MATH 242 - Quiz 8

03/28/2024

1. **[5 pts]** Use the Root Test to determine convergence/divergence:

$$\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$$

$$\lim_{n\to\infty}\sqrt[n]{|a_n|}=\lim_{n\to\infty}\left|\left(1+\frac{1}{n}\right)^{n^2}\right|^{1/n}$$

Let $y:=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ $\left(y=e\right)$

>1

diverges

$$\ln(y)=\lim_{n\to\infty}\frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}}$$

$$=\lim_{n\to\infty}\frac{\frac{1}{1+\frac{1}{n}}\cdot-\frac{1}{n^2}}{\frac{-1}{n^2}}=\lim_{n\to\infty}1+\frac{1}{n}=1$$

$$\therefore y=e^{\ln(y)}=e^1=e$$

2. [**5 pts**] Use the Ratio Test to determine convergence/divergence:

$$\sum_{n=1}^{\infty} \frac{n!}{n^{2n}}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{2n+2}} \cdot \frac{n^{2n}}{n!}$$

$$= \lim_{n\to\infty} \frac{(n+1)\,n^{2n}}{(n+1)^{2n+2}}$$

$$= \lim_{n\to\infty} \frac{n^{2n}}{(n+1)^{2n+1}} = 0 < 1$$

converges