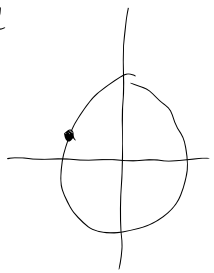
MATH 242 - Quiz 3

02/01/2024

1. **[4 pts]** True or False?

   (a) If $\sin(x) = \frac{1}{2}$ then $\cos(x) = \frac{\sqrt{3}}{2}$ False e.g. $x = \frac{5\pi}{6}$

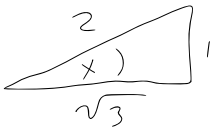
   then $\sin(x) = \frac{1}{2}$ but $\cos(x) = -\frac{\sqrt{3}}{2}$



   (b) If $\sin(x) = \frac{1}{2}$ then $x = \sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$ False e.g. $x = \frac{5\pi}{6}$

   then $\sin(x) = \frac{1}{2}$ but $x \neq \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

   (c) If $\sin(x) = \frac{1}{2}$ then $\sec(x) = 2$ False



$\sec(x) = \frac{2}{\sqrt{3}}$

   (d) If $\sin^{-1}(x) = \frac{\pi}{4}$ then $x = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$ True

$\frac{\sqrt{2}}{2} = \sin\left(\frac{\pi}{4}\right) = \sin\left(\sin^{-1}(x)\right) = x$

2. **[3 pts]** Evaluate the limit:

$$\lim_{x\to 0} \frac{x^4 - x^2}{1 - \cos(x)} \qquad \left(= \frac{0}{0}\right)$$

$$\overset{\text{L'H}}{=} \lim_{x\to 0} \frac{4x^3 - 2x}{\sin(x)} \qquad \left(= \frac{0}{0}\right)$$

$$\overset{\text{L'H}}{=} \lim_{x\to 0} \frac{12x^2 - 2}{\cos(x)} = \frac{0-2}{1} = \boxed{-2}$$

3. **[3 pts]** Evaluate the limit:

$$\lim_{x\to\infty} (\ln(x) - \ln(x+1))$$

$$= y = \lim_{x\to\infty} \ln\left(\frac{x}{x+1}\right)$$

$$\Rightarrow e^y = \lim_{x\to\infty} \frac{x}{x+1} = 1$$

$$\Rightarrow y = \ln(e^y) = \ln(1) = \boxed{0}$$