$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} \; !$$

If $f^{-1}(x) = \sin^{-1}(x)$, $f(x) = \sin(x)$, $f'(x) = \cos(x)$

$$\Rightarrow (f^{-1})'(x) = \frac{1}{\cos(\sin^{-1}(x))} = \frac{1}{\sqrt{1-\sin^2(\sin^{-1}(x))}} = \frac{1}{\sqrt{1-x^2}}$$

If $f^{-1}(x) = \cos^{-1}(x)$

$$\Rightarrow (f^{-1})'(x) = -\frac{1}{\sqrt{1-x^2}}$$

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If $f^{-1}(x) = \tan^{-1}(x)$

$$\Rightarrow (f^{-1})'(x) = \frac{1}{1+x^2}$$

$f^{-1}(x) = \cot^{-1}(x)$

$$\Rightarrow (f^{-1})'(x) = \frac{-1}{1+x^2}$$

01/25/2024

1. Find the derivative of each function

need generally, to use log. diff. or something similar to see that

$$\frac{d}{dx}\left[f(x)^{g(x)}\right] = \frac{d}{dx}\left[e^{g(x)\ln(f(x))}\right] = f(x)^{g(x)}\left(g'(x)\ln(f(x)) + \frac{g(x)f'(x)}{f(x)}\right)$$

(a) $f(x) = (\sin(2x))^{\sqrt{x}}$

$$f'(x) = (\sin(2x))^{\sqrt{x}}\left(\frac{\ln(\sin(2x))}{2\sqrt{x}} + \sqrt{x}\,\frac{2\cos(2x)}{\sin(2x)}\right)$$

$$\boxed{= (\sin(2x))^{\sqrt{x}}\left(\frac{\ln(\sin(2x))}{2\sqrt{x}} + 2\sqrt{x}\cot(2x)\right)}$$

(b) $g(x) = (\sin^{-1}(x^3 + 2x - 3))^2$

$$g'(x) = \boxed{2\sin^{-1}(x^3+2x-3) \cdot \frac{1}{\sqrt{1-(x^3+2x-3)^2}} \cdot (3x^2+2)}$$

(c) $3x^3 \sec^{-1}(\sqrt{x^2 + 1})$

$$h'(x) = 9x^2\sec^{-1}(\sqrt{x^2+1}) + 3x^3 \cdot \frac{1}{|x|\sqrt{x^2+1}} \cdot \frac{x}{\sqrt{x^2+1}}$$

$$\boxed{= 9x^2\sec^{-1}(\sqrt{x^2+1}) + \frac{3x^2|x|}{x^2+1}}$$

If $f^{-1}(x) = \csc^{-1}(x)$

$$\Rightarrow (f^{-1})'(x) = -\frac{1}{x\sqrt{x^2-1}}$$

If $f^{-1}(x) = \sec^{-1}(x)$, $f(x) = \sec(x)$, $f'(x) = \sec(x)\tan(x)$ $\Rightarrow$

$$(f^{-1})'(x) = \frac{1}{\sec(\sec^{-1}(x))\tan(\sec^{-1}(x))} = \frac{1}{x\sqrt{\sec^2(\sec^{-1}(x))-1}} = \frac{1}{x\sqrt{x^2-1}}$$

1

$u = x/a$

$a\,du = dx$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C, \quad \underbrace{\int \frac{1}{a^2+x^2}\,dx}_{(*)} = \frac{1}{a^2}\int \frac{1}{1+\left(\frac{x}{a}\right)^2}\,dx = \frac{a}{a^2}\int \frac{1}{1+u^2}\,du = \underline{\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C}$$

2. Evaluate each integral using an appropriate substitution:

(a) $\int \frac{3x}{x^4+25}dx$

$u = x^2$

$du = 2x\,dx$

$\frac{3}{2}du = 3x\,dx$

$$\frac{3}{2}\int \frac{1}{5^2+u^2}\,du \overset{*}{=} \frac{3}{2}\cdot\frac{1}{5}\tan^{-1}\left(\frac{u}{5}\right) + C$$

$$= \boxed{\frac{3\tan^{-1}\left(\frac{x^2}{5}\right)}{10} + C}$$

(b) $\int \frac{4e^{3x}}{\sqrt{1-e^{6x}}}dx$

$u = e^{3x}$

$du = 3e^{3x}dx$

$4/3\,du = 4e^{3x}dx$

$$\frac{4}{3}\int \frac{1}{\sqrt{1-u^2}}\,du = \frac{4}{3}\sin^{-1}(u) + C = \boxed{\frac{4}{3}\sin^{-1}\left(e^{3x}\right) + C}$$

removed