MATH 242 - WS9

03/14/2024

1. Test the series for convergence/divergence:

   (a)
$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}}$$

converges by AST

☑ $\lim_{n\to\infty} \frac{1}{\sqrt{n+1}} = 0$

☑ $\frac{1}{\sqrt{n+2}} \leq \frac{1}{\sqrt{n+1}}$ (since $\sqrt{n+2} \geq \sqrt{n+1}$)

$(a_{n+1} \leq a_n)$

(b)

$$\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{2n \text{[redacted]}}$$

converges by AST

$\boxed{\checkmark}$ $\lim_{n\to\infty} \frac{\sqrt{n}}{2n} = \lim_{n\to\infty} \frac{\frac{1}{2\sqrt{n}}}{2} = 0$ ✓

$\boxed{\checkmark}$ $\frac{\sqrt{n+1}}{2n+2} \overset{?}{\leq} \frac{\sqrt{n}}{2n}$ @ $n=1$: "$\overset{\text{LHS}}{\frac{\sqrt{2}}{4}} < \frac{2}{4} = \overset{\text{RHS}}{\frac{1}{2}}$"

so it holds!

(and continues to as function $f(x) = \frac{\sqrt{x}}{2x}$ is monotone as $f'(x) = -\frac{1}{4\sqrt{x^3}} < 0$, $x \geq 1$)

(c)

$$\sum_{n=1}^{\infty} (-1)^{n-1} \tan^{-1}(n)$$

$\boxed{\times}$ $\lim_{n\to\infty} \tan^{-1}(n) = \frac{\pi}{2}$

Diverges by test for Divergence

(d)

$$\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{\pi}{n}\right)$$

$\boxed{\times}$ $\lim_{n\to\infty} \cos\left(\frac{\pi}{n}\right) = \cos(0) = 1$

Diverges by test for Divergence

(e)

$$\sum_{n=1}^{\infty} \frac{n\cos(n\pi)}{2^n} = \sum_{n=1}^{\infty} \frac{(-1)^n n}{2^n}$$

converges by AST

$\boxed{\checkmark}$ $\lim_{n\to\infty} \frac{n}{2^n} = \lim_{n\to\infty} \frac{1}{\ln(2)2^n} = 0$

$\boxed{\checkmark}$ $\frac{n+1}{2^{n+1}} \overset{?}{\leq} \frac{n}{2^n}$ @ $n=1$: $\frac{2}{2^2} = \frac{1}{2}$ vs $\frac{1}{2}$ ✓

(and Keeps holding as $f(x) = \frac{x}{2^x}$ has $f'(x) = \frac{1 - x\ln(2)}{2^x} < 0$ for $x \geq 1$)