$$3^4 = 81 \Leftrightarrow 4 = \log_3 81$$

$$a^x = N \Leftrightarrow x = \log_a N$$

$$N > 0 : a > 0 : a \neq 1$$

$$(0.1)^3 = 0.001 \Leftrightarrow \log_{0.1} 0.001 = 3$$

$$10^1 = 10 \Leftrightarrow \log_{10} 10 = 1$$

$$\log_a a = 1$$

$$\log_a a = 1$$

$$a^0 = 1 \iff \log_a 1 = 0 \quad (a > 0 ; a \neq 1)$$

$$a^{-1} = \frac{1}{a} \iff \log_a (1/a) = -1$$

$$\log_{\frac{1}{a}} a = -1$$

1) $\log_a m + \log_a n = \log_a mn$

$m, n > 0$

2) $\log_a m - \log_a n = \log_a \left(\frac{m}{n}\right)$

3) $a^{\log_a m} = m$

$\Leftrightarrow \log_a m = \log_a m$

4) $\log_a x^m = m \cdot \log_a x$

$\log_a x$

$\boxed{x > 0}$

5) $\log_{a^n} x = \frac{1}{n} \log_a x$

6) $\log_{a^n} x^m = \frac{m}{n} \log_a x$

1) If $\log_4 m = 1.5 \Rightarrow m =$ ____.

$\log_4 m = 1.5$

$\Leftrightarrow \quad m = (4)^{\frac{3}{2}}$

$= (2^{\cancel{2}})^{3/\cancel{2}} = 2^3 = \boxed{8}$

2) $\log_{2\sqrt{3}} 1728 = x \Rightarrow x =$ ____

$1728 = (2\sqrt{3})^x$

$(12)^3 = (2\sqrt{3})^x$.

$(2^2 \cdot 3)^3 = (2\sqrt{3})^x$

$2^6 = 2^x \Rightarrow \boxed{x = 6}$

1). $\log_{1.4\bar{3}} \frac{43}{30} = \log_{1.4\bar{3}} 1.4\bar{3} = 1$

$$30 \overline{) 43} \left(1.4\bar{3}\ldots \right.$$

$$\begin{array}{r} 30 \\ \hline 130 \\ 120 \\ \hline 100 \\ 90 \\ \hline 10 \end{array}$$

2). $4^{\log_2 2x} = 36. \Rightarrow x = \_\_\_\_.$

$(2^2)^{\log_2 2x} = 36$

$2^{2\log_2 (2x)} = 36$

$2^{\log_2 4x^2} = 36$

$4x^2 = 36 \Rightarrow x^2 = 9 \Rightarrow \boxed{x = \pm 3}$

$(a^m)^n = a^{mn}$

$$\boxed{a^{\log_a N} = N}$$

$$\boxed{m\log_a x = \log_a x^m}$$

If $a^2+b^2=23ab$ then $\log\left(\frac{a+b}{5}\right)=$ ______

Sol: $a^2+b^2+2ab = 25ab$

$$\frac{(a+b)^2}{5^2} = ab.$$

$$\left(\frac{a+b}{5}\right)^2 = ab$$

$$2\log\left(\frac{a+b}{5}\right) = \log a + \log b.$$

$$\log\left(\frac{a+b}{5}\right) = \frac{1}{2}\left[\log a + \log b\right]$$

$$a^2 + b^2 = 23ab$$

$$\log\left(\frac{a+b}{5}\right) = \underline{\qquad\qquad}$$

$$a^2 + b^2 + 2ab = 25ab$$

$$(a+b)^2 = 5^2\, ab$$

$$\boxed{\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m}$$

$$\left(\frac{a+b}{5}\right)^2 = ab$$

$$2\log\left(\frac{a+b}{5}\right) = \log a + \log b$$

$$\boxed{\log\left(\frac{a+b}{5}\right) = \frac{1}{2}\left[\log a + \log b\right]}$$

$$\frac{1}{2}\log 9 + 2\log 6 + \frac{1}{4}\log 81 - \log 12 = \_\_\_\_\_\_$$

$$= \frac{1}{2}\log(3^2) + 2\left[\log(3\times 2)\right] + \frac{1}{4}\log 3^4 - \log(4\times 3)$$

$\log_a x^m$

$= m\log_a x$

$$= \frac{\cancel{2}}{\cancel{2}}\log 3 + 2\left[\log 3 + \log 2\right] + \frac{\cancel{4}}{\cancel{4}}\log 3 - \log 2^2 - \log 3$$

$$= \underline{3\log 3} + \cancel{2\log 2} + \cancel{\log 3} - \cancel{2\log 2} - \cancel{\log 3}$$

$$= 3\log 3 = \boxed{\log 27.}$$

If $\log_e x - \log_e y = a$

$\log_e y - \log_e z = b$

$\log_e z - \log_e x = c$

$$\left(\frac{x}{y}\right)^{b-c} \times \left(\frac{y}{z}\right)^{c-a} \times \left(\frac{z}{x}\right)^{a-b} = \_\_\_\_.$$

$$= e^{ab-ac} \times e^{bc-ba} \times e^{ca-cb}$$

$$= e^{0} = \boxed{1}$$

$$\log_e\left(\frac{x}{y}\right) = a \Rightarrow \frac{x}{y} = e^{a}$$

$$\left(\frac{x}{y}\right)^{b-c} = \left(e^{a}\right)^{b-c} = e^{ab-ac}$$

Base changing property

$$\log_b a = \frac{\log_p a}{\log_p b} = \frac{\log_q a}{\log_q b} = \frac{\log_r a}{\log_r b} = \cdots\cdots$$

$$\log_b a \cdot \log_e b = \log_e a$$

$$= \frac{\log_e a}{\log_e b} = \frac{\log a}{\log b}$$

$$\boxed{\log_b a = \frac{\log a}{\log b}}$$

$$\log_b a = \frac{1}{\log_a b}$$

$$\log_b a \cdot \log_a b = \log_a a = 1$$

$$\log_a b = \frac{1}{\log_b a}$$

$$\log_b a \cdot \log_c b = \log_c a$$

$$\log x = \log_e x = \ln x$$ Natural logarithm.

&

Naperian logarithm.

$\log_{10} x$ (Common logarithm & Brigg's logarithm)

$$\log_{10} x \cdot \underset{2.303}{\log_e 10} = \log_e x$$

$\log_{10} 2 = 0.3010$

$\log_{10} 2 \times \log_e 10 = \log_e 2$

$0.3010 \times 2.303 = \boxed{0.6303}$

$$\log_e x \times \log_{10} e = \log_{10} x$$

$$\log_e 10 = 2.303$$

$$\boxed{\log_e x \times 0.4342 = \log_{10} x}$$

$$\log_{10} e = \frac{1}{2.303}$$

$$= \boxed{0.4342}$$

$$\frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3} = \underline{\qquad}$$

$$\log_3 135 \cdot \log_3 15 - \log_3 5 \cdot \log_3 405$$

$$\log_3 (5 \times 27) \cdot \log_3 (3 \times 5) - \log_3 5 \cdot \log_3 (5 \times 81)$$

$$(\log_3 5 + \log_3 3^3) \times (1 + \log_3 5) - \log_3 5 \left(\log_3 5 + \log_3 3^4\right)$$

$$= \cancel{(\log_3 5)^2} + \log_3 5 + 3 - \cancel{(\log_3 5)^2} \ldots 3$$

$$= \ldots - 4.1$$

$$\log_3(5\times 3^3) \times \log_3(5\times 3) - \log_3 5\left(\log_3(5\times 81)\right)$$

$$= \left(\log_3 5 + 3\log_3 3\right)\left(\log_3 5 + 1\right) - \log_3 5\left(\log_3 5 + \log_3 3^4\right)$$

$$= \cancel{(\log_3 5)^2} + 3\cancel{\log_3 5} + \cancel{\log_3 5} + 3 - \cancel{(\log_3 5)^2} - 4\cancel{\log_3 5}$$

$$= \boxed{3}$$

$$\log_{9} 27 - \log_{27} 9 = \_\_\_\_\_\_$$

$$= \log_{3^2} 3^3 - \log_{3^3} 3^2$$

$$= \frac{3}{2} \cdot 1 - \frac{2}{3} \cdot 1$$

$$= \frac{3}{2} - \frac{2}{3} = \boxed{\frac{5}{6}}$$

$$\boxed{a^{\log_c b} = b^{\log_c a}} \quad (\text{Imp})$$

$$2^{\log_3 5} - 5^{\log_3 2} = \_\_\_\_$$

$$= 5^{\log_3 2} - 5^{\log_3 2} = \boxed{0}$$

$$\log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8 \cdot \log_8 9 = \_\_\_\_$$

$$= \log_3 9 = \log_3 3^2 = \boxed{2}$$

$\log_{10} 2 = x \quad : \quad \log_{10} 3 = y$

$\log_{10}(21\cdot 6) = \_\_\_\_\_\_$

Sol: $\log_{10}(21\cdot 6) = \log_{10}\left(\frac{216}{10}\right)$

$= \log_{10} 6^3 - \log_{10} 10$

$= 3\left[\log_{10}(3\times 2)\right] - 1.$

$\boxed{= 3[x+y] - 1.}$

Arrange the following <u>nos.</u> in the increasing order of magnitude

$\log_7 9$. $\log_8 16$ $\log_6 41$. $\log_2 10$.

$\log_7 9 > 1$

$9 > 7$

increasing order $\log_8 16$ $\log_7 9$. $\log_6 41$. $\log_2 10$

Solve $\log(x-1) + \log(x^2+x+1) = \log 999$

$x-1>0$ $\quad$ >0

$x>1.$

Sol: $\log(x-1)(x^2+x+1) = \log 999$

$$\log(x^3-1) = \log 999$$

$$x^3-1 = 999$$

$$x^3 = 1000$$

$$\boxed{x=10}$$

$$\log 27.91 = 1.4458 \qquad \log(2.791) = \_\_\_\_$$

$$\log_{10}(2.791) = \log_{10}\left(\frac{27.91}{10}\right)$$

$$= \log_{10}(27.91) - \log_{10} 10$$

$$= 1.4458 - 1$$

$$= \boxed{0.4458}$$

$\log_m a = b \iff a = \text{Antilog of } b$

with base m.

$10^2 = 100$

$\log_{10} 100 = 2 \iff 100 = \text{Antilog of } 2$ with base 10.

Find Anti log of $5/6$ to the base 64?

$= (64)^{5/6} = (2^{\cancel{6}})^{5/\cancel{6}} = 2^5 = \boxed{32}$

Find Anti log of $2/3$ with base 27.

$$= (27)^{2/3} = (3^3)^{2/3} = 3^2 = \boxed{9}$$

$$\log_{10} N = \underbrace{\text{Integer}}_{\text{Characteristic.}} + \underbrace{\text{Fractional part}}_{\text{Mantissa.}}$$

$\log_{10} 2 = 0.3010$

Char: 0 : Mantissa : 0.3010

$\log_{10} 20 = 1.3010$

Char: 1 : Mantissa = 0.3010

$\log_{10} 5 = 0.6989$

$\log_{10} 50 = 1.6989.$

Char: 1 Mantissa $= 0.6989$

$\log_{10} \boxed{500} = 2.6989.$

$\log_{10} \boxed{5000} = 3.6989.$

If characteristic is $-n$ then there will be $(n-1)$ cyphers

If characteristic is +ve integer n then there will be $(n+1)$ digits.

$10^{-2} = \frac{1}{100} = 0.01$

Characteristic of $\log_2(0.01)$

$= -2$

$\log_{10}(0.001)$

Charact: -3