1. Fill in the blank with the best fit:

   Often scientists and engineers simplify a problem by replacing a function with an approximation using the first few terms of a _____ that expresses it.

   (a) derivative
   (b) domain
   (c) limit
   (d) sequence
   (e) series

2. If $a_n = (n+t)^{\frac{1}{n+t}}$, $t \in \mathbb{R}$ and $b_n = \dfrac{n^2+1}{1-n}\sin\left(\dfrac{1}{n}\right)$, then $\lim\limits_{n\to\infty}(a_n \cdot b_n) = ?$

   (a) -2
   (b) -1
   (c) 0
   (d) 1
   (e) ∞

3. Which is true for the sequence whose nth term is $a_n = \dfrac{n+1}{n-1}\cos\left(\dfrac{n\pi}{2}\right) - 1$?

   (a) It converges to -2.
   (b) It converges to -1.
   (c) It converges to 0.
   (d) It converges to 1.
   (e) It diverges.

4. If $a_n = \dfrac{\ln n}{\sqrt{n}}$, then $\lim\limits_{n\to\infty} a_n = ?$

   (a) 0
   (b) 6
   (c) 12
   (d) 24
   (e) 48

5. Which of the following is always true?

   (a) A monotonic sequence is convergent.
   (b) A nonincreasing sequence is convergent if it is bounded above.
   (c) A bounded sequence is convergent.
   (d) A nondecreasing sequence is convergent if it is bounded below.
   (e) A bounded sequence is convergent if it is monotonic.

6. Which of the following is true for the series $\sum\limits_{n=0}^{\infty} a_n$ where $a_n = \begin{cases} r^n & , \text{ if } n \text{ is even} \\ 2r^n & , \text{ if } n \text{ is odd} \end{cases}$?

   (a) The series converges only for $r > 1$ and its sum is $\dfrac{1+2r}{1-r}$.
   (b) The series converges only for $|r| < 1$ and its sum is $\dfrac{1+2r}{1-r^2}$.
   (c) The series converges only for $|r| < 1$ and its sum is $\dfrac{1+2r}{1-r}$.
   (d) The series converges for all values of r and its sum is $\dfrac{1+2r}{1-r^2}$.
   (e) The series does not converge for any value of r.

7. By the Integral Test, which of the following can be concluded for the series $\sum\limits_{n=2}^{\infty} \dfrac{n}{n^2+4}$?

   (a) The series converges to 0.
   (b) The series converges but its sum is unknown.
   (c) The series is divergent.
   (d) The sum of the series is $\dfrac{\pi^2}{4}$.
   (e) The Integral Test cannot be applied as its conditions are not satisfied.

8. Which of the following statements are true?

I. The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+5}$ is convergent.

II. The series $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{\sqrt{n}\,e^n}$ is divergent by the Limit Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\,e^n}$.

III. The series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n!}$ is convergent by the Direct Comparison Test with $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$.

(a) I only
(b) I, II
(c) I, III
(d) II, III
(e) I, II, III

9. Which of the following is true for $\displaystyle\sum_{n=1}^{\infty} \frac{8^n}{(2n+1)5^{2n+1}}$?

(a) It converges by the nth-Term Test.
(b) It diverges by the nth-Term Test.
(c) It converges by the Root Test.
(d) It diverges by the Root Test.
(e) The Ratio Test is inconclusive.

10. Which of the following is true for $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)e^{2n}}{n!}$?

(a) It converges absolutely by the Ratio Test.
(b) It converges conditionally by the Ratio Test.
(c) It diverges by the Direct Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$.
(d) It diverges by the Root Test.
(e) It converges absolutely by the Alternating Series (Leibniz) Test.

11. Fill in the blank with the best fit:

A series $\displaystyle\sum_{n=1}^{\infty} a_n$ of nonnegative terms converges if and only if _____.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n a_n$ is convergent
(b) a_n is a monotonic sequence
(c) a_n has a limit
(d) its partial sums are bounded below
(e) its partial sums are bounded above

12. Which is true for the series

$$A:\sum_{n=2}^{\infty}(-1)^n\frac{1}{\ln n} \text{ and } B:\sum_{n=1}^{\infty}(-1)^n n^2\cos\left(\frac{2}{n^2}+\frac{\pi}{2}\right)?$$

(a) Both series converge conditionally.
(b) Both series diverge.
(c) Series A diverges and series B converges conditionally.
(d) Series A converges absolutely and series B diverges.
(e) Series A converges conditionally and series B diverges.

13. Find all the values of p for which the series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(n!)^p}{(n+2)!}$ converges absolutely.

(a) $p=1$
(b) $p<1$
(c) $p\le 1$
(d) $0<p<1$
(e) $0\le p\le 1$

14. What is the interval of convergence of the power series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2x-1)^n}{2^n n}$?

(a) $0 < x < \frac{3}{2}$

(b) $0 \le x < \frac{3}{2}$

(c) $-\frac{1}{2} < x < \frac{3}{2}$

(d) $-\frac{1}{2} < x \le \frac{3}{2}$

(e) $-\frac{1}{2} \le x \le \frac{3}{2}$

15. Which of the following is the power series representation of the function $f(x) = \frac{\tan^{-1} x}{x}$?

(a) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}, \quad -1 \le x \le 1, \ x \ne 0$

(b) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{2n-1}, \quad -1 \le x \le 1, \ x \ne 0$

(c) $\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}, \quad -1 \le x \le 1, \ x \ne 0$

(d) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{2n+1}, \quad -1 \le x \le 1, \ x \ne 0$

(e) $\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n}}{2n-1}, \quad -1 \le x \le 1, \ x \ne 0$

16. Which of the following statements are true for the function $f(x) = \ln(1+x^2)$?

I. The Maclaurin series is $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(2x)^n}{n}$.

II. The Taylor series at $x = 0$ is absolutely convergent on $(-1, 1)$.

III. The Taylor polynomials of orders $2n$ and $2n+1$ are identical.

(a) I only

(b) II only

(c) III only

(d) I, II

(e) II, III

17. Which of the following parametric equations does not specify a line, parabola, circle, or an ellipse?

(a) $x = 5\cos t, \ y = \sqrt{5}\sin t, \ 0 \le t \le 2\pi$

(b) $x = \sqrt{5} - 5t, \ y = \sqrt{5} + 5t, \ -\infty < t < \infty$

(c) $x = \cos(\sqrt{5}t), \ y = \sin(5t), \ 0 \le t \le 2\pi$

(d) $x = 5t, \ y = \sqrt{5}t^2, \ -\infty < t < \infty$

(e) $x = \sqrt{5}\cos(5t), \ y = \sqrt{5}\sin(5t), \ 0 \le t \le \frac{2\pi}{5}$

18. If $x = t - \sin t, \quad y = t + \cos t, \quad 0 \le t \le \frac{\pi}{2}$, then $\left.\frac{d^2y}{dx^2}\right|_{x=\frac{\pi-2\sqrt{2}}{4}} = ?$

(a) $-6\sqrt{2} - 8$

(b) $-3\sqrt{2} - 8$

(c) $3\sqrt{2} - 8$

(d) $6\sqrt{2} - 8$

(e) $3\sqrt{2} + 8$

19. What is the length of the parametric curve $x = 3\cos^3 t, \quad y = 3\sin^3 t, \quad 0 \le t \le \frac{\pi}{2}$?

(a) $\frac{3}{2}$

(b) 3

(c) $\frac{9}{2}$

(d) 6

(e) 9

20. What is the sum of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n(n+2)}$?

(a) $-\frac{1}{4}$

(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) 1

(e) 4

Answers

1. (e)
2. (b)
3. (e)
4. (a)
5. (e)
6. (b)
7. (c)
8. (c)
9. (c)
10. (a)
11. (e)
12. (e)
13. (c)
14. (d)
15. (d)
16. (e)
17. (c)
18. (a)
19. (c)
20. (b)

1. Boşluğu en uygun olanla doldurunuz:

   Bilim insanları ve mühendisler sıklıkla bir fonksiyonu, onu ifade eden bir _____ ilk birkaç terimini kullanan bir yaklaşımla değiştirerek problemi basitleştirirler.

   (a) türevin
   (b) tanım kümesinin
   (c) limitin
   (d) dizinin
   (e) serinin

2. Eğer $a_n = (n+t)^{\frac{1}{n+t}}$, $t \in \mathbb{R}$ ve $b_n = \dfrac{n^2+1}{1-n}\sin\left(\dfrac{1}{n}\right)$ ise $\lim\limits_{n\to\infty}(a_n \cdot b_n) = ?$

   (a) -2
   (b) -1
   (c) 0
   (d) 1
   (e) ∞

3. n. terimi $a_n = \dfrac{n+1}{n-1}\cos\left(\dfrac{n\pi}{2}\right) - 1$ olan dizi için hangisi doğrudur?

   (a) -2 'ye yakınsar.
   (b) -1 'e yakınsar.
   (c) 0 'a yakınsar.
   (d) 1 'e yakınsar.
   (e) Iraksar.

4. $a_n = \dfrac{\ln n}{\sqrt{n}}$ ise $\lim\limits_{n\to\infty} a_n = ?$

   (a) 0
   (b) 6
   (c) 12
   (d) 24
   (e) 48

5. Aşağıdakilerden hangisi her zaman doğrudur?

   (a) Monoton bir dizi yakınsaktır.
   (b) Artmayan bir dizi üstten sınırlı ise yakınsaktır.
   (c) Sınırlı bir dizi yakınsaktır.
   (d) Azalmayan bir dizi alttan sınırlı ise yakınsaktır.
   (e) Sınırlı bir dizi monoton ise yakınsaktır.

6. $a_n = \begin{cases} r^n & , \ n \text{ çift ise} \\ 2r^n & , \ n \text{ tek ise} \end{cases}$ olmak üzere $\sum\limits_{n=0}^{\infty} a_n$ serisi için aşağıdakilerden hangisi doğrudur?

   (a) Seri sadece $r > 1$ için yakınsar ve toplamı $\dfrac{1+2r}{1-r}$ 'dir.
   (b) Seri sadece $|r| < 1$ için yakınsar ve toplamı $\dfrac{1+2r}{1-r^2}$ 'dir.
   (c) Seri sadece $|r| < 1$ için yakınsar ve toplamı $\dfrac{1+2r}{1-r}$ 'dir.
   (d) Seri tüm r değerleri için yakınsar ve toplamı $\dfrac{1+2r}{1-r^2}$ 'dir.
   (e) Seri hiçbir r değeri için yakınsamaz.

7. İntegral Testi'ne göre, $\sum\limits_{n=2}^{\infty} \dfrac{n}{n^2+4}$ serisi için aşağıdaki sonuçlardan hangisine varılabilir?

   (a) Seri 0 'a yakınsar.
   (b) Seri yakınsaktır; ancak toplamı bilinmemektedir.
   (c) Seri ıraksaktır.
   (d) Serinin toplamı $\dfrac{\pi^2}{4}$ 'tür.
   (e) İntegral Testi uygulanamaz, çünkü testin koşulları sağlanmamaktadır.

8. Aşağıdaki ifadelerden hangileri doğrudur?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2+5}$ serisi yakınsaktır.

II. $\sum_{n=1}^{\infty} \frac{\ln(n)}{\sqrt{n}\,e^n}$ serisi, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\,e^n}$ ile yapılan Limit Karşılaştırma Testi'ne göre ıraksaktır.

III. $\sum_{n=2}^{\infty} \frac{1}{n!}$ serisi, $\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$ ile yapılan Doğrudan Karşılaştırma Testi'ne göre yakınsaktır.

(a) Yalnız I
(b) I, II
(c) I, III
(d) II, III
(e) I, II, III

9. $\sum_{n=1}^{\infty} \frac{8^n}{(2n+1)5^{2n+1}}$ için aşağıdakilerden hangisi doğrudur?

(a) n. Terim Testi'ne göre yakınsar.
(b) n. Terim Testi'ne göre ıraksar.
(c) Kök Testi'ne göre yakınsar.
(d) Kök Testi'ne göre ıraksar.
(e) Oran Testi sonuç vermez.

10. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)e^{2n}}{n!}$ için aşağıdakilerden hangisi doğrudur?

(a) Oran Testi'ne göre mutlak yakınsar.
(b) Oran Testi'ne göre koşullu yakınsar.
(c) $\sum_{n=1}^{\infty} \frac{1}{n}$ ile yapılan Doğrudan Karşılaştırma Testi'ne göre ıraksar.
(d) Kök Testi'ne göre ıraksar.
(e) Alterne Seri (Leibniz) Testi'ne göre mutlak yakınsar.

11. Boşluğu en uygun olanla doldurunuz:

Negatif olmayan terimli bir $\sum_{n=1}^{\infty} a_n$ serisi ancak ve ancak _____ ise yakınsar.

(a) $\sum_{n=1}^{\infty}(-1)^n a_n$ yakınsak
(b) a_n monoton bir dizi
(c) a_n 'nin bir limiti var
(d) kısmi toplamları alttan sınırlı
(e) kısmi toplamları üstten sınırlı

12. $A : \sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n}$ ve $B : \sum_{n=1}^{\infty} (-1)^n n^2 \cos\left(\frac{2}{n^2} + \frac{\pi}{2}\right)$ serileri için hangisi doğrudur?

(a) Her iki seri de koşullu yakınsar.
(b) Her iki seri de ıraksar.
(c) A serisi ıraksar ve B serisi koşullu yakınsar.
(d) A serisi mutlak yakınsar ve B serisi ıraksar.
(e) A serisi koşullu yakınsar ve B serisi ıraksar.

13. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(n!)^p}{(n+2)!}$ serisinin mutlak yakınsadığı tüm p değerlerini bulunuz.

(a) $p = 1$
(b) $p < 1$
(c) $p \leq 1$
(d) $0 < p < 1$
(e) $0 \leq p \leq 1$

14. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2x-1)^n}{2^n n}$ kuvvet serisinin yakınsaklık aralığı nedir?

(a) $0 < x < \frac{3}{2}$

(b) $0 \le x < \frac{3}{2}$

(c) $-\frac{1}{2} < x < \frac{3}{2}$

(d) $-\frac{1}{2} < x \le \frac{3}{2}$

(e) $-\frac{1}{2} \le x \le \frac{3}{2}$

15. $f(x) = \dfrac{\tan^{-1} x}{x}$ fonksiyonunun kuvvet serisi temsili aşağıdakilerden hangisidir?

(a) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}, \quad -1 \le x \le 1,\ x \ne 0$

(b) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{2n-1}, \quad -1 \le x \le 1,\ x \ne 0$

(c) $\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}, \quad -1 \le x \le 1,\ x \ne 0$

(d) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{2n+1}, \quad -1 \le x \le 1,\ x \ne 0$

(e) $\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n}}{2n-1}, \quad -1 \le x \le 1,\ x \ne 0$

16. $f(x) = \ln(1+x^2)$ fonksiyonu için aşağıdaki ifadelerden hangileri doğrudur?

I. Maclaurin serisi $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(2x)^n}{n}$ 'dir.

II. $x = 0$' daki Taylor serisi $(-1,1)$'de mutlak yakınsaktır.

III. $2n$ ve $2n+1$ mertebeli Taylor polinomları özdeştir.

(a) Yalnız I

(b) Yalnız II

(c) Yalnız III

(d) I, II

(e) II, III

17. Aşağıdaki parametrik denklemlerden hangisi herhangi bir doğru, parabol, çember ya da elips belirtmez?

(a) $x = 5\cos t,\ y = \sqrt{5}\sin t,\ 0 \le t \le 2\pi$

(b) $x = \sqrt{5} - 5t,\ y = \sqrt{5} + 5t,\ -\infty < t < \infty$

(c) $x = \cos(\sqrt{5}t),\ y = \sin(5t),\ 0 \le t \le 2\pi$

(d) $x = 5t,\ y = \sqrt{5}t^2,\ -\infty < t < \infty$

(e) $x = \sqrt{5}\cos(5t),\ y = \sqrt{5}\sin(5t),\ 0 \le t \le \frac{2\pi}{5}$

18. $x = t - \sin t, \quad y = t + \cos t, \quad 0 \le t \le \frac{\pi}{2}$ ise $\left.\frac{d^2y}{dx^2}\right|_{x=\frac{\pi-2\sqrt{2}}{4}} = ?$

(a) $-6\sqrt{2} - 8$

(b) $-3\sqrt{2} - 8$

(c) $3\sqrt{2} - 8$

(d) $6\sqrt{2} - 8$

(e) $3\sqrt{2} + 8$

19. $x = 3\cos^3 t, \quad y = 3\sin^3 t, \quad 0 \le t \le \frac{\pi}{2}$ parametrik eğrisinin uzunluğu nedir?

(a) $\frac{3}{2}$

(b) 3

(c) $\frac{9}{2}$

(d) 6

(e) 9

20. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n(n+2)}$ serisinin toplamı nedir?

(a) $-\frac{1}{4}$

(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) 1

(e) 4