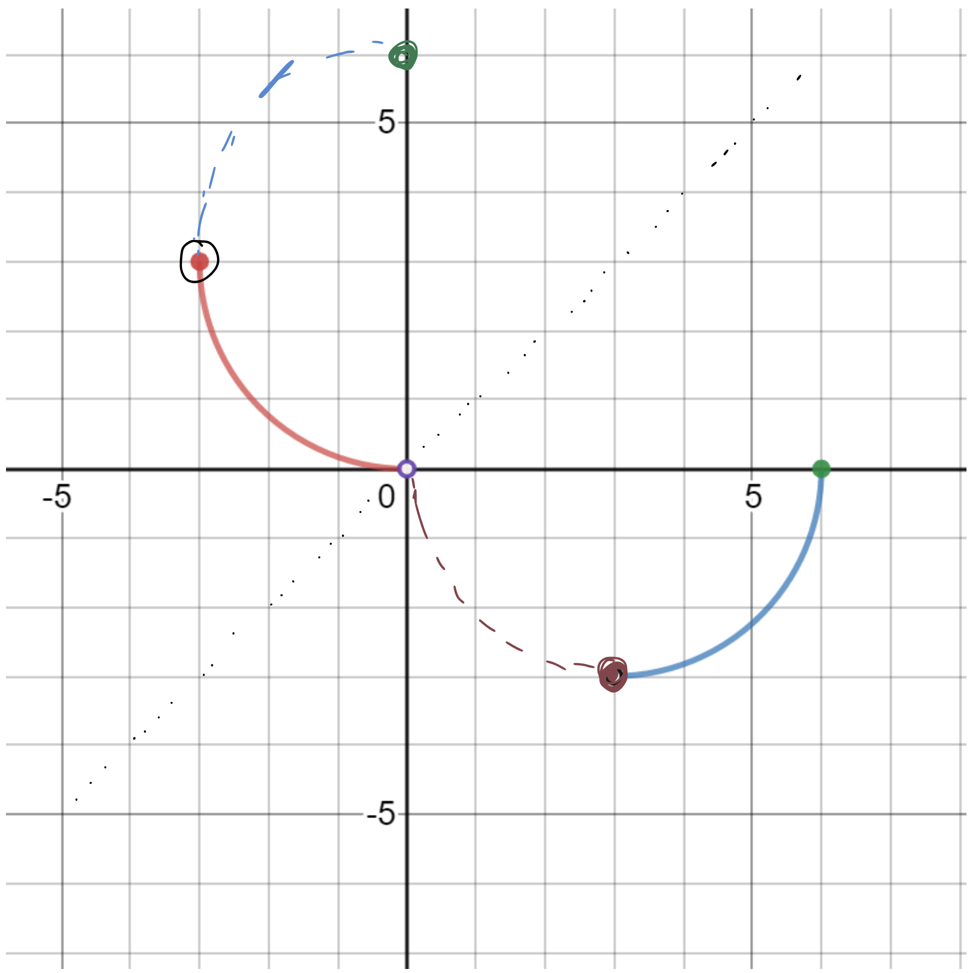
MATH 242 - WS1

01/11/2024

1. Below is the graph of a function $f(x)$. Answer the following questions:



   (a) Is the function 1-to-1? How can you visually tell? yes, it passes Horizontal Line Test

   (b) Graph $f^{-1}(x)$ on the same xy-plane.

   (c) What is the domain of f^{-1}? $\rightsquigarrow$ also range of f $\rightsquigarrow (-3, 3]$

   (d) What is the range or co-domain of f^{-1}? $\hookrightarrow$ also domain of f $\rightsquigarrow [-3, 0) \cup (3, 6]$

   (e) What is $f^{-1}(0)$?

$f^{-1}(0) = 6$ because

$f(6) = 0$

method #1

$y = \frac{3}{1-x}$

$x = \frac{3}{1-y}$

$\frac{1}{x} = \frac{1-y}{3}$

$\frac{3}{x} = 1 - y$

$y = f^{-1}(x) = 1 - \frac{3}{x}$

method #2

2. Let $f(x) = \dfrac{3}{1-x}$. Find $f^{-1}(x)$.

$f : x \overset{1}{\longmapsto} -x \overset{2}{\longmapsto} -x+1 \overset{3}{\longmapsto} \frac{1}{-x+1} \overset{4}{\longmapsto} \frac{3}{-x+1}$

To undo f, undo 4 $\div 3$

undo 3 reciprocate

undo 2 -1

undo 1 $\div -1$

$f^{-1} : x \longmapsto \frac{x}{3} \longmapsto \frac{3}{x} \longmapsto \frac{3}{x} - 1 \longmapsto 1 - \frac{3}{x}$

3. Verify your answer from the previous problem by checking that $f^{-1}(f(x)) = x$.

$f^{-1}(f(x)) = 1 - \frac{3}{f(x)} = 1 - \frac{3}{\frac{3}{1-x}}$

$= 1 - 3\left(\frac{1-x}{3}\right)$

$= 1 - (1-x) = x$ ✓

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method #1

$y = \sqrt[3]{5-2x}$

$x = \sqrt[3]{5-2y}$

$x^3 = 5 - 2y$

$2y = 5 - x^3$

$y = f^{-1}(x) = \frac{5-x^3}{2}$

method #2

4. Let $f(x) = \sqrt[3]{5-2x}$. Find $f^{-1}(x)$.

$f : x \overset{1}{\longmapsto} -2x \overset{2}{\longmapsto} -2x+5 \overset{3}{\longmapsto} \sqrt[3]{-2x+5}$

To undo f, undo 3 $(\cdot)^3$

undo 2 -5

undo 1 $\div -2$

$f^{-1} : x \longmapsto x^3 \longmapsto x^3 - 5 \longmapsto \frac{x^3-5}{-2} = \frac{5-x^3}{2}$

$y = \frac{1-x}{x+2} \Rightarrow x = \frac{1-y}{y+2}$

$xy + 2x = 1 - y$

$y(x+1) = 1 - 2x$

Domain is $x \in \mathbb{R}$: $x + 2 \neq 0$

$x \neq -2$

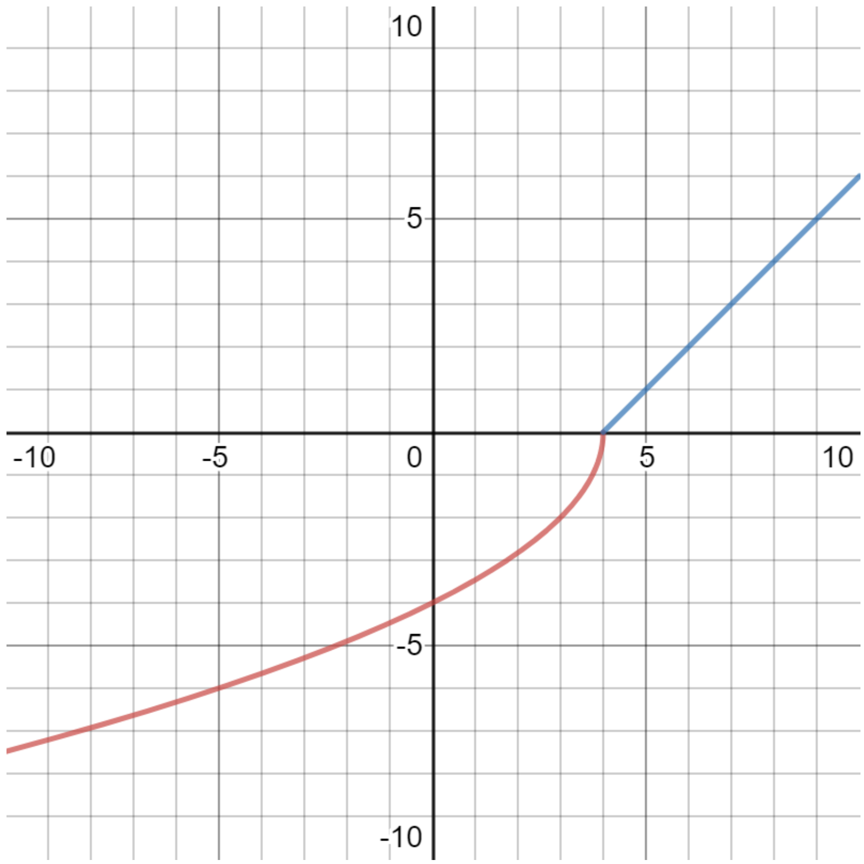
5. Find the domain and range of $f(x) = \dfrac{1-x}{x+2}$.

D. $(-\infty, -2) \cup (-2, \infty)$

Range is f^{-1} domain! (or use $\lim_{x \to \pm\infty} \frac{1-x}{x+2} = -1$)

$f^{-1}(x) = \frac{1-2x}{x+1}$ so R: $(-\infty, -1) \cup (-1, \infty)$

6. Consider this graph of $y = f(x)$ and answer the questions below:



(a) What is $f^{-1}(-4)$? $= 0$ because $f(0) = -4$

(b) What is $f^{-1}(-2)$? $= 3$ because $f(3) = -2$

(c) What is $f^{-1}(5)$? $= 9$ because $f(9) = 5$

7. Show the given functions are indeed inverses functions (i) analytically (by calculating $(f \circ g)(x)$ or $(g \circ f)(x)$) and (ii) graphically.

$f(2) = 2$ $\quad$ $g(2) = 2$

$f(-3) = -1$ $\quad$ $g(-1) = -3$

$$f(x) = \frac{3x+4}{5} \quad \text{and} \quad g(x) = \frac{5x-4}{3}$$

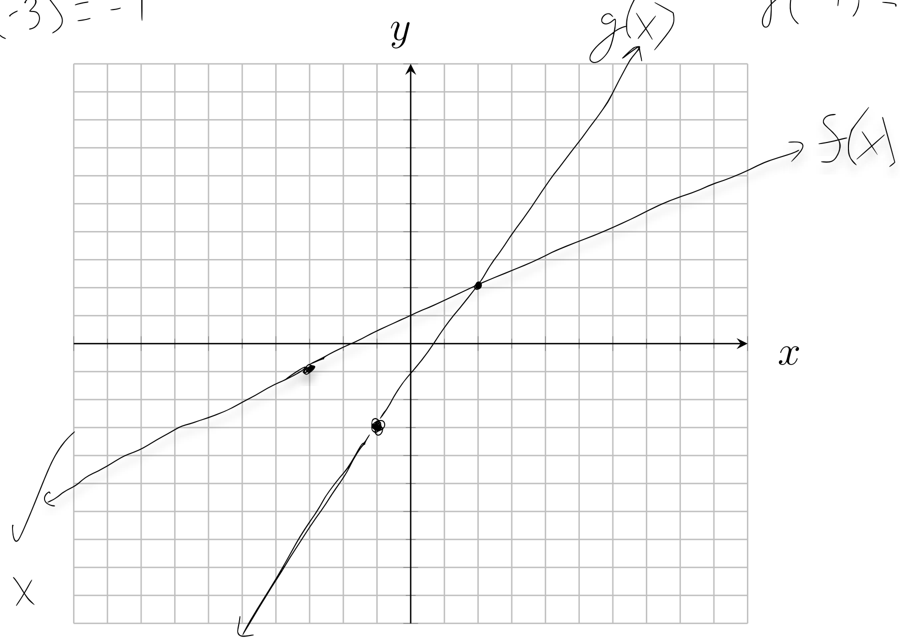
$(f \circ g)(x) =$

$f(g(x)) =$

$\dfrac{3g(x)+4}{5} =$

$\dfrac{3\left(\frac{5x-4}{3}\right)+4}{5} =$

$\dfrac{(5x-4)+4}{5} = \dfrac{5x}{5} = x$ ✓



8. Show the given functions are indeed inverses functions (i) analytically (by calculating $(f \circ g)(x)$ or $(g \circ f)(x)$) and (ii) graphically.

$$f(x) = 2x^2 + 3 \text{ for } x \in [0, \infty) \quad \text{and} \quad g(x) = \sqrt{\frac{x-3}{2}}$$

$(f \circ g)(x) =$

$f(g(x)) =$

$2(g(x))^2 + 3 =$

$2\left(\sqrt{\frac{x-3}{2}}\right)^2 + 3 =$

$2\left(\frac{x-3}{2}\right) + 3$

$= x - 3 + 3 = x$ ✓