1. Fill in the blanks with the best fit:

   Let $\{a_n\}$ be a sequence of positive terms. Suppose that $a_n = f(n)$, where f is a ... , ... and ... function of x for all $x \geq N$ (N a positive integer). Then the series $\sum_{n=N}^{\infty} a_n$ and the integral $\int_N^{\infty} f(x)dx$ both converge or both diverge.

   (a) composite, polynomial, decreasing
   (b) positive, composite, increasing
   (c) continuous, positive, decreasing
   (d) positive, continuous, increasing
   (e) polynomial, positive, decreasing

2. Let $a_1 = 1, \quad a_{n+1} = \left(\dfrac{n+1}{2n+3}\right)a_n, \quad (n \geq 1).$

   Which of the following is true for the sequence $\{a_n\}$ and the series $\sum_{n=1}^{\infty} a_n$?

   (a) $\{a_n\}$ converges, $\sum_{n=1}^{\infty} a_n$ diverges.
   (b) $\{a_n\}$ converges, $\sum_{n=1}^{\infty} a_n$ converges.
   (c) $\{a_n\}$ diverges, $\sum_{n=1}^{\infty} a_n$ diverges.
   (d) $\{a_n\}$ diverges, $\sum_{n=1}^{\infty} a_n$ converges.
   (e) $\{a_n\}$ converges, one cannot say anything about the convergence of the series $\sum_{n=1}^{\infty} a_n$.

3. Find all p values that make the series $\sum_{n=1}^{\infty} \dfrac{3^n}{1+9^{np}}$ convergent.

   (a) $p < \frac{1}{2}$
   (b) $p \leq \frac{1}{2}$
   (c) $0 < p \leq \frac{1}{2}$
   (d) $0 < p < \frac{1}{2}$
   (e) $p > \frac{1}{2}$

4. $\sum_{n=2}^{\infty} \dfrac{3}{n^2+3n+2} = ?$

   (a) $\frac{1}{3}$
   (b) $\frac{2}{3}$
   (c) 1
   (d) $\frac{3}{2}$
   (e) 3

5. Which of the following is true about the convergence of the following series?

   I. $\sum_{n=1}^{\infty} \dfrac{n^2}{2n^2+n+2}$

   II. $\sum_{n=1}^{\infty} \dfrac{\cos^2 n}{3^n}$

   III. $\sum_{n=1}^{\infty} \dfrac{e^n}{n!}$

   (a) I, II and III converge.
   (b) I, II and III diverge.
   (c) I and II diverge, III converges.
   (d) I diverges, II and III converge.
   (e) I and III diverge, II converges.

6. Which is the interval of the absolute convergence of the power series $\sum_{n=1}^{\infty} \frac{n}{4^n}(x-1)^{2n}$?

(a) $-1 < x < 3$
(b) $-1 \leq x < 3$
(c) $-1 < x \leq 3$
(d) $-3 < x < 5$
(e) $-3 \leq x \leq 5$

7. Which of the following series converge conditionally?

I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$

II. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\frac{1}{3}}}$

III. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[n]{n}}$

(a) I and III
(b) II and III
(c) I only
(d) II only
(e) III only

8. Which of the following is the Taylor series of the function $f(x) = 2^{2x}$ about $x = 1$?

(a) $\sum_{n=0}^{\infty} \frac{2^{n+2}(\ln 2)^n}{n!}(x-1)^n$

(b) $\sum_{n=0}^{\infty} \frac{2^{n+1}(\ln 2)^n}{n!}(x-1)^n$

(c) $\sum_{n=0}^{\infty} \frac{2^{n+2}(\ln 2)}{(n+1)!}(x-1)^{n+1}$

(d) $\sum_{n=0}^{\infty} \frac{4^n(\ln 2)^n}{n!}(x-1)^n$

(e) $\sum_{n=0}^{\infty} \frac{4^n(\ln 2)}{n!}(x-1)^n$

9. Find the Maclaurin series of $f(x) = x^2\sin(2x)$.
(Hint: You may use the substitution method.)

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n}}{(2n)!}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n+1}}{(2n+1)!}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} 2^{2n+1} x^{2n+1}}{(2n+1)!}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+3}}{(2n+1)!}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} 2^{2n} x^{2n+3}}{(2n+1)!}$

10. $\sum_{n=1}^{\infty} \frac{1}{3(n-1)!} = ?$

(Hint: You may differentiate the Maclaurin series of e^x term-by-term.)

(a) $\frac{e}{3}$
(b) 1
(c) $e^{1/3}$
(d) $3e$
(e) e^3

11. If the slope of the curve given by the polar form $r(\theta) = c + 2\sin\theta$ at $\theta = 0$ is 2, then $c = ?$

(a) $\frac{1}{2}$
(b) 1
(c) 2
(d) 3
(e) 4

12. Which of the following integrals gives the area of the region inside the circle $r = 3$ and outside the cardioid $r = 3(1 - \cos\theta)$?

(a) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (9\cos^2\theta + 18\cos\theta)d\theta$

(b) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (9\cos^2\theta - 18\cos\theta)d\theta$

(c) $\displaystyle\int_{0}^{\frac{\pi}{2}} (8\cos\theta + 18\cos^2\theta)d\theta$

(d) $\displaystyle\int_{0}^{\frac{\pi}{2}} (9\cos^2\theta - 18\cos\theta)d\theta$

(e) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(9\cos\theta - \frac{9}{2}\cos^2\theta\right) d\theta$

13. What is the length of the curve given by the polar form $r(\theta) = 2\theta^2$ for $0 \le \theta \le \sqrt{5}$?

(a) $\dfrac{16}{3}$

(b) $\dfrac{22}{3}$

(c) $\dfrac{38}{3}$

(d) 16

(e) 18

14. Given two vectors $\vec{A} = \vec{i} - \vec{j} + 2\vec{k}$ and $\vec{B} = 2\vec{i} + \vec{j} + \vec{k}$, $|\text{proj}_{\vec{B}}\vec{A}| = ?$

(a) $\dfrac{\sqrt{3}}{2}$

(b) $\dfrac{\sqrt{6}}{2}$

(c) $2\sqrt{3}$

(d) $2\sqrt{6}$

(e) $6\sqrt{2}$

15. If the volume of the parallelepiped determined by the vectors $\vec{A} = b\vec{i} + 2\vec{j} - \vec{k}$, $\vec{B} = \vec{i} + 3\vec{j} + \vec{k}$ and $\vec{C} = -\vec{i} + 3\vec{j} + 2\vec{k}$ is 9, where $b > 4$, then $b = ?$

(a) 5

(b) 6

(c) 7

(d) 8

(e) 9

16. Find the parametric equations for the line in which the planes $2x + y - z = 2$ and $3x + 2y + z = 1$ intersect. $(t \in \mathbb{R})$

(a) $x = 2t,\ y = 1 - 5t,\ z = -2 + t$

(b) $x = 3t,\ y = 1 - 5t,\ z = -1 + t$

(c) $x = 3t,\ y = -5t,\ z = 1 + t$

(d) $x = 3t,\ y = -5t,\ z = t$

(e) $x = 1 + 3t,\ y = -5t,\ z = t$

Answers

1. (c)
2. (b)
3. (e)
4. (c)
5. (d)
6. (a)
7. (d)
8. (a)
9. (d)
10. (a)
11. (e)
12. (e)
13. (c)
14. (b)
15. (c)
16. (b)