Constraint Satisfaction

CS5491: Artificial Intelligence
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Content Credits: **Prof. Wei**'s CS4486 Course
and **Prof. Boddeti**'s AI Course

WHAT ARE CONSTRAINT SATISFACTION PROBLEMS?

Special kind of search problems.

- N variables
- Values from domain D
- assignment satisfies constraints

States: partial assignment

Goal Test: satisfies all constraints

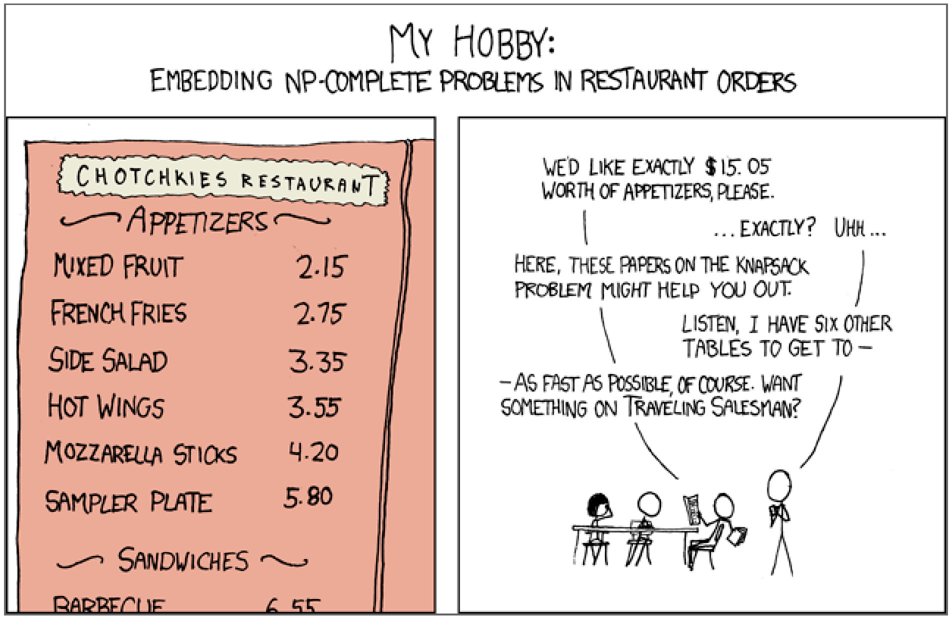
Successor Function: assign an unassigned variable

TODAY

Constraint Satisfaction

Reading

- Today's Lecture: RN Chapter 6



XKCD

WHAT IS SEARCH FOR?

Given assumptions about the world: a single agent, deterministic actions, fully observed state, discrete state space

- Planning: sequences of actions
- The path to the goal is the important thing
- Paths have various costs, depths
- Heuristics give problem-specific guidance
- Identification: assignments to variables
- The goal itself is important, not the path
- All paths at the same depth (for some formulations)
- CSPs are a specialized class of identification problems

CONSTRAINT SATISFACTION PROBLEMS

WHAT ARE CONSTRAINT SATISFACTION PROBLEMS?

Standard search problems:

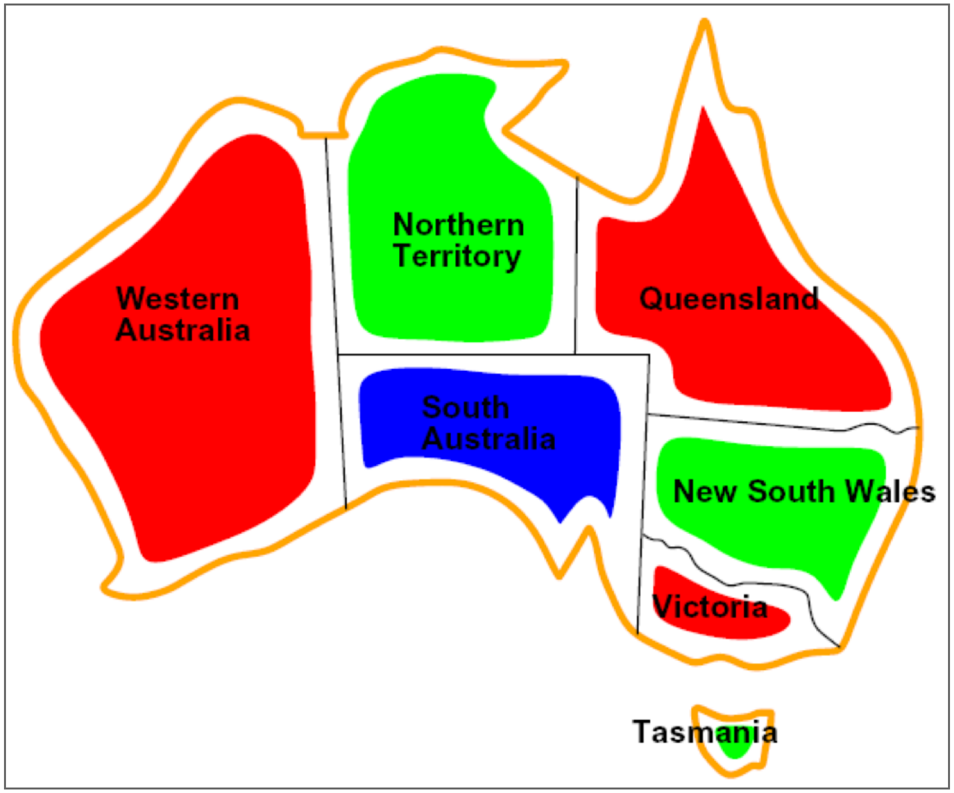
- State is a "black box": arbitrary data structure
- Goal test can be any function over states
- Successor function can also be anything

Constraint satisfaction problems (CSPs):

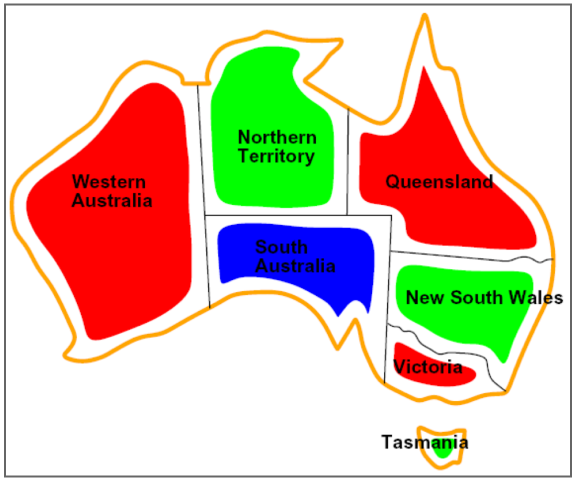
- A special subset of search problems
- State is defined by variables $\mathbf{X}_i$ with values from a domain D (sometimes D depends on i)
- Goal test is a set of constraints specifying allowable combinations of values for subsets of variables

Allows useful general-purpose algorithms with more power than standard search algorithms

CSP EXAMPLE


Western Australia
Northern Territory
South Australia
Queensland
New South Wales
Victoria
Tasmania

EXAMPLE: MAP COLORING



Variables: WA, NT, Q, NSW, V, SA, T

Domains: $D = \{red, green, blue\}$

Constraints: adjacent regions must have different colors

- Implicit: $WA \neq NT$
- Explicit: $(WA, NT) \in (red, green), (red, blue), \ldots$

Solutions: Assignments that satisfy all constraints, e.g.:

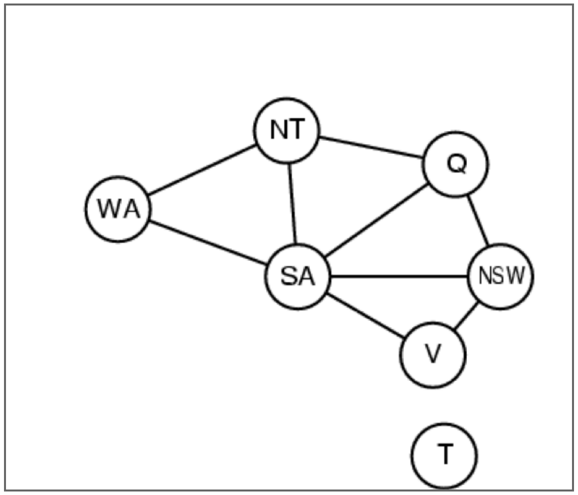
- {WA=red, NT=green, Q=red, NSW=green, V=red, SA=blue, T=green}

CONSTRAINT GRAPHS

Binary CSP: each constraint relates two variables

Constraint Graph: nodes are variables, arcs are constraints

General-purpose CSP algorithms use the graph structure to speed up search. E.g., Tasmania is an independent subproblem.

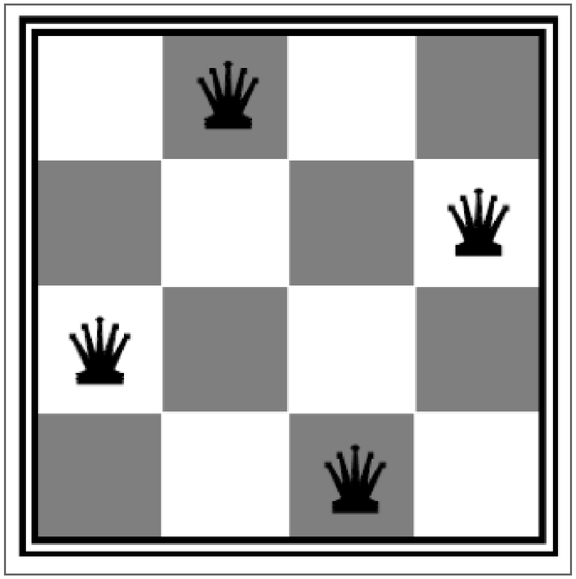


EXAMPLE: N QUEENS

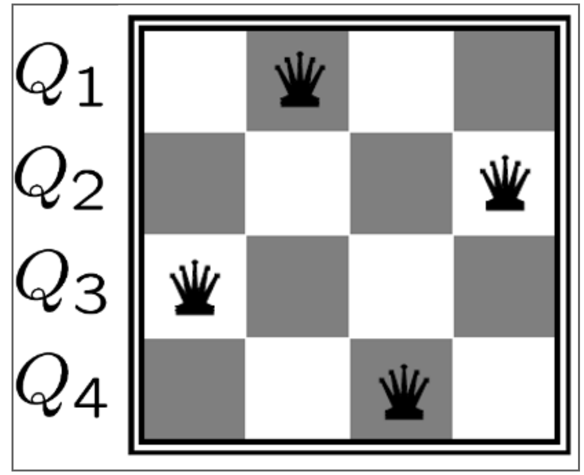
Formulation 1:

- Variables: X_{ij}
- Domains: $\{0,1\}$
- Constraints:

$$\forall\, i,j,k \;\; (X_{ij}, X_{ik}) \in (0,0),(0,1),(1,0)$$
$$\forall\, i,j,k \;\; (X_{ij}, X_{kj}) \in (0,0),(0,1),(1,0)$$
$$\forall\, i,j,k \;\; (X_{ij}, X_{i+k,j+k}) \in (0,0),(0,1),(1,0)$$
$$\forall\, i,j,k \;\; (X_{ij}, X_{i+k,j-k}) \in (0,0),(0,1),(1,0)$$
$$\sum_{ij} X_{ij} = N$$



EXAMPLE: N QUEENS



Formulation 2:

- Variables: Q_k
- Domains: $1, 2, 3, \ldots, N$
- Constraints:
- Implicit: $\forall(i, j)\ \mathbf{non - threatening}\ (Q_i, Q_j)$
- Explicit: $(Q_1, Q_2) \in \{(1,3), (1,4), \ldots\}$

VARIETY OF CSPS

Discrete Variables

- Finite domains
- Size d means $O(d^n)$ complete assignments
- E.g., Boolean CSPs, including Boolean satisfiability (NP-complete)
- Infinite domains (integers, strings, etc.)
- E.g., job scheduling, variables are start/end times for each job
- Linear constraints solvable, nonlinear undecidable

Continuous variables

- E.g., start/end times for Hubble Telescope observations
- Linear constraints solvable in polynomial time by LP methods

VARIETIES OF CONSTRAINTS

Varieties of Constraints

- Unary constraints involve a single variable (equivalent to reducing domains), e.g.: $SA \neq green$
- Binary constraints involve pairs of variables, e.g.: $SA \neq WA$
- Higher-order constraints involve 3 or more variables: e.g., cryptarithmetic column constraints

Preferences (soft constraints):

- E.g., red is better than green
- Often representable by a cost for each variable assignment
- Leads to constrained optimization problems

REAL-WORLD CSPS

Scheduling problems: e.g., when can we all meet?

Timetabling problems: e.g., which class is offered when and where?

Assignment problems: e.g., who teaches what class

Hardware configuration

Transportation scheduling

Factory scheduling

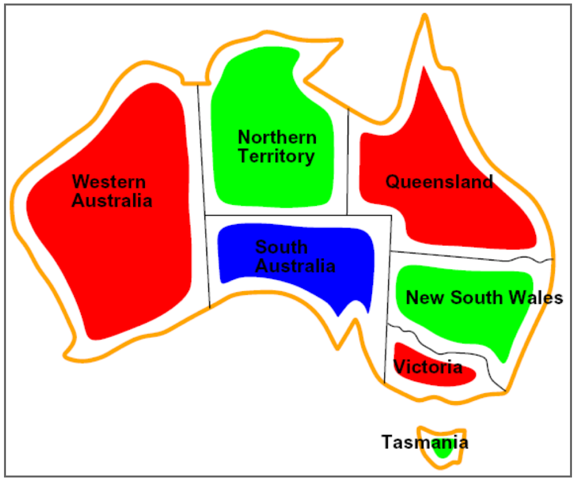
Circuit layout

Fault diagnosis

. . .

SOLVING CSPS

EXAMPLE: MAP COLORING



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STANDARD SEARCH FORMULATION

Standard search formulation of CSPs

States defined by the values assigned so far (partial assignments)

- Initial state: the empty assignment, {}
- Successor function: assign a value to an unassigned variable
- Goal test: the current assignment is complete and satisfies all constraints

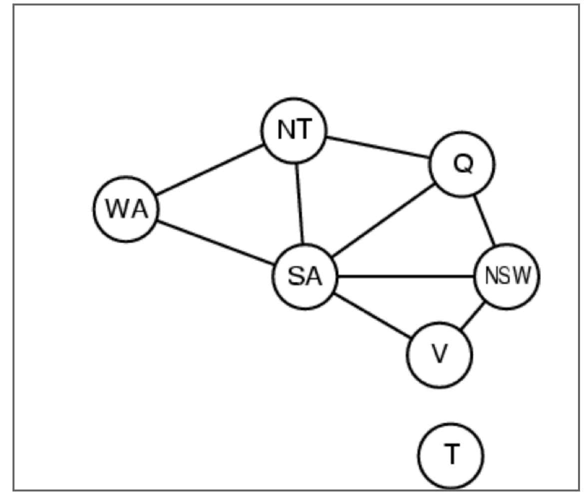
Start with straightforward search and then improve.

SEARCH METHODS

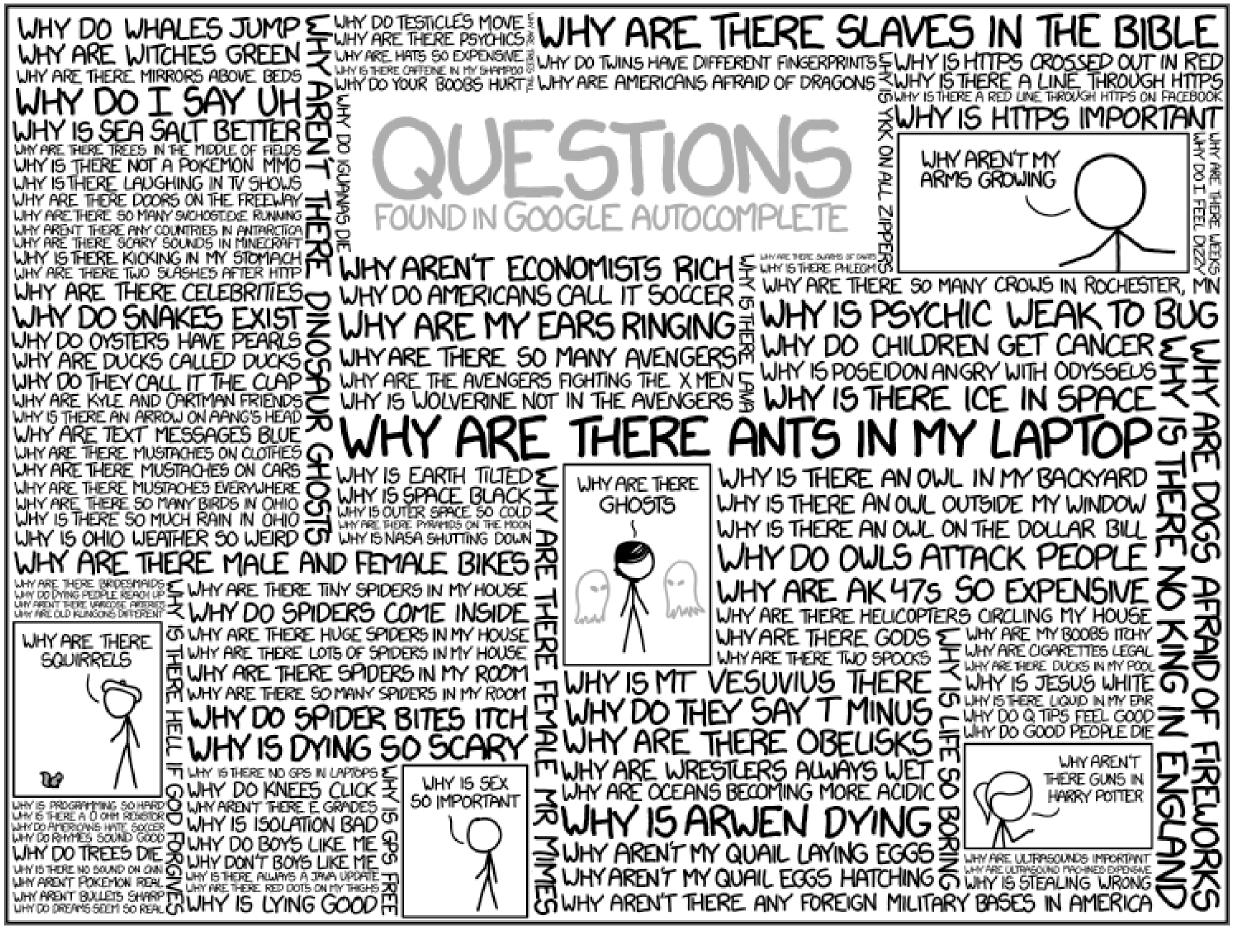
What would BFS do?

What would DFS do?

What problems does naïve search have?



Q & A


QUESTIONS
FOUND IN GOOGLE AUTOCOMPLETE
WHY DO WHALES JUMP
WHY ARE WITCHES GREEN
WHY ARE THERE MIRRORS ABOVE BEDS
WHY DO I SAY UH
WHY IS SEA SALT BETTER
WHY ARE THERE TREES IN THE MIDDLE OF FIELDS
WHY IS THERE NOT A POKEMON MMO
WHY IS THERE LAUGHING IN TV SHOWS
WHY ARE THERE DOORS ON THE FREEWAY
WHY ARE THERE SO MANY SVCHOST.EXE RUNNING
WHY AREN'T THERE ANY COUNTRIES IN ANTARCTICA
WHY ARE THERE SCARY SOUNDS IN MINECRAFT
WHY IS THERE KICKING IN MY STOMACH
WHY ARE THERE TWO SLASHES AFTER HTTP
WHY ARE THERE CELEBRITIES
WHY DO SNAKES EXIST
WHY DO OYSTERS HAVE PEARLS
WHY ARE DUCKS CALLED DUCKS
WHY DO THEY CALL IT THE CLAP
WHY ARE KYLE AND CARTMAN FRIENDS
WHY IS THERE AN ARROW ON AANG'S HEAD
WHY ARE TEXT MESSAGES BLUE
WHY ARE THERE MUSTACHES ON CLOTHES
WHY ARE THERE MUSTACHES ON CARS
WHY ARE THERE MUSTACHES EVERYWHERE
WHY ARE THERE SO MANY BIRDS IN OHIO
WHY IS THERE SO MUCH RAIN IN OHIO
WHY IS OHIO WEATHER SO WEIRD
WHY AREN'T THERE DINOSAUR GHOSTS
WHY DO IGUANAS DIE
WHY DO TESTICLES MOVE
WHY ARE THERE PSYCHICS
WHY ARE HATS SO EXPENSIVE
WHY IS THERE CAFFEINE IN MY SHAMPOO
WHY DO YOUR BOOBS HURT
WHY ARE THERE SLAVES IN THE BIBLE
WHY DO TWINS HAVE DIFFERENT FINGERPRINTS
WHY ARE AMERICANS AFRAID OF DRAGONS
WHY IS YKK ON ALL ZIPPERS
WHY IS HTTPS CROSSED OUT IN RED
WHY IS THERE A LINE THROUGH HTTPS
WHY IS THERE A RED LINE THROUGH HTTPS ON FACEBOOK
WHY IS HTTPS IMPORTANT
WHY AREN'T MY ARMS GROWING
WHY ARE THERE WEEKS
WHY DO I FEEL DIZZY
WHY AREN'T ECONOMISTS RICH
WHY DO AMERICANS CALL IT SOCCER
WHY ARE MY EARS RINGING
WHY ARE THERE SO MANY AVENGERS
WHY ARE THE AVENGERS FIGHTING THE X MEN
WHY IS WOLVERINE NOT IN THE AVENGERS
WHY IS THERE LAVA
WHY IS THERE PHLEGM
WHY ARE THERE SO MANY CROWS IN ROCHESTER, MN
WHY IS PSYCHIC WEAK TO BUG
WHY DO CHILDREN GET CANCER
WHY IS POSEIDON ANGRY WITH ODYSSEUS
WHY IS THERE ICE IN SPACE
WHY ARE THERE ANTS IN MY LAPTOP
WHY IS EARTH TILTED
WHY IS SPACE BLACK
WHY IS OUTER SPACE SO COLD
WHY ARE THERE PYRAMIDS ON THE MOON
WHY IS NASA SHUTTING DOWN
WHY ARE THERE GHOSTS
WHY IS THERE AN OWL IN MY BACKYARD
WHY IS THERE AN OWL OUTSIDE MY WINDOW
WHY IS THERE AN OWL ON THE DOLLAR BILL
WHY DO OWLS ATTACK PEOPLE
WHY ARE AK 47s SO EXPENSIVE
WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE
WHY ARE THERE GODS
WHY ARE THERE TWO SPOCKS
WHY ARE THERE MALE AND FEMALE BIKES
WHY ARE THERE TINY SPIDERS IN MY HOUSE
WHY DO SPIDERS COME INSIDE
WHY ARE THERE HUGE SPIDERS IN MY HOUSE
WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE
WHY ARE THERE SPIDERS IN MY ROOM
WHY ARE THERE SO MANY SPIDERS IN MY ROOM
WHY DO SPIDER BITES ITCH
WHY IS DYING SO SCARY
WHY ARE THERE SQUIRRELS
WHY IS THERE HELL IF GOD FORGIVES
WHY IS THERE NO GPS IN LAPTOPS
WHY DO KNEES CLICK
WHY AREN'T THERE E GRADES
WHY IS ISOLATION BAD
WHY DO BOYS LIKE ME
WHY DON'T BOYS LIKE ME
WHY IS THERE ALWAYS A JAVA UPDATE
WHY ARE THERE RED DOTS ON MY THIGHS
WHY IS LYING GOOD
WHY IS GPS FREE
WHY IS SEX SO IMPORTANT
WHY DO TREES DIE
WHY ARE THERE FEMALE MR MIMES
WHY IS MT VESUVIUS THERE
WHY DO THEY SAY T MINUS
WHY ARE THERE OBELISKS
WHY ARE WRESTLERS ALWAYS WET
WHY ARE OCEANS BECOMING MORE ACIDIC
WHY IS ARWEN DYING
WHY AREN'T MY QUAIL LAYING EGGS
WHY AREN'T MY QUAIL EGGS HATCHING
WHY AREN'T THERE ANY FOREIGN MILITARY BASES IN AMERICA
WHY IS LIFE SO BORING
WHY ARE MY BOOBS ITCHY
WHY ARE CIGARETTES LEGAL
WHY ARE THERE DUCKS IN MY POOL
WHY IS JESUS WHITE
WHY IS THERE LIQUID IN MY EAR
WHY DO Q TIPS FEEL GOOD
WHY DO GOOD PEOPLE DIE
WHY AREN'T THERE GUNS IN HARRY POTTER
WHY ARE ULTRASOUNDS IMPORTANT
WHY IS STEALING WRONG
WHY IS THERE NO KING IN ENGLAND
WHY ARE DOGS AFRAID OF FIREWORKS

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Speaker notes

Q & A

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