MATH 242 - Quiz 4 REMIX V2

04/04/2024

1. **[5 pts]** Evaluate the definite integral via Integration by Parts:

$$\int_1^2 \sqrt{x}\ln(x)dx$$

$u = \ln(x)$ | $dv = x^{1/2}dx$

$du = \frac{1}{x}dx$ | $v = \frac{2}{3}x^{3/2}$

$$= \left(\frac{2}{3}x^{3/2}\ln(x)\right|_1^2 - \frac{2}{3}\int_1^2 \sqrt{x}\,dx$$

$$= \frac{2}{3}\sqrt{8}\ln(2) - \left(\frac{2}{3}\cdot\frac{2}{3}x^{3/2}\right|_1^2$$

$$= \frac{2}{3}\sqrt{8}\ln(2) - \frac{4}{9}\sqrt{8} + \frac{4}{9}$$

$$= \frac{4}{9}\left(1 - 2\sqrt{2} + 3\sqrt{2}\ln(2)\right)$$

$\sin^2 + \cos^2 = 1$

$1 + \tan^2 = \sec^2$

2. **[5 pts]** Evaluate the definite integral

$$\int_0^{\pi/4} \tan^3(x)\sec^3(x)dx$$

$u = \sec(x)$

$du = \sec(x)\tan(x)$

$$\int (\sec^2(x) - 1)\sec^2(x)\underbrace{\sec\tan dx}$$

$(u^2 - 1) \quad u^2 \quad du$

$$\int u^4 - u^2\, du$$

$$\frac{u^5}{5} - \frac{u^3}{3}$$

$$\left(\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}\right)\Bigg|_0^{\frac{\pi}{4}}$$

$$\left(\frac{(\sqrt{2})^5}{5} - \frac{(\sqrt{2})^3}{3}\right) - \left(\frac{1}{5} - \frac{1}{3}\right)$$

$$= \frac{4\sqrt{2} - 1}{5} + \frac{1 - 2\sqrt{2}}{3} = \frac{12\sqrt{2} - 3 + 5 - 10\sqrt{2}}{15}$$

$$= \frac{2\sqrt{2} + 2}{15} = \boxed{\frac{2}{15}(\sqrt{2} + 1)}$$