

# MATH 242 - Quiz 4

02/08/2024

1. [5 pts] Evaluate the definite integral via Integration by Parts

$$\begin{aligned}
 I &= \int_0^{\pi/3} \sin(x) \ln(\cos(x)) dx \\
 u &= \ln(\cos(x)) \\
 du &= \frac{1}{\cos(x)} \cdot (-\sin(x)) dx = -\tan(x) dx \\
 dv &= \sin(x) dx \\
 v &= -\cos(x) \\
 I &= \left[ -\cos(x) \ln(\cos(x)) \right]_0^{\pi/3} - \int_0^{\pi/3} \cos(x) \tan(x) dx \\
 &= -\cos\left(\frac{\pi}{3}\right) \ln\left(\cos\left(\frac{\pi}{3}\right)\right) + \cos(0) \ln(\cos(0)) \\
 &= -\frac{1}{2} \ln\left(\frac{1}{2}\right) \\
 &= -\frac{1}{2} (\ln(1) - \ln(2)) \\
 &= \frac{\ln(2)}{2}
 \end{aligned}$$

$$\bar{J} = \int_0^{\pi/3} \cos(x) \tan(x) dx = \int_0^{\pi/3} \sin(x) dx = \left[ -\cos(x) \right]_0^{\pi/3} = -\frac{1}{2} + 1 = \frac{1}{2}$$

$$\boxed{I = \frac{\ln(2)}{2} - \bar{J} = \frac{\ln(2) - 1}{2}}$$

$\left. \begin{array}{l} \text{can put some/all on board} \\ \sin^2 + \cos^2 = 1 \end{array} \right\} \tan^2 + 1 = \sec^2 \quad (\tan^2)^2 = (\sec^2 - 1)^2$

2. [5 pts] Evaluate the definite integral

$$\int_0^{\pi/4} \tan^5(x) \sec^3(x) dx$$

$$u = \sec(x)$$

$$du = \sec(x) \tan(x) dx$$

$$\int_1^{\sqrt{2}} u^2 (u^2 - 1)^2 du = \int_1^{\sqrt{2}} u^2 (u^4 - 2u^2 + 1) du$$

$$= \int_1^{\sqrt{2}} u^6 - 2u^4 + u^2 du$$

$$= \left( \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} \right) \Big|_1^{\sqrt{2}}$$

$$= \left( u^3 \left( \frac{u^4}{7} - \frac{2u^2}{5} + \frac{1}{3} \right) \right) \Big|_1^{\sqrt{2}}$$

$$= 2\sqrt{2} \left( \frac{4}{7} - \frac{4}{5} + \frac{1}{3} \right) - \left( \frac{1}{7} - \frac{2}{5} + \frac{1}{3} \right)$$

$$= 2\sqrt{2} \left( \frac{60 - 84 + 35}{105} \right) - \left( \frac{15 - 42 + 35}{105} \right) = \boxed{\frac{22\sqrt{2} - 8}{105}}$$

maybe

stop  
after  
here ok?

or  
here?