1. The function

$$f(x,y) = 3xy^2 + x^3 - 3x^2 - 3y^2 + 9$$

has four critical points. Which of the following are two of these points?

(a) $(1,0), (0,1)$
(b) $(1,0), (-1,1)$
(c) $(0,0), (2,2)$
(d) $(2,0), (1,1)$
(e) $(2,0), (2,2)$

2. Given the vectors $\boldsymbol{a} = (a_1, a_2, a_3)$, $\boldsymbol{b} = (b_1, b_2, b_3)$ and $\boldsymbol{c} = (c_1, c_2, c_3)$, which of the following are always true?

I. $|\boldsymbol{a} \times \boldsymbol{b}|^2 = |\boldsymbol{a}|^2 \, |\boldsymbol{b}|^2 - (\boldsymbol{a} \cdot \boldsymbol{b})^2$

II. $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{c}$

III. $(\boldsymbol{a} \cdot \boldsymbol{b})\, \boldsymbol{c} = \boldsymbol{a}\, (\boldsymbol{b} \cdot \boldsymbol{c})$

(a) I only
(b) II only
(c) I, II
(d) II, III
(e) I, II, III

3. $\dfrac{\partial^{100}}{\partial x^{95}\, \partial y^2\, \partial x^3} (xye^x + \cos x) = ?$

(a) -1
(b) 0
(c) 1
(d) e^x
(e) $\sin x$

4. Which is the equation of the plane that contains the line given by $x = 1 - t$, $y = 1 + 2t$, $z = 2 - 3t$ where $t \in \mathbb{R}$ and the point $(-1, 1, 2)$?

(a) $-x + 2z = 3$
(b) $-x - 2z = -3$
(c) $3x + 2z = 1$
(d) $3y + 2z = 7$
(e) $-3y + 2z = 1$

5. Let L denote the line that passes through the point $(0, 0, 1)$ and is parallel to the line given by $y = 3x$ in the $xy-$plane. Which of the following is the equation of the plane that passes through the point $(0, 0, 1)$ and is perpendicular to the line L?

(a) $-x + 3y = 0$
(b) $3x - y = 0$
(c) $-3x - 3y = 0$
(d) $3x + y = 0$
(e) $x + 3y = 0$

6. Which of the following are extreme values of

$$f(x,y) = \frac{x}{1 + x^2 + y^2} \ ?$$

	maximum value	minimum value
(a)	$\frac{1}{2}$	$-\frac{1}{2}$
(b)	1	$-\frac{1}{2}$
(c)	$\frac{1}{2}$	-1
(d)	1	-1
(e)	1	$\frac{1}{2}$

7. Given $z = f(x,y) = xye^{-xy^2}$ and $x = u^2 + 3v$, $y = uv - 3v$, evaluate $\dfrac{\partial z}{\partial v}$ at the point $u = 2,\ v = -1$.

(a) e^{-1}

(b) 1

(c) 2

(d) e

(e) $2e$

8. Which of the following integrals gives the area of the region inside the cardioid $r = 1+\cos\theta$ and outside the circle $r = 3\cos\theta$ in polar coordinates?

(a) $2\cdot\dfrac{1}{2}\left[\displaystyle\int_{\pi/3}^{\pi/2} (1+\cos\theta)^2\, d\theta - 9\int_{\pi/3}^{\pi/2} (\cos\theta)^2\, d\theta\right]$

(b) $2\cdot\dfrac{1}{2}\left[\displaystyle\int_{\pi/3}^{\pi/2} (1+\cos\theta)^2\, d\theta - 9\int_{\pi/2}^{\pi} (\cos\theta)^2\, d\theta\right]$

(c) $2\cdot\dfrac{1}{2}\left[\displaystyle\int_{0}^{\pi/3} (1+\cos\theta)^2\, d\theta - 9\int_{\pi/3}^{\pi/2} (\cos\theta)^2\, d\theta\right]$

(d) $2\cdot\dfrac{1}{2}\left[\displaystyle\int_{\pi/3}^{\pi} (1+\cos\theta)^2\, d\theta - 9\int_{\pi/3}^{\pi/2} (\cos\theta)^2\, d\theta\right]$

(e) $2\cdot\dfrac{1}{2}\left[\displaystyle\int_{\pi/3}^{\pi} (1+\cos\theta)^2\, d\theta + 9\int_{\pi/3}^{\pi/2} (\cos\theta)^2\, d\theta\right]$

9. Two objects travel through space along two different curves. The trajectory of object A is given by $\boldsymbol{r_1}(t) = \big(t^2,\ 7t-12,\ t^2\big)$, $t \geq 0$ and the trajectory of object B is given by $\boldsymbol{r_2}(t) = \big(4t-3,\ t^2,\ 5t-6\big)$, $t \geq 0$. At which t value do these objects collide and which object is moving faster at the time of collision?

(a) $t = 2$, object A

(b) $t = 2$, object B

(c) $t = 3$, object B

(d) $t = 3$, object A

(e) $t = 4$, object B

10. $f(x,y)$ is a differentiable function whose exact formula is not known.
However, it is known that the intersection of the surface $z = f(x,y)$ and the plane $x = 1$ is given by the curve $\boldsymbol{r_1}(t) = \big(1,\ 1+t+t^2,\ 1-2t\big)$ and the intersection of the surface $z = f(x,y)$ and the plane $y = 1$ is given by the curve $\boldsymbol{r_2}(t) = \big(t^3,\ 1,\ 2-t\big)$. It is also known that $f(1,1) = 1$.
Which of the following are the values of the partial derivatives $f_x(1,1)$ and $f_y(1,1)$, respectively?

(a) $-\dfrac{1}{3}\ ,\ -2$

(b) $-\dfrac{1}{3}\ ,\ -1$

(c) $-\dfrac{1}{6}\ ,\ -1$

(d) $\dfrac{1}{3}\ ,\ 2$

(e) $3\ ,\ 2$

11. Let $f(x,y) = \begin{cases} xy\,\dfrac{y^2-x^2}{x^2+y^2} & ,\ (x,y) \neq (0,0) \\ 0 & ,\ (x,y) = (0,0). \end{cases}$

Which of the following are $\dfrac{\partial f}{\partial y}(x,0)$ and $\dfrac{\partial f}{\partial x}(0,y)$, respectively?

(a) $-x,\ -y$

(b) $-x,\ y$

(c) $x,\ -y$

(d) $y,\ -x$

(e) $x,\ y$

12. Which of the following is the condition provided by the constants a, b, and c that guarantees the existence of

$$\lim_{(x,y)\to(0,0)} \frac{xy}{ax^2+bxy+cy^2}\ ?$$

(a) $b = c = 0,\ a \neq 0$

(b) $a = c = 0,\ b \neq 0$

(c) $a = b = 0,\ c \neq 0$

(d) $a = c,\ b \neq 0$

(e) $a = c,\ b = 0$

13. Let $f(x,y)$ be a function whose directional derivative at $(1,0)$ in the direction $\frac{1}{\sqrt{2}}(\boldsymbol{i}+\boldsymbol{j})$ equals to $\frac{1}{2}$ and in the direction $\frac{1}{\sqrt{2}}(-\boldsymbol{i}+\boldsymbol{j})$ equals to 2. Which of the following is the gradient of f at $(1,0)$?

(a) $\frac{\sqrt{2}}{2}(-3\boldsymbol{i}+5\boldsymbol{j})$

(b) $\frac{\sqrt{2}}{3}(-3\boldsymbol{i}+5\boldsymbol{j})$

(c) $\frac{\sqrt{2}}{4}(-3\boldsymbol{i}+5\boldsymbol{j})$

(d) $\frac{2}{\sqrt{2}}(-3\boldsymbol{i}+5\boldsymbol{j})$

(e) $\frac{\sqrt{6}}{2}(-3\boldsymbol{i}+5\boldsymbol{j})$

14. Which of the following are the equations of the tangent plane and the normal line passing through the point $P(3,0,9)$ of the surface $z=f(x,y)=x^2e^{xy}$, respectively?

(a) $-4x-21y+z=-3$,
$\boldsymbol{r}(t)=(3,0,9)+t(3,9,1)$

(b) $-2x-27y+2z=12$,
$\boldsymbol{r}(t)=(3,0,9)+t(-1,-9,1)$

(c) $-6x-27y-z=-27$,
$\boldsymbol{r}(t)=(3,0,9)+t(-6,-27,1)$

(d) $-6x-27y+z=-9$,
$\boldsymbol{r}(t)=(3,0,9)+t(-6,-27,1)$

(e) $-6x+27y+z=-9$,
$\boldsymbol{r}(t)=(3,0,9)+t(-3,-9,1)$

15. Let $f(x,y)=\dfrac{x-y}{x+y}$ and $P\left(-\frac{1}{2},\frac{3}{2}\right)$. At P, which direction $\boldsymbol{u}$ yields $D_{\boldsymbol{u}}f=1$?

(a) $-\frac{3}{5}\boldsymbol{i}-\frac{4}{5}\boldsymbol{j}$

(b) $-\frac{3}{5}\boldsymbol{i}+\frac{4}{5}\boldsymbol{j}$

(c) $\frac{3}{4}\boldsymbol{i}-\frac{4}{5}\boldsymbol{j}$

(d) $\frac{3}{5}\boldsymbol{i}-\frac{4}{5}\boldsymbol{j}$

(e) $\frac{3}{4}\boldsymbol{i}+\frac{4}{5}\boldsymbol{j}$

16. Which of the following are extreme values of

$$f(x,y)=\frac{1}{x}+\frac{1}{y}$$

under the constraint $g(x,y)=\dfrac{1}{x^2}+\dfrac{1}{y^2}=1$?

	absolute maximum	absolute minimum
(a)	2	-2
(b)	$\sqrt{2}$	$-\sqrt{2}$
(c)	0	-1
(d)	2	0
(e)	$\sqrt{2}$	1

Answers

1. (d)
2. (c)
3. (b)
4. (d)
5. (e)
6. (a)
7. (a)
8. (d)
9. (d)
10. (a)
11. (b)
12. (b)
13. (c)
14. (d)
15. (d)
16. (b)