ELITE IIT ACADEMY

MATHEMATICS

MODULE 1

CONTENTS

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FUNDAMENTALS OF MATHEMATICS

1. NUMBER SYSTEM :

Natural Numbers : (N) = {1, 2, 3....∞}

Whole Numbers : (W) = {0, 1, 2, 3.....∞}

Integers : (I) = {–∞,........–3, –2, –1, 0, 1, 2, 3.....∞}

Positive Integers : (I^+) = {1, 2, 3 ... ∞}

Negative Integers : (I^-) = {–∞, ... –3, –2, –1}

Non-negative Integers : {0, 1, 2, 3........}

Non-positive Integers : {–∞, ... –3, –2, –1, 0}

Even Integers = {....–6, –4, –2, 0, 2, 4, 6 ...}

Odd Integers = {–5, –3, –1, 1, 3, 5 ......}

Note :

(i) Zero is neither positive nor negative.

(ii) Zero is even number.

(ii) Positive means > 0.

(iv) Non-negative means ≥ 0.

2. FRACTION $\left(\frac{p}{q}\right)$:

(a) Proper Fraction = $\frac{3}{5}$: $N^r < D^r$

(b) Improper Fraction = $\frac{5}{3}$: $N^r > D^r$

(c) Mixed Fraction : $2+\frac{3}{5}$

(d) Compound Fraction : $\dfrac{\frac{2}{3}}{\frac{5}{6}}$

(e) Complex Fraction : $2\frac{1}{3}$

(f) Continued Fraction : $2+\cfrac{2}{2+\cfrac{2}{+.....}}$

This is usually written in the more compact form $2+\frac{1}{2+}\,\frac{1}{2+}$........

3. RATIONAL NUMBERS (Q) :

All the numbers that can be represented in the form p/q, where p and q are integers and q ≠ 0, are called rational numbers. Integers, Fractions, Terminating decimal numbers, Non-terminating but repeating decimal numbers are all rational numbers. Q = $\left\{\frac{p}{q} : p, q \in I \text{ and } q \neq 0\right\}$

Note :

(i) Integers are rational numbers, but converse need not be true.

(ii) A rational number always exists between two distinct rational numbers, hence infinite rational numbers exist between two rational numbers.

4. IRRATIONAL NUMBERS (Q^C) :

There are real numbers which can not be expressed in p/q form. Non-Terminating non repeating decimal numbers are irrational number e.g. $\sqrt{2}, \sqrt{5}, \sqrt{3}, \sqrt[3]{10}$; e, π.

e ≈ 2.71 is called Napier's constant and π ≈ 3.14

Note :

(i) Sum of a rational number and an irrational number is an irrational number e.g. $2 + \sqrt{3}$

(ii) If $a \in Q$ and $b \notin Q$, then ab = rational number, only if $a = 0$.

(iii) Sum, difference, product and quotient of two irrational numbers need not be an irrational number or we can say, result may be a rational number also.

5. REAL NUMBERS (R) :

The complete set of rational and irrational number is the set of real numbers, $R = Q \cup Q^C$. The real numbers can be represented as a position of a point on the real number line.

6. COMPLEX NUMBERS. (C) :

A number of the form a + ib, where $a, b \in R$ and $i = \sqrt{-1}$ is called a complex number. Complex number is usually denoted by z and the set of all complex numbers is represented by
$C = \{(x + iy) : x, y \in R, i = \sqrt{-1}\}$

$\boxed{\text{N UW UI UQ UR UC}}$

7. EVEN NUMBERS :

Numbers divisible by 2, last digit 0, 2, 4, 6, 8 & represented by 2n.

8. ODD NUMBERS :

Not divisible by 2, last digit 1, 3, 5, 7, 9 represented by $(2n \pm 1)$

(a) even ± even = even

(b) even ± odd = odd

(c) odd ± odd = even

(d) even   any number = even number

(e) odd   odd = odd

9. PRIME NUMBERS :

Let 'p' be a natural number, 'p' is said to be prime if it has exactly two distinct positive integral factors, namely 1 and itself. e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 .....

10. COMPOSITE NUMBERS :

A number that has more than two divisors

Note :

(i) '1' is neither prime nor composite.

(ii) '2' is the only even prime number.

(iii) '4' is the smallest composite number.

(iv) Natural numbers which are not prime are composite numbers (except 1)

11. CO-PRIME NUMBERS/ RELATIVELY PRIME NUMBERS :

Two natural numbers (not necessarily prime) are coprime, if their H.C.F. is one
e.g. (1, 2), (1, 3), (3, 4) (5, 6) etc.

Note :

(i) Two prime number(s) are always co-prime but converse need not be true.

(ii) Consecutive natural numbers are always co-prime numbers.

12. TWIN PRIME NUMBERS :

If the difference between two prime numbers is two, then the numbers are twin prime numbers.
e.g. {3, 5}, {5, 7}, {11, 13} etc.

13. NUMBERS TO REMEMBER :

Number	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Square	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400
Cube	8	27	64	125	216	343	512	729	1000	1331	1728	2197	2744	3375	4096	4913	5832	6859	8000
Sq. Root	1.41	1.73	2	2.24	2.45	2.65	2.83	3	3.16										

Note :

(i) Square of a real number is always non negative (i.e. $x^2 \geq 0$)

(ii) Square root of a positive number is always positive e.g. $\sqrt{4} = 2$

(iii) $\sqrt{x^2} \neq \pm x$ but $\sqrt{x^2} = |x|$

14. DIVISIBILITY RULES :

Divisible by	Remark.
2	Last digit 0, 2, 4, 6, 8
3	Sum of digits divisible by 3 (Remainder will be same when number is divided by 3 or sum of digits is divided by 3.)
4	Last two digits divisible by 4 (Remainder will be same whether we divide the number or its last two digits)
5	Last digit 0 or 5
6	Divisible by 2 and 3 simultaneously.
8	Last three digits is divisible by 8 (Remainder will be same whether we divide the number or its last three digits)
9	Sum of digits divisible by 9. (Remainder will be same when number is divided by 9 or sum of digit is divided by 9)
10	Last digits 0
11	(Sum of digits at even places) – (sum of digits at odd places) = 0 or divisible by 11

15. LCM AND HCF :

(a) HCF is the highest common factor between any two or more numbers or algebraic expressions. When dealing only with numbers, it is also called "Greatest common divisor" (GCD).

(b) LCM is the lowest common multiple of two or more numbers or algebraic expressions.

(c) The product of HCF and LCM of two numbers (or expressions) is equal to the product of the numbers.

(d) LCM of $\left(\frac{a}{b}, \frac{p}{q}, \frac{l}{m}\right) = \frac{\text{L.C.M. of } (a, p, l)}{\text{H.C.F. of } (b, q, m)}$

16. FACTORIZATION :

Formulae :

(a) $(a \pm b)^2 = a^2 \pm 2ab + b^2 = (a \mp b)^2 \pm 4ab$

(b) $a^2 - b^2 = (a+b)(a-b)$;

• If $a^2 - b^2 = 1$ then $a + b = \frac{1}{a-b}$

For example : $\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$ or $\sqrt{3} + \sqrt{2} = \frac{1}{\sqrt{3} - \sqrt{2}}$

(c) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(d) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(e) $a^3 + b^3 = (a+b)(a^2 - ab + b^2) = (a+b)^3 - 3ab(a+b)$

(f) $a^3 - b^3 = (a-b)(a^2 + ab + b^2) = (a-b)^3 + 3ab(a-b)$

(g) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$$= \frac{1}{2}(a + b + c)\{(a - b)^2 + (b - c)^2 + (c - a)^2\}$$

(h) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(i) $(a + b + c)^3 = a^3+b^3+c^3 + 3(a + b)(b + c)(c + a)$

(j) $a^4 + a^2 + 1 = (a^2 + 1)^2 - a^2 = (1 + a + a^2)(1 - a + a^2)$

17. CYCLIC FACTORS :

If an expression remain same after replacing a by b, b by c & c by a, then it is called cyclic expression and its factors are called cyclic factors. e.g. $a(b - c) + b(c - a) + c(a - b)$

18. REMAINDER THEOREM :

If a polynomial $a_1x^n + a_2x^{n-1} + a_3x^{n-2} + \ldots + a_n$ is divided by $x-p$, then the remainder is obtained by putting $x = p$ in the polynomial.

19. FACTOR THEOREM :

A polynomial $a_1x^n + a_2x^{n-1} + a_3x^{n-2} + \ldots + a_n$ is divisible by $x-p$, if the remainder is zero i.e. if $a_1p^n + a_2p^{n-1} + \ldots + a_n = 0$ then $x - p$ will be a factor of polynomial.

20. RATIO AND PROPORTION :

(a) If $\frac{a}{b} = \frac{c}{d}$, then : $\frac{a+b}{b} = \frac{c+d}{d}$ (componendo); $\frac{a-b}{b} = \frac{c-d}{d}$ (dividendo);

$\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (componendo and dividendo); $\frac{a}{c} = \frac{b}{d}$ (alternendo); $\frac{b}{a} = \frac{d}{c}$ (invertendo)

(b) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots$, then each ratio $= \left(\frac{a^n + c^n + e^n}{b^n + d^n + f^n}\right)^{\frac{1}{n}}$

Example : $\frac{a}{b} = \frac{c}{d} = \frac{\sqrt{a^2 + c^2}}{\sqrt{b^2 + d^2}} = \frac{a+c}{b+d} = \frac{a-c}{b-d}$

21. INTERVALS :

Intervals are basically subsets of R. If there are two numbers a, b $\in$ R such that $a < b$, we can define four types of intervals as follows :

(a) Open interval : $(a, b) = \{x : a < x < b\}$ i.e. end points are not included.

(b) Closed interval : $[a, b] = \{x : a \le x \le b\}$ i.e. end points are also included.

This is possible only when both a and b are finite.

(c) Semi open or semi closed interval : $(a, b] = \{x : a < x \le b\}$; $[a, b) = \{x : a \le x < b\}$

(d) The infinite intervals are defined as follows :

(i) $(a, \infty) = \{x : x > a\}$ (ii) $[a, \infty) = \{x : x \ge a\}$

(iii) $(-\infty, b) = \{x : x < b\}$ (iv) $(\infty, b] = \{x : x \le b\}$

(v) $(-\infty, \infty) = R$

Note :

(i) For some particular values of x, we use symbol { } e.g. If x = 1, 2 we can write it as $x \in \{1, 2\}$

(ii) If their is no values of x, then we say $x \in \phi$ (null set)

22. BASIC CONCEPTS OF GEOMETRY :

(A) BASIC THEOREMS & RESULTS OF TRIANGLES :

(a) Two polygons are similar if (i) their corresponding angles are equal, (ii) the length of their corresponding sides are proportional. (Both conditions are independent & necessary)
In case of a triangle, any one of the conditions is sufficient, other satisfies automatically.

(b) **Thales Theorem (Basic Proportionality Theorem) :** In a triangle, a line drawn parallel to one side, to intersect the other sides in distinct points, divides the two sides in the same ratio.
Converse : If a line divides any two sides of a triangle in the same ratio then the line must be parallel to the third side.

(c) **Similarity Theorem :**

(i) **AAA similarity** : If in two triangles, corresponding angles are equal i.e. two triangles are equiangular, then the triangles are similar.

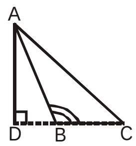
(ii) **SSS similarity** : If the corresponding sides of two triangles are proportional, then they are similar.

(iii) **SAS similarity :** If in two triangles, one pair of corresponding sides are proportional and the included angles are equal then the two triangles are similar.

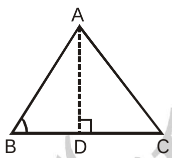
(iv) If two triangles are similar then
(1) They are equiangular
(2) The ratio of the corresponding (I) Sides (all), (II) Perimeters, (III) Medians, (IV) Angle bisector segments, (V) Altitudes are same (converse also true)
(3) The ratio of the areas is equal to the ratio of the squares of corresponding (I) Sides (all), (II) Perimeters, (III) Medians, (IV) Angle bisector segments, (V) Altitudes (converse also true)

(d) **Pythagoras theorem :**

(i) In a right triangle the square of hypotenuse is equal to the sum of square of the other two sides.
Converse : In a triangle if square of one side is equal to sum of the squares of the other two sides. then the angle opposite to the first side is a right angle.

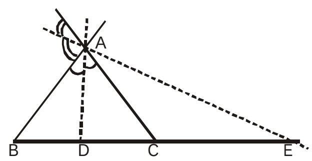
(ii) In obtuse Δ 

$AC^2 = AB^2 + BC^2 + 2BC \cdot BD$

(iii) In Acute Δ 

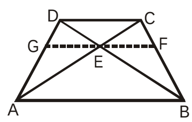
$AC^2 = AB^2 + BC^2 - 2BC \cdot BD$

(e) The internal/external bisector of an angle of a triangle divides the opposite side internally/externally in the ratio of sides containing the angle (converse is also true) i.e. $\dfrac{AB}{AC} = \dfrac{BD}{DC} = \dfrac{BE}{CE}$



(f) The line joining the mid points of two sides of a triangle is parallel & half of the third side. (It's converse is also true)

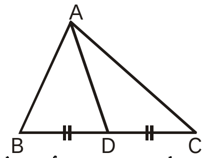
(g) (i) The diagonals of a trapezium divided each other proportionally. (converse is also true) i.e. $\dfrac{AE}{EC} = \dfrac{BE}{ED}$



(ii) Any line parallel to the parallel sides of a trapezium divides the non parallel sides proportionally i.e. $\dfrac{DG}{GA} = \dfrac{CF}{FB}$

(iii) If three or more parallel lines are intersected by two transversals, then intercepts made by them on transversals are proportional.

(h) In any triangle the sum of squares of any two sides is equal to twice the square of half of the third side together with twice the square of the median which bisects the third side. i.e. $AB^2 + AC^2 = 2\left(\frac{1}{2}BC\right)^2 + 2\,(AD)^2 = 2(AD^2 + BD^2)$

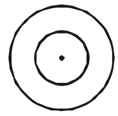


(i) In any triangle the three times the sum of squares of the sides of a triangle is equal to four times the sum of the square of the medians of the triangle.

(j) The altitudes, medians and angle bisectors of a triangle are concurrent among themselves.

(B) BASIC THEOREMS & RESULTS OF CIRCLES :

(a) **Concentric circles** : Circles having same centre.



(b) **Congruent circles :** Iff their radii are equal.

(c) **Congruent arcs :** Iff they have same degree measure at the centre.

Theorem 1 :

(i) If two arcs of a circle (or of congruent circles) are congruent, the corresponding chords are equal.
Converse : If two chords of a circle are equal then their corresponding arcs are congruent.

(ii) Equal chords of a circle (or of congruent circles) subtend equal angles at the centre.
Converse : If the angle subtended by two chords of a circle (or of congruent circles) at the centre are equal, the chords are equal.

Theorem 2 :

(i) The perpendicular from the centre of a circle to a chord bisects the chord.
Converse : The line joining the mid point of a chord to the centre of a circle is perpendicular to the chord.

(ii) Perpendicular bisectors of two chords of a circle intersect at its centre.

Theorem 3 :

(i) There is one and only one circle passing through three non collinear points.

(ii) If two circles intersects in two points, then the line joining the centres is perpendicular bisector of common chords.

Theorem 4 :

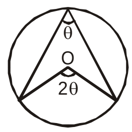
(i) Equal chords of a circle (or of congruent circles) are equidistant from the centre.
Converse : Chords of a circle (or of congruent circles) which are equidistant from the centre are equal.

(ii) If two equal chords are drawn from a point on the circle, then the centre of circle will lie on angle bisector of these two chords.

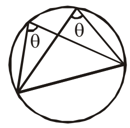
(iii) Of any two chords of a circle larger will be near to centre.

Theorem 5 :

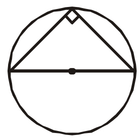
(i) The degree measure of an arc or angle subtended by an arc at the centre is double the angle subtended by it at any point of alternate segment.



(ii) Angle in the same segment of a circle are equal.



(iii) The angle in a semi circle is right angle.
Converse : The arc of a circle subtending a right angle in alternate segment is semi circle.



Theorem 6 :

Any angle subtended by a minor arc in the alternate segment is acute and any angle subtended by a major arc in the alternate segment is obtuse.

Theorem 7 :

If a line segment joining two points subtends equal angles at two other points lying on the same side of the line segment, the four points are concyclic, i.e. lie on the same circle.

(d) **Cyclic Quadrilaterals :**

A quadrilateral is called a cyclic quadrilateral if its all vertices lie on a circle.

Theorem 1 :

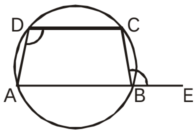
The sum of either pair of opposite angles of a cyclic quadrilateral is 180

OR

The opposite angles of a cyclic quadrilateral are supplementary.

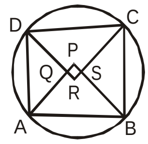
Converse : If the sum of any pair of opposite angle of a quadrilateral is 180 , then the quadrilateral is cyclic.

Theorem 2 :



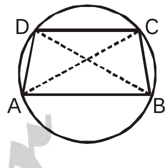
If a side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle. i.e. $\angle CBE = \angle ADC$

Theorem 3 :



The quadrilateral PQRS formed by angle bisectors of a cyclic quadrilateral is also cyclic.

Theorem 4 :



If two sides of a cyclic quadrilateral are parallel then the remaining two sides are equal and the diagonals are also equal. i.e. $AB \,||\, CD \Leftrightarrow AC = BD \;\&\; AD = BC$

OR

A cyclic trapezium is isosceles and its diagonals are equal.

Converse : If two non-parallel sides of a trapezium are equal, then it is cyclic.

OR

An isosceles trapezium is always cyclic.

Theorem 5 :

The bisectors of the angles formed by producing the opposite sides of a cyclic quadrilateral (provided that they are not parallel), intersect at right angle.

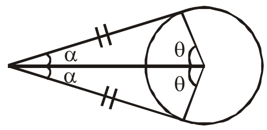
(C) TANGENTS TO A CIRCLE :

Theorem 1 :

A tangent to a circle is perpendicular to the radius through the point of contact.

Converse : A line drawn through the end point of a radius and perpendicular to it is a tangent to the circle.

Theorem 2 :



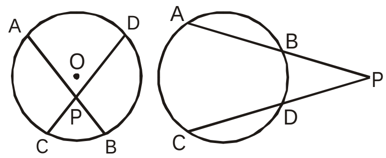
If two tangents are drawn to a circle from an external point, then :

(i) they are equal.

(ii) they subtend equal angles at the centre,

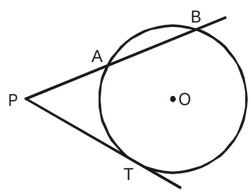
(iii) they are equally inclined to the segment, joining the centre to that point.

Theorem 3 :



If two chords of a circle intersect inside or outside the circle when produced, the rectangle formed by the two segments of one chord is equal in area to the rectangle formed by the two segments of the other chord. PA PB = PC PD

Theorem 4 :

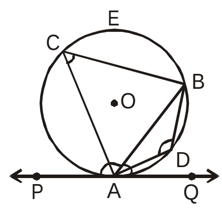


If PAB is a secant to a circle intersecting the circle at A and B and PT is tangent segment, then PA PB = PT^2

OR

Area of the rectangle formed by the two segments of a chord is equal to the area of the square of side equal to the length of the tangent from the point on the circle.

Theorem 5 :



If a chord is drawn through the point of contact of a tangent to a circle, then the angles which this chord makes with the given tangent are equal respectively to the angles formed in the corresponding alternate segments.

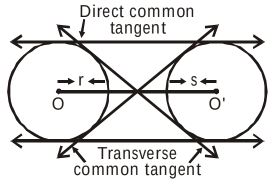
$\angle BAQ = \angle ACB$ and $\angle BAP = \angle ADB$

Converse :

If a line is drawn through an end point of a chord of a circle so that the angle formed with the chord is equal to the angle subtended by the chord in the alternate segment, then the line is a tangent to the circle.

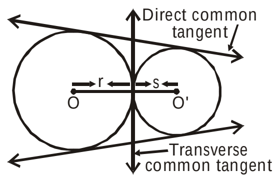
(D) COMMON TANGENTS OF TWO CIRCLES :

A common tangent is called direct tangent if both centres of circle lie on same side of it and called transverse tangent if centres lie on opposite side of it.



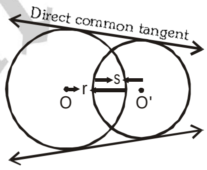
(a) When $OO' > r + s$ i.e. the distance between the centres is greater than the sum of the radii.

In this case, the two circles do not intersect with each other and four common tangents can be drawn to two circles. Two of them are called direct (external) common tangents and the other two are known as transverse (internal or indirect) common tangents



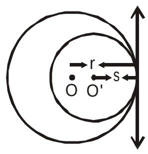
(b) When $OO' = r + s$ i.e. the distance between the centres is equal to the sum of the radii.

In this case, the two circles touch each other externally the common point of the two circles is called the point of contact and three common tangents can be drawn to the two circles. Two of them are direct common tangents and one transverse common tangent.



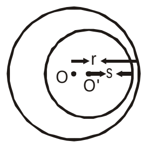
(c) When $|r - s| < OO' < r + s$ i.e. the distance between the centres is less than the sum of the radii and greater than their absolute difference.

In this case, the two circles intersect in two points and there are two direct common tangents only.



(d) When $OO' = r - s$, $r > s$ i.e. the distance between the centres is equal to the difference of the radii.

In this case the two circles touch internally. The common point of the two circles is called their point of contact and there is only one common tangent to the two circles.

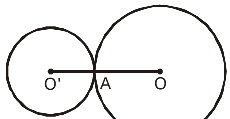


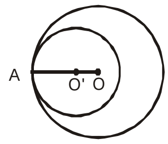
(e) When $OO' < r - s$, $r > s$ i.e. the distance between the centres is less than the difference of the radii.

In this case one circle lies inside the other and they do not touch. In such a case there is no common tangent.

Theorem 1 :

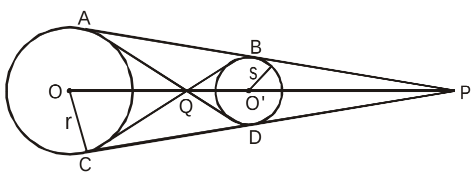
If two circles touch each other (internally or externally) the point of contact lies on the line through the centres.





Theorem 2 :

The points of intersection of direct common tangents and transverse common tangents to two circles divide the line segment joining the two centres externally and internally respectively in the ratio of their radii.



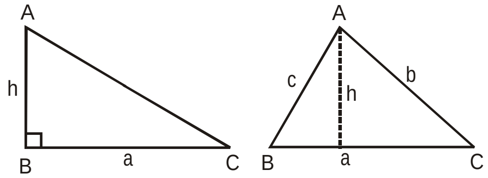
(i) P divides OO' externally in the ratio r : s i.e. $\frac{OP}{O'P} = \frac{r}{s}$

(ii) Q divides OO' internally in the ratio r : s i.e. $\frac{OQ}{O'Q} = \frac{r}{s}$

23. BASIC CONCEPT OF MENSURATION

PLANE

(A) TRIANGLE :



(a) Sum of three angle is 180

(b) Perimeter = Sum of three sides = a + b + c = 2s

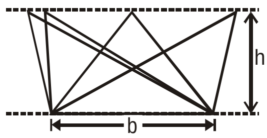
Semi perimeter s = (a + b + c)/2

(c) Area = 1/2 (Base Height)

$= \frac{1}{2}$ (Any side Altitude over it) = $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

Note : Area of triangles formed between two same parallel lines and on the same base is same

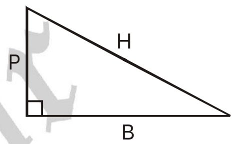
Area = $\frac{1}{2}bh$



(d) **Right Angle Triangle** : One angle 90 (Right angle)

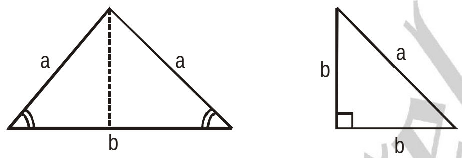
& Hypotenuse^2 = Perpendicular^2 + Base^2 (Pythagoras theorem)

Area = $\frac{1}{2}PB$



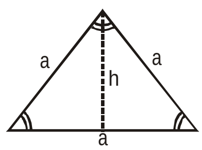
(e) **Isosceles Triangle** : Two sides equal hence two angle are equal.

Special case : Isosceles Right Triangle : Two sides equal and Base = Perpendicular.



(f) **Equilateral Triangle** : All three sides and angles (60) are equal; h = $\left(\frac{\sqrt{3}}{2}\right)$a;

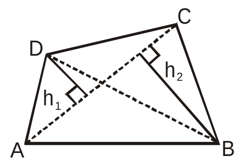
Area = $\left(\frac{1}{2}\right)$ base height = $\left(\frac{1}{2}\right)$(a) $\left(\frac{\sqrt{3}}{2}\right)$a = $\left(\frac{\sqrt{3}}{4}\right)a^2 = \frac{h^2}{\sqrt{3}}$



(B) QUADRILATERAL :

(a) Sum of all angles is 360

Area = $\frac{1}{2}(AC)(h_1 + h_2)$ i.e. sum of areas of Δ ACD + ΔABC $= \frac{1}{2}d_1d_2 \sin\theta$



(b) **Parallelogram** :

(i) Opposite sides are parallel and equal.

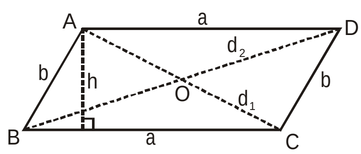
(ii) Opposite angles are equal. ($\angle B = \angle D$ and $\angle A = \angle C$)

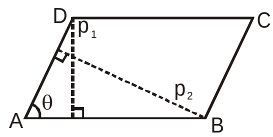
(iii) Diagonals bisects each other. AO = OC & BO = OD

(iv) Perimeter = 2(a + b) ;

(v) Area = $\frac{1}{2}$(ah) + $\frac{1}{2}$(ah) = ah i.e. sum of areas of Δ ACD + ΔABC

also, Area = $\frac{p_1p_2}{\sin\theta}$





(c) **Special cases of parallelogram :**

(i) **Rhombus :** All sides are equal and opposite angles are equal.

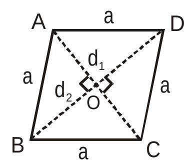
AB = BC = CD = DA = a

$\angle A = \angle C$ & $\angle B = \angle D$

Diagonals are not equal ($d_1 \neq d_2$) but bisects each other at 90

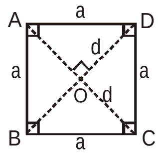
AC $\neq$ BD but AC $\perp$ BD

Area = $\frac{1}{2}(d_1 \quad d_2)$ i.e. sum of areas of Δ ACD + ΔABC



(ii) **Square :** All sides are equal and all angle are equal (90)

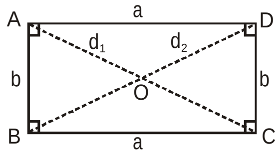
Diagonals are equal and perpendicular bisectors of each other



Area = $a^2 = \frac{d^2}{2}$

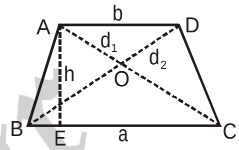
$AC \perp BD$ & $AO = OC$, $BO = OD$

(iii) **Rectangle :** Opposite sides are equal and parallel, all angles are equal (90) and diagonal are equal and bisects each other but not at 90 .



Area = a   b; Perimeter = 2(a + b)

(iv) **Trapezium** : Any two opposite sides are parallel but not equal. Diagonals cuts in same proportion. AD || BC ; AD ≠ BC ; $d_1 \neq d_2$

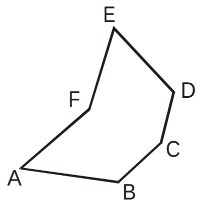


Area $=\left(\frac{1}{2}\right)(a + b)\, h$ i.e. sum of area of $\Delta ABC + \Delta ACD$

$\frac{AO}{OC} = \frac{OD}{OB}$ $(\because \Delta BOC \sim \Delta DOA)$

(C) POLYGON :

A plane figure enclosed by line segments (sides of polygon).

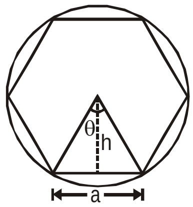


(a) **n sides polygon have n sides :** Triangle and quadrilaterals are polygon of three and four sides respectively. The polygons having 5 to 10 sides are called, PENTAGON, HEXAGON, HEPTAGON, OCTAGON, NANOGON and DECAGON respectively.

(b) **Regular polygon :** Polygon which has all equal sides and equal angles and can be inscribed in a circle whose center coincides with the center of polygon. Therefore the center is equidistant from all its vertices.

(i) A regular polygon can also circumscribe a circle.

(ii) **A 'n' sided regular polygon can be divided into 'n' Isosceles Congruent Triangles with a common vertex i.e. centre of polygon.**



(iii) Area = $n \left(\frac{1}{2}\right)$ a   h

(iv) Perimeter = na

(v) Each interior angle of polygon = $\left(\frac{n-2}{n}\right)$ 180

(vi) Angle subtended at the centre of inscribed/circumscribed circle by one side = 360 /n

(vii) Each exterior angle = $\left(\frac{360}{n}\right)^{o}$

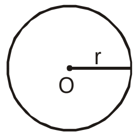
(viii) Sum of all interior angle = (n − 2)   180

(ix) Sum of all exterior angles = 360

(x) **Convex polygon :** If any two consecutive vertices are joined then remaining all other vertices will lie on same side.

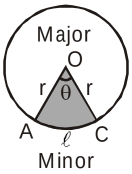
(D) CIRCLE :

Area $A = \pi r^2$; Circumference (perimeter) $= 2\pi r$



(a) **Sector of a circle :** Bounded by arc of circle (subtending angle 'θ' at center) and two radii. Circle is divided into minor (containing 'θ') and major sectors

(i) Arc length of sector : $\ell = \left(\dfrac{\theta^\circ}{360^\circ}\right) 2\pi r$



(ii) Area : $A = \left(\dfrac{\theta^\circ}{360^\circ}\right)\pi r^2 = \left(\dfrac{1}{2}\right)\ell r$

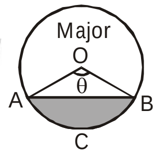
(iii) Perimeter of sector AOC $= 2r + \ell$

(b) **Segment of a circle :** Bounded by arc of the circle and the chord (determining the segment).

(i) Circle is divided into two segments minor segment and major segment.

(ii) When chord is diameter, sector coincides with segment.

(iii) Area (segment ACB) = Area of sector OACB - Area of ΔAOB



$$= \left(\frac{\theta^\circ}{360^\circ}\right)\pi r^2 - \left(\frac{1}{2}\right)\left(2r\sin\frac{\theta}{2}\right)\left(r\cos\frac{\theta}{2}\right)$$

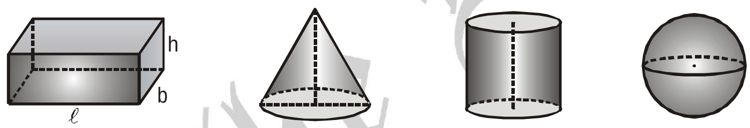
$$\text{Area} = \left(\frac{\theta^\circ}{360^\circ}\right)\pi r^2 - \left(\frac{1}{2}\right) r^2 \sin\theta$$

SOLIDS

Require three dimension to describe

(a) **Surfaces of solids :** Plane areas bounding the solid e.g. six rectangle faces bounding a brick. Surface area is measured in square units.

(b) **Volume of solids :** Space occupied by a solid and is measured in cubic units.



Cuboid   Cone   Cylinder   Sphere

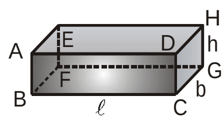
(A) CUBOID :

Rectangular shaped solid also known as rectangular parallelopiped (e.g. match box, brick)

(a) Have six rectangular faces with opposite faces parallel and congruent.

(b) Have twelve edges (Edge - The line segment where two adjacent faces meets).

(c) Three adjacent faces meet at a point called vertex and cuboid have eight vertices



(d) **Surface area :** $A = 2[\ell \quad b + b \quad h + h \quad \ell]$ square unit.

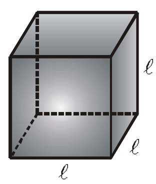
(e) **Volume :** $V = l \quad b \quad h$ cubic unit.

(B) CUBE :

Special case of cuboid having all sides equal.

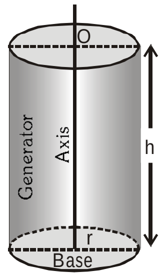
Area $= 6\ell^2$;   Volume $= \ell^3$ Unit cube : Side $\ell = 1$

Volume is 1 cubic unit (From this cubic unit is derived)



(C) CYLINDER :

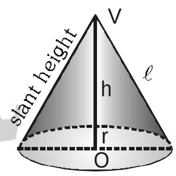
Having a lateral (curved) surface and two congruent circular cross section.
(e.g. Jar, Circular Pillars, Drums, Pipes etc.)



(a) **Axis :** Line joining the centers of two circular cross section.
(b) **Right circular cylinder :** When axis is perpendicular to circular cross section.
(c) **Generators :** Lines parallel to axis and lying on the lateral surface.
(d) **Base :** With cylinder in vertical position, the lower circular end is base.
(e) **Height (h) :** Distance between two circular faces.
(f) **Radius (r) :** Radius of base or top circle.
(g) **Total surface area :** Base area + curved surface area
$= 2\pi r^2 + 2\pi rh = 2\pi r(h + r)$(including two circular ends).
Without circular ends (Hollow cylinder) $= 2\pi rh$
(h) **Volume :** $V = \pi r^2 h$

(D) CONE :

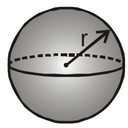
Have a curved surface with a vertex (V) and circular base radius : r and center O)



(a) **Axis :** Line joining vertex and center of base circle (VO)
(b) **Height of cone (h) :** Length of VO
(c) **Slant height (Q) :** Distance of vertex from any point of base circle
$\ell = \sqrt{r^2 + h^2}$
(d) **Right circular cone :** When axis is perpendicular to base.
(e) The cross section of a cone parallel to base is a circle and perpendicular to base is an isosceles triangle.
(f) **Volume :** $(1/3)\pi r^2 h$ (volume of a cone is 1/3rd of volume of a cylinder with same height and base radius).
(g) Curved surface Area : $\pi r\ell$
(h) Total surface Area : $\pi r\ell + \pi r^2 = \pi r(\ell + r)$
(i) A right circular cone can be generated by rotating a right angled triangle about its right angle forming side.

(E) SPHERE :

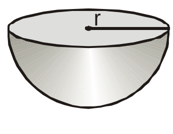
All point on its surface are equidistant from its center, the distance is called radius (r) and any line passing through center with end points on surface is called diameter.



(a) **Volume :** $(4/3)\ \pi r^3$
(b) **Surface area :** $4\pi r^2$

(F) HEMISPHERE :

A sphere is divided into two hemi spheres by a plane passing through center.



(a) **Volume :** $(2/3)\pi r^3$
(b) **Curved surface area :** $= 2\pi r^2$
(c) **Total surface area :** $= 2\pi r^2 + \pi r^2 = 3\pi r^2$

24. INDICES AND SURDS

Important Results :

1. $a\ a\ a\ \ldots.\ a$ (m times) $= a^m$
2. $a^m\ a^n = a^{m+n}$
3. $a^m \div a^n = a^{m-n}$
4. $(a^m)^n = a^{mn}$
5. $a^{-m} = \dfrac{1}{a^m}$
6. $\left(\dfrac{x}{y}\right)^m = \dfrac{x^m}{y^m}$
7. $(xy)^m = x^m.y^m$
8. $\sqrt[n]{x} = x^{1/n}$; $n \geq 2, n \in N$
9. $a^0 = 1$
10. $a^x = a^y \Rightarrow x = y$ or $a = 1$ or $a = 0$ if $x > 0$ & $y > 0$
11. $a^x = b^x \Rightarrow a = b$ or $x = 0$
12. $a^{p/q} = (a^p)^{1/q} = (a^{1/q})^p$
13. $(x^a)^b \neq x^{a^b}$ but $= x^{ab}$ e.g. $(2^3)^2 = 2^6 = 64$ & $2^{3^2} = 2^9 = 512$

LOGARITHM

1. DEFINITION :

Every positive real number N can be expressed in exponential form as $a^x = N$ where 'a' is also a positive real number different than unity and is called the base and 'x' is called an exponent.

We can write the relation $a^x = N$ in logarithmic form as $\log_a N = x$. Hence $a^x = N \Leftrightarrow \log_a N = x$.

Hence logarithm of a number to some base is the exponent by which the base must be raised in order to get that number.

Limitations of logarithm: $\log_a N$ is defined only when

(i) $N > 0$ (ii) $a > 0$ (iii) $a \neq 1$

Note :

(i) For a given value of N, $\log_a N$ will give us a unique value.

(ii) Logarithm of zero does not exist.

(iii) Logarithm of negative reals are not defined in the system of real numbers.

Illustration 1 : The value of N, satisfying $\log_a[1 + \log_b\{1 + \log_c(1 + \log_p N)\}] = 0$ is -

(A) 4 (B) 3 (C) 2 (D) 1

Solution : $1 + \log_b\{1 + \log_c(1 + \log_p N)\} = a^0 = 1$

$\Rightarrow \log_b\{1 + \log_c(1 + \log_p N)\} = 0 \quad \Rightarrow \quad 1 + \log_c(1 + \log_p N) = 1$

$\Rightarrow \log_c(1 + \log_p N) = 0 \quad \Rightarrow \quad 1 + \log_p N = 1$

$\Rightarrow \log_p N = 0 \quad \Rightarrow \quad N = 1$ **Ans. (D)**

Illustration 2 : If $\log_5 p = a$ and $\log_2 q = a$, then prove that $\frac{p^4q^4}{100} = 100^{2a-1}$

Solution : $\log_5 p = a \Rightarrow p = 5^a$

$\log_2 q = a \Rightarrow q = 2^a$

$$\Rightarrow \frac{p^4q^4}{100} = \frac{5^{4a}.2^{4a}}{100} = \frac{(10)^{4a}}{100} = \frac{(100)^{2a}}{100} = 100^{2a-1}$$

Do yourself - 1 :

(i) Express the following in logarithmic form :

(a) $81 = 3^4$ (b) $0.001 = 10^{-3}$ (c) $2 = 128^{1/7}$

(ii) Express the following in exponential form :

(a) $\log_2 32 = 5$ (b) $\log_{\sqrt{2}} 4 = 4$ (c) $\log_{10} 0.01 = -2$

(iii) If $\log_4 m = 1.5$, then find the value of m.

(iv) If $\log_{2\sqrt{3}} 1728 = x$, then find x.

2. FUNDAMENTAL IDENTITIES :

Using the basic definition of logarithm we have 3 important deductions :

(a) $\log_a 1 = 0$ i.e. logarithm of unity to any base is zero.

(b) $\log_N N = 1$ i.e. logarithm of a number to the same base is 1.

(c) $\log_{\frac{1}{N}} N = -1 = \log_N \frac{1}{N}$ i.e. logarithm of a number to the base as its reciprocal is –1.

Note : $N = (a)^{\log_a N}$ e.g. $2^{\log_2 7} = 7$

Do yourself - 2 :

(i) Find the value of the following :

(a) $\log_{\cot 22\frac{1}{2}^{\circ}}(\sec^2 x - \tan^2 x)$ **(b)** $\log_{1.4\bar{3}} \frac{43}{30}$ **(c)** $\left(\frac{1}{2}\right)^{\log_2 5}$

(ii) If E = $(\sin 10 + \cos 10)^2 + (\cos 10 - \sin 10)^2$, then find $\log_{0.5} E$

(iii) If $4^{\log_2 2x} = 36$, then find x.

3. THE PRINCIPAL PROPERTIES OF LOGARITHMS :

If m,n are arbitrary positive numbers where a > 0, a ≠ 1 and x is any real number, then-

(1) $\log_a mn = \log_a m + \log_a n$

(2) $\log_a \frac{m}{n} = \log_a m - \log_a n$

(3) $\log_a m^x = x \log_a m$

Illustration 3 : Prove that $7\log\frac{16}{15} + 5\log\frac{25}{24} + 3\log\frac{81}{80} = \log 2$

Solution :

$$7\log\frac{16}{15} + 5\log\frac{25}{24} + 3\log\frac{81}{80}$$

$$= \log\left(\frac{16}{15}\right)^7 + \log\left(\frac{25}{24}\right)^5 + \log\left(\frac{81}{80}\right)^3 = \log\left(\left(\frac{16}{15}\right)^7 \times \left(\frac{25}{24}\right)^5 \times \left(\frac{81}{80}\right)^3\right)$$

$$= \log\left[\left(\frac{2^4}{3\times5}\right)^7 \times \left(\frac{5^2}{2^3\times3}\right)^5 \times \left(\frac{3^4}{2^4\times5}\right)^3\right] = \log\left[\frac{2^{28}}{3^7\times5^7} \times \frac{5^{10}}{2^{15}\times3^5} \times \frac{3^{12}}{2^{12}\times5^3}\right]$$

$$= \log\left[2^{28-15-12} \times 5^{10-7-3} \times 3^{12-7-5}\right] = \log(2^1 \quad 5^0 \quad 3^0) = \log 2$$

Illustration 4 : If $a^2 + b^2 = 23ab$, then prove that $\log\frac{(a+b)}{5} = \frac{1}{2}(\log a + \log b)$.

Solution : $a^2 + b^2 = (a + b)^2 - 2ab = 23ab$

$\Rightarrow (a + b)^2 = 25ab \Rightarrow a+b = 5\sqrt{ab}$ ....(i)

Using (i)

L.H.S. = $\log\frac{(a+b)}{5} = \log\frac{5\sqrt{ab}}{5} = \frac{1}{2}\log ab = \frac{1}{2}(\log a + \log b) =$ R.H.S.

Illustration 5 : If $\log_a x = p$ and $\log_b x^2 = q$, then $\log_x \sqrt{ab}$ is equal to (where a, b, x ∈ $R^+ - \{1\}$) -

(A) $\frac{1}{p} + \frac{1}{q}$ (B) $\frac{1}{2p} + \frac{1}{q}$ (C) $\frac{1}{p} + \frac{1}{2q}$ (D) $\frac{1}{2p} + \frac{1}{2q}$

Solution : $\log_a x = p \Rightarrow a^p = x \Rightarrow a = x^{1/p}$.

similarly $b^q = x^2 \Rightarrow b = x^{2/q}$

Now, $\log_x \sqrt{ab} = \log_x \sqrt{x^{1/p}x^{2/q}} = \log_x x^{\left(\frac{1}{p}+\frac{2}{q}\right)\frac{1}{2}} = \frac{1}{2p} + \frac{1}{q}$

Do yourself - 3 :

(i) Show that $\frac{1}{2}\log 9 + 2\log 6 + \frac{1}{4}\log 81 - \log 12 = 3\log 3$

(ii) If $\log_e x - \log_e y = a$, $\log_e y - \log_e z = b$ & $\log_e z - \log_e x = c$, then find the value of $\left(\frac{x}{y}\right)^{b-c} \times \left(\frac{y}{z}\right)^{c-a} \times \left(\frac{z}{x}\right)^{a-b}$

4. BASE CHANGING THEOREM :

Can be stated as "quotient of the logarithm of two numbers is independent of their common base."

Symbolically, $\log_b m = \frac{\log_a m}{\log_a b}$ where $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$

Note :

(i) $\log_b a . \log_a b = \frac{\log a}{\log b} . \frac{\log b}{\log a} = 1$; hence $\boxed{\log_b a = \frac{1}{\log_a b}}$.

(ii) $a^{\log_b c} = c^{\log_b a}$

(iii) **Base power formula :** $\log_{a^k} m = \frac{1}{k}\log_a m$

(iv) The base of the logarithm can be any positive number other than 1, but in normal practice, only two bases are popular, these are 10 and e(=2.718 approx). Logarithms of numbers to the base 10 are named as 'common logarithm' and the logarithms of numbers to the base e are called Natural or Napierian logarithm. **We will consider logx as $\log_e x$ or ℓnx.**

(v) Conversion of base e to base 10 & viceversa :

$\log_e a = \frac{\log_{10} a}{\log_{10} e} = 2.303 \times \log_{10} a$; $\log_{10} a = \frac{\log_e a}{\log_e 10} = \log_{10} e \times \log_e a = 0.434 \log_e a$

Illustration 6 : If a, b, c are distinct positive real numbers different from 1 such that

$(\log_b a . \log_c a - \log_a a) + (\log_a b . \log_c b - \log_b b) + (\log_a c . \log_b c - \log_c c) = 0$, then abc is equal to -

(A) 0 (B) e (C) 1 (D) none of these

Solution : $(\log_b a \log_c a - 1) + (\log_a b . \log_c b - 1) + (\log_a c \log_b c - 1) = 0$

$\Rightarrow \frac{\log a}{\log b} . \frac{\log a}{\log c} + \frac{\log b}{\log a} . \frac{\log b}{\log c} + \frac{\log c}{\log a} . \frac{\log c}{\log b} = 3 \Rightarrow (\log a)^3 + (\log b)^3 + (\log c)^3 = 3\log a \log b \log c$

$\Rightarrow (\log a + \log b + \log c) = 0$ [$\because$ If $a^3 + b^3 + c^3 - 3abc = 0$, then $a + b + c = 0$ if $a \neq b \neq c$]

$\Rightarrow \log abc = \log 1 \Rightarrow abc = 1$

Illustration 7 : Evaluate : $81^{1/\log_5 3} + 27^{\log_9 36} + 3^{4/\log_7 9}$

Solution : $81^{\log_3 5} + 3^{3\log_9 36} + 3^{4\log_9 7}$

$= 3^{4\log_3 5} + 3^{\log_3 (36)^{3/2}} + 3^{\log_3 7^2}$

$= 625 + 216 + 49 = 890.$

Do yourself - 4 :

(i) Evaluate : $\dfrac{\log_3 135}{\log_{15} 3} - \dfrac{\log_3 5}{\log_{405} 3}$

(ii) Evaluate : $\log_9 27 - \log_{27} 9$

(iii) Evaluate : $2^{\log_3 5} - 5^{\log_3 2}$

(iv) Evaluate : $\log_3 4 \,.\, \log_4 5 \,.\, \log_5 6 \,.\, \log_6 7 \,.\, \log_7 8 \,.\, \log_8 9$

(v) If $\dfrac{1}{\log_3 \pi} + \dfrac{1}{\log_4 \pi} > x$, then x can be -

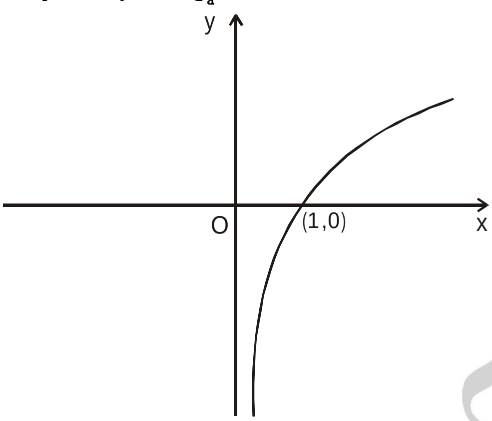
(A) 2 (B) 3 (C) 3.5 (D) π

(vi) If $\log_a 3 = 2$ and $\log_b 8 = 3$, then $\log_a b$ is -

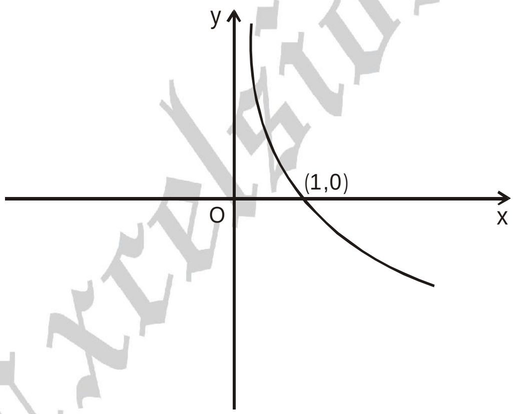
(A) $\log_3 2$ (B) $\log_2 3$ (C) $\log_3 4$ (D) $\log_4 3$

5. GRAPH OF LOGARITHMIC FUNCTIONS :

Graph of $y = \log_a x$:



When a > 1



When 0 < a < 1

Points to remember :

(i) If base of logarithm is greater than 1 then logarithm of greater number is greater. i.e. $\log_2 8 = 3$, $\log_2 4 = 2$ etc. and if base of logarithm is between 0 and 1 then logarithm of greater number is smaller. i.e. $\log_{1/2} 8 = -3$, $\log_{1/2} 4 = -2$ etc.

$$\log_a x < \log_a y \Leftrightarrow \begin{bmatrix} x < y & \text{if} & a > 1 \\ x > y & \text{if} & 0 < a < 1 \end{bmatrix}$$

(ii) It must be noted that whenever the number and the base are on the same side of unity then logarithm of that number to that base is positive, however if the number and the base are located on different side of unity then logarithm of that number to that base is negative.

e.g. $\log_{10} \sqrt[3]{10} = \dfrac{1}{3}$; $\log_{\sqrt{7}} 49 = 4$; $\log_{\frac{1}{2}}\left(\dfrac{1}{8}\right) = 3$; $\log_2\left(\dfrac{1}{32}\right) = -5$; $\log_{10}(0.001) = -3$

(iii) $x + \dfrac{1}{x} \ge 2$ if x is positive real number and $x + \dfrac{1}{x} \le -2$ if x is negative real number

(iv) $n \ge 2,\ n \in N$

$\sqrt[n]{a} = a^{1/n} \Rightarrow$ n^{th} root of 'a' ('a' is a non negative number)

Some important values : $\log_{10} 2 = 0.3010$; $\log_{10} 3 = 0.4771$; $\ell n 2 = 0.693$, $\ell n 10 = 2.303$

6. CHARACTERISTIC AND MANTISSA :

For any given number N, logarithm can be expressed as $\log_a N$ = Integer + Fraction

The integer part is called characteristic and the fractional part is called mantissa. When the value of log n is given, then to find digits of 'n' we use only the mantissa part. The characteristic is used only in determining the number of digits in the integral part (if $n \geq 1$) or the number of zeros after decimal & before first non-zero digit in the number (if $0 < n < 1$).

Note :

(i) The mantissa part of logarithm of a number is always positive $(0 \leq m < 1)$

(ii) If the characteristic of $\log_{10}N$ be n, then the number of digits in N is (n + 1)

(iii) If the characteristic of $\log_{10}N$ be (–n), then there exist (n – 1) zeros after decimal in N.

7. ANTILOGARITHM :

The positive real number 'n' is called the antilogarithm of a number 'm' if log n = m

Thus, **log n = m $\Leftrightarrow$ n = antilog m**

Do yourself - 5 :

(i) Evaluate : $\log(0.06)^6$

(ii) Find number of digits in 18^{20}

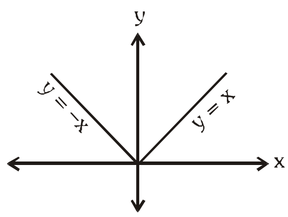
(iii) Determine number of cyphers (zeros) between decimal & first significant digit in $\left(\frac{1}{6}\right)^{200}$

(iv) Find antilog of $\frac{5}{6}$ to the base 64.

8. ABSOLUTE VALUE FUNCTION/MODULUS FUNCTION :

The symbol of modulus function is $|x|$

and is defined as : $y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$



y = |x|

Properties of Modulus :

For any a, b $\in$ R

(a) $|a| \geq 0$

(b) $|a| = |-a|$

(c) $|ab| = |a||b|$

(d) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

(e) $|a + b| \leq |a| + |b|$

(f) $|a| - |b| \leq |a - b|$

(g) $||a| - |b|| = |a - b|$ iff ab > 0 or b = 0

Illustration 8 : If $||x-1| - 2| = 5$, then find x.

Solution : $|x - 1| - 2 = \pm 5$

$|x - 1| = 7, -3$

Case-I : When $|x - 1| = 7 \Rightarrow x - 1 = \pm 7 \Rightarrow x = 8, -6$

Case-II : When $|x - 1| = -3$ (reject)

Illustration 9 : If $|x - 1| + |x + 1| = 2$, then find x.

Solution : **Case-I :** If $x \le -1$

$-(x - 1) - (x + 1) = 2$

$\Rightarrow\ -x + 1 - x - 1 = 2$

$\Rightarrow\ -2x = 2 \ \Rightarrow\ x = -1$ ....(i)

Case-II : If $-1 < x < 1$

$-(x - 1) + (x + 1) = 2$

$\Rightarrow\ -x + 1 + x + 1 = 2$

$\Rightarrow\ 2 = 2 \ \Rightarrow\ -1 < x < 1$ ....(ii)

Case-III : If $x \ge 1$

$x - 1 + x + 1 = 2$

$\Rightarrow\ x = 1$ ....(iii)

Thus from (i), (ii) and (iii) $-1 \le x \le 1$

Illustration 10 : Solve : $x\,|x + 3| + 2\,|x + 2| = 0$

Solution : **Case-I :** $x < -3$

$-x(x + 3) - 2(x + 2) = 0$

$x^2 + 5x + 4 = 0 \ \Rightarrow\ x = -1, -4$

$\Rightarrow x = -4. \quad \because \quad x = -1$ (reject)

Case- II : $-3 < x < -2$

$(x)(x + 3) - 2x - 4 = 0$

$x^2 + x - 4 = 0$

$$\Rightarrow \quad x = \frac{-1+\sqrt{17}}{2}, \frac{-1-\sqrt{17}}{2}$$

$$\Rightarrow \quad x = \frac{-1-\sqrt{17}}{2} \quad \because \quad x = \frac{-1+\sqrt{17}}{2} \text{ (reject)}$$

Case-III : $x > -2$

$x(x + 3) + 2x + 4 = 0$

$x^2 + 5x + 4 = 0$

$\Rightarrow\ x = -1, -4.$

$\Rightarrow\ x = -1 \quad \because \quad x = -4$ (reject)

Hence $x = -4, \dfrac{-1-\sqrt{17}}{2}, -1.$

Do yourself - 6 :

(i) Solve : $|x + 3| = 2(5 - x)$

(ii) Solve : $x|x| + 7x - 8 = 0$

Miscellaneous Illustrations :

Illustration 11 : Show that $\log_4 18$ is an irrational number.

Solution : $\log_4 18 = \log_4(3^2 \;\; 2) = 2\log_4 3 + \log_4 2 = 2\dfrac{\log_2 3}{\log_2 4} + \dfrac{1}{\log_2 4} = \log_2 3 + \dfrac{1}{2}$

assume the contrary, that this number $\log_2 3$ is rational number.

$\Rightarrow \quad \log_2 3 = \dfrac{p}{q}$. Since $\log_2 3 > 0$ both numbers p and q may be regarded as natural number

$\Rightarrow \quad 3 = 2^{p/q} \quad \Rightarrow \quad 2^p = 3^q$

But this is not possible for any natural number p and q. The resulting contradiction completes the proof.

Illustration 12 : If in a right angled triangle, a and b are the lengths of sides and c is the length of hypotenuse and $c - b \neq 1$, $c + b \neq 1$, then show that

$\log_{c+b} a + \log_{c-b} a = 2\log_{c+b} a \,.\, \log_{c-b} a$.

Solution : We know that in a right angled triangle

$c^2 = a^2 + b^2$

$c^2 - b^2 = a^2 \qquad \text{..........} \; (i)$

$$\text{LHS} = \frac{1}{\log_a(c+b)} + \frac{1}{\log_a(c-b)} = \frac{\log_a(c-b) + \log_a(c+b)}{\log_a(c+b).\log_a(c-b)}$$

$$= \frac{\log_a(c^2-b^2)}{\log_a(c+b).\log_a(c-b)} = \frac{\log_a a^2}{\log_a(c+b).\log_a(c-b)} \qquad \text{(using (i))}$$

$$= \frac{2}{\log_a(c+b).\log_a(c-b)} = 2\log_{(c+b)} a \,.\, \log_{(c-b)} a = \text{RHS}$$

Illustration 13 : Solve the following equation for x : $\dfrac{6}{5} a^{\log_a x . \log_{10} a . \log_a 5} - 3^{\log_{10}(x/10)} = 9^{\log_{100} x + \log_4 2}$

Solution : Let $A = \log_a x \,.\, \log_{10} a \,.\, \log_a 5 = \log_{10} x \,.\, \log_a 5 = \log_a (5)^{\log_{10} x}$

$\therefore \quad a^A = 5^{\log_{10} x} = 5^{\lambda}$ (say $\log_{10} x = \lambda$)

& $\quad$ let $B = \log_{10}(x/10) = \log_{10} x - 1 = \lambda - 1$

$\therefore \quad 3^B = 3^{\lambda - 1} = \dfrac{3^{\lambda}}{3}$

& $\quad$ let $C = \log_{100} x + \log_4 2 = \log_{10^2} x^1 + \log_{2^2} 2^1 = \dfrac{1}{2}\log_{10} x + \dfrac{1}{2} = \dfrac{\lambda + 1}{2}$

$\therefore \quad 9^C = 9^{\frac{\lambda+1}{2}} = 3^{\lambda+1} = 3.3^{\lambda}$

According to question $\dfrac{6}{5}.5^{\lambda} - \dfrac{3^{\lambda}}{3} = 3.3^{\lambda}$

$\Rightarrow \quad 6.5^{\lambda - 1} = 3^{\lambda}\left(\dfrac{1}{3} + 3\right) \quad \Rightarrow \quad 6.5^{\lambda - 1} = 3^{\lambda - 1}(10) \quad \Rightarrow 5^{\lambda - 2} = 3^{\lambda - 2}$

which is possible only when $\lambda = 2 \;\Rightarrow \log_{10} x = 2 \;\Rightarrow x = 10^2 = 100$ **Ans.**

Illustration 14: Solve : $\log_{(2x-1)}\left(\dfrac{x^4+2}{2x+1}\right)=1$

Solution :

$$\frac{x^4+2}{2x+1} = 2x - 1 \quad \Rightarrow \quad x^4 + 2 = 4x^2 - 1$$

$$\Rightarrow \quad x^4 - 4x^2 + 3 = 0 \quad \Rightarrow \quad x^2 = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm \sqrt{4}}{2} = \frac{4 \pm 2}{2} = 3,\ 1$$

$$\Rightarrow \quad x = \pm\sqrt{3}, \pm 1 \qquad \text{......... (i)}$$

Substituting $x = -\sqrt{3}$ and -1 in $\log_{2x-1}\left(\dfrac{x^4+2}{2x+1}\right)$ we get $2x - 1$ negative. And if $x = 1$ in $2x - 1$ we get base $= 1 \Rightarrow$ reject $x = \pm 1, -\sqrt{3}$

Hence $x = \sqrt{3}$ **Ans.**

ANSWERS FOR DO YOURSELF

1 : (i) **(a)** $\log_3 81 = 4$ **(b)** $\log_{10}(0.001) = -3$ **(c)** $\log_{128} 2 = 1/7$

(ii) **(a)** $32 = 2^5$ **(b)** $4 = (\sqrt{2})^4$ **(c)** $0.01 = 10^{-2}$

(iii) $m = 8$ **(iv)** 6

2 : (i) **(a)** 0 **(b)** 1 **(c)** $\dfrac{1}{5}$ **(ii)** -1 **(iii)** 3

3 : (ii) 1

4 : (i) 3 **(ii)** 5/6 **(iii)** 0 **(iv)** 2 **(v)** (A) **(vi)** (C)

5 : (i) $\bar{8}.6686$ **(ii)** 24 **(iii)** 155 **(iv)** 32

6 : (i) $\dfrac{7}{3}$ **(ii)** $x = 1$

EXERCISE - 01 CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. $\dfrac{1}{\log_{\sqrt{bc}} abc} + \dfrac{1}{\log_{\sqrt{ca}} abc} + \dfrac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to -

(A) 1/2 (B) 1 (C) 2 (D) 4

2. The ratio $\dfrac{2^{\log_{2^{1/4}} a} - 3^{\log_{27} (a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to -

(A) $a^2 - a - 1$ (B) $a^2 + a - 1$ (C) $a^2 - a + 1$ (D) $a^2 + a + 1$

3. The value of the expression, $\log_4\left(\dfrac{x^2}{4}\right) - 2\log_4(4x^4)$ when $x = -2$ is -

(A) – 6 (B) – 5 (C) – 4 (D) meaningless

4. $\dfrac{1}{1+\log_b a+\log_b c} + \dfrac{1}{1+\log_c a+\log_c b} + \dfrac{1}{1+\log_a b+\log_a c}$ is equal to-

(A) abc (B) $\dfrac{1}{abc}$ (C) 0 (D) 1

5. Which one of the following denotes the greatest positive proper fraction ?

(A) $\left(\dfrac{1}{4}\right)^{\log_2 6}$ (B) $\left(\dfrac{1}{3}\right)^{\log_3 5}$ (C) $3^{-\log_3 2}$ (D) $8^{\left(\frac{1}{-\log_3 2}\right)}$

6. If $p = \dfrac{s}{(1+k)^n}$, then n equals -

(A) $\log\dfrac{s}{p(1+k)}$ (B) $\dfrac{\log(s/p)}{\log(1+k)}$ (C) $\dfrac{\log s}{\log p(1+k)}$ (D) $\dfrac{\log p(1+k)}{\log(s/p)}$

7. Value of x satisfying $\log_{10}\sqrt{1+x} + 3\log_{10}\sqrt{1-x} = \log_{10}\sqrt{1-x^2} + 2$ is -

(A) $0 < x < 1$ (B) $-1 < x < 1$ (C) $-1 < x < 0$ (D) none of these

8. The number of real solution of the equation $\log_{10}(7x-9)^2 + \log_{10}(3x-4)^2 = 2$ is -

(A) 1 (B) 2 (C) 3 (D) 4

9. The equation, $\log_2(2x^2) + (\log_2 x).\,x^{\log_x(\log_2 x+1)} + \dfrac{1}{2}\log_4^2(x^4) + 2^{-3\log_{1/2}(\log_2 x)} = 1$ has -

(A) exactly one real solution (B) two real solutions

(C) 3 real solutions (D) no solution

10. Given system of simultaneous equations $2^x.\,5^y = 1$ and $5^{x+1}.\,2^y = 2$. Then -

(A) $x = \log_{10}5$ and $y = \log_{10}2$ (B) $x = \log_{10}2$ and $y = \log_{10}5$

(C) $x = \log_{10}\left(\dfrac{1}{5}\right)$ and $y = \log_{10}2$ (D) $x = \log_{10}5$ and $y = \log_{10}\left(\dfrac{1}{2}\right)$

11. The value of $3^{\log_4 5} + 4^{\log_5 3} - 5^{\log_4 3} - 3^{\log_5 4}$ is -

(A) 0 (B) 1 (C) 2 (D) none of these

12. Given that $\log_p x = \alpha$ and $\log_q x = \beta$, then value of $\log_{p/q} x$ equals -

(A) $\dfrac{\alpha\beta}{\beta-\alpha}$ (B) $\dfrac{\beta-\alpha}{\alpha\beta}$ (C) $\dfrac{\alpha-\beta}{\alpha\beta}$ (D) $\dfrac{\alpha\beta}{\alpha-\beta}$

13. $\log_A B$,where $B = \dfrac{12}{3+\sqrt{5}+\sqrt{8}}$ and $A = \sqrt{1}+\sqrt{2}+\sqrt{5}-\sqrt{10}$ is -

(A) a negative integer
(B) a prime integer
(C) a positive integer
(D) an even–natural number

14. If $\log_c 2 \,.\, \log_b 625 = \log_{10} 16 \,.\, \log_c 10$ where $c > 0$; $c \neq 1$; $b > 1$; $b \neq 1$ determine b -

(A) 25 (B) 5 (C) 625 (D) 16

15. Number of cyphers after decimal before a significant figure comes in $\left(\dfrac{5}{3}\right)^{-100}$ is -

(A) 21 (B) 22 (C) 23 (D) none

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

16. The solution set of the system of equations, $\log_{12} x \left(\dfrac{1}{\log_x 2}+\log_2 y\right) = \log_2 x$ and $\log_2 x.(\log_3(x+y)) = 3\log_3 x$ is -

(A) $x = 6$; $y = 2$ (B) $x = 4$; $y = 3$ (C) $x = 2$; $y = 6$ (D) $x = 3$; $y = 4$

17. If x_1 and x_2 are the solution of the equation $x^{3\log_{10}^{3} x-\frac{2}{3}\log_{10} x} = 100\sqrt[3]{10}$ then -

(A) $x_1 x_2 = 1$ (B) $x_1 \,.\, x_2 = x_1 + x_2$ (C) $\log_{x_2} x_1 = -1$ (D) $\log (x_1 \,.\, x_2) = 0$

18. If $a^x = b$, $b^y = c$, $c^z = a$ and $x = \log_b a^2$; $y = \log_c b^3$ & $z = \log_a c^k$, where a, b, c > 0 & a, b, c ≠ 1, then k is equal to -

(A) $\dfrac{1}{5}$ (B) $\dfrac{1}{6}$ (C) $\log_{64} 2$ (D) $\log_{32} 2$

19. If $\log_k x.\ \log_5 k = \log_x 5$, $k \neq 1$, $k > 0$, then x is is equal to -

(A) k (B) $\dfrac{1}{5}$ (C) 5 (D) none of these

20. If $x^{3/4(\log_3 x)^2+\log_3 x-5/4} = \sqrt{3}$, then x has -

(A) one positive integral value
(B) one irrational value
(C) two positive rational values
(D) none of these

CHECK YOUR GRASP				ANSWER KEY					EXERCISE-1
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	D	A	D	C	B	D	B	D	C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A	A	C	B	B	A,C	A,C,D	B,C	B,C	A,B,C

EXERCISE - 02 **BRAIN TEASERS**

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. Which of the following when simplified reduces to an integer ?

(A) $\dfrac{2\log 6}{\log 12 + \log 3}$ (B) $\dfrac{\log 32}{\log 4}$ (C) $\dfrac{\log_5 16 - \log_5 4}{\log_5 128}$ (D) $\log_{1/4}\left(\dfrac{1}{16}\right)^2$

2. The equation $\dfrac{\log_8\left(\dfrac{8}{x^2}\right)}{(\log_8 x)^2} = 3$ has -

(A) no integral solution (B) one natural solution (C) two real solution (D) one irrational solution

3. Which of the following when simplified, vanishes ?

(A) $\dfrac{1}{\log_3 2} + \dfrac{2}{\log_9 4} - \dfrac{3}{\log_{27} 8}$

(B) $\log_2\left(\dfrac{2}{3}\right) + \log_4\left(\dfrac{9}{4}\right)$

(C) $-\log_8 \log_4 \log_2 16$

(D) $\log_{10} \cot 1 + \log_{10} \cot 2 + \log_{10} \cot 3 + \ldots\ldots\ldots + \log_{10} \cot 89$

4. Which of the following when simplified, reduces to unity ?

(A) $\log_{10}5 \,.\, \log_{10}20 + \log_{10}^2 2$ (B) $\dfrac{2\log 2 + \log 3}{\log 48 - \log 4}$

(C) $-\log_5 \log_3 \sqrt{\sqrt[5]{9}}$ (D) $\dfrac{1}{6}\log_{\frac{\sqrt{3}}{2}}\left(\dfrac{64}{27}\right)$

5. The number $N = \dfrac{1 + 2\log_3 2}{(1 + \log_3 2)^2} + \log_6^2 2$ when simplified reduces to -

(A) a prime number (B) an irrational number

(C) a real which is less than $\log_3\pi$ (D) a real which is greater than $\log_7 6$

6. Which of the following are correct ?

(A) $\log_3 19 \,.\, \log_{1/7}3 . \log_4 1/7 > 2$

(B) $\log_5 (1/23)$ lies between -2 & -1

(C) if $m = 4^{\log_4 7}$ and $n = \left(\dfrac{1}{9}\right)^{-2\log_3 7}$ then $n = m^4$

(D) $\log_{\sqrt{5}} \sin\left(\dfrac{\pi}{5}\right) . \log_{\sqrt{\sin\frac{\pi}{5}}} 5$ simplifies to an irrational number

7. If p, q $\in$ N satisfy the equation $x^{\sqrt{x}} = \left(\sqrt{x}\right)^x$ then p & q are -

(A) relatively prime (B) twin prime

(C) coprime (D) if $\log_q p$ is defined then $\log_p q$ is not & vice versa

8. The expression, $\log_p \log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\ldots\ldots\sqrt[p]{p}}}}}_{\text{n radical sign}}$ where $p \ge 2$, $p \in N$, when simplified is -

(A) independent of p, but dependent on n (B) independent of n, but dependent on p

(C) dependent on both p & n (D) negative

9. Which of the following numbers are positive ?

(A) $\log_{\log_3 2}\left(\frac{1}{2}\right)$ (B) $\log_2\left(\frac{2}{3}\right)^{-2/3}$ (C) $\log_{10}\log_{10}9$ (D) $\log_{10}\sin 25$

10. If $\log_p q + \log_q r + \log_r p$ vanishes where p, q and r are positive reals different than unity then the value of $(\log_p q)^3 + (\log_q r)^3 + (\log_r p)^3$ is -

(A) an odd prime (B) an even prime (C) an odd composite (D) an irrational number

BRAIN TEASERS			ANSWER KEY						EXERCISE-2
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A,D	B,C	A,B,C,D	A,B,C	C,D	A,B,C	A,C,D	A,D	A,B	A

EXERCISE - 03

MISCELLANEOUS TYPE QUESTIONS

FILL IN THE BLANKS

1. The solution set of the equation $x^{\log_a x} = \left(a^{\pi}\right)^{\log_a^3 x}$ is_____________, (where $a > 0$ & $a \neq 1$) .

2. The value of b satisfying $\log_{\sqrt{8}} b = 3\frac{1}{3}$ is _____________.

3. Solution set of the equation, $\log_{10}^2 x + \log_{10}x^2 = \log_{10}^2 2 - 1$ is _____________ .

4. The expression $(0.05)^{\log_{\sqrt{20}}(0.\bar{1})}$ is a perfect square of the natural number _____________.

   (where $0.\bar{1}$ denotes 0.111111 .... ∞)

5. The expression $\sqrt{\log_{0.5}^2 8}$ has the value equal to _____________.

6. If $\log_7 2 = m$, then $\log_{49} 28$ in terms of m has the value equal to _____________.

7. Let p be the integral part of $\log_3 108$ and q be the integral part of $\log_5 375$ then $p + q - pq$ has the value equal to _____________.

8. $\log(a + b) = \log ab$ $(a, b > 0)$, then $\dfrac{(a^3-1)(b^3-1)-1}{ba(a+b)}$ is_____________.

MATCH THE COLUMN

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE OR MORE** statement(s) in **Column-II.**

1. Match the column for values of x which satisfy the equation in Column-I

	Column-I		Column-II
(A)	$\dfrac{\log_{10}(x-3)}{\log_{10}(x^2-21)} = \dfrac{1}{2}$	(p)	5
(B)	$x^{\log x + 4} = 32$, where base of logarithm is 2	(q)	100
(C)	$5^{\log x} - 3^{\log x - 1} = 3^{\log x + 1} - 5^{\log x - 1}$ where the base of logarithm is 10	(r)	2
(D)	$9^{1+\log x} - 3^{1+\log x} - 210 = 0$; where base of log is 3	(s)	$\dfrac{1}{32}$

ASSERTION & REASON

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

1. **Statement-I** : If $a = y^2$, $b = z^2$, $c = x^2$, then $8\log_a x^3 . \log_b y^3 . \log_c z^3 = 27$

   Becuase

   Statement-II : $\log_b a . \log_c b = \log_c a$, also $\log_b a = \dfrac{1}{\log_a b}$

   (A) A (B) B (C) C (D) D

2. **Statement-I** : If $\log_{(\log_5 x)} 5 = 2$, then $x = 5^{\sqrt{5}}$

Becuase

Statement-II : $\log_x a = b$, if $a > 0$, then $x = a^{1/b}$

(A) A (B) B (C) C (D) D

3. **Statement-I** : The equation $\log_{\frac{1}{2+|x|}}(5+x^2) = \log_{(3+x^2)}(15+\sqrt{x})$ has real solutions.

Becuase

Statement-II : $\log_{1/b} a = -\log_b a$ (where $a,b > 0$ and $b \neq 1$) and if number and base both are greater than unity then the number is positive.

(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1

In comparison of two numbers, logarithm of smaller number is smaller, if base of the logarithm is greater than one. Logarithm of smaller number is larger, if base of logarithm is in between zero and one. For example $\log_2 4$ is smaller than $\log_2 8$ and $\log_{\frac{1}{2}} 4$ is larger than $\log_{\frac{1}{2}} 8$.

On the basis of above information, answer the following questions :

1. Identify the correct order :-

(A) $\log_2 6 < \log_3 8 < \log_3 6 < \log_4 6$

(B) $\log_2 6 > \log_3 8 > \log_3 6 > \log_4 6$

(C) $\log_3 8 > \log_2 6 > \log_3 6 > \log_4 6$

(D) $\log_3 8 > \log_4 6 > \log_3 6 > \log_2 6$

2. $\log_{\frac{1}{20}} 40$ is-

(A) greater than one

(B) smaller than one

(C) greater than zero and smaller than one

(D) none of these

3. $\log_{\frac{2}{3}} \frac{5}{6}$ is-

(A) less than zero

(B) greater than zero and less than one

(C) greater than one

(D) none of these

MISCELLANEOUS TYPE QUESTION **ANSWER KEY** EXERCISE -3

- ***Fill in the Blanks***

1. $\{1, a^{1/\pi}\}$ 2. 32 3. $\frac{1}{20}, \frac{1}{5}$ 4. 9 5. 3 6. $\frac{2m+1}{2}$ 7. –5 8. 3

- ***Match the Column***

1. (A) → (p), (B) → (r, s), (C) → (q), (D) → (p)

- ***Assertion & Reason***

1. B 2. A 3. D

- ***Comprehension Based Questions***

Comprehension # 1 : 1. B 2. B 3. B

EXERCISE–04 [A] CONCEPTUAL SUBJECTIVE EXERCISE

1. Prove that $\frac{\log_a N}{\log_{ab} N} = 1 + \log_a b$ & indicate the permissible values of the letters :

2. Compute the following : (a) $\log_{1/3} \sqrt[4]{729.\sqrt[3]{9^{-1}.27^{-4/3}}}$ (b) $a^{\frac{\log_b(\log_b N)}{\log_b a}}$

3. Prove the identity ; $\log_a N.\log_b N + \log_b N.\log_c N + \log_c N.\log_a N = \frac{\log_a N.\log_b N.\log_c N}{\log_{abc} N}$

4. Which is smaller ? 2 or $\left(\log_{e-1} 2 + \log_2 e - 1\right)$.

5. Solve for x : $\log_4 \log_3 \log_2 x = 0$.

6. Find the value of $49^A + 5^B$ where $A = 1 - \log_7 2$ & $B = -\log_5 4$.

7. If $4^A + 9^B = 10^C$, where $A = \log_{16} 4, B = \log_3 9$ & $C = \log_x 83$ then find x.

8. Find the value of the expression $\frac{2}{\log_4 (2000)^6} + \frac{3}{\log_5 (2000)^6}$

9. Solve the system of equations : $\log_a x \log_a (xyz) = 48$
   $\log_a y \log_a (xyz) = 12,\ a > 0,\ a \neq 1$
   $\log_a z \log_a (xyz) = 84$

10. Compute the following :
   (a) $\frac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409} . \left(\left(\sqrt{7}\right)^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6} \right)$ (b) $5^{\log_{1/5}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}} \frac{4}{\sqrt{7}+\sqrt{3}} + \log_{1/2} \frac{1}{10+2\sqrt{21}}$
   (c) $4^{5\log_{4\sqrt{2}}\left(3-\sqrt{6}\right) - 6\log_8\left(\sqrt{3}-\sqrt{2}\right)}$

11. Solve for x : (a) $5^{\log x} + 5x^{\log 5} = 3$ $(a > 0)$; where base of log is a (b) $\log_x 2.\log_{2x} 2 = \log_{4x} 2$

12. Solve for x : (a) $\log_{x+1}(x^2+x-6)^2 = 4$ (b) $x + \log_{10}(1 + 2^x) = x.\log_{10}5 + \log_{10}6$

13. Given $a^2 + b^2 = c^2$ & $a > 0$; $b > 0$; $c > 0$, $c - b \neq 1$, $c + b \neq 1$,
   Prove that : $\log_{c+b} a + \log_{c-b} a = 2.\log_{c+b} a.\log_{c-b} a$

14. (a) Given : $\log_{10} 34.56 = 1.5386$, find $\log_{10} 3.456$; $\log_{10} 0.3456$ & $\log_{10} 0.003456$.
   (b) Find the number of positive integers which have the characteristic 3, when the base of the logarithm is 7.

15. If $\log_{10}2 = 0.3010$ & $\log_{10}3 = 0.4771$. Find the value of $\log_{10} (2.25)$.

16. If $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$. Find the number of integers in :
   (a) 5^{200} (b) 6^{15} & (c) the number of zeros after the decimal in 3^{-100}

17. Find the antilogarithm of 0.75, if the base of the logarithm is 2401.

CONCEPTUAL SUBJECTIVE EXERCISE **ANSWER KEY** EXERCISE-4(A)

1. $\begin{bmatrix} a > 0 ; & a \neq 1 \\ N > 0 ; & N \neq 1 \\ b > 0 ; & b \neq 1/a \end{bmatrix}$ **2.** (a) –1, (b) $\log_b N$ **4.** 2 **5.** 8 **6.** $\frac{25}{2}$ **7.** x = 10

8. $\frac{1}{6}$ **9.** (a^4, a, a^7) or $\left(\frac{1}{a^4}, \frac{1}{a}, \frac{1}{a^7}\right)$ **10.** (a) 1, (b) 6, (c) 9

11. (a) $x = 2^{-\log_5 a}$ (b) $x = 2^{\sqrt{2}}$ or $2^{-\sqrt{2}}$ **12.** (a) x = 1(b) x = 1

14. (a) 0.5386, $\bar{1}.5386, \bar{3}.5386$ (b) 2058 **15.** 0.3522 **16.** (a) 140, (b) 12, (c) 47 **17.** 343

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

1. Find a rational number which is 50 times its own logarithm to the base 10.

2. Prove that $a^x - b^y = 0$ where $x = \sqrt{\log_a b}$ & $y = \sqrt{\log_b a}$, a >0 , b > 0 & a , b $\neq$ 1.

3. If $a = \log_{12} 18$ & $b = \log_{24} 54$ then find the value of ab + 5 (a – b).

4. If $\dfrac{\log_a N}{\log_c N} = \dfrac{\log_a N - \log_b N}{\log_b N - \log_c N}$ where N > 0 & N ≠ 1, a, b, c > 0 & not equal to 1, then prove that $b^2 = ac$.

5. If $\log_b a.\log_c a + \log_a b.\log_c b + \log_a c.\log_b c = 3$ (where a, b, c are different positive real numbers ≠ 1), then find the value of abc.

6. Prove that $\log_7 10$ is greater than $\log_{11} 13$.

7. Solve the system of the equations $(ax)^{\log a} = (by)^{\log b}$; $b^{\log x} = a^{\log y}$ where a > 0, b > 0 and $a \neq b$, $ab \neq 1$

8. Solve for x : $\log_5 120 + (x - 3) - 2.\ \log_5(1 - 5^{x-3}) = -\log_5(0.2 - 5^{x-4})$.

9. Solve for x : $\log 4 + \left(1 + \dfrac{1}{2x}\right)\log 3 = \log\left(\sqrt[x]{3} + 27\right)$.

10. Find the real solutions to the system of equations

$$\log_{10}(2000xy) - \log_{10}x\ .\ \log_{10}y = 4$$
$$\log_{10}(2yz) - \log_{10}y\ .\ \log_{10}z = 1$$
and $$\log_{10}(zx) - \log_{10}z\ .\ \log_{10}x = 0$$

11. Find x satisfying the equation $\log^2\left(1 + \dfrac{4}{x}\right) + \log^2\left(1 - \dfrac{4}{x+4}\right) = 2\log^2\left(\dfrac{2}{x-1} - 1\right)$.

12. Solve for x : $\log^2(4 - x) + \log(4 - x).\log\left(x + \dfrac{1}{2}\right) - 2\log^2\left(x + \dfrac{1}{2}\right) = 0$.

13. Solve the following equation for x & y : $\log_{100}|x + y| = \dfrac{1}{2}$, $\log_{10}y - \log_{10}|x| = \log_{100}4$.

14. Find all real numbers x which satisfy the equation, $2\log_2 \log_2 x + \log_{1/2} \log_2\left(2\sqrt{2}x\right) = 1$.

15. Solve for x : $\log_{3/4} \log_8(x^2 + 7) + \log_{1/2} \log_{1/4}(x^2 + 7)^{-1} = -2$.

BRAIN STORMING SUBJECTIVE EXERCISE	ANSWER KEY	EXERCISE-4(B)

1. 100 **3.** 1 **5.** abc = 1 **7.** x = 1/a and y = 1/b **8.** x = 1 **9.** $x \in \phi$

10. x = 1, y = 5, z = 1 or x = 100, y = 20, z = 100 **11.** $x = \sqrt{2}$ or $\sqrt{6}$

12. $\left\{0, \dfrac{7}{4}, \dfrac{3 + \sqrt{24}}{2}\right\}$ **13.** {–10, 20}, {10/3, 20/3} **14.** x = 8 **15.** x = 3 or –3

Exercise-5

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Indicate all correct alternatives, where base of the log is 2. **[JEE '89]**

The equation $x^{(3/4)(\log x)^2 + \log x - (5/4)} = \sqrt{2}$ has :

(A) at least one real solution (B) exactly three real solutions
(C) exactly one irrational solution (D) complex roots

2. The number $\log_2 7$ is :

(A) an integer (B) a rational number
(C) an irrational number (D) a prime number **[JEE '90]**

3. Find all real numbers x which satisfy the equation

$2\log_2 \log_2 x + \log_{1/2} \log_2 (2\sqrt{2}\ x) = 1.$ **[REE – 1999]**

4. Solve the equation $\log_{3/4} \log_8 (x^2 + 7) + \log_{1/2} \log_{1/4} (x^2 + 7)^{-1} = -2.$ **[REE– 2000]**

5. The number of solution(s) of $\log_4(x - 1) = \log_2(x - 3)$ is/are **[IIT-JEE-2002]**

(A) 3 (B) 1 (C) 2 (D) 0

6. Let $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6}$ **[IIT-JEE 2007]**

Column – I		**Column – II**	
(A)	If $-1 < x < 1$, then f(x) satisfies	(p)	$0 < f(x) < 1$
(B)	If $1 < x < 2$, then f(x) satisfies	(q)	$f(x) < 0$
(C)	If $3 < x < 5$, then f(x) satisfies	(r)	$f(x) > 0$
(D)	If $x > 5$, then f(x) satisfies	(s)	$f(x) < 1$

7. Let (x_0, y_0) be the solution of the following equations

$(2x)^{\ell n 2} = (3y)^{\ell n 3}$

$3^{\ell n x} = 2^{\ell n y}$.

Then x_0 is **[IIT-JEE 2011]**

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

8. The value of $6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\cdots}}}\right)$ is

[IIT-JEE 2012]

9*. If $3^x = 4^{x-1}$, then x = **[JEE (Advanced) 2013]**

(A) $\dfrac{2\log_3 2}{2\log_3 2 - 1}$ (B) $\dfrac{2}{2 - \log_2 3}$ (C) $\dfrac{1}{1 - \log_4 3}$ (D) $\dfrac{2\log_2 3}{2\log_2 3 - 1}$

10. The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is ___________.

[JEE(Advanced) 2018]

PART - II : PREVIOUS YEARS PROBLEMS OF MAINS LEVEL

1. If $\log_p x = \alpha$ and $\log_q x = \beta$, then the value of $\log_{p/q} x$ is **[KCET-1997]**

(1) $\frac{\alpha-\beta}{\alpha\beta}$ (2) $\frac{\beta-\alpha}{\alpha\beta}$ (3) $\frac{\alpha\beta}{\alpha-\beta}$ (4) $\frac{\alpha\beta}{\beta-\alpha}$

2. If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in G.P. Then x is equal to **[KCET-1998]**

(1) $\log_a(\log_b a)$ (2) $\log_a(\log_e a) + \log_a \log_b b$

(3) $-\log_a(\log_a b)$ (4) none of these

3. If $\log_x 256 = 8/5$, then x is equal to **[KCET-2000]**

(1) 64 (2) 16 (3) 32 (4) 8

4. If log 2, $\log(2^x - 1)$ and $\log(2^x + 3)$ are in A.P., then x is equal to **[KCET-2000]**

(1) 5/2 (2) $\log_2 5$ (3) $\log_2 3$ (4) $\log_3 2$

5. The number $\log_2 7$ is **[DCE-2000]**

(1) an integer (2) a rational (3) an irrational (4) a prime number

6. The roots of the equation $\log_2(x^2 - 4x + 5) = (x - 2)$ are **[KCET-2001]**

(1) 4, 5 (2) 2, – 3 (3) 2, 3 (4) 3, 5

7. If x = 198 !, then value of the expression $\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + ... + \frac{1}{\log_{198} x}$ equals **[DCE-2005]**

(1) –1 (2) 0 (3) 1 (4) 198

8. If $\log_{0.3}(x - 1) < \log_{0.09}(x - 1)$, then x lies in the intervel **[DCE-2006]**

(1) $(2, \infty)$ (2) (1, 2) (3) (–2, –1) (4) none of these

9. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then **[AIEEE-2009,**

(1) A = C (2) B = C (3) $A \cap B = \phi$ (4) A = B

10. Let X = {1, 2, 3, 4, 5}. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is : **[AIEEE-2012]**

(1) 5^2 (2) 3^5 (3) 2^5 (4) 5^3

11. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to **[JEE(Main) 2014**

(1) X (2) Y (3) N (4) Y – X

12. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is **[JEE(Main) 2016**

(1) – 4 (2) 6 (3) 5 (4) 3

13. In a class 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible 5 opted Chemistry course. Then the number of student who **did not** opt for any of the three courses is : **[JEE(Main) 2019]**

(1) 38 (2) 42 (3) 102 (4) 1

14. Let $X = \{n \in N: 1 \le n \le 50\}$. If $A = \{n \in X: n$ is a multiple of $2\}$; $B = \{n \in X: n$ is a multiple of $7\}$, then the number of elements in the smallest subset of X containing both A and B is__________. **[JEE(Main) 2020]**

Answers

EXERCISE - 5

PART - I

1.	(ABCD)	**2.**	(C)	**3.**	x = 8	**4.**	x = 3 or – 3	**5.**	(B)

6. (A) → (p), (r), (s) ; (B) → (q), (s) ; (C) → (q), (s) ; (D) → (p), (r), (s)

7.	(C)	**8.**	(4)	**9.**	(ABC)	**10.**	(8)

PART - II

1.	(4)	**2.**	(1)	**3.**	(3)	**4.**	(2)	**5.**	(3)	**6.**	(3)	**7.**	(3)
8.	(1)	**9.**	(2)	**10.**	(2)	**11.**	(2)	**12.**	(4)	**13.**	(1)	**14.**	29

Exercise-6

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

✍ **Marked questions are recommended for Revision.**

*** Marked Questions may have more than one correct option.**

1. Draw the graph of $y = |x|^{1/2}$ for $-1 \le x \le 1$.

2. The number of real solutions of the equation $|x|^2 - 3|x| + 2 = 0$ is :
(A) 4 (B) 1 (C) 3 (D) 2

3.✍ If p, q, r are any real numbers, then
(A) max (p, q) < max (p, q, r) (B) min (p, q) = $\frac{1}{2}$ (p + q – |p – q|)
(C) max (p, q) < min(p, q, r) (D) None of these

4. Let $f(x) = |x - 1|$. Then
(A) $f(x^2) = (f(x))^2$ (B) $f(x + y) = f(x) + f(y)$ (C) $f(|x|) = |f(x)|$ (D) None of these

5. If x satisfies $|x - 1| + |x - 2| + |x - 3| \ge 6$, then
(A) $0 \le x \le 4$ (B) $x \le -2$ or $x \ge 4$ (C) $x \le 0$ or $x \ge 4$ (D) None of these

6. Solve $|x^2 + 4x + 3| + 2x + 5 = 0$.

7. If p, q, r are positive and are in A.P., then roots of the quadratic equation $px^2 + qx + r = 0$ are real for
(A) $\left|\frac{r}{p} - 7\right| \ge 4\sqrt{3}$ (B) $\left|\frac{r}{p} - 7\right| < 4\sqrt{3}$
(C) all p and r (D) no p and r

8. The function $f(x) = |ax - b| + c\,|x|\ \forall\ x \in (-\infty, \infty)$, where a > 0, b > 0, c > 0, assumes its minimum value only at one point if
(A) $a \ne b$ (B) $a \ne c$ (C) $b \ne c$ (D) a = b = c

9.✍ Find the set of all solutions of the equation $2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$

10. The sum of all the real roots of the equation $|x - 2|^2 + |x - 2| - 2 = 0$ is ______.

11. If α & β ($\alpha < \beta$) are the roots of the equation $x^2 + bx + c = 0$, where c < 0 < b, then
(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta < |\alpha|$ (C) $\alpha < \beta < 0$ (D) $\alpha < 0 < |\alpha| < \beta$

12. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that min f(x) > max g(x), then the relation between b and c, is
(A) no relation (B) 0 < c < b/2 (C) $|c| < \sqrt{2}\,|b|$ (D) $|c| > \sqrt{2}\,|b|$

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$
(1) is always positive (2) is always negative (3) does not exist (4) none of these

2. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is
(1) 3 (2) 2 (3) 4 (4) 1

3. The sum of the roots of the equation, $x^2 + |2x - 3| - 4 = 0$, is :

(1) $-\sqrt{2}$ (2) $\sqrt{2}$ (3) -2 (4) 2

4. The equation $\sqrt{3x^2 + x + 5} = x - 3$, where x is real , has :

(1) exactly four solutions (2) exactly one solutions

(3) exactly two solutions (4) no solution

5. The domain of the function $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is :

(1) $(-\infty, \infty)$ (2) $(0, \infty)$ (3) $(-\infty, 0)$ (4) $(-\infty, \infty) - \{0\}$

6. If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1, \left(x \ge \dfrac{1}{2}\right)$, then $\sqrt{4x^2 - 1}$ is equal to

(1) 2 (2) $\dfrac{3}{4}$ (3) $2\sqrt{2}$ (4) $\dfrac{1}{2}$

7. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \ne 0$. If p, q ,r are in the A.P. and $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is :

(1) $\dfrac{\sqrt{34}}{9}$ (2) $\dfrac{2\sqrt{13}}{9}$ (3) $\dfrac{\sqrt{61}}{9}$ (4) $\dfrac{2\sqrt{17}}{9}$

8. Let $S = \{x \in R : x \ge 0 \text{ and } 2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S :

(1) contains exactly two elements. (2) contains exactly four elements.

(3) is an empty set. (4) contains exactly one element

Answers

EXERCISE #6

PART-I

2.	(A)	**3.**	(B)	**4.**	(D)	**5.**	(C)	**6.**	$x = -1 - \sqrt{3}$	or	-4		
7.	(A)	**8.**	(B)	**9.**	$\{-1\} \cup [1, \infty)$	**10.**	4	**11.**	(B)	**12.**	(D)		

PART-I

1.	(3)	**2.**	(3)	**3.**	(2)	**4.**	(4)	**5.**	(3)	**6.**	(2)	**7.**	(2)
8.	(1)												