MATH 242 - Quiz 7

03/14/2024

1. **[5 pts]** Find S the sum of the series:

$$S = \sum_{n=1}^{\infty} \left(\frac{12}{n^2 + 2n} \right)$$

$$\frac{12}{n^2+2n} = \frac{12}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$$

$$\Rightarrow 12 = A(n+2) + Bn$$

$$\Rightarrow A = -B$$

$$12 = 2A$$

$$\Rightarrow A = 6$$

$$B = -6$$

$$S = \sum_{n=1}^{\infty} \frac{6}{n} - \frac{6}{n+2}$$

$$= 6 \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+2}$$

telescoping

$$= 1 + \frac{1}{2} = \frac{3}{2}$$

$$\Rightarrow 6 - \frac{3}{2} = \boxed{9}$$

$6S = \sum_{n=1}^{\infty} \frac{3^{n-1}}{4^n} = \sum_{i=0}^{\infty} \frac{1}{4}\left(\frac{3}{4}\right)^i$ or easier $\frac{\frac{1}{4}}{1-\frac{3}{4}}$

$\frac{1}{6}\left(\frac{3^0}{2^2}\right) + \frac{1}{6}\left(\frac{3^1}{2^4}\right)$

2. [**5 pts**] Find S the sum of the series:

$$S = \sum_{n=1}^{\infty} \frac{1}{6}\left(\frac{3^{n-1}}{2^{2n}}\right) = \frac{1}{24} + \frac{3}{96} + \cdots$$

$a = \frac{1}{24}$

$r = \frac{3}{4}$

$= \frac{a}{1-r}$

$= \frac{\frac{1}{24}}{1-\frac{3}{4}} \cdot \frac{24}{24}$

$= \frac{1}{24-18} = \boxed{\frac{1}{6}}$