$$\frac{1}{n^2+n} = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

MATH 242 - HW8

due: 03/13/2024

1. Sum the series

   (a)

$$\sum_{n=1}^{\infty}\left(\frac{1}{e^n} - \frac{1}{n^2+n}\right)$$

$$\sum_{n=1}^{\infty} \frac{1}{e^n} - \sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$\frac{\frac{1}{e}}{1-\frac{1}{e}} - 1$$

$$\boxed{\frac{1}{e-1} - 1} = \frac{1-(e-1)}{e-1} = \boxed{\frac{2-e}{e-1}}$$

(b)

$$\sum_{n=1}^{\infty} \frac{(2^{n+1})(3^{-n})}{5}$$

$$= \frac{4 \cdot \frac{1}{3}}{5} + \frac{8 \cdot \frac{1}{9}}{5}$$

$$= \frac{\frac{4 \cdot \frac{1}{3}}{5}}{1 - \frac{2}{3}} = \frac{\frac{4}{15}}{1 - \frac{2}{3}} \cdot \frac{15}{15} = \frac{4}{15 - 10} = \boxed{\frac{4}{5}}$$

2. Determine if the integral test for convergence applies (the comparable continuous function $f(x)$ must be continuous, positive, and decreasing for $x \in [1, \infty)$) and then if it does, show either convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$$

3. Find all of the values of x for which the series below converge. Provide you solution in interval notation e.g. $x \in (a, b) \cup (c, d)$:

(a)

$$\sum_{n=0}^{\infty} \frac{2^n}{4x^n}$$

$$\left|\frac{2}{x}\right| < 1$$

$$2 < |x|$$

$$x < -2 \qquad x > 2$$

$$(-\infty, -2) \cup (2, \infty)$$

(b)

$$\sum_{n=0}^{\infty} 5\frac{(x-2)^n}{3^{n+1}}$$

$$\left|\frac{x-2}{3}\right| < 1$$

$$|x-2| < 3$$

$$-3 < x-2 < 3$$

$$-1 < x < 5$$

$$(-1, 5)$$

4. Determine the Convergence/Divergence of the Series using the Direct Comparison Test:

$$\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^3}$$

5. Determine the Convergence/Divergence of the Series using the Limit Comparison Test:

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$$