MATH 242 - Quiz 2

01/25/2024

1. **[4 pts]** Evaluate the definite integral.

$$\int_0^1 x^2 e^{1-x^3}\,dx$$

$u = 1 - x^3$

$du = -3x^2\,dx$

$-\frac{1}{3}du = x^2\,dx$

$$-\frac{1}{3}\int_1^0 e^u\,du = \frac{1}{3}\int_0^1 e^u\,du$$

$$= \frac{1}{3}\left(e^x\Big|_0^1\right) = \boxed{\frac{e-1}{3}}$$

2. **[3 pts]** The equation below describes the Radioactive decay of a substance. If the half-life of the substance is $1,000$ years, determine the value of the constant k.

$$A(t) = A_0 e^{kt}$$

$$\frac{A_0}{2} = A(1,000) = A_0 e^{1000k}$$

$$\frac{1}{2} = e^{1000k}$$

$$\ln\left(\frac{1}{2}\right) = \ln(1) - \ln(2) = -\ln(2) = 1000k$$

$$k = \frac{-\ln(2)}{1000} = \frac{\ln\left(\frac{1}{2}\right)}{1000}$$

3. **[3 pts]** The equation below describes the Radioactive decay of a substance. If the constant $k = -\frac{2}{3}$, find the time required to have only 40 grams remaining after starting initially with 90 grams at $t = 0$.

$$A(t) = A_0 e^{kt}$$

$$40 = 90 e^{-2/3 t}$$

$$\frac{4}{9} = e^{-2/3 t}$$

$$\ln\left(\frac{4}{9}\right) = -\frac{2}{3}t$$

$$t = -\frac{3}{2}\ln\left(\frac{4}{9}\right) = \frac{3}{2}\ln\left(\frac{9}{4}\right) = 3\ln\left(\frac{3}{2}\right) = \ln\left(\frac{27}{8}\right)$$