MATH 242 - Quiz 6

03/07/2024

1. [**3 pts**] List the first three terms a_1, a_2, a_3 of the following sequence

$$\left\{ (-1)^{n+1} \frac{3}{n^2+1} \right\}_{n=1}^{\infty}$$

$$a_1 = (-1)^{1+1} \frac{3}{1^2+1} = (-1)^2 \cdot \frac{3}{1+1} = \boxed{\frac{3}{2}}$$

$$a_2 = (-1)^{2+1} \frac{3}{2^2+1} = (-1)^3 \frac{3}{4+1} = \boxed{-\frac{3}{5}}$$

$$a_3 = (-1)^{3+1} \frac{3}{3^2+1} = (-1)^4 \frac{3}{9+1} = \boxed{\frac{3}{10}}$$

2. [**3 pts**] Find a formula for the general nth term of the sequence:

$$\left\{ \frac{1}{3}, -\frac{2}{5}, \frac{3}{7}, -\frac{4}{9}, \frac{5}{11}, \dots \right\}$$

$$\boxed{a_n = \frac{(-1)^{n+1} n}{2n+1}} \quad \left(\text{for } n \geq 1 \right)$$

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3. **[4 pts]** Prove the sequence is monotone. (Hint: consider $f(x) = \frac{3x}{2x+5}$):

$$a_n = \frac{3n}{2n+5}$$

$$f(x) = \frac{3x}{2x+5}$$

$$f'(x) = \frac{3(2x+5) - 2(3x)}{(2x+5)^2}$$

$$= \frac{5}{(2x+5)^2} > 0 \text{ for } x \geq 1$$

hence $\quad f(x+1) > f(x) \ \forall x \geq 1$

$$f(n+1) > f(n)$$

$$\frac{3(n+1)}{2(n+1)+5} > \frac{3n}{2n+5}$$

$$a_{n+1} > a_n$$