 v
134     }
135 }
136 return 0;
137 }

```

3.4 伸展树 (splay tree)

3.4.1 例题: HDU1890

```

1  const int MAXN = 100010;
2  struct Node;
3  Node* null;
4  struct Node{
5      Node *ch[2], *fa;
6      int size;
7      int rev;
8      Node(){
9          ch[0] = ch[1] = fa = null; rev = 0;
10     }
11     inline void push_up(){
12         if(this == null) return;
13         size = ch[0] -> size + ch[1] -> size + 1;
14     }
15     inline void setc(Node* p, int d){
16         ch[d] = p;
17         p -> fa = this;
18     }
19     inline bool d(){
20         return fa -> ch[1] == this;
21     }

```

```

22     void clear(){
23         size = 1;
24         ch[0] = ch[1] = fa = null;
25         rev = 0;
26     }
27     void Update_Rev(){
28         if(this == null)return;
29         swap(ch[0],ch[1]);
30         rev ^= 1;
31     }
32     inline void push_down(){
33         if(this == null)return;
34         if(rev){
35             ch[0]→Update_Rev();
36             ch[1]→Update_Rev();
37             rev = 0;
38         }
39     }
40     inline bool isroot(){
41         return fa == null || this != fa→ch[0] && this != fa→ch
           [1];
42     }
43 };
44 inline void rotate(Node* x)
45 {
46     Node *f = x→fa, *ff = x→fa→fa;
47     f→push_down();
48     x→push_down();
49     int c = x→d(), cc = f→d();
50     f→setc(x→ch[!c],c);
51     x→setc(f,!c);
52     if(ff→ch[cc] == f)ff→setc(x,cc);
53     else x→fa = ff;
54     f→push_up();
55 }
56 inline void splay(Node* &root,Node* x,Node* goal)
57 {
58     while(x→fa != goal){
59         if(x→fa→fa == goal)rotate(x);
60         else {
61             x→fa→fa→push_down();
62             x→fa→push_down();
63             x→push_down();
64             bool f = x→fa→d();
65             x→d() == f ? rotate(x→fa):rotate(x);
66             rotate(x);
67         }
68     }
69     x→push_up();
70     if(goal == null)root = x;
71 }

```

```

72 Node* get_kth(Node* r,int k)
73 {
74     Node* x = r;
75     x->push_down();
76     while(x->ch[0]->size+1 != k){
77         if(k < x->ch[0]->size+1)x = x->ch[0];
78         else{
79             k -= x->ch[0]->size+1;
80             x = x->ch[1];
81         }
82         x->push_down();
83     }
84     return x;
85 }
86 Node* get_next(Node* p){
87     p->push_down();
88     p = p->ch[1];
89     p->push_down();
90     while(p->ch[0] != null){
91         p = p->ch[0];
92         p->push_down();
93     }
94     return p;
95 }
96 Node pool[MAXN],*tail;
97 Node *node[MAXN];
98 Node *root;
99 void build(Node* &x,int l,int r,Node* fa)
100 {
101     if(l > r)return;
102     int mid = (l+r)/2;
103     x = tail++;
104     x->clear();
105     x->fa = fa;
106     node[mid] = x;
107     build(x->ch[0],l,mid-1,x);
108     build(x->ch[1],mid+1,r,x);
109     x->push_up();
110 }
111 void init(int n)
112 {
113     tail = pool;
114     null = tail++;
115     null->fa = null->ch[0] = null->ch[1] = null;
116     null->size = 0; null->rev = 0;
117     Node *p = tail++;
118     p->clear();
119     root = p;
120     p = tail++;
121     p->clear();
122     root->setc(p,1);

```

```

123     build(root->ch[1]->ch[0],1,n,root->ch[1]);
124     root->ch[1]->push_up();
125     root->push_up();
126 }
127 int a[MAXN];
128 int b[MAXN];
129 bool cmp(int i,int j)
130 {
131     if(a[i] != a[j])return a[i] < a[j];
132     else return i < j;
133 }
134 int main()
135 {
136     int n;
137     while(scanf("%d",&n) == 1 && n){
138         for(int i = 1;i <= n;i++){
139             scanf("%d",&a[i]);
140             b[i] = i;
141         }
142         init(n);
143         sort(b+1,b+n+1,cmp);
144         for(int i = 1;i <= n;i++){
145             splay(root,node[b[i]],null);
146             int sz = root->ch[0]->size;
147             printf("%d",root->ch[0]->size);
148             if(i == n)printf("\n");
149             else printf(" ");
150             splay(root,get_kth(root,i),null);
151             splay(root,get_kth(root,sz+2),root);
152             root->ch[1]->ch[0]->Update_Rev();
153         }
154     }
155     return 0;
156 }

```

3.4.2 例题：HDU3726

```

1  const int MAXN = 20010;
2  struct Node;
3  Node* null;
4  struct Node
5  {
6      Node *ch[2],*fa;//指向儿子和父亲结点
7      int size,key;
8      Node(){
9          ch[0] = ch[1] = fa = null;
10     }
11     inline void setc(Node* p,int d){
12         ch[d] = p;
13         p->fa = this;
14     }
15     inline bool d(){

```

```

16         return fa->ch[1] == this;
17     }
18     void push_up(){
19         size = ch[0]->size + ch[1]->size + 1;
20     }
21     void clear(){
22         size = 1;
23         ch[0] = ch[1] = fa = null;
24     }
25     inline bool isroot()
26     {
27         return fa == null || this != fa->ch[0] && this != fa->ch
           [1];
28     }
29 };
30 inline void rotate(Node* x)
31 {
32     Node *f = x->fa, *ff = x->fa->fa;
33     int c = x->d(), cc = f->d();
34     f->setc(x->ch[!c],c);
35     x->setc(f,!c);
36     if(ff->ch[cc] == f) ff->setc(x,cc);
37     else x->fa = ff;
38     f->push_up();
39 }
40 inline void splay(Node* &root,Node* x,Node* goal)
41 {
42     while(x->fa != goal){
43         if(x->fa->fa == goal) rotate(x);
44         else {
45             bool f = x->fa->d();
46             x->d() == f ? rotate(x->fa) : rotate(x);
47             rotate(x);
48         }
49     }
50     x->push_up();
51     if(goal == null) root = x;
52 }
53 //找到 r 子树里面的第 k 个
54 Node* get_kth(Node* r,int k)
55 {
56     Node* x = r;
57     while(x->ch[0]->size+1 != k){
58         if(k < x->ch[0]->size+1) x = x->ch[0];
59         else {
60             k -= x->ch[0]->size+1;
61             x = x->ch[1];
62         }
63     }
64     return x;
65 }

```

```

66 //在 root 的树中删掉 x
67 void erase(Node* &root,Node* x)
68 {
69     splay(root,x,null);
70     Node* t = root;
71     if(t->ch[1] != null){
72         root = t->ch[1];
73         splay(root,get_kth(root,1),null);
74         root->setc(t->ch[0],0);
75     }
76     else{
77         root = root->ch[0];
78     }
79     root->fa = null;
80     if(root != null)root->push_up();
81 }
82 void insert(Node* &root,Node* x)
83 {
84     if(root == null){
85         root = x;
86         return;
87     }
88     Node* now = root;
89     Node* pre = root->fa;
90     while(now != null){
91         pre = now;
92         now = now->ch[x->key >= now->key];
93     }
94     x->clear();
95     pre->setc(x,x->key >= pre->key);
96     splay(root,x,null);
97 }
98 void merge(Node* &A,Node* B)
99 {
100     if(A->size <= B->size)swap(A,B);
101     queue<Node*>Q;
102     Q.push(B);
103     while(!Q.empty()){
104         Node* fr = Q.front();
105         Q.pop();
106         if(fr->ch[0] != null)Q.push(fr->ch[0]);
107         if(fr->ch[1] != null)Q.push(fr->ch[1]);
108         fr->clear();
109         insert(A,fr);
110     }
111 }
112 Node pool[MAXN],*tail;
113
114 struct Edge
115 {
116     int u,v;

```

```

117 }edge[60010];
118 int a[MAXN];
119 bool del[60010];
120 struct QUERY
121 {
122     char op[10];
123     int u,v;
124 }query[500010];
125 int y[500010];
126
127 Node* node[MAXN];
128 Node* root[MAXN];
129 int F[MAXN];
130 int find(int x)
131 {
132     if(F[x] == -1)return x;
133     return F[x] = find(F[x]);
134 }
135 void debug(Node *root)
136 {
137     if(root == null)return;
138     debug(root->ch[0]);
139     printf("size: %d, key= %d\n",root->size,root->key);
140     debug(root->ch[1]);
141 }
142
143 int main()
144 {
145     int n,m;
146     int iCase = 0;
147     while(scanf("%d%d",&n,&m) == 2)
148     {
149         if(n == 0 && m == 0)break;
150         iCase++;
151         memset(F,-1,sizeof(F));
152         tail = pool;
153         null = tail++;
154         null->size = 0; null->ch[0] = null->ch[1] = null->fa = null
            ;
155         null->key = 0;
156         for(int i = 1;i <= n;i++)scanf("%d",&a[i]);
157         for(int i = 0;i < m;i++)
158         {
159             scanf("%d%d",&edge[i].u,&edge[i].v);
160             del[i] = false;
161         }
162         int Q = 0;
163         while(1)
164         {
165             scanf("%s",&query[Q].op);
166             if(query[Q].op[0] == 'E')break;

```



```

167         if(query[Q].op[0] == 'D'){
168             scanf("%d",&query[Q].u);
169             query[Q].u--;
170             del[query[Q].u] = true;
171         }
172         else if(query[Q].op[0] == 'Q'){
173             scanf("%d%d",&query[Q].u,&query[Q].v);
174         }
175         else{
176             scanf("%d%d",&query[Q].u,&query[Q].v);
177             y[Q] = a[query[Q].u];
178             a[query[Q].u] = query[Q].v;
179         }
180         Q++;
181     }
182     for(int i = 1;i <= n;i++){
183         node[i] = tail++;
184         node[i]—>clear();
185         node[i]—>key = a[i];
186         root[i] = node[i];
187     }
188     for(int i = 0;i < m;i++){
189         if(!del[i]){
190             int u = edge[i].u;
191             int v = edge[i].v;
192             int t1 = find(u);
193             int t2 = find(v);
194             if(t1 == t2)continue;
195             F[t2] = t1;
196             merge(root[t1],root[t2]);
197         }
198     }
199     vector<int>ans;
200     for(int i = Q-1;i >= 0;i—){
201         if(query[i].op[0] == 'D'){
202             int u = edge[query[i].u].u;
203             int v = edge[query[i].u].v;
204             int t1 = find(u);
205             int t2 = find(v);
206             if(t1 == t2)continue;
207             F[t2] = t1;
208             merge(root[t1],root[t2]);
209         }
210         else if(query[i].op[0] == 'Q'){
211             int u = query[i].u;
212             int k = query[i].v;
213             u = find(u);
214             if(k <= 0 || k > root[u]—>size){
215                 ans.push_back(0);
216             }
217             else{
218                 k = root[u]—>size - k + 1;

```

```

218         Node* p = get_kth(root[u],k);
219         ans.push_back(p->key);
220     }
221 }
222 else{
223     int u = query[i].u;
224     int t1 = find(u);
225     Node* p = node[u];
226     erase(root[t1],p);
227     p->clear();
228     p->key = y[i];
229     a[u] = y[i];
230     insert(root[t1],p);
231 }
232 }
233 double ret = 0;
234 int sz = ans.size();
235 for(int i = 0;i < sz;i++)ret += ans[i];
236 if(sz)ret /= sz;
237 printf("Case_%d: %.6lf\n",iCase,ret);
238 }
239 return 0;
240 }

```

3.5 动态树

3.5.1 SPOJQTREE

给定一棵 n 个结点的树，树的边上有权。有两种操作：

1. 修改一条边上的权值。
2. 查询两个结点 x 和 y 之间的最短路径中经过的最大的边的权值。

其中 $n \leq 10^4$

```

1 // http://www.spoj.com/problems/QTREE/
2 const int MAXN = 10010;
3 struct Node *null;
4 struct Node{
5     Node *fa,*ch[2];
6     int Max,key;
7     inline void push_up(){
8         if(this == null)return;
9         Max = max(key,max(ch[0]->Max,ch[1]->Max));
10    }
11    inline void setc(Node *p,int d){
12        ch[d] = p;
13        p->fa = this;
14    }
15    inline bool d(){
16        return fa->ch[1] == this;
17    }
18    inline bool isroot() {

```

```

19         return fa == null || fa->ch[0] != this && fa->ch[1] != this
20         ;
21     }
22     inline void rot(){
23         Node *f = fa,*ff = fa->fa;
24         int c = d(), cc = fa->d();
25         f->setc(ch[!c],c);
26         this->setc(f,!c);
27         if(ff->ch[cc] == f)ff->setc(this,cc);
28         else this->fa = ff;
29         f->push_up();
30     }
31     inline Node* splay(){
32         while(!isroot()){
33             if(!fa->isroot())
34                 d()==fa->d() ? fa->rot() : rot();
35             rot();
36         }
37         push_up();
38         return this;
39     }
40     inline Node* access(){
41         for(Node *p = this,*q = null; p != null; q = p, p = p->fa){
42             p->splay()->setc(q,1);
43             p->push_up();
44         }
45         return splay();
46     };
47     Node pool[MAXN],*tail;
48     Node *node[MAXN];
49     void init(int n){
50         tail = pool;
51         null = tail++;
52         null->fa = null->ch[0] = null->ch[1] = null;
53         null->Max = null->key = 0;
54         for(int i = 1;i <= n;i++){
55             node[i] = tail++;
56             node[i]->fa = node[i]->ch[0] = node[i]->ch[1] = null;
57             node[i]->Max = node[i]->key = 0;
58         }
59     }
60     struct Edge{
61         int to,next;
62         int w,id;
63     }edge[MAXN*2];
64     int head[MAXN],tot;
65     inline int addedge(int u,int v,int w,int id){
66         edge[tot].to = v;
67         edge[tot].w = w;
68         edge[tot].id = id;

```

```

69     edge[tot].next = head[u];
70     head[u] = tot++;
71 }
72 Node *ee[MAXN];
73 bool vis[MAXN];
74 void bfs(int n){
75     for(int i = 1;i <= n;i++)vis[i] = false;
76     queue<int>q;
77     q.push(1);
78     vis[1] = true;
79     while(!q.empty()){
80         int u = q.front();
81         q.pop();
82         for(int i = head[u];i != -1;i = edge[i].next){
83             int v = edge[i].to;
84             if(vis[v])continue;
85             vis[v] = true;
86             q.push(v);
87             ee[edge[i].id] = node[v];
88             node[v]—>key = edge[i].w;
89             node[v]—>push_up();
90             node[v]—>fa = node[u];
91         }
92     }
93 }
94 inline int ask(Node *x,Node *y){
95     x—>access();
96     for(x = null; y != null; x = y, y = y—>fa){
97         y—>splay();
98         if(y—>fa == null)return max(y—>ch[1]—>Max,x—>Max);
99         y—>setc(x,1);
100        y—>push_up();
101    }
102 }
103 int main()
104 {
105     int T;
106     scanf("%d",&T);
107     int n;
108     while(T—){
109         scanf("%d",&n);
110         for(int i = 1;i <= n;i++)head[i] = -1;
111         tot = 0;
112         init(n);
113         int u,v,w;
114         for(int i = 1;i < n;i++){
115             scanf("%d%d%d",&u,&v,&w);
116             addedge(u,v,w,i);
117             addedge(v,u,w,i);
118         }
119         bfs(n);

```

```

120     char op[20];
121     int x,y;
122     while(scanf("%s",op) == 1){
123         if(strcmp(op,"DONE") == 0)break;
124         scanf("%d%d",&x,&y);
125         if(op[0] == 'Q'){
126             printf("%d\n",ask(node[x],node[y]));
127         }
128         else {
129             ee[x]→splay()→key = y;
130             ee[x]→push_up();
131         }
132     }
133 }
134 return 0;
135 }

```

3.5.2 SPOJQTREE2

给定一棵 n 个结点的树，树的边上有权。有两种操作：

1. 查询两个结点 x 和 y 之间的最短路径长度。
2. 查询从 x 到 y 的最短路径的第 K 条边的长度。

其中 $n \leq 10^4$

```

1 // http://www.spoj.com/problems/QTREE2/
2 const int MAXN = 10010;
3 struct Node *null;
4 struct Node{
5     Node *fa,*ch[2];
6     int sum,val;
7     int size;
8     int id;
9     void clear(){
10         fa = ch[0] = ch[1] = null;
11         sum = val = 0;
12         size = 1;
13     }
14     inline void push_up(){
15         if(this == null)return;
16         sum = val + ch[0]→sum + ch[1]→sum;
17         size = ch[0]→size + ch[1]→size + 1;
18     }
19     inline void setc(Node *p,int d){
20         ch[d] = p;
21         p→fa = this;
22     }
23     inline bool d(){
24         return fa→ch[1] == this;
25     }
26     inline bool isroot(){

```

```

27         return fa == null || fa->ch[0] != this && fa->ch[1] != this
28         ;
29     }
30     inline void rot(){
31         Node *f = fa, *ff = fa->fa;
32         int c = d(), cc = fa->d();
33         f->setc(ch[!c],c);
34         this->setc(f,!c);
35         if(ff->ch[cc] == f) ff->setc(this,cc);
36         else this->fa = ff;
37         f->push_up();
38     }
39     inline Node* splay(){
40         while(!isroot()){
41             if(!fa->isroot())
42                 d()==fa->d() ? fa->rot() : rot();
43             rot();
44         }
45         push_up();
46         return this;
47     }
48     inline Node* access(){
49         for(Node *p = this,*q = null; p != null; q = p, p = p->fa){
50             p->splay()->setc(q,1);
51             p->push_up();
52         }
53         return splay();
54     };
55     Node pool[MAXN],*tail;
56     Node *node[MAXN];
57     void init(int n){
58         tail = pool;
59         null = tail++;
60         null->fa = null->ch[0] = null->ch[1] = null;
61         null->size = null->sum = null->val = 0;
62         for(int i = 1;i <= n;i++){
63             node[i] = tail++;
64             node[i]->id = i;
65             node[i]->clear();
66         }
67     }
68     struct Edge{
69         int to,next;
70         int w;
71     }edge[MAXN*2];
72     int head[MAXN],tot;
73     inline void addedge(int u,int v,int w){
74         edge[tot].to = v;
75         edge[tot].w = w;
76         edge[tot].next = head[u];

```

```

77     head[u] = tot++;
78 }
79 void dfs(int u,int pre){
80     for(int i = head[u];i != -1;i = edge[i].next){
81         int v = edge[i].to;
82         if(v == pre)continue;
83         dfs(v,u);
84         node[v]→val = edge[i].w;
85         node[v]→push_up();
86         node[v]→fa = node[u];
87     }
88 }
89 //查询 x→y 的距离
90 inline int query_sum(Node *x,Node *y){
91     x→access();
92     for(x = null; y != null; x = y, y = y→fa){
93         y→splay();
94         if(y→fa == null)
95             return y→ch[1]→sum + x→sum;
96         y→setc(x,1);
97         y→push_up();
98     }
99 }
100 //在 splay 中得到第 k 个点
101 Node* get_kth(Node* r,int k){
102     Node *x = r;
103     while(x→ch[0]→size+1 != k){
104         if(k < x→ch[0]→size+1)x = x→ch[0];
105         else {
106             k -= x→ch[0]→size+1;
107             x = x→ch[1];
108         }
109     }
110     return x;
111 }
112 //查询 x→y 路径上的第 k 个点
113 inline int query_kth(Node *x,Node *y,int k){
114     x→access();
115     for(x = null; y != null; x = y, y = y→fa){
116         y→splay();
117         if(y→fa == null){
118             if(y→ch[1]→size+1 == k)return y→id;
119             else if(y→ch[1]→size+1 > k)
120                 return get_kth(y→ch[1],y→ch[1]→size+1-k)→id;
121             else return get_kth(x,k-(y→ch[1]→size+1))→id;
122         }
123         y→setc(x,1);
124         y→push_up();
125     }
126 }
127 int main()

```

```

128 {
129     int T,n;
130     scanf("%d",&T);
131     while(T--){
132         scanf("%d",&n);
133         for(int i = 1;i <= n;i++)head[i] = -1;
134         tot = 0;
135         init(n);
136         int u,v,w;
137         for(int i = 1;i < n;i++){
138             scanf("%d%d%d",&u,&v,&w);
139             addedge(u,v,w);
140             addedge(v,u,w);
141         }
142         dfs(1,1);
143         char op[20];
144         while(scanf("%s",op) == 1){
145             if(strcmp(op,"DONE") == 0)break;
146             if(op[0] == 'D'){
147                 scanf("%d%d",&u,&v);
148                 printf("%d\n",query_sum(node[u],node[v]));
149             }
150             else {
151                 int k; scanf("%d%d%d",&u,&v,&k);
152                 printf("%d\n",query_kth(node[u],node[v],k));
153             }
154         }
155     }
156     return 0;
157 }

```

3.5.3 SPOJQTREE4

给定一棵 n 个结点的树，树的边上有权，每个结点有黑白两色，初始时所有的结点都是白的。有两种操作：

1. 对一个结点执行反色操作（白变黑，黑变白）
2. 查询树中距离最远的两个白点的距离。

其中 $n \leq 10^5$ ，查询数目不超过 10^5 。

```

1 //http://www.spoj.com/problems/QTREE4/
2 const int MAXN = 100010;
3 const int INF = 0x3f3f3f3f;
4 struct Node *null;
5 struct Node{
6     Node *fa,*ch[2];
7     multiset<int>st0,st1;//st0 是链，st1 是路径
8     int dd,d0;//d0 是该点对应边的长度，dd 是重链长度
9     int w0;//白点值为 0，黑点值为 -INF
10    int ls,rs,ms;
11    inline void clear(){
12        fa = ch[0] = ch[1] = null;

```



```

13         st0.clear(); st1.clear();
14         st0.insert(-INF);
15         st0.insert(-INF);
16         st1.insert(-INF);
17         w0 = 0; dd = d0 = 0;
18         ls = rs = ms = -INF;
19     }
20     inline void push_up(){
21         if(this == null) return;
22         dd = d0 + ch[0]→dd + ch[1]→dd;
23         int m0 = max(w0,*st0.rbegin()), ml = max(m0,ch[0]→rs+d0),
            mr = max(m0,ch[1]→ls);
24         ls = max(ch[0]→ls,ch[0]→dd + d0 + mr);
25         rs = max(ch[1]→rs,ch[1]→dd + ml);
26         multiset<int>::reverse_iterator it = st0.rbegin();
27         ++it;
28         int t0 = max((*st0.rbegin()) + (*it) , *st1.rbegin());
29         if(w0 == 0)
30             t0 = max(t0,max(0,*st0.rbegin()));
31         ms = max(max(max(ml+ch[1]→ls,mr+d0+ch[0]→rs),max(ch[0]→
            ms,ch[1]→ms)),t0);
32     }
33     inline void setc(Node *p,int d){
34         ch[d] = p;
35         p→fa = this;
36     }
37     inline bool d(){
38         return fa→ch[1] == this;
39     }
40     inline bool isroot(){
41         return fa == null || fa→ch[0] != this && fa→ch[1] != this
            ;
42     }
43     inline void rot(){
44         Node *f = fa, *ff = fa→fa;
45         int c = d(), cc = fa→d();
46         f→setc(ch[!c],c);
47         this→setc(f,!c);
48         if(ff→ch[cc] == f) ff→setc(this,cc);
49         else this→fa = ff;
50         f→push_up();
51     }
52     inline Node* splay(){
53         while(!isroot()){
54             if(!fa→isroot())
55                 d()==fa→d() ? fa→rot() : rot();
56             rot();
57         }
58         push_up();
59         return this;
60     }

```

```

61     inline Node* access(){
62         for(Node *p = this,*q = null; p != null; q = p, p = p->fa){
63             p->splay();
64             if(p->ch[1] != null){
65                 p->st0.insert(p->ch[1]->ls);
66                 p->st1.insert(p->ch[1]->ms);
67             }
68             if(q != null){
69                 p->st0.erase(p->st0.find(q->ls));
70                 p->st1.erase(p->st1.find(q->ms));
71             }
72             p->setc(q,1);
73             p->push_up();
74         }
75         return splay();
76     }
77 };
78 Node pool[MAXN],*tail;
79 Node *node[MAXN];
80 inline void init(int n){
81     tail = pool;
82     null = tail++;
83     null->fa = null->ch[0] = null->ch[1] = null;
84     null->st0.clear(); null->st1.clear();
85     null->ls = null->rs = null->ms = -INF;
86     null->w0 = -INF;
87     null->d0 = null->dd = 0;
88     for(int i = 1;i <= n;i++){
89         node[i] = tail++;
90         node[i]->clear();
91     }
92 }
93 struct Edge{
94     int to,next,w;
95 }edge[MAXN*2];
96 int head[MAXN],tot;
97 inline void addedge(int u,int v,int w){
98     edge[tot].to = v;
99     edge[tot].w = w;
100    edge[tot].next = head[u];
101    head[u] = tot++;
102 }
103 inline void dfs(int u,int pre){
104     for(int i = head[u];i != -1;i = edge[i].next){
105         int v = edge[i].to;
106         if(v == pre)continue;
107         node[v]->fa = node[u];
108         node[v]->d0 = edge[i].w;
109         dfs(v,u);
110         node[u]->st0.insert(node[v]->ls);
111         node[u]->st1.insert(node[v]->ms);

```

```

112     }
113     node[u]→push_up();
114 }
115 template <class T>
116 inline bool scan_d(T &ret) {
117     char c; int sgn;
118     if(c=getchar(),c==EOF) return 0;
119     while(c!='-'&&(c<'0' || c>'9')) c=getchar();
120     sgn=(c=='-')?-1:1;
121     ret=(c=='-')?0:(c-'0');
122     while(c=getchar(),c>='0'&&c<='9') ret=ret*10+(c-'0');
123     ret*=sgn;
124     return 1;
125 }
126 int main()
127 {
128     int n;
129     while(scanf("%d",&n) == 1){
130         for(int i = 1;i <= n;i++)head[i] = -1;
131         tot = 0;
132         init(n);
133         int u,v,w;
134         for(int i = 1;i < n;i++){
135             scan_d(u); scan_d(v);scan_d(w);
136             addedge(u,v,w);
137             addedge(v,u,w);
138         }
139         dfs(1,1);
140         int ans = node[1]→ms;
141         int Q;
142         char op[10];
143         scanf("%d",&Q);
144         while(Q—){
145             scanf("%s",op);
146             if(op[0] == 'C'){
147                 scan_d(u);
148                 node[u]→access();
149                 node[u]→splay();
150                 if(node[u]→w0 == 0)node[u]→w0 = -INF;
151                 else node[u]→w0 = 0;
152                 node[u]→push_up();
153                 ans = node[u]→ms;
154             }
155             else{
156                 if(ans < 0)puts("They have disappeared.");
157                 else printf("%d\n",ans);
158             }
159         }
160     }
161     return 0;
162 }

```

3.5.4 SPOJQTREE5

给定一棵 n 个结点的树，边权均为 1。每个结点有黑白两色，初始时所有结点都是黑的。两种查询操作：

1. 对一个结点执行反色操作（白变黑，黑变白）
2. 查询距离某个特定结点 i 最远的白点的距离。

其中 $\leq 10^5$ ，查询数目不超过 10^5 。

```

1 //http://www.spoj.com/problems/QTREE5/
2 const int MAXN = 100010;
3 const int INF = 0x3f3f3f3f;
4 struct Node *null;
5 struct Node{
6     Node *fa,*ch[2];
7     multiset<int>st;
8     int dd,d0;
9     int w0;
10    int ls,rs;
11    inline void clear(){
12        fa = ch[0] = ch[1] = null;
13        st.clear();
14        st.insert(INF);
15        w0 = INF; dd = d0 = 0;
16        ls = rs = INF;
17    }
18    inline void push_up(){
19        if(this == null)return;
20        dd = d0 + ch[0]->dd + ch[1]->dd;
21        int m0 = min(w0,*st.begin()), ml = min(m0,ch[0]->rs+d0), mr
            = min(m0,ch[1]->ls);
22        ls = min(ch[0]->ls,ch[0]->dd + d0 + mr);
23        rs = min(ch[1]->rs,ch[1]->dd + ml);
24    }
25    inline void setc(Node *p,int d){
26        ch[d] = p;
27        p->fa = this;
28    }
29    inline bool d(){
30        return fa->ch[1] == this;
31    }
32    inline bool isroot(){
33        return fa == null || fa->ch[0] != this && fa->ch[1] != this
            ;
34    }
35    inline void rot(){
36        Node *f = fa, *ff = fa->fa;
37        int c = d(), cc = fa->d();
38        f->setc(ch[!c],c);
39        this->setc(f,!c);
40        if(ff->ch[cc] == f)ff->setc(this,cc);
41        else this->fa = ff;

```

```

42         f->push_up();
43     }
44     inline Node* splay(){
45         while(!isroot()){
46             if(!fa->isroot())
47                 d()==fa->d() ? fa->rot() : rot();
48             rot();
49         }
50         push_up();
51         return this;
52     }
53     inline Node* access(){
54         for(Node *p = this,*q = null; p != null; q = p, p = p->fa){
55             p->splay();
56             if(p->ch[1] != null){
57                 p->st.insert(p->ch[1]->ls);
58             }
59             if(q != null){
60                 p->st.erase(p->st.find(q->ls));
61             }
62             p->setc(q,1);
63             p->push_up();
64         }
65         return splay();
66     }
67 };
68 Node pool[MAXN],*tail;
69 Node *node[MAXN];
70 inline void init(int n){
71     tail = pool;
72     null = tail++;
73     null->fa = null->ch[0] = null->ch[1] = null;
74     null->st.clear();
75     null->ls = null->rs = INF;
76     null->w0 = INF;
77     null->dd = null->d0 = 0;
78     for(int i = 1;i <= n;i++){
79         node[i] = tail++;
80         node[i]->clear();
81     }
82 }
83 struct Edge{
84     int to,next;
85 }edge[MAXN*2];
86 int head[MAXN],tot;
87 inline void addedge(int u,int v){
88     edge[tot].to = v;
89     edge[tot].next = head[u];
90     head[u] = tot++;
91 }
92 inline void dfs(int u,int pre){

```

```

93     for(int i = head[u]; i != -1; i = edge[i].next){
94         int v = edge[i].to;
95         if(v == pre)continue;
96         node[v]→fa = node[u];
97         node[v]→d0 = 1;
98         dfs(v,u);
99         node[u]→st.insert(node[v]→ls);
100     }
101     node[u]→push_up();
102 }
103 int main()
104 {
105     int n;
106     while(scanf("%d",&n) == 1){
107         init(n);
108         for(int i = 1; i <= n; i++)head[i] = -1;
109         tot = 0;
110         int u,v;
111         for(int i = 1; i < n; i++){
112             scanf("%d%d",&u,&v);
113             addedge(u,v);
114             addedge(v,u);
115         }
116         dfs(1,1);
117         int Q;
118         scanf("%d",&Q);
119         int op;
120         while(Q—){
121             scanf("%d%d",&op,&v);
122             if(op == 0){
123                 node[v]→access();
124                 node[v]→splay();
125                 if(node[v]→w0 == 0)node[v]→w0 = INF;
126                 else node[v]→w0 = 0;
127                 node[v]→push_up();
128             }
129             else {
130                 node[v]→access();
131                 node[v]→splay();
132                 if(node[v]→rs < INF)printf("%d\n",node[v]→rs);
133                 else printf("-1\n");
134             }
135         }
136     }
137     return 0;
138 }

```

3.5.5 SPOJQTREE6

给定一棵 n 个结点的树，每个结点有黑白两色，初始时所有结点都是黑的。你被要求支持：

1. 对一个结点执行反色操作（白变黑，黑变白）

2. 询问有多少个点与 u 相连。两个结点 u, v 相连当且仅当 u, v 路径上所有点的颜色相同。
其中 $n \leq 10^5$, 查询数目不超过 10^5 .

```

1 //http://www.spoj.com/problems/QTREE6/
2 const int MAXN = 100010;
3 struct Node *null;
4 struct Node{
5     Node *fa,*ch[2];
6     int co;//0 is black, 1 is white
7     int lco,rco;
8     int ls,rs;
9     int s[2];
10    int sum[2];//the sum of black and white
11    inline void clear(){
12        fa = ch[0] = ch[1] = null;
13        co = lco = rco = 0;
14        ls = rs = 1;
15        s[0] = s[1] = 0;
16        sum[0] = 1; sum[1] = 0;
17    }
18    inline void push_up(){
19        if(this == null)return;
20        if(ch[0] != null)lco = ch[0]->lco;
21        else lco = co;
22        if(ch[1] != null)rco = ch[1]->rco;
23        else rco = co;
24        sum[0] = ch[0]->sum[0] + ch[1]->sum[0] + (co == 0);
25        sum[1] = ch[0]->sum[1] + ch[1]->sum[1] + (co == 1);
26        int ml = 1 + s[co] + (co==ch[0]->rco?ch[0]->rs:0);
27        int mr = 1 + s[co] + (co==ch[1]->lco?ch[1]->ls:0);
28        ls = ch[0]->ls;
29        if(lco == co && ch[0]->sum[!co] == 0)ls += mr;
30        rs = ch[1]->rs;
31        if(rco == co && ch[1]->sum[!co] == 0)rs += ml;
32    }
33    inline void setc(Node *p,int d){
34        ch[d] = p;
35        p->fa = this;
36    }
37    inline bool d(){
38        return fa->ch[1] == this;
39    }
40    inline bool isroot(){
41        return fa == null || fa->ch[0] != this && fa->ch[1] != this;
42    }
43    inline void rot(){
44        Node *f = fa, *ff = fa->fa;
45        int c = d(), cc = fa->d();
46        f->setc(ch[!c],c);
47        this->setc(f,!c);

```

```

48     if(ff->ch[cc] == f)ff->setc(this,cc);
49     else this->fa = ff;
50     f->push_up();
51 }
52 inline Node* splay(){
53     while(!isroot()){
54         if(!fa->isroot())
55             d()==fa->d() ? fa->rot() : rot();
56         rot();
57     }
58     push_up();
59     return this;
60 }
61 inline Node* access(){
62     for(Node *p = this,*q = null; p != null; q = p, p = p->fa){
63         p->splay();
64         if(p->ch[1] != null)
65             p->s[p->ch[1]->lco] += p->ch[1]->ls;
66         if(q != null)
67             p->s[q->lco] -= q->ls;
68         p->setc(q,1);
69         p->push_up();
70     }
71     return splay();
72 }
73 };
74 Node pool[MAXN],*tail;
75 Node *node[MAXN];
76 void init(int n){
77     tail = pool;
78     null = tail++;
79     null->fa = null->ch[0] = null->ch[1] = null;
80     null->s[0] = null->s[1] = 0;
81     null->ls = null->rs = 0;
82     null->sum[0] = null->sum[1] = 0;
83     null->co = null->lco = null->rco = 0;
84     for(int i = 1;i <= n;i++){
85         node[i] = tail++;
86         node[i]->clear();
87     }
88 }
89 struct Edge{
90     int to,next;
91 }edge[MAXN*2];
92 int head[MAXN],tot;
93 inline void addedge(int u,int v){
94     edge[tot].to = v; edge[tot].next = head[u]; head[u] = tot++;
95 }
96 void dfs(int u,int pre){
97     for(int i = head[u];i != -1;i = edge[i].next){
98         int v = edge[i].to;

```



```

99         if(v == pre)continue;
100        node[v]→fa = node[u];
101        dfs(v,u);
102        node[u]→s[node[v]→lco] += node[v]→ls;
103    }
104    node[u]→push_up();
105 }
106 int main()
107 {
108     int n;
109     while(scanf("%d",&n) == 1){
110         init(n);
111         for(int i = 1;i <= n;i++)head[i] = -1;
112         tot = 0;
113         int u,v;
114         for(int i = 1;i < n;i++){
115             scanf("%d%d",&u,&v);
116             addedge(u,v);
117             addedge(v,u);
118         }
119         dfs(1,1);
120         int Q;
121         int op;
122         scanf("%d",&Q);
123         while(Q—){
124             scanf("%d%d",&op,&u);
125             if(op == 0){
126                 node[u]→access();
127                 node[u]→splay();
128                 printf("%d\n",node[u]→rs);
129             }
130             else{
131                 node[u]→access();
132                 node[u]→splay();
133                 node[u]→co ^= 1;
134                 node[u]→push_up();
135             }
136         }
137         return 0;
138     }
139     return 0;
140 }

```

3.5.6 SPOJQTREE7

给定一棵 n 个结点的树，每个结点有黑白两色和权值。三种操作：

1. 对一个结点执行反色操作（白变黑，黑变白）
2. 询问与 u 相连的点中点权的最大值。两个结点 u,v 相连当且仅当 u,v 路径上所有点的颜色相同。
3. 改变一个点的点权。

其中 $n \leq 10^5$ ，查询数目不超过 10^5 。

```

1 //http://www.spoj.com/problems/QTREE7/
2 const int MAXN = 100010;
3 const int INF = 0x3f3f3f3f;
4 struct Node *null;
5 struct Node{
6     Node *fa,*ch[2];
7     int co;
8     int lco,rco;
9     int ls,rs;
10    int w0;
11    multiset<int>st[2];
12    int sum[2];
13    inline void clear(int _co = 0, int _w0 = 0){
14        fa = ch[0] = ch[1] = null;
15        co = lco = rco = _co;
16        w0 = _w0;
17        ls = rs = _w0;
18        st[0].clear(); st[1].clear();
19        st[0].insert(-INF); st[1].insert(-INF);
20        sum[0] = sum[1] = 0; sum[_co]++;
21    }
22    inline void push_up(){
23        if(this == null)return;
24        if(ch[0] != null)lco = ch[0]->lco;
25        else lco = co;
26        if(ch[1] != null)rco = ch[1]->rco;
27        else rco = co;
28        sum[0] = ch[0]->sum[0] + ch[1]->sum[0] + (co == 0);
29        sum[1] = ch[0]->sum[1] + ch[1]->sum[1] + (co == 1);
30        int ml = max(w0,max(*st[co].rbegin(),co==ch[0]->rco?ch[0]->
31            rs:-INF));
32        int mr = max(w0,max(*st[co].rbegin(),co==ch[1]->lco?ch[1]->
33            ls:-INF));
34        ls = ch[0]->ls;
35        if(lco == co && ch[0]->sum[!co] == 0)ls = max(ls,mr);
36        rs = ch[1]->rs;
37        if(rco == co && ch[1]->sum[!co] == 0)rs = max(rs,ml);
38    }
39    inline void setc(Node *p,int d){
40        ch[d] = p;
41        p->fa = this;
42    }
43    inline bool d(){
44        return fa->ch[1] == this;
45    }
46    inline bool isroot(){
47        return fa == null || fa->ch[0] != this && fa->ch[1] != this
48            ;
49    }
50    inline void rot(){

```

```

48     Node *f = fa, *ff = fa->fa;
49     int c = d(), cc = fa->d();
50     f->setc(ch[!c],c);
51     this->setc(f,!c);
52     if(ff->ch[cc] == f)ff->setc(this,cc);
53     else this->fa = ff;
54     f->push_up();
55 }
56 inline Node* splay(){
57     while(!isroot()){
58         if(!fa->isroot())
59             d()==fa->d() ? fa->rot() : rot();
60         rot();
61     }
62     push_up();
63     return this;
64 }
65 inline Node* access(){
66     for(Node *p = this,*q = null; p != null; q = p, p = p->fa){
67         p->splay();
68         if(p->ch[1] != null)
69             p->st[p->ch[1]->lco].insert(p->ch[1]->ls);
70         if(q != null)
71             p->st[q->lco].erase(p->st[q->lco].find(q->ls));
72         p->setc(q,1);
73         p->push_up();
74     }
75     return splay();
76 }
77 };
78 Node pool[MAXN],*tail;
79 Node *node[MAXN];
80 int color[MAXN],val[MAXN];
81 void init(int n){
82     tail = pool;
83     null = tail++;
84     null->fa = null->ch[0] = null->ch[1] = null;
85     null->st[0].clear(); null->st[1].clear();
86     null->ls = null->rs = -INF;
87     null->sum[0] = null->sum[1] = 0;
88     null->co = null->lco = null->rco = 0;
89     for(int i = 1;i <= n;i++){
90         node[i] = tail++;
91         node[i]->clear(color[i],val[i]);
92     }
93 }
94 struct Edge{
95     int to,next;
96 }edge[MAXN*2];
97 int head[MAXN],tot;
98 inline void addedge(int u,int v){

```

```

99     edge[tot].to = v; edge[tot].next = head[u]; head[u] = tot++;
100 }
101 void dfs(int u,int pre){
102     for(int i = head[u];i != -1;i = edge[i].next){
103         int v = edge[i].to;
104         if(v == pre)continue;
105         node[v]→fa = node[u];
106         dfs(v,u);
107         node[u]→st[node[v]→lco].insert(node[v]→ls);
108     }
109     node[u]→push_up();
110 }
111 int main()
112 {
113     int n;
114     while(scanf("%d",&n) == 1){
115         for(int i = 1;i <= n;i++)head[i] = -1;
116         tot = 0;
117         int u,v;
118         for(int i = 1;i < n;i++){
119             scanf("%d%d",&u,&v);
120             addedge(u,v);
121             addedge(v,u);
122         }
123         for(int i = 1;i <= n;i++)scanf("%d",&color[i]);
124         for(int i = 1;i <= n;i++)scanf("%d",&val[i]);
125         init(n);
126         dfs(1,1);
127         int Q;
128         int w,op;
129         scanf("%d",&Q);
130         while(Q—){
131             scanf("%d",&op);
132             if(op == 0){
133                 scanf("%d",&u);
134                 node[u]→access(); node[u]→splay();
135                 printf("%d\n",node[u]→rs);
136             }
137             else if(op == 1){
138                 scanf("%d",&u);
139                 node[u]→access(); node[u]→splay();
140                 node[u]→co ^= 1;
141                 node[u]→push_up();
142             }
143             else {
144                 scanf("%d%d",&u,&w);
145                 node[u]→access(); node[u]→splay();
146                 node[u]→w0 = w;
147                 node[u]→push_up();
148             }
149         }

```

```

150     }
151     return 0;
152 }

```

3.5.7 HDU4010

支持:

- 1 x y: 如果 x,y 不在同一颗子树中, 则通过在 x,y 之间连边的方式, 连接这两颗子树
 - 2 x y: 如果 x,y 在同一颗子树中, 且 $x \neq y$, 则将 x 视为这颗子树的根以后, 切断 y 与其父亲结点的连接
 - 3 w x y: 如果 x,y 在同一颗子树中, 则将 x,y 之间路径上所有点的点权增加 w
 - 4 x y: 如果 x,y 在同一颗子树中, 输出 x,y 之间路径上点权的最大值
- 非法操作输出 -1

```

1  const int MAXN = 300010;
2  const int INF = 0x3f3f3f3f;
3  struct Node *null;
4  struct Node{
5      Node *fa,*ch[2];
6      int Max,val;
7      int rev;//旋转标记
8      int add;
9      inline void clear(int _val){
10         fa = ch[0] = ch[1] = null;
11         val = Max = _val;
12         rev = 0;
13         add = 0;
14     }
15     inline void push_up(){
16         Max = max(val,max(ch[0]->Max,ch[1]->Max));
17     }
18     inline void setc(Node *p,int d){
19         ch[d] = p;
20         p->fa = this;
21     }
22     inline bool d(){
23         return fa->ch[1] == this;
24     }
25     inline bool isroot(){
26         return fa == null || fa->ch[0] != this && fa->ch[1] != this
27             ;
28     }
29     //翻转
30     inline void flip(){
31         if(this == null)return;
32         swap(ch[0],ch[1]);
33         rev ^= 1;
34     }
35     inline void update_add(int w){
36         if(this == null)return;
37         val += w;

```

```

37         add += w;
38         Max += w;
39     }
40     inline void push_down(){
41         if(rev){
42             ch[0]→flip(); ch[1]→flip(); rev = 0;
43         }
44         if(add){
45             ch[0]→update_add(add); ch[1]→update_add(add);
46             add = 0;
47         }
48     }
49     //直接标记下放
50     inline void go(){
51         if(!isroot())fa→go();
52         push_down();
53     }
54     inline void rot(){
55         Node *f = fa, *ff = fa→fa;
56         int c = d(), cc = fa→d();
57         f→setc(ch[!c],c);
58         this→setc(f,!c);
59         if(ff→ch[cc] == f)ff→setc(this,cc);
60         else this→fa = ff;
61         f→push_up();
62     }
63     inline Node* splay(){
64         go();
65         while(!isroot()){
66             if(!fa→isroot())
67                 d()==fa→d() ? fa→rot() : rot();
68             rot();
69         }
70         push_up();
71         return this;
72     }
73     inline Node* access(){
74         for(Node *p = this,*q = null; p != null; q = p, p = p→fa){
75             p→splay()→setc(q,1);
76             p→push_up();
77         }
78         return splay();
79     }
80     //找该点的根
81     inline Node* find_root(){
82         Node *x;
83         for(x = access(); x→push_down(), x→ch[0] != null; x = x→ch[0]);
84         return x;
85     }
86     //变为树根 (换根操作)

```

```

87     void make_root(){
88         access()->flip();
89     }
90     //切断该点和父亲结点的边
91     void cut(){
92         access();
93         ch[0]->fa = null;
94         ch[0] = null;
95         push_up();
96     }
97     //切断该点以 x 为根时, 该点和父亲结点的根
98     //要求这个点和 x 在同一颗树而且不能相同
99     //x 变为所在树的树根
100    void cut(Node* x){
101        if(this == x || find_root() != x->find_root())
102            puts("-1");
103        else {
104            x->make_root();
105            cut();
106        }
107    }
108    //该点连接到 x
109    //假如是有虚边信息的, 需要先 x->access() 再连接
110    void link(Node *x){
111        if(find_root() == x->find_root())
112            puts("-1");
113        else {
114            make_root(); fa = x;
115        }
116    }
117 };
118 Node pool[MAXN],*tail;
119 Node *node[MAXN];
120 struct Edge{
121     int to,next;
122 }edge[MAXN*2];
123 int head[MAXN],tot;
124 inline void addedge(int u,int v){
125     edge[tot].to = v;
126     edge[tot].next = head[u];
127     head[u] = tot++;
128 }
129 void dfs(int u,int pre){
130     for(int i = head[u];i != -1;i = edge[i].next){
131         int v = edge[i].to;
132         if(v == pre)continue;
133         node[v]->fa = node[u];
134         dfs(v,u);
135     }
136 }
137 void ADD(Node *x,Node *y,int w){

```

```

138     x->access();
139     for(x = null; y != null; x = y, y = y->fa){
140         y->splay();
141         if(y->fa == null){
142             y->ch[1]->update_add(w);
143             x->update_add(w);
144             y->val += w;
145             y->push_up();
146             return;
147         }
148         y->setc(x,1);
149         y->push_up();
150     }
151 }
152 int ask(Node *x,Node *y){
153     x->access();
154     for(x = null; y != null; x = y, y = y->fa){
155         y->splay();
156         if(y->fa == null)
157             return max(y->val,max(y->ch[1]->Max,x->Max));
158         y->setc(x,1);
159         y->push_up();
160     }
161 }
162 int main()
163 {
164     int n;
165     while(scanf("%d",&n) == 1){
166         for(int i = 1;i <= n;i++)head[i] = -1;
167         tot = 0;
168         int u,v;
169         for(int i = 1;i < n;i++){
170             scanf("%d%d",&u,&v);
171             addedge(u,v);
172             addedge(v,u);
173         }
174         tail = pool;
175         null = tail++;
176         null->clear(-INF);
177         for(int i = 1;i <= n;i++){
178             node[i] = tail++;
179             scanf("%d",&v);
180             node[i]->clear(v);
181         }
182         dfs(1,1);
183         int m,op;
184         int x,y,w;
185         scanf("%d",&m);
186         while(m--){
187             scanf("%d",&op);
188             if(op == 1){

```



```

189         scanf("%d%d",&x,&y);
190         node[x]→link(node[y]);
191     }
192     else if(op == 2){
193         scanf("%d%d",&x,&y);
194         node[y]→cut(node[x]);
195     }
196     else if(op == 3){
197         scanf("%d%d%d",&w,&x,&y);
198         if(node[x]→find_root() != node[y]→find_root())
199             printf("-1\n");
200         else ADD(node[x],node[y],w);
201     }
202     else{
203         scanf("%d%d",&x,&y);
204         if(node[x]→find_root() != node[y]→find_root())
205             printf("-1\n");
206         else printf("%d\n",ask(node[x],node[y]));
207     }
208 }
209 printf("\n");
210 }
211 return 0;
212 }

```

3.6 主席树

3.6.1 查询区间多少个不同的数

查询区间有多少个不同的数 (SPOJ DQUERY)

```

1  /*
2   *  给出一个序列，查询区间内有多少个不相同的数
3   */
4  const int MAXN = 30010;
5  const int M = MAXN * 100;
6  int n,q,tot;
7  int a[MAXN];
8  int T[MAXN],lson[M],rson[M],c[M];
9  int build(int l,int r){
10     int root = tot++;
11     c[root] = 0;
12     if(l != r){
13         int mid = (l+r)>>1;
14         lson[root] = build(l,mid);
15         rson[root] = build(mid+1,r);
16     }
17     return root;
18 }
19 int update(int root,int pos,int val){
20     int newroot = tot++, tmp = newroot;
21     c[newroot] = c[root] + val;

```

```

22     int l = 1, r = n;
23     while(l < r){
24         int mid = (l+r)>>1;
25         if(pos <= mid){
26             lson[newroot] = tot++; rson[newroot] = rson[root];
27             newroot = lson[newroot]; root = lson[root];
28             r = mid;
29         }
30         else{
31             rson[newroot] = tot++; lson[newroot] = lson[root];
32             newroot = rson[newroot]; root = rson[root];
33             l = mid+1;
34         }
35         c[newroot] = c[root] + val;
36     }
37     return tmp;
38 }
39 int query(int root,int pos){
40     int ret = 0;
41     int l = 1, r = n;
42     while(pos < r){
43         int mid = (l+r)>>1;
44         if(pos <= mid){
45             r = mid;
46             root = lson[root];
47         }
48         else{
49             ret += c[lson[root]];
50             root = rson[root];
51             l = mid+1;
52         }
53     }
54     return ret + c[root];
55 }
56 int main(){
57     while(scanf("%d",&n) == 1){
58         tot = 0;
59         for(int i = 1;i <= n;i++){
60             scanf("%d",&a[i]);
61             T[n+1] = build(1,n);
62             map<int,int>mp;
63             for(int i = n;i >= 1;i--){
64                 if(mp.find(a[i]) == mp.end()){
65                     T[i] = update(T[i+1],i,1);
66                 }
67                 else{
68                     int tmp = update(T[i+1],mp[a[i]],-1);
69                     T[i] = update(tmp,i,1);
70                 }
71                 mp[a[i]] = i;
72             }

```

```

73         scanf("%d",&q);
74         while(q--){
75             int l,r;
76             scanf("%d%d",&l,&r);
77             printf("%d\n",query(T[l],r));
78         }
79     }
80     return 0;
81 }

```

3.6.2 静态区间第 k 大

POJ2104

```

1  const int MAXN = 100010;
2  const int M = MAXN * 30;
3  int n,q,m,tot;
4  int a[MAXN], t[MAXN];
5  int T[MAXN], lson[M], rson[M], c[M];
6  void Init_hash(){
7      for(int i = 1; i <= n;i++)
8          t[i] = a[i];
9      sort(t+1,t+1+n);
10     m = unique(t+1,t+1+n)-t-1;
11 }
12 int build(int l,int r){
13     int root = tot++;
14     c[root] = 0;
15     if(l != r){
16         int mid = (l+r)>>1;
17         lson[root] = build(l,mid);
18         rson[root] = build(mid+1,r);
19     }
20     return root;
21 }
22 int hash(int x){
23     return lower_bound(t+1,t+1+m,x) - t;
24 }
25 int update(int root,int pos,int val){
26     int newroot = tot++, tmp = newroot;
27     c[newroot] = c[root] + val;
28     int l = 1, r = m;
29     while(l < r){
30         int mid = (l+r)>>1;
31         if(pos <= mid){
32             lson[newroot] = tot++; rson[newroot] = rson[root];
33             newroot = lson[newroot]; root = lson[root];
34             r = mid;
35         }
36         else{
37             rson[newroot] = tot++; lson[newroot] = lson[root];
38             newroot = rson[newroot]; root = rson[root];

```

```

39         l = mid+1;
40     }
41     c[newroot] = c[root] + val;
42 }
43 return tmp;
44 }
45 int query(int left_root,int right_root,int k){
46     int l = 1, r = m;
47     while( l < r){
48         int mid = (l+r)>>1;
49         if(c[lson[left_root]]-c[lson[right_root]] >= k ){
50             r = mid;
51             left_root = lson[left_root];
52             right_root = lson[right_root];
53         }
54         else{
55             l = mid + 1;
56             k -= c[lson[left_root]] - c[lson[right_root]];
57             left_root = rson[left_root];
58             right_root = rson[right_root];
59         }
60     }
61     return l;
62 }
63 int main(){
64     while(scanf("%d%d",&n,&q) == 2){
65         tot = 0;
66         for(int i = 1;i <= n;i++){
67             scanf("%d",&a[i]);
68             Init_hash();
69             T[n+1] = build(1,m);
70             for(int i = n;i ;i--){
71                 int pos = hash(a[i]);
72                 T[i] = update(T[i+1],pos,1);
73             }
74             while(q--){
75                 int l,r,k;
76                 scanf("%d%d%d",&l,&r,&k);
77                 printf("%d\n",t[query(T[l],T[r+1],k)]);
78             }
79         }
80         return 0;
81     }

```

3.6.3 树上路径点权第 k 大

树上路径点权第 k 大 (SPOJ COT)

LCA + 主席树

```

1 //主席树部分 *****
2 const int MAXN = 200010;
3 const int M = MAXN * 40;

```

```

4  int n,q,m,TOT;
5  int a[MAXN], t[MAXN];
6  int T[MAXN], lson[M], rson[M], c[M];
7  void Init_hash(){
8      for(int i = 1; i <= n;i++)
9          t[i] = a[i];
10     sort(t+1,t+1+n);
11     m = unique(t+1,t+n+1)-t-1;
12 }
13 int build(int l,int r){
14     int root = TOT++;
15     c[root] = 0;
16     if(l != r){
17         int mid = (l+r)>>1;
18         lson[root] = build(l,mid);
19         rson[root] = build(mid+1,r);
20     }
21     return root;
22 }
23 int hash(int x){
24     return lower_bound(t+1,t+1+m,x) - t;
25 }
26 int update(int root,int pos,int val){
27     int newroot = TOT++, tmp = newroot;
28     c[newroot] = c[root] + val;
29     int l = 1, r = m;
30     while( l < r){
31         int mid = (l+r)>>1;
32         if(pos <= mid){
33             lson[newroot] = TOT++; rson[newroot] = rson[root];
34             newroot = lson[newroot]; root = lson[root];
35             r = mid;
36         }
37         else{
38             rson[newroot] = TOT++; lson[newroot] = lson[root];
39             newroot = rson[newroot]; root = rson[root];
40             l = mid+1;
41         }
42         c[newroot] = c[root] + val;
43     }
44     return tmp;
45 }
46 int query(int left_root,int right_root,int LCA,int k){
47     int lca_root = T[LCA];
48     int pos = hash(a[LCA]);
49     int l = 1, r = m;
50     while(l < r){
51         int mid = (l+r)>>1;
52         int tmp = c[lson[left_root]] + c[lson[right_root]] - 2*c[
                    lson[lca_root]] + (pos >= l && pos <= mid);
53         if(tmp >= k){

```

```

54         left_root = lson[left_root];
55         right_root = lson[right_root];
56         lca_root = lson[lca_root];
57         r = mid;
58     }
59     else{
60         k -= tmp;
61         left_root = rson[left_root];
62         right_root = rson[right_root];
63         lca_root = rson[lca_root];
64         l = mid + 1;
65     }
66 }
67 return l;
68 }
69
70 //LCA 部分
71 int rmq[2*MAXN]; //rmq 数组, 就是欧拉序列对应的深度序列
72 struct ST{
73     int mm[2*MAXN];
74     int dp[2*MAXN][20]; //最小值对应的下标
75     void init(int n){
76         mm[0] = -1;
77         for(int i = 1; i <= n; i++){
78             mm[i] = ((i&(i-1)) == 0)?mm[i-1]+1:mm[i-1];
79             dp[i][0] = i;
80         }
81         for(int j = 1; j <= mm[n]; j++){
82             for(int i = 1; i + (1<<j) - 1 <= n; i++){
83                 dp[i][j] = rmq[dp[i][j-1]] < rmq[dp[i+(1<<(j-1))][j-1]]?dp[i][j-1]:dp[i+(1<<(j-1))][j-1];
84             }
85             //查询 [a,b] 之间最小值的下标
86             int query(int a,int b){
87                 if(a > b)swap(a,b);
88                 int k = mm[b-a+1];
89                 return rmq[dp[a][k]] <= rmq[dp[b-(1<<k)+1][k]]?dp[a][k]:dp[b-(1<<k)+1][k];
90             }
91 };
92 //边的结构体定义
93 struct Edge{
94     int to,next;
95 };
96 Edge edge[MAXN*2];
97 int tot,head[MAXN];
98
99 int F[MAXN*2]; //欧拉序列, 就是 dfs 遍历的顺序, 长度为 2*n-1, 下标从 1 开始
100 int P[MAXN]; //P[i] 表示点 i 在 F 中第一次出现的位置
101 int cnt;
102

```

```

103 ST st;
104 void init(){
105     tot = 0;
106     memset(head,-1,sizeof(head));
107 }
108 //加边, 无向边需要加两次
109 void addedge(int u,int v){
110     edge[tot].to = v;
111     edge[tot].next = head[u];
112     head[u] = tot++;
113 }
114 void dfs(int u,int pre,int dep){
115     F[++cnt] = u;
116     rmq[cnt] = dep;
117     P[u] = cnt;
118     for(int i = head[u]; i != -1; i = edge[i].next){
119         int v = edge[i].to;
120         if(v == pre)continue;
121         dfs(v,u,dep+1);
122         F[++cnt] = u;
123         rmq[cnt] = dep;
124     }
125 }
126 //查询 LCA 前的初始化
127 void LCA_init(int root,int node_num){
128     cnt = 0;
129     dfs(root,root,0);
130     st.init(2*node_num-1);
131 }
132 //查询 u,v 的 lca 编号
133 int query_lca(int u,int v){
134     return F[st.query(P[u],P[v])];
135 }
136 void dfs_build(int u,int pre){
137     int pos = hash(a[u]);
138     T[u] = update(T[pre],pos,1);
139     for(int i = head[u]; i != -1; i = edge[i].next){
140         int v = edge[i].to;
141         if(v == pre)continue;
142         dfs_build(v,u);
143     }
144 }
145 int main(){
146     while(scanf("%d%d",&n,&q) == 2){
147         for(int i = 1; i <= n; i++){
148             scanf("%d",&a[i]);
149             Init_hash();
150             init();
151             TOT = 0;
152             int u,v;
153             for(int i = 1; i < n; i++){

```

```

154         scanf("%d%d",&u,&v);
155         addedge(u,v);
156         addedge(v,u);
157     }
158     LCA_init(1,n);
159     T[n+1] = build(1,m);
160     dfs_build(1,n+1);
161     int k;
162     while(q--){
163         scanf("%d%d%d",&u,&v,&k);
164         printf("%d\n",t[query(T[u],T[v],query_lca(u,v),k)]);
165     }
166     return 0;
167 }
168 return 0;
169 }

```

3.6.4 动态第 k 大

树状数组套主席树 ZOJ 2112

```

1  const int MAXN = 60010;
2  const int M = 2500010;
3  int n,q,m,tot;
4  int a[MAXN], t[MAXN];
5  int T[MAXN], lson[M], rson[M],c[M];
6  int S[MAXN];
7  struct Query{
8      int kind;
9      int l,r,k;
10 }query[10010];
11
12 void Init_hash(int k){
13     sort(t,t+k);
14     m = unique(t,t+k) - t;
15 }
16 int hash(int x){
17     return lower_bound(t,t+m,x)-t;
18 }
19 int build(int l,int r){
20     int root = tot++;
21     c[root] = 0;
22     if(l != r){
23         int mid = (l+r)/2;
24         lson[root] = build(l,mid);
25         rson[root] = build(mid+1,r);
26     }
27     return root;
28 }
29
30 int Insert(int root,int pos,int val){
31     int newroot = tot++, tmp = newroot;

```



```

32     int l = 0, r = m-1;
33     c[newroot] = c[root] + val;
34     while(l < r){
35         int mid = (l+r)>>1;
36         if(pos <= mid){
37             lson[newroot] = tot++; rson[newroot] = rson[root];
38             newroot = lson[newroot]; root = lson[root];
39             r = mid;
40         }
41         else{
42             rson[newroot] = tot++; lson[newroot] = lson[root];
43             newroot = rson[newroot]; root = rson[root];
44             l = mid+1;
45         }
46         c[newroot] = c[root] + val;
47     }
48     return tmp;
49 }
50
51 int lowbit(int x){
52     return x&(-x);
53 }
54 int use[MAXN];
55 void add(int x,int pos,int val){
56     while(x <= n){
57         S[x] = Insert(S[x],pos,val);
58         x += lowbit(x);
59     }
60 }
61 int sum(int x){
62     int ret = 0;
63     while(x > 0){
64         ret += c[lson[use[x]]];
65         x -= lowbit(x);
66     }
67     return ret;
68 }
69 int Query(int left,int right,int k){
70     int left_root = T[left-1];
71     int right_root = T[right];
72     int l = 0, r = m-1;
73     for(int i = left-1;i;i -= lowbit(i)) use[i] = S[i];
74     for(int i = right;i;i -= lowbit(i)) use[i] = S[i];
75     while(l < r){
76         int mid = (l+r)/2;
77         int tmp = sum(right) - sum(left-1) + c[lson[right_root]] -
              c[lson[left_root]];
78         if(tmp >= k){
79             r = mid;
80             for(int i = left-1; i ;i -= lowbit(i))
81                 use[i] = lson[use[i]];

```

```

82         for(int i = right; i; i -= lowbit(i))
83             use[i] = lson[use[i]];
84         left_root = lson[left_root];
85         right_root = lson[right_root];
86     }
87     else{
88         l = mid+1;
89         k -= tmp;
90         for(int i = left-1; i; i -= lowbit(i))
91             use[i] = rson[use[i]];
92         for(int i = right; i; i -= lowbit(i))
93             use[i] = rson[use[i]];
94         left_root = rson[left_root];
95         right_root = rson[right_root];
96     }
97 }
98 return l;
99 }
100 void Modify(int x,int p,int d){
101     while(x <= n){
102         S[x] = Insert(S[x],p,d);
103         x += lowbit(x);
104     }
105 }
106
107 int main(){
108     int Tcase;
109     scanf("%d",&Tcase);
110     while(Tcase--){
111         scanf("%d%d",&n,&q);
112         tot = 0;
113         m = 0;
114         for(int i = 1; i <= n; i++){
115             scanf("%d",&a[i]);
116             t[m++] = a[i];
117         }
118         char op[10];
119         for(int i = 0; i < q; i++){
120             scanf("%s",op);
121             if(op[0] == 'Q'){
122                 query[i].kind = 0;
123                 scanf("%d%d%d",&query[i].l,&query[i].r,&query[i].k)
124                     ;
125             }
126             else{
127                 query[i].kind = 1;
128                 scanf("%d%d",&query[i].l,&query[i].r);
129                 t[m++] = query[i].r;
130             }
131         }
132         Init_hash(m);

```

```

132     T[0] = build(0,m-1);
133     for(int i = 1;i <= n;i++)
134         T[i] = Insert(T[i-1],hash(a[i]),1);
135     for(int i = 1;i <= n;i++)
136         S[i] = T[0];
137     for(int i = 0;i < q;i++){
138         if(query[i].kind == 0)
139             printf("%d\n",t[Query(query[i].l,query[i].r,query[i]
140                                     ].k)]);
141         else{
142             Modify(query[i].l,hash(a[query[i].l]),-1);
143             Modify(query[i].l,hash(query[i].r),1);
144             a[query[i].l] = query[i].r;
145         }
146     }
147     return 0;
148 }

```

3.7 Treap

ZOJ3765

```

1 long long gcd(long long a,long long b){
2     if(b == 0)return a;
3     else return gcd(b,a%b);
4 }
5 const int MAXN = 300010;
6 int num[MAXN],st[MAXN];
7 struct Treap{
8     int tot1;
9     int s[MAXN],tot2;//内存池和容量
10    int ch[MAXN][2];
11    int key[MAXN],size[MAXN];
12    int sum0[MAXN],sum1[MAXN];
13    int status[MAXN];
14    void Init(){
15        tot1 = tot2 = 0;
16        size[0] = 0;
17        ch[0][0] = ch[0][1] = 0;
18        sum0[0] = sum1[0] = 0;
19    }
20    bool random(double p){
21        return (double)rand() / RAND_MAX < p;
22    }
23    int newnode(int val,int _status){
24        int r;
25        if(tot2)r = s[tot2--];
26        else r = ++tot1;
27        size[r] = 1;
28        key[r] = val;
29        status[r] = _status;

```

```

30     ch[r][0] = ch[r][1] = 0;
31     sum0[r] = sum1[r] = 0; //需要push_up
32     return r;
33 }
34 void del(int r){
35     if(!r) return;
36     s[++tot2] = r;
37     del(ch[r][0]);
38     del(ch[r][1]);
39 }
40 void push_up(int r){
41     int lson = ch[r][0], rson = ch[r][1];
42     size[r] = size[lson] + size[rson] + 1;
43     sum0[r] = gcd(sum0[lson], sum0[rson]);
44     sum1[r] = gcd(sum1[lson], sum1[rson]);
45     if(status[r] == 0)
46         sum0[r] = gcd(sum0[r], key[r]);
47     else sum1[r] = gcd(sum1[r], key[r]);
48 }
49 void merge(int &p, int x, int y){
50     if(!x || !y)
51         p = x | y;
52     else if(random((double) size[x] / (size[x] + size[y]))) {
53         merge(ch[x][1], ch[x][1], y);
54         push_up(p=x);
55     }
56     else{
57         merge(ch[y][0], x, ch[y][0]);
58         push_up(p=y);
59     }
60 }
61 void split(int p, int &x, int &y, int k){
62     if(!k){
63         x = 0; y = p;
64         return;
65     }
66     if(size[ch[p][0]] >= k){
67         y = p;
68         split(ch[p][0], x, ch[y][0], k);
69         push_up(y);
70     }
71     else{
72         x = p;
73         split(ch[p][1], ch[x][1], y, k - size[ch[p][0]] - 1);
74         push_up(x);
75     }
76 }
77 void build(int &p, int l, int r){
78     if(l > r) return;
79     int mid = (l + r) / 2;
80     p = newnode(num[mid], st[mid]);

```

```

81     build(ch[p][0],l,mid-1);
82     build(ch[p][1],mid+1,r);
83     push_up(p);
84 }
85 void debug(int root){
86     if(root == 0)return;
87     printf("%d左儿子: %d右儿子: %dsize=%dkey=%d\n",
88         root,ch[root][0],ch[root][1],size[root],key[root]);
89     debug(ch[root][0]);
90     debug(ch[root][1]);
91 }
92 Treap T;
93 char op[10];
94 int main(){
95     int n,q;
96     while(scanf("%d%d",&n,&q) == 2){
97         int root = 0;
98         T.Init();
99         for(int i = 1;i <= n;i++){
100             scanf("%d%d",&num[i],&st[i]);
101             T.build(root,1,n);
102             while(q--){
103                 scanf("%s",op);
104                 if(op[0] == 'Q'){
105                     int l,r,s;
106                     scanf("%d%d%d",&l,&r,&s);
107                     int x,y,z;
108                     T.split(root,x,z,r);
109                     T.split(x,x,y,l-1);
110                     if(s == 0)
111                         printf("%d\n",T.sum0[y] == 0? -1:T.sum0[y]);
112                     else
113                         printf("%d\n",T.sum1[y] == 0?-1:T.sum1[y]);
114                     T.merge(x,x,y);
115                     T.merge(root,x,z);
116                 }
117                 else if(op[0] == 'I'){
118                     int v,s,loc;
119                     scanf("%d%d%d",&loc,&v,&s);
120                     int x,y;
121                     T.split(root,x,y,loc);
122                     T.merge(x,x,T.newnode(v,s));
123                     T.merge(root,x,y);
124                 }
125                 else if(op[0] == 'D'){
126                     int loc;
127                     scanf("%d",&loc);
128                     int x,y,z;
129                     T.split(root,x,z,loc);
130                     T.split(x,x,y,loc-1);

```

```

131         T.del(y);
132         T.merge(root,x,z);
133     }
134     else if(op[0] == 'R'){
135         int loc;
136         scanf("%d",&loc);
137         int x,y,z;
138         T.split(root,x,z,loc);
139         T.split(x,x,y,loc-1);
140         T.status[y] = 1-T.status[y];
141         T.push_up(y);
142         T.merge(x,x,y);
143         T.merge(root,x,z);
144     }
145     else{
146         int loc,v;
147         scanf("%d%d",&loc,&v);
148         int x,y,z;
149         T.split(root,x,z,loc);
150         T.split(x,x,y,loc-1);
151         T.key[y] = v;
152         T.push_up(y);
153         T.merge(x,x,y);
154         T.merge(root,x,z);
155     }
156 }
157 }
158 return 0;
159 }

```

3.8 KD 树

3.8.1 HDU4347 K 近邻

模板题，求出最近的 K 个点。

```

1  const int MAXN = 50010;
2  const int DIM = 10;
3  inline double sqr(double x){return x*x;}
4  namespace KDTree{
5      int K;//维数
6      struct Point{
7          int x[DIM];
8          double distance(const Point &b)const{
9              double ret = 0;
10             for(int i = 0;i < K;i++)
11                 ret += sqr(x[i]-b.x[i]);
12             return ret;
13         }
14         void input(){
15             for(int i = 0;i < K;i++)scanf("%d",&x[i]);
16         }

```

```

17     void output(){
18         for(int i = 0;i < K;i++)
19             printf("%d%c",x[i],i < K-1?' ':'\n');
20     }
21 };
22 struct qnode{
23     Point p;
24     double dis;
25     qnode(){}
26     qnode(Point _p,double _dis){
27         p = _p; dis = _dis;
28     }
29     bool operator <(const qnode &b)const{
30         return dis < b.dis;
31     }
32 };
33 priority_queue<qnode>q;
34 struct cmpx{
35     int div;
36     cmpx(const int &_div){div = _div;}
37     bool operator()(const Point &a,const Point &b){
38         for(int i = 0;i < K;i++)
39             if(a.x[(div+i)%K] != b.x[(div+i)%K])
40                 return a.x[(div+i)%K] < b.x[(div+i)%K];
41         return true;
42     }
43 };
44 bool cmp(const Point &a,const Point &b,int div){
45     cmpx cp = cmpx(div);
46     return cp(a,b);
47 }
48 struct Node{
49     Point e;
50     Node *lc,*rc;
51     int div;
52 }pool[MAXN],*tail,*root;
53 void init(){
54     tail = pool;
55 }
56 Node* build(Point *a,int l,int r,int div){
57     if(l >= r)return NULL;
58     Node *p = tail++;
59     p->div = div;
60     int mid = (l+r)/2;
61     nth_element(a+l,a+mid,a+r,cmpx(div));
62     p->e = a[mid];
63     p->lc = build(a,l,mid,(div+1)%K);
64     p->rc = build(a,mid+1,r,(div+1)%K);
65     return p;
66 }
67 void search(Point p,Node *x,int div,int m){

```

```

68         if(!x)return;
69         if(cmp(p,x->e,div)){
70             search(p,x->lc,(div+1)%K,m);
71             if(q.size() < m){
72                 q.push(qnode(x->e,p.distance(x->e)));
73                 search(p,x->rc,(div+1)%K,m);
74             }
75             else {
76                 if(p.distance(x->e) < q.top().dis){
77                     q.pop();
78                     q.push(qnode(x->e,p.distance(x->e)));
79                 }
80                 if(sqr(x->e.x[div]-p.x[div]) < q.top().dis)
81                     search(p,x->rc,(div+1)%K,m);
82             }
83         }
84         else {
85             search(p,x->rc,(div+1)%K,m);
86             if(q.size() < m){
87                 q.push(qnode(x->e,p.distance(x->e)));
88                 search(p,x->lc,(div+1)%K,m);
89             }
90             else {
91                 if(p.distance(x->e) < q.top().dis){
92                     q.pop();
93                     q.push(qnode(x->e,p.distance(x->e)));
94                 }
95                 if(sqr(x->e.x[div]-p.x[div]) < q.top().dis)
96                     search(p,x->lc,(div+1)%K,m);
97             }
98         }
99     }
100     void search(Point p,int m){
101         while(!q.empty())q.pop();
102         search(p,root,0,m);
103     }
104 };
105 KDTree::Point p[MAXN];
106 int main()
107 {
108     int n,k;
109     while(scanf("%d%d",&n,&k) == 2){
110         KDTree::K = k;
111         for(int i = 0;i < n;i++)p[i].input();
112         KDTree::init();
113         KDTree::root = KDTree::build(p,0,n,0);
114         int Q;
115         scanf("%d",&Q);
116         KDTree::Point o;
117         while(Q--){
118             o.input();

```



```

119         int m;
120         scanf("%d",&m);
121         KDTree::search(o,m);
122         printf("the closest %d points are:\n",m);
123         int cnt = 0;
124         while(!KDTree::q.empty()){
125             p[cnt++] = KDTree::q.top().p;
126             KDTree::q.pop();
127         }
128         for(int i = 0;i < m;i++)p[m-1-i].output();
129     }
130 }
131 return 0;
132 }

```

3.8.2 CF44G

给定若干个靶子 (xl, xr, yl, yr, z) , z 为该靶子离射击位置的距离, 所有靶子都可以看成是二维平面上平行于坐标轴的矩形。然后按顺序给定若干子弹的射击位置 (x, y) , 子弹射到一个靶子就会将靶子打碎, 并掉落到地上。问每个子弹射到的靶子是谁。保证靶子的 z 值不相同。

找矩形内权值最小的点, 支持删除操作

```

1  const int MAXN = 100010;
2  const int INF = 0x3f3f3f3f;
3  struct Point{
4      int x,y,id;
5      bool operator ==(const Point &b)const{
6          return x == b.x && y == b.y && id == b.id;
7      }
8  };
9  struct Node{
10     Point e;
11     Node *lc,*rc;
12     bool div;
13     int sub,cur;
14     int size;
15     bool exist;
16     void push_up(){
17         size = lc->size + rc->size + exist;
18         sub = min(cur,min(lc->sub,rc->sub));
19     }
20 }pool[MAXN],*root,*tail,*null;
21 inline bool cmpX(const Point &a,const Point &b){return a.x < b.x ||
    (a.x == b.x && a.y < b.y) || (a.x == b.x && a.y == b.y && a.id
    < b.id);}
22 inline bool cmpY(const Point &a,const Point &b){return a.y < b.y ||
    (a.y == b.y && a.x < b.x) || (a.y == b.y && a.x == b.x && a.id
    < b.id);}
23 inline bool cmp(const Point &a,const Point &b,bool div){return div?
    cmpY(a,b):cmpX(a,b);}
24 Node* build(Point *a,int l,int r,bool div){

```

```

25     if(l >= r)return null;
26     Node *p = tail++;
27     p->div = div;
28     int mid = (l+r)/2;
29     nth_element(a+l,a+mid,a+r,div?cmpY:cmpX);
30     p->e = a[mid];
31     p->lc = build(a,l,mid,!div);
32     p->rc = build(a,mid+1,r,!div);
33     p->exist = 1;
34     p->cur = p->e.id;
35     p->push_up();
36     return p;
37 }
38 void remove(Node *p,Point o){
39     if(p->e == o){
40         p->exist = 0;
41         p->cur = INF;
42         p->size--;
43     }
44     else {
45         if(cmp(p->e,o,p->div))remove(p->rc,o);
46         else remove(p->lc,o);
47     }
48     p->push_up();
49 }
50 int getMin(Node *p,int xl,int xr,int yl,int yr,int minx,int maxx,
51 int miny,int maxy){
52     if(p == null || p->size == 0)return INF;
53     if(xl <= minx && xr >= maxx && yl <= miny && yr >= maxy)return
54         p->sub;
55     if(xl > maxx || xr < minx || yl > maxy || yr < miny)return INF;
56     int ret = INF;
57     if(p->e.x >= xl && p->e.x <= xr && p->e.y >= yl && p->e.y <= yr
58         )
59         ret = min(ret,p->cur);
60     if(p->div){
61         if(yl <= p->e.y)
62             ret = min(ret,getMin(p->lc,xl,xr,yl,min(yr,p->e.y),minx
63                 ,maxx,miny,min(maxy,p->e.y)));
64         if(yr >= p->e.y)
65             ret = min(ret,getMin(p->rc,xl,xr,max(yl,p->e.y),yr,minx
66                 ,maxx,max(miny,p->e.y),maxy));
67     }
68     else {
69         if(xl <= p->e.x)
70             ret = min(ret,getMin(p->lc,xl,min(xr,p->e.x),yl,yr,minx
71                 ,min(maxx,p->e.x),miny,maxy));
72         if(xr >= p->e.x)
73             ret = min(ret,getMin(p->rc,max(xl,p->e.x),xr,yl,yr,max(
74                 minx,p->e.x),maxx,miny,maxy));
75     }
76 }

```

```

69     return ret;
70 }
71 Point pp[MAXN],pp2[MAXN];
72 struct REC{
73     int xl,xr,yl,yr,z;
74     int id;
75     void input(){
76         scanf("%d%d%d%d%d",&xl,&xr,&yl,&yr,&z);
77     }
78     bool operator <(const REC &b)const{
79         return z < b.z;
80     }
81 }rec[MAXN];
82 int ans[MAXN];
83 int main()
84 {
85     int n,m;
86     while(scanf("%d",&n) == 1){
87         for(int i = 0;i < n;i++){
88             rec[i].input();
89             rec[i].id = i+1;
90         }
91         sort(rec,rec+n);
92         scanf("%d",&m);
93         for(int i = 0;i < m;i++){
94             scanf("%d%d",&pp[i].x,&pp[i].y);
95             pp[i].id = i;
96             pp2[i] = pp[i]; //备份
97         }
98         tail = pool;
99         null = tail++;
100        null->size = 0;
101        null->sub = null->cur = INF;
102        null->lc = null->rc = null;
103        root = build(pp,0,m,0);
104        memset(ans,0,sizeof(ans));
105        for(int i = 0;i < n;i++){
106            int tmp = getMin(root,rec[i].xl,rec[i].xr,rec[i].yl,rec
                [i].yr,-INF,INF,-INF,INF);
107            if(tmp == INF)continue;
108            ans[tmp] = rec[i].id;
109            remove(root,pp2[tmp]);
110        }
111        for(int i = 0;i < m;i++)printf("%d\n",ans[i]);
112    }
113    return 0;
114 }

```

3.8.3 HDU4742

三维 LIS。即每个点有个三维坐标，两个点能放在一前一后当且仅当 $x_i < x_j, y_i < y_j, z_i < z_j$ ，求最长的序列，并该条件下的方案数。

```

1  const int MAXN = 100010;
2  const int MOD = 1<<30;
3  const int INF = 0x7fffffff;//这个一定要够大
4  struct Node{
5      pair<int,int>e,sub,cur;
6      bool div;
7      Node *lc,*rc;
8  };
9  Node pool[MAXN],*tail;
10 Node *root;
11 bool cmpX(const pair<int,int> &a,const pair<int,int> &b){return a.
    first < b.first || (a.first == b.first && a.second < b.second)
    };
12 bool cmpY(const pair<int,int> &a,const pair<int,int> &b){return a.
    second < b.second || (a.second == b.second && a.first < b.first)
    };
13 bool cmp(const pair<int,int> &a,const pair<int,int> &b,bool div){
    return div?cmpY(a,b):cmpX(a,b);}
14 Node* build(pair<int,int> *a,int l,int r,bool div){
15     if(l >= r)return NULL;
16     Node *p = tail++;
17     p->div = div;
18     int mid = (l+r)/2;
19     nth_element(a+l,a+mid,a+r,div?cmpY:cmpX);
20     p->e = a[mid];
21     p->cur = p->sub = make_pair(0,0);
22     p->lc = build(a,l,mid,!div);
23     p->rc = build(a,mid+1,r,!div);
24     return p;
25 }
26 inline void update(pair<int,int> &a,pair<int,int> b){
27     if(a.first < b.first)a = b;
28     else if(a.first == b.first){
29         a.second += b.second;
30         if(a.second >= MOD)a.second -= MOD;
31     }
32 }
33 void add(Node *p,pair<int,int> e,pair<int,int> v){
34     update(p->sub,v);
35     if(e == p->e){
36         update(p->cur,v);
37         return;
38     }
39     else {
40         if(cmp(p->e,e,p->div))add(p->rc,e,v);
41         else add(p->lc,e,v);
42     }

```

```

43 }
44 pair<int,int>ans;
45 //查询最大值
46 void get(Node *p,pair<int,int>e,int maxx,int maxy){
47     if(!p)return;
48     if(p->sub.first < ans.first)return;
49     if(maxx <= e.first && maxy <= e.second)
50         update(ans,p->sub);
51     else {
52         if(p->e.first <= e.first && p->e.second <= e.second)update(
53             ans,p->cur);
54         if(p->div){
55             if(p->e.second <= e.second)get(p->rc,e,maxx,maxy);
56             get(p->lc,e,maxx,min(maxy,p->e.second));
57         }
58         else {
59             if(p->e.first <= e.first)get(p->rc,e,maxx,maxy);
60             get(p->lc,e,min(maxx,p->e.first),maxy);
61         }
62     }
63 struct TNode{
64     int x,y,z;
65     void input(){
66         scanf("%d%d%d",&x,&y,&z);
67     }
68     bool operator < (const TNode &b)const{
69         if(x != b.x)return x < b.x;
70         else if(y != b.y)return y < b.y;
71         else return z < b.z;
72     }
73 }node[MAXN];
74 pair<int,int>p[MAXN];
75 pair<int,int>dp[MAXN];
76 int main()
77 {
78     int T;
79     int n;
80     scanf("%d",&T);
81     while(T--){
82         scanf("%d",&n);
83         int cnt = 0;
84         for(int i = 0;i < n;i++){
85             node[i].input();
86             p[cnt++] = make_pair(node[i].y,node[i].z);
87         }
88         sort(node,node+n);
89         sort(p,p+cnt);
90         cnt = unique(p,p+cnt)-p;
91         tail = pool;
92         root = build(p,0,cnt,0);

```

```

93     for(int i = 0;i < n;i++)dp[i] = make_pair(1,1);
94     for(int i = 0;i < n;i++){
95         ans = make_pair(0,0);
96         get(root,make_pair(node[i].y,node[i].z),INF,INF);
97         ans.first++;
98         update(dp[i],ans);
99         add(root,make_pair(node[i].y,node[i].z),dp[i]);
100    }
101    printf("%d_%d\n",root->sub.first,root->sub.second);
102 }
103 return 0;
104 }

```

3.9 替罪羊树 (ScapeGoat Tree)

3.9.1 CF455D

<http://codeforces.com/contest/455/problem/D>

题意：给了一个序列，1 操作把一个区间的末尾的数插入到头部，2 操作是询问一个区间里面等于某个数的个数。

使用替罪羊树，里面套一个 map 来统计区间的个数。

```

1  const int MAXN = 200010;
2  const double alpha = 0.75;
3  struct Node{
4      Node *ch[2];
5      int size,key,nodeCount;
6      bool exist;
7      map<int,int>mp;
8      bool isBad(){
9          return ch[0]->nodeCount > alpha*nodeCount+5 || ch[1]->
              nodeCount > alpha*nodeCount + 5;
10     }
11     void push_up(){
12         size = exist + ch[0]->size + ch[1]->size;
13         nodeCount = 1 + ch[0]->nodeCount + ch[1]->nodeCount;
14         mp.clear();
15         if(exist)mp[key]++;
16         for(map<int,int>::iterator it = ch[0]->mp.begin();it != ch
            [0]->mp.end();it++)
17             mp[(*it).first] += (*it).second;
18         for(map<int,int>::iterator it = ch[1]->mp.begin();it != ch
            [1]->mp.end();it++)
19             mp[(*it).first] += (*it).second;
20     }
21 };
22 struct ScapeGoatTree{
23     Node pool[MAXN];
24     Node *tail,*root,*null;
25     Node *bc[MAXN]; //内存回收
26     int bc_top;
27     void init(){

```

```

28     tail = pool;
29     null = tail++;
30     null->ch[0] = null->ch[1] = null;
31     null->size = null->key = null->nodeCount = 0;
32     null->mp.clear();
33     root = null;
34     bc_top = 0;
35 }
36 inline Node *newNode(int key){
37     Node *p;
38     if(bc_top)p = bc[--bc_top];
39     else p = tail++;
40     p->ch[0] = p->ch[1] = null;
41     p->size = p->nodeCount = 1;
42     p->key = key;
43     p->exist = true;
44     p->mp.clear();
45     p->mp[key] = 1;
46     return p;
47 }
48 Node *buildTree(int *a,int l,int r){
49     if(l >= r)return null;
50     int mid = (l+r)>>1;
51     Node *p = newNode(a[mid]);
52     p->ch[0] = buildTree(a,l,mid);
53     p->ch[1] = buildTree(a,mid+1,r);
54     p->push_up();
55     return p;
56 }
57 inline void Travel(Node *p,vector<Node *>&v){
58     if(p == null)return;
59     Travel(p->ch[0],v);
60     if(p->exist)v.push_back(p);
61     else bc[bc_top++] = p;
62     Travel(p->ch[1],v);
63 }
64 inline Node *divide(vector<Node *>&v,int l,int r){
65     if(l >= r)return null;
66     int mid = (l+r)/2;
67     Node *p = v[mid];
68     p->ch[0] = divide(v,l,mid);
69     p->ch[1] = divide(v,mid+1,r);
70     p->push_up();
71     return p;
72 }
73 //重构, 注意 p 要引用
74 inline void rebuild(Node *&p){
75     vector<Node *>v;
76     Travel(p,v);
77     p = divide(v,0,v.size());
78 }

```

```

79 //删除第 id 个元素, 返回第 id 个元素的值
80 inline int erase(Node *p,int id){
81     if(p->exist && id == p->ch[0]->size + 1){
82         p->exist = 0;
83         p->mp[p->key]--;
84         p->size--;
85         return p->key;
86     }
87     p->size--;
88     int res;
89     if(p->ch[0]->size >= id)
90         res = erase(p->ch[0],id);
91     else res = erase(p->ch[1],id - p->ch[0]->size - p->exist);
92     p->mp[res]--;
93     return res;
94 }
95 //删除一定的点以后重构
96 void check_erase(){
97     if(root->size < 0.5*root->nodeCount)
98         rebuild(root);
99 }
100 Node **insert(Node *&p,int id,int val){
101     if(p == null){
102         p = newNode(val);
103         return &null;
104     }
105     else {
106         p->size++;
107         p->nodeCount++;
108         p->mp[val]++;
109         Node ** res;
110         if(id <= p->ch[0]->size+p->exist)
111             res = insert(p->ch[0],id,val);
112         else res = insert(p->ch[1],id-p->ch[0]->size-p->exist,
113             val);
114         if(p->isBad())res = &p;
115         return res;
116     }
117 }
118 //在第 id 个位置插入数 val
119 void insert(int id,int val){
120     Node **p = insert(root,id,val);
121     if(*p != null)rebuild(*p);
122 }
123 //查询 [l,r] 之间值为 val 的数的个数
124 int query(Node *p,int l,int r,int val){
125     if(p == null)return 0;
126     if(l <= 1 && p->size <= r)
127         return p->mp.count(val)?p->mp[val]:0;
128     else {
129         int ans = 0;

```



```

129         if(l <= p->ch[0]->size)
130             ans += query(p->ch[0],l,r,val);
131         if(r > p->ch[0]->size+p->exist)
132             ans += query(p->ch[1],l - p->ch[0]->size - p->exist
133                         , r - p->ch[0]->size - p->exist,val);
134         if(p->exist && p->key == val && l <= p->ch[0]->size+1
135            && r >= p->ch[0]->size+1)
136             ans++;
137         return ans;
138     }
139 }tree;
140 int a[MAXN];
141 int main()
142 {
143     int n;
144     while(scanf("%d",&n) == 1){
145         tree.init();
146         for(int i = 0;i < n;i++)scanf("%d",&a[i]);
147         tree.root = tree.buildTree(a,0,n);
148         int m;
149         int op,l,r,k;
150         scanf("%d",&m);
151         int ans = 0;
152         while(m--){
153             scanf("%d",&op);
154             if(op == 1){
155                 scanf("%d%d",&l,&r);
156                 l = ((l+ans-1)%n)+1;
157                 r = ((r+ans-1)%n)+1;
158                 if(l > r)swap(l,r);
159                 int v = tree.erase(tree.root,r);
160                 //tree.check_erase(); //有时候可以加上删除重构
161                 tree.insert(l,v);
162             }
163             else {
164                 scanf("%d%d%d",&l,&r,&k);
165                 l = ((l+ans-1)%n)+1;
166                 r = ((r+ans-1)%n)+1;
167                 k = ((k+ans-1)%n)+1;
168                 if(l > r)swap(l,r);
169                 ans = tree.query(tree.root,l,r,k);
170                 printf("%d\n",ans);
171             }
172         }
173     }
174     return 0;
175 }

```

3.10 动态 KD 树

动态 KD 树就是结合了 KD 树和替罪羊树。支持 KD 树的插入删除操作，用替罪羊树的思想来保存平衡。

UVALive6045

题意：给了二维平面上的 N 个整点 ($N \leq 50000$)。每次操作给了点 (x_i, y_i) ，需要曼哈顿距离小于 E 的点进行一个变换。输出最后的点的坐标，保证变换次数不超过 50000。

```

1  const int MAXN = 100010;
2  const double alpha = 0.75;
3  struct Point{
4      int x,y,id;
5  };
6  struct Node{
7      Point e;
8      int size,nodeCount;
9      Node *lc,*rc;
10     bool div;
11     bool exist;
12     bool isBad(){
13         return lc->nodeCount > alpha*nodeCount+5 || rc->nodeCount >
            alpha*nodeCount+5;
14     }
15     inline void push_up(){
16         size = exist + lc->size + rc->size;
17         nodeCount = 1+lc->nodeCount+rc->nodeCount;
18     }
19 };
20 Node pool[MAXN],*tail,*root,*null;
21 Node *bc[MAXN];
22 int bc_top;
23 void init(){
24     tail = pool;
25     null = tail++;
26     null->lc = null->rc = null;
27     null->size = null->nodeCount = 0;
28     root = null;
29     bc_top = 0;
30 }
31 Node *newNode(Point e){
32     Node *p;
33     if(bc_top)p = bc[--bc_top];
34     else p = tail++;
35     p->e = e;
36     p->lc = p->rc = null;
37     p->size = p->nodeCount = 1;
38     p->exist = true;
39     return p;
40 }
41 inline bool cmpX(const Point &a,const Point &b){
42     return a.x < b.x || (a.x == b.x && a.y < b.y) || (a.x == b.x &&
        a.y == b.y && a.id < b.id);

```

```

43 }
44 inline bool cmpY(const Point &a,const Point &b){
45     return a.y < b.y || (a.y == b.y && a.x < b.x) || (a.y == b.y &&
        a.x == b.x && a.id < b.id);
46 }
47 inline bool cmp(const Point &a,const Point &b,bool div){
48     return div?cmpY(a,b):cmpX(a,b);
49 }
50 //注意 a 需要备份, 否则就乱序
51 Node *build(Point *a,int l,int r,bool div){
52     if(l >= r)return null;
53     int mid = (l+r)/2;
54     nth_element(a+l,a+mid,a+r,div?cmpY:cmpX);
55     Node *p = newNode(a[mid]);
56     p->div = div;
57     p->lc = build(a,l,mid,!div);
58     p->rc = build(a,mid+1,r,!div);
59     p->push_up();
60     return p;
61 }
62 void Travel(Node *p,vector<Point>&v){
63     if(p == null)return;
64     Travel(p->lc,v);
65     if(p->exist)v.push_back(p->e);
66     bc[bc_top++] = p;
67     Travel(p->rc,v);
68 }
69 Node *divide(vector<Point>&v,int l,int r,bool div){
70     if(l >= r)return null;
71     int mid = (l+r)/2;
72     nth_element(v.begin()+l,v.begin()+mid,v.begin()+r,div?cmpY:cmpX);
73     Node *p = newNode(v[mid]);
74     p->div = div;
75     p->lc = divide(v,l,mid,!div);
76     p->rc = divide(v,mid+1,r,!div);
77     p->push_up();
78     return p;
79 }
80 inline void rebuild(Node *&p){
81     vector<Point>v;
82     Travel(p,v);
83     p = divide(v,0,v.size(),p->div);
84 }
85 Node **insert(Node *&p,Point a,bool div){
86     if(p == null){
87         p = newNode(a);
88         p->div = div;
89         return &null;
90     }
91     else {

```

```

92         p->nodeCount++;
93         p->size++;
94         Node **res;
95         if(cmp(a,p->e,div))
96             res = insert(p->lc,a,!div);
97         else res = insert(p->rc,a,!div);
98         if(p->isBad())res = &p;
99         return res;
100     }
101 }
102 void insert(Point e){
103     Node **p = insert(root,e,0);
104     if(*p != null)rebuild(*p);
105 }
106 vector<int>vec;
107 void getvec(Node *p,int minx,int maxx,int miny,int maxy){
108     if(p->size == 0)return;
109     if(p->exist && minx <= p->e.x && p->e.x <= maxx && miny <= p->e
        .y && p->e.y <= maxy){
110         vec.push_back(p->e.id);
111         p->exist = 0;
112         p->size--;
113     }
114     if(p->div? miny <= p->e.y : minx <= p->e.x)getvec(p->lc,minx,
        maxx,miny,maxy);
115     if(p->div? maxy >= p->e.y : maxx >= p->e.x)getvec(p->rc,minx,
        maxx,miny,maxy);
116     p->push_up();
117 }
118 Point p[MAXN],p2[MAXN];
119 Point p3[MAXN];
120 int main()
121 {
122     int T;
123     scanf("%d",&T);
124     int iCase = 0;
125     int N,Q,W,H;
126     while(T--){
127         iCase++;
128         scanf("%d%d%d%d",&N,&Q,&W,&H);
129         init();
130         for(int i = 0;i < N;i++){
131             scanf("%d%d",&p[i].x,&p[i].y);
132             p[i].id = p2[i].id = i;
133             p2[i].x = p[i].x+p[i].y;
134             p2[i].y = p[i].x-p[i].y;
135             p3[i] = p2[i];
136         }
137         root = build(p3,0,N,0);
138         int X,Y,E,a,b,c,d,e,f;
139         while(Q--){

```

```

140         scanf("%d%d%d%d%d%d%d%d", &X, &Y, &E, &a, &b, &c, &d, &e, &f);
141         vec.clear();
142         int minx = X+Y-E;
143         int maxx = X+Y+E;
144         int miny = X-Y-E;
145         int maxy = X-Y+E;
146         getvec(root, minx, maxx, miny, maxy);
147         int sz = vec.size();
148         for(int i = 0; i < sz; i++){
149             int id = vec[i];
150             long long tx = p[id].x;
151             long long ty = p[id].y;
152             p[id].x = (tx*a+ty*b+(long long)(id+1)*c)%W;
153             p[id].y = (tx*d+ty*e+(long long)(id+1)*f)%H;
154             p2[id].x = p[id].x+p[id].y;
155             p2[id].y = p[id].x-p[id].y;
156             insert(p2[id]);
157         }
158     }
159     printf("Case_#%d:\n", iCase);
160     for(int i = 0; i < N; i++)
161         printf("%d_ %d\n", p[i].x, p[i].y);
162 }
163 return 0;
164 }

```

3.11 树套树

3.11.1 替罪羊树套 splay

BZOJ 3065: 带插入区间 K 小值

带插入、修改的区间 k 小值在线查询。

1. Q x y k: 询问从左至右第 x 只跳蚤到从左至右第 y 只跳蚤中，弹跳力第 k 小的跳蚤的弹跳力是多少。 ($1 \leq x \leq y \leq m, 1 \leq k \leq y - x + 1$)
2. M x val: 将从左至右第 x 只跳蚤的弹跳力改为 val。 ($1 \leq x \leq m$)
3. I x val: 在从左至右第 x 只跳蚤的前面插入一只弹跳力为 val 的跳蚤。即插入后从左至右第 x 只跳蚤是我刚插入的跳蚤。 ($1 \leq x \leq m + 1$)

```

1  const int MAXN = 70010;
2  namespace Splay{
3      struct Node *null;
4      struct Node{
5          Node *ch[2], *fa;
6          int size, key, cnt;
7          inline void setc(Node *p, int d){
8              ch[d] = p;
9              p->fa = this;
10         }
11         inline bool d(){
12             return fa->ch[1] == this;
13         }
14         inline void push_up(){

```

```

15         size = ch[0]→size + ch[1]→size + cnt;
16     }
17     void clear(int _key){
18         size = cnt = 1;
19         key = _key;
20         ch[0] = ch[1] = fa = null;
21     }
22     inline bool isroot(){
23         return fa == null || this != fa→ch[0] && this != fa→
            ch[1];
24     }
25 };
26 Node pool[MAXN*20],*tail;
27 Node *bc[MAXN*20];
28 int bc_top;//内存回收
29 void init(){
30     tail = pool;
31     bc_top = 0;
32     null = tail++;
33     null→size = null→cnt = 0;
34     null→ch[0] = null→ch[1] = null→fa = null;
35 }
36 inline void rotate(Node *x){
37     Node *f = x→fa, *ff = x→fa→fa;
38     int c = x→d(), cc = f→d();
39     f→setc(x→ch[!c],c);
40     x→setc(f,!c);
41     if(ff→ch[cc] == f)ff→setc(x,cc);
42     else x→fa = ff;
43     f→push_up();
44 }
45 inline void splay(Node* &root,Node* x,Node* goal){
46     while(x→fa != goal){
47         if(x→fa→fa == goal)rotate(x);
48         else {
49             bool f = x→fa→d();
50             x→d() == f ? rotate(x→fa) : rotate(x);
51             rotate(x);
52         }
53     }
54     x→push_up();
55     if(goal == null)root = x;
56 }
57 //找到 r 子树里面的最左边那个
58 Node* get_left(Node* r){
59     Node* x = r;
60     while(x→ch[0] != null)x = x→ch[0];
61     return x;
62 }
63 //在 root 的树中删掉 x
64 void erase(Node* &root,Node* x){

```

```

65     splay(root,x,null);
66     Node* t = root;
67     if(t->ch[1] != null){
68         root = t->ch[1];
69         splay(root,get_left(t->ch[1]),null);
70         root->setc(t->ch[0],0);
71     }
72     else root = root->ch[0];
73     bc[bc_top++] = x;
74     root->fa = null;
75     if(root != null)root->push_up();
76 }
77 Node* newNode(int key){
78     Node* p;
79     if(bc_top)p = bc[--bc_top];
80     else p = tail++;
81     p->clear(key);
82     return p;
83 }
84 //插入一个值 key
85 void insert(Node* &root,int key){
86     if(root == null){
87         root = newNode(key);
88         return;
89     }
90     Node* now = root;
91     Node* pre = root->fa;
92     while(now != null){
93         if(now->key == key){
94             now->cnt++;
95             splay(root,now,null);
96             return;
97         }
98         pre = now;
99         now = now->ch[key >= now->key];
100     }
101     Node *x = newNode(key);
102     pre->setc(x,key >= pre->key);
103     splay(root,x,null);
104 }
105 //删除一个值 key
106 void erase(Node* &root,int key){
107     Node* now = root;
108     while(now->key != key){
109         now = now->ch[key >= now->key];
110     }
111     now->cnt--;
112     if(now->cnt == 0)erase(root,now);
113     else splay(root,now,null);
114 }
115 void Travel(Node* r){

```

```

116         if(r == null)return;
117         Travel(r->ch[0]);
118         bc[bc_top++] = r;
119         Travel(r->ch[1]);
120     }
121     void CLEAR(Node* &root){
122         Travel(root);
123         root = null;
124     }
125     //查询小于等于 val 的个数
126     int query(Node *root,int val){
127         int ans = 0;
128         Node *x = root;
129         while(x != null){
130             if(val < x->key)x = x->ch[0];
131             else{
132                 ans += x->ch[0]->size + x->cnt;
133                 x = x->ch[1];
134             }
135         }
136         return ans;
137     }
138 };
139 namespace ScapeGoatTree{
140     const double alpha = 0.75;
141     struct Node{
142         Node *ch[2];
143         int size,nodeCount,key;
144         Splay::Node *root;
145         bool isBad(){
146             return ch[0]->nodeCount > alpha*nodeCount+5 || ch[1]->
                nodeCount > alpha*nodeCount+5;
147         }
148         void push_up(){
149             size = 1+ch[0]->size+ch[1]->size;
150             nodeCount = 1+ch[0]->nodeCount+ch[1]->nodeCount;
151         }
152     };
153     Node pool[MAXN];
154     Node *tail,*root,*null;
155     Node *bc[MAXN];
156     int bc_top;
157     void init(){
158         tail = pool;
159         null = tail++;
160         null->ch[0] = null->ch[1] = null;
161         null->size = null->nodeCount = 0;
162         null->root = Splay::null;
163         bc_top = 0;
164     }
165     inline Node* newNode(int key){

```



```

166     Node *p;
167     if(bc_top)p = bc[--bc_top];
168     else p = tail++;
169     p->key = key;
170     p->ch[0] = p->ch[1] = null;
171     p->size = p->nodeCount = 1;
172     p->root = Splay::null;
173     return p;
174 }
175 Node *buildTree(int *a,int l,int r){
176     if(l >= r)return null;
177     int mid = (l+r)/2;
178     Node *p = newNode(a[mid]);
179     for(int i = l;i < r;i++)
180         Splay::insert(p->root,a[i]);
181     p->ch[0] = buildTree(a,l,mid);
182     p->ch[1] = buildTree(a,mid+1,r);
183     p->push_up();
184     return p;
185 }
186 void Travel(Node *p,vector<int>&v){
187     if(p == null)return;
188     Travel(p->ch[0],v);
189     v.push_back(p->key);
190     Splay::CLEAR(p->root);
191     bc[bc_top++] = p;
192     Travel(p->ch[1],v);
193 }
194 Node *divide(vector<int>&v,int l,int r){
195     if(l == r)return null;
196     int mid = (l+r)/2;
197     Node *p = newNode(v[mid]);
198     for(int i = l;i < r;i++)
199         Splay::insert(p->root,v[i]);
200     p->ch[0] = divide(v,l,mid);
201     p->ch[1] = divide(v,mid+1,r);
202     p->push_up();
203     return p;
204 }
205 inline void rebuild(Node *&p){
206     vector<int>v;
207     Travel(p,v);
208     p = divide(v,0,v.size());
209 }
210 //将第 id 个值修改为 val
211 int Modify(Node *p,int id,int val){
212     if(id == p->ch[0]->size+1){
213         int v = p->key;
214         Splay::erase(p->root,v);
215         Splay::insert(p->root,val);
216         p->key = val;

```

```

217         return v;
218     }
219     int res;
220     if(p->ch[0]->size >= id)
221         res = Modify(p->ch[0],id,val);
222     else res = Modify(p->ch[1],id-p->ch[0]->size-1,val);
223     Splay::erase(p->root,res);
224     Splay::insert(p->root,val);
225     return res;
226 }
227 Node **insert(Node *&p,int id,int val){
228     if(p == null){
229         p = newNode(val);
230         Splay::insert(p->root,val);
231         return &null;
232     }
233     else {
234         p->size++;
235         p->nodeCount++;
236         Splay::insert(p->root,val);
237         Node ** res;
238         if(id <= p->ch[0]->size+1)
239             res = insert(p->ch[0],id,val);
240         else res = insert(p->ch[1],id-p->ch[0]->size-1,val);
241         if(p->isBad())res = &p;
242         return res;
243     }
244 }
245 void insert(int id,int val){
246     Node **p = insert(root,id,val);
247     if(*p != null)rebuild(*p);
248 }
249 //查询在 [l,r] 区间, 值小于等于 val 的个数
250 int query(Node *p,int l,int r,int val){
251     if(p == null)return 0;
252     if(l <= 1 && p->size <= r)
253         return Splay::query(p->root,val);
254     else {
255         int ans = 0;
256         if(l <= p->ch[0]->size)
257             ans += query(p->ch[0],l,r,val);
258         if(r > p->ch[0]->size+1)
259             ans += query(p->ch[1],l-p->ch[0]->size-1,r-p->ch[0]->size-1,val);
260         if(p->key <= val && l <= p->ch[0]->size+1 && p->ch[0]->size+1 <= r)
261             ans++;
262         return ans;
263     }
264 }
265 int query(int L,int R,int k){

```

```

266     int ans;
267     int l = 0, r = 100000;
268     while(l <= r){
269         int mid = (l+r)/2;
270         if(query(root,L,R,mid) >= k){
271             ans = mid;
272             r = mid-1;
273         }
274         else l = mid+1;
275     }
276     return ans;
277 }
278 };
279 int a[MAXN];
280 int main()
281 {
282     int n;
283     while(scanf("%d",&n) == 1){
284         Splay::init();
285         ScapeGoatTree::init();
286         for(int i = 0; i < n; i++) scanf("%d",&a[i]);
287         ScapeGoatTree::root = ScapeGoatTree::buildTree(a,0,n);
288         int m;
289         char op[10];
290         scanf("%d",&m);
291         int ans = 0;
292         while(m--){
293             scanf("%s",op);
294             if(op[0] == 'Q'){
295                 int x,y,k;
296                 scanf("%d%d%d",&x,&y,&k);
297                 x ^= ans; y ^= ans; k ^= ans;
298                 ans = ScapeGoatTree::query(x,y,k);
299                 printf("%d\n",ans);
300             }
301             else if(op[0] == 'M'){
302                 int x,val;
303                 scanf("%d%d",&x,&val);
304                 x ^= ans; val ^= ans;
305                 ScapeGoatTree::Modify(ScapeGoatTree::root,x,val);
306             }
307             else if(op[0] == 'I'){
308                 int x,val;
309                 scanf("%d%d",&x,&val);
310                 x ^= ans; val ^= ans;
311                 ScapeGoatTree::insert(x,val);
312             }
313         }
314     }
315     return 0;
316 }

```

4 图论

4.1 最短路

4.1.1 Dijkstra 单源最短路

权值必须是非负

```

1  /*
2   * 单源最短路径, Dijkstra 算法, 邻接矩阵形式, 复杂度为 $O(n^2)$ 
3   * 求出源 beg 到所有点的最短路径, 传入图的顶点数, 和邻接矩阵 cost[][]
4   * 返回各点的最短路径 lowcost[], 路径 pre[].pre[i] 记录 beg 到 i 路径上的
      父结点, pre[beg]=-1
5   * 可更改路径权类型, 但是权值必须为非负
6   */
7  const int MAXN=1010;
8  #define typec int
9  const typec INF=0x3f3f3f3f;//防止后面溢出, 这个不能太大
10 bool vis[MAXN];
11 int pre[MAXN];
12 void Dijkstra(typec cost[][MAXN],typec lowcost[],int n,int beg){
13     for(int i=0;i<n;i++){
14         lowcost[i]=INF;vis[i]=false;pre[i]=-1;
15     }
16     lowcost[beg]=0;
17     for(int j=0;j<n;j++){
18         int k=-1;
19         int Min=INF;
20         for(int i=0;i<n;i++){
21             if(!vis[i]&&lowcost[i]<Min){
22                 Min=lowcost[i];
23                 k=i;
24             }
25             if(k==-1)break;
26             vis[k]=true;
27             for(int i=0;i<n;i++){
28                 if(!vis[i]&&lowcost[k]+cost[k][i]<lowcost[i]){
29                     lowcost[i]=lowcost[k]+cost[k][i];
30                     pre[i]=k;
31                 }
32             }
33 }

```

4.1.2 Dijkstra 算法 + 堆优化

使用优先队列优化, 复杂度 $O(E \log E)$

```

1  /*
2   * 使用优先队列优化 Dijkstra 算法
3   * 复杂度  $O(E \log E)$ 
4   * 注意对 vector<Edge>E[MAXN] 进行初始化后加边
5   */
6  const int INF=0x3f3f3f3f;

```

```

7  const int MAXN=1000010;
8  struct qnode{
9      int v;
10     int c;
11     qnode(int _v=0,int _c=0):v(_v),c(_c){}
12     bool operator <(const qnode &r)const{
13         return c>r.c;
14     }
15 };
16 struct Edge{
17     int v,cost;
18     Edge(int _v=0,int _cost=0):v(_v),cost(_cost){}
19 };
20 vector<Edge>E[MAXN];
21 bool vis[MAXN];
22 int dist[MAXN];
23 //点的编号从 1 开始
24 void Dijkstra(int n,int start){
25     memset(vis,false,sizeof(vis));
26     for(int i=1;i<=n;i++)dist[i]=INF;
27     priority_queue<qnode>que;
28     while(!que.empty())que.pop();
29     dist[start]=0;
30     que.push(qnode(start,0));
31     qnode tmp;
32     while(!que.empty()){
33         tmp=que.top();
34         que.pop();
35         int u=tmp.v;
36         if(vis[u])continue;
37         vis[u]=true;
38         for(int i=0;i<E[u].size();i++){
39             int v=E[u][i].v;
40             int cost=E[u][i].cost;
41             if(!vis[v]&&dist[v]>dist[u]+cost){
42                 dist[v]=dist[u]+cost;
43                 que.push(qnode(v,dist[v]));
44             }
45         }
46     }
47 }
48 void addedge(int u,int v,int w){
49     E[u].push_back(Edge(v,w));
50 }

```

4.1.3 单源最短路 bellman_ford 算法

```

1  /*
2  * 单源最短路 bellman_ford 算法, 复杂度 O(VE)
3  * 可以处理负边权图。
4  * 可以判断是否存在负环回路。返回 true, 当且仅当图中不包含从源点可达的负权回路

```

```

5  * vector<Edge>E; 先 E.clear() 初始化, 然后加入所有边
6  * 点的编号从 1 开始 (从 0 开始简单修改就可以了)
7  */
8  const int INF=0x3f3f3f3f;
9  const int MAXN=550;
10 int dist[MAXN];
11 struct Edge{
12     int u,v;
13     int cost;
14     Edge(int _u=0,int _v=0,int _cost=0):u(_u),v(_v),cost(_cost){}
15 };
16 vector<Edge>E;
17 //点的编号从 1 开始
18 bool bellman_ford(int start,int n){
19     for(int i=1;i<=n;i++)dist[i]=INF;
20     dist[start]=0;
21     //最多做 n-1 次
22     for(int i=1;i<n;i++){
23         bool flag=false;
24         for(int j=0;j<E.size();j++){
25             int u=E[j].u;
26             int v=E[j].v;
27             int cost=E[j].cost;
28             if(dist[v]>dist[u]+cost){
29                 dist[v]=dist[u]+cost;
30                 flag=true;
31             }
32         }
33         if(!flag)return true;//没有负环回路
34     }
35     for(int j=0;j<E.size();j++)
36         if(dist[E[j].v]>dist[E[j].u]+E[j].cost)
37             return false;//有负环回路
38     return true;//没有负环回路
39 }

```

4.1.4 单源最短路 SPFA

```

1  /*
2  * 单源最短路 SPFA
3  * 时间复杂度  $O(kE)$ 
4  * 这个是队列实现, 有时候改成栈实现会更加快, 很容易修改
5  * 这个复杂度是不定的
6  */
7  const int MAXN=1010;
8  const int INF=0x3f3f3f3f;
9  struct Edge{
10     int v;
11     int cost;
12     Edge(int _v=0,int _cost=0):v(_v),cost(_cost){}
13 };
14 vector<Edge>E[MAXN];

```

```

15 void addedge(int u,int v,int w){
16     E[u].push_back(Edge(v,w));
17 }
18 bool vis[MAXN]; //在队列标志
19 int cnt[MAXN]; //每个点的入队列次数
20 int dist[MAXN];
21 bool SPFA(int start,int n){
22     memset(vis,false,sizeof(vis));
23     for(int i=1;i<=n;i++) dist[i]=INF;
24     vis[start]=true;
25     dist[start]=0;
26     queue<int> que;
27     while(!que.empty()) que.pop();
28     que.push(start);
29     memset(cnt,0,sizeof(cnt));
30     cnt[start]=1;
31     while(!que.empty()){
32         int u=que.front();
33         que.pop();
34         vis[u]=false;
35         for(int i=0;i<E[u].size();i++){
36             int v=E[u][i].v;
37             if(dist[v]>dist[u]+E[u][i].cost){
38                 dist[v]=dist[u]+E[u][i].cost;
39                 if(!vis[v]){
40                     vis[v]=true;
41                     que.push(v);
42                     if(++cnt[v]>n) return false;
43                     //cnt[i] 为入队列次数，用来判定是否存在负环回路
44                 }
45             }
46         }
47     }
48     return true;
49 }

```

4.2 最小生成树

4.2.1 Prim 算法

```

1  /*
2   * Prim 求 MST
3   * 耗费矩阵 cost[][], 标号从 0 开始, 0~n-1
4   * 返回最小生成树的权值, 返回 -1 表示原图不连通
5   */
6  const int INF=0x3f3f3f3f;
7  const int MAXN=110;
8  bool vis[MAXN];
9  int lowc[MAXN];
10 //点是 0 n-1
11 int Prim(int cost[][MAXN],int n){
12     int ans=0;

```

```

13     memset(vis, false, sizeof(vis));
14     vis[0]=true;
15     for(int i=1; i<n; i++) lowc[i]=cost[0][i];
16     for(int i=1; i<n; i++){
17         int minc=INF;
18         int p=-1;
19         for(int j=0; j<n; j++){
20             if(!vis[j]&&minc>lowc[j]){
21                 minc=lowc[j];
22                 p=j;
23             }
24             if(minc==INF) return -1; //原图不连通
25             ans+=minc;
26             vis[p]=true;
27             for(int j=0; j<n; j++){
28                 if(!vis[j]&&lowc[j]>cost[p][j])
29                     lowc[j]=cost[p][j];
30             }
31         }
32     }

```

4.2.2 Kruskal 算法

```

1  /*
2   * Kruskal 算法求 MST
3   */
4  const int MAXN=110; //最大点数
5  const int MAXM=10000; //最大边数
6  int F[MAXN]; //并查集使用
7  struct Edge{
8      int u,v,w;
9  }edge[MAXM]; //存储边的信息, 包括起点/终点/权值
10 int tol; //边数, 加边前赋值为 0
11 void addedge(int u, int v, int w){
12     edge[tol].u=u;
13     edge[tol].v=v;
14     edge[tol++].w=w;
15 }
16 //排序函数, 讲边按照权值从小到大排序
17 bool cmp(Edge a, Edge b){
18     return a.w<b.w;
19 }
20 int find(int x){
21     if(F[x]==-1) return x;
22     else return F[x]=find(F[x]);
23 }
24 //传入点数, 返回最小生成树的权值, 如果不连通返回 -1
25 int Kruskal(int n){
26     memset(F, -1, sizeof(F));
27     sort(edge, edge+tol, cmp);
28     int cnt=0; //计算加入的边数
29     int ans=0;

```



```

30     for(int i=0;i<tol;i++){
31         int u=edge[i].u;
32         int v=edge[i].v;
33         int w=edge[i].w;
34         int t1=find(u);
35         int t2=find(v);
36         if(t1!=t2){
37             ans+=w;
38             F[t1]=t2;
39             cnt++;
40         }
41         if(cnt==n-1)break;
42     }
43     if(cnt<n-1)return -1;//不连通
44     else return ans;
45 }

```

4.3 次小生成树

```

1  /*
2  * 次小生成树
3  * 求最小生成树时,用数组 Max[i][j] 来表示 MST 中 i 到 j 最大边权
4  * 求完后,直接枚举所有不在 MST 中的边,替换掉最大边权的边,更新答案
5  * 点的编号从 0 开始
6  */
7  const int MAXN=110;
8  const int INF=0x3f3f3f3f;
9  bool vis[MAXN];
10 int lowc[MAXN];
11 int pre[MAXN];
12 int Max[MAXN][MAXN]; //Max[i][j] 表示在最小生成树中从 i 到 j 的路径中的最
    大边权
13 bool used[MAXN][MAXN];
14 int Prim(int cost[][MAXN],int n){
15     int ans=0;
16     memset(vis,false,sizeof(vis));
17     memset(Max,0,sizeof(Max));
18     memset(used,false,sizeof(used));
19     vis[0]=true;
20     pre[0]=-1;
21     for(int i=1;i<n;i++){
22         lowc[i]=cost[0][i];
23         pre[i]=0;
24     }
25     lowc[0]=0;
26     for(int i=1;i<n;i++){
27         int minc=INF;
28         int p=-1;
29         for(int j=0;j<n;j++){
30             if(!vis[j]&&minc>lowc[j]){
31                 minc=lowc[j];
32                 p=j;

```

```

33         }
34         if(minc==INF)return -1;
35         ans+=minc;
36         vis[p]=true;
37         used[p][pre[p]]=used[pre[p]][p]=true;
38         for(int j=0;j<n;j++){
39             if(vis[j] && j != p)Max[j][p]=Max[p][j]=max(Max[j][pre[
                p]],lowc[p]);
40             if(!vis[j]&&lowc[j]>cost[p][j]){
41                 lowc[j]=cost[p][j];
42                 pre[j]=p;
43             }
44         }
45     }
46     return ans;
47 }

```

4.4 有向图的强连通分量

4.4.1 Tarjan

```

1  /*
2   * Tarjan 算法
3   * 复杂度 O(N+M)
4   */
5  const int MAXN = 20010;//点数
6  const int MAXM = 50010;//边数
7  struct Edge{
8      int to,next;
9  }edge[MAXN];
10 int head[MAXN],tot;
11 int Low[MAXN],DFN[MAXN],Stack[MAXN],Belong[MAXN];//Belong 数组的值是
    1 ~ scc
12 int Index,top;
13 int scc;//强连通分量的个数
14 bool Instack[MAXN];
15 int num[MAXN];//各个强连通分量包含点的个数，数组编号 1 ~ scc
16 //num 数组不一定需要，结合实际情况
17
18 void addedge(int u,int v){
19     edge[tot].to = v;edge[tot].next = head[u];head[u] = tot++;
20 }
21 void Tarjan(int u){
22     int v;
23     Low[u] = DFN[u] = ++Index;
24     Stack[top++] = u;
25     Instack[u] = true;
26     for(int i = head[u];i != -1;i = edge[i].next){
27         v = edge[i].to;
28         if( !DFN[v] ){
29             Tarjan(v);
30             if( Low[u] > Low[v] )Low[u] = Low[v];

```

```

31     }
32     else if(Instack[v] && Low[u] > DFN[v])
33         Low[u] = DFN[v];
34 }
35 if(Low[u] == DFN[u]){
36     scc++;
37     do{
38         v = Stack[--top];
39         Instack[v] = false;
40         Belong[v] = scc;
41         num[scc]++;
42     }
43     while( v != u);
44 }
45 }
46 void solve(int N){
47     memset(DFN,0,sizeof(DFN));
48     memset(Instack,false,sizeof(Instack));
49     memset(num,0,sizeof(num));
50     Index = scc = top = 0;
51     for(int i = 1;i <= N;i++)
52         if(!DFN[i])
53             Tarjan(i);
54 }
55 void init(){
56     tot = 0;
57     memset(head,-1,sizeof(head));
58 }

```

4.4.2 Kosaraju

```

1  /*
2   * Kosaraju 算法, 复杂度 O(N+M)
3   */
4  const int MAXN = 20010;
5  const int MAXM = 50010;
6  struct Edge{
7      int to,next;
8  }edge1[MAXN],edge2[MAXN];
9  //edge1 是原图 G, edge2 是逆图 GT
10 int head1[MAXN],head2[MAXN];
11 bool mark1[MAXN],mark2[MAXN];
12 int tot1,tot2;
13 int cnt1,cnt2;
14 int st[MAXN]; //对原图进行 dfs, 点的结束时间从小到大排序
15 int Belong[MAXN]; //每个点属于哪个连通分量 (0~cnt2-1)
16 int num; //中间变量, 用来数某个连通分量中点的个数
17 int setNum[MAXN]; //强连通分量中点的个数, 编号 0~cnt2-1
18 void addedge(int u,int v){
19     edge1[tot1].to = v; edge1[tot1].next = head1[u]; head1[u] = tot1++;

```

```

20     edge2[tot2].to = u; edge2[tot2].next = head2[v]; head2[v] = tot2
      ++;
21 }
22 void DFS1(int u){
23     mark1[u] = true;
24     for(int i = head1[u]; i != -1; i = edge1[i].next)
25         if(!mark1[edge1[i].to])
26             DFS1(edge1[i].to);
27     st[cnt1++] = u;
28 }
29 void DFS2(int u){
30     mark2[u] = true;
31     num++;
32     Belong[u] = cnt2;
33     for(int i = head2[u]; i != -1; i = edge2[i].next)
34         if(!mark2[edge2[i].to])
35             DFS2(edge2[i].to);
36 }
37 //点的编号从 1 开始
38 void solve(int n){
39     memset(mark1, false, sizeof(mark1));
40     memset(mark2, false, sizeof(mark2));
41     cnt1 = cnt2 = 0;
42     for(int i = 1; i <= n; i++)
43         if(!mark1[i])
44             DFS1(i);
45     for(int i = cnt1-1; i >= 0; i--)
46         if(!mark2[st[i]]){
47             num = 0;
48             DFS2(st[i]);
49             setNum[cnt2++] = num;
50         }
51 }

```

4.5 图的割点、桥和双连通分支的基本概念

[点连通度与边连通度] 在一个无向连通图中，如果有一个顶点集合，删除这个顶点集合，以及这个集合中所有顶点相关联的边以后，原图变成多个连通块，就称这个点集为割点集合。一个图的点连通度的定义为，最小割点集合中的顶点数。

类似的，如果有一个边集合，删除这个边集合以后，原图变成多个连通块，就称这个点集为割边集合。一个图的边连通度的定义为，最小割边集合中的边数。

[双连通图、割点与桥]

如果一个无向连通图的点连通度大于 1，则称该图是点双连通的 (point biconnected)，简称双连通或重连通。一个图有割点，当且仅当这个图的点连通度为 1，则割点集合的唯一元素被称为割点 (cut point)，又叫关节点 (articulation point)。

如果一个无向连通图的边连通度大于 1，则称该图是边双连通的 (edge biconnected)，简称双连通或重连通。一个图有桥，当且仅当这个图的边连通度为 1，则割边集合的唯一元素被称为桥 (bridge)，又叫关节边 (articulation edge)。

可以看出，点双连通与边双连通都可以简称为双连通，它们之间是有着某种联系的，下文中提到的双连通，均既可指点双连通，又可指边双连通。

[双连通分支]

在图 G 的所有子图 G' 中，如果 G' 是双连通的，则称 G' 为双连通子图。如果一个双连

通子图 G' 它不是任何一个双连通子图的真子集, 则 G' 为极大双连通子图。双连通分支 (biconnected component), 或重连通分支, 就是图的极大双连通子图。特殊的, 点双连通分支又叫做块。[求割点与桥]

该算法是 R.Tarjan 发明的。对图深度优先搜索, 定义 $DFS(u)$ 为 u 在搜索树 (以下简称为树) 中被遍历到的次序号。定义 $Low(u)$ 为 u 或 u 的子树中能通过非父子边追溯到的最早的节点, 即 DFS 序号最小的节点。根据定义, 则有:

$Low(u) = \min DFS(u) DFS(v)$ (u, v 为后向边 (返祖边) 等价于 $DFS(v) < DFS(u)$ 且 v 不为 u 的父亲节点 $Low(v)$ (u, v 为树枝边 (父子边) 一个顶点 u 是割点, 当且仅当满足 (1) 或 (2) (1) u 为树根, 且 u 有多于一个子树。(2) u 不为树根, 且满足存在 (u, v) 为树枝边 (或称父子边, 即 u 为 v 在搜索树中的父亲), 使得 $DFS(u) \leq Low(v)$ 。

一条无向边 (u, v) 是桥, 当且仅当 (u, v) 为树枝边, 且满足 $DFS(u) < Low(v)$ 。

[求双连通分支]

下面要分开讨论点双连通分支与边双连通分支的求法。

对于点双连通分支, 实际上在求割点的过程中就能顺便把每个点双连通分支求出。建立一个栈, 存储当前双连通分支, 在搜索图时, 每找到一条树枝边或后向边 (非横叉边), 就把这条边加入栈中。如果遇到某时满足 $DFS(u) \leq Low(v)$, 说明 u 是一个割点, 同时把边从栈顶一个个取出, 直到遇到了边 (u, v), 取出的这些边与其关联的点, 组成一个点双连通分支。割点可以属于多个点双连通分支, 其余点和每条边只属于且属于一个点双连通分支。

对于边双连通分支, 求法更为简单。只需在求出所有的桥以后, 把桥边删除, 原图变成了多个连通块, 则每个连通块就是一个边双连通分支。桥不属于任何一个边双连通分支, 其余的边和每个顶点都属于且只属于一个边双连通分支。

[构造双连通图]

一个有桥的连通图, 如何把它通过加边变成边双连通图? 方法为首先求出所有的桥, 然后删除这些桥边, 剩下的每个连通块都是一个双连通子图。把每个双连通子图收缩为一个顶点, 再把桥边加回来, 最后的这个图一定是一棵树, 边连通度为 1。

统计出树中度为 1 的节点的个数, 即为叶节点的个数, 记为 $leaf$ 。则至少在树上添加 $(leaf+1)/2$ 条边, 就能使树达到边二连通, 所以至少添加的边数就是 $(leaf+1)/2$ 。具体方法为, 首先把两个最近公共祖先最远的两个叶节点之间连接一条边, 这样可以把这两个点到祖先的路径上所有点收缩到一起, 因为一个形成的环一定是双连通的。然后再找两个最近公共祖先最远的两个叶节点, 这样一对一对找完, 恰好是 $(leaf+1)/2$ 次, 把所有点收缩到了一起。

4.6 割点与桥

4.6.1 模板

```

1  /*
2  *   求无向图的割点和桥
3  *   可以找出割点和桥, 求删掉每个点后增加的连通块。
4  *   需要注意重边的处理, 可以先用矩阵存, 再转邻接表, 或者进行判重
5  */
6  const int MAXN = 10010;
7  const int MAXM = 100010;
8  struct Edge{
9      int to,next;
10     bool cut;//是否为桥的标记
11 }edge[MAXN];
12 int head[MAXN],tot;
13 int Low[MAXN],DFN[MAXN],Stack[MAXN];
14 int Index,top;
15 bool Instack[MAXN];

```

```

16 bool cut[MAXN];
17 int add_block[MAXN]; //删除一个点后增加的连通块
18 int bridge;
19 void addedge(int u,int v){
20     edge[tot].to = v;edge[tot].next = head[u];edge[tot].cut = false
21     ;
22     head[u] = tot++;
23 }
24 void Tarjan(int u,int pre){
25     int v;
26     Low[u] = DFN[u] = ++Index;
27     Stack[top++] = u;
28     Instack[u] = true;
29     int son = 0;
30     int pre_cnt = 0; //处理重边, 如果不需要可以去掉
31     for(int i = head[u]; i != -1; i = edge[i].next){
32         v = edge[i].to;
33         if(v == pre && pre_cnt == 0){pre_cnt++;continue;}
34         if( !DFN[v] ){
35             son++;
36             Tarjan(v,u);
37             if(Low[u] > Low[v])Low[u] = Low[v];
38             //桥
39             //一条无向边 (u,v) 是桥, 当且仅当 (u,v) 为树枝边, 且满足
40             DFS(u)<Low(v)。
41             if(Low[v] > DFN[u]){
42                 bridge++;
43                 edge[i].cut = true;
44                 edge[i^1].cut = true;
45             }
46             //割点
47             //一个顶点 u 是割点, 当且仅当满足 (1) 或 (2) (1) u 为树根, 且
48             u 有多于一个子树。
49             //(2) u 不为树根, 且满足存在 (u,v) 为树枝边 (或称父子边,
50             //即 u 为 v 在搜索树中的父亲), 使得 DFS(u)<=Low(v)
51             if(u != pre && Low[v] >= DFN[u]){ //不是树根
52                 cut[u] = true;
53                 add_block[u]++;
54             }
55         }
56         else if( Low[u] > DFN[v])
57             Low[u] = DFN[v];
58     }
59     //树根, 分支数大于 1
60     if(u == pre && son > 1)cut[u] = true;
61     if(u == pre)add_block[u] = son - 1;
62     Instack[u] = false;
63     top--;
64 }

```

4.6.2 调用

1) UVA 796 Critical Links 给出一个无向图，按顺序输出桥

```

1 void solve(int N){
2     memset(DFN,0,sizeof(DFN));
3     memset(Instack,false,sizeof(Instack));
4     memset(add_block,0,sizeof(add_block));
5     memset(cut,false,sizeof(cut));
6     Index = top = 0;
7     bridge = 0;
8     for(int i = 1;i <= N;i++){
9         if( !DFN[i] )
10             Tarjan(i,i);
11     }
12     printf("%d critical links\n",bridge);
13     vector<pair<int,int> >ans;
14     for(int u = 1;u <= N;u++){
15         for(int i = head[u];i != -1;i = edge[i].next)
16             if(edge[i].cut && edge[i].to > u)
17                 ans.push_back(make_pair(u,edge[i].to));
18     }
19     sort(ans.begin(),ans.end());
20     //按顺序输出桥
21     for(int i = 0;i < ans.size();i++){
22         printf("%d - %d\n",ans[i].first-1,ans[i].second-1);
23     }
24     printf("\n");
25 }
26 void init(){
27     tot = 0;
28     memset(head,-1,sizeof(head));
29 }
30 //处理重边
31 map<int,int>mapit;
32 inline bool isHash(int u,int v){
33     if(mapit[u*MAXN+v])return true;
34     if(mapit[v*MAXN+u])return true;
35     mapit[u*MAXN+v] = mapit[v*MAXN+u] = 1;
36     return false;
37 }
38 int main(){
39     int n;
40     while(scanf("%d",&n) == 1){
41         init();
42         int u;
43         int k;
44         int v;
45         //mapit.clear();
46         for(int i = 1;i <= n;i++){
47             scanf("%d %d",&u,&k);
48             u++;
49             //这样加边，要保证正边和反边是相邻的，建无向图

```

```

49         while(k—){
50             scanf("%d",&v);
51             v++;
52             if(v <= u)continue;
53             //if(isHash(u,v))continue;
54             addedge(u,v);
55             addedge(v,u);
56         }
57     }
58     solve(n);
59 }
60 return 0;
61 }

```

2) POJ 2117 求删除一个点后，图中最多有多少个连通块

```

1 void solve(int N){
2     memset(DFN,0,sizeof(DFN));
3     memset(Instack,0,sizeof(Instack));
4     memset(add_block,0,sizeof(add_block));
5     memset(cut,false,sizeof(cut));
6     Index = top = 0;
7     int cnt = 0;//原来的连通块数
8     for(int i = 1;i <= N;i++){
9         if( !DFN[i] ){
10             Tarjan(i,i);//找割点调用必须是 Tarjan(i,i)
11             cnt++;
12         }
13         int ans = 0;
14         for(int i = 1;i <= N;i++){
15             ans = max(ans,cnt+add_block[i]);
16         }
17         printf("%d\n",ans);
18     }
19 void init(){
20     tot = 0;
21     memset(head,-1,sizeof(head));
22 }
23 int main(){
24     int n,m;
25     int u,v;
26     while(scanf("%d%d",&n,&m)==2){
27         if(n==0 && m == 0)break;
28         init();
29         while(m—){
30             scanf("%d%d",&u,&v);
31             u++;v++;
32             addedge(u,v);
33             addedge(v,u);
34         }
35         solve(n);
36     }
37     return 0;
38 }

```


4.7 边双连通分支

去掉桥，其余的连通分支就是边双连通分支了。一个有桥的连通图要变成边双连通图的话，把双连通子图收缩为一个点，形成一颗树。需要加的边为 $(leaf+1)/2$ (leaf 为叶子结点个数)
POJ 3177 给定一个连通的无向图 G，至少要添加几条边，才能使其变为双连通图。

```

1  const int MAXN = 5010; //点数
2  const int MAXM = 20010; //边数，因为是无向图，所以这个值要 *2
3  struct Edge{
4      int to,next;
5      bool cut; //是否是桥标记
6  }edge[MAXN];
7  int head[MAXN],tot;
8  int Low[MAXN],DFN[MAXN],Stack[MAXN],Belong[MAXN]; //Belong 数组的值是
    1 ~ block
9  int Index,top;
10 int block; //边双连通块数
11 bool Instack[MAXN];
12 int bridge; //桥的数目
13 void addedge(int u,int v){
14     edge[tot].to = v; edge[tot].next = head[u]; edge[tot].cut=false;
15     head[u] = tot++;
16 }
17 void Tarjan(int u,int pre){
18     int v;
19     Low[u] = DFN[u] = ++Index;
20     Stack[top++] = u;
21     Instack[u] = true;
22     int pre_cnt = 0;
23     for(int i = head[u]; i != -1; i = edge[i].next){
24         v = edge[i].to;
25         if(v == pre && pre_cnt == 0){ pre_cnt++; continue; }
26         if( !DFN[v] ){
27             Tarjan(v,u);
28             if( Low[u] > Low[v] ) Low[u] = Low[v];
29             if( Low[v] > DFN[u] ){
30                 bridge++;
31                 edge[i].cut = true;
32                 edge[i^1].cut = true;
33             }
34         }
35         else if( Instack[v] && Low[u] > DFN[v] )
36             Low[u] = DFN[v];
37     }
38     if(Low[u] == DFN[u]){
39         block++;
40         do{
41             v = Stack[--top];
42             Instack[v] = false;
43             Belong[v] = block;
44         }

```

```

45     while( v!=u );
46 }
47 }
48 void init(){
49     tot = 0;
50     memset(head,-1,sizeof(head));
51 }
52 int du[MAXN]; //缩点后形成树, 每个点的度数
53 void solve(int n){
54     memset(DFN,0,sizeof(DFN));
55     memset(Instack,false,sizeof(Instack));
56     Index = top = block = 0;
57     Tarjan(1,0);
58     int ans = 0;
59     memset(du,0,sizeof(du));
60     for(int i = 1; i <= n; i++){
61         for(int j = head[i]; j != -1; j = edge[j].next){
62             if(edge[j].cut)
63                 du[Belong[i]]++;
64         }
65         for(int i = 1; i <= block; i++){
66             if(du[i]==1)
67                 ans++;
68         }
69         //找叶子结点的个数 ans, 构造边双连通图需要加边 (ans+1)/2
70         printf("%d\n", (ans+1)/2);
71     }
72 }
73 int main(){
74     int n,m;
75     int u,v;
76     while(scanf("%d%d",&n,&m)==2){
77         init();
78         while(m--){
79             scanf("%d%d",&u,&v);
80             addedge(u,v);
81             addedge(v,u);
82         }
83         solve(n);
84     }
85     return 0;
86 }

```

4.8 点双连通分支

对于点双连通分支, 实际上在求割点的过程中就能顺便把每个点双连通分支求出。建立一个栈, 存储当前双连通分支, 在搜索图时, 每找到一条树枝边或后向边 (非横叉边), 就把这条边加入栈中。如果遇到某时满足 $DFS(u) \leq Low(v)$, 说明 u 是一个割点, 同时把边从栈顶一个个取出, 直到遇到了边 (u,v) , 取出的这些边与其关联的点, 组成一个点双连通分支。割点可以属于多个点双连通分支, 其余点和每条边只属于且属于一个点双连通分支。

POJ 2942

奇圈, 二分图判断的染色法, 求点双连通分支

```

1  /*
2  POJ 2942 Knights of the Round Table

```

```

3 | 亚瑟王要在圆桌上召开骑士会议，为了不引发骑士之间的冲突，
4 | 并且能够让会议的议题有令人满意的结果，每次开会前都必须对出席会议的骑士有如下要
   | 求：
5 | 1、相互憎恨的两个骑士不能坐在直接相邻的 2 个位置；
6 | 2、出席会议的骑士数必须是奇数，这是为了让投票表决议题时都能有结果。
7 |
8 | 注意：1、所给出的憎恨关系一定是双向的，不存在单向憎恨关系。
9 | 2、由于是圆桌会议，则每个出席的骑士身边必定刚好有 2 个骑士。
10 | 即每个骑士的座位两边都必定各有一个骑士。
11 | 3、一个骑士无法开会，就是说至少有 3 个骑士才可能开会。
12 |
13 | 首先根据给出的互相憎恨的图中得到补图。
14 | 然后就相当于找出不能形成奇圈的点。
15 | 利用下面两个定理：
16 | （1）如果一个双连通分量内的某些顶点在一个奇圈中（即双连通分量含有奇圈），
17 | 那么这个双连通分量的其他顶点也在某个奇圈中；
18 | （2）如果一个双连通分量含有奇圈，则他必定不是一个二分图。反过来也成立，这是一个
   | 充要条件。
19 |
20 | 所以本题的做法，就是对补图求点双连通分量。
21 | 然后对于求得的点双连通分量，使用染色法判断是不是二分图，不是二分图，这个双连通分
   | 量的点是可以存在的
22 | */
23 |
24 | const int MAXN = 1010;
25 | const int MAXM = 2000010;
26 | struct Edge{
27 |     int to,next;
28 | }edge[MAXN];
29 | int head[MAXN],tot;
30 | int Low[MAXN],DFN[MAXN],Stack[MAXN],Belong[MAXN];
31 | int Index,top;
32 | int block;//点双连通分量的个数
33 | bool Instack[MAXN];
34 | bool can[MAXN];
35 | bool ok[MAXN]);//标记
36 | int tmp[MAXN]);//暂时存储双连通分量中的点
37 | int cc;//tmp 的计数
38 | int color[MAXN]);//染色
39 | void addedge(int u,int v){
40 |     edge[tot].to = v;edge[tot].next = head[u];head[u] = tot++;
41 | }
42 | //染色判断二分图
43 | bool dfs(int u,int col){
44 |     color[u] = col;
45 |     for(int i = head[u];i != -1;i = edge[i].next){
46 |         int v = edge[i].to;
47 |         if( !ok[v] )continue;
48 |         if(color[v] != -1){
49 |             if(color[v]==col)return false;
50 |             continue;

```

```

51     }
52     if(!dfs(v,!col))return false;
53 }
54 return true;
55 }
56 void Tarjan(int u,int pre){
57     int v;
58     Low[u] = DFN[u] = ++Index;
59     Stack[top++] = u;
60     Instack[u] = true;
61     int pre_cnt = 0;
62     for(int i = head[u];i != -1;i = edge[i].next){
63         v = edge[i].to;
64         if(v == pre && pre_cnt == 0){pre_cnt++; continue;}
65         if( !DFN[v] ){
66             Tarjan(v,u);
67             if(Low[u] > Low[v])Low[u] = Low[v];
68             if( Low[v] >= DFN[u]){
69                 block++;
70                 int vn;
71                 cc = 0;
72                 memset(ok,false,sizeof(ok));
73                 do{
74                     vn = Stack[--top];
75                     Belong[vn] = block;
76                     Instack[vn] = false;
77                     ok[vn] = true;
78                     tmp[cc++] = vn;
79                 }
80                 while( vn!=v );
81                 ok[u] = 1;
82                 memset(color,-1,sizeof(color));
83                 if( !dfs(u,0) ){
84                     can[u] = true;
85                     while(cc--)can[tmp[cc]]=true;
86                 }
87             }
88         }
89         else if(Instack[v] && Low[u] > DFN[v])
90             Low[u] = DFN[v];
91     }
92 }
93 void solve(int n){
94     memset(DFN,0,sizeof(DFN));
95     memset(Instack,false,sizeof(Instack));
96     Index = block = top = 0;
97     memset(can,false,sizeof(can));
98     for(int i = 1;i <= n;i++){
99         if(!DFN[i])
100             Tarjan(i,-1);
101     }
102     int ans = n;

```

```

102     for(int i = 1;i <= n;i++)
103         if(can[i])
104             ans—;
105     printf("%d\n",ans);
106 }
107 void init(){
108     tot = 0;
109     memset(head,-1,sizeof(head));
110 }
111 int g[MAXN][MAXN];
112 int main(){
113     int n,m;
114     int u,v;
115     while(scanf("%d%d",&n,&m)==2){
116         if(n==0 && m==0)break;
117         init();
118         memset(g,0,sizeof(g));
119         while(m—){
120             scanf("%d%d",&u,&v);
121             g[u][v]=g[v][u]=1;
122         }
123         for(int i = 1;i <= n;i++)
124             for(int j = 1;j <= n;j++)
125                 if(i != j && g[i][j]==0)
126                     addedge(i,j);
127         solve(n);
128     }
129     return 0;
130 }

```

4.9 最小树形图

```

1  /*
2  * 最小树形图
3  * int 型
4  * 复杂度 O(NM)
5  * 点从 0 开始
6  */
7  const int INF = 0x3f3f3f3f;
8  const int MAXN = 1010;
9  const int MAXM = 40010;
10 struct Edge{
11     int u,v,cost;
12 };
13 Edge edge[MAXM];
14 int pre[MAXN],id[MAXN],visit[MAXN],in[MAXN];
15 int zhuliu(int root,int n,int m,Edge edge[]){
16     int res = 0,u,v;
17     while(1){
18         for(int i = 0;i < n;i++)
19             in[i] = INF;

```

```

20     for(int i = 0;i < m;i++)
21         if(edge[i].u != edge[i].v && edge[i].cost < in[edge[i].
22             v]){
23             pre[edge[i].v] = edge[i].u;
24             in[edge[i].v] = edge[i].cost;
25         }
26     for(int i = 0;i < n;i++)
27         if(i != root && in[i] == INF)
28             return -1;//不存在最小树形图
29     int tn = 0;
30     memset(id,-1,sizeof(id));
31     memset(visit,-1,sizeof(visit));
32     in[root] = 0;
33     for(int i = 0;i < n;i++){
34         res += in[i];
35         v = i;
36         while( visit[v] != i && id[v] == -1 && v != root){
37             visit[v] = i;
38             v = pre[v];
39         }
40         if( v != root && id[v] == -1 ){
41             for(int u = pre[v]; u != v ;u = pre[u])
42                 id[u] = tn;
43             id[v] = tn++;
44         }
45     }
46     if(tn == 0)break;//没有有向环
47     for(int i = 0;i < n;i++)
48         if(id[i] == -1)
49             id[i] = tn++;
50     for(int i = 0;i < m;){
51         v = edge[i].v;
52         edge[i].u = id[edge[i].u];
53         edge[i].v = id[edge[i].v];
54         if(edge[i].u != edge[i].v)
55             edge[i].cost -= in[v];
56         else
57             swap(edge[i],edge[--m]);
58     }
59     n = tn;
60     root = id[root];
61 }
62 return res;
63 }
64 int g[MAXN][MAXN];
65 int main(){
66     int n,m;
67     int iCase = 0;
68     int T;
69     scanf("%d",&T);
70     while( T-- ){

```

```

70         iCase ++;
71         scanf("%d%d",&n,&m);
72         for(int i = 0;i < n;i++)
73             for(int j = 0;j < n;j++)
74                 g[i][j] = INF;
75         int u,v,cost;
76         while(m--){
77             scanf("%d%d%d",&u,&v,&cost);
78             if(u == v)continue;
79             g[u][v] = min(g[u][v],cost);
80         }
81         int L = 0;
82         for(int i = 0;i < n;i++)
83             for(int j = 0;j < n;j++)
84                 if(g[i][j] < INF){
85                     edge[L].u = i;
86                     edge[L].v = j;
87                     edge[L++].cost = g[i][j];
88                 }
89         int ans = zhuliu(0,n,L,edge);
90         printf("Case_#%d:",iCase);
91         if(ans == -1)printf("Possums!\n");
92         else printf("%d\n",ans);
93     }
94     return 0;
95 }

```

4.10 二分图匹配

4.10.1 邻接矩阵（匈牙利算法）

```

1  /* *****
2  //二分图匹配（匈牙利算法的 DFS 实现）（邻接矩阵形式）
3  //初始化: g[][] 两边顶点的划分情况
4  //建立 g[i][j] 表示 i->j 的有向边就可以了，是左边向右边的匹配
5  //g 没有边相连则初始化为 0
6  //uN 是匹配左边的顶点数，vN 是匹配右边的顶点数
7  //调用: res=hungary(); 输出最大匹配数
8  //优点: 适用于稠密图，DFS 找增广路，实现简洁易于理解
9  //时间复杂度:O(VE)
10 //*****/
11 //顶点编号从 0 开始的
12 const int MAXN = 510;
13 int uN,vN;//u,v 的数目，使用前面必须赋值
14 int g[MAXN][MAXN];//邻接矩阵
15 int linker[MAXN];
16 bool used[MAXN];
17 bool dfs(int u){
18     for(int v = 0; v < vN;v++){
19         if(g[u][v] && !used[v]){
20             used[v] = true;
21             if(linker[v] == -1 || dfs(linker[v])){

```

```

22         linker[v] = u;
23         return true;
24     }
25 }
26 return false;
27 }
28 int hungary(){
29     int res = 0;
30     memset(linker,-1,sizeof(linker));
31     for(int u = 0;u < uN;u++){
32         memset(used,false,sizeof(used));
33         if(dfs(u))res++;
34     }
35     return res;
36 }

```

4.10.2 邻接表（匈牙利算法）

```

1  /*
2   * 匈牙利算法邻接表形式
3   * 使用前用 init() 进行初始化, 给 uN 赋值
4   * 加边使用函数 addedge(u,v)
5   *
6   */
7  const int MAXN = 5010;//点数的最大值
8  const int MAXM = 50010;//边数的最大值
9  struct Edge{
10     int to,next;
11 }edge[MAXM];
12 int head[MAXN],tot;
13 void init(){
14     tot = 0;
15     memset(head,-1,sizeof(head));
16 }
17 void addedge(int u,int v){
18     edge[tot].to = v; edge[tot].next = head[u];
19     head[u] = tot++;
20 }
21 int linker[MAXN];
22 bool used[MAXN];
23 int uN;
24 bool dfs(int u){
25     for(int i = head[u]; i != -1 ;i = edge[i].next){
26         int v = edge[i].to;
27         if(!used[v]){
28             used[v] = true;
29             if(linker[v] == -1 || dfs(linker[v])){
30                 linker[v] = u;
31                 return true;
32             }
33         }
34     }
35 }

```



```

35     return false;
36 }
37 int hungary(){
38     int res = 0;
39     memset(linker,-1,sizeof(linker));
40     //点的编号 0~uN-1
41     for(int u = 0; u < uN;u++){
42         memset(used,false,sizeof(used));
43         if(dfs(u))res++;
44     }
45     return res;
46 }

```

4.10.3 Hopcroft-Karp 算法

```

1  /* *****
2  * 二分图匹配 (Hopcroft-Karp 算法)
3  * 复杂度  $O(\sqrt{n} * E)$ 
4  * 邻接表存图, vector 实现
5  * vector 先初始化, 然后假如边
6  * uN 为左端的顶点数, 使用前赋值 (点编号 0 开始)
7  */
8  const int MAXN = 3000;
9  const int INF = 0x3f3f3f3f;
10 vector<int>G[MAXN];
11 int uN;
12 int Mx[MAXN],My[MAXN];
13 int dx[MAXN],dy[MAXN];
14 int dis;
15 bool used[MAXN];
16 bool SearchP(){
17     queue<int>Q;
18     dis = INF;
19     memset(dx,-1,sizeof(dx));
20     memset(dy,-1,sizeof(dy));
21     for(int i = 0 ; i < uN; i++){
22         if(Mx[i] == -1){
23             Q.push(i);
24             dx[i] = 0;
25         }
26     }
27     while(!Q.empty()){
28         int u = Q.front();
29         Q.pop();
30         if(dx[u] > dis)break;
31         int sz = G[u].size();
32         for(int i = 0;i < sz;i++){
33             int v = G[u][i];
34             if(dy[v] == -1){
35                 dy[v] = dx[u] + 1;
36                 if(My[v] == -1)dis = dy[v];
37                 else{
38                     dx[My[v]] = dy[v] + 1;

```

```

38         Q.push(My[v]);
39     }
40 }
41 }
42 }
43     return dis != INF;
44 }
45 bool DFS(int u){
46     int sz = G[u].size();
47     for(int i = 0; i < sz; i++){
48         int v = G[u][i];
49         if(!used[v] && dy[v] == dx[u] + 1){
50             used[v] = true;
51             if(My[v] != -1 && dy[v] == dis) continue;
52             if(My[v] == -1 || DFS(My[v])){
53                 My[v] = u;
54                 Mx[u] = v;
55                 return true;
56             }
57         }
58     }
59     return false;
60 }
61 int MaxMatch(){
62     int res = 0;
63     memset(Mx, -1, sizeof(Mx));
64     memset(My, -1, sizeof(My));
65     while(SearchP()){
66         memset(used, false, sizeof(used));
67         for(int i = 0; i < uN; i++){
68             if(Mx[i] == -1 && DFS(i))
69                 res++;
70         }
71     }
72     return res;
73 }

```

4.11 二分图多重匹配

```

1  const int MAXN = 1010;
2  const int MAXM = 510;
3  int uN, vN;
4  int g[MAXN][MAXM];
5  int linker[MAXM][MAXN];
6  bool used[MAXM];
7  int num[MAXM]; //右边最大的匹配数
8  bool dfs(int u){
9      for(int v = 0; v < vN; v++){
10         if(g[u][v] && !used[v]){
11             used[v] = true;
12             if(linker[v][0] < num[v]){
13                 linker[v][++linker[v][0]] = u;
14                 return true;

```

```

15         }
16         for(int i = 1; i <= num[v]; i++)
17             if(dfs(linker[v][i])){
18                 linker[v][i] = u;
19                 return true;
20             }
21     }
22     return false;
23 }
24 int hungary(){
25     int res = 0;
26     for(int i = 0; i < vN; i++){
27         linker[i][0] = 0;
28         for(int u = 0; u < uN; u++){
29             memset(used, false, sizeof(used));
30             if(dfs(u)) res++;
31         }
32     }
33     return res;
34 }

```

4.12 二分图最大权匹配 (KM 算法)

```

1  /* KM 算法
2   *   复杂度  $O(n_x \times n_x \times n_y)$ 
3   *   求最大权匹配
4   *   若求最小权匹配, 可将权值取相反数, 结果取相反数
5   *   点的编号从 0 开始
6   */
7  const int N = 310;
8  const int INF = 0x3f3f3f3f;
9  int nx, ny; // 两边的点数
10 int g[N][N]; // 二分图描述
11 int linker[N], lx[N], ly[N]; // y 中各点匹配状态, x, y 中的点标号
12 int slack[N];
13 bool visx[N], visy[N];
14 bool DFS(int x){
15     visx[x] = true;
16     for(int y = 0; y < ny; y++){
17         if(visy[y]) continue;
18         int tmp = lx[x] + ly[y] - g[x][y];
19         if(tmp == 0){
20             visy[y] = true;
21             if(linker[y] == -1 || DFS(linker[y])){
22                 linker[y] = x;
23                 return true;
24             }
25         }
26         else if(slack[y] > tmp)
27             slack[y] = tmp;
28     }
29     return false;
30 }

```

```

31 int KM(){
32     memset(linker,-1,sizeof(linker));
33     memset(ly,0,sizeof(ly));
34     for(int i = 0;i < nx;i++){
35         lx[i] = -INF;
36         for(int j = 0;j < ny;j++){
37             if(g[i][j] > lx[i])
38                 lx[i] = g[i][j];
39         }
40         for(int x = 0;x < nx;x++){
41             for(int i = 0;i < ny;i++){
42                 slack[i] = INF;
43                 while(true){
44                     memset(visx,false,sizeof(visx));
45                     memset(visy,false,sizeof(visy));
46                     if(DFS(x))break;
47                     int d = INF;
48                     for(int i = 0;i < ny;i++){
49                         if(!visy[i] && d > slack[i])
50                             d = slack[i];
51                     }
52                     for(int i = 0;i < nx;i++){
53                         if(visx[i])
54                             lx[i] -= d;
55                     }
56                     for(int i = 0;i < ny;i++){
57                         if(visy[i])ly[i] += d;
58                         else slack[i] -= d;
59                     }
60                 }
61             }
62         }
63         int res = 0;
64         for(int i = 0;i < ny;i++){
65             if(linker[i] != -1)
66                 res += g[linker[i]][i];
67         }
68         return res;
69     }
70 }
71 //HDU 2255
72 int main(){
73     int n;
74     while(scanf("%d",&n) == 1){
75         for(int i = 0;i < n;i++){
76             for(int j = 0;j < n;j++){
77                 scanf("%d",&g[i][j]);
78             }
79             nx = ny = n;
80             printf("%d\n",KM());
81         }
82     }
83     return 0;
84 }

```

4.13 一般图匹配带花树

URAL 1099

```
1 const int MAXN = 250;
```

```

2  int N; //点的个数, 点的编号从 1 到 N
3  bool Graph[MAXN][MAXN];
4  int Match[MAXN];
5  bool InQueue[MAXN], InPath[MAXN], InBlossom[MAXN];
6  int Head, Tail;
7  int Queue[MAXN];
8  int Start, Finish;
9  int NewBase;
10 int Father[MAXN], Base[MAXN];
11 int Count; //匹配数, 匹配对数是 Count/2
12 void CreateGraph(){
13     int u, v;
14     memset(Graph, false, sizeof(Graph));
15     scanf("%d", &N);
16     while(scanf("%d%d", &u, &v) == 2){
17         Graph[u][v] = Graph[v][u] = true;
18     }
19 }
20 void Push(int u){
21     Queue[Tail] = u;
22     Tail++;
23     InQueue[u] = true;
24 }
25 int Pop(){
26     int res = Queue[Head];
27     Head++;
28     return res;
29 }
30 int FindCommonAncestor(int u, int v){
31     memset(InPath, false, sizeof(InPath));
32     while(true){
33         u = Base[u];
34         InPath[u] = true;
35         if(u == Start) break;
36         u = Father[Match[u]];
37     }
38     while(true){
39         v = Base[v];
40         if(InPath[v]) break;
41         v = Father[Match[v]];
42     }
43     return v;
44 }
45 void ResetTrace(int u){
46     int v;
47     while(Base[u] != NewBase){
48         v = Match[u];
49         InBlossom[Base[u]] = InBlossom[Base[v]] = true;
50         u = Father[v];
51         if(Base[u] != NewBase) Father[u] = v;
52     }

```

```

53 }
54 void BlossomContract(int u,int v){
55     NewBase = FindCommonAncestor(u,v);
56     memset(InBlossom,false,sizeof(InBlossom));
57     ResetTrace(u);
58     ResetTrace(v);
59     if(Base[u] != NewBase) Father[u] = v;
60     if(Base[v] != NewBase) Father[v] = u;
61     for(int tu = 1; tu <= N; tu++){
62         if(InBlossom[Base[tu]]){
63             Base[tu] = NewBase;
64             if(!InQueue[tu]) Push(tu);
65         }
66     }
67 void FindAugmentingPath(){
68     memset(InQueue,false,sizeof(InQueue));
69     memset(Father,0,sizeof(Father));
70     for(int i = 1;i <= N;i++){
71         Base[i] = i;
72     }
73     Head = Tail = 1;
74     Push(Start);
75     Finish = 0;
76     while(Head < Tail){
77         int u = Pop();
78         for(int v = 1; v <= N; v++){
79             if(Graph[u][v] && (Base[u] != Base[v]) && (Match[u] !=
              v)){
80                 if((v == Start) || ((Match[v] > 0) && Father[Match[
              v]] > 0))
81                     BlossomContract(u,v);
82                 else if(Father[v] == 0){
83                     Father[v] = u;
84                     if(Match[v] > 0)
85                         Push(Match[v]);
86                     else{
87                         Finish = v;
88                         return;
89                     }
90                 }
91             }
92         }
93     }
94 void AugmentPath(){
95     int u,v,w;
96     u = Finish;
97     while(u > 0){
98         v = Father[u];
99         w = Match[v];
100         Match[v] = u;
101         Match[u] = v;
102         u = w;

```

```

102     }
103 }
104 void Edmonds(){
105     memset(Match,0,sizeof(Match));
106     for(int u = 1; u <= N; u++){
107         if(Match[u] == 0){
108             Start = u;
109             FindAugmentingPath();
110             if(Finish > 0)AugmentPath();
111         }
112     }
113 void PrintMatch(){
114     Count = 0;
115     for(int u = 1; u <= N;u++){
116         if(Match[u] > 0)
117             Count++;
118     }
119     printf("%d\n",Count);
120     for(int u = 1; u <= N; u++){
121         if(u < Match[u])
122             printf("%d_ %d\n",u,Match[u]);
123 }
124 int main(){
125     CreateGraph();//建图
126     Edmonds();//进行匹配
127     PrintMatch();//输出匹配数和匹配
128     return 0;
129 }

```

4.14 一般图最大加权匹配

```

1 //一般图的最大加权匹配模板
2 //注意 G 的初始化，需要偶数个点，刚好可以形成 n/2 个匹配
3 //如果要求最小权匹配，可以取相反数，或者稍加修改就可以了
4 //点的编号从 0 开始的
5 const int MAXN = 110;
6 const int INF = 0x3f3f3f3f;
7 int G[MAXN][MAXN];
8 int cnt_node;//点的个数
9 int dis[MAXN];
10 int match[MAXN];
11 bool vis[MAXN];
12 int sta[MAXN],top;//堆栈
13 bool dfs(int u){
14     sta[top++] = u;
15     if(vis[u])return true;
16     vis[u] = true;
17     for(int i = 0;i < cnt_node;i++){
18         if(i != u && i != match[u] && !vis[i]){
19             int t = match[i];
20             if(dis[t] < dis[u] + G[u][i] - G[i][t]){
21                 dis[t] = dis[u] + G[u][i] - G[i][t];

```

```

22         if(dfs(t))return true;
23     }
24 }
25 top--;
26 vis[u] = false;
27 return false;
28 }
29 int P[MAXN];
30 //返回最大匹配权值
31 int get_Match(int N){
32     cnt_node = N;
33     for(int i = 0;i < cnt_node;i++)P[i] = i;
34     for(int i = 0;i < cnt_node;i += 2){
35         match[i] = i+1;
36         match[i+1] = i;
37     }
38     int cnt = 0;
39     while(1){
40         memset(dis,0,sizeof(dis));
41         memset(vis,false,sizeof(vis));
42         top = 0;
43         bool update = false;
44         for(int i = 0;i < cnt_node;i++){
45             if(dfs(P[i])){
46                 update = true;
47                 int tmp = match[sta[top-1]];
48                 int j = top-2;
49                 while(sta[j] != sta[top-1]){
50                     match[tmp] = sta[j];
51                     swap(tmp,match[sta[j]]);
52                     j--;
53                 }
54                 match[tmp] = sta[j];
55                 match[sta[j]] = tmp;
56                 break;
57             }
58             if(!update){
59                 cnt++;
60                 if(cnt >= 3)break;
61                 random_shuffle(P,P+cnt_node);
62             }
63         }
64         int ans = 0;
65         for(int i = 0;i < cnt_node;i++){
66             int v = match[i];
67             if(i < v)ans += G[i][v];
68         }
69         return ans;
70     }

```


4.15 生成树计数

Matrix-Tree 定理 (Kirchhoff 矩阵-树定理)

1、G 的度数矩阵 $D[G]$ 是一个 $n \times n$ 的矩阵，并且满足：当 $i \neq j$ 时, $d_{ij}=0$ ；当 $i=j$ 时, d_{ij} 等于 v_i 的度数。

2、G 的邻接矩阵 $A[G]$ 也是一个 $n \times n$ 的矩阵，并且满足：如果 v_i 、 v_j 之间有边直接相连，则 $a_{ij}=1$ ，否则为 0。

我们定义 G 的 Kirchhoff 矩阵 (也称为拉普拉斯算子) $C[G]$ 为 $C[G]=D[G]-A[G]$ ，则 Matrix-Tree 定理可以描述为：

G 的所有不同的生成树的个数等于其 Kirchhoff 矩阵 $C[G]$ 任何一个 $n-1$ 阶主子式的行列式的绝对值。

所谓 $n-1$ 阶主子式，就是对于 $r(1 \leq r \leq n)$ ，将 $C[G]$ 的第 r 行、第 r 列同时去掉后得到的新矩阵，用 $Cr[G]$ 表示。

//HDU 4305

//求生成树计数部分代码，计数对 10007 取模

```

1  const int MOD = 10007;
2  int INV[MOD];
3  //求 ax = 1( mod m) 的 x 值，就是逆元 (0<a<m)
4  long long inv(long long a,long long m){
5      if(a == 1)return 1;
6      return inv(m%a,m)*(m-m/a)%m;
7  }
8  struct Matrix{
9      int mat[330][330];
10     void init(){
11         memset(mat,0,sizeof(mat));
12     }
13     //求行列式的值模上，需要使用逆元MOD
14     int det(int n){
15         for(int i = 0;i < n;i++){
16             for(int j = 0;j < n;j++){
17                 mat[i][j] = (mat[i][j]%MOD+MOD)%MOD;
18             }
19             int res = 1;
20             for(int i = 0;i < n;i++){
21                 for(int j = i;j < n;j++){
22                     if(mat[j][i]!=0){
23                         for(int k = i;k < n;k++){
24                             swap(mat[i][k],mat[j][k]);
25                         }
26                         if(i != j)
27                             res = (-res+MOD)%MOD;
28                         break;
29                     }
30                 }
31                 if(mat[i][i] == 0){
32                     res = -1;//不存在也就是行列式值为(0)
33                     break;
34                 }
35                 for(int j = i+1;j < n;j++){
36                     //int mut = (mat[j][i]*INV[mat[i][i]])%MOD打表逆元;
37                     int mut = (mat[j][i]*inv(mat[i][i],MOD))%MOD;

```

```

35         for(int k = i;k < n;k++)
36             mat[j][k] = (mat[j][k]-(mat[i][k]*mut)%MOD+MOD)
37                 %MOD;
38     }
39     res = (res * mat[i][i])%MOD;
40 }
41 return res;
42 }
43 };
44
45 Matrix ret;
46 ret.init();
47 for(int i = 0;i < n;i++)
48     for(int j = 0;j < n;j++)
49         if(i != j && g[i][j]){
50             ret.mat[i][j] = -1;
51             ret.mat[i][i]++;
52         }
53 printf("%d\n",ret.det(n-1));

```

计算生成树个数，不取模，SPOJ 104

```

1  const double eps = 1e-8;
2  const int MAXN = 110;
3  int sgn(double x){
4      if(fabs(x) < eps)return 0;
5      if(x < 0)return -1;
6      else return 1;
7  }
8  double b[MAXN][MAXN];
9  double det(double a[][MAXN],int n){
10     int i, j, k, sign = 0;
11     double ret = 1;
12     for(i = 0;i < n;i++)
13         for(j = 0;j < n;j++)
14             b[i][j] = a[i][j];
15     for(i = 0;i < n;i++){
16         if(sgn(b[i][i]) == 0){
17             for(j = i + 1; j < n;j++)
18                 if(sgn(b[j][i]) != 0)
19                     break;
20             if(j == n)return 0;
21             for(k = i;k < n;k++)
22                 swap(b[i][k],b[j][k]);
23             sign++;
24         }
25         ret *= b[i][i];
26         for(k = i + 1;k < n;k++)
27             b[i][k]/=b[i][i];
28         for(j = i+1;j < n;j++)
29             for(k = i+1;k < n;k++)
30                 b[j][k] -= b[j][i]*b[i][k];
31     }

```

```

32     if(sign & 1)ret = -ret;
33     return ret;
34 }
35 double a[MAXN][MAXN];
36 int g[MAXN][MAXN];
37 int main(){
38     int T;
39     int n,m;
40     int u,v;
41     scanf("%d",&T);
42     while(T--){
43         scanf("%d%d",&n,&m);
44         memset(g,0,sizeof(g));
45         while(m--){
46             scanf("%d%d",&u,&v);
47             u--;v--;
48             g[u][v] = g[v][u] = 1;
49         }
50         memset(a,0,sizeof(a));
51         for(int i = 0;i < n;i++)
52             for(int j = 0;j < n;j++)
53                 if(i != j && g[i][j]){
54                     a[i][i]++;
55                     a[i][j] = -1;
56                 }
57         double ans = det(a,n-1);
58         printf("%.0lf\n",ans);
59     }
60     return 0;
61 }

```

4.16 最大流

4.16.1 SAP 邻接矩阵形式

```

1  /*
2   * SAP 算法（矩阵形式）
3   * 结点编号从 0 开始
4   */
5  const int MAXN=1100;
6  int maze[MAXN][MAXN];
7  int gap[MAXN],dis[MAXN],pre[MAXN],cur[MAXN];
8  int sap(int start,int end,int nodenum){
9      memset(cur,0,sizeof(cur));
10     memset(dis,0,sizeof(dis));
11     memset(gap,0,sizeof(gap));
12     int u=pre[start]=start,maxflow=0,aug=-1;
13     gap[0]=nodenum;
14     while(dis[start]<nodenum){
15         loop:
16         for(int v=cur[u];v<nodenum;v++)

```

```

17         if(maze[u][v] && dis[u]==dis[v]+1){
18             if(aug== -1 || aug>maze[u][v]) aug=maze[u][v];
19             pre[v]=u;
20             u=cur[u]=v;
21             if(v==end){
22                 maxflow+=aug;
23                 for(u=pre[u];v!=start;v=u,u=pre[u]){
24                     maze[u][v]-=aug;
25                     maze[v][u]+=aug;
26                 }
27                 aug=-1;
28             }
29             goto loop;
30         }
31         int mindis=nodenum-1;
32         for(int v=0;v<nodenum;v++)
33             if(maze[u][v]&&mindis>dis[v]){
34                 cur[u]=v;
35                 mindis=dis[v];
36             }
37         if((--gap[dis[u]])==0) break;
38         gap[dis[u]=mindis+1]++;
39         u=pre[u];
40     }
41     return maxflow;
42 }

```

4.16.2 SAP 邻接矩阵形式 2

保留原矩阵，可用于多次使用最大流

```

1  /*
2   * SAP 邻接矩阵形式
3   * 点的编号从 0 开始
4   * 增加个 flow 数组，保留原矩阵 maze，可用于多次使用最大流
5   */
6  const int MAXN=1100;
7  int maze[MAXN][MAXN];
8  int gap[MAXN],dis[MAXN],pre[MAXN],cur[MAXN];
9  int flow[MAXN][MAXN]; //存最大流的容量
10 int sap(int start,int end,int nodenum){
11     memset(cur,0,sizeof(cur));
12     memset(dis,0,sizeof(dis));
13     memset(gap,0,sizeof(gap));
14     memset(flow,0,sizeof(flow));
15     int u=pre[start]=start,maxflow=0,aug=-1;
16     gap[0]=nodenum;
17     while(dis[start]<nodenum){
18         loop:
19         for(int v=cur[u];v<nodenum;v++)
20             if(maze[u][v]-flow[u][v] && dis[u]==dis[v]+1){
21                 if(aug== -1 || aug>maze[u][v]-flow[u][v]) aug=maze[u][v]-flow[u][v];

```

```

22         pre[v]=u;
23         u=cur[u]=v;
24         if(v==end){
25             maxflow+=aug;
26             for(u=pre[u];v!=start;v=u,u=pre[u]){
27                 flow[u][v]+=aug;
28                 flow[v][u]-=aug;
29             }
30             aug=-1;
31         }
32         goto loop;
33     }
34     int mindis=nodenum-1;
35     for(int v=0;v<nodenum;v++)
36         if(maze[u][v]-flow[u][v]&&mindis>dis[v]){
37             cur[u]=v;
38             mindis=dis[v];
39         }
40     if((--gap[dis[u]])==0)break;
41     gap[dis[u]=mindis+1]++;
42     u=pre[u];
43 }
44 return maxflow;
45 }

```

4.16.3 ISAP 邻接表形式

```

1  const int MAXN = 100010; //点数的最大值
2  const int MAXM = 400010; //边数的最大值
3  const int INF = 0x3f3f3f3f;
4  struct Edge{
5      int to,next,cap,flow;
6  }edge[MAXM]; //注意是 MAXM
7  int tol;
8  int head[MAXN];
9  int gap[MAXN],dep[MAXN],pre[MAXN],cur[MAXN];
10 void init(){
11     tol = 0;
12     memset(head,-1,sizeof(head));
13 }
14 //加边, 单向图三个参数, 双向图四个参数
15 void addedge(int u,int v,int w,int rw=0){
16     edge[tol].to = v;edge[tol].cap = w;edge[tol].next = head[u];
17     edge[tol].flow = 0;head[u] = tol++;
18     edge[tol].to = u;edge[tol].cap = rw;edge[tol].next = head[v];
19     edge[tol].flow = 0;head[v]=tol++;
20 }
21 //输入参数: 起点、终点、点的总数
22 //点的编号没有影响, 只要输入点的总数
23 int sap(int start,int end,int N){
24     memset(gap,0,sizeof(gap));

```

```

25     memset(dep,0,sizeof(dep));
26     memcpy(cur,head,sizeof(head));
27     int u = start;
28     pre[u] = -1;
29     gap[0] = N;
30     int ans = 0;
31     while(dep[start] < N){
32         if(u == end){
33             int Min = INF;
34             for(int i = pre[u]; i != -1; i = pre[edge[i^1].to])
35                 if(Min > edge[i].cap - edge[i].flow)
36                     Min = edge[i].cap - edge[i].flow;
37             for(int i = pre[u]; i != -1; i = pre[edge[i^1].to]){
38                 edge[i].flow += Min;
39                 edge[i^1].flow -= Min;
40             }
41             u = start;
42             ans += Min;
43             continue;
44         }
45         bool flag = false;
46         int v;
47         for(int i = cur[u]; i != -1; i = edge[i].next){
48             v = edge[i].to;
49             if(edge[i].cap - edge[i].flow && dep[v]+1 == dep[u])
50             {
51                 flag = true;
52                 cur[u] = pre[v] = i;
53                 break;
54             }
55         }
56         if(flag){
57             u = v;
58             continue;
59         }
60         int Min = N;
61         for(int i = head[u]; i != -1; i = edge[i].next)
62             if(edge[i].cap - edge[i].flow && dep[edge[i].to] < Min)
63             {
64                 Min = dep[edge[i].to];
65                 cur[u] = i;
66             }
67         gap[dep[u]]--;
68         if(!gap[dep[u]])return ans;
69         dep[u] = Min+1;
70         gap[dep[u]]++;
71         if(u != start) u = edge[pre[u]^1].to;
72     }
73     return ans;
74 }

```

4.16.4 ISAP+bfs 初始化 + 栈优化

```

1  const int MAXN = 100010; //点数的最大值
2  const int MAXM = 400010; //边数的最大值
3  const int INF = 0x3f3f3f3f;
4  struct Edge{
5      int to,next,cap,flow;
6  }edge[MAXM]; //注意是 MAXM
7  int tol;
8  int head[MAXN];
9  int gap[MAXN],dep[MAXN],cur[MAXN];
10 void init(){
11     tol = 0;
12     memset(head,-1,sizeof(head));
13 }
14 void addedge(int u,int v,int w,int rw = 0){
15     edge[tol].to = v; edge[tol].cap = w; edge[tol].flow = 0;
16     edge[tol].next = head[u]; head[u] = tol++;
17     edge[tol].to = u; edge[tol].cap = rw; edge[tol].flow = 0;
18     edge[tol].next = head[v]; head[v] = tol++;
19 }
20 int Q[MAXN];
21 void BFS(int start,int end){
22     memset(dep,-1,sizeof(dep));
23     memset(gap,0,sizeof(gap));
24     gap[0] = 1;
25     int front = 0, rear = 0;
26     dep[end] = 0;
27     Q[rear++] = end;
28     while(front != rear){
29         int u = Q[front++];
30         for(int i = head[u]; i != -1; i = edge[i].next){
31             int v = edge[i].to;
32             if(dep[v] != -1)continue;
33             Q[rear++] = v;
34             dep[v] = dep[u] + 1;
35             gap[dep[v]]++;
36         }
37     }
38 }
39 int S[MAXN];
40 int sap(int start,int end,int N){
41     BFS(start,end);
42     memcpy(cur,head,sizeof(head));
43     int top = 0;
44     int u = start;
45     int ans = 0;
46     while(dep[start] < N){
47         if(u == end){
48             int Min = INF;
49             int inser;

```

```

50         for(int i = 0; i < top; i++)
51             if(Min > edge[S[i]].cap - edge[S[i]].flow){
52                 Min = edge[S[i]].cap - edge[S[i]].flow;
53                 inser = i;
54             }
55         for(int i = 0; i < top; i++){
56             edge[S[i]].flow += Min;
57             edge[S[i]^1].flow -= Min;
58         }
59         ans += Min;
60         top = inser;
61         u = edge[S[top]^1].to;
62         continue;
63     }
64     bool flag = false;
65     int v;
66     for(int i = cur[u]; i != -1; i = edge[i].next){
67         v = edge[i].to;
68         if(edge[i].cap - edge[i].flow && dep[v]+1 == dep[u]){
69             flag = true;
70             cur[u] = i;
71             break;
72         }
73     }
74     if(flag){
75         S[top++] = cur[u];
76         u = v;
77         continue;
78     }
79     int Min = N;
80     for(int i = head[u]; i != -1; i = edge[i].next)
81         if(edge[i].cap - edge[i].flow && dep[edge[i].to] < Min)
82             {
83                 Min = dep[edge[i].to];
84                 cur[u] = i;
85             }
86     gap[dep[u]]--;
87     if(!gap[dep[u]]) return ans;
88     dep[u] = Min + 1;
89     gap[dep[u]]++;
90     if(u != start) u = edge[S[top]^1].to;
91 }
92 return ans;
93 }

```

4.16.5 dinic

```

1 const int MAXN = 2010; //点数的最大值
2 const int MAXM = 1200010; //边数的最大值
3 const int INF = 0x3f3f3f3f;
4 struct Edge{
5     int to, next, cap, flow;

```



```

6 }edge[MAXM]; //注意是 MAXM
7 int tol;
8 int head[MAXN];
9 void init(){
10     tol = 2;
11     memset(head, -1, sizeof(head));
12 }
13 void addedge(int u, int v, int w, int rw = 0){
14     edge[tol].to = v; edge[tol].cap = w; edge[tol].flow = 0;
15     edge[tol].next = head[u]; head[u] = tol++;
16     edge[tol].to = u; edge[tol].cap = rw; edge[tol].flow = 0;
17     edge[tol].next = head[v]; head[v] = tol++;
18 }
19 int Q[MAXN];
20 int dep[MAXN], cur[MAXN], sta[MAXN];
21 bool bfs(int s, int t, int n){
22     int front = 0, tail = 0;
23     memset(dep, -1, sizeof(dep[0]) * (n + 1));
24     dep[s] = 0;
25     Q[tail++] = s;
26     while(front < tail){
27         int u = Q[front++];
28         for(int i = head[u]; i != -1; i = edge[i].next){
29             int v = edge[i].to;
30             if(edge[i].cap > edge[i].flow && dep[v] == -1){
31                 dep[v] = dep[u] + 1;
32                 if(v == t) return true;
33                 Q[tail++] = v;
34             }
35         }
36     }
37     return false;
38 }
39 int dinic(int s, int t, int n){
40     int maxflow = 0;
41     while(bfs(s, t, n)){
42         for(int i = 0; i < n; i++) cur[i] = head[i];
43         int u = s, tail = 0;
44         while(cur[s] != -1){
45             if(u == t){
46                 int tp = INF;
47                 for(int i = tail - 1; i >= 0; i--)
48                     tp = min(tp, edge[sta[i]].cap - edge[sta[i]].flow);
49                 maxflow += tp;
50                 for(int i = tail - 1; i >= 0; i--){
51                     edge[sta[i]].flow += tp;
52                     edge[sta[i]^1].flow -= tp;
53                     if(edge[sta[i]].cap - edge[sta[i]].flow == 0)
54                         tail = i;
55                 }

```

```

56         u = edge[sta[tail]^1].to;
57     }
58     else if(cur[u] != -1 && edge[cur[u]].cap > edge[cur[u]
59         ].flow && dep[u] + 1 == dep[edge[cur[u]].to]){
60         sta[tail++] = cur[u];
61         u = edge[cur[u]].to;
62     }
63     else {
64         while(u != s && cur[u] == -1)
65             u = edge[sta[--tail]^1].to;
66         cur[u] = edge[cur[u]].next;
67     }
68 }
69 return maxflow;
70 }

```

4.16.6 最大流判断多解

```

1 //判断最大流多解就是在残留网络中找正环
2 bool vis[MAXN],no[MAXN];
3 int Stack[MAXN],top;
4 bool dfs(int u,int pre,bool flag){
5     vis[u] = 1;
6     Stack[top++] = u;
7     for(int i = head[u];i != -1;i = edge[i].next)
8     {
9         int v = edge[i].to;
10        if(edge[i].cap <= edge[i].flow)continue;
11        if(v == pre)continue;
12        if(!vis[v]){
13            if(dfs(v,u,edge[i^1].flow < edge[i^1].cap))return true;
14        }
15        else if(!no[v])return true;
16    }
17    if(!flag){
18        while(1){
19            int v = Stack[--top];
20            no[v] = true;
21            if(v == u)break;
22        }
23    }
24    return false;
25 }
26 //执行完最大流后可进行调用
27 memset(vis,false,sizeof(vis));
28 memset(no,false,sizeof(no));
29 top = 0;
30 bool flag = dfs(end,end,0);

```

4.17 最小费用最大流

4.17.1 SPFA 版费用流

最小费用最大流，求最大费用只需要取相反数，结果取相反数即可。

点的总数为 N ，点的编号 $0 \sim N-1$

```

1  const int MAXN = 10000;
2  const int MAXM = 100000;
3  const int INF = 0x3f3f3f3f;
4  struct Edge{
5      int to,next,cap,flow,cost;
6  }edge[MAXM];
7  int head[MAXN],tol;
8  int pre[MAXN],dis[MAXN];
9  bool vis[MAXN];
10 int N;//节点总个数，节点编号从 0~N-1
11 void init(int n){
12     N = n;
13     tol = 0;
14     memset(head,-1,sizeof(head));
15 }
16 void addedge(int u,int v,int cap,int cost){
17     edge[tol].to = v;
18     edge[tol].cap = cap;
19     edge[tol].cost = cost;
20     edge[tol].flow = 0;
21     edge[tol].next = head[u];
22     head[u] = tol++;
23     edge[tol].to = u;
24     edge[tol].cap = 0;
25     edge[tol].cost = -cost;
26     edge[tol].flow = 0;
27     edge[tol].next = head[v];
28     head[v] = tol++;
29 }
30 bool spfa(int s,int t){
31     queue<int>q;
32     for(int i = 0;i < N;i++){
33         dis[i] = INF;
34         vis[i] = false;
35         pre[i] = -1;
36     }
37     dis[s] = 0;
38     vis[s] = true;
39     q.push(s);
40     while(!q.empty()){
41         int u = q.front();
42         q.pop();
43         vis[u] = false;
44         for(int i = head[u]; i != -1;i = edge[i].next){
45             int v = edge[i].to;

```

```

46         if(edge[i].cap > edge[i].flow && dis[v] > dis[u] + edge
           [i].cost )
47         {
48             dis[v] = dis[u] + edge[i].cost;
49             pre[v] = i;
50             if(!vis[v]){
51                 vis[v] = true;
52                 q.push(v);
53             }
54         }
55     }
56 }
57 if(pre[t] == -1)return false;
58 else return true;
59 }
60 //返回的是最大流, cost 存的是最小费用
61 int minCostMaxflow(int s,int t,int &cost){
62     int flow = 0;
63     cost = 0;
64     while(spfa(s,t)){
65         int Min = INF;
66         for(int i = pre[t];i != -1;i = pre[edge[i^1].to]){
67             if(Min > edge[i].cap - edge[i].flow)
68                 Min = edge[i].cap - edge[i].flow;
69         }
70         for(int i = pre[t];i != -1;i = pre[edge[i^1].to]){
71             edge[i].flow += Min;
72             edge[i^1].flow -= Min;
73             cost += edge[i].cost * Min;
74         }
75         flow += Min;
76     }
77     return flow;
78 }

```

4.17.2 zkw 费用流

对于二分图类型的比较高效

```

1  const int MAXN = 100;
2  const int MAXM = 20000;
3  const int INF = 0x3f3f3f3f;
4  struct Edge{
5      int to,next,cap,flow,cost;
6      Edge(int _to = 0,int _next = 0,int _cap = 0,int _flow = 0,int
          _cost = 0):
7          to(_to),next(_next),cap(_cap),flow(_flow),cost(_cost){}
8  }edge[MAXN];
9  struct ZKW_MinCostMaxFlow{
10     int head[MAXN],tot;
11     int cur[MAXN];
12     int dis[MAXN];

```

```

13  bool vis[MAXN];
14  int ss,tt,N;//源点、汇点和点的总个数（编号是 0~N-1），不需要额外赋值，
    调用会直接赋值
15  int min_cost, max_flow;
16  void init(){
17      tot = 0;
18      memset(head,-1,sizeof(head));
19  }
20  void addedge(int u,int v,int cap,int cost){
21      edge[tot] = Edge(v,head[u],cap,0,cost);
22      head[u] = tot++;
23      edge[tot] = Edge(u,head[v],0,0,-cost);
24      head[v] = tot++;
25  }
26  int aug(int u,int flow){
27      if(u == tt)return flow;
28      vis[u] = true;
29      for(int i = cur[u];i != -1;i = edge[i].next){
30          int v = edge[i].to;
31          if(edge[i].cap > edge[i].flow && !vis[v] && dis[u] ==
              dis[v] + edge[i].cost){
32              int tmp = aug(v,min(flow,edge[i].cap-edge[i].flow))
              ;
33              edge[i].flow += tmp;
34              edge[i^1].flow -= tmp;
35              cur[u] = i;
36              if(tmp)return tmp;
37          }
38      }
39      return 0;
40  }
41  bool modify_label(){
42      int d = INF;
43      for(int u = 0;u < N;u++){
44          if(vis[u])
45              for(int i = head[u];i != -1;i = edge[i].next){
46                  int v = edge[i].to;
47                  if(edge[i].cap>edge[i].flow && !vis[v])
48                      d = min(d,dis[v]+edge[i].cost-dis[u]);
49              }
50      }
51      if(d == INF)return false;
52      for(int i = 0;i < N;i++){
53          if(vis[i]){
54              vis[i] = false;
55              dis[i] += d;
56          }
57      }
58      return true;
59  }
60  /*
    * 直接调用获取最小费用和最大流

```

```

61     * 输入: start-源点, end-汇点, n-点的总个数 (编号从 0 开始)
62     * 返回值: pair<int,int> 第一个是最小费用, 第二个是最大流
63     */
64     pair<int,int> mincostmaxflow(int start,int end,int n){
65         ss = start, tt = end, N = n;
66         min_cost = max_flow = 0;
67         for(int i = 0;i < n;i++)dis[i] = 0;
68         while(1){
69             for(int i = 0;i < n;i++)cur[i] = head[i];
70             while(1){
71                 for(int i = 0;i < n;i++)vis[i] = false;
72                 int tmp = aug(ss,INF);
73                 if(tmp == 0)break;
74                 max_flow += tmp;
75                 min_cost += tmp*dis[ss];
76             }
77             if(!modify_label())break;
78         }
79         return make_pair(min_cost,max_flow);
80     }
81 }solve;

```

4.18 2-SAT

4.18.1 染色法 (可以得到字典序最小的解)

HDU 1814

```

1  const int MAXN = 20020;
2  const int MAXM = 100010;
3  struct Edge
4  {
5      int to,next;
6  }edge[MAXN];
7  int head[MAXN],tot;
8  void init()
9  {
10     tot = 0;
11     memset(head,-1,sizeof(head));
12 }
13 void addedge(int u,int v)
14 {
15     edge[tot].to = v;edge[tot].next = head[u];head[u] = tot++;
16 }
17 bool vis[MAXN]; //染色标记, 为 true 表示选择
18 int S[MAXN],top; //栈
19 bool dfs(int u)
20 {
21     if(vis[u^1])return false;
22     if(vis[u])return true;
23     vis[u] = true;
24     S[top++] = u;

```

```

25     for(int i = head[u]; i != -1; i = edge[i].next)
26         if(!dfs(edge[i].to))
27             return false;
28     return true;
29 }
30 bool Twosat(int n)
31 {
32     memset(vis, false, sizeof(vis));
33     for(int i = 0; i < n; i += 2)
34     {
35         if(vis[i] || vis[i^1]) continue;
36         top = 0;
37         if(!dfs(i))
38         {
39             while(top) vis[S[--top]] = false;
40             if(!dfs(i^1)) return false;
41         }
42     }
43     return true;
44 }
45 int main()
46 {
47     int n, m;
48     int u, v;
49     while(scanf("%d%d", &n, &m) == 2)
50     {
51         init();
52         while(m--)
53         {
54             scanf("%d%d", &u, &v);
55             u--; v--;
56             addedge(u, v^1);
57             addedge(v, u^1);
58         }
59         if(Twosat(2*n))
60         {
61             for(int i = 0; i < 2*n; i++)
62                 if(vis[i])
63                     printf("%d\n", i+1);
64         }
65         else printf("NIE\n");
66     }
67     return 0;
68 }

```

4.18.2 强连通缩点法（拓扑排序只能得到任意解）

POJ 3648 Wedding

```

1 //*****
2 //2-SAT 强连通缩点
3 const int MAXN = 1010;

```

```

4  const int MAXM = 100010;
5  struct Edge
6  {
7      int to,next;
8  }edge[MAXM];
9  int head[MAXN],tot;
10 void init()
11 {
12     tot = 0;
13     memset(head,-1,sizeof(head));
14 }
15 void addedge(int u,int v)
16 {
17     edge[tot].to = v; edge[tot].next = head[u]; head[u] = tot++;
18 }
19 int Low[MAXN],DFN[MAXN],Stack[MAXN],Belong[MAXN]; //Belong 数组的值 1
    ~ scc
20 int Index,top;
21 int scc;
22 bool Instack[MAXN];
23 int num[MAXN];
24 void Tarjan(int u)
25 {
26     int v;
27     Low[u] = DFN[u] = ++Index;
28     Stack[top++] = u;
29     Instack[u] = true;
30     for(int i = head[u];i != -1;i = edge[i].next)
31     {
32         v = edge[i].to;
33         if( !DFN[v] )
34         {
35             Tarjan(v);
36             if(Low[u] > Low[v])Low[u] = Low[v];
37         }
38         else if(Instack[v] && Low[u] > DFN[v])
39             Low[u] = DFN[v];
40     }
41     if(Low[u] == DFN[u])
42     {
43         scc++;
44         do
45         {
46             v = Stack[--top];
47             Instack[v] = false;
48             Belong[v] = scc;
49             num[scc]++;
50         }
51         while(v != u);
52     }
53 }

```



```

54 bool solvable(int n)//n 是总个数, 需要选择一半
55 {
56     memset(DFN,0,sizeof(DFN));
57     memset(Instack,false,sizeof(Instack));
58     memset(num,0,sizeof(num));
59     Index = scc = top = 0;
60     for(int i = 0;i < n;i++)
61         if(!DFN[i])
62             Tarjan(i);
63     for(int i = 0;i < n;i += 2)
64     {
65         if(Belong[i] == Belong[i^1])
66             return false;
67     }
68     return true;
69 }
70 //*****
71
72 //拓扑排序求任意一组解部分
73 queue<int>q1,q2;
74 vector<vector<int> > dag;//缩点后的逆向 DAG 图
75 char color[MAXN];//染色, 为'R' 是选择的
76 int indeg[MAXN];//入度
77 int cf[MAXN];
78 void solve(int n)
79 {
80     dag.assign(scc+1,vector<int>());
81     memset(indeg,0,sizeof(indeg));
82     memset(color,0,sizeof(color));
83     for(int u = 0;u < n;u++)
84         for(int i = head[u];i != -1;i = edge[i].next)
85         {
86             int v = edge[i].to;
87             if(Belong[u] != Belong[v])
88             {
89                 dag[Belong[v]].push_back(Belong[u]);
90                 indeg[Belong[u]]++;
91             }
92         }
93     for(int i = 0;i < n;i += 2)
94     {
95         cf[Belong[i]] = Belong[i^1];
96         cf[Belong[i^1]] = Belong[i];
97     }
98     while(!q1.empty())q1.pop();
99     while(!q2.empty())q2.pop();
100    for(int i = 1;i <= scc;i++)
101        if(indeg[i] == 0)
102            q1.push(i);
103    while(!q1.empty())
104    {

```

```

105     int u = q1.front();
106     q1.pop();
107     if(color[u] == 0)
108     {
109         color[u] = 'R';
110         color[cf[u]] = 'B';
111     }
112     int sz = dag[u].size();
113     for(int i = 0; i < sz; i++)
114     {
115         indeg[dag[u][i]]--;
116         if(indeg[dag[u][i]] == 0)
117             q1.push(dag[u][i]);
118     }
119 }
120 }
121
122 int change(char s[])
123 {
124     int ret = 0;
125     int i = 0;
126     while(s[i] >= '0' && s[i] <= '9')
127     {
128         ret *= 10;
129         ret += s[i] - '0';
130         i++;
131     }
132     if(s[i] == 'w') return 2*ret;
133     else return 2*ret+1;
134 }
135 int main()
136 {
137     int n,m;
138     char s1[10],s2[10];
139     while(scanf("%d%d",&n,&m) == 2)
140     {
141         if(n == 0 && m == 0) break;
142         init();
143         while(m--)
144         {
145             scanf("%s%s",s1,s2);
146             int u = change(s1);
147             int v = change(s2);
148             addedge(u^1,v);
149             addedge(v^1,u);
150         }
151         addedge(1,0);
152         if(solvable(2*n))
153         {
154             solve(2*n);
155             for(int i = 1; i < n; i++)

```

```

156         {
157             //注意这一定是判断 color[Belong]
158             if(color[Belong[2*i]] == 'R')printf("%dw",i);
159             else printf("%dh",i);
160             if(i < n-1)printf("_");
161             else printf("\n");
162         }
163     }
164     else printf("bad luck\n");
165 }
166 return 0;
167 }

```

4.19 曼哈顿最小生成树

POJ 3241 求曼哈顿最小生成树上第 k 大的边

```

1  const int MAXN = 100010;
2  const int INF = 0x3f3f3f3f;
3  struct Point{
4      int x,y,id;
5  }p[MAXN];
6  bool cmp(Point a,Point b){
7      if(a.x != b.x) return a.x < b.x;
8      else return a.y < b.y;
9  }
10 //树状数组，找 y-x 大于当前的，但是 y+x 最小的
11 struct BIT{
12     int min_val,pos;
13     void init()
14     {
15         min_val = INF;
16         pos = -1;
17     }
18 }bit[MAXN];
19 //所有有效边
20 struct Edge{
21     int u,v,d;
22 }edge[MAXN<<2];
23 bool cmpedge(Edge a,Edge b){
24     return a.d < b.d;
25 }
26 int tot;
27 int n;
28 int F[MAXN];
29 int find(int x){
30     if(F[x] == -1) return x;
31     else return F[x] = find(F[x]);
32 }
33 void addedge(int u,int v,int d){
34     edge[tot].u = u;
35     edge[tot].v = v;

```

```

36     edge[tot++].d = d;
37 }
38 int lowbit(int x){
39     return x&(-x);
40 }
41 void update(int i,int val,int pos){
42     while(i > 0){
43         if(val < bit[i].min_val){
44             bit[i].min_val = val;
45             bit[i].pos = pos;
46         }
47         i -= lowbit(i);
48     }
49 }
50 //查询 [i,m] 的最小值位置
51 int ask(int i,int m){
52     int min_val = INF,pos = -1;
53     while(i <= m){
54         if(bit[i].min_val < min_val){
55             min_val = bit[i].min_val;
56             pos = bit[i].pos;
57         }
58         i += lowbit(i);
59     }
60     return pos;
61 }
62 int dist(Point a,Point b){
63     return abs(a.x - b.x) + abs(a.y - b.y);
64 }
65 void Manhattan_minimum_spanning_tree(int n,Point p[]){
66     int a[MAXN],b[MAXN];
67     tot = 0;
68     for(int dir = 0; dir < 4;dir++){
69         //4 种坐标变换
70         if(dir == 1 || dir == 3){
71             for(int i = 0;i < n;i++){
72                 swap(p[i].x,p[i].y);
73             }
74         }
75         else if(dir == 2){
76             for(int i = 0;i < n;i++){
77                 p[i].x = -p[i].x;
78             }
79         }
80         sort(p,p+n,cmp);
81         for(int i = 0;i < n;i++){
82             a[i] = b[i] = p[i].y - p[i].x;
83         }
84         sort(b,b+n);
85         int m = unique(b,b+n) - b;
86         for(int i = 1;i <= m;i++){
87             bit[i].init();
88         }
89         for(int i = n-1 ;i >= 0;i--){
90             int pos = lower_bound(b,b+m,a[i]) - b + 1;
91             update(i,a[i],pos);
92         }
93     }
94 }

```

```

87         int ans = ask(pos,m);
88         if(ans != -1)
89             addedge(p[i].id,p[ans].id,dist(p[i],p[ans]));
90         update(pos,p[i].x+p[i].y,i);
91     }
92 }
93 }
94 int solve(int k){
95     Manhattan_minimum_spanning_tree(n,p);
96     memset(F,-1,sizeof(F));
97     sort(edge,edge+tot,cmpedge);
98     for(int i = 0;i < tot;i++){
99         int u = edge[i].u;
100        int v = edge[i].v;
101        int t1 = find(u), t2 = find(v);
102        if(t1 != t2){
103            F[t1] = t2;
104            k--;
105            if(k == 0)return edge[i].d;
106        }
107    }
108 }
109 int main()
110 {
111     //freopen("in.txt","r",stdin);
112     //freopen("out.txt","w",stdout);
113     int k;
114     while(scanf("%d%d",&n,&k)==2 && n){
115         for(int i = 0;i < n;i++){
116             scanf("%d%d",&p[i].x,&p[i].y);
117             p[i].id = i;
118         }
119         printf("%d\n",solve(n-k));
120     }
121     return 0;
122 }

```

4.20 LCA

4.20.1 dfs+ST 在线算法

```

1  /*
2   * LCA (POJ 1330)
3   * 在线算法 DFS + ST
4   */
5  const int MAXN = 10010;
6  int rmq[2*MAXN]; //rmq 数组, 就是欧拉序列对应的深度序列
7  struct ST{
8      int mm[2*MAXN];
9      int dp[2*MAXN][20]; //最小值对应的下标
10     void init(int n){

```

```

11     mm[0] = -1;
12     for(int i = 1; i <= n; i++){
13         mm[i] = ((i & (i-1)) == 0) ? mm[i-1] + 1 : mm[i-1];
14         dp[i][0] = i;
15     }
16     for(int j = 1; j <= mm[n]; j++){
17         for(int i = 1; i + (1 << j) - 1 <= n; i++){
18             dp[i][j] = rmq[dp[i][j-1]] < rmq[dp[i+(1<<(j-1))][j-1]] ? dp[i][j-1] : dp[i+(1<<(j-1))][j-1];
19         }
20         //查询 [a,b] 之间最小值的下标
21         int query(int a, int b)
22         {
23             if(a > b) swap(a, b);
24             int k = mm[b-a+1];
25             return rmq[dp[a][k]] <= rmq[dp[b-(1<<k)+1][k]] ? dp[a][k] : dp[b-(1<<k)+1][k];
26         }
27     };
28     //边的结构体定义
29     struct Edge{
30         int to, next;
31     };
32     Edge edge[MAXN*2];
33     int tot, head[MAXN];
34
35     int F[MAXN*2]; //欧拉序列, 就是 dfs 遍历的顺序, 长度为 2*n-1, 下标从 1 开始
36     int P[MAXN]; //P[i] 表示点 i 在 F 中第一次出现的位置
37     int cnt;
38     ST st;
39     void init(){
40         tot = 0;
41         memset(head, -1, sizeof(head));
42     }
43     //加边, 无向边需要加两次
44     void addedge(int u, int v){
45         edge[tot].to = v;
46         edge[tot].next = head[u];
47         head[u] = tot++;
48     }
49     void dfs(int u, int pre, int dep){
50         F[++cnt] = u;
51         rmq[cnt] = dep;
52         P[u] = cnt;
53         for(int i = head[u]; i != -1; i = edge[i].next){
54             int v = edge[i].to;
55             if(v == pre) continue;
56             dfs(v, u, dep+1);
57             F[++cnt] = u;
58             rmq[cnt] = dep;
59         }

```

```

60 }
61 //查询 LCA 前的初始化
62 void LCA_init(int root,int node_num){
63     cnt = 0;
64     dfs(root,root,0);
65     st.init(2*node_num-1);
66 }
67 //查询 u,v 的 lca 编号
68 int query_lca(int u,int v){
69     return F[st.query(P[u],P[v])];
70 }
71 bool flag[MAXN];
72 int main(){
73     int T;
74     int N;
75     int u,v;
76     scanf("%d",&T);
77     while(T--){
78         scanf("%d",&N);
79         init();
80         memset(flag,false,sizeof(flag));
81         for(int i = 1; i < N;i++){
82             scanf("%d%d",&u,&v);
83             addedge(u,v);
84             addedge(v,u);
85             flag[v] = true;
86         }
87         int root;
88         for(int i = 1; i <= N;i++){
89             if(!flag[i]){
90                 root = i;
91                 break;
92             }
93         }
94         LCA_init(root,N);
95         scanf("%d%d",&u,&v);
96         printf("%d\n",query_lca(u,v));
97     }
98     return 0;
99 }

```

4.20.2 离线 Tarjan 算法

```

1  /*
2   * POJ 1470
3   * 给出一颗有向树，Q 个查询
4   * 输出查询结果中每个点出现次数
5   */
6  /*
7   * 离线算法，LCATarjan
8   * 复杂度O(n+Q);
9   */
10 const int MAXN = 1010;

```

```

11 const int MAXQ = 500010;//查询数的最大值
12
13 //并查集部分
14 int F[MAXN];//需要初始化为 -1
15 int find(int x){
16     if(F[x] == -1)return x;
17     return F[x] = find(F[x]);
18 }
19 void bing(int u,int v){
20     int t1 = find(u);
21     int t2 = find(v);
22     if(t1 != t2)
23         F[t1] = t2;
24 }
25 //*****
26 bool vis[MAXN];//访问标记
27 int ancestor[MAXN];//祖先
28 struct Edge{
29     int to,next;
30 }edge[MAXN*2];
31 int head[MAXN],tot;
32 void addedge(int u,int v){
33     edge[tot].to = v;
34     edge[tot].next = head[u];
35     head[u] = tot++;
36 }
37
38 struct Query{
39     int q,next;
40     int index;//查询编号
41 }query[MAXQ*2];
42 int answer[MAXQ];//存储最后的查询结果, 下标 0 Q-1
43 int h[MAXQ];
44 int tt;
45 int Q;
46
47 void add_query(int u,int v,int index){
48     query[tt].q = v;
49     query[tt].next = h[u];
50     query[tt].index = index;
51     h[u] = tt++;
52     query[tt].q = u;
53     query[tt].next = h[v];
54     query[tt].index = index;
55     h[v] = tt++;
56 }
57
58 void init(){
59     tot = 0;
60     memset(head,-1,sizeof(head));
61     tt = 0;

```



```

62     memset(h,-1,sizeof(h));
63     memset(vis,false,sizeof(vis));
64     memset(F,-1,sizeof(F));
65     memset(ancestor,0,sizeof(ancestor));
66 }
67 void LCA(int u){
68     ancestor[u] = u;
69     vis[u] = true;
70     for(int i = head[u];i != -1;i = edge[i].next){
71         int v = edge[i].to;
72         if(vis[v])continue;
73         LCA(v);
74         bing(u,v);
75         ancestor[find(u)] = u;
76     }
77     for(int i = h[u];i != -1;i = query[i].next){
78         int v = query[i].q;
79         if(vis[v]){
80             answer[query[i].index] = ancestor[find(v)];
81         }
82     }
83 }
84 bool flag[MAXN];
85 int Count_num[MAXN];
86 int main(){
87     int n;
88     int u,v,k;
89     while(scanf("%d",&n) == 1){
90         init();
91         memset(flag,false,sizeof(flag));
92         for(int i = 1;i <= n;i++){
93             scanf("%d:(%d",&u,&k);
94             while(k--){
95                 scanf("%d",&v);
96                 flag[v] = true;
97                 addedge(u,v);
98                 addedge(v,u);
99             }
100         }
101         scanf("%d",&Q);
102         for(int i = 0;i < Q;i++){
103             char ch;
104             cin>>ch;
105             scanf("%d_%d",&u,&v);
106             add_query(u,v,i);
107         }
108         int root;
109         for(int i = 1;i <= n;i++){
110             if(!flag[i]){
111                 root = i;
112                 break;

```

```

113         }
114         LCA(root);
115         memset(Count_num,0,sizeof(Count_num));
116         for(int i = 0;i < Q;i++)
117             Count_num[answer[i]]++;
118         for(int i = 1;i <= n;i++)
119             if(Count_num[i] > 0)
120                 printf("%d:%d\n",i,Count_num[i]);
121     }
122     return 0;
123 }

```

4.20.3 LCA 倍增法

```

1  /*
2   * POJ 1330
3   * LCA 在线算法
4   */
5  const int MAXN = 10010;
6  const int DEG = 20;
7
8  struct Edge{
9      int to,next;
10 }edge[MAXN*2];
11 int head[MAXN],tot;
12 void addedge(int u,int v){
13     edge[tot].to = v;
14     edge[tot].next = head[u];
15     head[u] = tot++;
16 }
17 void init(){
18     tot = 0;
19     memset(head,-1,sizeof(head));
20 }
21 int fa[MAXN][DEG]; //fa[i][j] 表示结点 i 的第2^j个祖先
22 int deg[MAXN]; //深度数组
23
24 void BFS(int root){
25     queue<int>que;
26     deg[root] = 0;
27     fa[root][0] = root;
28     que.push(root);
29     while(!que.empty()){
30         int tmp = que.front();
31         que.pop();
32         for(int i = 1;i < DEG;i++){
33             fa[tmp][i] = fa[fa[tmp][i-1]][i-1];
34         }
35         for(int i = head[tmp]; i != -1;i = edge[i].next){
36             int v = edge[i].to;
37             if(v == fa[tmp][0]) continue;
38             deg[v] = deg[tmp] + 1;
39             fa[v][0] = tmp;

```

```

39         que.push(v);
40     }
41
42 }
43 }
44 int LCA(int u,int v){
45     if(deg[u] > deg[v])swap(u,v);
46     int hu = deg[u], hv = deg[v];
47     int tu = u, tv = v;
48     for(int det = hv-hu, i = 0; det ;det>>=1, i++)
49         if(det&1)
50             tv = fa[tv][i];
51     if(tu == tv)return tu;
52     for(int i = DEG-1; i >= 0; i--){
53         if(fa[tu][i] == fa[tv][i])
54             continue;
55         tu = fa[tu][i];
56         tv = fa[tv][i];
57     }
58     return fa[tu][0];
59 }
60 bool flag[MAXN];
61 int main(){
62     int T;
63     int n;
64     int u,v;
65     scanf("%d",&T);
66     while(T--){
67         scanf("%d",&n);
68         init();
69         memset(flag,false,sizeof(flag));
70         for(int i = 1;i < n;i++){
71             scanf("%d%d",&u,&v);
72             addedge(u,v);
73             addedge(v,u);
74             flag[v] = true;
75         }
76         int root;
77         for(int i = 1;i <= n;i++){
78             if(!flag[i]){
79                 root = i;
80                 break;
81             }
82         }
83         BFS(root);
84         scanf("%d%d",&u,&v);
85         printf("%d\n",LCA(u,v));
86     }
87     return 0;
88 }

```

4.21 欧拉路

欧拉回路：每条边只经过一次，而且回到起点

欧拉路径：每条边只经过一次，不要求回到起点

欧拉回路判断：

无向图：连通（不考虑度为 0 的点），每个顶点度数都为偶数。

有向图：基图连通（把边当成无向边，同样不考虑度为 0 的点），每个顶点出度等于入度。

混合图（有无向边和有向边）：首先是基图连通（不考虑度为 0 的点），然后需要借助网络流判定。

首先给原图中的每条无向边随便指定一个方向（称为初始定向），将原图改为有向图 G' ，然后任务就是改变 G' 中某些边的方向（当然是无向边转化来的，原混合图中的有向边不能动）使其满足每个点的入度等于出度。

设 $D[i]$ 为 G' 中（点 i 的出度 - 点 i 的入度）。可以发现，在改变 G' 中边的方向的过程中，任何点的 D 值的奇偶性都不会发生改变（设将边 $\langle i, j \rangle$ 改为 $\langle j, i \rangle$ ，则 i 入度加 1 出度减 1， j 入度减 1 出度加 1，两者之差加 2 或减 2，奇偶性不变）！而最终要求的是每个点的入度等于出度，即每个点的 D 值都为 0，是偶数，故可得：若初始定向得到的 G' 中任意一个点的 D 值是奇数，那么原图中一定不存在欧拉环！

若初始 D 值都是偶数，则将 G' 改装成网络：设立源点 S 和汇点 T ，对于每个 $D[i] > 0$ 的点 i ，连边 $\langle S, i \rangle$ ，容量为 $D[i]/2$ ；对于每个 $D[j] < 0$ 的点 j ，连边 $\langle j, T \rangle$ ，容量为 $-D[j]/2$ ； G' 中的每条边在网络中仍保留，容量为 1（表示该边最多只能被改变方向一次）。求这个网络的最大流，若 S 引出的所有边均满流，则原混合图是欧拉图，将网络中所有流量为 1 的中间边（就是不与 S 或 T 关联的边）在 G' 中改变方向，形成的新图 G'' 一定是有向欧拉图；若 S 引出的边中有的没有满流，则原混合图不是欧拉图。

欧拉路径的判断：

无向图：连通（不考虑度为 0 的点），每个顶点度数都为偶数或者仅有两个点的度数为偶数。

有向图：基图连通（把边当成无向边，同样不考虑度为 0 的点），每个顶点出度等于入度或者有且仅有一个点的出度比入度多 1，有且仅有一个点的出度比入度少 1，其余出度等于入度。

混合图：如果存在欧拉回路，一点存在欧拉路径了。否则如果有且仅有两个点的（出度 - 入度）是奇数，那么给这两个点加边，判断是否存在欧拉回路。

4.21.1 有向图

POJ 2337

给出 n 个小写字母组成的单词，要求将 n 个单词连接起来，使得前一个单词的最后一个字母和后一个单词的第一个字母相同。输出字典序最小的解。

```

1 struct Edge{
2     int to,next;
3     int index;
4     bool flag;
5 }edge[2010];
6 int head[30],tot;
7 void init(){
8     tot = 0;
9     memset(head,-1,sizeof(head));
10 }
11 void addedge(int u,int v,int index){
12     edge[tot].to = v;
```

```

13     edge[tot].next = head[u];
14     edge[tot].index = index;
15     edge[tot].flag = false;
16     head[u] = tot++;
17 }
18 string str[1010];
19 int in[30],out[30];
20 int cnt;
21 int ans[1010];
22 void dfs(int u){
23     for(int i = head[u] ;i != -1;i = edge[i].next)
24         if(!edge[i].flag)
25         {
26             edge[i].flag = true;
27             dfs(edge[i].to);
28             ans[cnt++] = edge[i].index;
29         }
30 }
31 int main(){
32     int T,n;
33     scanf("%d",&T);
34     while(T--){
35         scanf("%d",&n);
36         for(int i = 0;i < n;i++)
37             cin>>str[i];
38         sort(str,str+n);//要输出字典序最小的解，先按照字典序排序
39         init();
40         memset(in,0,sizeof(in));
41         memset(out,0,sizeof(out));
42         int start = 100;
43         for(int i = n-1;i >= 0;i--)//字典序大的先加入
44         {
45             int u = str[i][0] - 'a';
46             int v = str[i][str[i].length() - 1] - 'a';
47             addedge(u,v,i);
48             out[u]++;
49             in[v]++;
50             if(u < start)start = u;
51             if(v < start)start = v;
52         }
53         int cc1 = 0, cc2 = 0;
54         for(int i = 0;i < 26;i++){
55             if(out[i] - in[i] == 1){
56                 cc1++;
57                 start = i;//如果有一个出度比入度大 1 的点，就从这个点出发，
否则从最小的点出发
58             }
59             else if(out[i] - in[i] == -1)
60                 cc2++;
61             else if(out[i] - in[i] != 0)
62                 cc1 = 3;

```

```

63     }
64     if(! ( (cc1 == 0 && cc2 == 0) || (cc1 == 1 && cc2 == 1) )){
65         printf("***\n");
66         continue;
67     }
68     cnt = 0;
69     dfs(start);
70     if(cnt != n)//判断是否连通
71     {
72         printf("***\n");
73         continue;
74     }
75     for(int i = cnt-1; i >= 0; i--){
76         cout<<str[ans[i]];
77         if(i > 0)printf(".");
78         else printf("\n");
79     }
80 }
81 return 0;
82 }

```

4.21.2 无向图

SGU 101

```

1 struct Edge{
2     int to,next;
3     int index;
4     int dir;
5     bool flag;
6 }edge[220];
7 int head[10],tot;
8 void init(){
9     memset(head,-1,sizeof(head));
10    tot = 0;
11 }
12 void addedge(int u,int v,int index){
13     edge[tot].to = v;
14     edge[tot].next = head[u];
15     edge[tot].index = index;
16     edge[tot].dir = 0;
17     edge[tot].flag = false;
18     head[u] = tot++;
19     edge[tot].to = u;
20     edge[tot].next = head[v];
21     edge[tot].index = index;
22     edge[tot].dir = 1;
23     edge[tot].flag = false;
24     head[v] = tot++;
25 }
26 int du[10];
27 vector<int>ans;

```

```

28 void dfs(int u){
29     for(int i = head[u]; i != -1; i = edge[i].next)
30         if(!edge[i].flag ){
31             edge[i].flag = true;
32             edge[i^1].flag = true;
33             dfs(edge[i].to);
34             ans.push_back(i);
35         }
36 }
37 int main(){
38     int n;
39     while(scanf("%d",&n) == 1){
40         init();
41         int u,v;
42         memset(du,0,sizeof(du));
43         for(int i = 1;i <= n;i++){
44             scanf("%d%d",&u,&v);
45             addedge(u,v,i);
46             du[u]++;
47             du[v]++;
48         }
49         int s = -1;
50         int cnt = 0;
51         for(int i = 0;i <= 6;i++){
52             if(du[i]&1) {cnt++; s = i;}
53             if(du[i] > 0 && s == -1)
54                 s = i;
55         }
56         bool ff = true;
57         if(cnt != 0 && cnt != 2){
58             printf("No solution\n");
59             continue;
60         }
61         ans.clear();
62         dfs(s);
63         if(ans.size() != n){
64             printf("No solution\n");
65             continue;
66         }
67         for(int i = 0;i < ans.size();i++){
68             printf("%d_",edge[ans[i]].index);
69             if(edge[ans[i]].dir == 0)printf("-\n");
70             else printf("+\n");
71         }
72     }
73     return 0;
74 }

```

4.21.3 混合图

POJ 1637 （本题保证了连通，故不需要判断连通，否则要判断连通）

```

1  const int MAXN = 210;
2  //最大流部分ISAP
3  const int MAXM = 20100;
4  const int INF = 0x3f3f3f3f;
5  struct Edge
6  {
7      int to,next,cap,flow;
8  }edge[MAXN];
9  int tol;
10 int head[MAXN];
11 int gap[MAXN],dep[MAXN],pre[MAXN],cur[MAXN];
12 void init()
13 {
14     tol = 0;
15     memset(head,-1,sizeof(head));
16 }
17 void addedge(int u,int v,int w,int rw = 0)
18 {
19     edge[tol].to = v;
20     edge[tol].cap = w;
21     edge[tol].next = head[u];
22     edge[tol].flow = 0;
23     head[u] = tol++;
24     edge[tol].to = u;
25     edge[tol].cap = rw;
26     edge[tol].next = head[v];
27     edge[tol].flow = 0;
28     head[v] = tol++;
29 }
30 int sap(int start,int end,int N)
31 {
32     memset(gap,0,sizeof(gap));
33     memset(dep,0,sizeof(dep));
34     memcpy(cur,head,sizeof(head));
35     int u = start;
36     pre[u] = -1;
37     gap[0] = N;
38     int ans = 0;
39     while(dep[start] < N)
40     {
41         if(u == end)
42         {
43             int Min = INF;
44             for(int i = pre[u]; i != -1; i = pre[edge[i^1].to])
45                 if(Min > edge[i].cap - edge[i].flow)
46                     Min = edge[i].cap - edge[i].flow;
47             for(int i = pre[u]; i != -1; i = pre[edge[i^1].to])
48             {
49                 edge[i].flow += Min;
50                 edge[i^1].flow -= Min;
51             }

```



```

52         u = start;
53         ans += Min;
54         continue;
55     }
56     bool flag = false;
57     int v;
58     for(int i = cur[u]; i != -1; i = edge[i].next)
59     {
60         v = edge[i].to;
61         if(edge[i].cap - edge[i].flow && dep[v] + 1 == dep[u])
62         {
63             flag = true;
64             cur[u] = pre[v] = i;
65             break;
66         }
67     }
68     if(flag)
69     {
70         u = v;
71         continue;
72     }
73     int Min = N;
74     for(int i = head[u]; i != -1; i = edge[i].next)
75         if(edge[i].cap - edge[i].flow && dep[edge[i].to] < Min)
76         {
77             Min = dep[edge[i].to];
78             cur[u] = i;
79         }
80     gap[dep[u]]--;
81     if(!gap[dep[u]]) return ans;
82     dep[u] = Min+1;
83     gap[dep[u]]++;
84     if(u != start) u = edge[pre[u]^1].to;
85 }
86 return ans;
87 }
88 //the end of 最大流部分
89
90 int in[MAXN], out[MAXN]; //每个点的出度和入度
91
92 int main()
93 {
94     //freopen("in.txt", "r", stdin);
95     //freopen("out.txt", "w", stdout);
96     int T;
97     int n, m;
98     scanf("%d", &T);
99     while(T--)
100     {
101         scanf("%d%d", &n, &m);
102         init();

```

```

103     int u,v,w;
104     memset(in,0,sizeof(in));
105     memset(out,0,sizeof(out));
106     while(m--)
107     {
108         scanf("%d%d%d",&u,&v,&w);
109         out[u]++; in[v]++;
110         if(w == 0)//双向
111             addedge(u,v,1);
112     }
113     bool flag = true;
114     for(int i = 1;i <= n;i++)
115     {
116         if(out[i] - in[i] > 0)
117             addedge(0,i,(out[i] - in[i])/2);
118         else if(in[i] - out[i] > 0)
119             addedge(i,n+1,(in[i] - out[i])/2);
120         if((out[i] - in[i]) & 1)
121             flag = false;
122     }
123     if(!flag)
124     {
125         printf("impossible\n");
126         continue;
127     }
128     sap(0,n+1,n+2);
129     for(int i = head[0]; i != -1;i = edge[i].next)
130         if(edge[i].cap > 0 && edge[i].cap > edge[i].flow)
131         {
132             flag = false;
133             break;
134         }
135     if(flag)printf("possible\n");
136     else printf("impossible\n");
137 }
138 return 0;
139 }

```

4.22 树分治

4.22.1 点分治 -HDU5016

HDU 5016 给定一个边权树，初始时，一些结点上已经建立了市场。每个结点会被距离自己最近的市场所支配（距离相同时，被标号最小的市场支配）可以新建一个市场，问新建的市场最多可以支配多少点。

```

1  const int MAXN = 100010;
2  const int INF = 0x3f3f3f3f;
3  struct Edge{
4      int to,next,w;
5  }edge[MAXN*2];
6  int head[MAXN],tot;

```

```

7 void init(){
8     tot = 0;
9     memset(head,-1,sizeof(head));
10 }
11 inline void addedge(int u,int v,int w){
12     edge[tot].to = v;
13     edge[tot].w = w;
14     edge[tot].next = head[u];
15     head[u] = tot++;
16 }
17 int size[MAXN],vis[MAXN],fa[MAXN],que[MAXN];
18 int TT;//时间戳
19 //找重心
20 inline int getroot(int u){
21     int Min = MAXN, root = 0;
22     int l,r;
23     que[l = r = 1] = u;
24     fa[u] = 0;
25     for(;l <= r;l++){
26         for(int i = head[que[l]]; i != -1;i = edge[i].next){
27             int v = edge[i].to;
28             if(v == fa[que[l]] || vis[v] == TT)continue;
29             que[++r] = v;
30             fa[v] = que[l];
31         }
32         for(l--;l;l--){
33             int x = que[l], Max = 0;
34             size[x] = 1;
35             for(int i = head[x];i != -1;i = edge[i].next){
36                 int v = edge[i].to;
37                 if(v == fa[x] || vis[v] == TT)continue;
38                 Max = max(Max,size[v]);
39                 size[x] += size[v];
40             }
41             Max = max(Max,r - size[x]);
42             if(Max < Min){
43                 Min = Max; root = x;
44             }
45         }
46     }
47     return root;
48 }
49 int ans[MAXN];
50 pair<int,int>pp[MAXN];
51 pair<int,int>np[MAXN];
52 int dis[MAXN];
53 int type[MAXN];
54 inline void go(int u,int pre,int w,int tt){
55     int l,r;
56     que[l = r = 1] = u;
57     fa[u] = pre; dis[u] = w;

```

```

58     for(;l <= r;l++)
59         for(int i = head[que[l]];i != -1;i = edge[i].next){
60             int v = edge[i].to;
61             if(v == fa[que[l]] || vis[v] == TT)continue;
62             que[++r] = v;
63             fa[v] = que[l];
64             dis[v] = dis[que[l]] + edge[i].w;
65         }
66     int cnt = 0;
67     for(int i = 1;i <= r;i++){
68         int x = que[i];
69         pp[cnt++] = make_pair(np[x].first-dis[x],np[x].second);
70     }
71     sort(pp,pp+cnt);
72     for(int i = 1;i <= r;i++){
73         int x = que[i];
74         if(type[x])continue;
75         int id = lower_bound(pp,pp+cnt,make_pair(dis[x],x)) - pp;
76         ans[x] += (cnt-id)*tt;
77     }
78 }
79 void solve(int u){
80     int root = getroot(u);
81     vis[root] = TT;
82     go(root,0,0,1);
83     for(int i = head[root];i != -1;i = edge[i].next){
84         int v = edge[i].to;
85         if(vis[v] == TT)continue;
86         go(v,root,edge[i].w,-1);
87     }
88     for(int i = head[root];i != -1;i = edge[i].next){
89         int v = edge[i].to;
90         if(vis[v] == TT)continue;
91         solve(v);
92     }
93 }
94 bool ff[MAXN];
95 int main()
96 {
97     int n;
98     memset(vis,0,sizeof(vis));
99     TT = 0;
100    while(scanf("%d",&n) == 1){
101        init();
102        int u,v,w;
103        for(int i = 1;i < n;i++){
104            scanf("%d%d%d",&u,&v,&w);
105            addedge(u,v,w);
106            addedge(v,u,w);
107        }
108        for(int i = 1;i <= n;i++)scanf("%d",&type[i]);

```

```

109     queue<int>q;
110     for(int i = 1;i <= n;i++){
111         if(type[i]){
112             np[i] = make_pair(0,i);
113             ff[i] = true;
114             q.push(i);
115         }
116         else{
117             np[i] = make_pair(INF,0);
118             ff[i] = false;
119         }
120     }
121     while(!q.empty()){
122         u = q.front();
123         q.pop();
124         ff[u] = false;
125         for(int i = head[u];i != -1;i = edge[i].next){
126             v = edge[i].to;
127             pair<int,int>tmp = make_pair(np[u].first+edge[i].w,
128                                     np[u].second);
129             if(tmp < np[v]){
130                 np[v] = tmp;
131                 if(!ff[v]){
132                     ff[v] = true;
133                     q.push(v);
134                 }
135             }
136         }
137         TT++;
138         for(int i = 1;i <= n;i++)ans[i] = 0;
139         solve(1);
140         int ret = 0;
141         for(int i = 1;i <= n;i++)ret = max(ret,ans[i]);
142         printf("%d\n",ret);
143     }
144     return 0;
145 }

```

4.22.2 * 点分治 -HDU4918

HDU 4918

题意：给出一颗 n 个点的树，每个点有一个权值，有两种操作，一种是将某个点的权值修改为 v ，另一种是查询距离点 u 不超过 d 的点的权值和。

```

1  const int MAXN = 100010;
2  const int MAXD = 40;
3  int cc[MAXN*MAXD];
4  int *cc_tail; //记得初始化 cc_tail = cc
5  //0-based BinaryIndexTree
6  struct BIT{
7      int *c;

```

```

8      int n;
9      void init(int _n){
10         n = _n;
11         c = cc_tail;
12         cc_tail = cc_tail + n;
13         memset(c,0,sizeof(int)*n);
14     }
15     void add(int i,int val){
16         while(i < n){
17             c[i] += val;
18             i += ~i & i+1;
19         }
20     }
21     int sum(int i){
22         i = min(i,n-1);
23         int s = 0;
24         while(i >= 0){
25             s += c[i];
26             i -= ~i & i+1;
27         }
28         return s;
29     }
30 }bits[MAXN<<1];
31 struct Edge{
32     int to,next;
33 }edge[MAXN*2];
34 int head[MAXN],tot;
35 void init(){
36     tot = 0;
37     memset(head,-1,sizeof(head));
38 }
39 inline void addedge(int u,int v){
40     edge[tot].to = v;
41     edge[tot].next = head[u];
42     head[u] = tot++;
43 }
44 int size[MAXN],vis[MAXN],fa[MAXN],que[MAXN];
45 int TT;
46 inline int getroot(int u,int &tot){
47     int Min = MAXN, root = 0;
48     int l,r;
49     que[l = r = 1] = u;
50     fa[u] = 0;
51     for(;l <= r;l++){
52         for(int i = head[que[l]];i != -1;i = edge[i].next){
53             int v = edge[i].to;
54             if(v == fa[que[l]] || vis[v] == TT)continue;
55             que[++r] = v;
56             fa[v] = que[l];
57         }
58     }
    tot = r;

```

```

59     for(l--;l;l++){
60         int x = que[l], Max = 0;
61         size[x] = 1;
62         for(int i = head[x];i != -1;i = edge[i].next){
63             int v = edge[i].to;
64             if(v == fa[x] || vis[v] == TT)continue;
65             Max = max(Max,size[v]);
66             size[x] += size[v];
67         }
68         Max = max(Max,r - size[x]);
69         if(Max < Min){
70             Min = Max, root = x;
71         }
72     }
73     return root;
74 }
75 struct Node{
76     int root,subtree,dis;
77     Node(int _root = 0, int _subtree = 0,int _dis = 0){
78         root = _root;
79         subtree = _subtree;
80         dis = _dis;
81     }
82 };
83 vector<Node>vec[MAXN];
84 int id[MAXN];
85 int dist[MAXN];
86 int val[MAXN];
87 int cnt;
88 inline void go(int u,int root,int subtree){
89     int l,r;
90     que[l = r = 1] = u;
91     fa[u] = 0; dist[u] = 1;
92     for(; l <= r;l++){
93         u = que[l];
94         vec[u].push_back(Node(id[root],subtree,dist[u]));
95         for(int i = head[u];i != -1;i = edge[i].next){
96             int v = edge[i].to;
97             if(v == fa[u] || vis[v] == TT)continue;
98             que[++r] = v;
99             fa[v] = u;
100             dist[v] = dist[u] + 1;
101         }
102     }
103     bits[subtree].init(r+1);
104     for(int i = 1;i <= r;i++){
105         u = que[i];
106         bits[id[root]].add(dist[u],val[u]);
107         bits[subtree].add(dist[u],val[u]);
108     }
109 }

```

```

110 void solve(int u){
111     int tot;
112     int root = getroot(u,tot);
113     vis[root] = TT;
114     id[root] = cnt++;
115     vec[root].push_back(Node(id[root],-1,0));
116     bits[id[root]].init(tot);
117     bits[id[root]].add(0,val[root]);
118     for(int i = head[root];i != -1;i = edge[i].next){
119         int v = edge[i].to;
120         if(vis[v] == TT)continue;
121         go(v,root,cnt);
122         cnt++;
123     }
124     for(int i = head[root];i != -1;i = edge[i].next){
125         int v = edge[i].to;
126         if(vis[v] == TT)continue;
127         solve(v);
128     }
129 }
130 int main(){
131     int n,m;
132     memset(vis,0,sizeof(vis));
133     TT = 0;
134     while(scanf("%d%d",&n,&m) == 2){
135         init();
136         TT++;
137         cc_tail = cc;
138         cnt = 0;
139         for(int i = 1;i <= n;i++)vec[i].clear();
140         for(int i = 1;i <= n;i++)scanf("%d",&val[i]);
141         int u,v;
142         for(int i = 1;i < n;i++){
143             scanf("%d%d",&u,&v);
144             addedge(u,v);
145             addedge(v,u);
146         }
147         solve(1);
148         char op[10];
149         int d;
150         while(m--){
151             scanf("%s%d%d",op,&u,&d);
152             if(op[0] == '!'){
153                 int dv = d - val[u];
154                 int sz = vec[u].size();
155                 for(int i = 0;i < sz;i++){
156                     Node tmp = vec[u][i];
157                     bits[tmp.root].add(tmp.dis,dv);
158                     if(tmp.subtree != -1)
159                         bits[tmp.subtree].add(tmp.dis,dv);
160                 }

```



```

161         val[u] += dv;
162     }
163     else {
164         int ans = 0;
165         int sz = vec[u].size();
166         for(int i = 0; i < sz; i++){
167             Node tmp = vec[u][i];
168             ans += bits[tmp.root].sum(d-tmp.dis);
169             if(tmp.subtree != -1)
170                 ans -= bits[tmp.subtree].sum(d-tmp.dis);
171         }
172         printf("%d\n", ans);
173     }
174 }
175 }
176 return 0;
177 }

```

4.22.3 链分治 -HDU5039

HDU 5039 一颗树，每条边的属性为 0 或 1，求有多少条路径经过奇数条属性为 1 的边。一种是查询操作，一种是修改边的属性。

虽然这题有更加简单的方法，但是用来练习链分治还是不错的。

```

1  const int MAXN = 30010;
2  const int INF = 0x3f3f3f3f;
3  struct Edge{
4      int to,next;
5      int f;
6  }edge[MAXN*2];
7  int head[MAXN],tot;
8  void init(){
9      tot = 0;
10     memset(head,-1,sizeof(head));
11 }
12 void addedge(int u,int v,int f){
13     edge[tot].to = v;
14     edge[tot].next = head[u];
15     edge[tot].f = f;
16     head[u] = tot++;
17 }
18 long long ans;
19 int num0[MAXN],num1[MAXN];
20 long long tnum[MAXN];
21 struct Node{
22     int l0,l1;
23     int r0,r1;
24     int cc;
25     long long sum;
26     Node gao(int u){
27         l0 = r0 = num0[u];
28         l1 = r1 = num1[u];

```

```

29         sum = tnum[u];
30         cc = 0;
31         return *this;
32     }
33 };
34 int pos[MAXN];
35 int val[MAXN];
36 int fa[MAXN];
37 int cnt[MAXN];
38 int col[MAXN];
39 int link[MAXN];
40 int CHANGEU;
41 struct chain{
42     vector<int>uu;
43     vector<Node>nde;
44     int n;
45     void init(){
46         n = uu.size();
47         nde.resize(n << 2);
48         for(int i = 0;i < n;i++)pos[uu[i]] = i;
49         build(0,n-1,1);
50     }
51     void up(int l,int r,int p){
52         int mid = (l+r)>>1;
53         nde[p].cc = nde[p<<1].cc ^ nde[(p<<1)|1].cc ^ val[uu[mid]];
54         nde[p].l0 = nde[p<<1].l0;
55         nde[p].l1 = nde[p<<1].l1;
56         if(nde[p<<1].cc^val[uu[mid]]){
57             nde[p].l0 += nde[(p<<1)|1].l1;
58             nde[p].l1 += nde[(p<<1)|1].l0;
59         }
60         else {
61             nde[p].l0 += nde[(p<<1)|1].l0;
62             nde[p].l1 += nde[(p<<1)|1].l1;
63         }
64         nde[p].r0 = nde[(p<<1)|1].r0;
65         nde[p].r1 = nde[(p<<1)|1].r1;
66         if(nde[(p<<1)|1].cc^val[uu[mid]]){
67             nde[p].r0 += nde[p<<1].r1;
68             nde[p].r1 += nde[p<<1].r0;
69         }
70         else {
71             nde[p].r0 += nde[p<<1].r0;
72             nde[p].r1 += nde[p<<1].r1;
73         }
74         if(val[uu[mid]] == 0){
75             nde[p].sum = nde[p<<1].sum + nde[(p<<1)|1].sum +
76                 (long long)nde[p<<1].r0*nde[(p<<1)|1].l1 +
77                 (long long)nde[p<<1].r1*nde[(p<<1)|1].l0;
78         }
79         else {

```

```

80         nde[p].sum = nde[p<<1].sum + nde[(p<<1)|1].sum +
81         (long long)nde[p<<1].r0*nde[(p<<1)|1].l0 +
82         (long long)nde[p<<1].r1*nde[(p<<1)|1].l1;
83     }
84 }
85 void build(int l,int r,int p){
86     if(l == r){
87         nde[p].gao(uu[l]);
88         return;
89     }
90     int mid = (l+r)/2;
91     build(l,mid,p<<1);
92     build(mid+1,r,(p<<1)|1);
93     up(l,r,p);
94 }
95 void update(int k,int l,int r,int p){
96     if(l == r){
97         nde[p].gao(uu[k]);
98         return;
99     }
100    int mid = (l+r)/2;
101    if(k <= mid)update(k,l,mid,p<<1);
102    else update(k,mid+1,r,(p<<1)|1);
103    up(l,r,p);
104 }
105 int change(int y){
106     int x = uu.back();
107     int p = fa[x];
108     if(p){
109         if(x == CHANGEU)val[x] ^= 1;
110         if(val[x]){
111             tnum[p] -= (long long)nde[1].r0*(num0[p]-nde[1].r1)
112             ;
113             tnum[p] -= (long long)nde[1].r1*(num1[p]-nde[1].r0)
114             ;
115             num0[p] -= nde[1].r1;
116             num1[p] -= nde[1].r0;
117         }
118         else {
119             tnum[p] -= (long long)nde[1].r1*(num0[p]-nde[1].r0)
120             ;
121             tnum[p] -= (long long)nde[1].r0*(num1[p]-nde[1].r1)
122             ;
123             num0[p] -= nde[1].r0;
124             num1[p] -= nde[1].r1;
125         }
126         if(x == CHANGEU)val[x] ^= 1;
127     }
128     ans -= nde[1].sum;
129     update(pos[y],0,n-1,1);
130     if(p){

```

```

127         if(val[x]){
128             tnum[p] += (long long)nde[1].r0*num0[p];
129             tnum[p] += (long long)nde[1].r1*num1[p];
130             num0[p] += nde[1].r1;
131             num1[p] += nde[1].r0;
132         }
133         else {
134             tnum[p] += (long long)nde[1].r0*num1[p];
135             tnum[p] += (long long)nde[1].r1*num0[p];
136             num0[p] += nde[1].r0;
137             num1[p] += nde[1].r1;
138         }
139     }
140     ans += nde[1].sum;
141     return p;
142 }
143 }ch[MAXN];
144 void dfs1(int u,int pre){
145     chain &c = ch[u];
146     c.uu.clear();
147     int v, x = 0;
148     cnt[u] = 1;
149     for(int i = head[u];i != -1;i = edge[i].next){
150         v = edge[i].to;
151         if(v == pre)continue;
152         dfs1(v,u);
153         link[i/2] = v;
154         val[v] = edge[i].f;
155         cnt[u] += cnt[v];
156         fa[v] = u;
157         if(cnt[v] > cnt[x]) x = v;
158     }
159     if(!x)col[u] = u;
160     else col[u] = col[x];
161     ch[col[u]].uu.push_back(u);
162     num0[u] = 1;
163     num1[u] = 0;
164     tnum[u] = 0;
165 }
166 }
167 void dfs2(int x){
168     x = col[x];
169     chain &c = ch[x];
170     int n = c.uu.size();
171     int u,v;
172     for(int i = 1;i < n;i++){
173         u = c.uu[i];
174         for(int j = head[u];j != -1;j = edge[j].next){
175             v = edge[j].to;
176             if(v == c.uu[i-1] || fa[u] == v)continue;
177             dfs2(v);

```

```

178         if(val[v]){
179             tnum[u] += (long long)num0[u]*ch[col[v]].nde[1].r0
                  + (long long)num1[u]*ch[col[v]].nde[1].r1;
180             num0[u] += ch[col[v]].nde[1].r1;
181             num1[u] += ch[col[v]].nde[1].r0;
182         }
183         else {
184             tnum[u] += (long long)num1[u]*ch[col[v]].nde[1].r0
                  + (long long)num0[u]*ch[col[v]].nde[1].r1;
185             num0[u] += ch[col[v]].nde[1].r0;
186             num1[u] += ch[col[v]].nde[1].r1;
187         }
188     }
189 }
190 c.init();
191 ans += c.nde[1].sum;
192 }
193 char str[100];
194 char str1[100],str2[100];
195 int main()
196 {
197     int T;
198     int iCase = 0;
199     scanf("%d",&T);
200     int n;
201     while(T--){
202         ans = 0;
203         iCase++;
204         scanf("%d",&n);
205         map<string,int>mp;
206         init();
207         for(int i = 1;i <= n;i++){
208             scanf("%s",str);
209             mp[str] = i;
210         }
211         int u,v,f;
212         for(int i = 1;i < n;i++){
213             scanf("%s%s%d",str1,str2,&f);
214             addedge(mp[str1],mp[str2],f);
215             addedge(mp[str2],mp[str1],f);
216         }
217         int Q;
218         char op[10];
219         scanf("%d",&Q);
220         printf("Case_#%d:\n",iCase);
221         val[1] = 0;
222         fa[1] = 0;
223         dfs1(1,1);
224         dfs2(1);
225         while(Q--){
226             scanf("%s",op);

```

```
227         if(op[0] == 'Q'){
228             printf("%I64d\n",ans*2);
229         }
230         else {
231             int id ;
232             scanf("%d",&id);
233             id--;
234             u = link[id];
235             val[u] ^= 1;
236             CHANGEU = u;
237             while(u)
238                 u = ch[col[u]].change(u);
239         }
240     }
241 }
242 return 0;
243 }
```

5 搜索

5.1 Dancing Links

5.1.1 精确覆盖

```

1  /*
2   * POJ3074
3   */
4  const int N = 9; //3*3 数独
5  const int MaxN = N*N*N + 10;
6  const int MaxM = N*N*4 + 10;
7  const int maxnode = MaxN*4 + MaxM + 10;
8  char g[MaxN];
9  struct DLX{
10     int n,m,size;
11     int U[maxnode],D[maxnode],R[maxnode],L[maxnode],Row[maxnode],
        Col[maxnode];
12     int H[MaxN],S[MaxM];
13     int ansd,ans[MaxN];
14     void init(int _n,int _m){
15         n = _n;
16         m = _m;
17         for(int i = 0;i <= m;i++){
18             S[i] = 0;
19             U[i] = D[i] = i;
20             L[i] = i-1;
21             R[i] = i+1;
22         }
23         R[m] = 0; L[0] = m;
24         size = m;
25         for(int i = 1;i <= n;i++)H[i] = -1;
26     }
27     void Link(int r,int c){
28         ++S[Col[++size]=c];
29         Row[size] = r;
30         D[size] = D[c];
31         U[D[c]] = size;
32         U[size] = c;
33         D[c] = size;
34         if(H[r] < 0)H[r] = L[size] = R[size] = size;
35         else{
36             R[size] = R[H[r]];
37             L[R[H[r]]] = size;
38             L[size] = H[r];
39             R[H[r]] = size;
40         }
41     }
42     void remove(int c){
43         L[R[c]] = L[c]; R[L[c]] = R[c];
44         for(int i = D[c];i != c;i = D[i])
45             for(int j = R[i];j != i;j = R[j]){

```

```

46         U[D[j]] = U[j];
47         D[U[j]] = D[j];
48         —S[Col[j]];
49     }
50 }
51 void resume(int c){
52     for(int i = U[c];i != c;i = U[i])
53         for(int j = L[i];j != i;j = L[j])
54             ++S[Col[U[D[j]]=D[U[j]]=j]];
55     L[R[c]] = R[L[c]] = c;
56 }
57 bool Dance(int d){
58     if(R[0] == 0){
59         for(int i = 0;i < d;i++)g[(ans[i]-1)/9] = (ans[i]-1)%9
60             + '1';
61         for(int i = 0;i < N*N;i++)printf("%c",g[i]);
62         printf("\n");
63         return true;
64     }
65     int c = R[0];
66     for(int i = R[0];i != 0;i = R[i])
67         if(S[i] < S[c])
68             c = i;
69     remove(c);
70     for(int i = D[c];i != c;i = D[i]){
71         ans[d] = Row[i];
72         for(int j = R[i];j != i;j = R[j])remove(Col[j]);
73         if(Dance(d+1))return true;
74         for(int j = L[i];j != i;j = L[j])resume(Col[j]);
75     }
76     resume(c);
77     return false;
78 };
79 void place(int &r,int &c1,int &c2,int &c3,int &c4,int i,int j,int k
80 ) {
81     r = (i*N+j)*N + k; c1 = i*N+j+1; c2 = N*N+i*N+k;
82     c3 = N*N*2+j*N+k; c4 = N*N*3+((i/3)*3+(j/3))*N+k;
83 }
84 DLX dlx;
85 int main(){
86     while(scanf("%s",g) == 1){
87         if(strcmp(g,"end") == 0)break;
88         dlx.init(N*N*N,N*N*4);
89         int r,c1,c2,c3,c4;
90         for(int i = 0;i < N;i++)
91             for(int j = 0;j < N;j++)
92                 for(int k = 1;k <= N;k++)
93                     if(g[i*N+j] == '.' || g[i*N+j] == '0'+k){
94                         place(r,c1,c2,c3,c4,i,j,k);
95                         dlx.Link(r,c1);

```



```

95         dlx.Link(r,c2);
96         dlx.Link(r,c3);
97         dlx.Link(r,c4);
98     }
99     dlx.Dance(0);
100 }
101 return 0;
102 }

```

5.1.2 可重复覆盖

```

1  /*
2   * FZU1686
3   */
4  const int MaxM = 15*15+10;
5  const int MaxN = 15*15+10;
6  const int maxnode = MaxN * MaxM;
7  const int INF = 0x3f3f3f3f;
8  struct DLX{
9      int n,m,size;
10     int U[maxnode],D[maxnode],R[maxnode],L[maxnode],Row[maxnode],
        Col[maxnode];
11     int H[MaxN],S[MaxM];
12     int ansd;
13     void init(int _n,int _m){
14         n = _n;
15         m = _m;
16         for(int i = 0;i <= m;i++){
17             S[i] = 0;
18             U[i] = D[i] = i;
19             L[i] = i-1;
20             R[i] = i+1;
21         }
22         R[m] = 0; L[0] = m;
23         size = m;
24         for(int i = 1;i <= n;i++)H[i] = -1;
25     }
26     void Link(int r,int c){
27         ++S[Col[++size]=c];
28         Row[size] = r;
29         D[size] = D[c];
30         U[D[c]] = size;
31         U[size] = c;
32         D[c] = size;
33         if(H[r] < 0)H[r] = L[size] = R[size] = size;
34         else{
35             R[size] = R[H[r]];
36             L[R[H[r]]] = size;
37             L[size] = H[r];
38             R[H[r]] = size;
39         }
40     }

```

```

41 void remove(int c){
42     for(int i = D[c];i != c;i = D[i])
43         L[R[i]] = L[i], R[L[i]] = R[i];
44 }
45 void resume(int c){
46     for(int i = U[c];i != c;i = U[i])
47         L[R[i]] = R[L[i]] = i;
48 }
49 bool v[MaxM];
50 int f(){
51     int ret = 0;
52     for(int c = R[0]; c != 0;c = R[c])v[c] = true;
53     for(int c = R[0]; c != 0;c = R[c])
54         if(v[c])
55             {
56                 ret++;
57                 v[c] = false;
58                 for(int i = D[c];i != c;i = D[i])
59                     for(int j = R[i];j != i;j = R[j])
60                         v[Col[j]] = false;
61             }
62     return ret;
63 }
64 void Dance(int d){
65     if(d + f() >= ansd)return;
66     if(R[0] == 0){
67         if(d < ansd)ansd = d;
68         return;
69     }
70     int c = R[0];
71     for(int i = R[0];i != 0;i = R[i])
72         if(S[i] < S[c])
73             c = i;
74     for(int i = D[c];i != c;i = D[i]){
75         remove(i);
76         for(int j = R[i];j != i;j = R[j])remove(j);
77         Dance(d+1);
78         for(int j = L[i];j != i;j = L[j])resume(j);
79         resume(i);
80     }
81 }
82 };
83 DLX g;
84 int a[20][20];
85 int id[20][20];
86 int main(){
87     int n,m;
88     while(scanf("%d%d",&n,&m) == 2){
89         int sz = 0;
90         memset(id,0,sizeof(id));
91         for(int i = 0;i < n;i++)

```

```

92         for(int j = 0;j < m;j++){
93             scanf("%d",&a[i][j]);
94             if(a[i][j] == 1)id[i][j] = (++sz);
95         }
96         g.init(n*m,sz);
97         sz = 1;
98         int n1,m1;
99         scanf("%d%d",&n1,&m1);
100        for(int i = 0;i < n;i++){
101            for(int j = 0;j < m;j++){
102                for(int x = 0;x < n1 && i + x < n;x++)
103                    for(int y = 0;y < m1 && j + y < m;y++)
104                        if(id[i+x][j+y])
105                            g.Link(sz,id[i+x][j+y]);
106                sz++;
107            }
108            g.ansd = INF;
109            g.Dance(0);
110            printf("%d\n",g.ansd);
111        }
112        return 0;
113    }

```

5.2 八数码

5.2.1 HDU1043 反向搜索

```

1  /*
2  HDU 1043 Eight
3  八数码，输出路径
4  思路：反向搜索，从目标状态找回状态对应的路径
5  用康托展开判重
6  */
7  const int MAXN=1000000;//最多是 9!/2
8  int fac[]={1,1,2,6,24,120,720,5040,40320,362880}; //康拖展开判重
9  //      0! 1! 2! 3! 4! 5! 6! 7! 8! 9!
10 bool vis[MAXN]; //标记
11 string path[MAXN]; //记录路径
12 //康拖展开求该序列的 hash 值
13 int cantor(int s[]){
14     int sum=0;
15     for(int i=0;i<9;i++){
16         int num=0;
17         for(int j=i+1;j<9;j++)
18             if(s[j]<s[i])num++;
19         sum+=(num*fac[9-i-1]);
20     }
21     return sum+1;
22 }
23 struct Node{
24     int s[9];
25     int loc;//'0' 的位置

```

```

26     int status;//康拖展开的 hash 值
27     string path;//路径
28 };
29 int move[4][2]={{-1,0},{1,0},{0,-1},{0,1}};//u,d,l,r
30 char indexs[5]="durl";//和上面的要相反，因为是反向搜索
31 int aim=46234;//123456780 对应的康拖展开的 hash 值
32 void bfs(){
33     memset(vis,false,sizeof(vis));
34     Node cur,next;
35     for(int i=0;i<8;i++)cur.s[i]=i+1;
36     cur.s[8]=0;
37     cur.loc=8;
38     cur.status=aim;
39     cur.path="";
40     queue<Node>q;
41     q.push(cur);
42     path[aim]="";
43     while(!q.empty()){
44         cur=q.front();
45         q.pop();
46         int x=cur.loc/3;
47         int y=cur.loc%3;
48         for(int i=0;i<4;i++){
49             int tx=x+move[i][0];
50             int ty=y+move[i][1];
51             if(tx<0||tx>2||ty<0||ty>2)continue;
52             next=cur;
53             next.loc=tx*3+ty;
54             next.s[cur.loc]=next.s[next.loc];
55             next.s[next.loc]=0;
56             next.status=cantor(next.s);
57             if(!vis[next.status]){
58                 vis[next.status]=true;
59                 next.path=indexs[i]+next.path;
60                 q.push(next);
61                 path[next.status]=next.path;
62             }
63         }
64     }
65 }
66
67 int main(){
68     char ch;
69     Node cur;
70     bfs();
71     while(cin>>ch){
72         if(ch=='x') {cur.s[0]=0;cur.loc=0;}
73         else cur.s[0]=ch-'0';
74         for(int i=1;i<9;i++){
75             cin>>ch;
76             if(ch=='x'){

```

```
77         cur.s[i]=0;
78         cur.loc=i;
79     }
80     else cur.s[i]=ch-'0';
81 }
82 cur.status=cantor(cur.s);
83 if(vis[cur.status]){
84     cout<<path[cur.status]<<endl;
85 }
86 else cout<<"unsolvable"<<endl;
87 }
88 return 0;
89 }
```

6 动态规划

6.1 最长上升子序列 $O(n\log n)$

```

1  const int MAXN=500010;
2  int a[MAXN],b[MAXN];
3
4  //用二分查找的方法找到一个位置,使得 num>b[i-1] 并且 num<b[i],并用 num 代
   替 b[i]
5  int Search(int num,int low,int high){
6      int mid;
7      while(low<=high){
8          mid=(low+high)/2;
9          if(num>=b[mid]) low=mid+1;
10         else high=mid-1;
11     }
12     return low;
13 }
14 int DP(int n){
15     int i,len,pos;
16     b[1]=a[1];
17     len=1;
18     for(i=2;i<=n;i++){
19         if(a[i]>=b[len])//如果 a[i] 比 b[] 数组中最大还大直接插入到后面即可
20         {
21             len=len+1;
22             b[len]=a[i];
23         }
24         else//用二分的方法在 b[] 数组中找出第一个比 a[i] 大的位置并且让
           a[i] 替代这个位置
25         {
26             pos=Search(a[i],1,len);
27             b[pos]=a[i];
28         }
29     }
30     return len;
31 }

```

6.2 背包

```

1  int nValue,nKind;
2
3  //0-1 背包,代价为 cost,获得的价值为 weight
4  void ZeroOnePack(int cost,int weight){
5      for(int i=nValue;i>=cost;i--)
6          dp[i]=max(dp[i],dp[i-cost]+weight);
7  }
8
9  //完全背包,代价为 cost,获得的价值为 weight
10 void CompletePack(int cost,int weight){
11     for(int i=cost;i<=nValue;i++)

```

```

12     dp[i]=max(dp[i],dp[i-cost]+weight);
13 }
14
15 //多重背包
16 void MultiplePack(int cost,int weight,int amount){
17     if(cost*amount>=nValue) CompletePack(cost,weight);
18     else{
19         int k=1;
20         while(k<amount){
21             ZeroOnePack(k*cost,k*weight);
22             amount-=k;
23             k<<=1;
24         }
25         ZeroOnePack(amount*cost,amount*weight); //这个不要忘了, 经常
           掉了
26     }
27 }

```

分组背包:

for k = 1 to K

for v = V to 0

for item i in group k

$F[v] = \max(F[v], F[v-C_i] + W_i)$

6.3 插头 DP

6.3.1 HDU 4285

求 K 个回路的方案数。而且不能是环套环。

增加个标志位来记录形成的回路个数。而且注意避免环套环的情况。不形成环套环的话就是在形成新的回路时，两边的插头个数要为偶数。

```

1  /*
2  HDU 4285
3  要形成刚好 K 条回路的方法数
4  要避免环套环的情况。
5  所以形成回路时，要保证两边的插头数是偶数
6  G++ 11265ms 11820K
7  C++ 10656ms 11764K
8  */
9  const int MAXD=15;
10 const int STATE=1000010;
11 const int HASH=300007; //这个大一点可以防止 TLE，但是容易 MLE
12 const int MOD=1000000007;
13 int N,M,K;
14 int maze[MAXD][MAXD];
15 int code[MAXD];
16 int ch[MAXD];
17 int num; //圈的个数
18 struct HASHMAP
19 {
20     int head[HASH],next[STATE],size;
21     long long state[STATE];

```

```

22     int f[STATE];
23     void init()
24     {
25         size=0;
26         memset(head,-1,sizeof(head));
27     }
28     void push(long long st,int ans)
29     {
30         int i;
31         int h=st%HASH;
32         for(i=head[h];i!=-1;i=next[i])
33             if(state[i]==st)
34             {
35                 f[i]+=ans;
36                 f[i]%=MOD;
37                 return;
38             }
39         state[size]=st;
40         f[size]=ans;
41         next[size]=head[h];
42         head[h]=size++;
43     }
44 }hm[2];
45 void decode(int *code,int m,long long st)
46 {
47     num=st&63;
48     st>>=6;
49     for(int i=m;i>=0;i--)
50     {
51         code[i]=st&7;
52         st>>=3;
53     }
54 }
55 long long encode(int *code,int m)//最小表示法
56 {
57     int cnt=1;
58     memset(ch,-1,sizeof(ch));
59     ch[0]=0;
60     long long st=0;
61     for(int i=0;i<=m;i++)
62     {
63         if(ch[code[i]]==-1)ch[code[i]]=cnt++;
64         code[i]=ch[code[i]];
65         st<<=3;
66         st|=code[i];
67     }
68     st<<=6;
69     st|=num;
70     return st;
71 }
72 void shift(int *code,int m)

```



```

73 {
74     for(int i=m;i>0;i--)code[i]=code[i-1];
75     code[0]=0;
76 }
77 void dpblank(int i,int j,int cur)
78 {
79     int k,left,up;
80     for(k=0;k<hm[cur].size;k++)
81     {
82         decode(code,M,hm[cur].state[k]);
83         left=code[j-1];
84         up=code[j];
85         if(left&&up)
86         {
87             if(left==up)
88             {
89                 if(num>=K)continue;
90                 int t=0;
91                 //要避免环套环的情况，需要两边插头数为偶数
92                 for(int p=0;p<j-1;p++)
93                     if(code[p])t++;
94                 if(t&1)continue;
95                 if(num<K)
96                 {
97                     num++;
98                     code[j-1]=code[j]=0;
99                     hm[cur^1].push(encode(code,j==M?M-1:M),hm[cur].
100                                     f[k]);
101                 }
102             }
103             else
104             {
105                 code[j-1]=code[j]=0;
106                 for(int t=0;t<=M;t++)
107                     if(code[t]==up)
108                         code[t]=left;
109                 hm[cur^1].push(encode(code,j==M?M-1:M),hm[cur].f[k
110                                 ]);
111             }
112         }
113         else if(left||up)
114         {
115             int t;
116             if(left)t=left;
117             else t=up;
118             if(maze[i][j+1])
119             {
120                 code[j-1]=0;
121                 code[j]=t;
122                 hm[cur^1].push(encode(code,M),hm[cur].f[k]);
123             }
124         }
125     }
126 }

```

```

122         if(maze[i+1][j])
123         {
124             code[j]=0;
125             code[j-1]=t;
126             hm[cur^1].push(encode(code,j==M?M-1:M),hm[cur].f[k
                ]));
127         }
128     }
129     else
130     {
131         if(maze[i][j+1]&&maze[i+1][j])
132         {
133             code[j-1]=code[j]=13;
134             hm[cur^1].push(encode(code,j==M?M-1:M),hm[cur].f[k
                ]));
135         }
136     }
137 }
138 }
139 void dpblock(int i,int j,int cur)
140 {
141     int k;
142     for(k=0;k<hm[cur].size;k++)
143     {
144         decode(code,M,hm[cur].state[k]);
145         code[j-1]=code[j]=0;
146         hm[cur^1].push(encode(code,j==M?M-1:M),hm[cur].f[k]);
147     }
148 }
149 char str[20];
150 void init()
151 {
152     scanf("%d%d%d",&N,&M,&K);
153     memset(maze,0,sizeof(maze));
154     for(int i=1;i<=N;i++)
155     {
156         scanf("%s",&str);
157         for(int j=1;j<=M;j++)
158             if(str[j-1]=='.')
159                 maze[i][j]=1;
160     }
161 }
162 void solve()
163 {
164     int i,j,cur=0;
165     hm[cur].init();
166     hm[cur].push(0,1);
167     for(i=1;i<=N;i++)
168         for(j=1;j<=M;j++)
169         {
170             hm[cur^1].init();

```

```

171         if(maze[i][j])dpblank(i,j,cur);
172         else dpblock(i,j,cur);
173         cur^=1;
174     }
175     int ans=0;
176     for(i=0;i<hm[cur].size;i++)
177         if(hm[cur].state[i]==K)
178         {
179             ans+=hm[cur].f[i];
180             ans%=MOD;
181         }
182     printf("%d\n",ans);
183 }
184 }
185 int main()
186 {
187     int T;
188     scanf("%d",&T);
189     while(T——)
190     {
191         init();
192         solve();
193     }
194     return 0;
195 }
196
197 /*
198 Sample Input
199 4 4 1
200 **..
201 ....
202 ....
203 ....
204 4 1
205 ....
206 ....
207 ....
208 ....
209
210 Sample Output
211 6
212
213 */

```

7 计算几何

7.1 二维几何

```

1 // 计算几何模板
2 const double eps = 1e-8;
3 const double inf = 1e20;
4 const double pi = acos(-1.0);
5 const int maxp = 1010;
6 //Compares a double to zero
7 int sgn(double x){
8     if(fabs(x) < eps)return 0;
9     if(x < 0)return -1;
10    else return 1;
11 }
12 //square of a double
13 inline double sqr(double x){return x*x;}
14 /*
15  * Point
16  * Point()                - Empty constructor
17  * Point(double _x,double _y) - constructor
18  * input()                - double input
19  * output()               - %.2f output
20  * operator ==            - compares x and y
21  * operator <             - compares first by x, then by y
22  * operator -             - return new Point after subtracting
23                          - currepsonging x and y
24  * operator ^            - cross product of 2d points
25  * operator *            - dot product
26  * len()                 - gives length from origin
27  * len2()                - gives square of length from origin
28  * distance(Point p)     - gives distance from p
29  * operator + Point b   - returns new Point after adding
30                          - currepsonging x and y
31  * operator * double k  - returns new Point after multiplieing x and
32                          - y by k
33  * operator / double k  - returns new Point after divideing x and y
34                          - by k
35  * rad(Point a,Point b)- returns the angle of Point a and Point b
36                          - from this Point
37  * trunc(double r)       - return Point that if truncated the
38                          - distance from center to r
39  * rotleft()             - returns 90 degree ccw rotated point
40  * rotright()            - returns 90 degree cw rotated point
41  * rotate(Point p,double angle) - returns Point after rotateing the
42                          - Point centering at p by angle radian ccw
43 */
44 struct Point{
45     double x,y;
46     Point(){}
47     Point(double _x,double _y){

```

```

41         x = _x;
42         y = _y;
43     }
44     void input(){
45         scanf("%lf%lf",&x,&y);
46     }
47     void output(){
48         printf("%.2f□%.2f\n",x,y);
49     }
50     bool operator == (Point b)const{
51         return sgn(x-b.x) == 0 && sgn(y-b.y) == 0;
52     }
53     bool operator < (Point b)const{
54         return sgn(x-b.x)== 0?sgn(y-b.y)<0:x<b.x;
55     }
56     Point operator -(const Point &b)const{
57         return Point(x-b.x,y-b.y);
58     }
59     //叉积
60     double operator ^(const Point &b)const{
61         return x*b.y - y*b.x;
62     }
63     //点积
64     double operator *(const Point &b)const{
65         return x*b.x + y*b.y;
66     }
67     //返回长度
68     double len(){
69         return hypot(x,y); //库函数
70     }
71     //返回长度的平方
72     double len2(){
73         return x*x + y*y;
74     }
75     //返回两点的距离
76     double distance(Point p){
77         return hypot(x-p.x,y-p.y);
78     }
79     Point operator +(const Point &b)const{
80         return Point(x+b.x,y+b.y);
81     }
82     Point operator *(const double &k)const{
83         return Point(x*k,y*k);
84     }
85     Point operator /(const double &k)const{
86         return Point(x/k,y/k);
87     }
88     //计算 pa 和 pb 的夹角
89     //就是求这个点看 a,b 所成的夹角
90     //测试 LightOJ1203
91     double rad(Point a,Point b){

```

```

92     Point p = *this;
93     return fabs(atan2( fabs((a-p)^(b-p)),(a-p)*(b-p) ));
94 }
95 //化为长度为 r 的向量
96 Point trunc(double r){
97     double l = len();
98     if(!sgn(l))return *this;
99     r /= l;
100    return Point(x*r,y*r);
101 }
102 //逆时针旋转 90 度
103 Point rotleft(){
104     return Point(-y,x);
105 }
106 //顺时针旋转 90 度
107 Point rotright(){
108     return Point(y,-x);
109 }
110 //绕着 p 点逆时针旋转 angle
111 Point rotate(Point p,double angle){
112     Point v = (*this) - p;
113     double c = cos(angle), s = sin(angle);
114     return Point(p.x + v.x*c - v.y*s,p.y + v.x*s + v.y*c);
115 }
116 };
117 /*
118  * Stores two points
119  * Line()                    - Empty constructor
120  * Line(Point _s,Point _e)    - Line through _s and _e
121  * operator ==                - checks if two points are same
122  * Line(Point p,double angle) - one end p , another end at
123                                angle degree
124  * Line(double a,double b,double c) - Line of equation ax + by + c
125                                = 0
126  * input()                   - inputs s and e
127  * adjust()                   - orders in such a way that s < e
128  * length()                   - distance of se
129  * angle()                    - return 0 <= angle < pi
130  * relation(Point p)          - 3 if point is on line
131                                1 if point on the left of line
132                                2 if point on the right of line
133  * pointonseg(double p)       - return true if point on segment
134  * parallel(Line v)            - return true if they are
135                                parallel
136  * segcrossseg(Line v)        - returns 0 if does not intersect
137                                returns 1 if non-standard
138                                intersection
139  *                            - returns 2 if intersects
140  * linecrossseg(Line v)       - line and seg
141  * linecrossline(Line v)      - 0 if parallel
142  *                            - 1 if coincides

```

```

139 *                2 if intersects
140 * crosspoint(Line v)          - returns intersection point
141 * dispointtoline(Point p)     - distance from point p to the
    line
142 * dispointtoseg(Point p)      - distance from p to the segment
143 * dissegtoseg(Line v)         - distance of two segment
144 * lineprog(Point p)           - returns projected point p on se
    line
145 * symmetrypoint(Point p)      - returns reflection point of p
    over se
146 *
147 */
148 struct Line{
149     Point s,e;
150     Line(){}
151     Line(Point _s,Point _e){
152         s = _s;
153         e = _e;
154     }
155     bool operator ==(Line v){
156         return (s == v.s)&&(e == v.e);
157     }
158     //根据一个点和倾斜角 angle 确定直线,0<=angle<pi
159     Line(Point p,double angle){
160         s = p;
161         if(sgn(angle-pi/2) == 0){
162             e = (s + Point(0,1));
163         }
164         else{
165             e = (s + Point(1,tan(angle)));
166         }
167     }
168     //ax+by+c=0
169     Line(double a,double b,double c){
170         if(sgn(a) == 0){
171             s = Point(0,-c/b);
172             e = Point(1,-c/b);
173         }
174         else if(sgn(b) == 0){
175             s = Point(-c/a,0);
176             e = Point(-c/a,1);
177         }
178         else{
179             s = Point(0,-c/b);
180             e = Point(1,(-c-a)/b);
181         }
182     }
183     void input(){
184         s.input();
185         e.input();
186     }

```

```

187 void adjust(){
188     if(e < s)swap(s,e);
189 }
190 //求线段长度
191 double length(){
192     return s.distance(e);
193 }
194 //返回直线倾斜角 0<=angle<pi
195 double angle(){
196     double k = atan2(e.y-s.y,e.x-s.x);
197     if(sgn(k) < 0)k += pi;
198     if(sgn(k-pi) == 0)k -= pi;
199     return k;
200 }
201 //点和直线关系
202 //1 在左侧
203 //2 在右侧
204 //3 在直线上
205 int relation(Point p){
206     int c = sgn((p-s)^(e-s));
207     if(c < 0)return 1;
208     else if(c > 0)return 2;
209     else return 3;
210 }
211 // 点在线段上的判断
212 bool pointonseg(Point p){
213     return sgn((p-s)^(e-s)) == 0 && sgn((p-s)*(p-e)) <= 0;
214 }
215 //两向量平行 (对应直线平行或重合)
216 bool parallel(Line v){
217     return sgn((e-s)^(v.e-v.s)) == 0;
218 }
219 //两线段相交判断
220 //2 规范相交
221 //1 非规范相交
222 //0 不相交
223 int segcrossseg(Line v){
224     int d1 = sgn((e-s)^(v.s-s));
225     int d2 = sgn((e-s)^(v.e-s));
226     int d3 = sgn((v.e-v.s)^(s-v.s));
227     int d4 = sgn((v.e-v.s)^(e-v.s));
228     if( (d1^d2)==-2 && (d3^d4)==-2 )return 2;
229     return (d1==0 && sgn((v.s-s)*(v.s-e))<=0) ||
230         (d2==0 && sgn((v.e-s)*(v.e-e))<=0) ||
231         (d3==0 && sgn((s-v.s)*(s-v.e))<=0) ||
232         (d4==0 && sgn((e-v.s)*(e-v.e))<=0);
233 }
234 //直线和线段相交判断
235 //-*this line -v seg
236 //2 规范相交
237 //1 非规范相交

```



```

238 //0 不相交
239 int linecrossseg(Line v){
240     int d1 = sgn((e-s)^(v.s-s));
241     int d2 = sgn((e-s)^(v.e-s));
242     if((d1^d2)==-2) return 2;
243     return (d1==0 || d2==0);
244 }
245 //两直线关系
246 //0 平行
247 //1 重合
248 //2 相交
249 int linecrossline(Line v){
250     if((*this).parallel(v))
251         return v.relation(s)==3;
252     return 2;
253 }
254 //求两直线的交点
255 //要保证两直线不平行或重合
256 Point crosspoint(Line v){
257     double a1 = (v.e-v.s)^(s-v.s);
258     double a2 = (v.e-v.s)^(e-v.s);
259     return Point((s.x*a2-e.x*a1)/(a2-a1), (s.y*a2-e.y*a1)/(a2-a1));
260 }
261 //点到直线的距离
262 double dispointtoline(Point p){
263     return fabs((p-s)^(e-s))/length();
264 }
265 //点到线段的距离
266 double dispointtoseg(Point p){
267     if(sgn((p-s)*(e-s))<0 || sgn((p-e)*(s-e))<0)
268         return min(p.distance(s), p.distance(e));
269     return dispointtoline(p);
270 }
271 //返回线段到线段的距离
272 //前提是两线段不相交，相交距离就是 0 了
273 double dissegtoseg(Line v){
274     return min(min(dispointtoseg(v.s), dispointtoseg(v.e)), min(v
        .dispointtoseg(s), v.dispointtoseg(e)));
275 }
276 //返回点 p 在直线上的投影
277 Point lineprog(Point p){
278     return s + ( ((e-s)*((e-s)*(p-s)))/((e-s).len2()) );
279 }
280 //返回点 p 关于直线的对称点
281 Point symmetrypoint(Point p){
282     Point q = lineprog(p);
283     return Point(2*q.x-p.x, 2*q.y-p.y);
284 }
285 };
286 //圆

```

```

287 struct circle{
288     Point p;//圆心
289     double r;//半径
290     circle(){}
291     circle(Point _p,double _r){
292         p = _p;
293         r = _r;
294     }
295     circle(double x,double y,double _r){
296         p = Point(x,y);
297         r = _r;
298     }
299     //三角形的外接圆
300     //需要 Point 的 + / rotate() 以及 Line 的 crosspoint()
301     //利用两条边的中垂线得到圆心
302     //测试: UVA12304
303     circle(Point a,Point b,Point c){
304         Line u = Line((a+b)/2,((a+b)/2)+((b-a).rotleft()));
305         Line v = Line((b+c)/2,((b+c)/2)+((c-b).rotleft()));
306         p = u.crosspoint(v);
307         r = p.distance(a);
308     }
309     //三角形的内切圆
310     //参数 bool t 没有作用, 只是为了和上面外接圆函数区别
311     //测试: UVA12304
312     circle(Point a,Point b,Point c,bool t){
313         Line u,v;
314         double m = atan2(b.y-a.y,b.x-a.x), n = atan2(c.y-a.y,c.x-a.
315             x);
316         u.s = a;
317         u.e = u.s + Point(cos((n+m)/2),sin((n+m)/2));
318         v.s = b;
319         m = atan2(a.y-b.y,a.x-b.x) , n = atan2(c.y-b.y,c.x-b.x);
320         v.e = v.s + Point(cos((n+m)/2),sin((n+m)/2));
321         p = u.crosspoint(v);
322         r = Line(a,b).dispointtoseg(p);
323     }
324     //输入
325     void input(){
326         p.input();
327         scanf("%lf",&r);
328     }
329     //输出
330     void output(){
331         printf("%.2lf_%.2lf_%.2lf\n",p.x,p.y,r);
332     }
333     bool operator == (circle v){
334         return (p==v.p) && sgn(r-v.r)==0;
335     }
336     bool operator < (circle v)const{
337         return ((p<v.p) || ((p==v.p)&&sgn(r-v.r)<0));

```

```

337     }
338     //面积
339     double area(){
340         return pi*r*r;
341     }
342     //周长
343     double circumference(){
344         return 2*pi*r;
345     }
346     //点和圆的关系
347     //0 圆外
348     //1 圆上
349     //2 圆内
350     int relation(Point b){
351         double dst = b.distance(p);
352         if(sgn(dst-r) < 0)return 2;
353         else if(sgn(dst-r)==0)return 1;
354         return 0;
355     }
356     //线段和圆的关系
357     //比较的是圆心到线段的距离和半径的关系
358     int relationseg(Line v){
359         double dst = v.dispointtoseg(p);
360         if(sgn(dst-r) < 0)return 2;
361         else if(sgn(dst-r) == 0)return 1;
362         return 0;
363     }
364     //直线和圆的关系
365     //比较的是圆心到直线的距离和半径的关系
366     int relationline(Line v){
367         double dst = v.dispointtoline(p);
368         if(sgn(dst-r) < 0)return 2;
369         else if(sgn(dst-r) == 0)return 1;
370         return 0;
371     }
372     //两圆的关系
373     //5 相离
374     //4 外切
375     //3 相交
376     //2 内切
377     //1 内含
378     //需要 Point 的 distance
379     //测试: UVA12304
380     int relationcircle(circle v){
381         double d = p.distance(v.p);
382         if(sgn(d-r-v.r) > 0)return 5;
383         if(sgn(d-r-v.r) == 0)return 4;
384         double l = fabs(r-v.r);
385         if(sgn(d-r-v.r)<0 && sgn(d-l)>0)return 3;
386         if(sgn(d-l)==0)return 2;
387         if(sgn(d-l)<0)return 1;

```

```

388 }
389 //求两个圆的交点, 返回 0 表示没有交点, 返回 1 是一个交点, 2 是两个交点
390 //需要 relationcircle
391 //测试: UVA12304
392 int pointcrosscircle(circle v, Point &p1, Point &p2){
393     int rel = relationcircle(v);
394     if(rel == 1 || rel == 5) return 0;
395     double d = p.distance(v.p);
396     double l = (d*d+r*r-v.r*v.r)/(2*d);
397     double h = sqrt(r*r-l*l);
398     Point tmp = p + (v.p-p).trunc(l);
399     p1 = tmp + ((v.p-p).rotleft().trunc(h));
400     p2 = tmp + ((v.p-p).rotright().trunc(h));
401     if(rel == 2 || rel == 4)
402         return 1;
403     return 2;
404 }
405 //求直线和圆的交点, 返回交点个数
406 int pointcrossline(Line v, Point &p1, Point &p2){
407     if(!(*this).relationline(v)) return 0;
408     Point a = v.lineprog(p);
409     double d = v.dispointtoline(p);
410     d = sqrt(r*r-d*d);
411     if(sgn(d) == 0){
412         p1 = a;
413         p2 = a;
414         return 1;
415     }
416     p1 = a + (v.e-v.s).trunc(d);
417     p2 = a - (v.e-v.s).trunc(d);
418     return 2;
419 }
420 //得到过 a,b 两点, 半径为 r1 的两个圆
421 int gercircle(Point a, Point b, double r1, circle &c1, circle &c2){
422     circle x(a, r1), y(b, r1);
423     int t = x.pointcrosscircle(y, c1.p, c2.p);
424     if(!t) return 0;
425     c1.r = c2.r = r;
426     return t;
427 }
428 //得到与直线 u 相切, 过点 q, 半径为 r1 的圆
429 //测试: UVA12304
430 int getcircle(Line u, Point q, double r1, circle &c1, circle &c2){
431     double dis = u.dispointtoline(q);
432     if(sgn(dis-r1*2)>0) return 0;
433     if(sgn(dis) == 0){
434         c1.p = q + ((u.e-u.s).rotleft().trunc(r1));
435         c2.p = q + ((u.e-u.s).rotright().trunc(r1));
436         c1.r = c2.r = r1;
437         return 2;
438     }

```

```

439     Line u1 = Line((u.s + (u.e-u.s).rotleft().trunc(r1)),(u.e +
        (u.e-u.s).rotleft().trunc(r1)));
440     Line u2 = Line((u.s + (u.e-u.s).rotright().trunc(r1)),(u.e
        + (u.e-u.s).rotright().trunc(r1)));
441     circle cc = circle(q,r1);
442     Point p1,p2;
443     if(!cc.pointcrossline(u1,p1,p2))cc.pointcrossline(u2,p1,p2)
        ;
444     c1 = circle(p1,r1);
445     if(p1 == p2){
446         c2 = c1;
447         return 1;
448     }
449     c2 = circle(p2,r1);
450     return 2;
451 }
452 //同时与直线 u,v 相切, 半径为 r1 的圆
453 //测试: UVA12304
454 int getcircle(Line u,Line v,double r1,circle &c1,circle &c2,
    circle &c3,circle &c4){
455     if(u.parallel(v))return 0;//两直线平行
456     Line u1 = Line(u.s + (u.e-u.s).rotleft().trunc(r1),u.e + (u
        .e-u.s).rotleft().trunc(r1));
457     Line u2 = Line(u.s + (u.e-u.s).rotright().trunc(r1),u.e + (
        u.e-u.s).rotright().trunc(r1));
458     Line v1 = Line(v.s + (v.e-v.s).rotleft().trunc(r1),v.e + (v
        .e-v.s).rotleft().trunc(r1));
459     Line v2 = Line(v.s + (v.e-v.s).rotright().trunc(r1),v.e + (
        v.e-v.s).rotright().trunc(r1));
460     c1.r = c2.r = c3.r = c4.r = r1;
461     c1.p = u1.crosspoint(v1);
462     c2.p = u1.crosspoint(v2);
463     c3.p = u2.crosspoint(v1);
464     c4.p = u2.crosspoint(v2);
465     return 4;
466 }
467 //同时与不相交圆 cx,cy 相切, 半径为 r1 的圆
468 //测试: UVA12304
469 int getcircle(circle cx,circle cy,double r1,circle &c1,circle &
    c2){
470     circle x(cx.p,r1+cx.r),y(cy.p,r1+cy.r);
471     int t = x.pointcrosscircle(y,c1.p,c2.p);
472     if(!t)return 0;
473     c1.r = c2.r = r1;
474     return t;
475 }
476
477 //过一点作圆的切线 (先判断点和圆的关系)
478 //测试: UVA12304
479 int tangentline(Point q,Line &u,Line &v){
480     int x = relation(q);

```

```

481         if(x == 2)return 0;
482         if(x == 1){
483             u = Line(q,q + (q-p).rotleft());
484             v = u;
485             return 1;
486         }
487         double d = p.distance(q);
488         double l = r*r/d;
489         double h = sqrt(r*r-l*l);
490         u = Line(q,p + ((q-p).trunc(l) + (q-p).rotleft().trunc(h)))
491             ;
492         v = Line(q,p + ((q-p).trunc(l) + (q-p).rotright().trunc(h))
493             );
494         return 2;
495     }
496 //求两圆相交的面积
497 double areacircle(circle v){
498     int rel = relationcircle(v);
499     if(rel >= 4)return 0.0;
500     if(rel <= 2)return min(area(),v.area());
501     double d = p.distance(v.p);
502     double hf = (r+v.r+d)/2.0;
503     double ss = 2*sqrt(hf*(hf-r)*(hf-v.r)*(hf-d));
504     double a1 = acos((r*r+d*d-v.r*v.r)/(2.0*r*d));
505     a1 = a1*r*r;
506     double a2 = acos((v.r*v.r+d*d-r*r)/(2.0*v.r*d));
507     a2 = a2*v.r*v.r;
508     return a1+a2-ss;
509 }
510 //求圆和三角形 pab 的相交面积
511 //测试: POJ3675 HDU3982 HDU2892
512 double areatriangle(Point a,Point b){
513     if(sgn((p-a)^(p-b)) == 0)return 0.0;
514     Point q[5];
515     int len = 0;
516     q[len++] = a;
517     Line l(a,b);
518     Point p1,p2;
519     if(pointcrossline(l,q[1],q[2])==2){
520         if(sgn((a-q[1])*(b-q[1]))<0)q[len++] = q[1];
521         if(sgn((a-q[2])*(b-q[2]))<0)q[len++] = q[2];
522     }
523     q[len++] = b;
524     if(len == 4 && sgn((q[0]-q[1])*(q[2]-q[1]))>0)swap(q[1],q
525         [2]);
526     double res = 0;
527     for(int i = 0;i < len-1;i++){
528         if(relation(q[i])==0||relation(q[i+1])==0){
529             double arg = p.rad(q[i],q[i+1]);
530             res += r*r*arg/2.0;
531         }
532     }
533 }

```

```

529         else{
530             res += fabs((q[i]-p)^(q[i+1]-p))/2.0;
531         }
532     }
533     return res;
534 }
535 };
536
537 /*
538  * n,p   Line l for each side
539  * input(int _n)                - inputs _n size polygon
540  * add(Point q)                 - adds a point at end of
    the list
541  * getline()                    - populates line array
542  * cmp                          - comparision in
    convex_hull order
543  * norm()                       - sorting in convex_hull
    order
544  * getconvex(polygon &convex)   - returns convex hull in
    convex
545  * Graham(polygon &convex)      - returns convex hull in
    convex
546  * isconvex()                   - checks if convex
547  * relationpoint(Point q)       - returns 3 if q is a
    vertex
548  *                               2 if on a side
549  *                               1 if inside
550  *                               0 if outside
551  * convexcute(Line u,polygon &po) - left side of u in po
552  * gercircumference()           - returns side length
553  * getarea()                    - returns area
554  * getdir()                     - returns 0 for cw, 1 for
    ccw
555  * getbarycentre()              - returns barycenter
556  *
557 */
558 struct polygon{
559     int n;
560     Point p[maxp];
561     Line l[maxp];
562     void input(int _n){
563         n = _n;
564         for(int i = 0;i < n;i++){
565             p[i].input();
566         }
567     void add(Point q){
568         p[n++] = q;
569     }
570     void getline(){
571         for(int i = 0;i < n;i++){
572             l[i] = Line(p[i],p[(i+1)%n]);

```

```

573     }
574 }
575 struct cmp{
576     Point p;
577     cmp(const Point &p0){p = p0;}
578     bool operator()(const Point &aa,const Point &bb){
579         Point a = aa, b = bb;
580         int d = sgn((a-p)^(b-p));
581         if(d == 0){
582             return sgn(a.distance(p)-b.distance(p)) < 0;
583         }
584         return d > 0;
585     }
586 };
587 //进行极角排序
588 //首先需要找到最左下角的点
589 //需要重载号好 Point 的 < 操作符 (min 函数要用)
590 void norm(){
591     Point mi = p[0];
592     for(int i = 1;i < n;i++)mi = min(mi,p[i]);
593     sort(p,p+n,cmp(mi));
594 }
595 //得到凸包
596 //得到的凸包里面的点编号是 0~n-1 的
597 //两种凸包的方法
598 //注意如果有影响, 要特判下所有点共点, 或者共线的特殊情况
599 //测试 LightOJ1203 LightOJ1239
600 void getconvex(polygon &convex){
601     sort(p,p+n);
602     convex.n = n;
603     for(int i = 0;i < min(n,2);i++){
604         convex.p[i] = p[i];
605     }
606     if(convex.n == 2 && (convex.p[0] == convex.p[1]))convex.n
        —;//特
        判
607     if(n <= 2)return;
608     int &top = convex.n;
609     top = 1;
610     for(int i = 2;i < n;i++){
611         while(top && sgn((convex.p[top]-p[i])^(convex.p[top-1]-
            p[i])) <= 0)
612             top—;
613         convex.p[++top] = p[i];
614     }
615     int temp = top;
616     convex.p[++top] = p[n-2];
617     for(int i = n-3;i >= 0;i—){
618         while(top != temp && sgn((convex.p[top]-p[i])^(convex.p
            [top-1]-p[i])) <= 0)
619             top—;

```



```

620         convex.p[++top] = p[i];
621     }
622     if(convex.n == 2 && (convex.p[0] == convex.p[1]))convex.n
        —; //特
        判
623     convex.norm(); //原来得到的是顺时针的点，排序后逆时针
624 }
625 //得到凸包的另外一种方法
626 //测试 LightOJ1203 LightOJ1239
627 void Graham(polygon &convex){
628     norm();
629     int &top = convex.n;
630     top = 0;
631     if(n == 1){
632         top = 1;
633         convex.p[0] = p[0];
634         return;
635     }
636     if(n == 2){
637         top = 2;
638         convex.p[0] = p[0];
639         convex.p[1] = p[1];
640         if(convex.p[0] == convex.p[1])top—;
641         return;
642     }
643     convex.p[0] = p[0];
644     convex.p[1] = p[1];
645     top = 2;
646     for(int i = 2; i < n; i++){
647         while( top > 1 && sgn((convex.p[top-1]-convex.p[top-2])
            ^ (p[i]-convex.p[top-2])) <= 0 )
648             top—;
649         convex.p[top++] = p[i];
650     }
651     if(convex.n == 2 && (convex.p[0] == convex.p[1]))convex.n
        —; //特
        判
652 }
653 //判断是不是凸的
654 bool isconvex(){
655     bool s[2];
656     memset(s, false, sizeof(s));
657     for(int i = 0; i < n; i++){
658         int j = (i+1)%n;
659         int k = (j+1)%n;
660         s[sgn((p[j]-p[i])^(p[k]-p[i]))+1] = true;
661         if(s[0] && s[2])return false;
662     }
663     return true;
664 }
665 //判断点和任意多边形的关系

```

```

666 // 3 点上
667 // 2 边上
668 // 1 内部
669 // 0 外部
670 int relationpoint(Point q){
671     for(int i = 0;i < n;i++){
672         if(p[i] == q)return 3;
673     }
674     getline();
675     for(int i = 0;i < n;i++){
676         if(l[i].pointonseg(q))return 2;
677     }
678     int cnt = 0;
679     for(int i = 0;i < n;i++){
680         int j = (i+1)%n;
681         int k = sgn((q-p[j])^(p[i]-p[j]));
682         int u = sgn(p[i].y-q.y);
683         int v = sgn(p[j].y-q.y);
684         if(k > 0 && u < 0 && v >= 0)cnt++;
685         if(k < 0 && v < 0 && u >= 0)cnt--;
686     }
687     return cnt != 0;
688 }
689 //直线 u 切割凸多边形左侧
690 //注意直线方向
691 //测试: HDU3982
692 void convexcut(Line u,polygon &po){
693     int &top = po.n;//注意引用
694     top = 0;
695     for(int i = 0;i < n;i++){
696         int d1 = sgn((u.e-u.s)^(p[i]-u.s));
697         int d2 = sgn((u.e-u.s)^(p[(i+1)%n]-u.s));
698         if(d1 >= 0)po.p[top++] = p[i];
699         if(d1*d2 < 0)po.p[top++] = u.crosspoint(Line(p[i],p[(i+1)%n]));
700     }
701 }
702 //得到周长
703 //测试 LightOJ1239
704 double getcircumference(){
705     double sum = 0;
706     for(int i = 0;i < n;i++){
707         sum += p[i].distance(p[(i+1)%n]);
708     }
709     return sum;
710 }
711 //得到面积
712 double getarea(){
713     double sum = 0;
714     for(int i = 0;i < n;i++){
715         sum += (p[i]^p[(i+1)%n]);

```

```

716     }
717     return fabs(sum)/2;
718 }
719 //得到方向
720 // 1 表示逆时针, 0 表示顺时针
721 bool getdir(){
722     double sum = 0;
723     for(int i = 0; i < n; i++){
724         sum += (p[i]^p[(i+1)%n]);
725     }
726     if(sgn(sum) > 0) return 1;
727     return 0;
728 }
729 //得到重心
730 Point getbarycentre(){
731     Point ret(0,0);
732     double area = 0;
733     for(int i = 1; i < n-1; i++){
734         double tmp = (p[i]-p[0])^(p[i+1]-p[0]);
735         if(sgn(tmp) == 0) continue;
736         area += tmp;
737         ret.x += (p[0].x+p[i].x+p[i+1].x)/3*tmp;
738         ret.y += (p[0].y+p[i].y+p[i+1].y)/3*tmp;
739     }
740     if(sgn(area)) ret = ret/area;
741     return ret;
742 }
743 //多边形和圆交的面积
744 //测试: POJ3675 HDU3982 HDU2892
745 double areacircle(circle c){
746     double ans = 0;
747     for(int i = 0; i < n; i++){
748         int j = (i+1)%n;
749         if(sgn( (p[j]-c.p)^(p[i]-c.p) ) >= 0)
750             ans += c.reatriangle(p[i],p[j]);
751         else ans -= c.reatriangle(p[i],p[j]);
752     }
753     return fabs(ans);
754 }
755 //多边形和圆关系
756 // 2 圆完全在多边形内
757 // 1 圆在多边形里面, 碰到了多边形边界
758 // 0 其它
759 int relationcircle(circle c){
760     getline();
761     int x = 2;
762     if(relationpoint(c.p) != 1) return 0; //圆心不在内部
763     for(int i = 0; i < n; i++){
764         if(c.relationseg(l[i]) == 2) return 0;
765         if(c.relationseg(l[i]) == 1) x = 1;
766     }
767     return x;

```

```

767     }
768 };
769 //AB X AC
770 double cross(Point A,Point B,Point C){
771     return (B-A)^(C-A);
772 }
773 //AB*AC
774 double dot(Point A,Point B,Point C){
775     return (B-A)*(C-A);
776 }
777 //最小矩形面积覆盖
778 // A 必须是凸包 (而且是逆时针顺序)
779 // 测试 UVA 10173
780 double minRectangleCover(polygon A){
781     //要特判 A.n < 3 的情况
782     if(A.n < 3)return 0.0;
783     A.p[A.n] = A.p[0];
784     double ans = -1;
785     int r = 1, p = 1, q;
786     for(int i = 0;i < A.n;i++){
787         //卡出离边 A.p[i] - A.p[i+1] 最远的点
788         while( sgn( cross(A.p[i],A.p[i+1],A.p[r+1]) - cross(A.p[i],
789             A.p[i+1],A.p[r]) ) >= 0 )
790             r = (r+1)%A.n;
791         //卡出 A.p[i] - A.p[i+1] 方向上正向 n 最远的点
792         while(sgn( dot(A.p[i],A.p[i+1],A.p[p+1]) - dot(A.p[i],A.p[i
793             +1],A.p[p]) ) >= 0 )
794             p = (p+1)%A.n;
795         if(i == 0)q = p;
796         //卡出 A.p[i] - A.p[i+1] 方向上负向最远的点
797         while(sgn(dot(A.p[i],A.p[i+1],A.p[q+1]) - dot(A.p[i],A.p[i
798             +1],A.p[q])) <= 0)
799             q = (q+1)%A.n;
800         double d = (A.p[i] - A.p[i+1]).len2();
801         double tmp = cross(A.p[i],A.p[i+1],A.p[r]) *
802             (dot(A.p[i],A.p[i+1],A.p[p]) - dot(A.p[i],A.p[i+1],A.p[
803             q]))/d;
804         if(ans < 0 || ans > tmp)ans = tmp;
805     }
806     return ans;
807 }
808 //直线切凸多边形
809 //多边形是逆时针的, 在 q1q2 的左侧
810 //测试:HDU3982
811 vector<Point> convexCut(const vector<Point> &ps,Point q1,Point q2){
812     vector<Point>q;
813     int n = ps.size();
814     for(int i = 0;i < n;i++){
815         Point p1 = ps[i], p2 = ps[(i+1)%n];
816         int d1 = sgn((q2-q1)^(p1-q1)), d2 = sgn((q2-q1)^(p2-q1));

```

```

814         if(d1 >= 0)
815             qs.push_back(p1);
816         if(d1 * d2 < 0)
817             qs.push_back(Line(p1,p2).crosspoint(Line(q1,q2)));
818     }
819     return qs;
820 }
821 //半平面交
822 //测试 POJ3335 POJ1474 POJ1279
823 //*****
824 struct halfplane:public Line{
825     double angle;
826     halfplane(){}
827     //表示向量 s->e 逆时针 (左侧) 的半平面
828     halfplane(Point _s,Point _e){
829         s = _s;
830         e = _e;
831     }
832     halfplane(Line v){
833         s = v.s;
834         e = v.e;
835     }
836     void calcangle(){
837         angle = atan2(e.y-s.y,e.x-s.x);
838     }
839     bool operator <(const halfplane &b)const{
840         return angle < b.angle;
841     }
842 };
843 struct halfplanes{
844     int n;
845     halfplane hp[maxp];
846     Point p[maxp];
847     int que[maxp];
848     int st,ed;
849     void push(halfplane tmp){
850         hp[n++] = tmp;
851     }
852     //去重
853     void unique(){
854         int m = 1;
855         for(int i = 1;i < n;i++){
856             if(sgn(hp[i].angle-hp[i-1].angle) != 0)
857                 hp[m++] = hp[i];
858             else if(sgn( (hp[m-1].e-hp[m-1].s)^(hp[i].s-hp[m-1].s)
859                 ) > 0)
860                 hp[m-1] = hp[i];
861         }
862         n = m;
863     }
864     bool halfplaneinsert(){

```

```

864     for(int i = 0;i < n;i++)hp[i].calcangle();
865     sort(hp,hp+n);
866     unique();
867     que[st=0] = 0;
868     que[ed=1] = 1;
869     p[1] = hp[0].crosspoint(hp[1]);
870     for(int i = 2;i < n;i++){
871         while(st<ed && sgn((hp[i].e-hp[i].s)^(p[ed]-hp[i].s))
            <0)ed--;
872         while(st<ed && sgn((hp[i].e-hp[i].s)^(p[st+1]-hp[i].s))
            <0)st++;
873         que[++ed] = i;
874         if(hp[i].parallel(hp[que[ed-1]]))return false;
875         p[ed]=hp[i].crosspoint(hp[que[ed-1]]);
876     }
877     while(st<ed && sgn((hp[que[st]].e-hp[que[st]].s)^(p[ed]-hp[
        que[st]].s))<0)ed--;
878     while(st<ed && sgn((hp[que[ed]].e-hp[que[ed]].s)^(p[st+1]-
        hp[que[ed]].s))<0)st++;
879     if(st+1>=ed)return false;
880     return true;
881 }
882 //得到最后半平面交得到的凸多边形
883 //需要先调用 halfplaneinsert() 且返回 true
884 void getconvex(polygon &con){
885     p[st] = hp[que[st]].crosspoint(hp[que[ed]]);
886     con.n = ed-st+1;
887     for(int j = st,i = 0;j <= ed;i++,j++)
888         con.p[i] = p[j];
889 }
890 };
891 //*****
892
893 const int maxn = 1010;
894 struct circles{
895     circle c[maxn];
896     double ans[maxn]; //ans[i] 表示被覆盖了 i 次的面积
897     double pre[maxn];
898     int n;
899     circles(){}
900     void add(circle cc){
901         c[n++] = cc;
902     }
903     //x 包含在 y 中
904     bool inner(circle x,circle y){
905         if(x.relationcircle(y) != 1)return 0;
906         return sgn(x.r-y.r)<=0?1:0;
907     }
908     //圆的面积并去掉内含的圆
909     void init_or(){
910         bool mark[maxn] = {0};

```

```

911     int i,j,k=0;
912     for(i = 0;i < n;i++){
913         for(j = 0;j < n;j++){
914             if(i != j && !mark[j]){
915                 if( (c[i]==c[j])||inner(c[i],c[j]) )break;
916             }
917             if(j < n)mark[i] = 1;
918         }
919         for(i = 0;i < n;i++){
920             if(!mark[i])
921                 c[k++] = c[i];
922         }
923         n = k;
924     }
925     //圆的面积交去掉内含的圆
926     void init_add(){
927         int i,j,k;
928         bool mark[maxn] = {0};
929         for(i = 0;i < n;i++){
930             for(j = 0;j < n;j++){
931                 if(i != j && !mark[j]){
932                     if( (c[i]==c[j])||inner(c[j],c[i]) )break;
933                 }
934                 if(j < n)mark[i] = 1;
935             }
936             for(i = 0;i < n;i++){
937                 if(!mark[i])
938                     c[k++] = c[i];
939             }
940             n = k;
941         }
942         //半径为 r 的圆，弧度为 th 对应的弓形的面积
943         double areaarc(double th,double r){
944             return 0.5*r*r*(th-sin(th));
945         }
946         //测试 SPOJVCIRCLES SPOJCIRUT
947         //SPOJVCIRCLES 求 n 个圆并的面积，需要加上 init_or() 去掉重复圆（否则
948         //WA）
949         //SPOJCIRUT 是求被覆盖 k 次的面积，不能加 init_or()
950         //对于求覆盖多少次面积的问题，不能解决相同圆，而且不能 init_or()
951         //求多圆面积并，需要 init_or，其中一个目的就是去掉相同圆
952         void getarea(){
953             memset(ans,0,sizeof(ans));
954             vector<pair<double,int> >v;
955             for(int i = 0;i < n;i++){
956                 v.clear();
957                 v.push_back(make_pair(-pi,1));
958                 v.push_back(make_pair(pi,-1));
959                 for(int j = 0;j < n;j++){
960                     if(i != j){
961                         Point q = (c[j].p - c[i].p);
962                         double ab = q.len(),ac = c[i].r, bc = c[j].r;
963                         if(sgn(ab+ac-bc)<=0){

```

```

961         v.push_back(make_pair(-pi,1));
962         v.push_back(make_pair(pi,-1));
963         continue;
964     }
965     if(sgn(ab+bc-ac)<=0)continue;
966     if(sgn(ab-ac-bc)>0)continue;
967     double th = atan2(q.y,q.x), fai = acos((ac*ac+
        ab*ab-bc*bc)/(2.0*ac*ab));
968     double a0 = th-fai;
969     if(sgn(a0+pi)<0)a0+=2*pi;
970     double a1 = th+fai;
971     if(sgn(a1-pi)>0)a1-=2*pi;
972     if(sgn(a0-a1)>0){
973         v.push_back(make_pair(a0,1));
974         v.push_back(make_pair(pi,-1));
975         v.push_back(make_pair(-pi,1));
976         v.push_back(make_pair(a1,-1));
977     }
978     else{
979         v.push_back(make_pair(a0,1));
980         v.push_back(make_pair(a1,-1));
981     }
982 }
983 sort(v.begin(),v.end());
984 int cur = 0;
985 for(int j = 0;j < v.size();j++){
986     if(cur && sgn(v[j].first-pre[cur])){
987         ans[cur] += areaarc(v[j].first-pre[cur],c[i].r)
988         ;
989         ans[cur] += 0.5*(Point(c[i].p.x+c[i].r*cos(pre[
            cur]),c[i].p.y+c[i].r*sin(pre[cur]))^Point(c
            [i].p.x+c[i].r*cos(v[j].first),c[i].p.y+c[i
            ].r*sin(v[j].first)));
990     }
991     cur += v[j].second;
992     pre[cur] = v[j].first;
993 }
994 for(int i = 1;i < n;i++)
995     ans[i] -= ans[i+1];
996 }
997 };

```

7.2 三维几何

```

1  const double eps = 1e-8;
2  int sgn(double x){
3      if(fabs(x) < eps)return 0;
4      if(x < 0)return -1;
5      else return 1;
6  }
7  struct Point3{

```



```

8     double x,y,z;
9     Point3(double _x = 0,double _y = 0,double _z = 0){
10         x = _x;
11         y = _y;
12         z = _z;
13     }
14     void input(){
15         scanf("%lf%lf%lf",&x,&y,&z);
16     }
17     void output(){
18         scanf("%.2lf□%.2lf□%.2lf\n",x,y,z);
19     }
20     bool operator ==(const Point3 &b)const{
21         return sgn(x-b.x) == 0 && sgn(y-b.y) == 0 && sgn(z-b.z) ==
            0;
22     }
23     bool operator <(const Point3 &b)const{
24         return sgn(x-b.x)==0?(sgn(y-b.y)==0?sgn(z-b.z)<0:y<b.y):x<b
            .x;
25     }
26     double len(){
27         return sqrt(x*x+y*y+z*z);
28     }
29     double len2(){
30         return x*x+y*y+z*z;
31     }
32     double distance(const Point3 &b)const{
33         return sqrt((x-b.x)*(x-b.x)+(y-b.y)*(y-b.y)+(z-b.z)*(z-b.z)
            );
34     }
35     Point3 operator -(const Point3 &b)const{
36         return Point3(x-b.x,y-b.y,z-b.z);
37     }
38     Point3 operator +(const Point3 &b)const{
39         return Point3(x+b.x,y+b.y,z+b.z);
40     }
41     Point3 operator *(const double &k)const{
42         return Point3(x*k,y*k,z*k);
43     }
44     Point3 operator /(const double &k)const{
45         return Point3(x/k,y/k,z/k);
46     }
47     //点乘
48     double operator *(const Point3 &b)const{
49         return x*b.x+y*b.y+z*b.z;
50     }
51     //叉乘
52     Point3 operator ^(const Point3 &b)const{
53         return Point3(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);
54     }
55     double rad(Point3 a,Point3 b){

```

```

56     Point3 p = (*this);
57     return acos( ( (a-p)*(b-p) ) / (a.distance(p)*b.distance(p))
58         );
59 }
60 //变换长度
61 Point3 trunc(double r){
62     double l = len();
63     if(!sgn(l))return *this;
64     r /= l;
65     return Point3(x*r,y*r,z*r);
66 };
67 struct Line3
68 {
69     Point3 s,e;
70     Line3(){}
71     Line3(Point3 _s,Point3 _e)
72     {
73         s = _s;
74         e = _e;
75     }
76     bool operator ==(const Line3 v)
77     {
78         return (s==v.s)&&(e==v.e);
79     }
80     void input()
81     {
82         s.input();
83         e.input();
84     }
85     double length()
86     {
87         return s.distance(e);
88     }
89     //点到直线距离
90     double dispointtoline(Point3 p)
91     {
92         return ((e-s)^(p-s)).len()/s.distance(e);
93     }
94     //点到线段距离
95     double dispointtoseg(Point3 p)
96     {
97         if(sgn((p-s)*(e-s)) < 0 || sgn((p-e)*(s-e)) < 0)
98             return min(p.distance(s),e.distance(p));
99         return dispointtoline(p);
100     }
101     //返回点 p 在直线上的投影
102     Point3 lineprog(Point3 p)
103     {
104         return s + ( ((e-s)*((e-s)*(p-s)))/((e-s).len2()) );
105     }

```

```

106 //p 绕此向量逆时针 arg 角度
107 Point3 rotate(Point3 p,double ang)
108 {
109     if(sgn(((s-p)^(e-p)).len()) == 0)return p;
110     Point3 f1 = (e-s)^(p-s);
111     Point3 f2 = (e-s)^(f1);
112     double len = ((s-p)^(e-p)).len()/s.distance(e);
113     f1 = f1.trunc(len); f2 = f2.trunc(len);
114     Point3 h = p+f2;
115     Point3 pp = h+f1;
116     return h + ((p-h)*cos(ang)) + ((pp-h)*sin(ang));
117 }
118 //点在直线上
119 bool pointonseg(Point3 p)
120 {
121     return sgn( ((s-p)^(e-p)).len() ) == 0 && sgn((s-p)*(e-p))
        == 0;
122 }
123 };
124 struct Plane
125 {
126     Point3 a,b,c,o;//平面上的三个点, 以及法向量
127     Plane(){}
128     Plane(Point3 _a,Point3 _b,Point3 _c)
129     {
130         a = _a;
131         b = _b;
132         c = _c;
133         o = pvec();
134     }
135     Point3 pvec()
136     {
137         return (b-a)^(c-a);
138     }
139     //ax+by+cz+d = 0
140     Plane(double _a,double _b,double _c,double _d)
141     {
142         o = Point3(_a,_b,_c);
143         if(sgn(_a) != 0)
144             a = Point3((-_d-_c-_b)/_a,1,1);
145         else if(sgn(_b) != 0)
146             a = Point3(1,(-_d-_c-_a)/_b,1);
147         else if(sgn(_c) != 0)
148             a = Point3(1,1,(-_d-_a-_b)/_c);
149     }
150     //点在平面上的判断
151     bool pointonplane(Point3 p)
152     {
153         return sgn((p-a)*o) == 0;
154     }
155     //两平面夹角

```

```

156     double angleplane(Plane f)
157     {
158         return acos(o*f.o)/(o.len()*f.o.len());
159     }
160     //平面和直线的交点，返回值是交点个数
161     int crossline(Line3 u,Point3 &p)
162     {
163         double x = o*(u.e-a);
164         double y = o*(u.s-a);
165         double d = x-y;
166         if(sgn(d) == 0)return 0;
167         p = ((u.s*x)-(u.e*y))/d;
168         return 1;
169     }
170     //点到平面最近点（也就是投影）
171     Point3 pointtoplane(Point3 p)
172     {
173         Line3 u = Line3(p,p+o);
174         crossline(u,p);
175         return p;
176     }
177     //平面和平面的交线
178     int crossplane(Plane f,Line3 &u)
179     {
180         Point3 oo = o^f.o;
181         Point3 v = o^oo;
182         double d = fabs(f.o*v);
183         if(sgn(d) == 0)return 0;
184         Point3 q = a + (v*(f.o*(f.a-a))/d);
185         u = Line3(q,q+oo);
186         return 1;
187     }
188 };

```

7.3 平面最近点对

HDU1007/ZOJ2107

```

1  const int MAXN = 100010;
2  const double eps = 1e-8;
3  const double INF = 1e20;
4  struct Point{
5      double x,y;
6      void input(){
7          scanf("%lf%lf",&x,&y);
8      }
9  };
10 double dist(Point a,Point b){
11     return sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
12 }
13 Point p[MAXN];
14 Point tmp[MAXN];
15 bool cmpx(Point a,Point b){

```

```

16     return a.x < b.x || (a.x == b.x && a.y < b.y);
17 }
18 bool cmpy(Point a, Point b){
19     return a.y < b.y || (a.y == b.y && a.x < b.x);
20 }
21 double Closest_Pair(int left, int right){
22     double d = INF;
23     if(left == right) return d;
24     if(left+1 == right) return dist(p[left], p[right]);
25     int mid = (left+right)/2;
26     double d1 = Closest_Pair(left, mid);
27     double d2 = Closest_Pair(mid+1, right);
28     d = min(d1, d2);
29     int cnt = 0;
30     for(int i = left; i <= right; i++){
31         if(fabs(p[mid].x - p[i].x) <= d)
32             tmpt[cnt++] = p[i];
33     }
34     sort(tmpt, tmpt+cnt, cmpy);
35     for(int i = 0; i < cnt; i++){
36         for(int j = i+1; j < cnt && tmpt[j].y - tmpt[i].y < d; j++){
37             d = min(d, dist(tmpt[i], tmpt[j]));
38         }
39     }
40     return d;
41 }
42 int main(){
43     int n;
44     while(scanf("%d", &n) == 1 && n){
45         for(int i = 0; i < n; i++) p[i].input();
46         sort(p, p+n, cmpx);
47         printf("%.2lf\n", Closest_Pair(0, n-1));
48     }
49     return 0;
50 }

```

7.4 三维凸包

7.4.1 HDU4273

HDU 4273 给一个三维凸包，求重心到表面的最短距离。

```

1  const double eps = 1e-8;
2  const int MAXN = 550;
3  int sgn(double x){
4      if(fabs(x) < eps) return 0;
5      if(x < 0) return -1;
6      else return 1;
7  }
8  struct Point3{
9      double x, y, z;
10     Point3(double _x = 0, double _y = 0, double _z = 0){
11         x = _x;

```

```

12     y = _y;
13     z = _z;
14 }
15 void input(){
16     scanf("%lf%lf%lf",&x,&y,&z);
17 }
18 bool operator ==(const Point3 &b)const{
19     return sgn(x-b.x) == 0 && sgn(y-b.y) == 0 && sgn(z-b.z) ==
        0;
20 }
21 double len(){
22     return sqrt(x*x+y*y+z*z);
23 }
24 double len2(){
25     return x*x+y*y+z*z;
26 }
27 double distance(const Point3 &b)const{
28     return sqrt((x-b.x)*(x-b.x)+(y-b.y)*(y-b.y)+(z-b.z)*(z-b.z)
        );
29 }
30 Point3 operator -(const Point3 &b)const{
31     return Point3(x-b.x,y-b.y,z-b.z);
32 }
33 Point3 operator +(const Point3 &b)const{
34     return Point3(x+b.x,y+b.y,z+b.z);
35 }
36 Point3 operator *(const double &k)const{
37     return Point3(x*k,y*k,z*k);
38 }
39 Point3 operator /(const double &k)const{
40     return Point3(x/k,y/k,z/k);
41 }
42 //点乘
43 double operator *(const Point3 &b)const{
44     return x*b.x + y*b.y + z*b.z;
45 }
46 //叉乘
47 Point3 operator ^(const Point3 &b)const{
48     return Point3(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);
49 }
50 };
51 struct CH3D{
52     struct face{
53         //表示凸包一个面上的三个点的编号
54         int a,b,c;
55         //表示该面是否属于最终的凸包上的面
56         bool ok;
57     };
58     //初始顶点数
59     int n;
60     Point3 P[MAXN];

```

```

61 //凸包表面的三角形数
62 int num;
63 //凸包表面的三角形
64 face F[8*MAXN];
65 int g[MAXN][MAXN];
66 //叉乘
67 Point3 cross(const Point3 &a,const Point3 &b,const Point3 &c){
68     return (b-a)^(c-a);
69 }
70 //三角形面积 *2
71 double area(Point3 a,Point3 b,Point3 c){
72     return ((b-a)^(c-a)).len();
73 }
74 //四面体有向面积 *6
75 double volume(Point3 a,Point3 b,Point3 c,Point3 d){
76     return ((b-a)^(c-a))*(d-a);
77 }
78 //正: 点在面同向
79 double dblcmp(Point3 &p,face &f){
80     Point3 p1 = P[f.b] - P[f.a];
81     Point3 p2 = P[f.c] - P[f.a];
82     Point3 p3 = p - P[f.a];
83     return (p1^p2)*p3;
84 }
85 void deal(int p,int a,int b){
86     int f = g[a][b];
87     face add;
88     if(F[f].ok){
89         if(dblcmp(P[p],F[f]) > eps)
90             dfs(p,f);
91         else {
92             add.a = b;
93             add.b = a;
94             add.c = p;
95             add.ok = true;
96             g[p][b] = g[a][p] = g[b][a] = num;
97             F[num++] = add;
98         }
99     }
100 }
101 //递归搜索所有应该从凸包内删除的面
102 void dfs(int p,int now){
103     F[now].ok = false;
104     deal(p,F[now].b,F[now].a);
105     deal(p,F[now].c,F[now].b);
106     deal(p,F[now].a,F[now].c);
107 }
108 bool same(int s,int t){
109     Point3 &a = P[F[s].a];
110     Point3 &b = P[F[s].b];
111     Point3 &c = P[F[s].c];

```

```

112         return fabs(volume(a,b,c,P[F[t].a])) < eps &&
113             fabs(volume(a,b,c,P[F[t].b])) < eps &&
114             fabs(volume(a,b,c,P[F[t].c])) < eps;
115     }
116     //构建三维凸包
117     void create(){
118         num = 0;
119         face add;
120
121         //*****
122         //此段是为了保证前四个点不共面
123         bool flag = true;
124         for(int i = 1;i < n;i++){
125             if(!(P[0] == P[i])){
126                 swap(P[1],P[i]);
127                 flag = false;
128                 break;
129             }
130         }
131         if(flag)return;
132         flag = true;
133         for(int i = 2;i < n;i++){
134             if( ((P[1]-P[0])^(P[i]-P[0])).len() > eps ){
135                 swap(P[2],P[i]);
136                 flag = false;
137                 break;
138             }
139         }
140         if(flag)return;
141         flag = true;
142         for(int i = 3;i < n;i++){
143             if(fabs( ((P[1]-P[0])^(P[2]-P[0]))*(P[i]-P[0]) ) > eps)
144                 {
145                 swap(P[3],P[i]);
146                 flag = false;
147                 break;
148             }
149         }
150         //*****
151
152         for(int i = 0;i < 4;i++){
153             add.a = (i+1)%4;
154             add.b = (i+2)%4;
155             add.c = (i+3)%4;
156             add.ok = true;
157             if(dblcmp(P[i],add) > 0)swap(add.b,add.c);
158             g[add.a][add.b] = g[add.b][add.c] = g[add.c][add.a] =
159                 num;
160             F[num++] = add;
161         }

```



```

161     for(int i = 4;i < n;i++)
162         for(int j = 0;j < num;j++){
163             if(F[j].ok && dblcmp(P[i],F[j]) > eps){
164                 dfs(i,j);
165                 break;
166             }
167         int tmp = num;
168         num = 0;
169         for(int i = 0;i < tmp;i++){
170             if(F[i].ok)
171                 F[num++] = F[i];
172         }
173     //表面积
174     //测试: HDU3528
175     double area(){
176         double res = 0;
177         if(n == 3){
178             Point3 p = cross(P[0],P[1],P[2]);
179             return p.len()/2;
180         }
181         for(int i = 0;i < num;i++){
182             res += area(P[F[i].a],P[F[i].b],P[F[i].c]);
183         }
184         return res/2.0;
185     }
186     double volume(){
187         double res = 0;
188         Point3 tmp = Point3(0,0,0);
189         for(int i = 0;i < num;i++){
190             res += volume(tmp,P[F[i].a],P[F[i].b],P[F[i].c]);
191         }
192         return fabs(res/6);
193     }
194     //表面三角形个数
195     int triangle(){
196         return num;
197     }
198     //表面多边形个数
199     //测试: HDU3662
200     int polygon(){
201         int res = 0;
202         for(int i = 0;i < num;i++){
203             bool flag = true;
204             for(int j = 0;j < i;j++){
205                 if(same(i,j)){
206                     flag = 0;
207                     break;
208                 }
209             }
210             res += flag;
211         }
212         return res;
213     }
214     //重心

```

```

212 //测试: HDU4273
213 Point3 barycenter(){
214     Point3 ans = Point3(0,0,0);
215     Point3 o = Point3(0,0,0);
216     double all = 0;
217     for(int i = 0; i < num; i++){
218         double vol = volume(o, P[F[i].a], P[F[i].b], P[F[i].c]);
219         ans = ans + (((o + P[F[i].a] + P[F[i].b] + P[F[i].c]) / 4.0) *
220             vol);
221         all += vol;
222     }
223     ans = ans / all;
224     return ans;
225 }
226 //点到面的距离
227 //测试: HDU4273
228 double ptoface(Point3 p, int i){
229     double tmp1 = fabs(volume(P[F[i].a], P[F[i].b], P[F[i].c], p));
230     ;
231     double tmp2 = ((P[F[i].b] - P[F[i].a]) ^ (P[F[i].c] - P[F[i].a]))
232         .len();
233     return tmp1 / tmp2;
234 }
235 };
236 CH3D hull;
237 int main()
238 {
239     while(scanf("%d", &hull.n) == 1){
240         for(int i = 0; i < hull.n; i++) hull.P[i].input();
241         hull.create();
242         Point3 p = hull.barycenter();
243         double ans = 1e20;
244         for(int i = 0; i < hull.num; i++)
245             ans = min(ans, hull.ptoface(p, i));
246         printf("%.3lf\n", ans);
247     }
248     return 0;
249 }

```

8 其他

8.1 高精度

高精度，支持乘法和加法

```

1  /*
2  * 高精度，支持乘法和加法
3  */
4  struct BigInt{
5      const static int mod = 10000;
6      const static int DLEN = 4;
7      int a[600],len;
8      BigInt(){
9          memset(a,0,sizeof(a));
10         len = 1;
11     }
12     BigInt(int v){
13         memset(a,0,sizeof(a));
14         len = 0;
15         do{
16             a[len++] = v%mod;
17             v /= mod;
18         }while(v);
19     }
20     BigInt(const char s[]){
21         memset(a,0,sizeof(a));
22         int L = strlen(s);
23         len = L/DLEN;
24         if(L%DLEN)len++;
25         int index = 0;
26         for(int i = L-1;i >= 0;i -= DLEN){
27             int t = 0;
28             int k = i - DLEN + 1;
29             if(k < 0)k = 0;
30             for(int j = k;j <= i;j++){
31                 t = t*10 + s[j] - '0';
32                 a[index++] = t;
33             }
34         }
35         BigInt operator +(const BigInt &b)const{
36             BigInt res;
37             res.len = max(len,b.len);
38             for(int i = 0;i <= res.len;i++){
39                 res.a[i] = 0;
40             }
41             for(int i = 0;i < res.len;i++){
42                 res.a[i] += ((i < len)?a[i]:0)+((i < b.len)?b.a[i]:0);
43                 res.a[i+1] += res.a[i]/mod;
44                 res.a[i] %= mod;
45             }
46             if(res.a[res.len] > 0)res.len++;
47             return res;

```

```

47     }
48     BigInt operator *(const BigInt &b) const{
49         BigInt res;
50         for(int i = 0; i < len; i++){
51             int up = 0;
52             for(int j = 0; j < b.len; j++){
53                 int temp = a[i]*b.a[j] + res.a[i+j] + up;
54                 res.a[i+j] = temp%mod;
55                 up = temp/mod;
56             }
57             if(up != 0)
58                 res.a[i + b.len] = up;
59         }
60         res.len = len + b.len;
61         while(res.a[res.len - 1] == 0 && res.len > 1) res.len--;
62         return res;
63     }
64     void output(){
65         printf("%d", a[len-1]);
66         for(int i = len-2; i >= 0; i--)
67             printf("%04d", a[i]);
68         printf("\n");
69     }
70 };

```

8.2 完全高精度

HDU 1134 求卡特兰数

```

1  /*
2   * 完全大数模板
3   * 输入 cin>>a
4   * 输出 a.print();
5   * 注意这个输入不能自动去掉前导 0 的，可以先读入到 char 数组，去掉前导 0，再用
   构造函数。
6   */
7  #define MAXN 9999
8  #define MAXSIZE 1010
9  #define DLEN 4
10 class BigInt{
11 private:
12     int a[500]; //可以控制大数的位数
13     int len;
14 public:
15     BigInt(){len=1; memset(a, 0, sizeof(a));} //构造函数
16     BigInt(const int); //将一个 int 类型的变量转化成大数
17     BigInt(const char*); //将一个字符串类型的变量转化为大数
18     BigInt(const BigInt &); //拷贝构造函数
19     BigInt &operator=(const BigInt &); //重载赋值运算符，大数之间进行赋值运算
20     friend istream& operator>>(istream&, BigInt&); //重载输入运算符
21     friend ostream& operator<<(ostream&, BigInt&); //重载输出运算符

```

```

22
23     BigNum operator+(const BigNum &)const;    //重载加法运算符，两个大数
        之间的相加运算
24     BigNum operator-(const BigNum &)const;    //重载减法运算符，两个大数
        之间的相减运算
25     BigNum operator*(const BigNum &)const;    //重载乘法运算符，两个大数
        之间的相乘运算
26     BigNum operator/(const int &)const;        //重载除法运算符，大数对一
        个整数进行相除运算
27
28     BigNum operator^(const int &)const;        //大数的 n 次方运算
29     int operator%(const int &)const;          //大数对一个类型的变量进行
        取模运算int
30     bool operator>(const BigNum &T)const;      //大数和另一个大数的大小比
        较
31     bool operator>(const int &t)const;         //大数和一个 int 类型的变
        量的大小比较
32
33     void print();          //输出大数
34 };
35 //将一个 int 类型的变量转化为大数
36 BigNum::BigNum(const int b){
37     int c,d=b;
38     len=0;
39     memset(a,0,sizeof(a));
40     while(d>MAXN){
41         c=d-(d/(MAXN+1))*(MAXN+1);
42         d=d/(MAXN+1);
43         a[len++]=c;
44     }
45     a[len++]=d;
46 }
47 //将一个字符串类型的变量转化为大数
48 BigNum::BigNum(const char *s){
49     int t,k,index,L,i;
50     memset(a,0,sizeof(a));
51     L=strlen(s);
52     len=L/DLEN;
53     if(L%DLEN)len++;
54     index=0;
55     for(i=L-1;i>=0;i-=DLEN){
56         t=0;
57         k=i-DLEN+1;
58         if(k<0)k=0;
59         for(int j=k;j<=i;j++){
60             t=t*10+s[j]-'0';
61             a[index++]=t;
62         }
63     }
64 //拷贝构造函数
65 BigNum::BigNum(const BigNum &T):len(T.len){

```

```

66     int i;
67     memset(a,0,sizeof(a));
68     for(i=0;i<len;i++)
69         a[i]=T.a[i];
70 }
71 //重载赋值运算符, 大数之间赋值运算
72 BigNum & BigNum::operator=(const BigNum &n){
73     int i;
74     len=n.len;
75     memset(a,0,sizeof(a));
76     for(i=0;i<len;i++)
77         a[i]=n.a[i];
78     return *this;
79 }
80 istream& operator>>(istream &in,BigNum &b){
81     char ch[MAXSIZE*4];
82     int i=-1;
83     in>>ch;
84     int L=strlen(ch);
85     int count=0,sum=0;
86     for(i=L-1;i>=0;){
87         sum=0;
88         int t=1;
89         for(int j=0;j<4&& i>=0;j++,i--,t*=10){
90             sum+=(ch[i]-'0')*t;
91         }
92         b.a[count]=sum;
93         count++;
94     }
95     b.len=count++;
96     return in;
97 }
98 //重载输出运算符
99 ostream& operator<<(ostream& out,BigNum& b){
100     int i;
101     cout<<b.a[b.len-1];
102     for(i=b.len-2;i>=0;i--){
103         printf("%04d",b.a[i]);
104     }
105     return out;
106 }
107 //两个大数之间的相加运算
108 BigNum BigNum::operator+(const BigNum &T)const{
109     BigNum t(*this);
110     int i,big;
111     big=T.len>len?T.len:len;
112     for(i=0;i<big;i++){
113         t.a[i]+=T.a[i];
114         if(t.a[i]>MAXN){
115             t.a[i+1]++;
116             t.a[i]-=MAXN+1;

```

```

117     }
118 }
119 if(t.a[big]!=0)
120     t.len=big+1;
121 else t.len=big;
122 return t;
123 }
124 //两个大数之间的相减运算
125 BigNum BigNum::operator-(const BigNum &T)const{
126     int i,j,big;
127     bool flag;
128     BigNum t1,t2;
129     if(*this>T){
130         t1=*this;
131         t2=T;
132         flag=0;
133     }
134     else{
135         t1=T;
136         t2=*this;
137         flag=1;
138     }
139     big=t1.len;
140     for(i=0;i<big;i++){
141         if(t1.a[i]<t2.a[i]){
142             j=i+1;
143             while(t1.a[j]==0)
144                 j++;
145             t1.a[j-]--;
146             while(j>i)
147                 t1.a[j-]+=MAXN;
148             t1.a[i]+=MAXN+1-t2.a[i];
149         }
150         else t1.a[i]-=t2.a[i];
151     }
152     t1.len=big;
153     while(t1.a[t1.len-1]==0 && t1.len>1){
154         t1.len--;
155         big--;
156     }
157     if(flag)
158         t1.a[big-1]=0-t1.a[big-1];
159     return t1;
160 }
161 //两个大数之间的相乘
162 BigNum BigNum::operator*(const BigNum &T)const{
163     BigNum ret;
164     int i,j,up;
165     int temp,templ;
166     for(i=0;i<len;i++){
167         up=0;

```

```

168         for(j=0;j<T.len;j++){
169             temp=a[i]*T.a[j]+ret.a[i+j]+up;
170             if(temp>MAXN){
171                 temp1=temp-temp/(MAXN+1)*(MAXN+1);
172                 up=temp/(MAXN+1);
173                 ret.a[i+j]=temp1;
174             }
175             else{
176                 up=0;
177                 ret.a[i+j]=temp;
178             }
179         }
180         if(up!=0)
181             ret.a[i+j]=up;
182     }
183     ret.len=i+j;
184     while(ret.a[ret.len-1]==0 && ret.len>1)ret.len--;
185     return ret;
186 }
187 //大数对一个整数进行相除运算
188 BigNum BigNum::operator/(const int &b)const{
189     BigNum ret;
190     int i,down=0;
191     for(i=len-1;i>=0;i--){
192         ret.a[i]=(a[i]+down*(MAXN+1))/b;
193         down=a[i]+down*(MAXN+1)-ret.a[i]*b;
194     }
195     ret.len=len;
196     while(ret.a[ret.len-1]==0 && ret.len>1)
197         ret.len--;
198     return ret;
199 }
200 //大数对一个 int 类型的变量进行取模
201 int BigNum::operator%(const int &b)const{
202     int i,d=0;
203     for(i=len-1;i>=0;i--){
204         d=((d*(MAXN+1))%b+a[i])%b;
205     }
206     return d;
207 }
208 //大数的 n 次方运算
209 BigNum BigNum::operator^(const int &n)const{
210     BigNum t,ret(1);
211     int i;
212     if(n<0)exit(-1);
213     if(n==0)return 1;
214     if(n==1)return *this;
215     int m=n;
216     while(m>1){
217         t=*this;
218         for(i=1;(i<<1)<=m;i<=1)

```



```

219         m-=i;
220         ret=ret*t;
221         if(m==1)ret=ret*(*this);
222     }
223     return ret;
224 }
225 //大数和另一个大数的大小比较
226 bool BigNum::operator>(const BigNum &T)const{
227     int ln;
228     if(len>T.len)return true;
229     else if(len==T.len){
230         ln=len-1;
231         while(a[ln]==T.a[ln]&&ln>=0)
232             ln--;
233         if(ln>=0 && a[ln]>T.a[ln])
234             return true;
235         else
236             return false;
237     }
238     else
239         return false;
240 }
241 //大数和一个 int 类型的变量的大小比较
242 bool BigNum::operator>(const int &t)const{
243     BigNum b(t);
244     return *this>b;
245 }
246 //输出大数
247 void BigNum::print(){
248     int i;
249     printf("%d",a[len-1]);
250     for(i=len-2;i>=0;i--)
251         printf("%04d",a[i]);
252     printf("\n");
253 }
254 BigNum f[110];//卡特兰数
255
256 int main(){
257     f[0]=1;
258     for(int i=1;i<=100;i++)
259         f[i]=f[i-1]*(4*i-2)/(i+1);//卡特兰数递推式
260     int n;
261     while(scanf("%d",&n)==1){
262         if(n==-1)break;
263         f[n].print();
264     }
265     return 0;
266 }

```

8.3 strtok 和 sscanf 结合输入

空格作为分隔输入，读取一行的整数：

```

1      gets(buf);
2      int v;
3      char *p = strtok(buf," ");
4      while(p)
5      {
6          sscanf(p,"%d",&v);
7          p = strtok(NULL," ");
8      }

```

8.4 解决爆栈，手动加栈

防止爆栈最好方法是改变写法，弄成 bfs，或者模拟栈。加栈都是旁门左道，需谨慎！

C++

放在头文件前面

```

1 #pragma comment(linker, "/STACK:1024000000,1024000000")

```

G++

放在主函数里面（汇编开栈不一定适用，和系统有关。需谨慎!）

```

1 int __size__ = 256<<20;
2 char *__p__ = (char *)malloc(__size__)+__size__;
3 __asm__("movl %0, %%esp\n": "r"(__p__));

```

8.5 STL

8.5.1 优先队列 priority_queue

empty() 如果队列为空返回真

pop() 删除对顶元素

push() 加入一个元素

size() 返回优先队列中拥有的元素个数

top() 返回优先队列队顶元素

在默认的优先队列中，优先级高的先出队。在默认的 int 型中先出队的为较大的数。

```

1 priority_queue<int>q1;//大的先出对
2 priority_queue<int,vector<int>,greater<int> >q2; //小的先出队

```

自定义比较函数：

```

1 struct cmp
2 {
3     bool operator()(int x, int y)
4     {
5         return x > y; // x 小的优先级高
6         //也可以写成其他方式，如：return p[x] > p[y]; 表示 p[i] 小的优先级高
7     }
8 };
9 priority_queue<int, vector<int>, cmp>q;//定义方法
10 //其中，第二个参数为容器类型。第三个参数为比较函数。

```

```

1 struct node
2 {
3     int x, y;
4     friend bool operator < (node a, node b)
5     {
6         return a.x > b.x; //结构体中, x 小的优先级高
7     }
8 };
9 priority_queue<node>q; //定义方法
10 //在该结构中, y 为值, x 为优先级。
11 //通过自定义 operator< 操作符来比较元素中的优先级。
12 //在重载" <" 时, 最好不要重载" >", 可能会发生编译错误

```

8.5.2 set 和 multiset

set 和 multiset 用法一样, 就是 multiset 允许重复元素。

元素放入容器时, 会按照一定的排序法则自动排序, 默认是按照 less<> 排序规则来排序。不能修改容器里面的元素值, 只能插入和删除。

自定义 int 排序函数: (默认的是从小到大的, 下面这个从大到小)

```

1 struct classcomp {
2     bool operator() (const int& lhs, const int& rhs) const
3     {return lhs>rhs;}
4 }; //这里有个逗号的, 注意
5 multiset<int,classcomp> fifth; // class as Compare

```

上面这样就定义成了从大到小排列了。

结构体自定义排序函数:

(定义 set 或者 multiset 的时候定义了排序函数, 定义迭代器时一样带上排序函数)

```

1 struct Node
2 {
3     int x,y;
4 };
5 struct classcomp//先按照 x 从小到大排序, x 相同则按照 y 从大到小排序
6 {
7     bool operator()(const Node &a,const Node &b)const
8     {
9         if(a.x!=b.x)return a.x<b.x;
10        else return a.y>b.y;
11    }
12 }; //注意这里有个逗号
13 multiset<Node,classcomp>mt;
14 multiset<Node,classcomp>::iterator it;

```

```

1
2 主要函数:
3 begin() 返回指向第一个元素的迭代器
4 clear() 清除所有元素
5 count() 返回某个值元素的个数
6 empty() 如果集合为空, 返回 true

```

```

7 end() 返回指向最后一个元素的迭代器
8 erase() 删除集合中的元素（参数是一个元素值，或者迭代器）
9 find() 返回一个指向被查找到元素的迭代器
10 insert() 在集合中插入元素
11 size() 集合中元素的数目
12 lower_bound() 返回指向大于（或等于）某值的第一个元素的迭代器
13 upper_bound() 返回大于某个值元素的迭代器
14 equal_range() 返回集合中与给定值相等的上下限的两个迭代器
15 （注意对于 multiset 删除操作之间删除值会把所以这个值的都删掉，删除一个要用迭代器）

```

8.6 输入输出外挂

```

1
2 //适用于正负整数
3 template <class T>
4 inline bool scan_d(T &ret) {
5     char c; int sgn;
6     if(c=getchar(),c==EOF) return 0; //EOF
7     while(c!='-'&&(c<'0' || c>'9')) c=getchar();
8     sgn=(c=='-')?-1:1;
9     ret=(c=='-')?0:(c-'0');
10    while(c=getchar(),c>='0'&&c<='9') ret=ret*10+(c-'0');
11    ret*=sgn;
12    return 1;
13 }
14
15 inline void out(int x) {
16     if(x>9) out(x/10);
17     putchar(x%10+'0');
18 }

```

8.7 莫队算法

莫队算法，可以解决一类静态，离线区间查询问题。

BZOJ 2038: [2009 国家集训队] 小 Z 的袜子 (hose)

Description

作为一个生活散漫的人，小 Z 每天早上都要耗费很久从一堆五颜六色的袜子中找出一双来穿。终于有一天，小 Z 再也无法忍受这恼人的找袜子过程，于是他决定听天由命……具体来说，小 Z 把这 N 只袜子从 1 到 N 编号，然后从编号 L 到 R (L

Input

输入文件第一行包含两个正整数 N 和 M 。 N 为袜子的数量， M 为小 Z 所提的询问的数量。接下来一行包含 N 个正整数 C_i ，其中 C_i 表示第 i 只袜子的颜色，相同的颜色用相同的数字表示。再接下来 M 行，每行两个正整数 L, R 表示一个询问。

Output

包含 M 行，对于每个询问在一行中输出分数 A/B 表示从该询问的区间 $[L,R]$ 中随机抽出两只袜子颜色相同的概率。若该概率为 0 则输出 0/1，否则输出的 A/B 必须为最简分数。（详见样例）

Sample Input

```

6 4
1 2 3 3 3 2

```

```

2 6
1 3
3 5
1 6
Sample Output
2/5
0/1
1/1
4/15

```

题解:

只需要统计区间内各个数出现次数的平方和

莫队算法，两种方法，一种是直接分成 \sqrt{n} 块，分块排序。

另外一种求得曼哈顿距离最小生成树，根据 manhattan MST 的 dfs 序求解。

8.7.1 分块

```

1  const int MAXN = 50010;
2  const int MAXM = 50010;
3  struct Query
4  {
5      int L,R,id;
6  }node[MAXN];
7  long long gcd(long long a,long long b){
8      if(b == 0)return a;
9      return gcd(b,a%b);
10 }
11 struct Ans{
12     long long a,b;//分数 a/b
13     void reduce()//分数化简
14     {
15         long long d = gcd(a,b);
16         a /= d; b /= d;
17     }
18 }ans[MAXM];
19 int a[MAXN];
20 int num[MAXN];
21 int n,m,unit;
22 bool cmp(Query a,Query b){
23     if(a.L/unit != b.L/unit)return a.L/unit < b.L/unit;
24     else return a.R < b.R;
25 }
26 void work(){
27     long long temp = 0;
28     memset(num,0,sizeof(num));
29     int L = 1;
30     int R = 0;
31     for(int i = 0;i < m;i++){
32         while(R < node[i].R){

```

```

33         R++;
34         temp -= (long long) num[a[R]] * num[a[R]];
35         num[a[R]]++;
36         temp += (long long) num[a[R]] * num[a[R]];
37     }
38     while(R > node[i].R){
39         temp -= (long long) num[a[R]] * num[a[R]];
40         num[a[R]]--;
41         temp += (long long) num[a[R]] * num[a[R]];
42         R--;
43     }
44     while(L < node[i].L){
45         temp -= (long long) num[a[L]] * num[a[L]];
46         num[a[L]]--;
47         temp += (long long) num[a[L]] * num[a[L]];
48         L++;
49     }
50     while(L > node[i].L){
51         L--;
52         temp -= (long long) num[a[L]] * num[a[L]];
53         num[a[L]]++;
54         temp += (long long) num[a[L]] * num[a[L]];
55     }
56     ans[node[i].id].a = temp - (R-L+1);
57     ans[node[i].id].b = (long long) (R-L+1) * (R-L);
58     ans[node[i].id].reduce();
59 }
60 }
61 int main(){
62     while(scanf("%d%d",&n,&m) == 2){
63         for(int i = 1; i <= n; i++){
64             scanf("%d",&a[i]);
65         }
66         for(int i = 0; i < m; i++){
67             node[i].id = i;
68             scanf("%d%d",&node[i].L,&node[i].R);
69         }
70         unit = (int) sqrt(n);
71         sort(node,node+m,cmp);
72         work();
73         for(int i = 0; i < m; i++){
74             printf("%lld/%lld\n",ans[i].a,ans[i].b);
75         }
76         return 0;
77     }
78 }

```

8.7.2 Manhattan MST 的 dfs 顺序求解

```

1  const int MAXN = 50010;
2  const int MAXM = 50010;
3  const int INF = 0x3f3f3f3f;
4  struct Point{
5      int x,y,id;

```

```

6  }p[MAXN],pp[MAXN];
7  bool cmp(Point a,Point b)
8  {
9      if(a.x != b.x) return a.x < b.x;
10     else return a.y < b.y;
11 }
12 //树状数组, 找 y-x 大于当前的, 但是 y+x 最小的
13 struct BIT{
14     int min_val,pos;
15     void init()
16     {
17         min_val = INF;
18         pos = -1;
19     }
20 }bit[MAXN];
21 struct Edge{
22     int u,v,d;
23 }edge[MAXN<<2];
24 bool cmpedge(Edge a,Edge b){
25     return a.d < b.d;
26 }
27 int tot;
28 int n;
29 int F[MAXN];
30 int find(int x){
31     if(F[x] == -1) return x;
32     else return F[x] = find(F[x]);
33 }
34 void addedge(int u,int v,int d){
35     edge[tot].u = u;
36     edge[tot].v = v;
37     edge[tot++].d = d;
38 }
39 struct Graph{
40     int to,next;
41 }e[MAXN<<1];
42 int total,head[MAXN];
43 void _addedge(int u,int v){
44     e[total].to = v;
45     e[total].next = head[u];
46     head[u] = total++;
47 }
48 int lowbit(int x){
49     return x&(-x);
50 }
51 void update(int i,int val,int pos){
52     while(i > 0){
53         if(val < bit[i].min_val){
54             bit[i].min_val = val;
55             bit[i].pos = pos;
56         }

```

```

57     i -= lowbit(i);
58 }
59 }
60 int ask(int i,int m){
61     int min_val = INF,pos = -1;
62     while(i <= m){
63         if(bit[i].min_val < min_val){
64             min_val = bit[i].min_val;
65             pos = bit[i].pos;
66         }
67         i += lowbit(i);
68     }
69     return pos;
70 }
71 int dist(Point a,Point b){
72     return abs(a.x - b.x) + abs(a.y - b.y);
73 }
74 void Manhattan_minimum_spanning_tree(int n,Point p[]){
75     int a[MAXN],b[MAXN];
76     tot = 0;
77     for(int dir = 0;dir < 4;dir++){
78         if(dir == 1 || dir == 3){
79             for(int i = 0;i < n;i++)
80                 swap(p[i].x,p[i].y);
81         }
82         else if(dir == 2){
83             for(int i = 0;i < n;i++)
84                 p[i].x = -p[i].x;
85         }
86         sort(p,p+n,cmp);
87         for(int i = 0;i < n;i++)
88             a[i] = b[i] = p[i].y - p[i].x;
89         sort(b,b+n);
90         int m = unique(b,b+n) - b;
91         for(int i = 1;i <= m;i++){
92             bit[i].init();
93         }
94         for(int i = n-1;i >= 0;i--){
95             int pos = lower_bound(b,b+m,a[i]) - b + 1;
96             int ans = ask(pos,m);
97             if(ans != -1)
98                 addedge(p[i].id,p[ans].id,dist(p[i],p[ans]));
99             update(pos,p[i].x+p[i].y,i);
100         }
101     }
102     memset(F,-1,sizeof(F));
103     sort(edge,edge+tot,cmpedge);
104     total = 0;
105     memset(head,-1,sizeof(head));
106     for(int i = 0;i < tot;i++){
107         int u = edge[i].u, v = edge[i].v;
108         int t1 = find(u), t2 = find(v);

```



```

108         if(t1 != t2){
109             F[t1] = t2;
110             _addedge(u,v);
111             _addedge(v,u);
112         }
113     }
114 }
115 int m;
116 int a[MAXN];
117 struct Ans{
118     long long a,b;
119 }ans[MAXM];
120 long long temp ;
121 int num[MAXN];
122 void add(int l,int r){
123     for(int i = l;i <= r;i++){
124         temp -= (long long)num[a[i]]*num[a[i]];
125         num[a[i]]++;
126         temp += (long long)num[a[i]]*num[a[i]];
127     }
128 }
129 void del(int l,int r){
130     for(int i = l;i <= r;i++){
131         temp -= (long long)num[a[i]]*num[a[i]];
132         num[a[i]]--;
133         temp += (long long)num[a[i]]*num[a[i]];
134     }
135 }
136 void dfs(int l1,int r1,int l2,int r2,int idx,int pre){
137     if(l2 < l1) add(l2,l1-1);
138     if(r2 > r1) add(r1+1,r2);
139     if(l2 > l1) del(l1,l2-1);
140     if(r2 < r1) del(r2+1,r1);
141     ans[pp[idx].id].a = temp - (r2-l2+1);
142     ans[pp[idx].id].b = (long long)(r2-l2+1)*(r2-l2);
143     for(int i = head[idx];i != -1;i = e[i].next){
144         int v = e[i].to;
145         if(v == pre) continue;
146         dfs(l2,r2,pp[v].x,pp[v].y,v,idx);
147     }
148     if(l2 < l1)del(l2,l1-1);
149     if(r2 > r1)del(r1+1,r2);
150     if(l2 > l1)add(l1,l2-1);
151     if(r2 < r1)add(r2+1,r1);
152 }
153 long long gcd(long long a,long long b){
154     if(b == 0) return a;
155     else return gcd(b,a%b);
156 }
157 int main(){
158     while(scanf("%d%d",&n,&m) == 2){

```

```

159     for(int i = 1;i <= n;i++)
160         scanf("%d",&a[i]);
161     for(int i = 0;i < m;i++){
162         scanf("%d%d",&p[i].x,&p[i].y);
163         p[i].id = i;
164         pp[i] = p[i];
165     }
166     Manhattan_minimum_spanning_tree(m,p);
167     memset(num,0,sizeof(num));
168     temp = 0;
169     dfs(1,0,pp[0].x,pp[0].y,0,-1);
170     for(int i = 0;i < m;i++){
171         long long d = gcd(ans[i].a,ans[i].b);
172         printf("%lld/%lld\n",ans[i].a/d,ans[i].b/d);
173     }
174 }
175 return 0;
176 }

```

8.8 VIM 配置

```

1 set nu
2 set history=1000000
3
4 set tabstop=4
5 set shiftwidth=4
6 set smarttab
7
8 set cindent
9
10 colo evening
11
12 set nobackup
13 set noswapfile
14
15 set mouse=a
16 map <F6> :call CR()<CR>
17 func! CR()
18   exec "w"
19   exec "!g++_%-o_%"
20   exec "!_./%"
21 endfunc
22
23 imap <c-]> {<cr>}<c-o>0<left><right>
24 map <F2> :call SetTitle()<CR>
25 func SetTitle()
26   let l = 0
27   let l = l + 1 | call setline(l,'#include<stdio.h>')
28   let l = l + 1 | call setline(l,'#include<string.h>')
29   let l = l + 1 | call setline(l,'#include<iostream>')
30   let l = l + 1 | call setline(l,'#include<algorithm>')
31   let l = l + 1 | call setline(l,'#include<vector>')

```

```

32 let l = l + 1 | call setline(l,'#include<queue>')
33 let l = l + 1 | call setline(l,'#include<set>')
34 let l = l + 1 | call setline(l,'#include<map>')
35 let l = l + 1 | call setline(l,'#include<string>')
36 let l = l + 1 | call setline(l,'#include<math.h>')
37 let l = l + 1 | call setline(l,'#include<stdlib.h>')
38 let l = l + 1 | call setline(l,'#include<time.h>')
39 let l = l + 1 | call setline(l,'using namespace std;')
40 let l = l + 1 | call setline(l,'')
41 let l = l + 1 | call setline(l,'int main()')
42 let l = l + 1 | call setline(l,'{')
43 let l = l + 1 | call setline(l,'    //freopen("in.txt","r",stdin);'
    )
44 let l = l + 1 | call setline(l,'    //freopen("out.txt","w",stdout)
    ;')
45 let l = l + 1 | call setline(l,'')
46 let l = l + 1 | call setline(l,'    return 0;')
47 let l = l + 1 | call setline(l,'}')
48 endfunc

```

现场赛配置:

```

1 syntax on
2 set nu
3 set tabstop=4
4 set shiftwidth=4
5 set cin
6 colo evening
7 set mouse=a

```