MATH 242 - HW10

due: 04/03/2024

1. Find the interval of convergence of the power series:

   (a)
$$\sum_{n=1}^{\infty} \frac{(-1)^n(x-1)^n}{2^n(2n-1)}$$

$2(n+1)-1$

$2n+2-1$

$2n+1$

$$\lim_{n\to\infty}\left|\frac{\dfrac{(-1)^{n+1}(x-1)^{n+1}}{2^{n+1}(2n+1)}}{\dfrac{(-1)^n(x-1)^n}{2^n(2n-1)}}\right|$$

$$=\lim_{n\to\infty}\left(\frac{1}{2^{n+1}(2n+1)}\cdot 2^n(2n-1)\right)|x-1|<1$$

$$=\underbrace{\lim_{n\to\infty}\frac{2n-1}{4n+2}}_{=\frac{1}{2}}|x-1|<1$$

$|x-1|<2$

$x\in(-1,3)$

@ $x=3$ $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}$ converges!

@ $x=-1$ $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2n-1}$ diverges

$$\boxed{x\in(-1,3]}$$

(b)

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{2^n \ln(n)}$$

$$\lim_{n\to\infty} \left| \frac{\frac{(x+2)^{n+1}}{2^{n+1}\ln(n+1)}}{\frac{(x+2)^n}{2^n \ln(n)}} \right| = \lim_{n\to\infty} \frac{\ln(n)}{2\ln(n+1)} |x+2|$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{2}{n+1}}$$

$$= \lim_{n\to\infty} \underbrace{\frac{n+1}{2n}}_{\frac{1}{2}}$$

$|x+2| < 2$

$x \in (-4, 0)$

@ $x = -4$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n)}$ Converges!

@ $x = 0$ $\sum_{n=1}^{\infty} \frac{1}{\ln(n)}$ diverges

$$\boxed{x \in [-4, 0)}$$

(c)

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n}(x+6)^n$$

$$\lim_{n\to\infty} \left| \frac{\frac{\sqrt{(n+1)}}{8^{n+1}}(x+6)^{n+1}}{\frac{\sqrt{n}}{8^n}(x+6)^n} \right| = \lim_{n\to\infty} \frac{\sqrt{n+1}}{8\sqrt{n}} |x+6| < 1$$

$$= \lim_{n\to\infty} \underbrace{\frac{1}{8}\sqrt{\frac{n+1}{n}}}_{=\frac{1}{8}}$$

$|x+6| < 8$

$$\boxed{x \in (-14, 2)}$$

@ $x = -14$ diverges

$\sum \sqrt{n}(-1)^n$

@ $x = 2$ $\sum \sqrt{n}$ diverges

(d)

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$$

$$\lim_{n\to\infty}\left|\frac{\frac{(x-2)^{n+1}}{(n+1)^{n+1}}}{\frac{(x-2)^n}{n^n}}\right| = \underbrace{\lim_{n\to\infty} \frac{n^n}{(n+1)^{n+1}}}_{=0} |x-2| < 1$$

$$\boxed{(-\infty, \infty)}$$

(e)

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n\sqrt{n}}$$

$$\lim_{n\to\infty}\left|\frac{\frac{(2x-1)^{n+1}}{5^{n+1}\sqrt{n+1}}}{\frac{(2x-1)^n}{5^n\sqrt{n}}}\right| = \lim \frac{\sqrt{n}}{5\sqrt{n+1}}|2x-1| = \lim \frac{\sqrt{n}}{\underbrace{5\sqrt{n}\sqrt{1+\frac{1}{n}}}}|2x-1| = \frac{1}{5}|2x-1|$$

$$|2x-1| < 5$$

$$-4 < 2x < 6$$

$$x \in (-2, 3)$$

@ $x=-2$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges!

@ $x=3$ $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges

$$\boxed{x \in [-2, 3)}$$