MATH 242 - Quiz 5 REMIX

04/04/2024

1. **[7 pts]** Use the Midpoint Rule with $n = 6$ to approximate the integral:

$$\int_0^{12} \frac{x^3}{3} dx$$

For clarity in grading, first fill in the following:

(a) $a =$ 0

(b) $b =$ 12

(c) $\Delta x = \frac{12-0}{6} = 2$

(d) the seven ~~four~~ $x_i = \{$ 0, 2, 4, 6 $\}$ 8 10 12

(e) the six ~~three~~ midpoints $\bar{x}_i = \{$ 1, 3, 5 $\}$, 7, 9, 11

(f) the six ~~three~~ $f(\bar{x}_i) = \{$ $\frac{1}{3}$, 9, $\frac{125}{3}$, $\frac{7^3}{3}$, $\frac{9^3}{3}$, $\frac{11^3}{3}$

(g) Therefore $\int_0^{12} \frac{x^3}{3} dx \approx$

$$2\left(\frac{1}{3}+9+\frac{125}{3}+\frac{7^3}{3}+\frac{9^3}{3}+\frac{11^3}{3}\right)$$

2. **[3 pts]** The Error associated with the Midpoint Rule is

$$|E_{M_n}| \leq \frac{K(b-a)^3}{24n^2}$$

where $K \geq |f''(x)|$ for all $x \in [a,b]$

(a) What is the appropriate K for the integral above?

$$\frac{x^3}{3} \overset{\frac{d}{dx}}{\mapsto} x^2 \mapsto 2x \Rightarrow K = 24$$

(b) What is the worst magnitude error $|E_{M_n}|$ we can expect using $n = 6$? Keep in mind your answer in (1.) may have been closer to the truth, but this provides an upper bound.

$$\frac{24 \cdot 12^3}{24 \cdot 6^2} = \frac{12 \cdot 12 \cdot 12}{6 \cdot 6} = \boxed{48}$$