Logarithms

$3^4 = 81 \Leftrightarrow \log_3 81 = 4$

$N = a^x.$

$N > 0 : a > 0 : a \neq 1.$

$a^x = N \Leftrightarrow x = \log_a N$

$a^x = N \Leftrightarrow x = \log_a N$

$10^2 = 100$

$\Leftrightarrow \log_{10} 100 = 2$

$(0.1)^3 = 0.001$

$\Leftrightarrow \log_{0.1} 0.001 = 3$

$$2^4 = 16 \Leftrightarrow \log_2 16 = 4$$

$$3^3 = 27 \Leftrightarrow \log_3 27 = 3$$

$$a^x = N > 0 \Leftrightarrow \log_a N = x$$

$a > 0 : a \neq 1$

$N > 0 : a > 0 : a \neq 1$

$$10^1 = 10 \Leftrightarrow \log_{10} 10 = 1$$

$$a^1 = a \Leftrightarrow \log_a a = 1$$

$$\log_a a = 1$$

$a > 0 : a \neq 1$

$$a^0 = 1 \iff \log_a 1 = 0$$

$$\log_a 1 = 0 \quad (a > 0 : a \neq 1)$$

$$\log_a a = 1.$$

$$a^{-1} = \frac{1}{a}.$$

$$\iff \log_a (1/a) = -1.$$

$$\iff \log_{1/a} a = -1$$

$$\log_{1/a} a =$$

$$\boxed{\begin{array}{l} 1)\ \log_N N = 1 \\ 2)\ \log_N 1 = 0 \\ 3)\ \log_{1/N} N = \log_N \frac{1}{N} = -1 \end{array}}$$

$a^x = N \Rightarrow \log_a N = x$

then N is Anti log of x to the base a.

$2^3 = 8 \Leftrightarrow \log_2 8 = 3$

$8 =$ Anti log of 3 with base 2.

<u>Prob.:</u> Anti log of $\frac{5}{6}$ with base 64.

$= (64)^{5/6}$

$= (2^6)^{5/6} = 2^5 = \boxed{32}$

Find Anti log of $\frac{2}{3}$ with base 27

$$(27)^{2/3} = (3^{\cancel{3}})^{2/\cancel{3}} = 3^2 = \boxed{9}$$

<u>Properties</u>:

1) $\log_a m + \log_a n = \log_a (mn)$

$m, n > 0 \; : a > 0 ; a \neq 1$

2) $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

3) $\log_b a^x = x \cdot \log_b a$

$$\log_b a^x = x \log_b a$$

$$L.H.S = \log_b a^x = K.$$

$$a^x = b^K.$$

$$a = (b^K)^{1/x}.$$

$$a = b^{\frac{K}{x}}.$$

$$L.H.S = R.H.S.$$

$$R.H.S = x.\log_b b^{K/x}.$$

$$= \cancel{x}.\frac{K}{\cancel{x}}$$

$$= \boxed{K.}$$

$$\log_{a^m} N = \frac{1}{m} \log_a N$$

Ex: $\log_{16} 2^4 = \log_{16} 16 = 1$

$\log_{16} 2 = \log_{2^4} 2 = \frac{1}{4} \log_2 2 = \boxed{\frac{1}{4}}$

$\log_3 3^5 = 5 \cdot \log_3 3$

$= \boxed{5}$

$$\boxed{a^{\log_a N} = N} \qquad (a^m)^n = a^{mn}.$$

$$\Leftrightarrow \log_a N = \log_a N$$

$$a^{\log_c b} = b^{\log_c a}.$$

Take $\log_c b = \alpha.$

$$\Rightarrow b = c^\alpha$$

$$L.H.S = a^\alpha$$

$$R.H.S = b^{\log_c a}$$

$$= (c^\alpha)^{\log_c a}$$

$$= c^{\alpha \log_c a}$$

$$= c^{\log_c (a^\alpha)}$$

$$= a^\alpha$$

$$\therefore L.H.S = R.H.S.$$

1) $\log_4 m = 1.5 \Rightarrow m = \_\_\_\_$

$1728 = (12)^3$

$= (2^2 \cdot 3)^3$

$m = 4^{1.5}$

$= (2^{\cancel{2}})^{3/\cancel{2}} = 2^3 = \boxed{8}$

2) $\log_{2\sqrt{3}} 1728 = x \Rightarrow x = \_\_\_\_$

$1728 = (2\sqrt{3})^x$

$(2^2 \cdot 3)^3 = (2\sqrt{3})^x \Rightarrow \boxed{x = 6}$

$$\log_a\left[1+\log_b\left\{1+\log_c\left(1+\log_p N\right)\right\}\right]=0$$

then $N =$ ____

<u>Sol:</u>

$$\cancel{1}+\log_b\left\{1+\log_c\left(1+\log_p N\right)\right\}=\cancel{1}$$

$$\cancel{1}+\log_c\left(1+\log_p N\right)=b^0=\cancel{1}$$

$$(\cancel{1}+\log_p N)=\cancel{1}$$

$$N=p^0=1.$$

$$\log_a\left[1+\log_b\left\{1+\log_c\left(1+\log_p N\right)\right\}\right]=0$$

$$\cancel{1}+\log_b\left\{1+\log_c\left(1+\log_p N\right)\right\}=a^0=\cancel{1}$$

$$\log_b\left\{1+\log_c\left(1+\log_p N\right)\right\}=0$$

$$\cancel{1}+\log_c\left(1+\log_p N\right)=\cancel{1}$$

$$\log_c\left(1+\log_p N\right)=0$$

$$\cancel{1}+\log_p N=\cancel{1}$$

$$\log_p N=0$$

$$N=p^0$$

$$=1$$

$$4^{\log_2 2x} = 36 \Rightarrow x = \_\_\_$$

$$\left(2^2\right)^{\log_2 2x} = 36$$

$$2^{2\log_2 2x} = 36$$

$$2^{\log_2 4x^2} = 36$$

$$4x^2 = 36$$

$$\Rightarrow \boxed{x = 3}$$

$$\left(\frac{1}{2}\right)^{\log_2 5} = \underline{\qquad\qquad}.$$

$$\left(2^{-1}\right)^{\log_2 5}$$

$$= 2^{-\log_2 5}$$

$$= 2^{\log_2 5^{-1}}$$

$$= 5^{-1} = \boxed{1/5}$$

1) If $a^2+b^2=23ab$ then $\log\left(\frac{a+b}{5}\right)=$ ______

$a^2+b^2+2ab = 25ab.$

$(a+b)^2 = 5^2\,ab.$

$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

$\left(\frac{a+b}{5}\right)^2 = ab. \Rightarrow \log\left(\frac{a+b}{5}\right)^2 = \log ab$

$2\log\left(\frac{a+b}{5}\right) = \log a + \log b$

$\log\left(\frac{a+b}{5}\right) = \boxed{\frac{1}{2}\left[\log a + \log b\right]}$

Simplify

$$\frac{1}{2}\log 9 + 2\log 6 + \frac{1}{4}\log 81 - \log 12.$$

$$= \underline{\qquad}$$

<u>Sol:</u>

$$\frac{1}{2}\log 3^2 + 2\left[\log 3 + \log 2\right] + \frac{1}{4}\log 3^4$$

$$- \left[\log 2^2 \cdot 3\right]$$

$$= \cancel{2} \times \frac{1}{\cancel{2}}\log 3 + 2\log 3 + 2\log 2 + \cancel{4} \times \frac{1}{\cancel{4}}\log 3$$

$$- \log 2^2 - \log 3$$

$$= 4\log 3 + \cancel{2\log 2} - \cancel{2\log 2} - \log 3$$

$$= 3\log 3$$

If $\log_e x - \log_e y = a$; $\log_e y - \log_e z = b$;

$\log_e z - \log_e x = c$

then $\left(\frac{x}{y}\right)^{b-c} \cdot \left(\frac{y}{z}\right)^{c-a} \cdot \left(\frac{z}{x}\right)^{a-b} = \_\_\_\_\_$

Sol: $\log_e\left(\frac{x}{y}\right) = a \Rightarrow \frac{x}{y} = e^a$

$\left(\frac{x}{y}\right)^{b-c} = \left(e^a\right)^{b-c} = e^{ab-ac}$

|||ly $\left(\frac{y}{z}\right)^{c-a} = e^{bc-ba}$; $\left(\frac{z}{x}\right)^{a-b} = e^{ac-bc}$

$e^{\cancel{ab}-\cancel{ac}+\cancel{bc}-\cancel{ba}+\cancel{ca}-\cancel{bc}}$

$= e^0 = 1$

Base changing property

$$\log_{a} N = \frac{\log_b N}{\log_b a} = \frac{\log_c N}{\log_c a} = \frac{\log_d N}{\log_d a}$$

$b, c, d \ldots > 0$ - - - - - - .

$b, c, d \neq 1.$

'e'   '10'.

$\log x = \log_e x = \ln x$ (Natural logarithm)

$\log_{10} x$ (Commen logarithms)

$\log_b a \cdot \log_c b$

$= \log_c a$

$\log_e x \times \log_{10} e = \log_{10} x$

$\log_e x \times 0.4342$

$= \log_{10} x$

$\log_a N = \dfrac{\log_b N}{\log_b a} = \dfrac{\log_e N}{\log_e a} = \dfrac{\log N}{\log a}$

$\log_{10} x \cdot \log_e 10$

$= \dfrac{\log x}{\cancel{\log 10}} \times \dfrac{\cancel{\log 10}}{\log e}$

$= \dfrac{\log x}{\log e} = \log_e x$

$\log_{10} x \cdot \log_e 10 = \log_e x$

2.303 ✓

$\log_e x \times \log_{10} e = \log_{10} x$

$\log_e x \times 0.4342 = \log_{10} x$