MATH 242 - Quiz 1 REMIX

02/15/2024

1. **[3 pts]** Consider $g : \mathbb{R} \to \mathbb{R}$ given by the rule $x \mapsto g(x) = \sin(x)$.

   (a) Notice the function $g(x)$ is NOT onto. Explain why by providing a new, different co-domain other than $\mathbb{R}$ that would make $g(x)$ an onto function.

   $[-1, 1]$

   (b) Notice the function $g(x)$ is NOT one-to-one. Explain why by providing a new, different domain other than $\mathbb{R}$ that would make $g(x)$ a one-to-one function.

   $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

   (c) By the previous two questions you have made g invertible (as it is now onto and one-to-one). Consider the rule for its inverse function $g^{-1}(x)$ and tell me what is $g^{-1}(-\frac{1}{2}) =?$

   $-\frac{\pi}{6}$

2. **[3 pts]** Let $f(x) = -2x^5 - \frac{x^3}{3} - 3x + 1$. Without computing the inverse function directly, compute the derivative of the inverse function $(f^{-1})'(+1)$.

$f'(x) = -10x^4 - x - 3$

$f^{-1}(1) = 0$ (since $f(0) = 1$)

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(0)} = -\frac{1}{3}$$

3. **[4 pts]** Use "Logarithmic Differentiation" to find the derivative $h'(x)$ given that

$$h(x) = \frac{(3x^2 - 5)^6 \sin(2x^3)}{(4x^5 + 5x)^4}$$

$$\ln\left(h(x)\right) = 6\ln\left(3x^2 - 5\right) + \ln\left(\sin(2x^3)\right) - 4\ln\left(4x^5 + 5x\right)$$

$$\frac{h'(x)}{h(x)} = \frac{36x}{3x^2 - 5} + 6x^2\cot(2x^3) - \frac{80x^4 + 20}{4x^5 + 5x}$$

$$h'(x) = \frac{(3x^2 - 5)^6 \sin(2x^3)}{(4x^5 + 5x)^4}\left(\frac{36x}{3x^2 - 5} + 6x^2\cot(2x^3) - \frac{80x^4 + 20}{4x^5 + 5x}\right)$$