Constraint Satisfaction - II

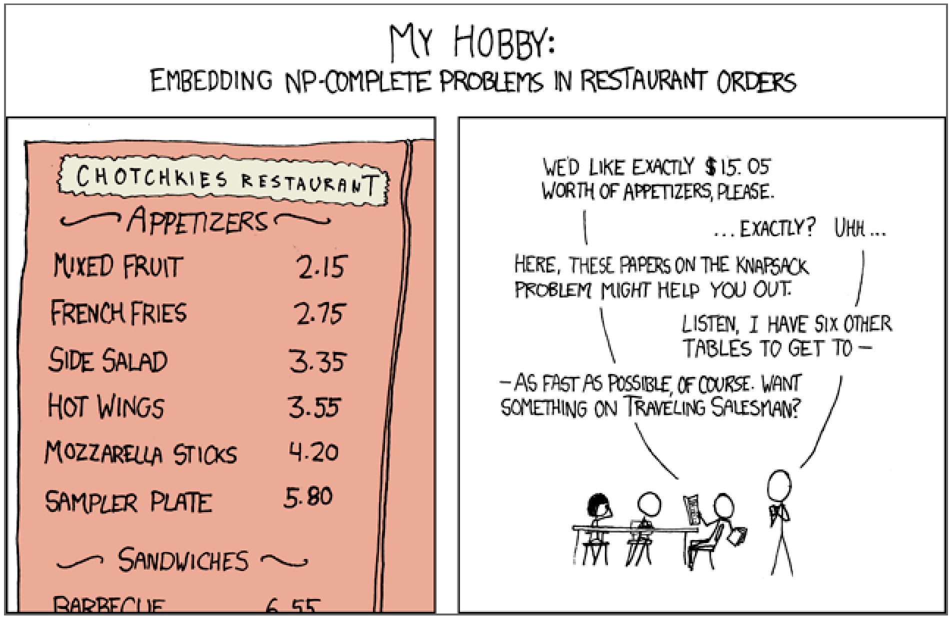
CS5491: Artificial Intelligence
ZHICHAO LU

Content Credits: **Prof. Wei**'s CS4486 Course and **Prof. Boddeti**'s AI Course

TODAY

Improvements to Backtracking Search:

- Filtering
- Ordering
- Problem Structure



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Reading

- Today's Lecture: RN Chapter 6

BACKTRACKING SEARCH

BACKTRACKING SEARCH

Backtracking search is the basic uninformed algorithm for solving CSPs

Idea 1: One variable at a time

- Variable assignments are commutative, so fix ordering
- Problem is commutative if order of application of any given set of actions has no effect on outcome.
- i.e., [WA = red then NT = green] same as [NT = green then WA = red]
- Only need to consider assignments to a single variable at each step

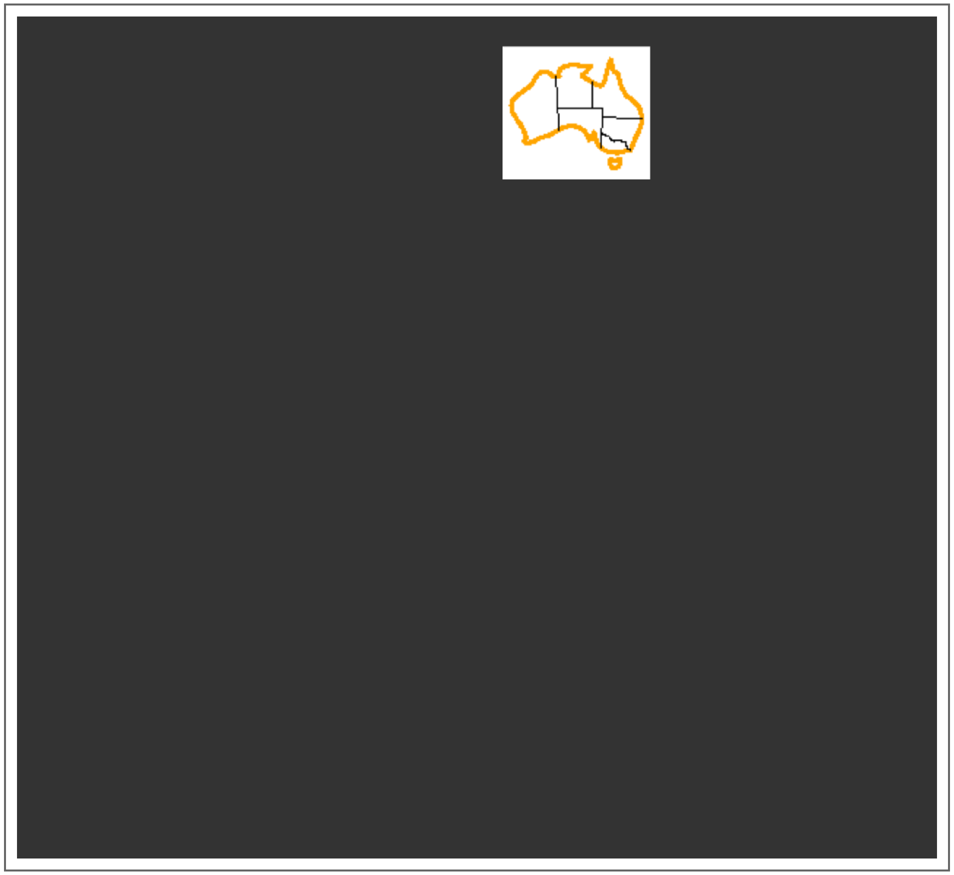
Idea 2: Check constraints as you go

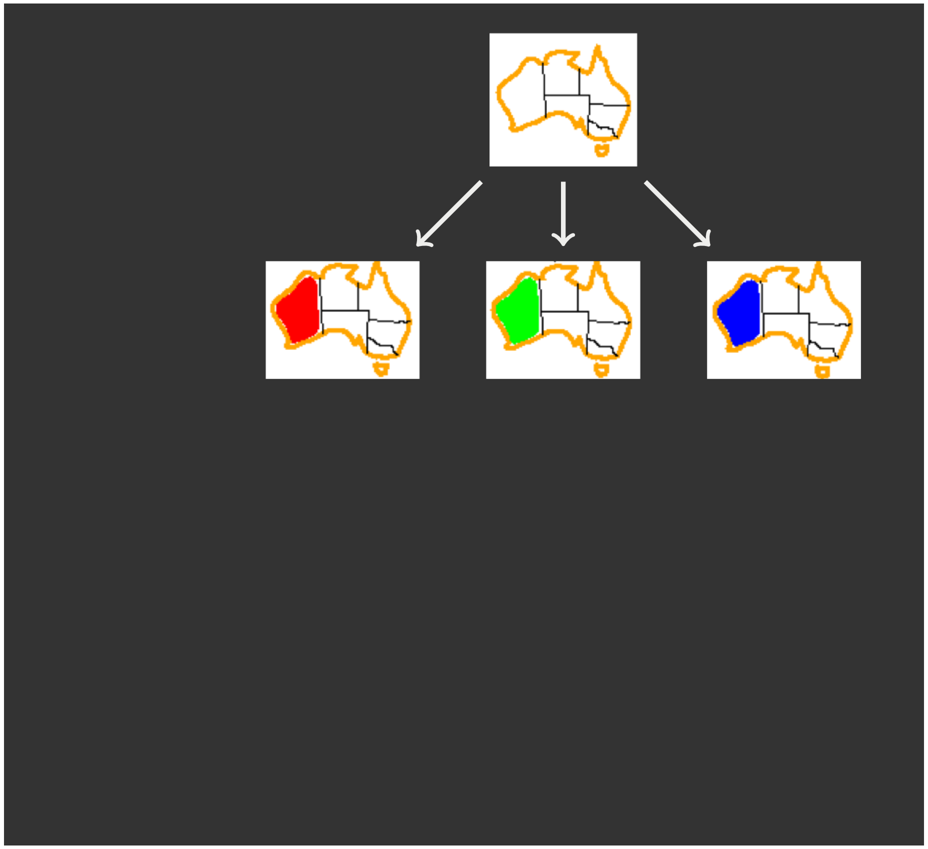
- i.e. consider only values which do not conflict with previous assignments
- Might have to do some computation to check the constraints
- "Incremental goal test"

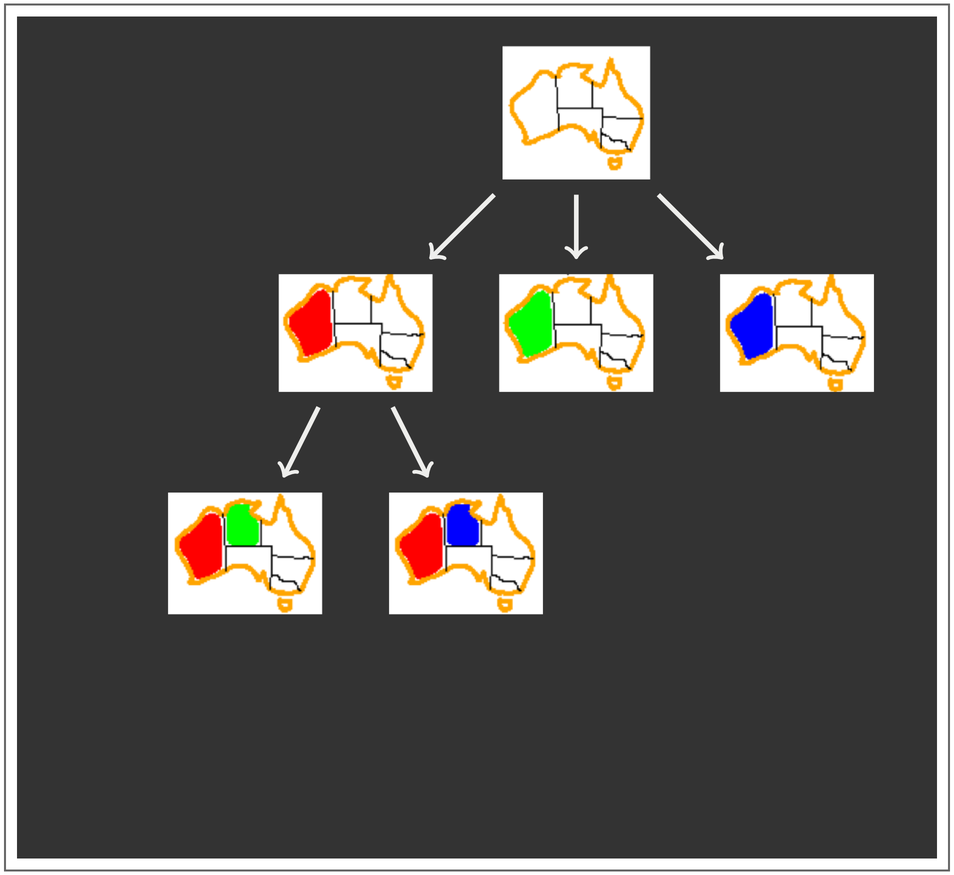
DFS with these two improvements is called backtracking search (not the best name)

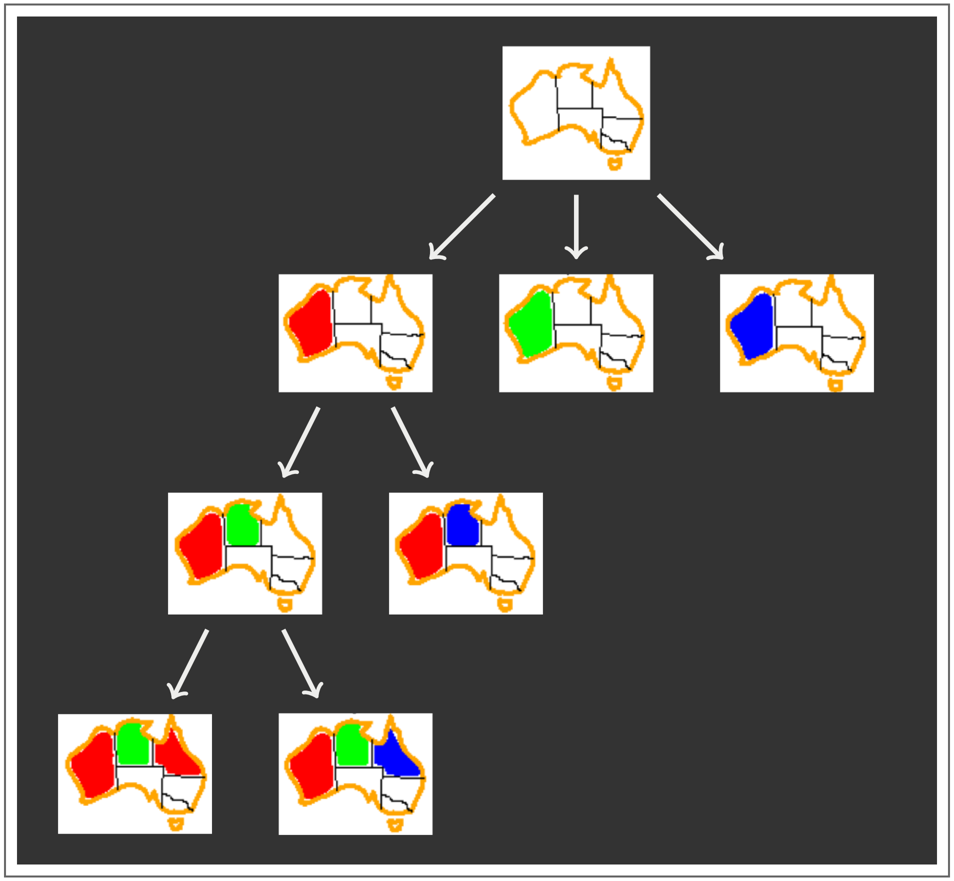
Can solve n-queens for $n \approx 25$

BACKTRACKING EXAMPLE









BACKTRACKING SEARCH

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
   return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
   if assignment is complete then return assignment
   var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
   for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
       if value is consistent with assignment given CONSTRAINTS[csp] then
           add {var = value} to assignment
           result ← RECURSIVE-BACKTRACKING(assignment, csp)
           if result ≠ failure then return result
           remove {var = value} from assignment
   return failure
```

- Backtracking = DFS + variable-ordering + fail-on-violation

IMPROVING BACKTRACKING SEARCH

General-purpose ideas give huge gains in speed

- Backtracking is an uninformed algorithm. So we don't expect it to be very effective for large problems. We know the informed search can improve the efficiency and effectiveness.

Ordering:

- Which variable should be assigned next?
- In what order should its values be tried?

Filtering: Can we detect inevitable failure early?

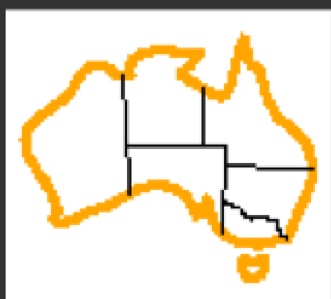
Structure: Can we exploit the problem structure?

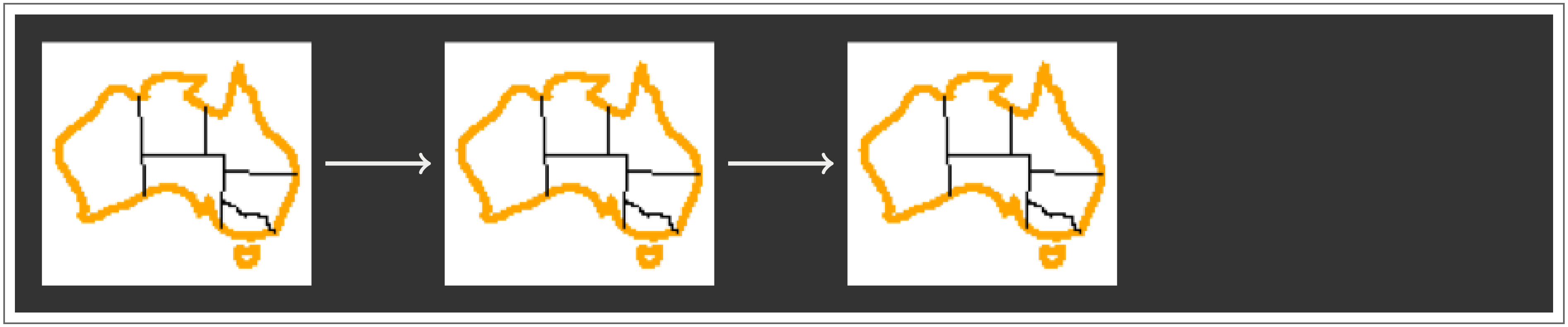
ORDERING

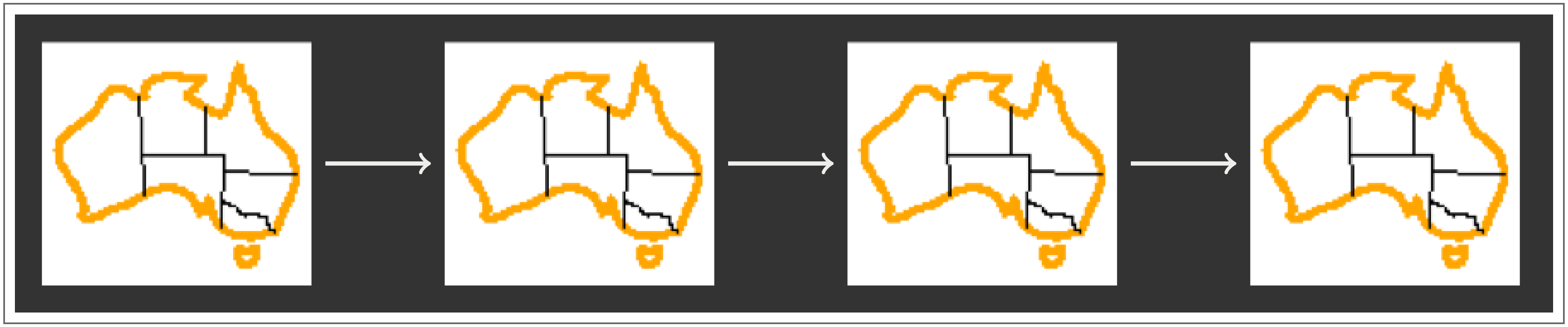
ORDERING: MINIMUM REMAINING VALUES

Variable Ordering: Minimum remaining values (MRV):

- Choose the variable with the fewest legal values left in its domain





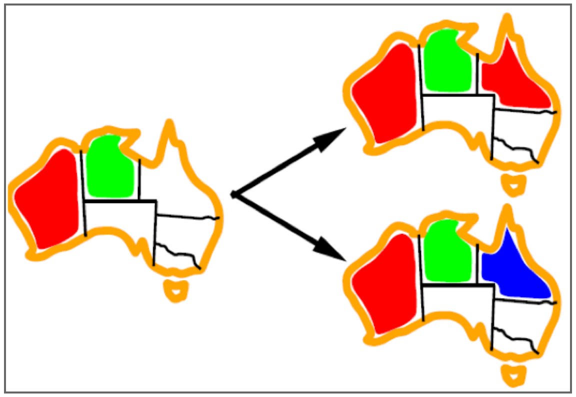


Why min rather than max?

Also called "most constrained variable"

"Fail-fast" ordering

ORDERING: LEAST CONSTRAINING VALUE



Value Ordering: Least Constraining Value

- Given a choice of variable, choose the least constraining value
- i.e., the one that rules out the fewest values in the remaining variables

Why least rather than most?

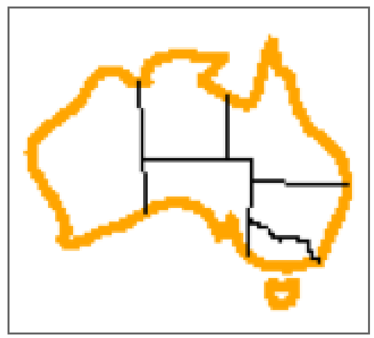
Combining these ordering ideas makes 1000 queens feasible

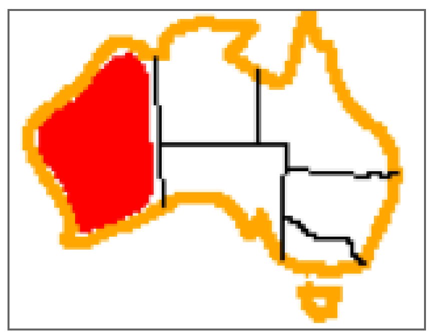
FILTERING

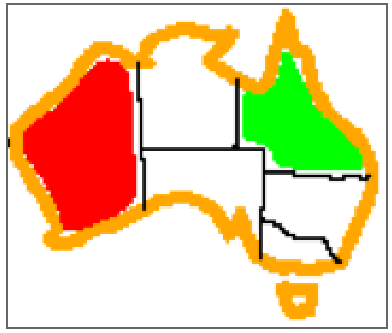
FILTERING: FORWARD CHECKING

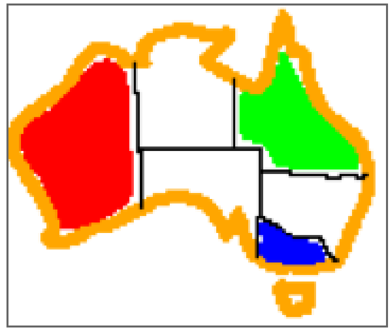
Filtering: Keep track of domains for unassigned variables and cross off bad options.

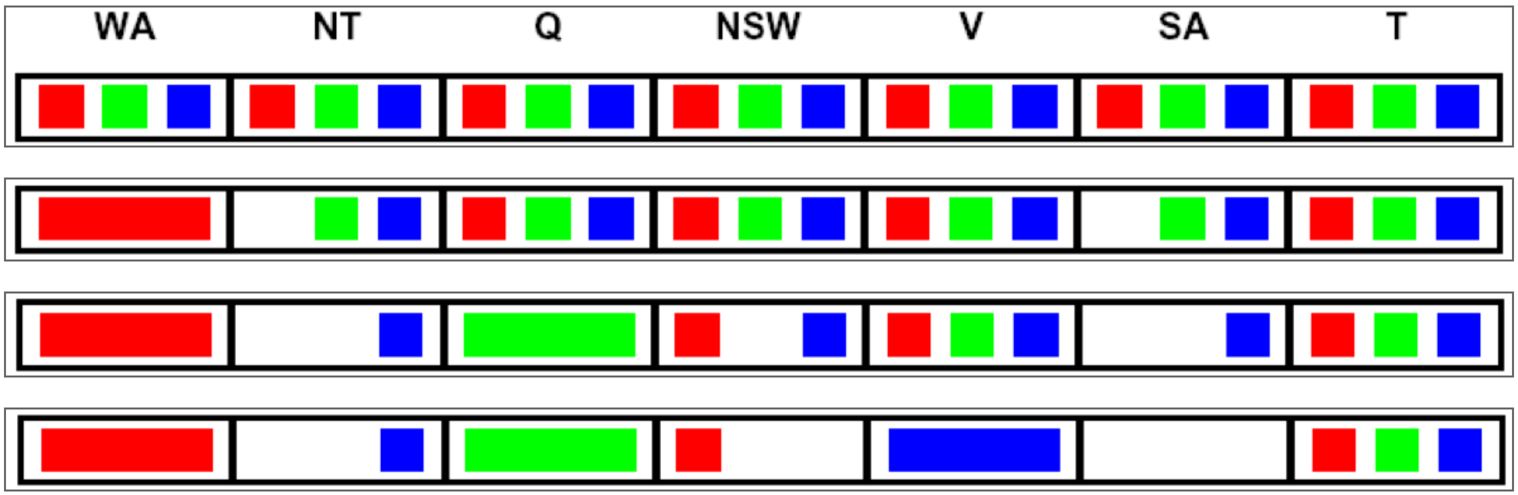
Forward checking: Cross off values that violate a constraint when added to the existing assignment





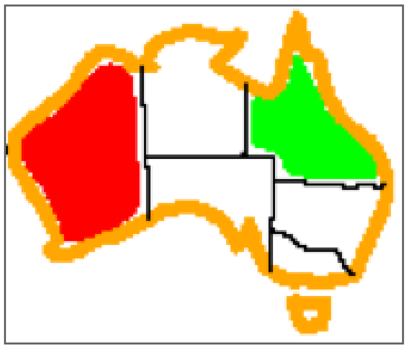


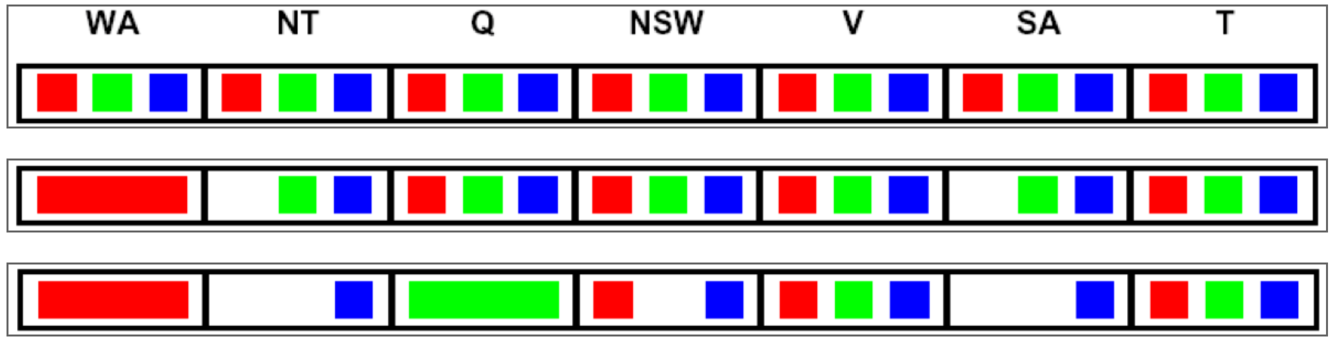



WA
NT
Q
NSW
V
SA
T

FILTERING: CONSTRAINT PROPAGATION

Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:





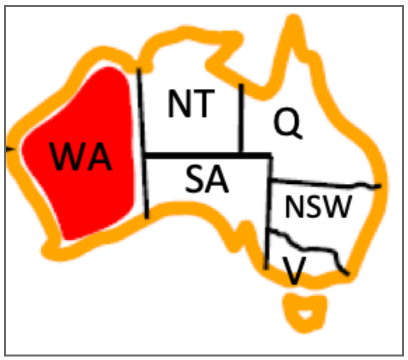
NT and SA cannot both be blue!

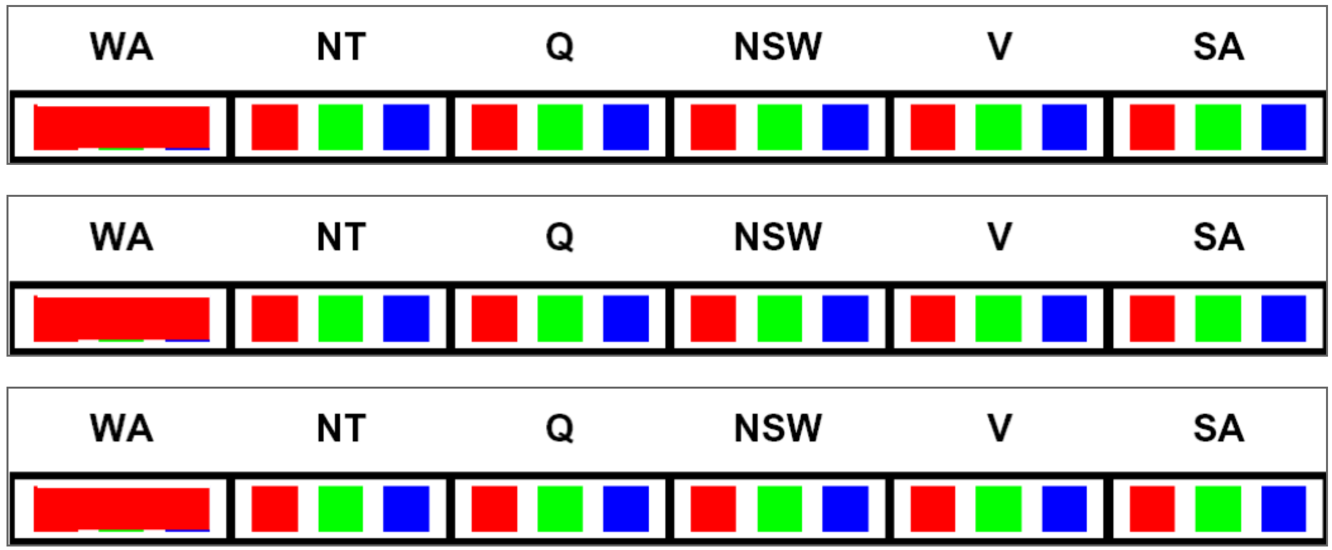
Why didn't we detect this yet?

Constraint propagation: reason from constraint to constraint

CONSISTENCY OF A SINGLE ARC

An arc $X \rightarrow Y$ is consistent iff for *every* x in the tail there is some y in the head which could be assigned without violating a constraint.



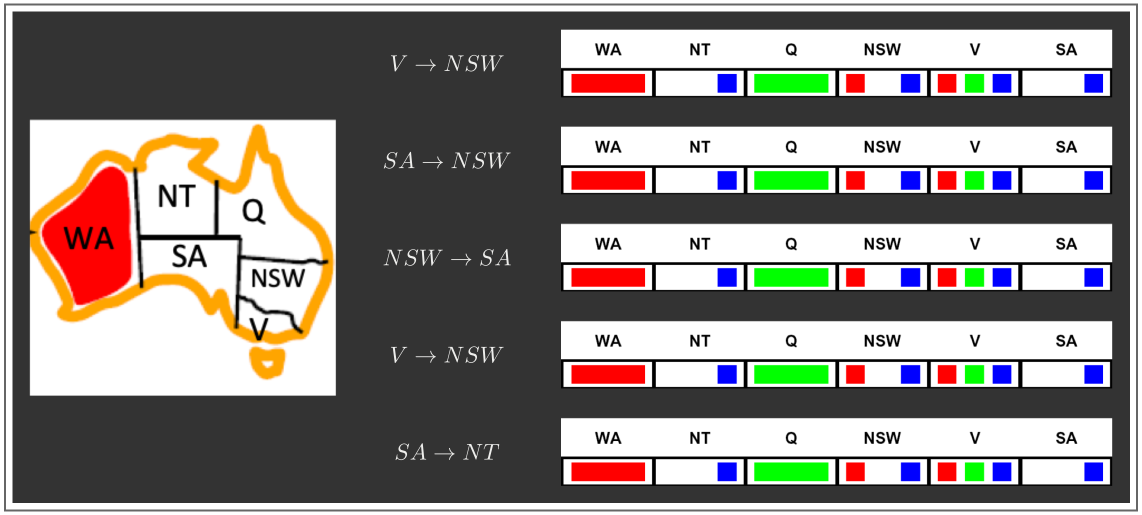


Forward checking: Enforcing consistency of arcs pointing to each new assignment.

Delete from the tail !!

ARC CONSISTENCY OF AN ENTIRE CSP

A simple form of propagation makes sure all arcs are consistent:



Important: If X loses a value, neighbors of X need to be rechecked.

Arc consistency detects failure earlier than forward checking

Can be run as a preprocessor or after each assignment

What is the downside of enforcing arc consistency?

ENFORCING ARC CONSISTENCY IN A CSP

function AC-3(*csp*) **returns** the CSP, possibly with reduced domains
  inputs: *csp*, a binary CSP with variables $\{X_1,\ X_2,\ \ldots,\ X_n\}$
  local variables: *queue*, a queue of arcs, initially all the arcs in *csp*

  while *queue* is not empty **do**
    $(X_i,\ X_j) \leftarrow$ REMOVE-FIRST(*queue*)
    if REMOVE-INCONSISTENT-VALUES($X_i,\ X_j$) **then**
      for each X_k **in** NEIGHBORS[X_i] **do**
        add $(X_k,\ X_i)$ to *queue*

---

function REMOVE-INCONSISTENT-VALUES($X_i,\ X_j$) **returns** true iff succeeds
  removed $\leftarrow$ *false*
  for each x **in** DOMAIN[X_i] **do**
    if no value y in DOMAIN[X_j] allows (x,y) to satisfy the constraint $X_i \leftrightarrow X_j$
      then delete x from DOMAIN[X_i]; *removed* $\leftarrow$ *true*
  return *removed*

Runtime: $\mathcal{O}(n^2 d^3)$, can be reduced to $\mathcal{O}(n^2 d^2)$

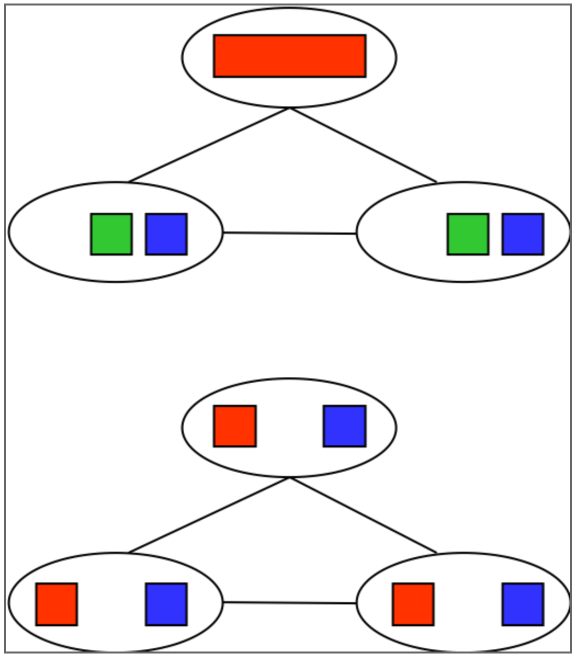
But detecting all possible future problems is NP-hard.

LIMITATIONS OF ARC CONSISTENCY

After enforcing arc consistency:

- Can have one solution left
- Can have multiple solutions left
- Can have no solutions left (and not know it)

Arc consistency still runs inside a backtracking search.



PROBLEM STRUCTURE

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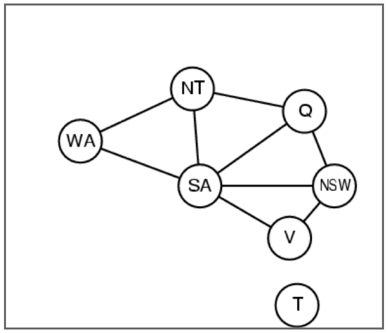


Real world problem can be decomposed into many subproblems.

Extreme case: independent subproblems

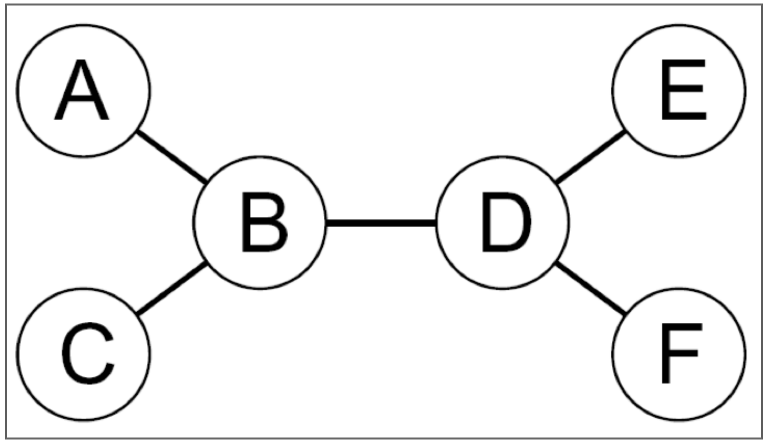
- Tasmania and mainland are independent subproblems

Independent subproblems are identifiable as connected components of constraint graph

Suppose a graph of n variables can be broken into subproblems of only c variables:

- Worst-case solution cost is $\mathcal{O}((n/c)(d^c))$, linear in n
- E.g., $n = 80$, $d = 2$, $c = 20$
- $2^{80} = 4$ billion years at 10 million nodes/sec
- $(4)(2^{20}) = 0.4$ seconds at 10 million nodes/sec

PROBLEM STRUCTURE



Theorem: if the constraint graph has no loops, the CSP can be solved in $\mathcal{O}(nd^2)$ time

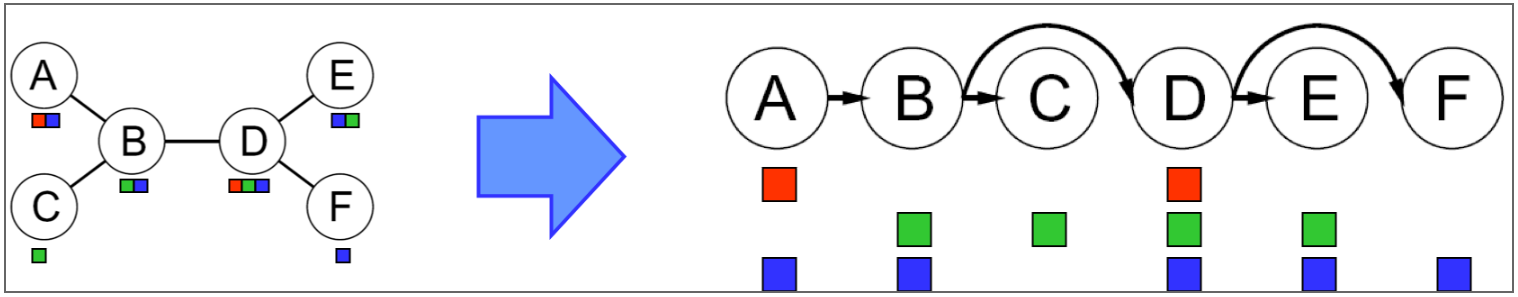
- Compare to general CSPs, where worst-case time is $\mathcal{O}(d^n)$

This property also applies to probabilistic reasoning (later): an example of the relation between syntactic restrictions and the complexity of reasoning.

PROBLEM STRUCTURE

Algorithm for tree-structured CSPs:

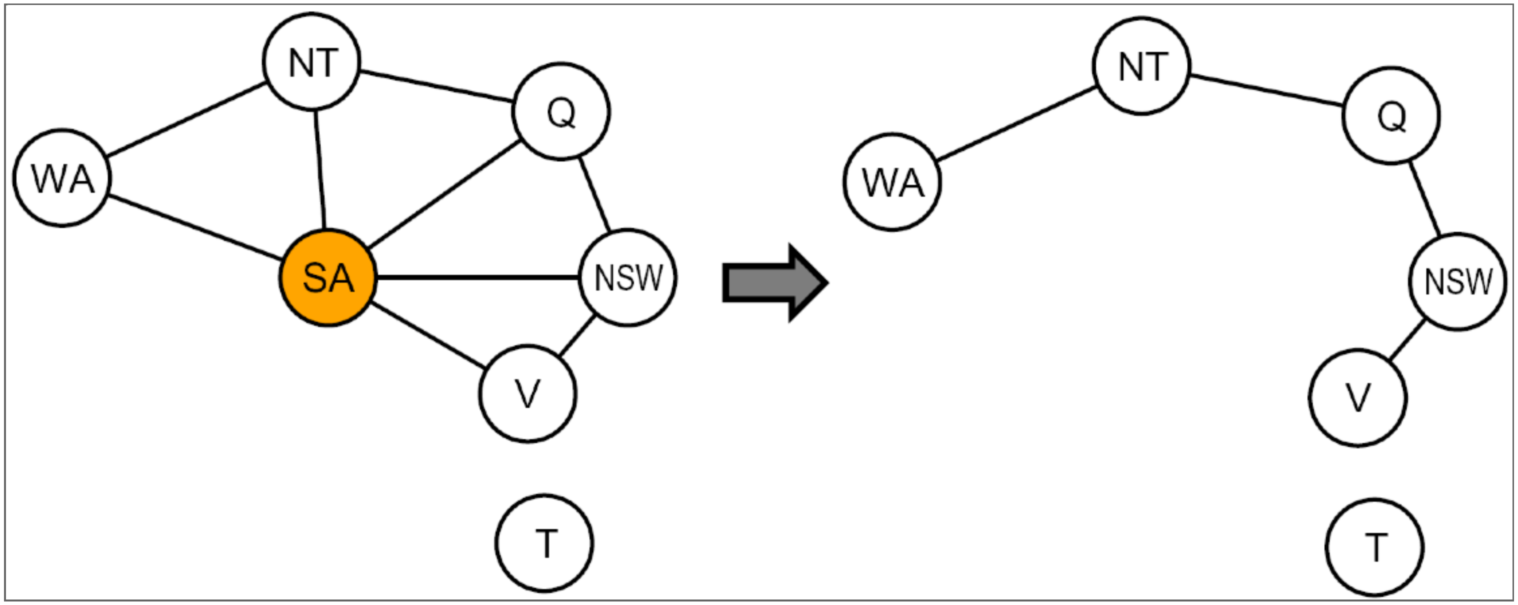
- Order: Choose a root variable, order variables so that parents precede children



- Remove backward: For $i = n : 2$, apply RemoveInconsistent(Parent(X_i),X_i)
- Assign forward: For $i = 1 : n$, assign X_i consistently with Parent(X_i)

Runtime: $\mathcal{O}(nd^2)$

PROBLEM STRUCTURE

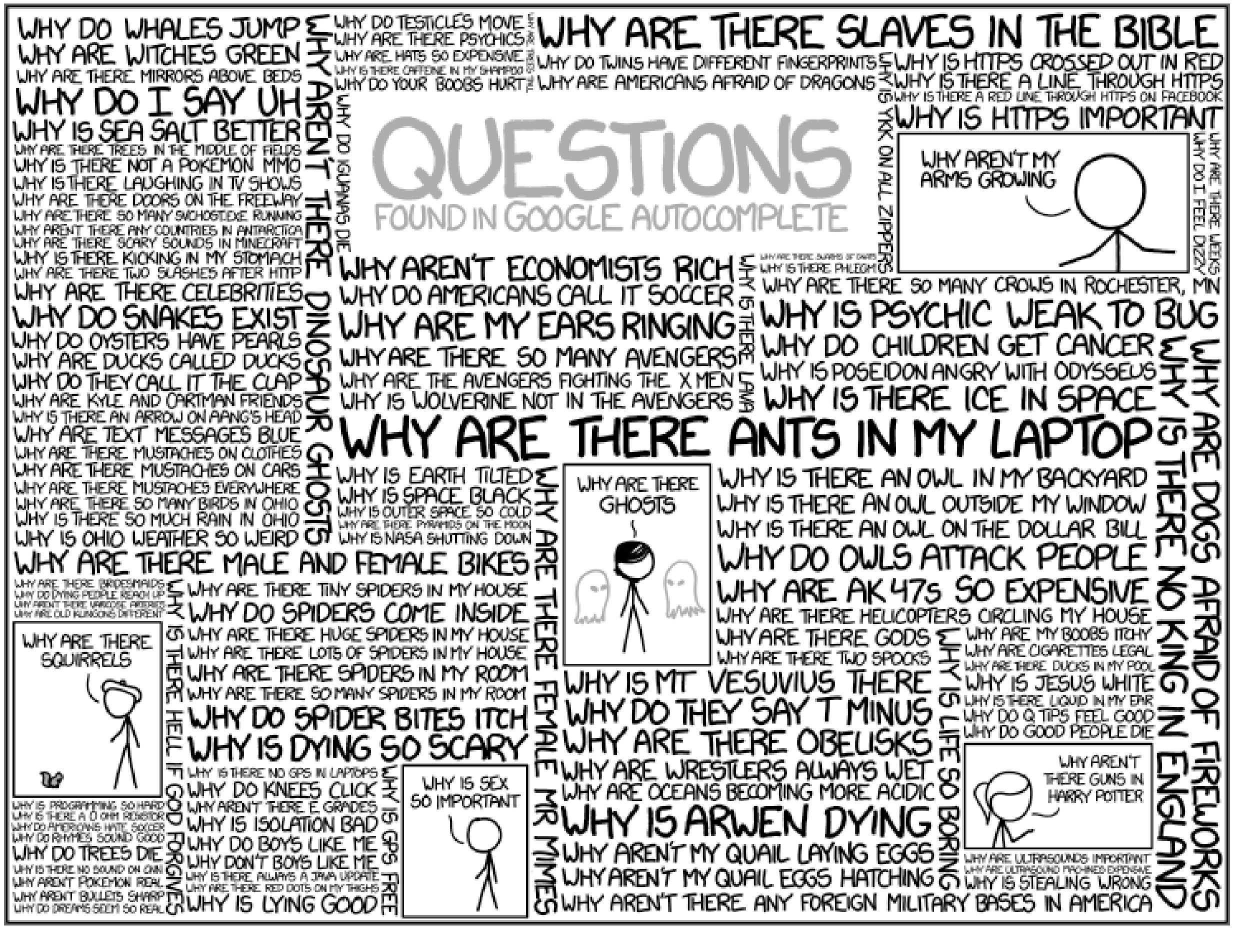


Conditioning: instantiate a variable, prune its neighbors' domains

Cutset conditioning: instantiate (in all ways) a set of variables such that the remaining constraint graph is a tree

Cutset size c gives runtime $\mathcal{O}((d^c)(n-c)d^2)$, very fast for small c.

Q & A


QUESTIONS
FOUND IN GOOGLE AUTOCOMPLETE
WHY DO WHALES JUMP
WHY ARE WITCHES GREEN
WHY ARE THERE MIRRORS ABOVE BEDS
WHY DO I SAY UH
WHY IS SEA SALT BETTER
WHY ARE THERE TREES IN THE MIDDLE OF FIELDS
WHY IS THERE NOT A POKEMON MMO
WHY IS THERE LAUGHING IN TV SHOWS
WHY ARE THERE DOORS ON THE FREEWAY
WHY ARE THERE SO MANY SVCHOST.EXE RUNNING
WHY AREN'T THERE ANY COUNTRIES IN ANTARCTICA
WHY ARE THERE SCARY SOUNDS IN MINECRAFT
WHY IS THERE KICKING IN MY STOMACH
WHY ARE THERE TWO SLASHES AFTER HTTP
WHY ARE THERE CELEBRITIES
WHY DO SNAKES EXIST
WHY DO OYSTERS HAVE PEARLS
WHY ARE DUCKS CALLED DUCKS
WHY DO THEY CALL IT THE CLAP
WHY ARE KYLE AND CARTMAN FRIENDS
WHY IS THERE AN ARROW ON AANG'S HEAD
WHY ARE TEXT MESSAGES BLUE
WHY ARE THERE MUSTACHES ON CLOTHES
WHY ARE THERE MUSTACHES ON CARS
WHY ARE THERE MUSTACHES EVERYWHERE
WHY ARE THERE SO MANY BIRDS IN OHIO
WHY IS THERE SO MUCH RAIN IN OHIO
WHY IS OHIO WEATHER SO WEIRD
WHY AREN'T THERE DINOSAUR GHOSTS
WHY DO IGUANAS DIE
WHY DO TESTICLES MOVE
WHY ARE THERE PSYCHICS
WHY ARE HATS SO EXPENSIVE
WHY IS THERE CAFFEINE IN MY SHAMPOO
WHY DO YOUR BOOBS HURT
WHY ARE THERE SLAVES IN THE BIBLE
WHY DO TWINS HAVE DIFFERENT FINGERPRINTS
WHY ARE AMERICANS AFRAID OF DRAGONS
WHY IS YKK ON ALL ZIPPERS
WHY IS HTTPS CROSSED OUT IN RED
WHY IS THERE A LINE THROUGH HTTPS
WHY IS THERE A RED LINE THROUGH HTTPS ON FACEBOOK
WHY IS HTTPS IMPORTANT
WHY AREN'T MY ARMS GROWING
WHY ARE THERE WEEKS
WHY DO I FEEL DIZZY
WHY AREN'T ECONOMISTS RICH
WHY DO AMERICANS CALL IT SOCCER
WHY ARE MY EARS RINGING
WHY ARE THERE SO MANY AVENGERS
WHY ARE THE AVENGERS FIGHTING THE X MEN
WHY IS WOLVERINE NOT IN THE AVENGERS
WHY IS THERE LAVA
WHY IS THERE PHLEGM
WHY ARE THERE SO MANY CROWS IN ROCHESTER, MN
WHY IS PSYCHIC WEAK TO BUG
WHY DO CHILDREN GET CANCER
WHY IS POSEIDON ANGRY WITH ODYSSEUS
WHY IS THERE ICE IN SPACE
WHY ARE THERE ANTS IN MY LAPTOP
WHY IS EARTH TILTED
WHY IS SPACE BLACK
WHY IS OUTER SPACE SO COLD
WHY ARE THERE PYRAMIDS ON THE MOON
WHY IS NASA SHUTTING DOWN
WHY ARE THERE GHOSTS
WHY IS THERE AN OWL IN MY BACKYARD
WHY IS THERE AN OWL OUTSIDE MY WINDOW
WHY IS THERE AN OWL ON THE DOLLAR BILL
WHY DO OWLS ATTACK PEOPLE
WHY ARE AK 47s SO EXPENSIVE
WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE
WHY ARE THERE GODS
WHY ARE THERE TWO SPOCKS
WHY ARE MY BOOBS ITCHY
WHY ARE CIGARETTES LEGAL
WHY ARE THERE DUCKS IN MY POOL
WHY IS JESUS WHITE
WHY IS THERE LIQUID IN MY EAR
WHY DO Q TIPS FEEL GOOD
WHY DO GOOD PEOPLE DIE
WHY IS THERE NO KING IN ENGLAND
WHY ARE DOGS AFRAID OF FIREWORKS
WHY ARE THERE MALE AND FEMALE BIKES
WHY ARE THERE BRIDESMAIDS
WHY DO DYING PEOPLE REACH UP
WHY ARE OLD KLINGONS DIFFERENT
WHY ARE THERE SQUIRRELS
WHY IS THERE HELL IF GOD FORGIVES
WHY ARE THERE TINY SPIDERS IN MY HOUSE
WHY DO SPIDERS COME INSIDE
WHY ARE THERE HUGE SPIDERS IN MY HOUSE
WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE
WHY ARE THERE SPIDERS IN MY ROOM
WHY ARE THERE SO MANY SPIDERS IN MY ROOM
WHY DO SPIDER BITES ITCH
WHY IS DYING SO SCARY
WHY ARE THERE FEMALE MR MIMES
WHY IS PROGRAMMING SO HARD
WHY IS THERE A 0 OHM RESISTOR
WHY DO AMERICANS HATE SOCCER
WHY DO RHYMES SOUND GOOD
WHY DO TREES DIE
WHY IS THERE NO SOUND ON CNN
WHY AREN'T POKEMON REAL
WHY AREN'T BULLETS SHARP
WHY DO DREAMS SEEM SO REAL
WHY IS THERE NO GPS IN LAPTOPS
WHY DO KNEES CLICK
WHY AREN'T THERE E GRADES
WHY IS ISOLATION BAD
WHY DO BOYS LIKE ME
WHY DON'T BOYS LIKE ME
WHY IS THERE ALWAYS A JAVA UPDATE
WHY ARE THERE RED DOTS ON MY THIGHS
WHY IS LYING GOOD
WHY IS GPS FREE
WHY IS SEX SO IMPORTANT
WHY IS MT VESUVIUS THERE
WHY DO THEY SAY T MINUS
WHY ARE THERE OBELISKS
WHY ARE WRESTLERS ALWAYS WET
WHY ARE OCEANS BECOMING MORE ACIDIC
WHY IS ARWEN DYING
WHY AREN'T MY QUAIL LAYING EGGS
WHY AREN'T MY QUAIL EGGS HATCHING
WHY AREN'T THERE ANY FOREIGN MILITARY BASES IN AMERICA
WHY IS LIFE SO BORING
WHY AREN'T THERE GUNS IN HARRY POTTER
WHY ARE ULTRASOUNDS IMPORTANT
WHY ARE ULTRASOUND MACHINES EXPENSIVE
WHY IS STEALING WRONG

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Speaker notes

Q & A

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