MATH 242 - WS7

02/29/2024

1. Calculate each improper integral.

   (a)

$$I = \int_0^9 \frac{1}{\sqrt[3]{x-1}}dx = \underbrace{\int_0^1 \frac{1}{\sqrt[3]{x-1}}dx}_{J_1} + \underbrace{\int_1^9 \frac{1}{\sqrt[3]{x-1}}dx}_{J_2}$$

$$J_1 = \lim_{t \to 1^-} \int_0^t \frac{1}{\sqrt[3]{x-1}}dx = \lim_{t \to 1^-} \left(\frac{3}{2}(x-1)^{2/3}\right)\Bigg|_0^t$$

$$= \frac{3}{2}(1-1)^{2/3} - \frac{3}{2}(0-1)^{2/3} = -\frac{3}{2}$$

$$J_2 = \lim_{t \to 1^+} \int_t^9 \frac{1}{\sqrt[3]{x-1}}dx = \lim_{t \to 1^+} \left(\frac{3}{2}(x-1)^{2/3}\right)\Bigg|_t^9$$

$$= \frac{3}{2}(9-1)^{2/3} - \frac{3}{2}(1-1)^{2/3} = 6$$

$$\therefore I = J_1 + J_2 = -\frac{3}{2} + 6 = \boxed{\frac{9}{2}}$$

(b)

$$\int_1^3 \frac{1}{\sqrt{3-x}}dx$$

$$= \lim_{t\to 3^-}\int_1^t \frac{1}{\sqrt{3-x}} = \lim_{t\to 3^-}\left(-2\sqrt{3-x}\right|_1^t$$

$$= -2\sqrt{3-3} + 2\sqrt{3-1}$$

$$= \boxed{2\sqrt{2}}$$

(c)

$$\int_0^\infty 4x^2e^{-x^3}dx$$

$$u = -x^3$$

$$-\frac{4}{3}du = -\frac{4}{3}\left(-3x^2dx\right)$$

$$= 4x^2dx$$

$$= -\frac{4}{3}\int_0^{-\infty} e^u du$$

$$= \frac{4}{3}\int_{-\infty}^0 e^u du = \lim_{t\to-\infty}\frac{4}{3}\left(e^u\right|_t^0$$

$$= \frac{4}{3} - \lim_{t\to-\infty} e^t = \boxed{\frac{4}{3}}$$

(d)

$$\int_{-\infty}^{1} xe^{4x}dx$$

$u = x \quad dv = e^{4x}dx$

$du = dx \quad v = \frac{1}{4}e^{4x}$

$$= \left(\frac{x}{4}e^{4x}\right|_{-\infty}^{1} - \frac{1}{4}\int_{-\infty}^{1} e^{4x}dx$$

$$= \left(\frac{x}{4}e^{4x} - \frac{1}{16}e^{4x}\right|_{-\infty}^{1}$$

$$= \frac{e^4}{4} - \frac{e^4}{16} - \lim_{t\to-\infty} \frac{4t-1}{16e^{-4t}} \left(\text{"}= \frac{-\infty}{\infty}\text{"}\right)$$

$$= \frac{3e^4}{16} - \underbrace{\lim_{t\to-\infty} \frac{4}{-4\cdot 16e^{-4t}}}_{=0}$$

$$= \boxed{\frac{3e^4}{16}}$$