MATH 242 - WS10

03/28/2024

1. Find the interval of convergence of the power series:

   (a)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{\sqrt[3]{n}}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \underbrace{\lim_{n\to\infty} \frac{\sqrt[3]{n}}{\sqrt[3]{n+1}}}_{=1} |x| \overset{?}{<} 1$$

$$x \in (-1, 1)$$

@ $x=1$ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n}}$ converges!

@ $x=-1$ $\sum_{n=1}^{\infty} \frac{(-1)^{2n+1}}{\sqrt[3]{n}}$ diverges

I.O.C

$$\boxed{(-1, 1]}$$

(b)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n 5^n}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \underbrace{\lim_{n\to\infty} \frac{n 5^n}{(n+1) 5^{n+1}}}_{= 1/5} |x| \overset{?}{<} 1$$

$$|x| < 5$$

$$x \in (-5, 5)$$

@ $x = 5$ $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ converges!

@ $x = -5$ $\sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{n}$ diverges

I.O.C

$$\boxed{(-5, 5]}$$

(c)

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{2n+2}}{(n+1)!} \cdot \frac{n!}{x^{2n}}\right|$$

$$= \lim_{n\to\infty} \frac{1}{n} |x|^2 = 0$$

I.O.C

$$\boxed{(-\infty, \infty)}$$