MATH 242 - Quiz 5

02/29/2024

1. **[7 pts]** Use the Midpoint Rule with $n = 3$ to approximate the integral:

$$\int_0^1 x^3 dx \quad = \frac{1}{4}$$

For clarity in grading, first fill in the following:

(a) $a = 0$

(b) $b = 1$

(c) $\Delta x = \frac{b-a}{n} = \frac{1-0}{3} = \frac{1}{3}$

(d) the four $x_i = \{ 0, \frac{1}{3}, \frac{2}{3}, 1 \}$

(e) the three midpoints $\bar{x}_i = \{ \frac{1}{6}, \frac{1}{2}, \frac{5}{6} \}$

(f) the three $f(\bar{x}_i) = \{ \frac{1}{216}, \frac{1}{8}, \frac{125}{216} \}$

$$\begin{array}{r} {}^{3} \\ 36 \\ \times\ 6 \\ \hline 216 \end{array}$$

(g) Therefore $\int_0^1 x^3 dx \approx$ $\frac{1}{3}\left[\frac{1}{216} + \frac{1}{8} + \frac{125}{216}\right]$

2. **[3 pts]** The Error associated with the Midpoint Rule is

$$|E_{M_n}| \leq \frac{K(b-a)^3}{24n^2}$$

where $K \geq |f''(x)|$ for all $x \in [a,b]$

(a) What is the appropriate K for the integral above?

$f(x) = x^3$

$f'(x) = 3x^2$

$f''(x) = 6x \Rightarrow K = 6$

(b) What is the worst magnitude error $|E_{M_n}|$ we can expect using $n = 3$? Keep in mind your answer in (1.) may have been closer to the truth, but this provides an upper bound.

$$|E_{M_3}| \leq \frac{6 \cdot 1^3}{24 \cdot 3^2} = \frac{6}{4 \cdot 6 \cdot 9} = \frac{1}{36}$$