Adversarial Search

CS5491: Artificial Intelligence
ZHICHAO LU

Content Credits: **Prof. Wei**'s CS4486 Course and **Prof. Boddeti**'s AI Course

TODAY

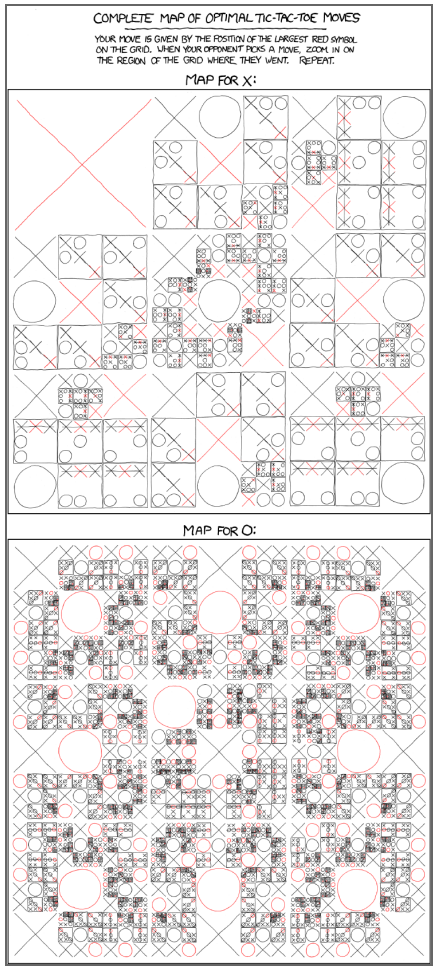
Adversarial Games

Minimax Games

$\alpha - \beta$ pruning

Reading

- Today's Lecture: RN Chapter 5.1-5.5



XKCD

AGENTS INTERACTING WITH AGENTS

GAME PLAYING STATE-OF-THE-ART

Checkers: 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved.

Chess: 1997: Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 layers. Current programs are even better, if less historic.

Go: Go: Human champions are now starting to be challenged by machines. In go, $b > 300$. Classic programs use pattern knowledge bases, but big recent advances use Monte Carlo (randomized) expansion methods.

- 2016: Alpha Go defeats human champion. Uses Monte Carlo Tree Search, learned evaluation function.

ADVERSARIAL GAMES

TYPES OF GAMES

Many different kinds of games.

Axes:

- Deterministic or stochastic?
- One, two, or more players?
- Zero sum?
- Perfect information (can you see the state)?

Want algorithms for calculating a strategy (policy) which recommends a move from each state.

DETERMINISTIC GAMES

Many possible formalizations, one is:

- **States** S (start at s_0)
- **Players** $P = \{1, \ldots, N\}$ (usually take turns)
- **Actions** A (may depend on player / state)
- **Transition Function** $S \times A \to S$
- **Terminal Test** $S \to \{t, f\}$
- **Terminal Utilities** $S \times P \to R$

Solution for a player is a policy: $S \to A$

GAME THEORY

Zero-Sum Games

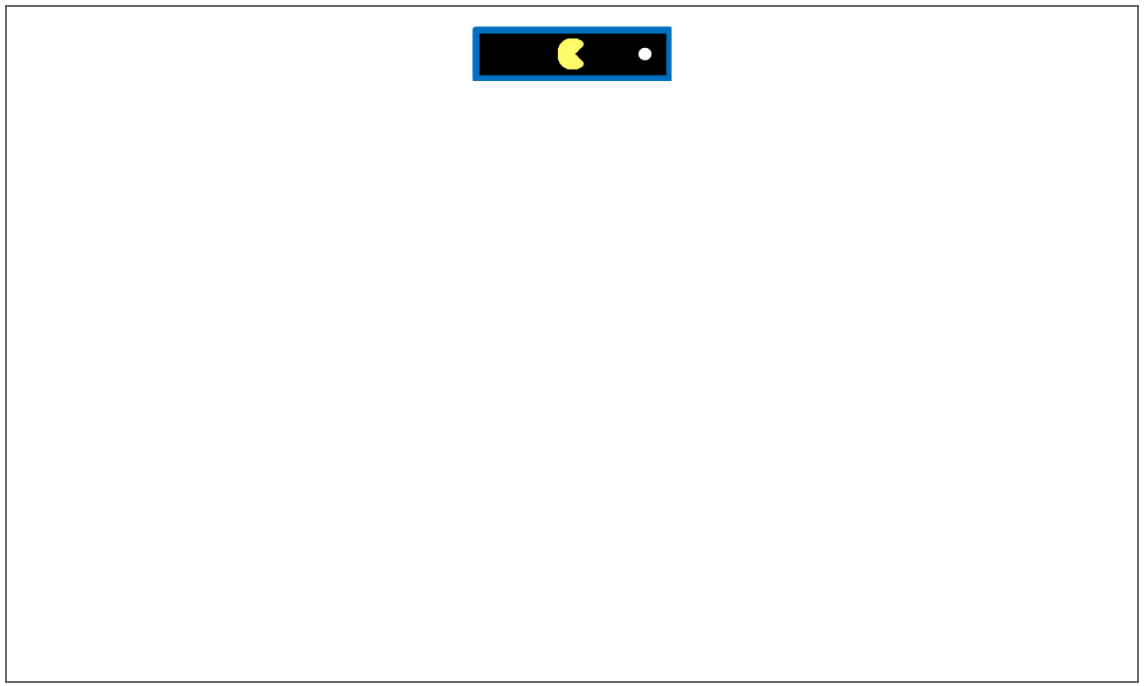
- Agents have opposite utilities (values on outcomes)
- Lets us think of a single value that one maximizes and the other minimizes
- Adversarial, pure competition

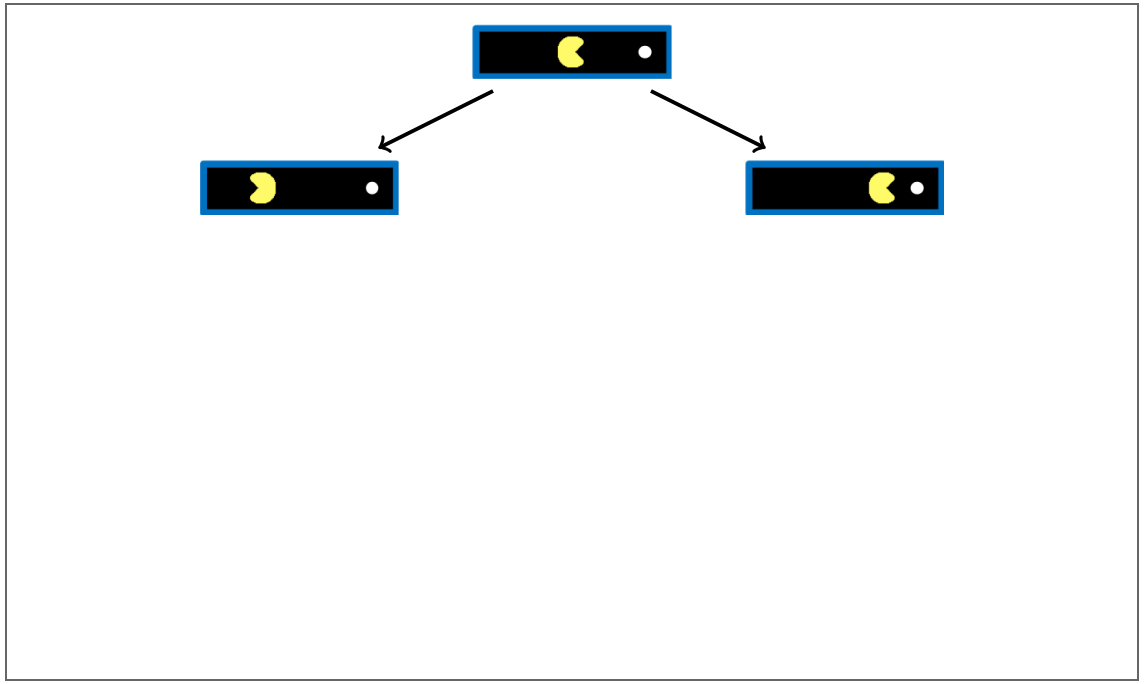
General Games

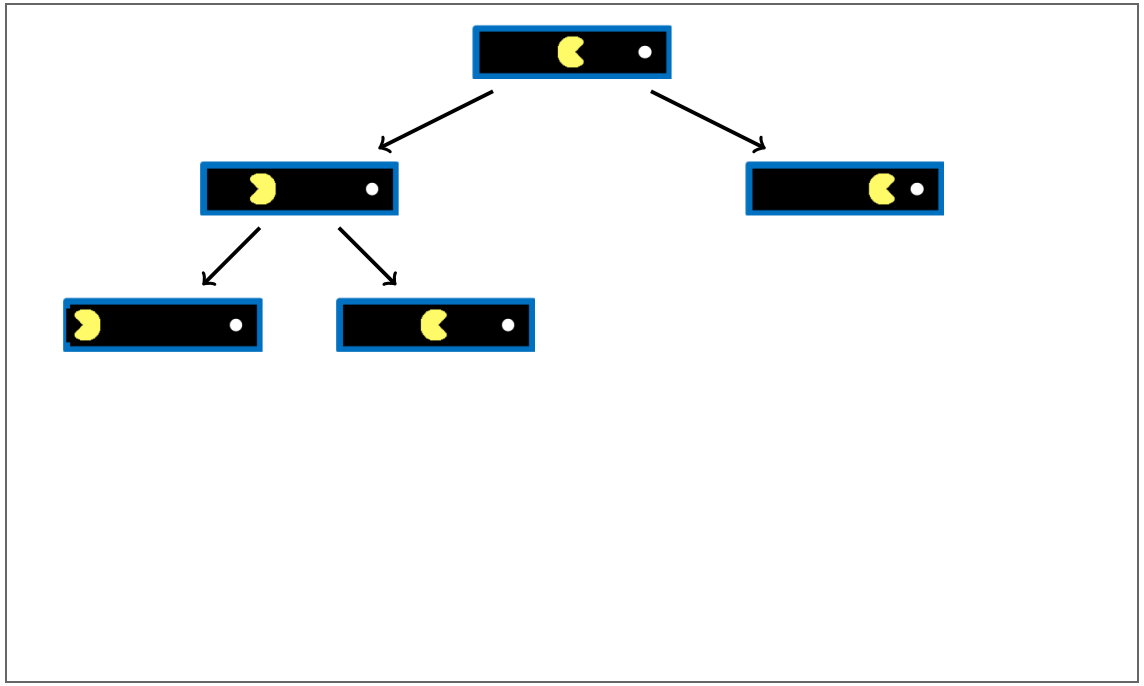
- Agents have independent utilities (values on outcomes)
- Cooperation, indifference, competition, and more are all possible
- More later on non-zero-sum games

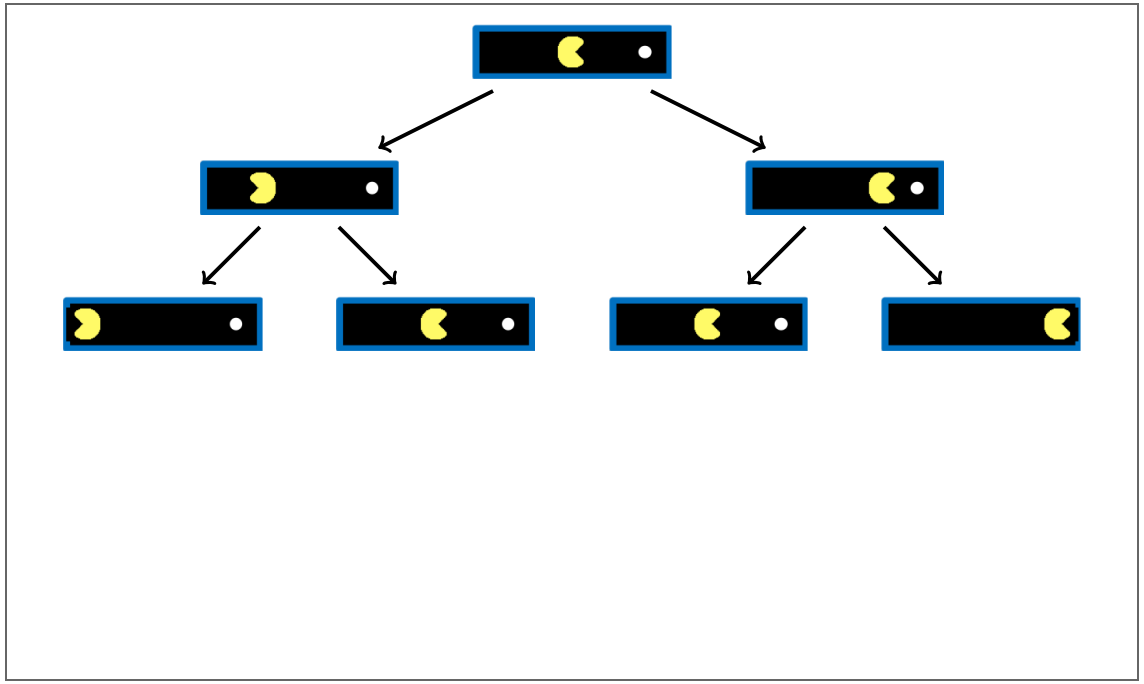
ADVERSARIAL SEARCH

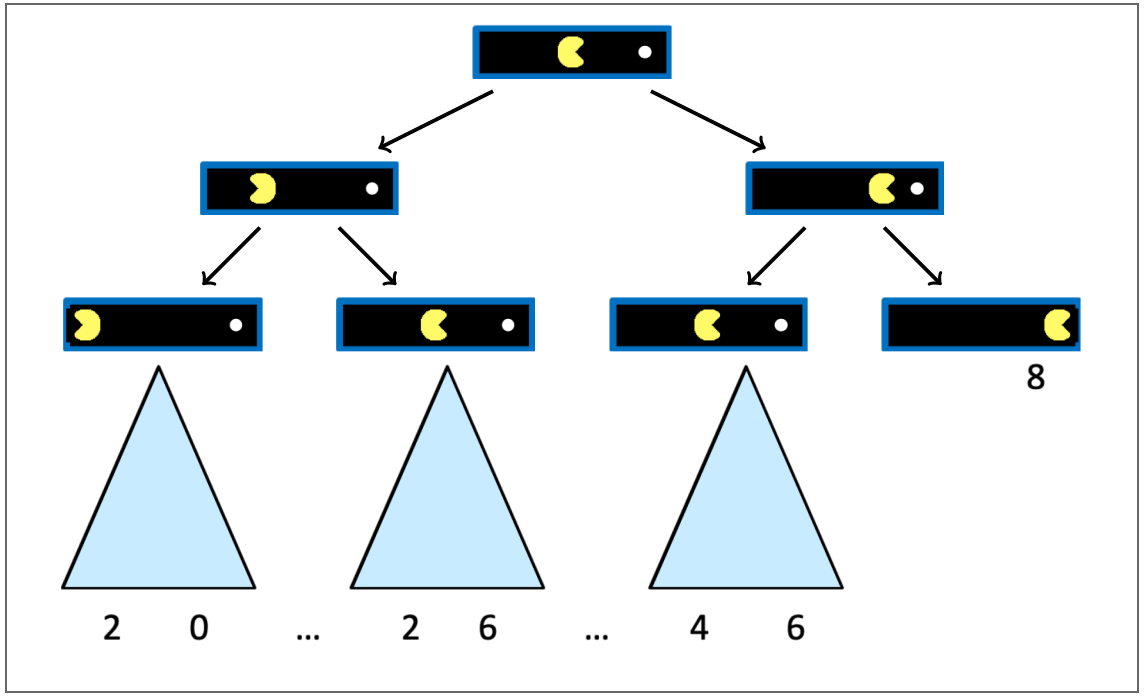
SINGLE-AGENT TREES







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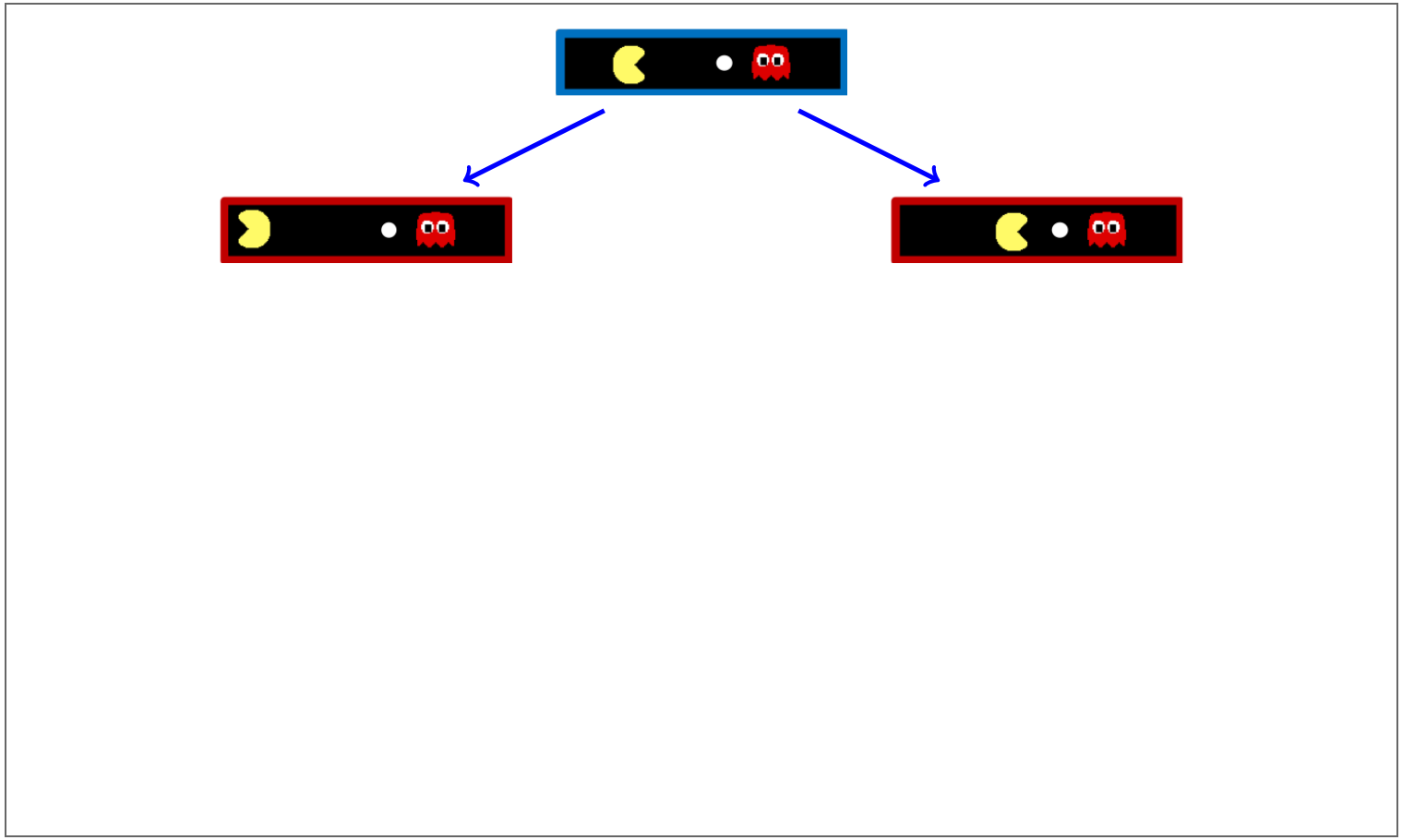
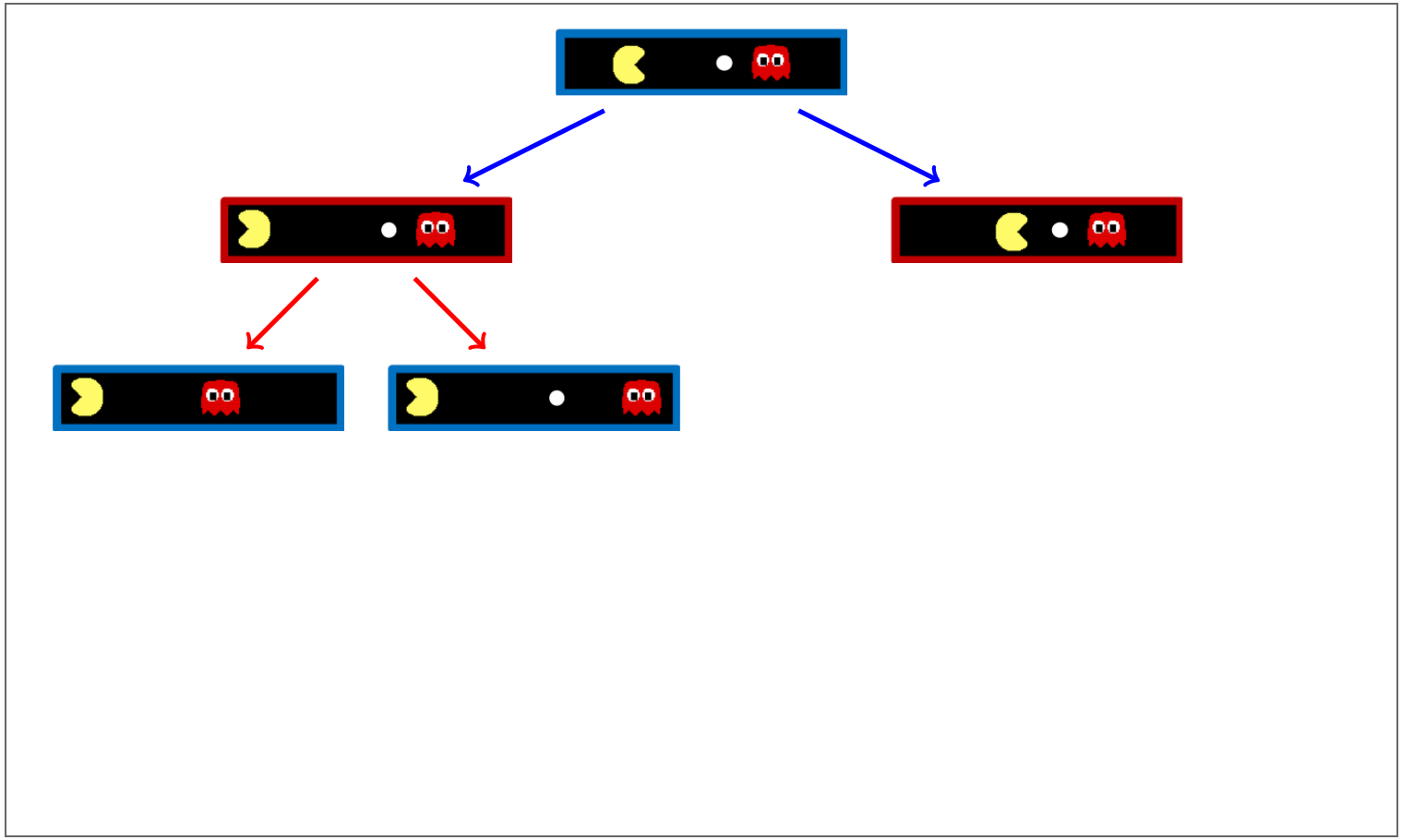
Value of State: The best achievable outcome (utility) from that state

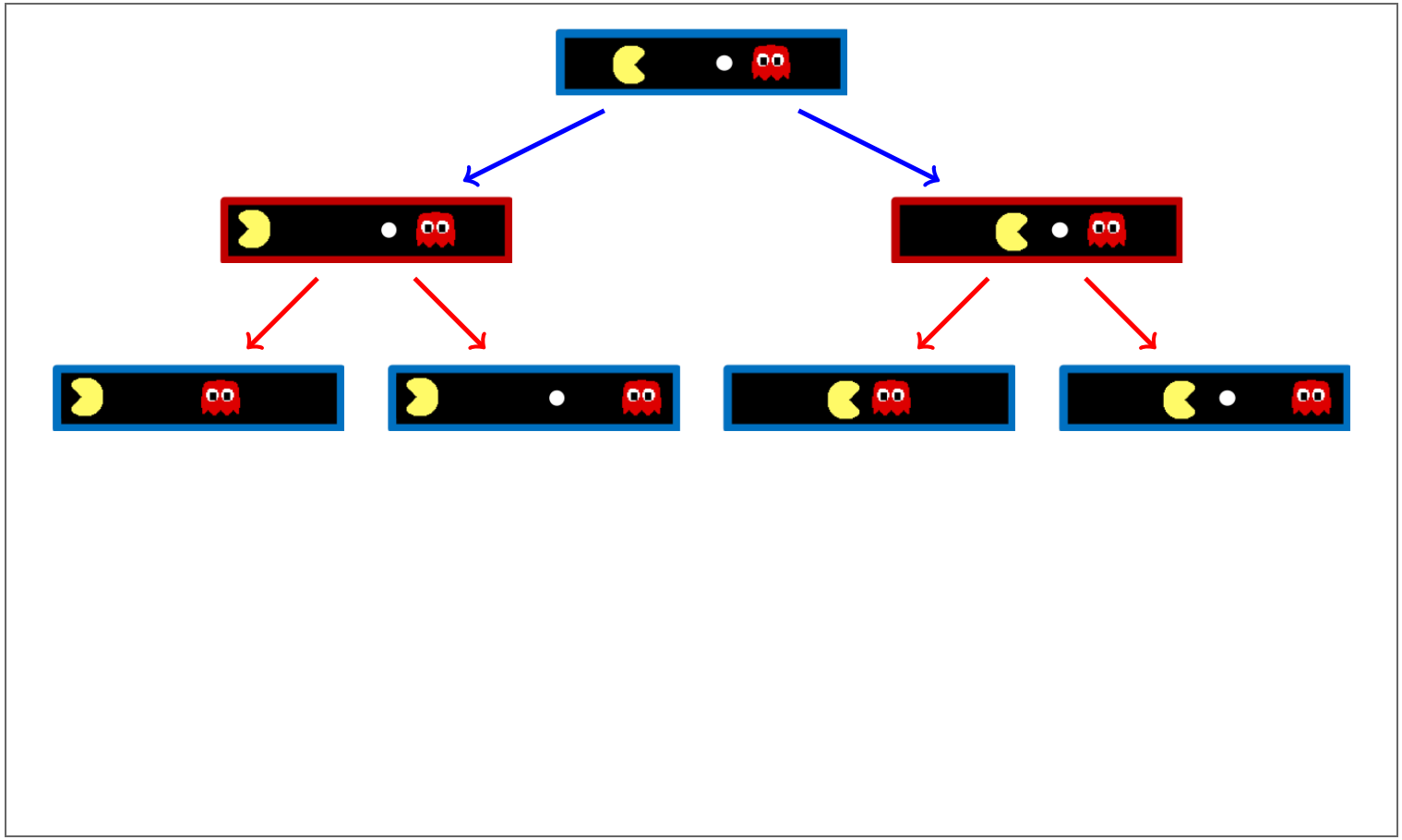
Terminal States: $V(s)$ known

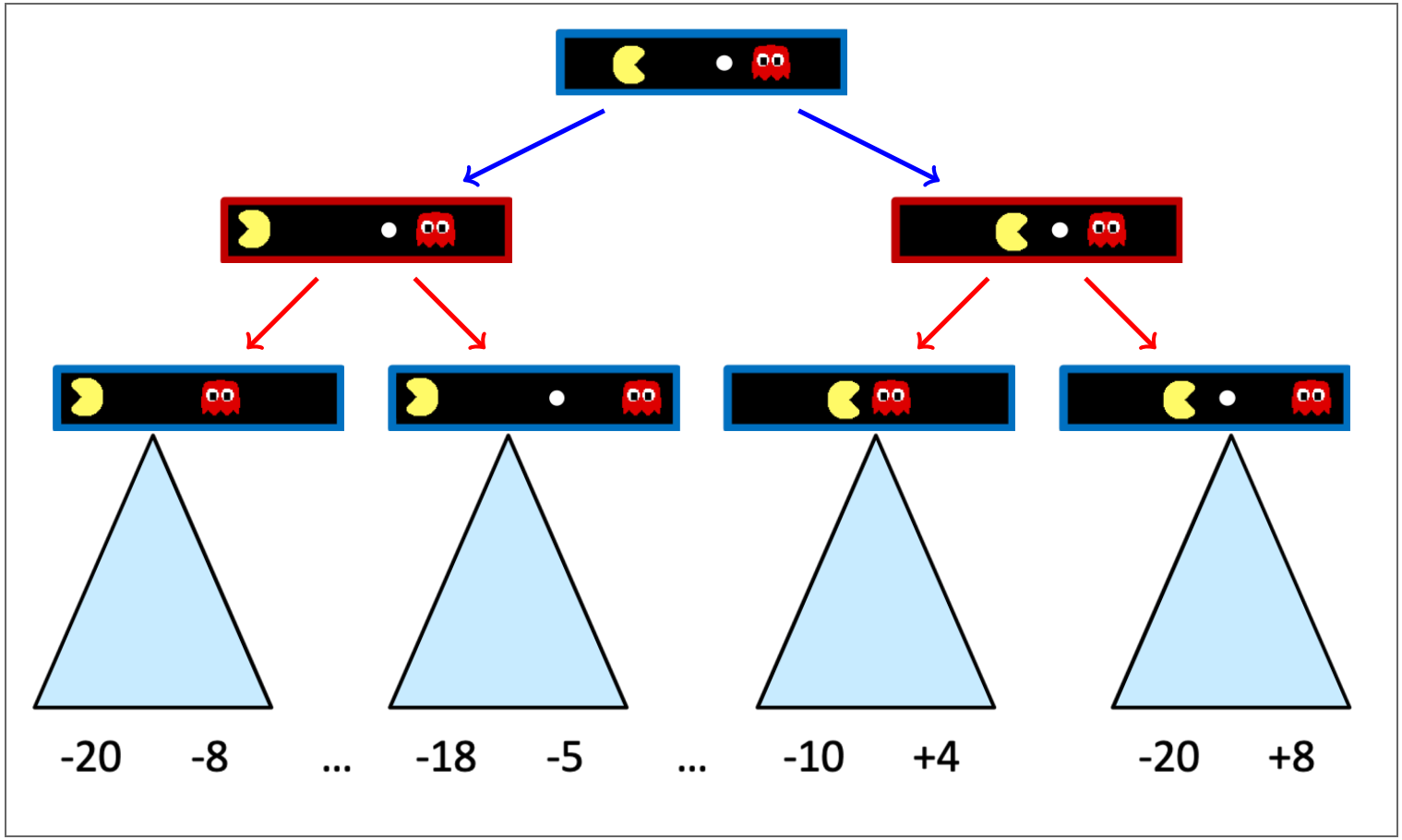
Non-Terminal States: $V(s) = \max_{s' \in children(s)} V(s')$

ADVERSARIAL GAME TREES





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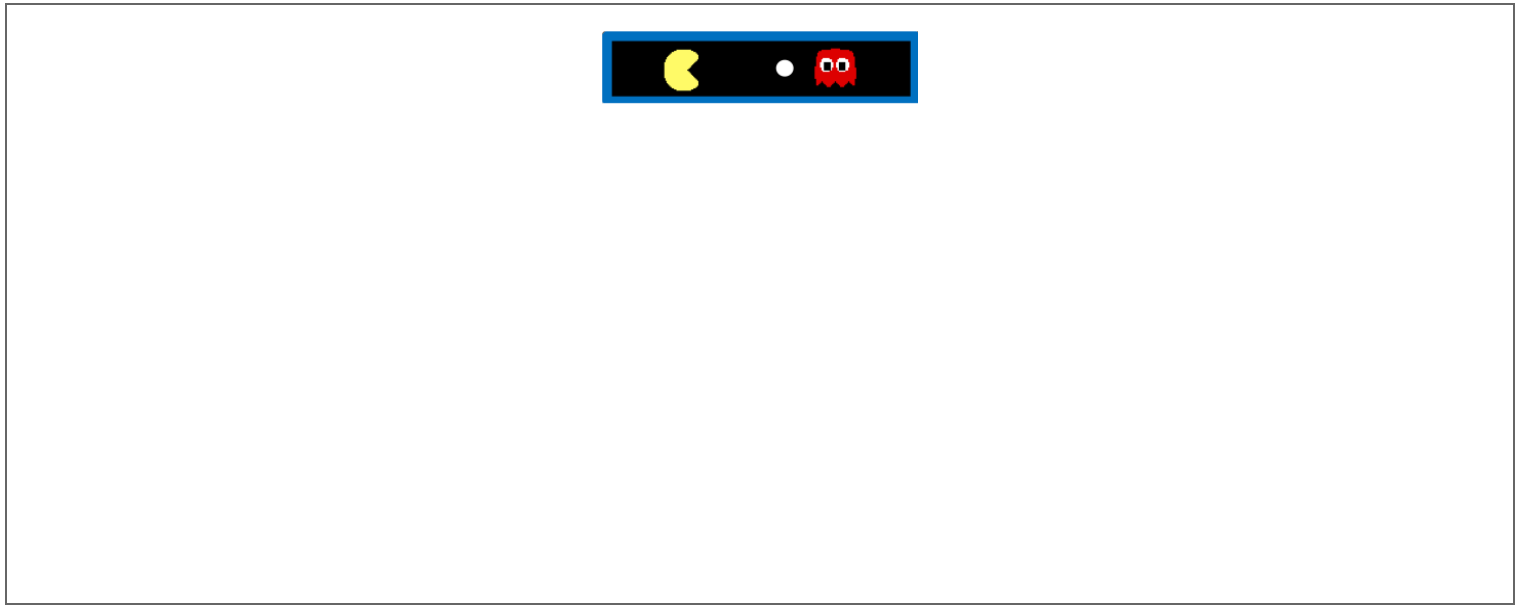
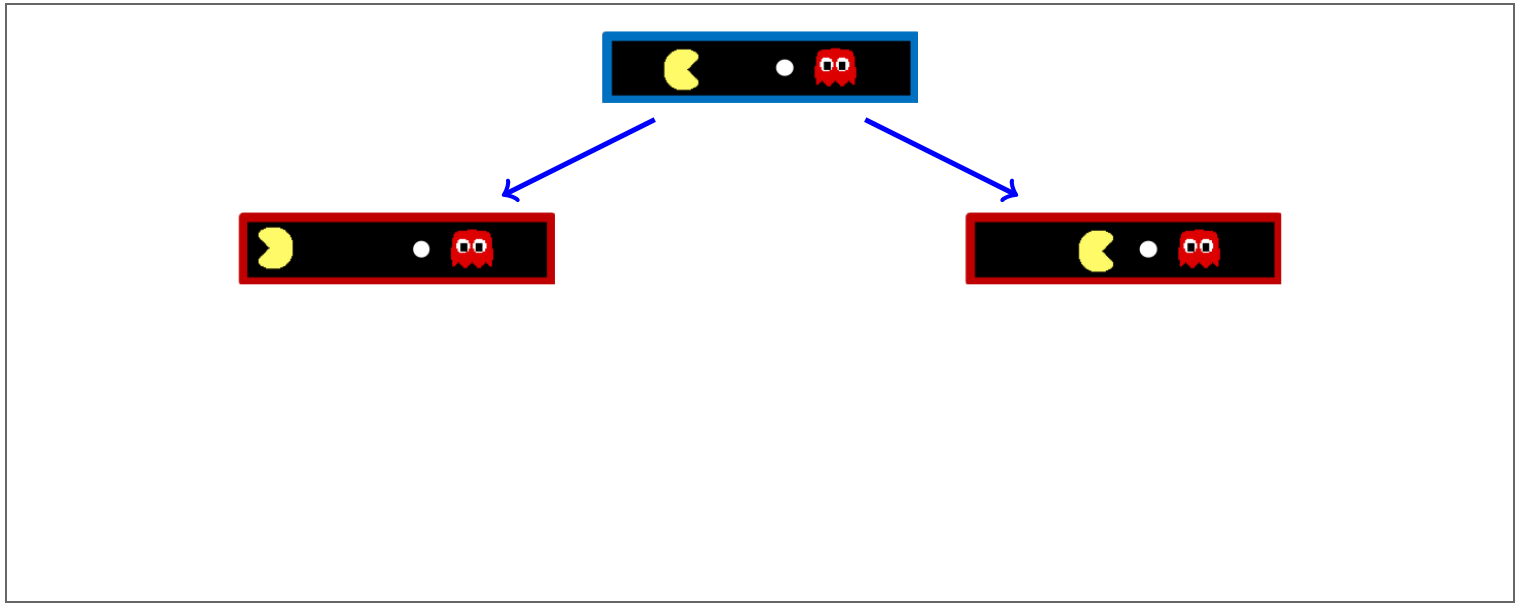
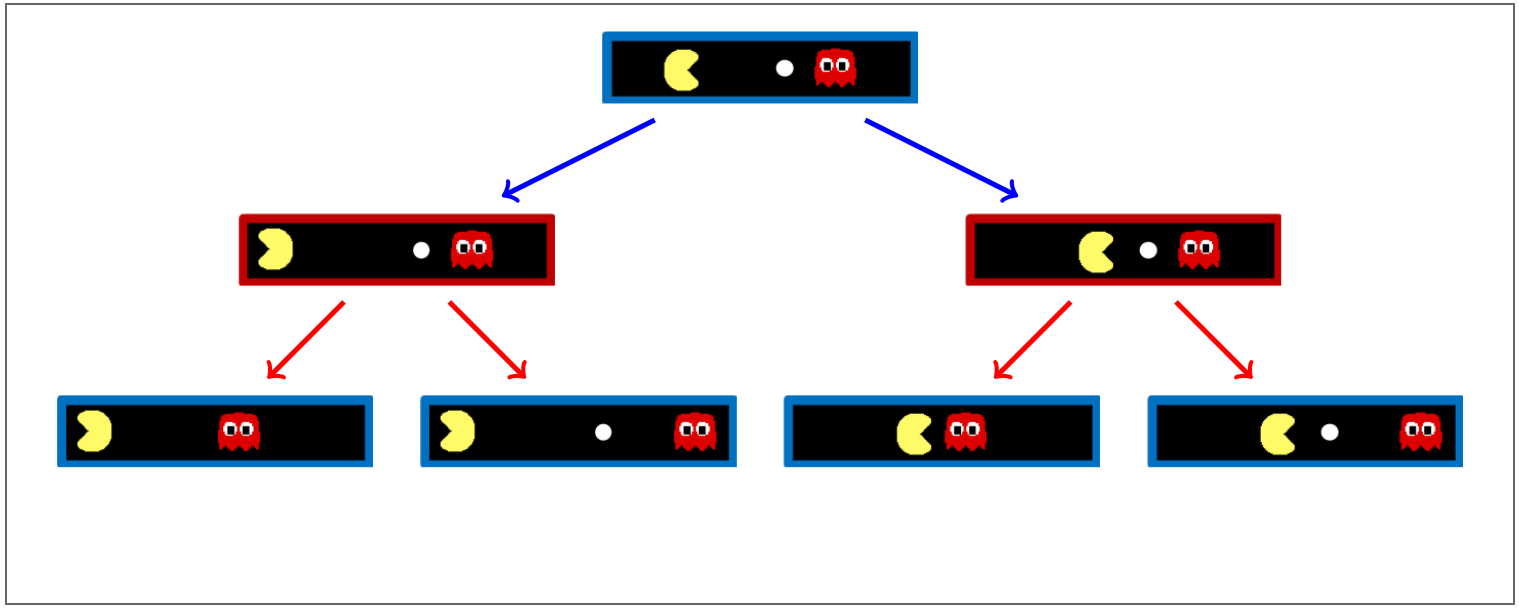
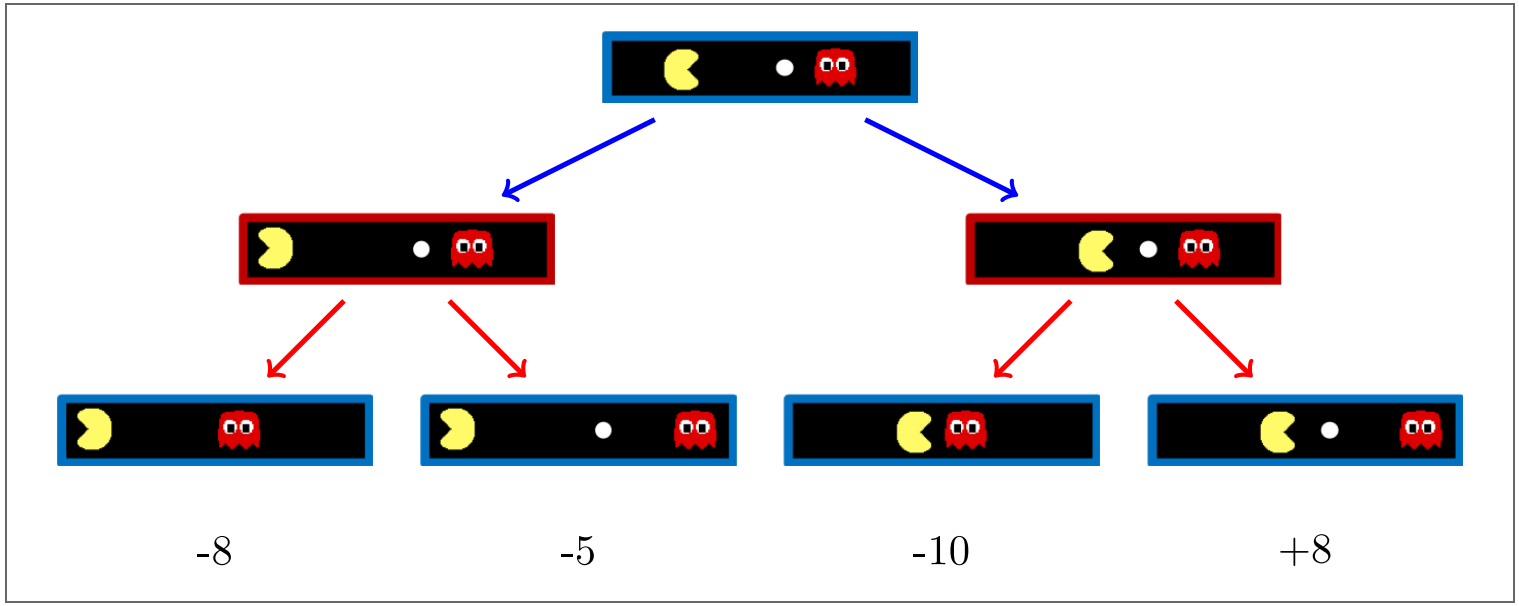
MINIMAX VALUES







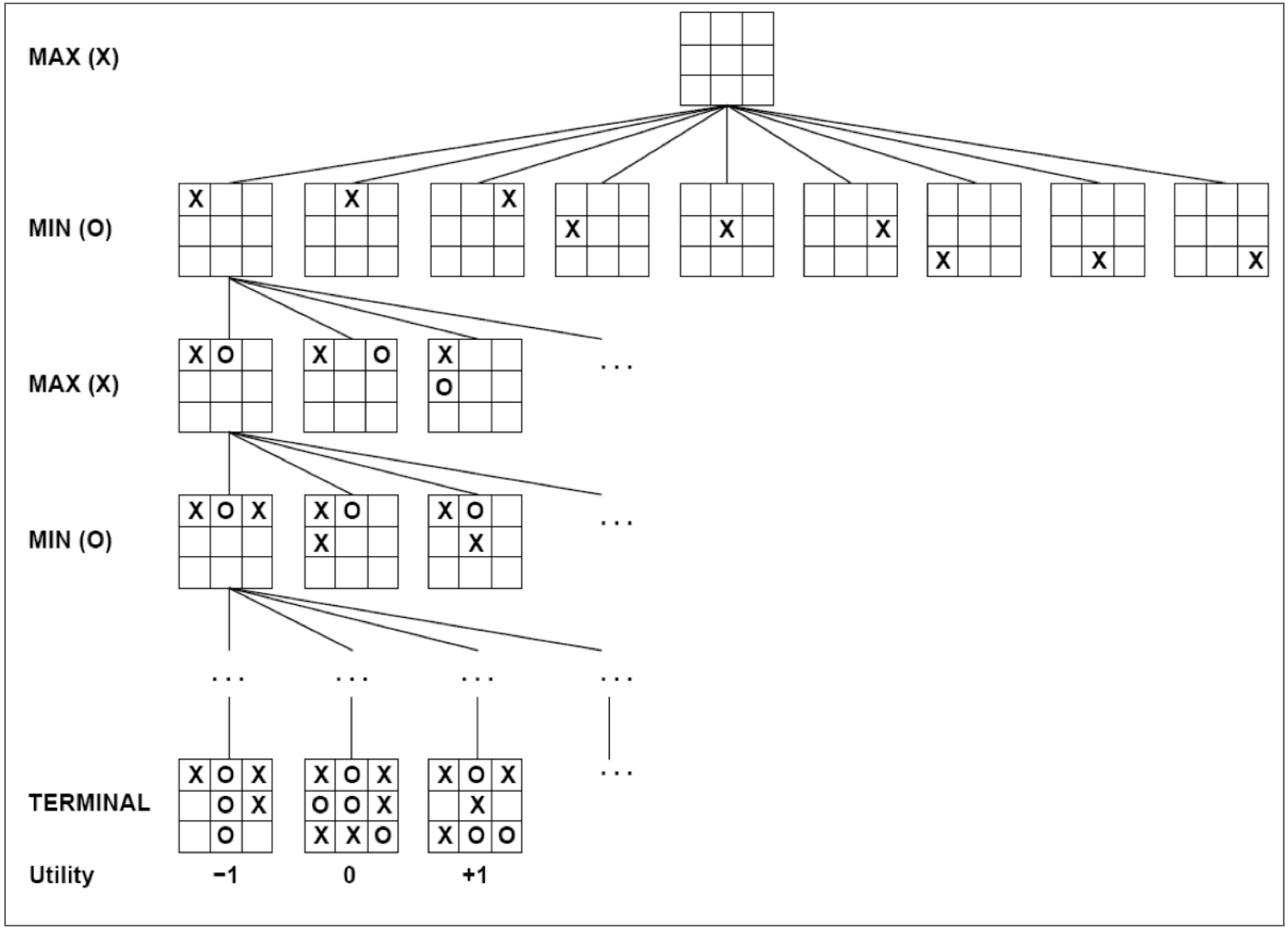


States Under Agent's Control: $V(s) = \max_{s' \in successors(s)} V(s')$

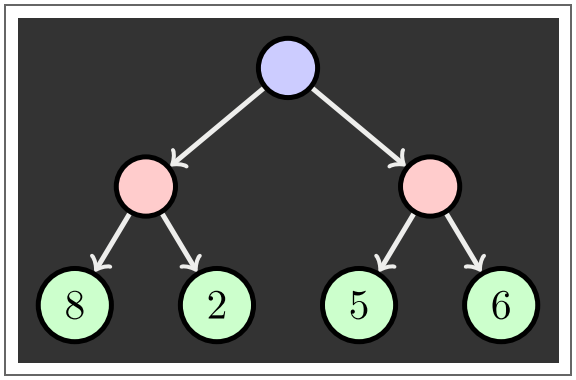
States Under Opponent's Control: $V(s') = \min_{s \in successors(s')} V(s)$

Terminal States: $V(s)$ known

TIC-TAC TOE


MAX (X)
MIN (O)
MAX (X)
MIN (O)
TERMINAL
Utility
−1
0
+1

ADVERSARIAL SEARCH (MINIMAX)



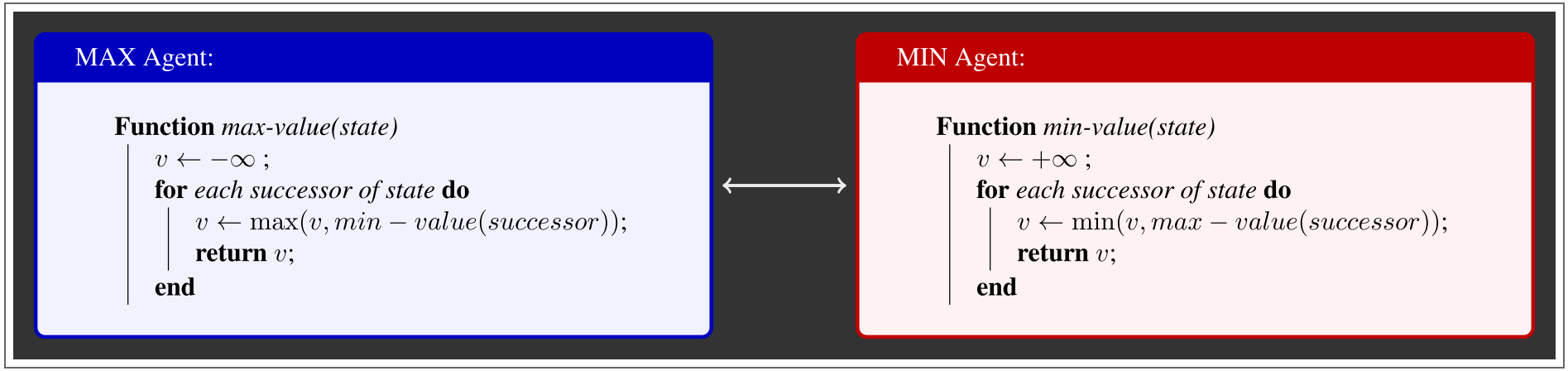
Deterministic, zero-sum games:

- Tic-tac-toe, chess, checkers
- One player maximizes result
- The other minimizes result

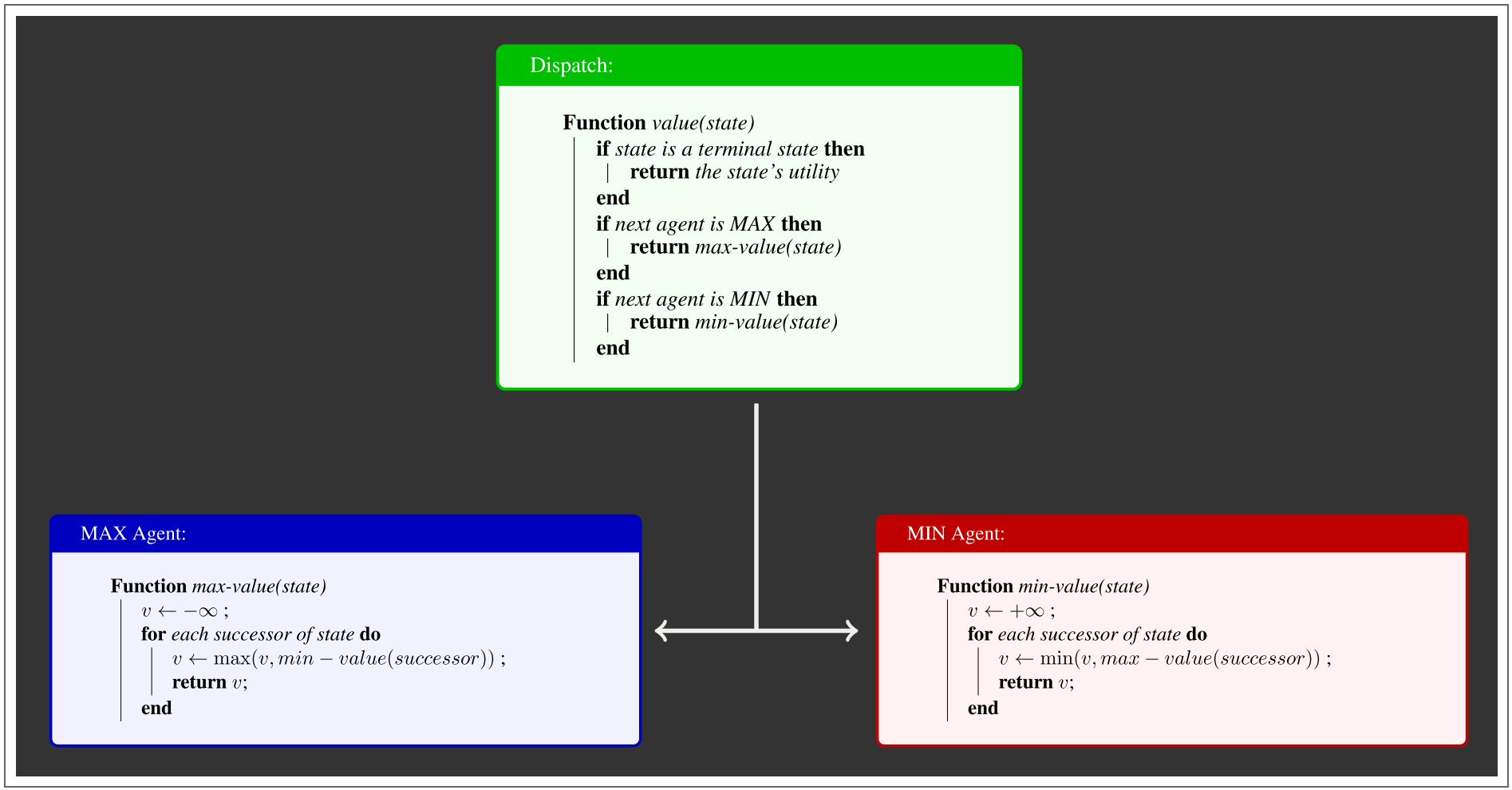
Minimax Search:

- A state-space search tree
- Players alternate turns
- Compute each node's minimax value: the best achievable utility against a rational (optimal) adversary

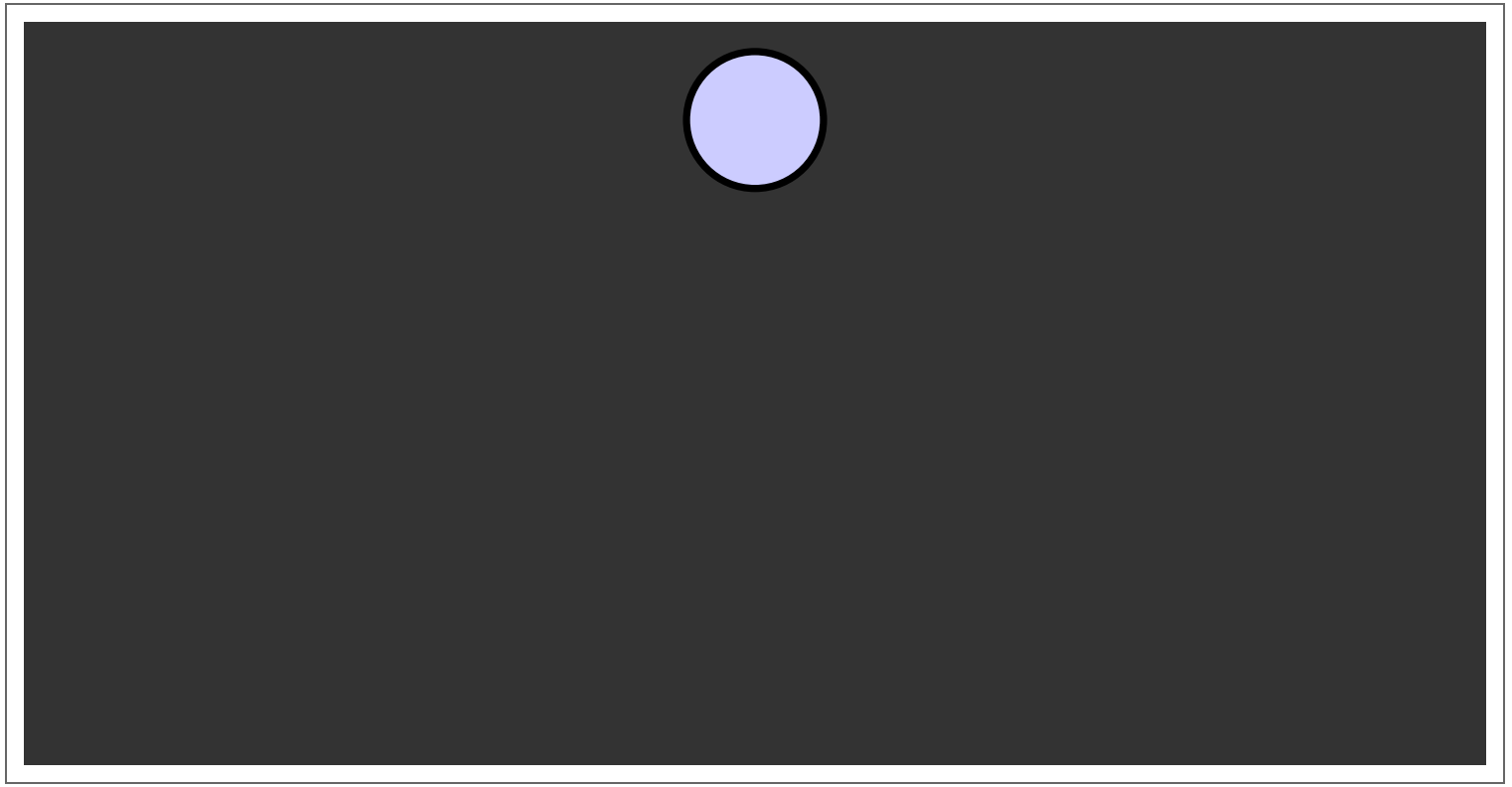
MINIMAX IMPLEMENTATION

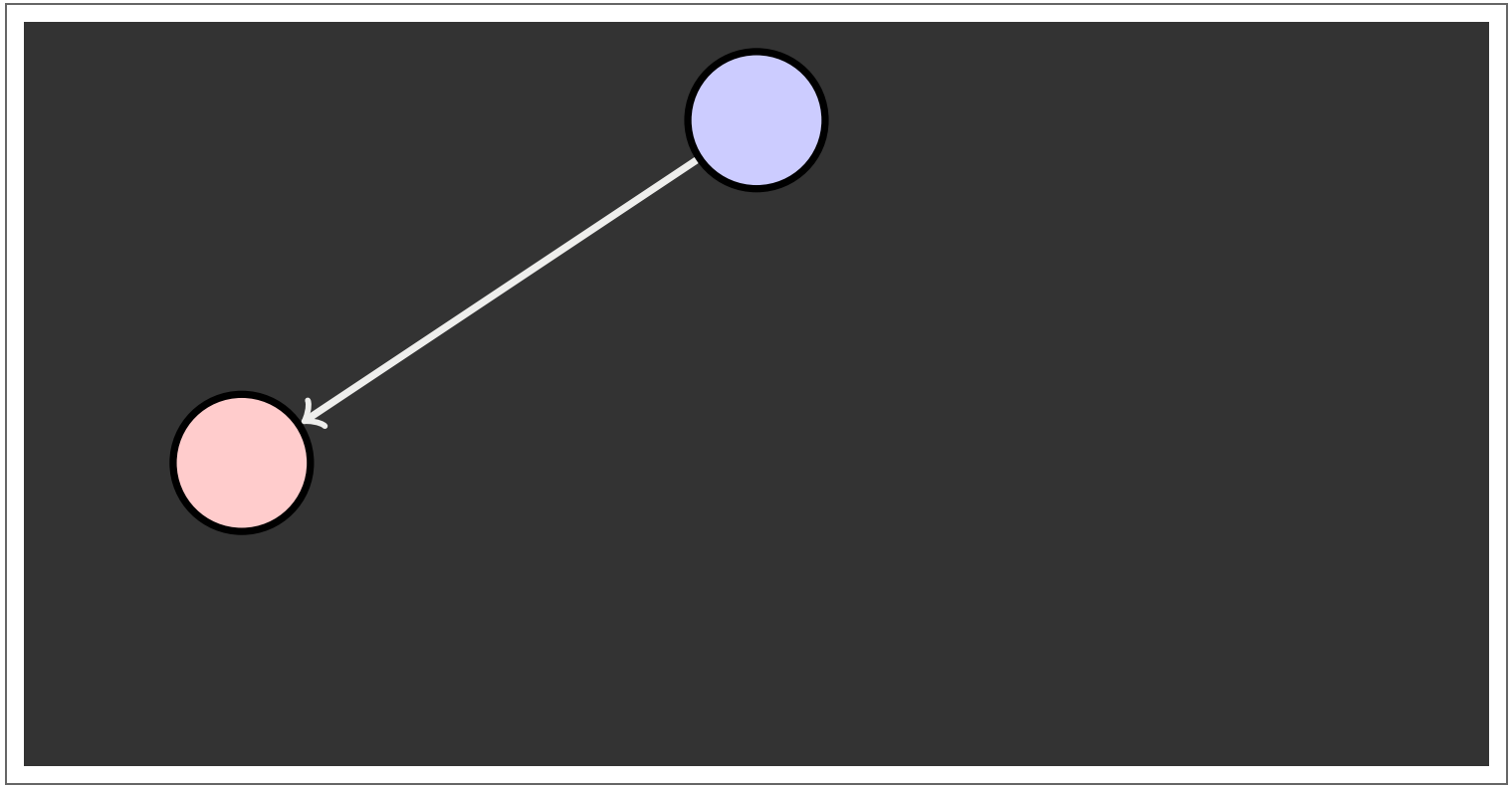


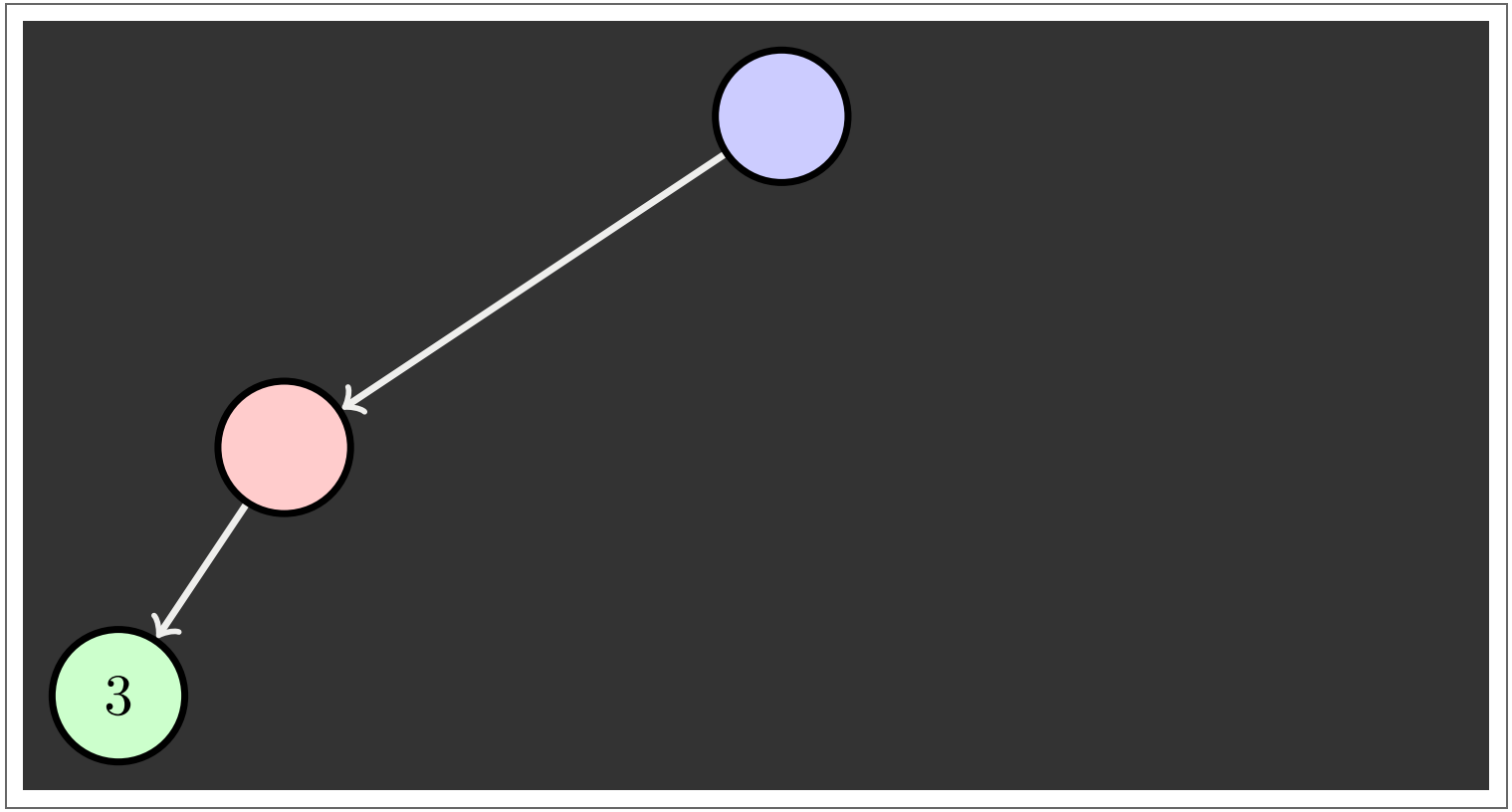
MINIMAX IMPLEMENTATION (DISPATCH)



MINIMAX EXAMPLE



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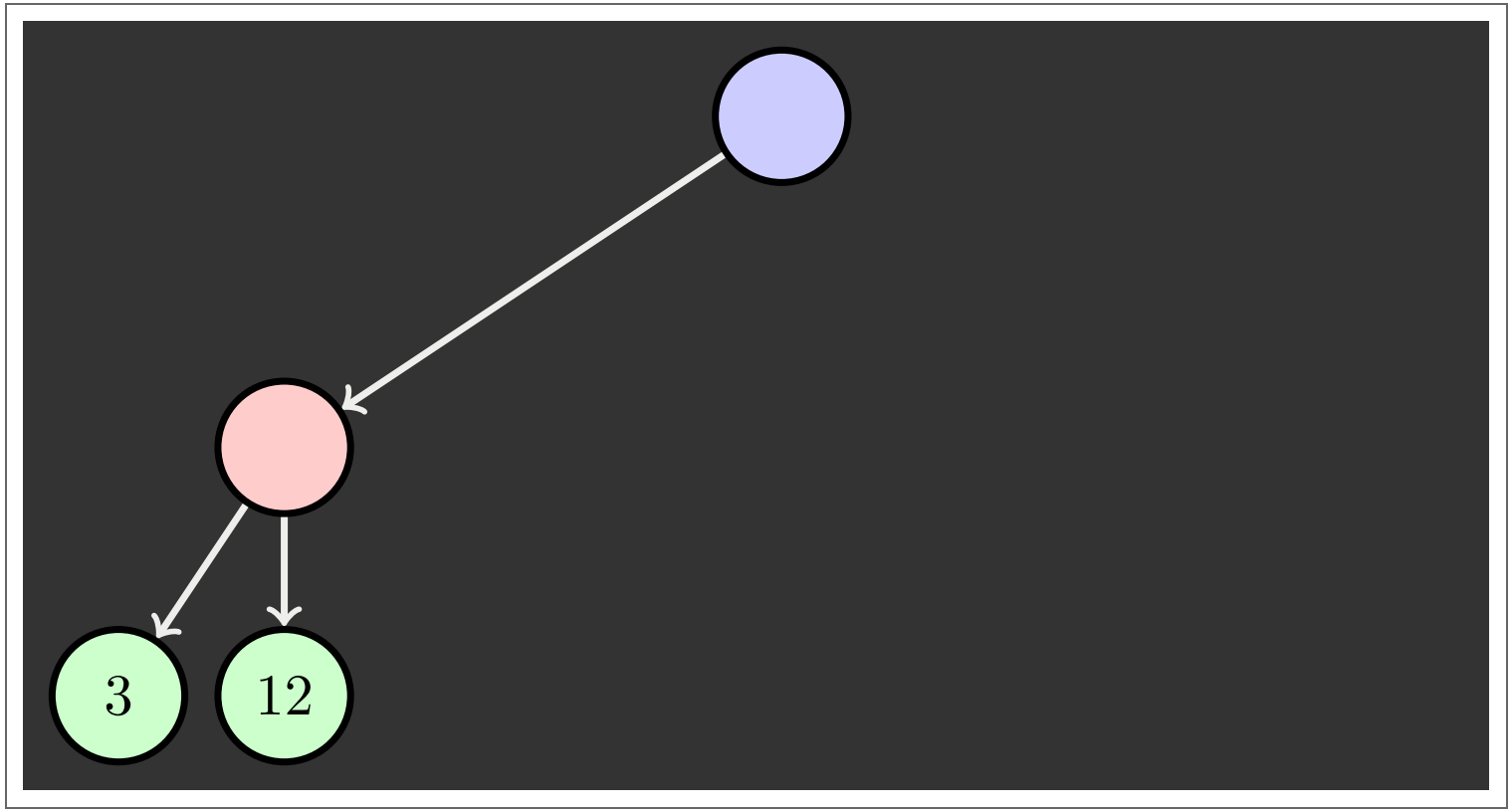
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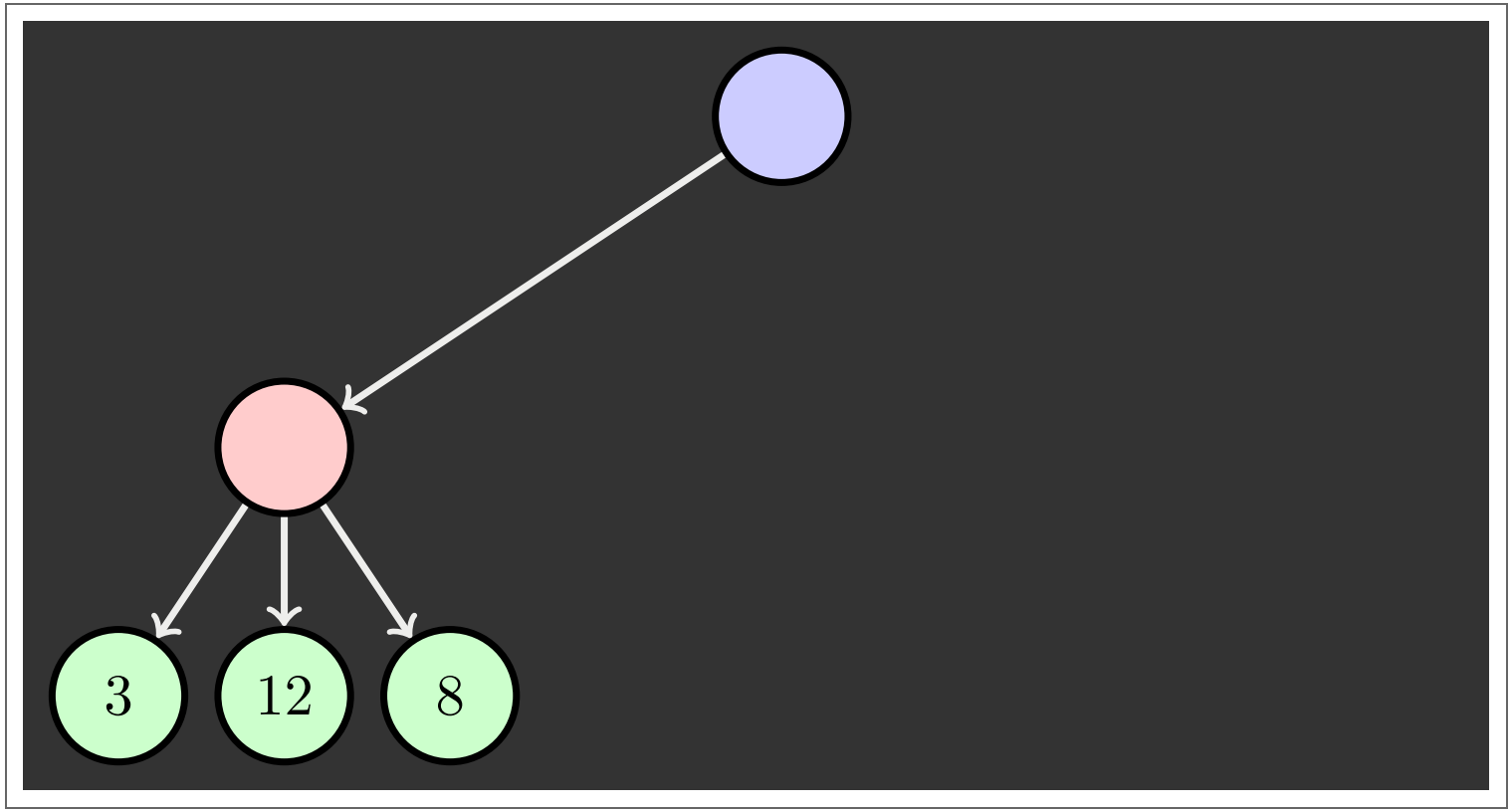
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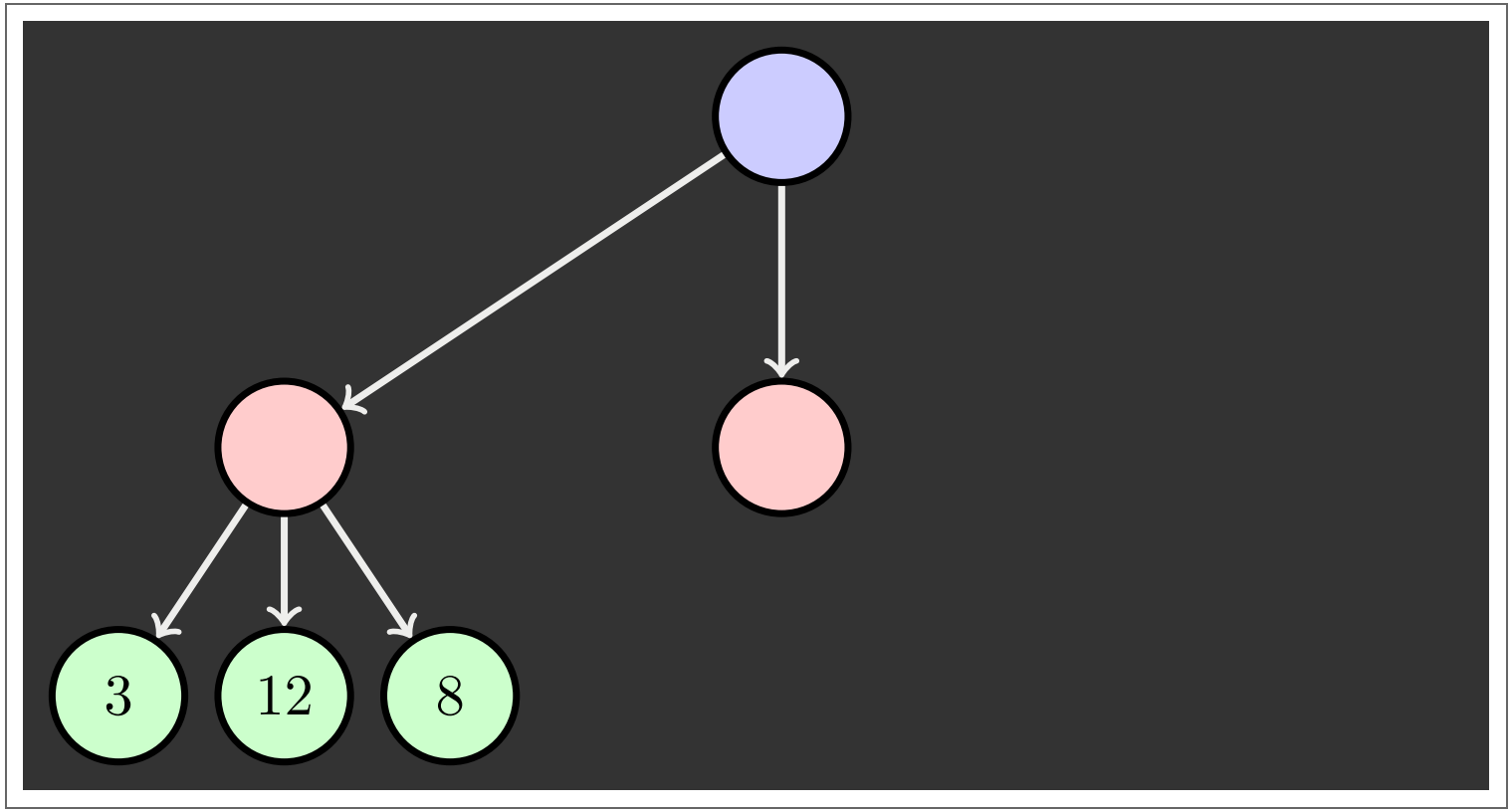
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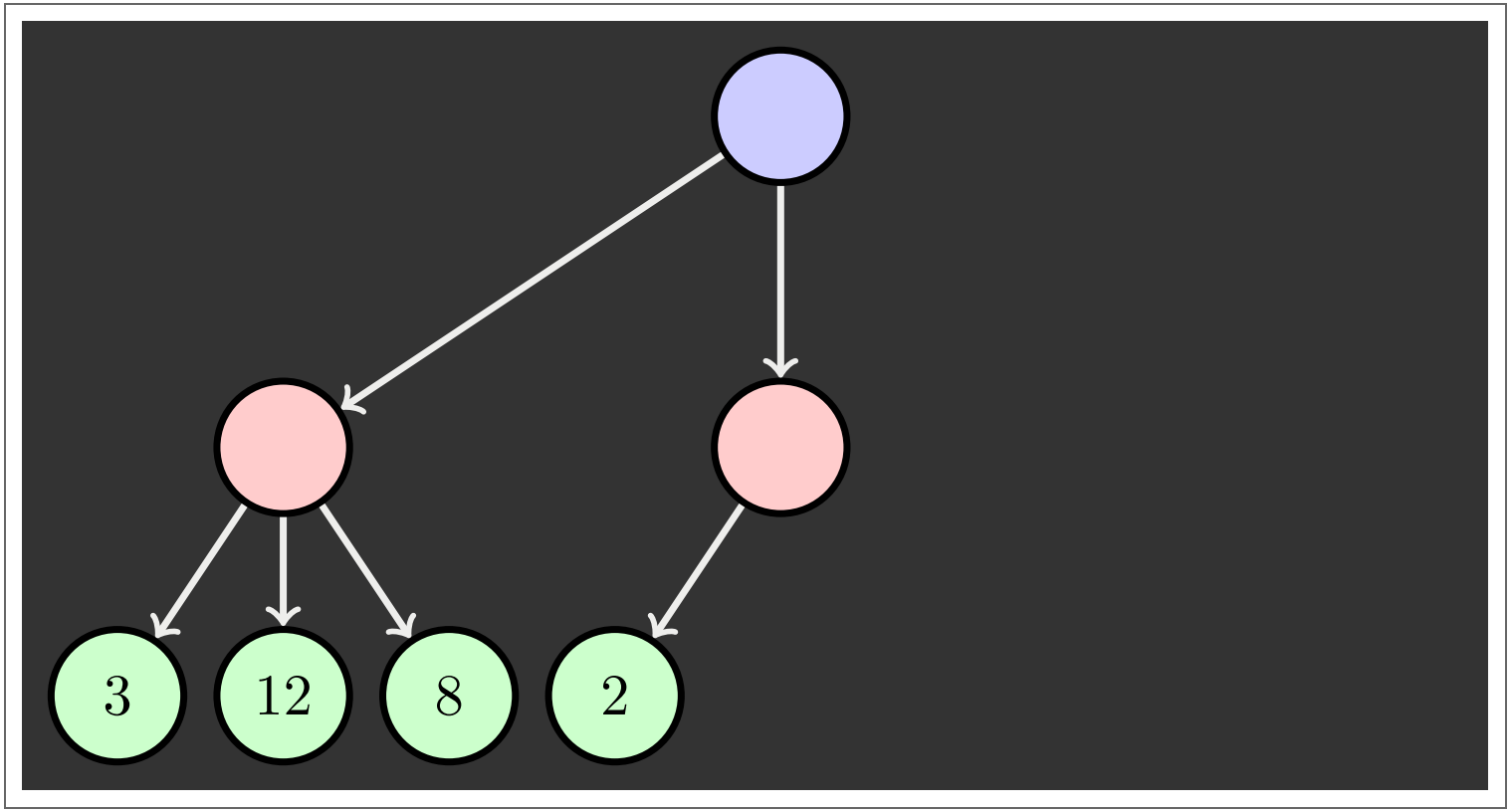
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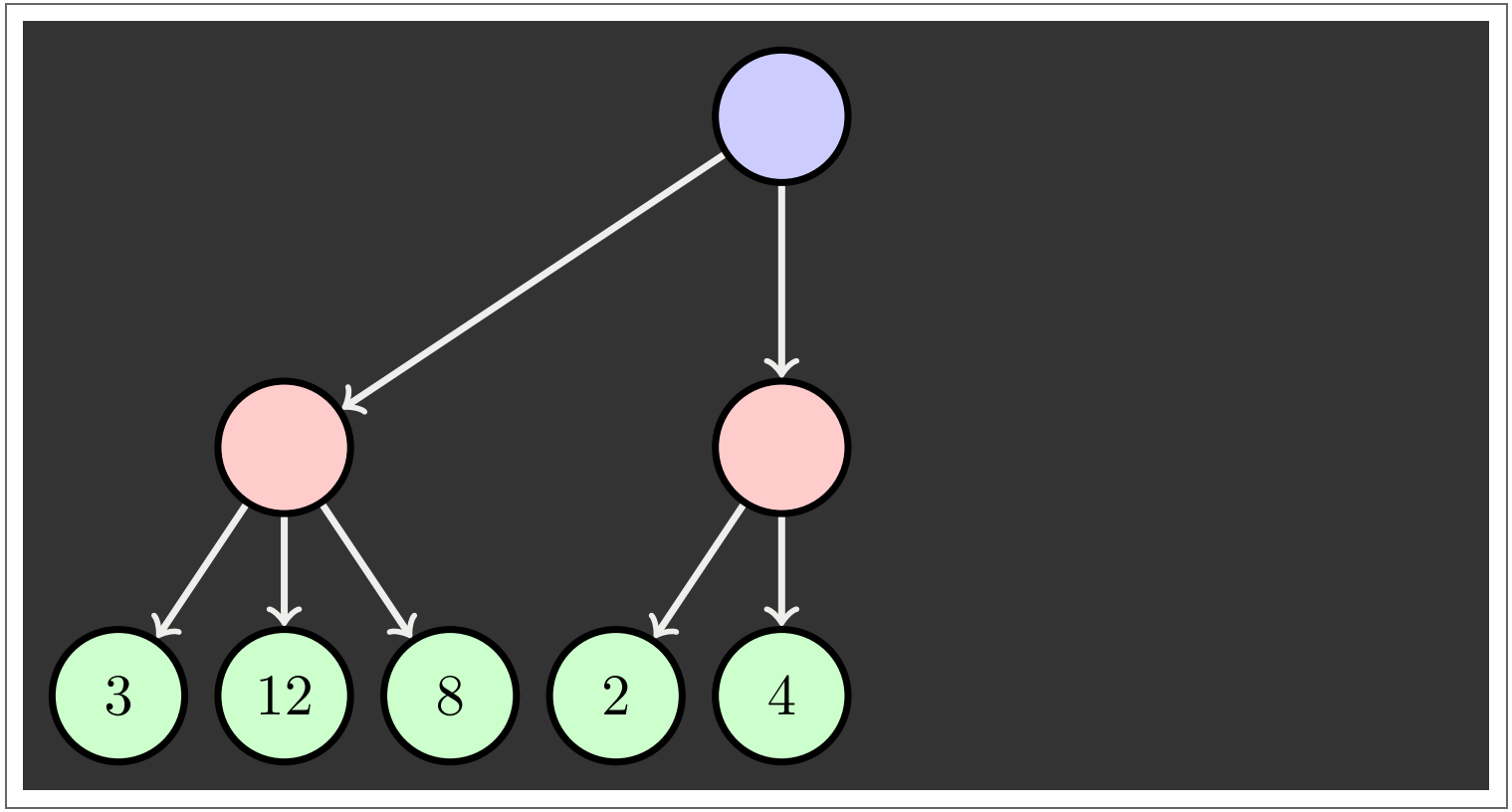
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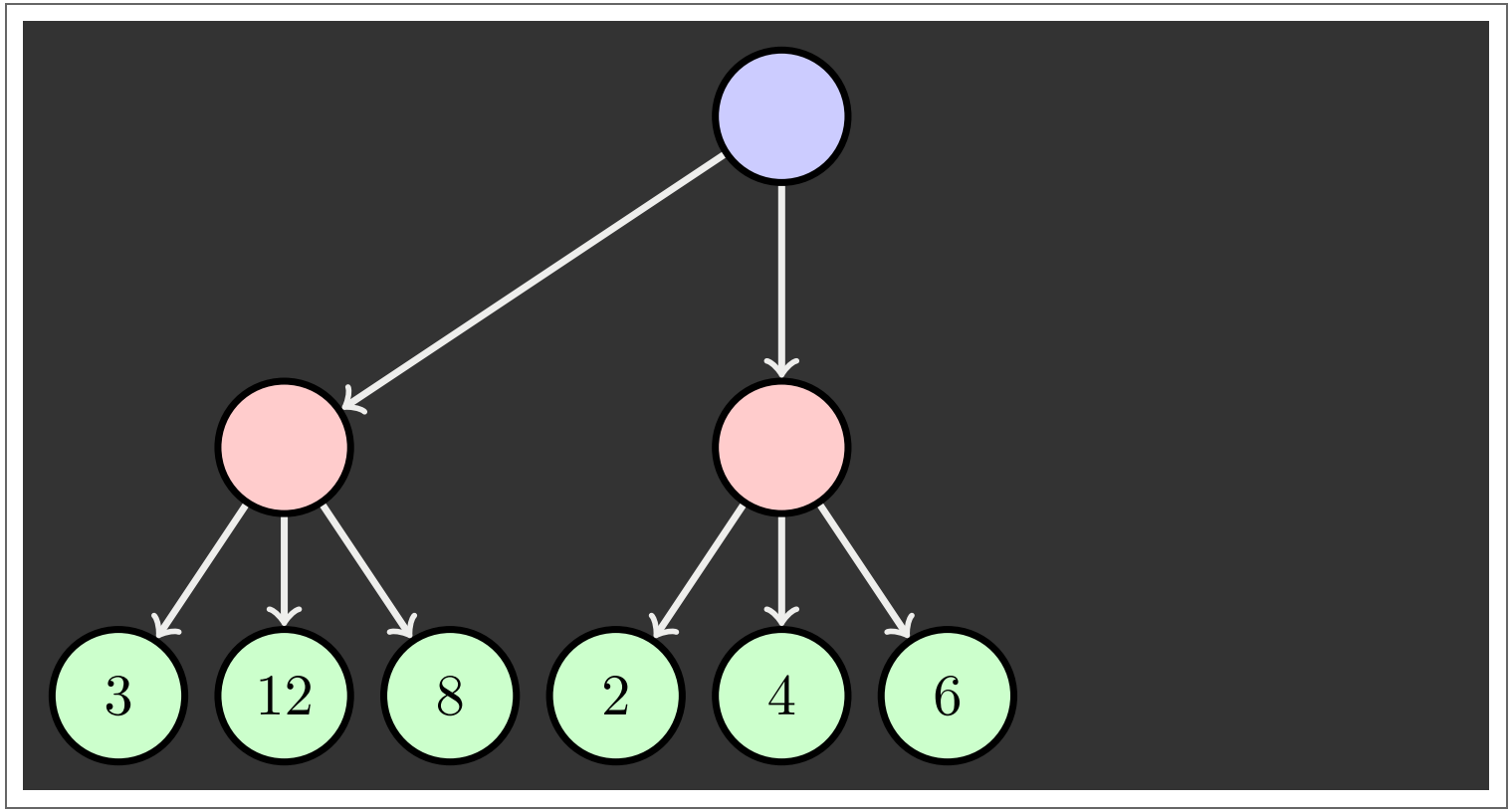
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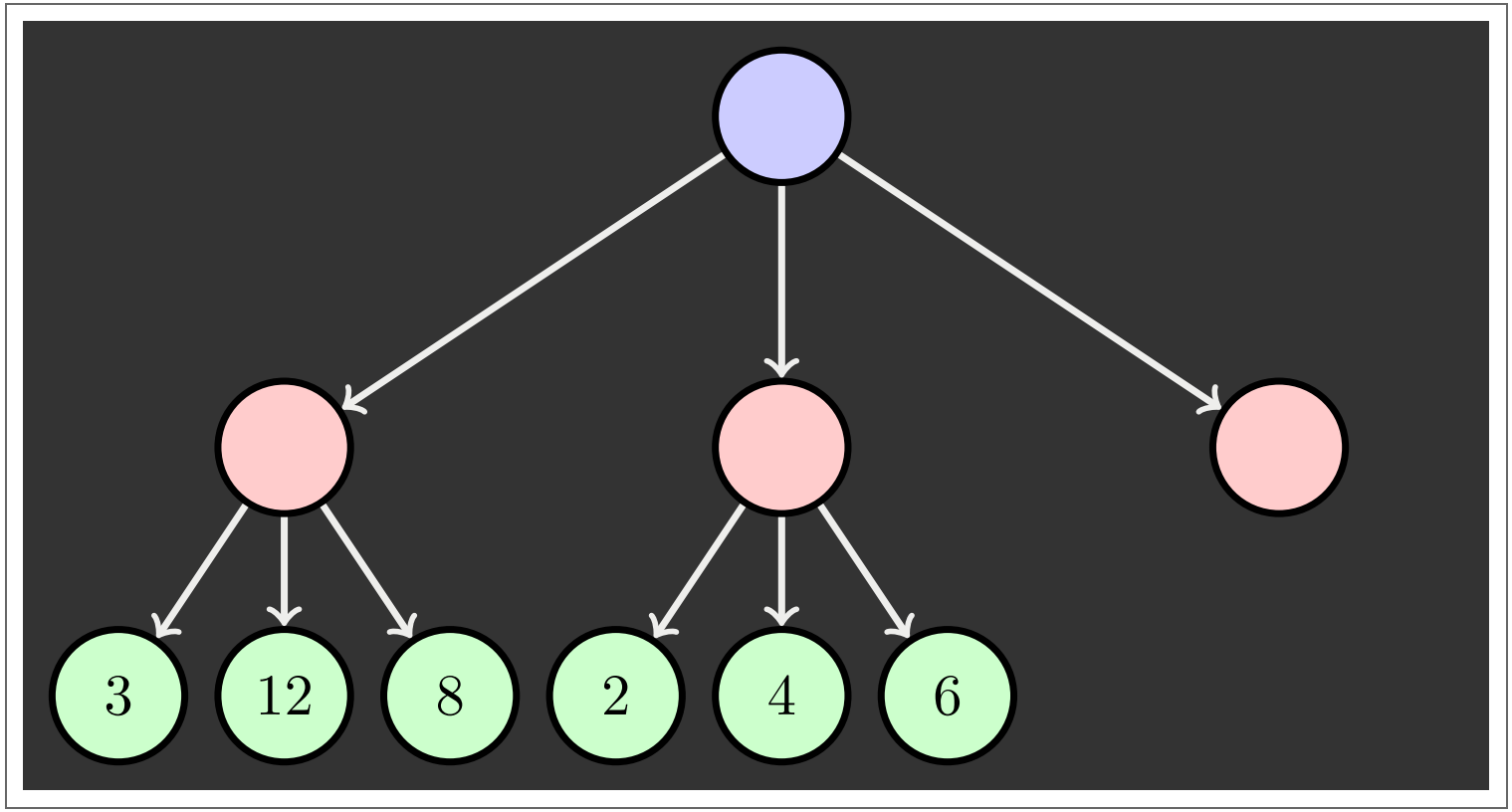
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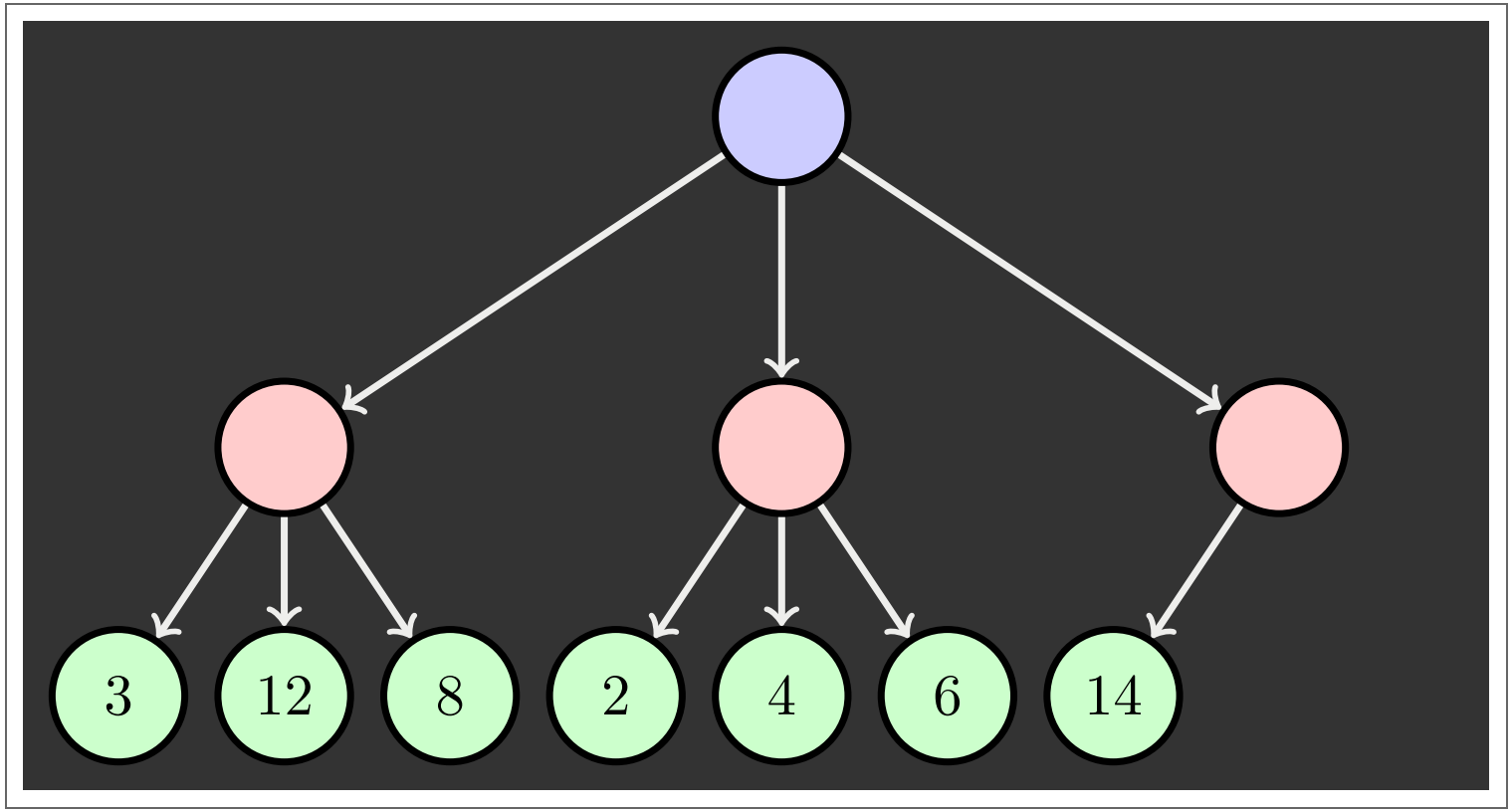
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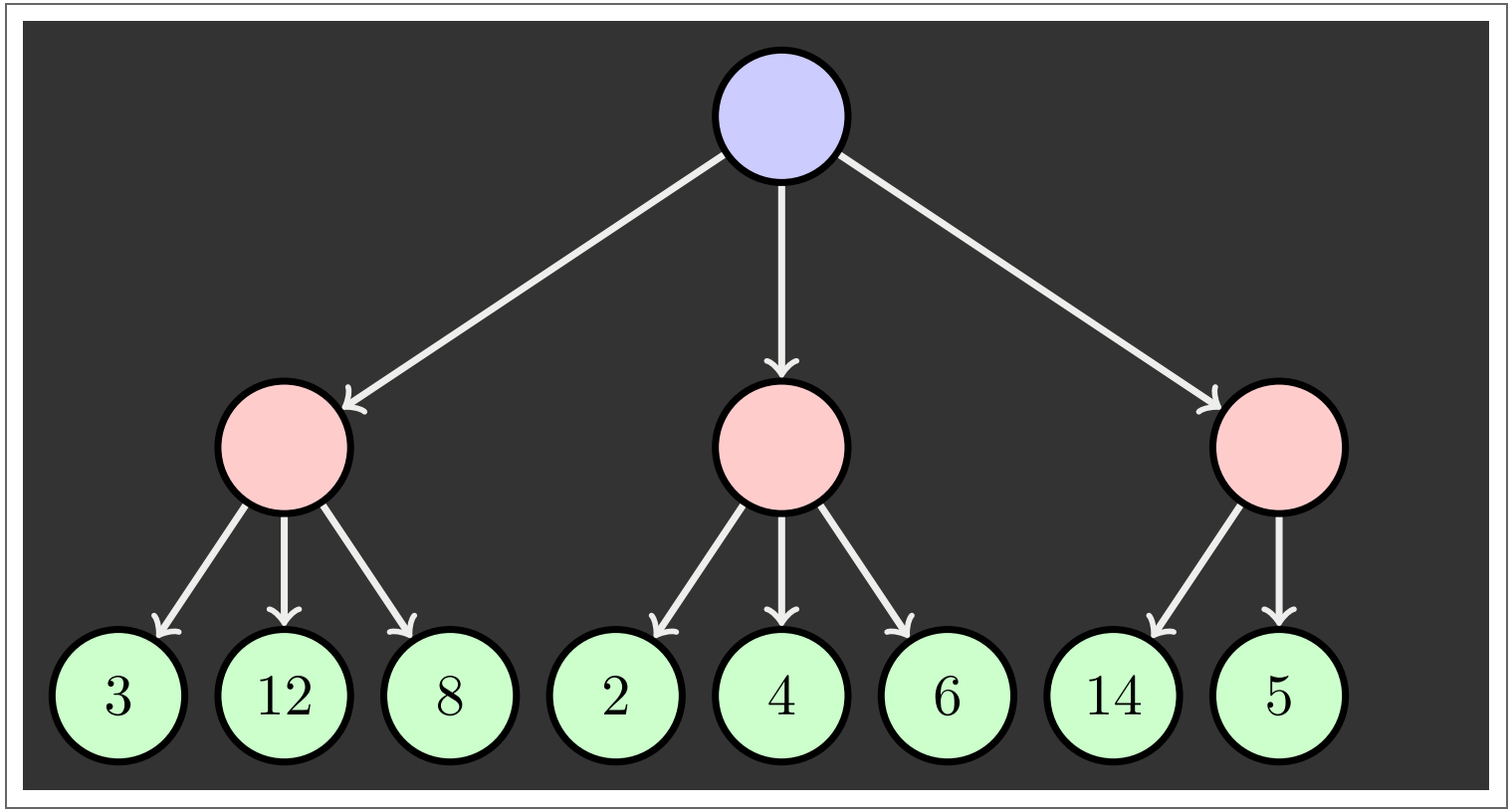
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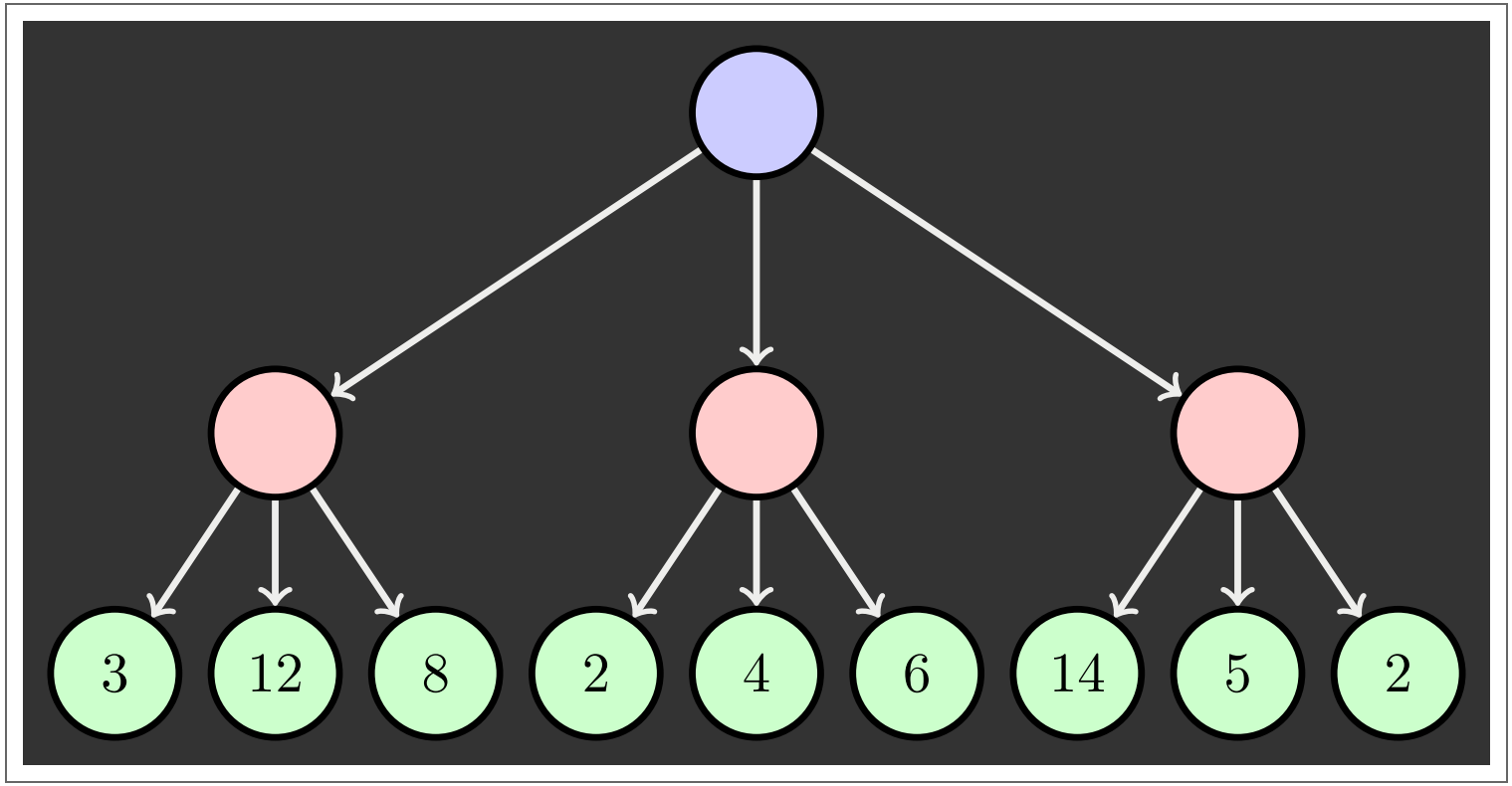
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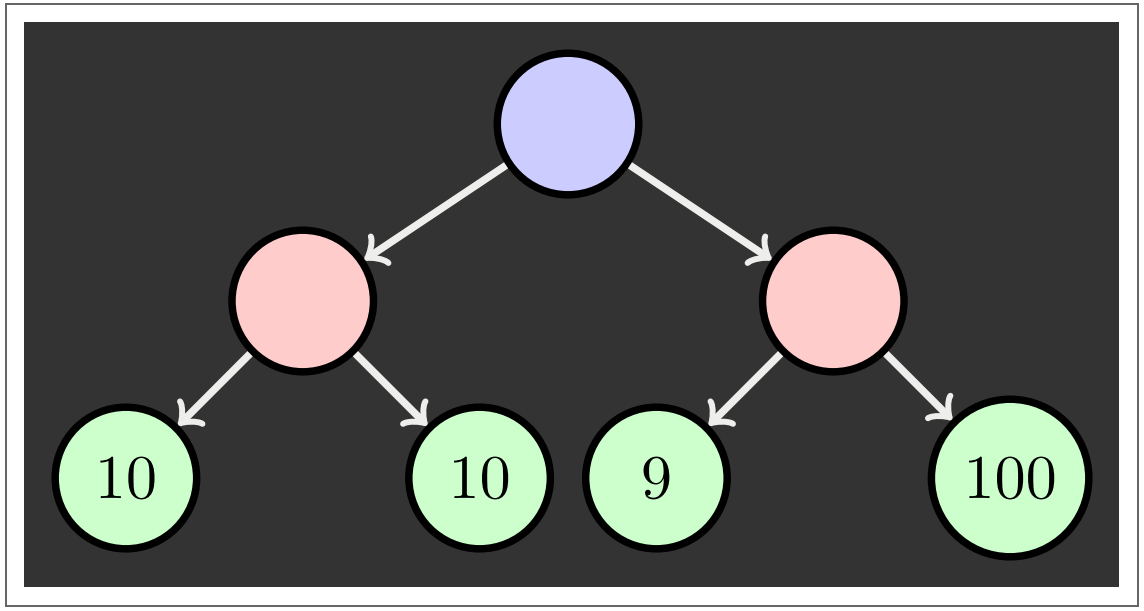
MINIMAX PROPERTIES



Optimal against a perfect player. Otherwise?

MINIMAX EFFICIENCY

How efficient is minimax?

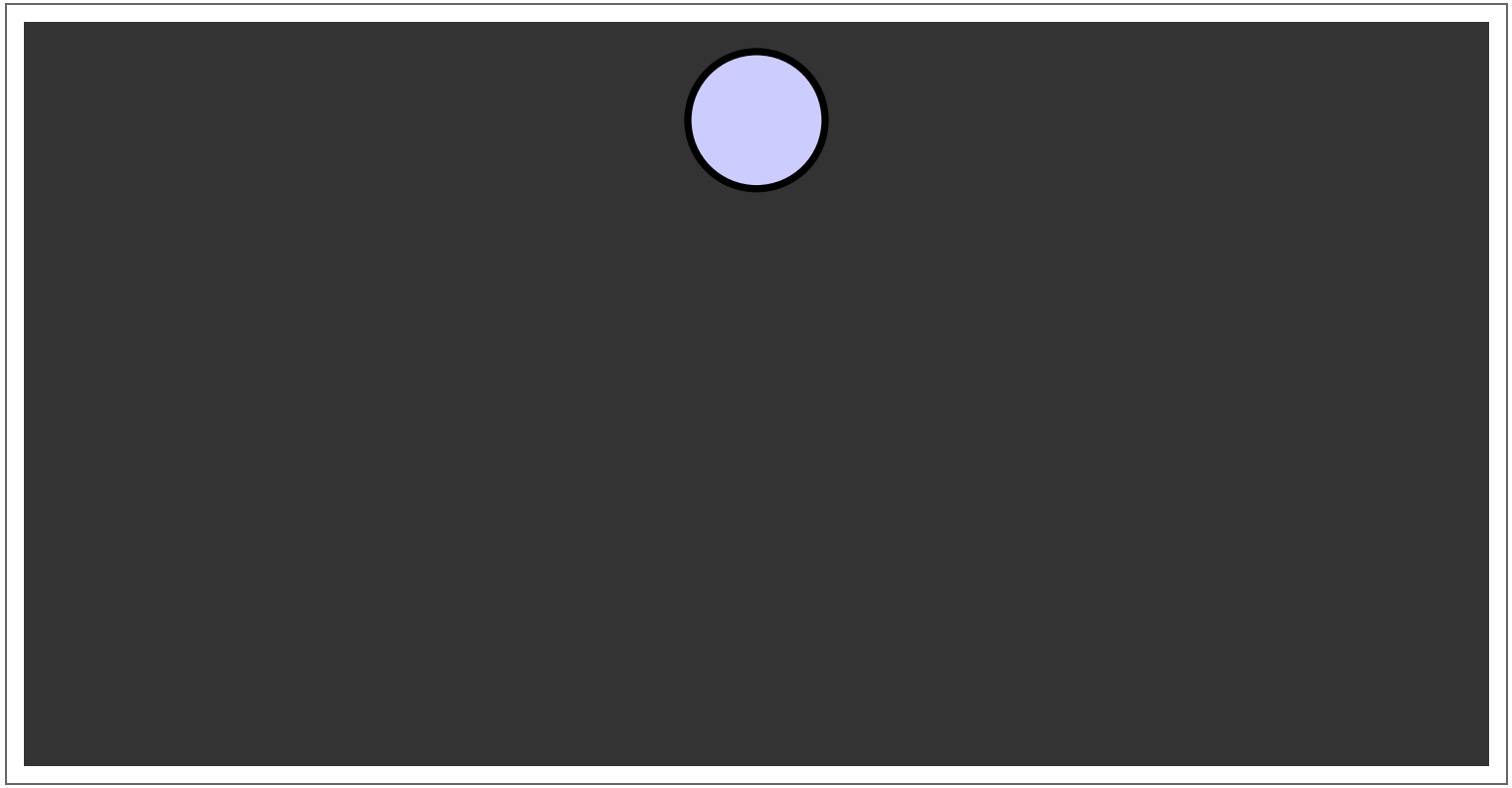
- Just like (exhaustive) DFS
- Time: $\mathcal{O}(b^m)$
- Space: $\mathcal{O}(bm)$

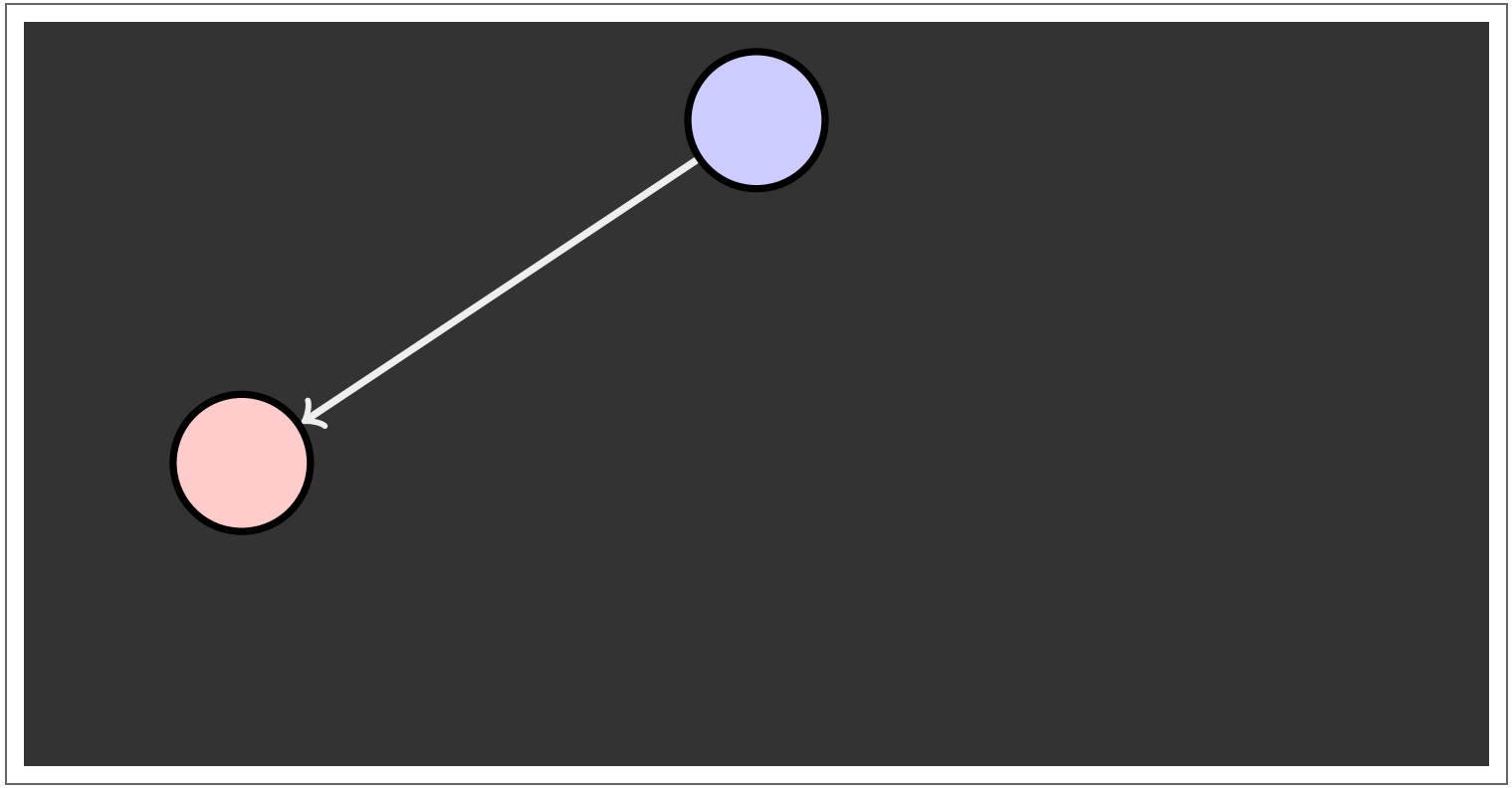
Example: For chess, $b \approx 35, m \approx 100$

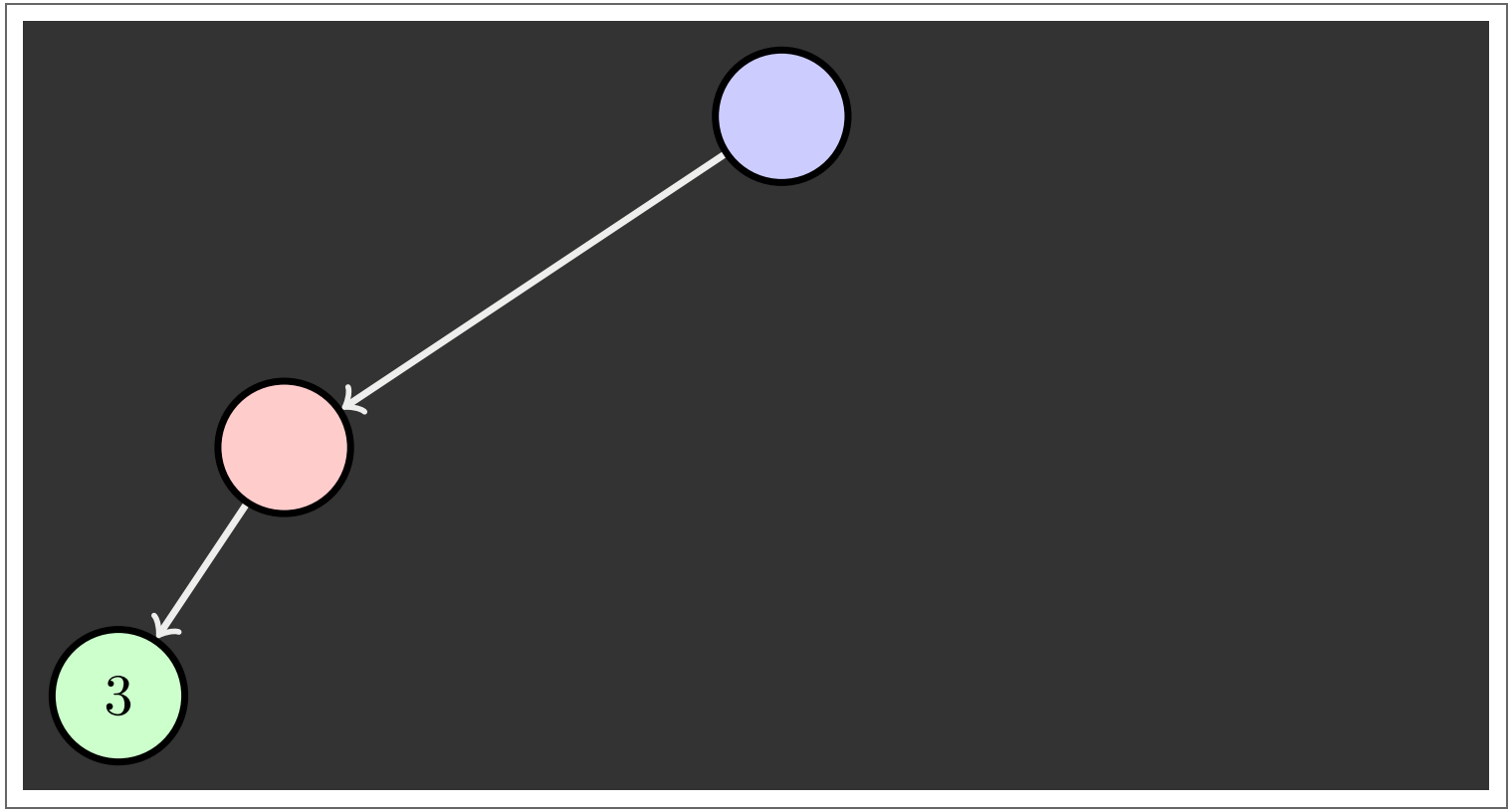
- Exact solution is completely infeasible
- But, do we need to explore the whole tree?

GAME TREE PRUNING

MINIMAX PRUNING



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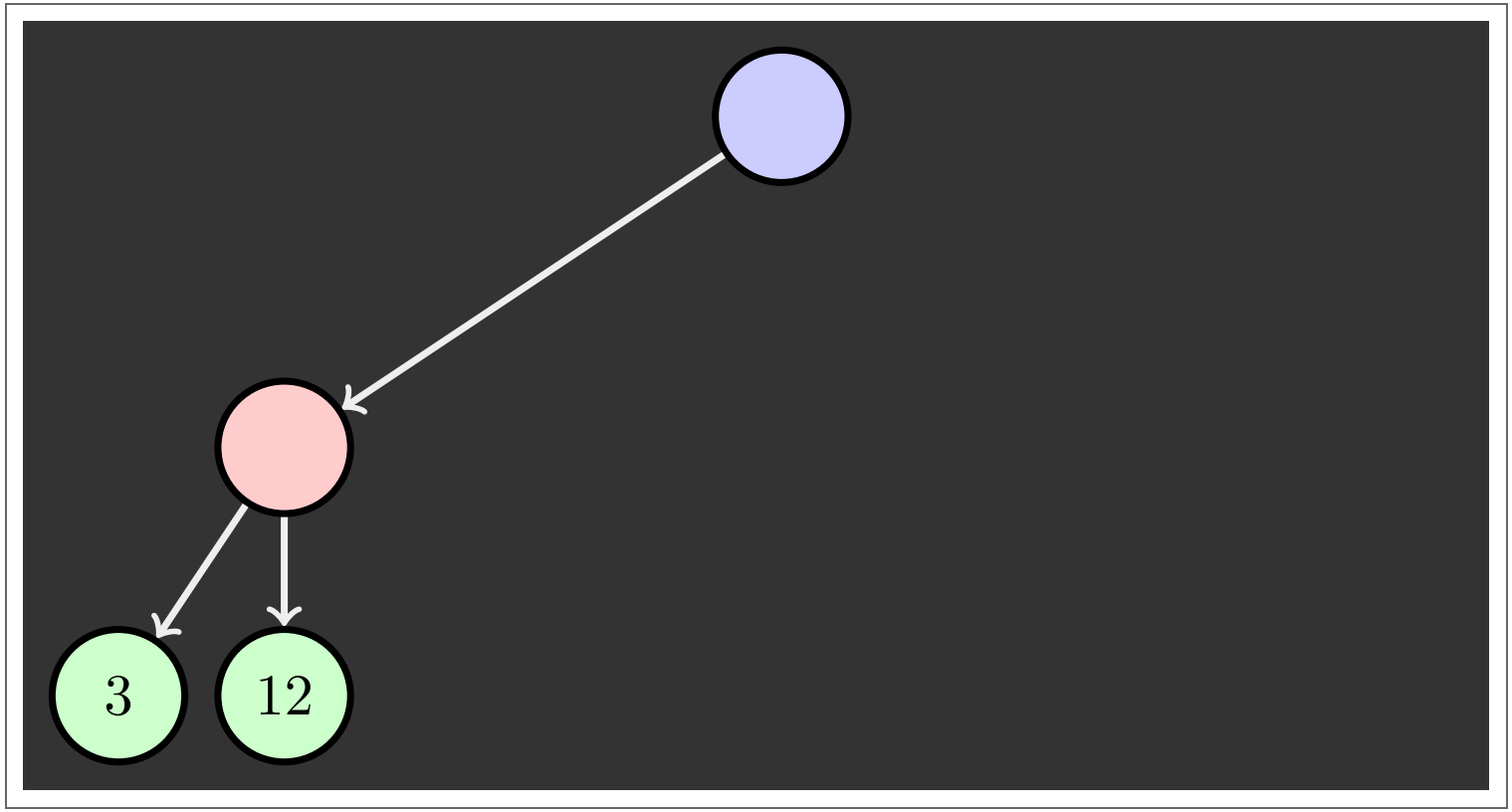
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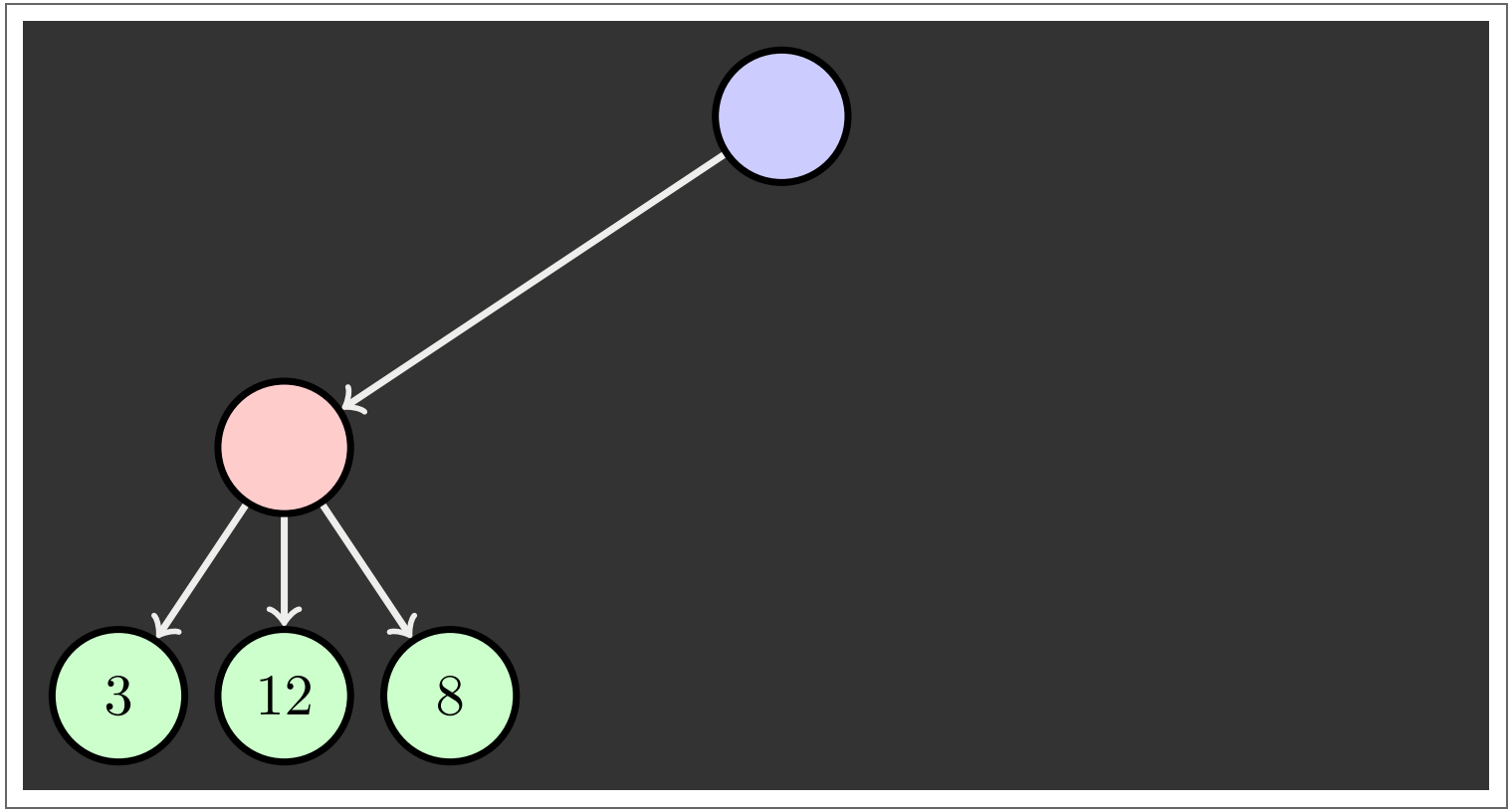
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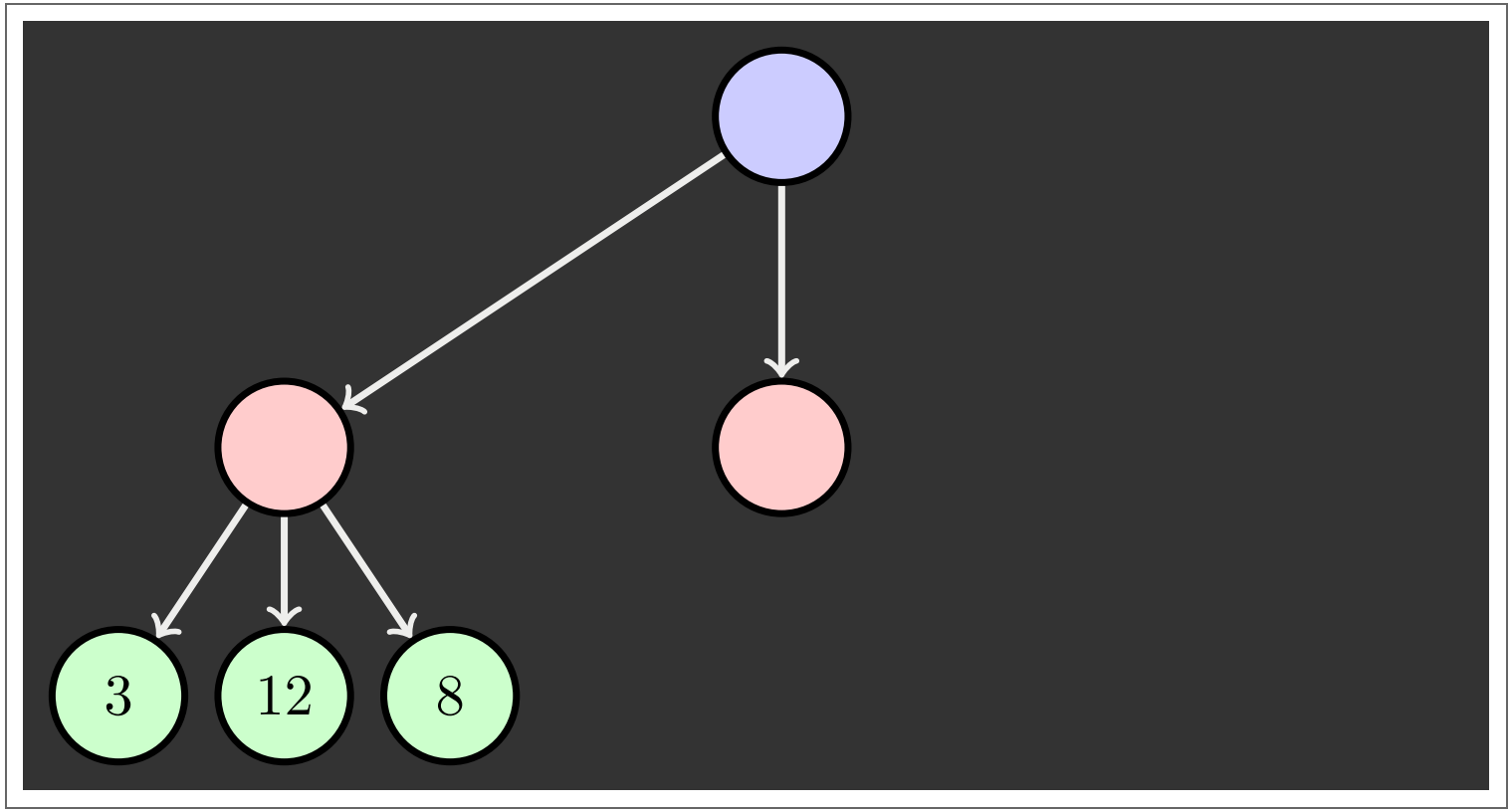
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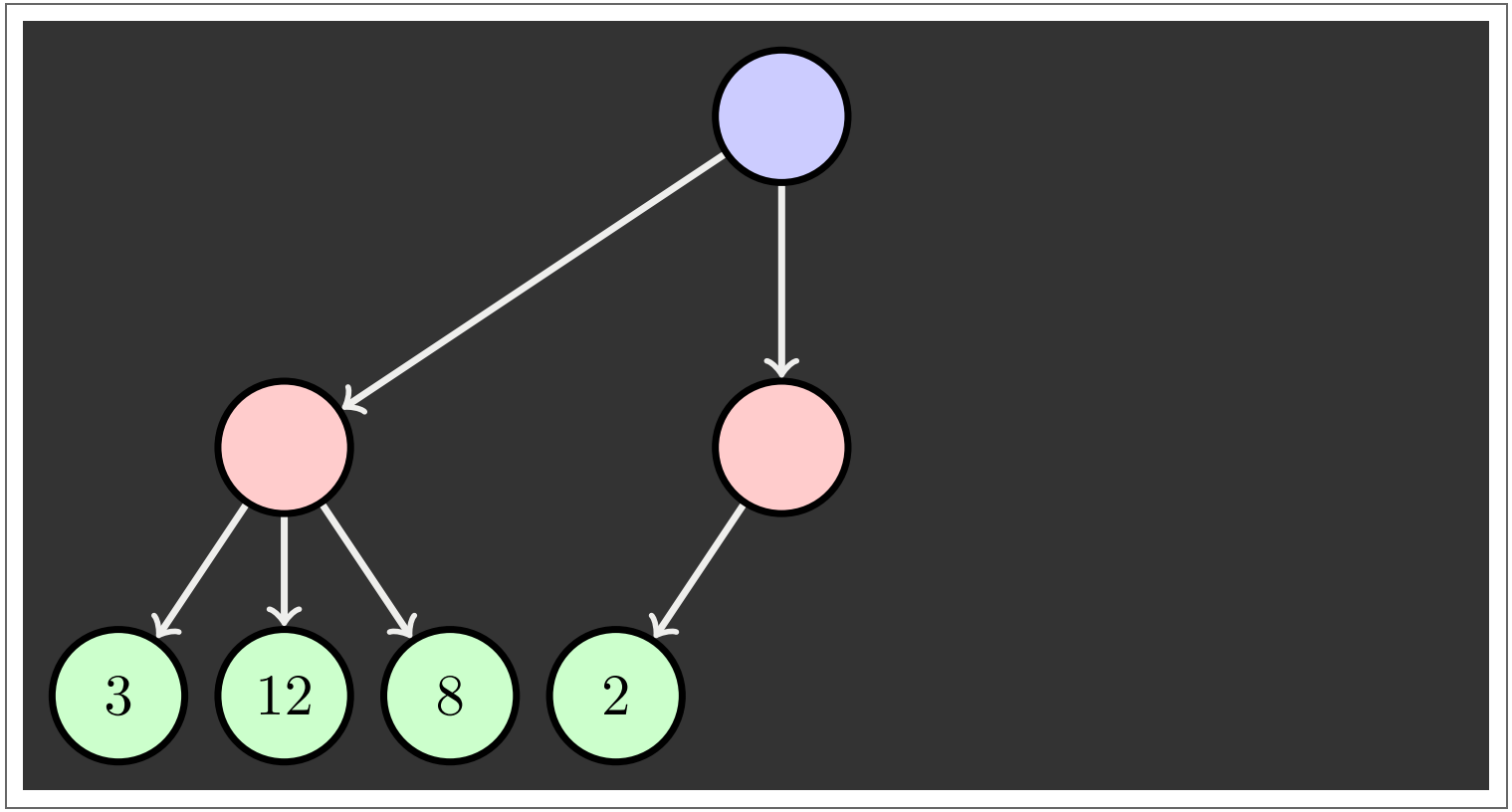
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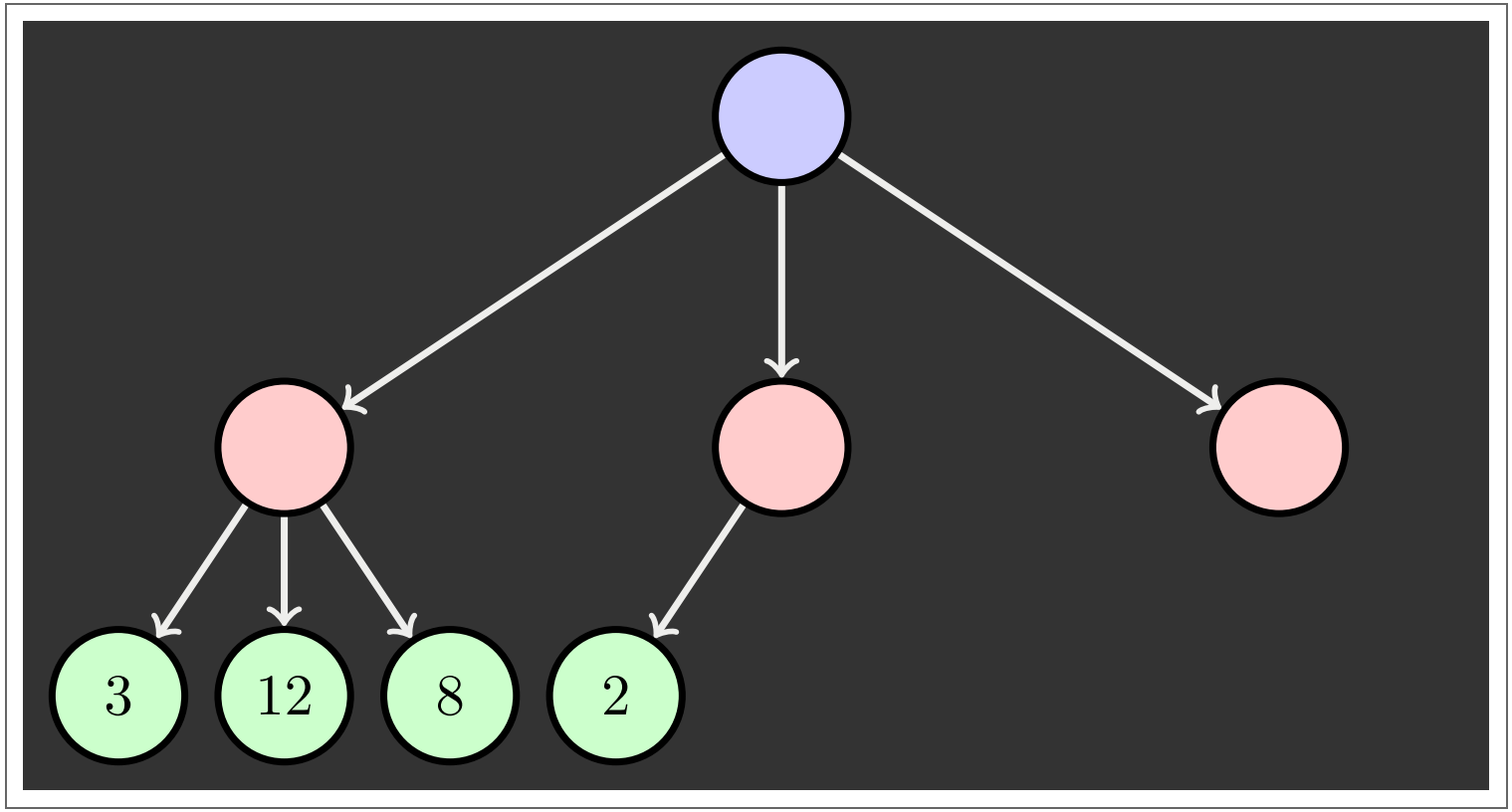
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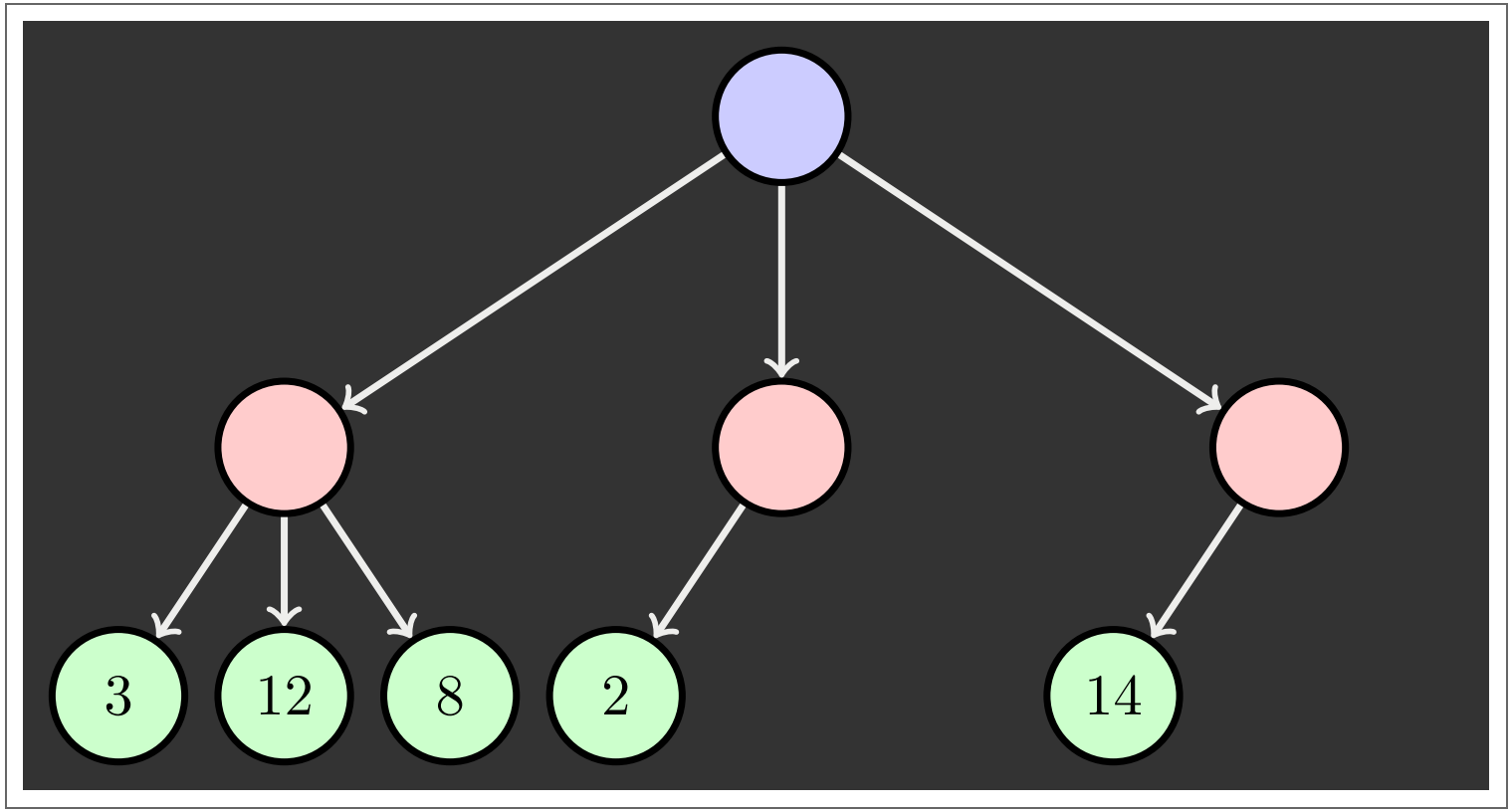
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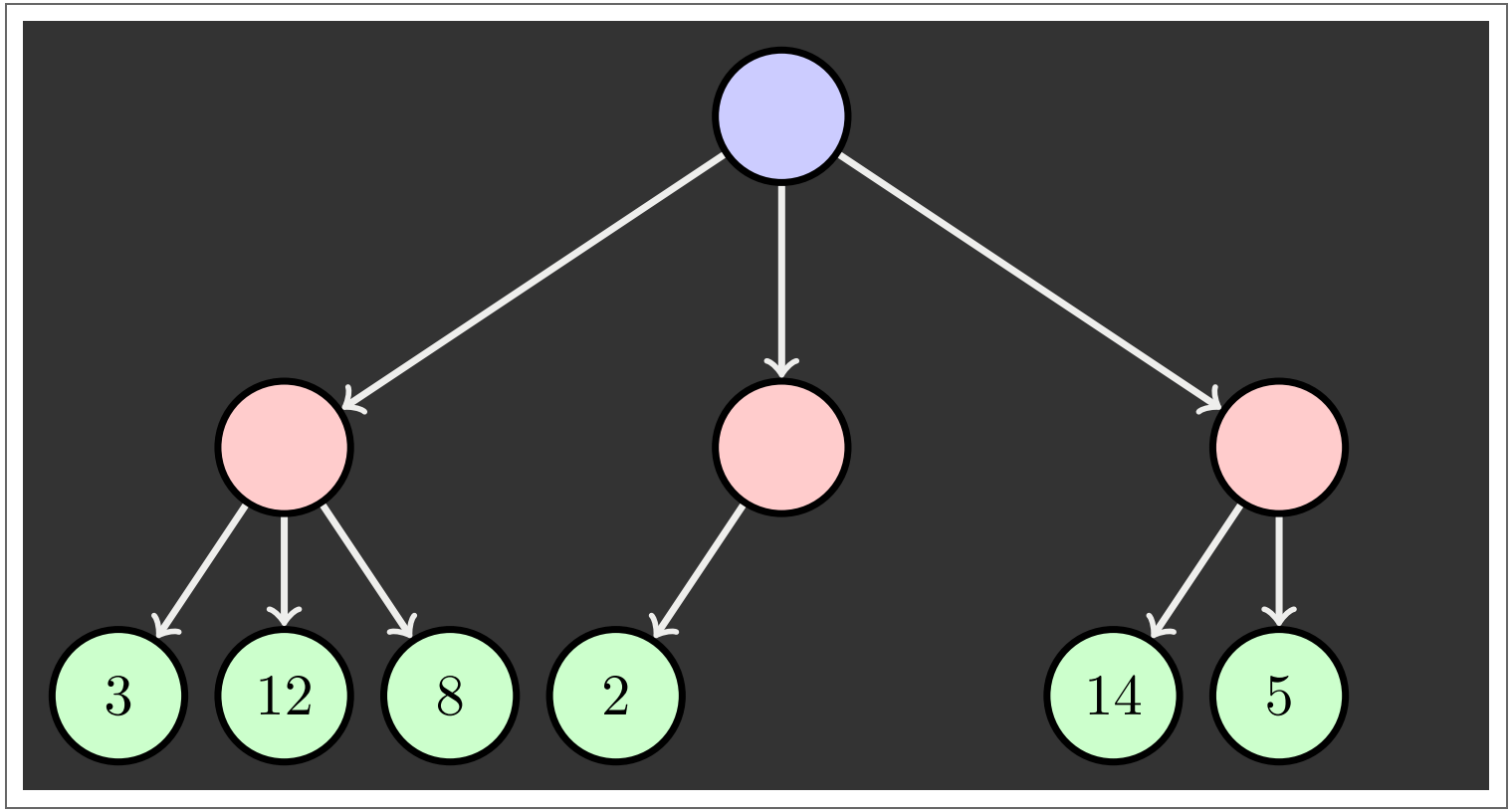
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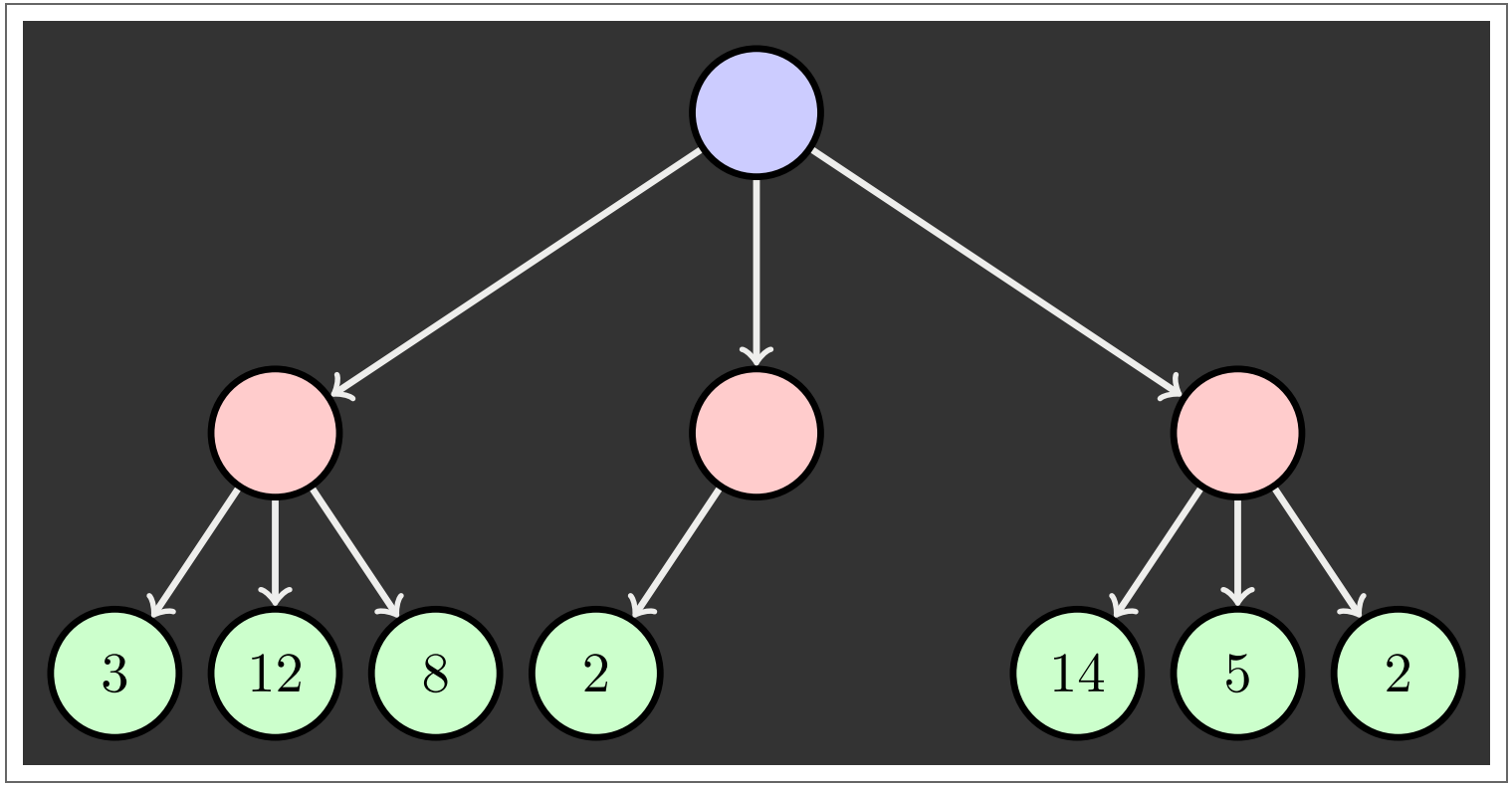
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$A-B$ PRUNING

General configuration (MIN version)

- We are computing the $MIN-VALUE$ at some node n
- We are looping over n's children
- n's estimate of the childrens' min is dropping
- Who cares about n's value? MAX
- Let α be the best value that MAX can get at any choice point along the current path from the root.
- If n becomes worse than α, MAX will avoid it, so we can stop considering n's other children (it is already bad enough that it won't be played)

MAX version is symmetric

$A-B$ IMPLEMENTATION

Dispatch:

α: MAX's best option on path to root

β: MIN's best option on path to root

MAX Agent:

```
def max-value(state, α, β):
    initialize v = −∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v
```

MIN Agent:

```
def min-value(state, α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v
```

$A - B$ PRUNING PROPERTIES

This pruning has no effect on minimax value computed for the root.

Values of intermediate nodes might be wrong

- Important: children of the root may have the wrong value
- So the most naïve version won't let you do action selection

Good child ordering improves effectiveness of pruning

With "perfect ordering":

- Time complexity drops to $\mathcal{O}(b^{\frac{m}{2}})$
- Doubles solvable depth
- Full search of, e.g. chess, is still hopeless . . .

This is a simple example of metareasoning (computing about what to compute)

CONSTRAINED RESOURCES

Problem: In realistic games, cannot search to leaves.

Solution: Depth-limited search

- Instead, search only to a limited depth in the tree
- Replace terminal utilities with an evaluation function for non-terminal positions

Example:

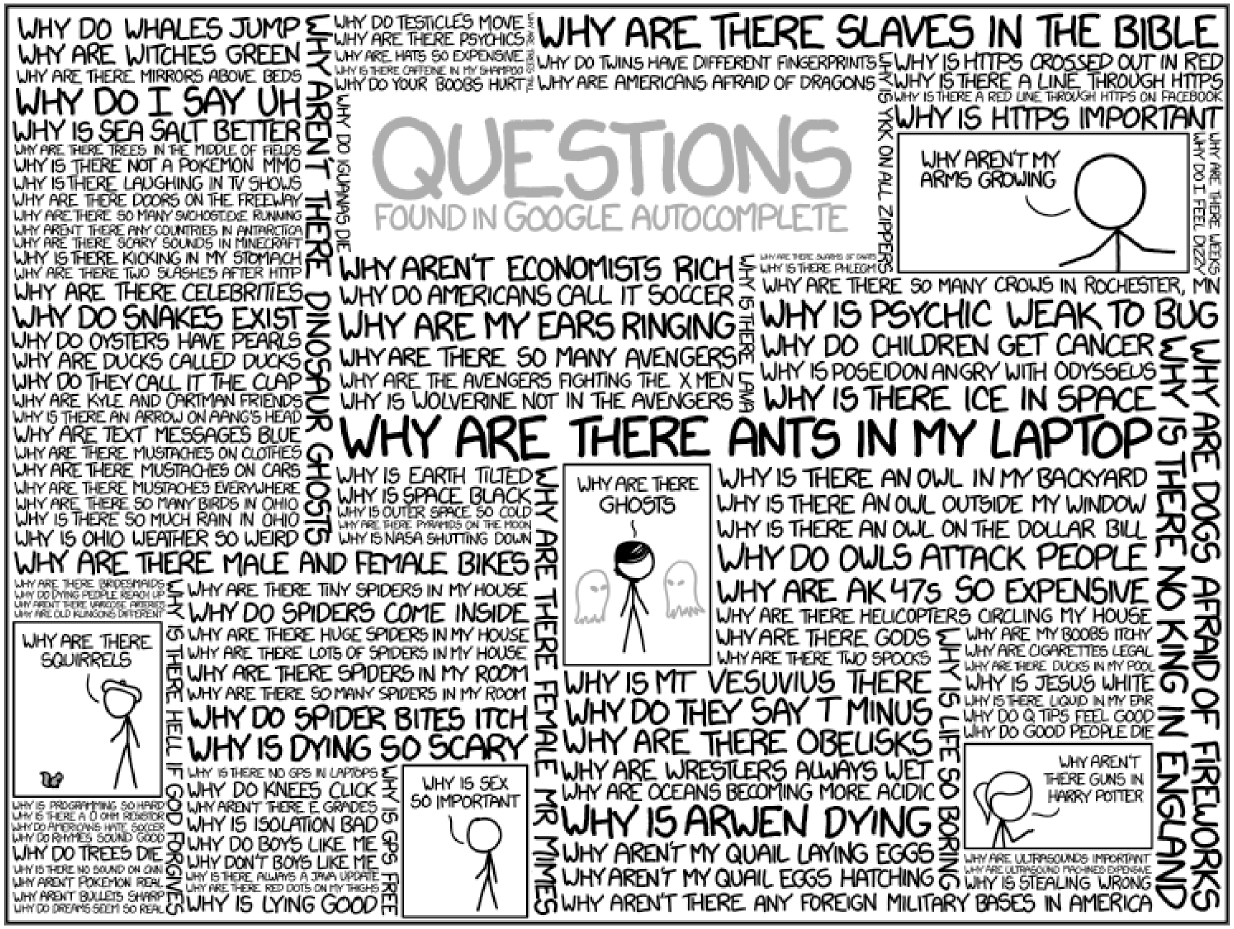
- Suppose we have 100 seconds, can explore 10K nodes / sec
- So can check 1M nodes per move
- $\alpha - \beta$ reaches about depth 8 – decent chess program

Guarantee of optimal play is gone

More depth makes a BIG difference

Use iterative deepening for an anytime algorithm

Q & A


QUESTIONS
FOUND IN GOOGLE AUTOCOMPLETE
WHY DO WHALES JUMP
WHY ARE WITCHES GREEN
WHY ARE THERE MIRRORS ABOVE BEDS
WHY DO I SAY UH
WHY IS SEA SALT BETTER
WHY ARE THERE TREES IN THE MIDDLE OF FIELDS
WHY IS THERE NOT A POKEMON MMO
WHY IS THERE LAUGHING IN TV SHOWS
WHY ARE THERE DOORS ON THE FREEWAY
WHY ARE THERE SO MANY SVCHOST.EXE RUNNING
WHY AREN'T THERE ANY COUNTRIES IN ANTARCTICA
WHY ARE THERE SCARY SOUNDS IN MINECRAFT
WHY IS THERE KICKING IN MY STOMACH
WHY ARE THERE TWO SLASHES AFTER HTTP
WHY ARE THERE CELEBRITIES
WHY DO SNAKES EXIST
WHY DO OYSTERS HAVE PEARLS
WHY ARE DUCKS CALLED DUCKS
WHY DO THEY CALL IT THE CLAP
WHY ARE KYLE AND CARTMAN FRIENDS
WHY IS THERE AN ARROW ON AANG'S HEAD
WHY ARE TEXT MESSAGES BLUE
WHY ARE THERE MUSTACHES ON CLOTHES
WHY ARE THERE MUSTACHES ON CARS
WHY ARE THERE MUSTACHES EVERYWHERE
WHY ARE THERE SO MANY BIRDS IN OHIO
WHY IS THERE SO MUCH RAIN IN OHIO
WHY IS OHIO WEATHER SO WEIRD
WHY AREN'T THERE DINOSAUR GHOSTS
WHY DO IGUANAS DIE
WHY DO TESTICLES MOVE
WHY ARE THERE PSYCHICS
WHY ARE HATS SO EXPENSIVE
WHY IS THERE CAFFEINE IN MY SHAMPOO
WHY DO YOUR BOOBS HURT
WHY ARE THERE SLAVES IN THE BIBLE
WHY DO TWINS HAVE DIFFERENT FINGERPRINTS
WHY ARE AMERICANS AFRAID OF DRAGONS
WHY IS HTTPS CROSSED OUT IN RED
WHY IS THERE A LINE THROUGH HTTPS
WHY IS THERE A RED LINE THROUGH HTTPS ON FACEBOOK
WHY IS HTTPS IMPORTANT
WHY IS YKK ON ALL ZIPPERS
WHY AREN'T MY ARMS GROWING
WHY ARE THERE WEEKS
WHY DO I FEEL DIZZY
WHY AREN'T ECONOMISTS RICH
WHY DO AMERICANS CALL IT SOCCER
WHY ARE MY EARS RINGING
WHY ARE THERE SO MANY AVENGERS
WHY ARE THE AVENGERS FIGHTING THE X MEN
WHY IS WOLVERINE NOT IN THE AVENGERS
WHY IS THERE LAVA
WHY ARE THERE SWARMS OF GNATS
WHY IS THERE PHLEGM
WHY ARE THERE SO MANY CROWS IN ROCHESTER, MN
WHY IS PSYCHIC WEAK TO BUG
WHY DO CHILDREN GET CANCER
WHY IS POSEIDON ANGRY WITH ODYSSEUS
WHY IS THERE ICE IN SPACE
WHY ARE THERE ANTS IN MY LAPTOP
WHY IS EARTH TILTED
WHY IS SPACE BLACK
WHY IS OUTER SPACE SO COLD
WHY ARE THERE PYRAMIDS ON THE MOON
WHY IS NASA SHUTTING DOWN
WHY ARE THERE GHOSTS
WHY IS THERE AN OWL IN MY BACKYARD
WHY IS THERE AN OWL OUTSIDE MY WINDOW
WHY IS THERE AN OWL ON THE DOLLAR BILL
WHY DO OWLS ATTACK PEOPLE
WHY ARE AK47s SO EXPENSIVE
WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE
WHY ARE THERE GODS
WHY ARE THERE TWO SPOCKS
WHY ARE THERE MALE AND FEMALE BIKES
WHY ARE THERE BRIDESMAIDS
WHY DO DYING PEOPLE REACH UP
WHY AREN'T THERE VARICOSE ARTERIES
WHY ARE OLD KLINGONS DIFFERENT
WHY ARE THERE SQUIRRELS
WHY IS THERE HELL IF GOD FORGIVES
WHY ARE THERE TINY SPIDERS IN MY HOUSE
WHY DO SPIDERS COME INSIDE
WHY ARE THERE HUGE SPIDERS IN MY HOUSE
WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE
WHY ARE THERE SPIDERS IN MY ROOM
WHY ARE THERE SO MANY SPIDERS IN MY ROOM
WHY DO SPIDER BITES ITCH
WHY IS DYING SO SCARY
WHY IS THERE NO GPS IN LAPTOPS
WHY DO KNEES CLICK
WHY AREN'T THERE E GRADES
WHY IS ISOLATION BAD
WHY DO BOYS LIKE ME
WHY DON'T BOYS LIKE ME
WHY IS THERE ALWAYS A JAVA UPDATE
WHY ARE THERE RED DOTS ON MY THIGHS
WHY IS LYING GOOD
WHY IS GPS FREE
WHY IS SEX SO IMPORTANT
WHY IS PROGRAMMING SO HARD
WHY IS THERE A 0 OHM RESISTOR
WHY DO AMERICANS HATE SOCCER
WHY DO RHYMES SOUND GOOD
WHY DO TREES DIE
WHY IS THERE NO SOUND ON CNN
WHY AREN'T POKEMON REAL
WHY AREN'T BULLETS SHARP
WHY DO DREAMS SEEM SO REAL
WHY ARE THERE FEMALE MR MIMES
WHY IS MT VESUVIUS THERE
WHY DO THEY SAY T MINUS
WHY ARE THERE OBELISKS
WHY ARE WRESTLERS ALWAYS WET
WHY ARE OCEANS BECOMING MORE ACIDIC
WHY IS ARWEN DYING
WHY AREN'T MY QUAIL LAYING EGGS
WHY AREN'T MY QUAIL EGGS HATCHING
WHY AREN'T THERE ANY FOREIGN MILITARY BASES IN AMERICA
WHY IS LIFE SO BORING
WHY ARE MY BOOBS ITCHY
WHY ARE CIGARETTES LEGAL
WHY ARE THERE DUCKS IN MY POOL
WHY IS JESUS WHITE
WHY IS THERE LIQUID IN MY EAR
WHY DO Q TIPS FEEL GOOD
WHY DO GOOD PEOPLE DIE
WHY AREN'T THERE GUNS IN HARRY POTTER
WHY ARE ULTRASOUNDS IMPORTANT
WHY ARE ULTRASOUND MACHINES EXPENSIVE
WHY IS STEALING WRONG
WHY IS THERE NO KING IN ENGLAND
WHY ARE DOGS AFRAID OF FIREWORKS

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Speaker notes

Q & A


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