MATH 242 - Quiz 1 REMIX V2

04/04/2024

1. **[3 pts]** Consider $f : \mathbb{R} \to \mathbb{R}$ given by $x \mapsto f(x) = 2x^4$.

   (a) Notice the function $f(x)$ is NOT onto as it is written. Explain why by providing a new, different co-domain other than $\mathbb{R}$ that would make $f(x)$ an onto function.

   $[0, \infty)$

   (b) Notice the function $f(x)$ is NOT one-to-one. Explain why by providing a new, different domain other than $\mathbb{R}$ that would make $f(x)$ a one-to-one function.

   $f^{-1}(x) = \sqrt[4]{\frac{x}{2}}$

   $[0, \infty)$

   (c) By the previous two questions you have made f invertible (as it is now onto and one-to-one). Consider the rule for its inverse function $f^{-1}(x)$ and tell me what is $f^{-1}(2) =?$

   1

   $f(0) = 2$ so $f^{-1}(2) = 0$

2. **[3 pts]** Let $f(x) = -x^7 + \frac{x^3}{3} - 3x + 2$. Without computing the inverse function directly, compute the derivative of the inverse function $(f^{-1})'(2)$.

   $(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$

   $f' = -7x^6 + x^2 - 3$

   $f'(f^{-1}(2)) =$

   $f'(0) = -3$

   $= -\frac{1}{3}$

3. **[4 pts]** Use "Logarithmic Differentiation" to find the derivative $h'(x)$ given that

$$h(x) = \frac{(2x^3 - 6)^5 \cos(2x^3)}{(2x^5 - 6x)^4}$$

$$\ln(h) = 5\ln(2x^3 - 6) + \ln(\cos(2x^3)) - 4\ln(2x^5 - 6x)$$

$$\frac{h'}{h} = 5 \cdot \frac{1}{2x^3 - 6} \cdot 6x^2 + \frac{1}{\cos(2x^3)} \cdot -\sin(2x^3) \cdot 6x^2 - 4 \cdot \frac{1}{2x^5 - 6x} \cdot 10x^4 \cdot 6$$

$$h' = \frac{(2x^3 - 6)^5 \cos(2x^3)}{(2x^5 - 6x)^4} \left(\frac{30x^2}{2x^3 - 6} - 6x^2 \tan(2x^3) - \frac{4(10x^4 - 6)}{2x^5 - 6x} \right)$$