MATH 242 - WS5

02/08/2024

1. Evaluate each integral:

   (a) $\int \frac{e^x}{1+16e^{2x}}dx$

$u = 4e^x \Rightarrow u^2 = 16e^{2x}$

$\frac{1}{4}du = e^x dx$

$\frac{1}{4}\int \frac{1}{1+u^2}du = \frac{1}{4}\tan^{-1}(u) + C$

$= \frac{1}{4}\tan^{-1}(4e^x) + C$

   (b) $\int \frac{1}{\sqrt{9-4x^2}}dx$

for $\theta \in (\frac{-\pi}{2}, \frac{\pi}{2})$

$x = \frac{3}{2}\sin(\theta) \Rightarrow \sqrt{9-4(\frac{3}{2}\sin(\theta))^2} = 3\cos(\theta)$

$dx = \frac{3}{2}\cos(\theta)d\theta$

$= \int \frac{1}{3\cos(\theta)} \cdot \frac{3}{2}\cos(\theta)d\theta = \frac{1}{2}\int d\theta = \frac{\theta}{2} + C = \frac{1}{2}\sin^{-1}(\frac{2x}{3}) + C$

(c) $\int \frac{1}{x^2(x-1)}dx$

$$\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$$\Rightarrow 1 = Ax(x-1) + B(x-1) + Cx^2$$
$$= Ax^2 - Ax + Bx - B + Cx^2$$
$$= x^2(A+C) + x(B-A) - B$$

$$\therefore \quad 0 = A + C$$
$$0 = B - A$$
$$1 = -B \Rightarrow B = -1 \Rightarrow A = -1 \Rightarrow C = 1$$

$$\int \frac{1}{x^2(x-1)}dx = \int \left(\frac{1}{x-1} - \frac{1}{x^2} - \frac{1}{x}\right)dx$$
$$= \ln|x-1| + \frac{1}{x} - \ln|x| + C$$

Check: $\frac{1}{x-1} - \frac{1}{x^2} - \frac{1}{x} = \frac{x^2 - (x-1) - x(x-1)}{x^2(x-1)} = \frac{1}{x^2(x-1)}$ ✓