

# Informed Search

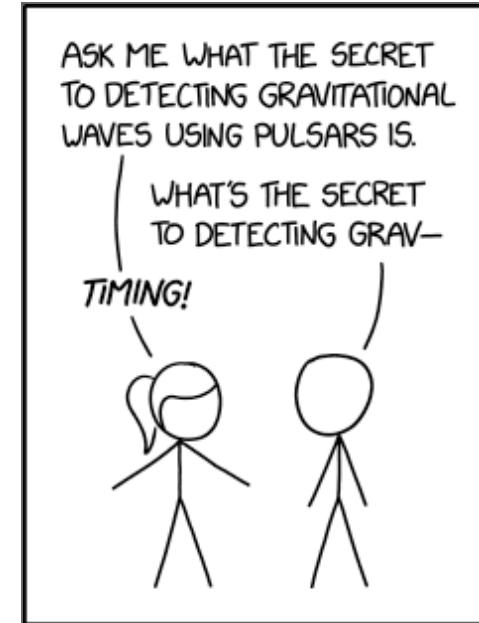
CS5491: Artificial Intelligence  
ZHICHAO LU

Content Credits: Prof. Wei's CS4486 Course  
and Prof. Boddeti's AI Course

# TODAY (PART 2)

## Informed Search Methods

- › Heuristics
- › Greedy Search
- ›  $A^*$  Search



**XKCD**

## Reading

- › Today's Lecture: RN Chapter 3.5-3.6, 4.1-4.2

# SEARCH PROBLEMS

---

## Uninformed Search

Methods we saw:

- › DFS
- › BFS
- › UCS

Searching with your **EYES CLOSED**

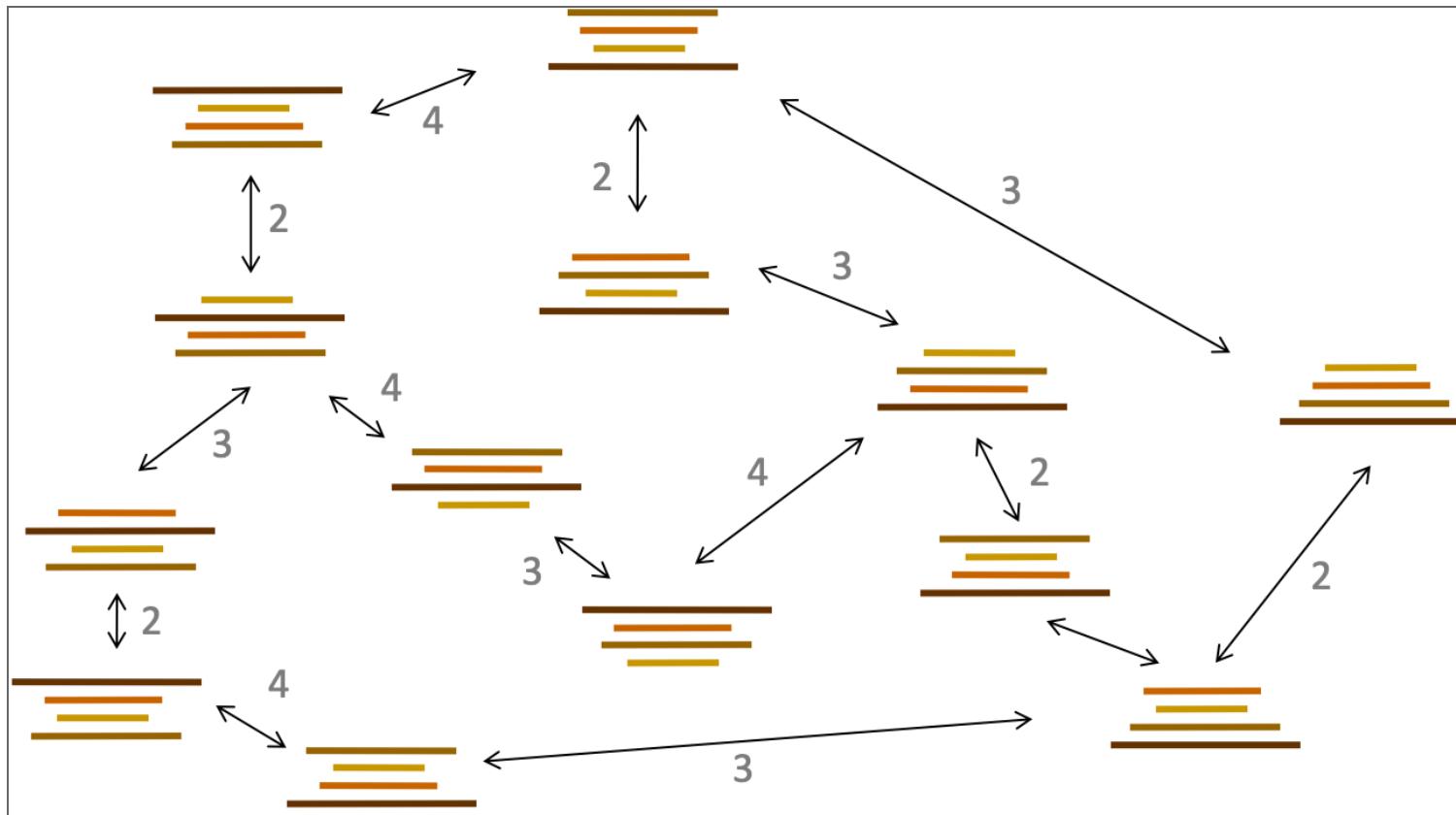
## Informed Search

Methods we will see:

- Greedy
- $A^*$

Searching with your **EYES OPEN**

## EXAMPLE: PANCAKE PROBLEM



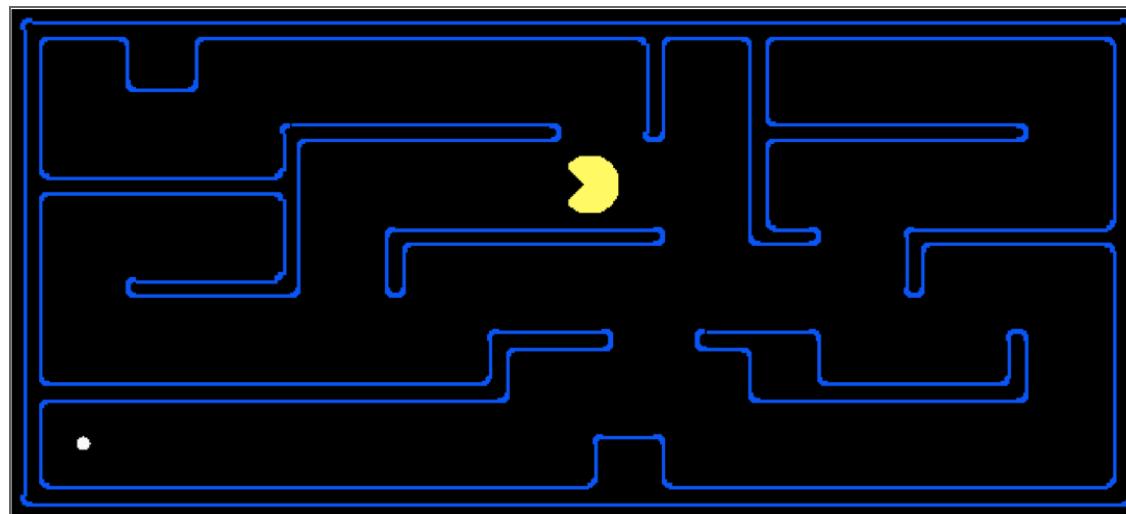
# HEURISTICS

---

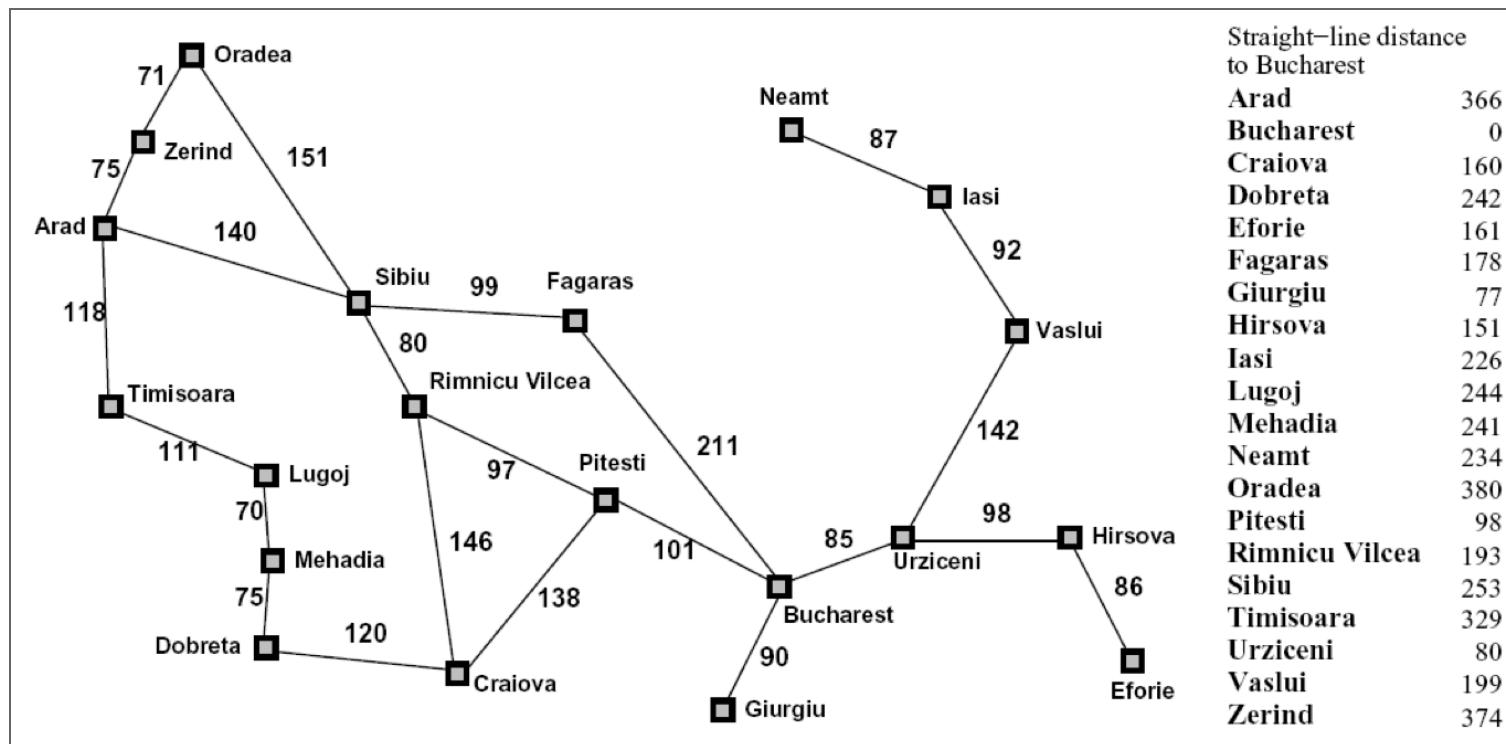
# SEARCH HEURISTICS

A heuristic is:

- › A function that estimates how close a state is to a goal
- › Designed for a particular search problem
- › Examples: Manhattan distance, Euclidean distance

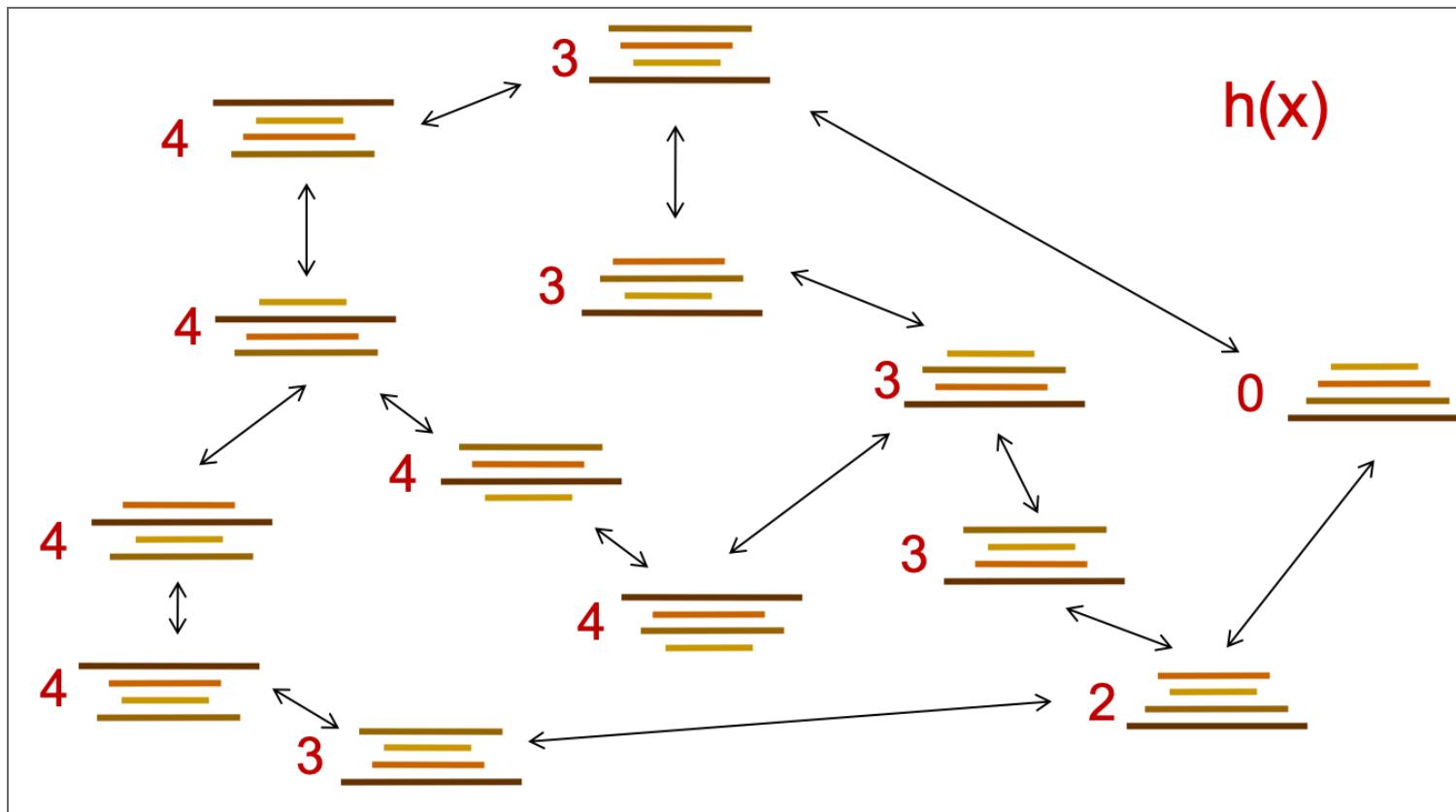


# EXAMPLE: HEURISTIC FUNCTION



## EXAMPLE: HEURISTIC FUNCTION

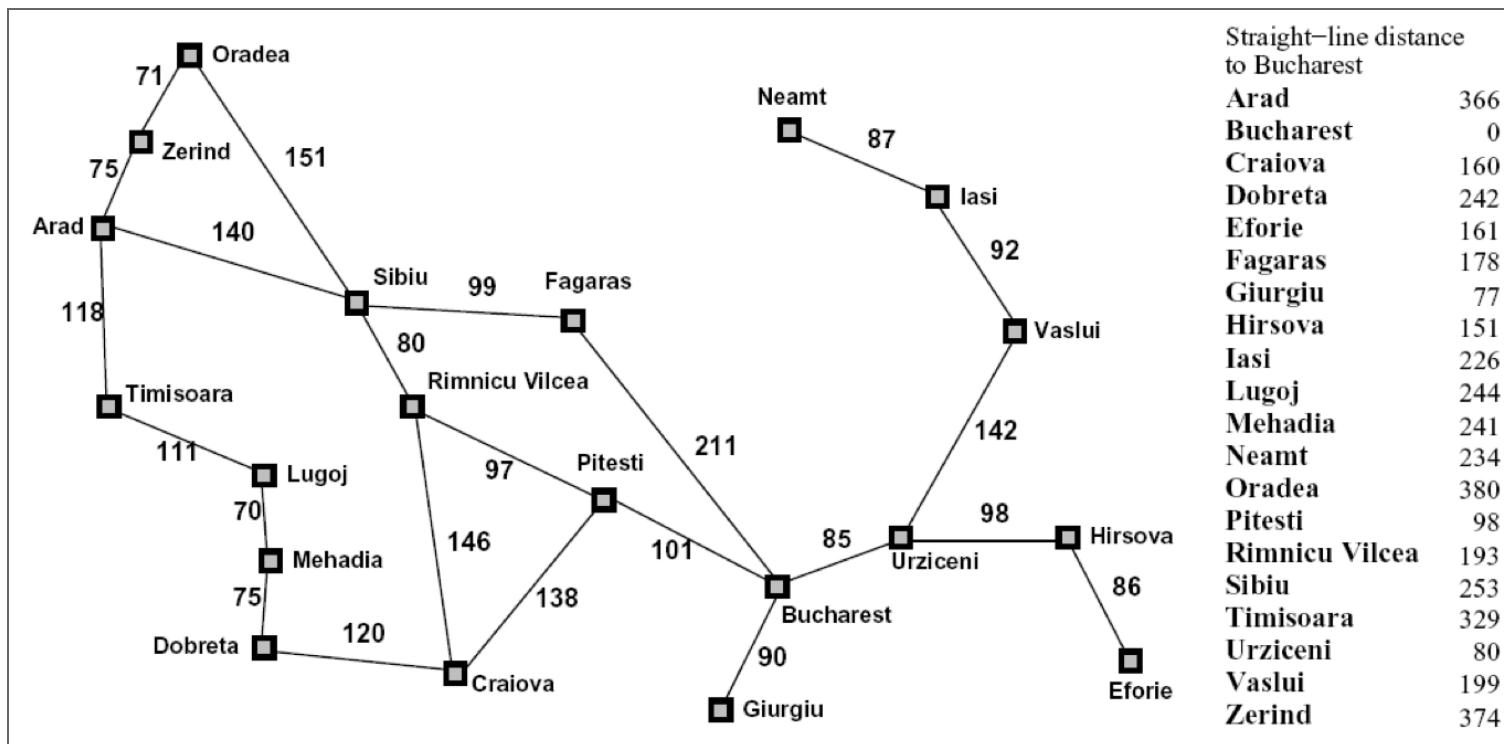
Eg: index of the largest pancake that is still out of place



# GREEDY SEARCH

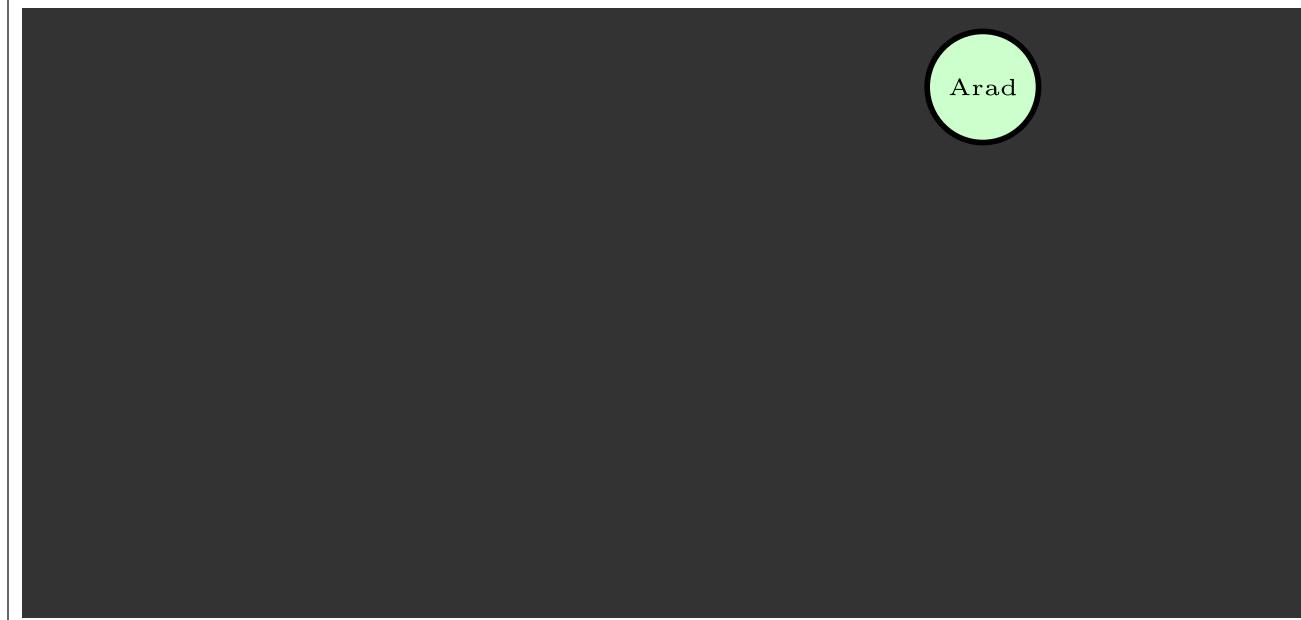
---

# EXAMPLE: HEURISTIC FUNCTION



# GREEDY SEARCH: ROMANIA

Expand the node that seems closest...



Straight-line distance to Bucharest	
<b>Arad</b>	366
<b>Bucharest</b>	0
<b>Craiova</b>	160
<b>Dobreta</b>	242
<b>Eforie</b>	161
<b>Fagaras</b>	178
<b>Giurgiu</b>	77
<b>Hirsova</b>	151
<b>Iasi</b>	226
<b>Lugoj</b>	244
<b>Mehadia</b>	241
<b>Neamt</b>	234
<b>Oradea</b>	380
<b>Pitesti</b>	98
<b>Rimnicu Vilcea</b>	193
<b>Sibiu</b>	253
<b>Timisoara</b>	329
<b>Urziceni</b>	80
<b>Vaslui</b>	199
<b>Zerind</b>	374

## GREEDY SEARCH

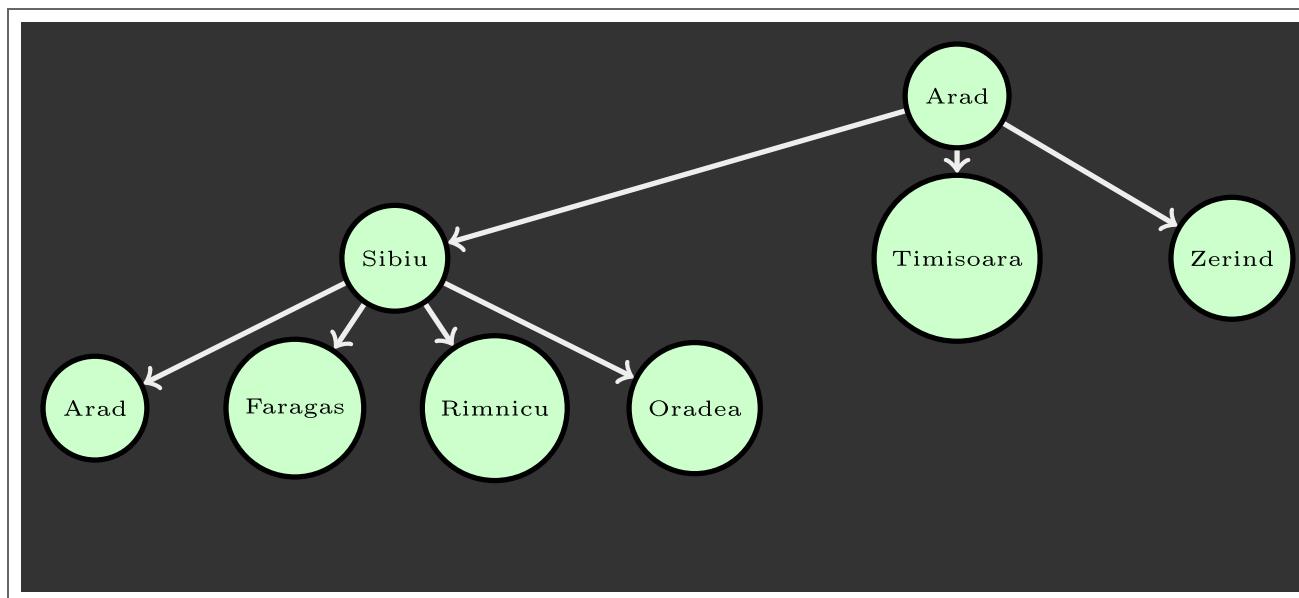
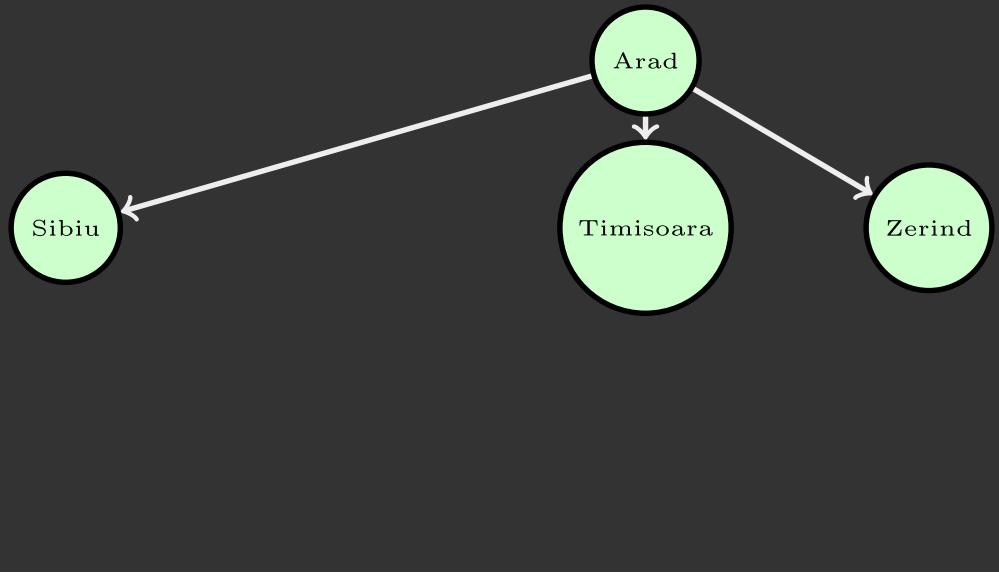
Strategy: expand a node that you think is closest to a goal state

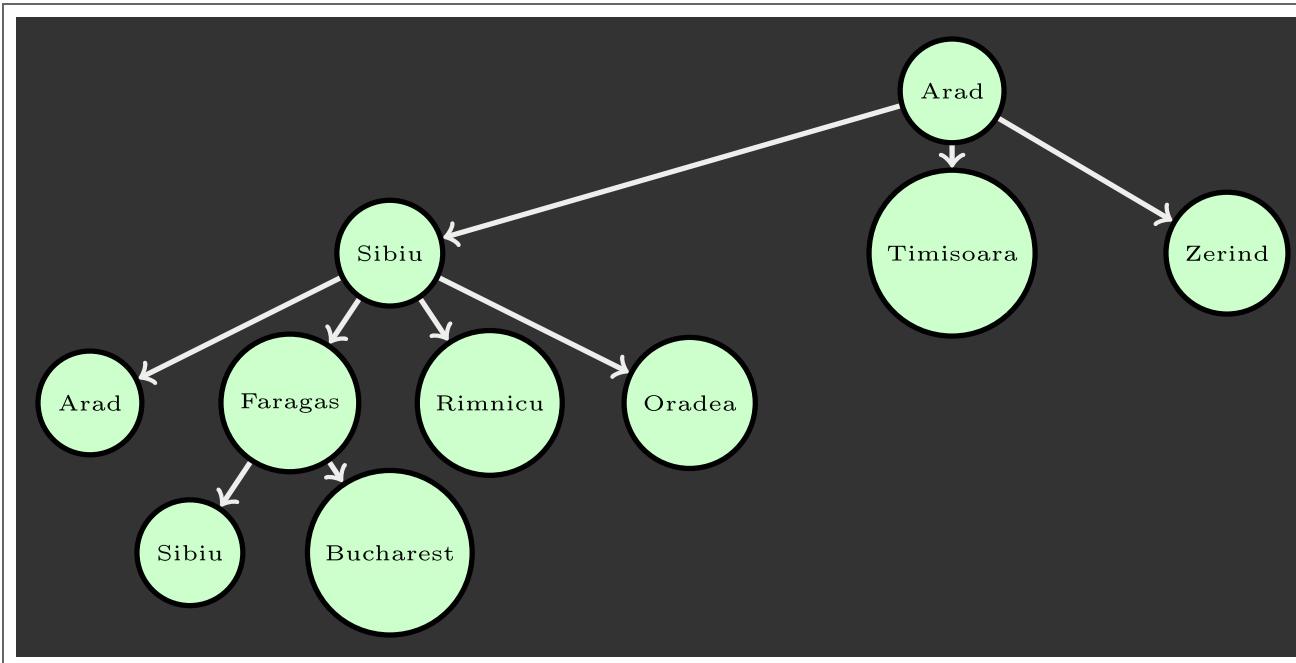
Heuristic: distance estimate to nearest goal for each state

A common case:

Best-first takes you straight to the (wrong) goal

Worst-case: like a badly-guided DFS





## GREEDY DEMO

What can go wrong?



# *A\** SEARCH

---

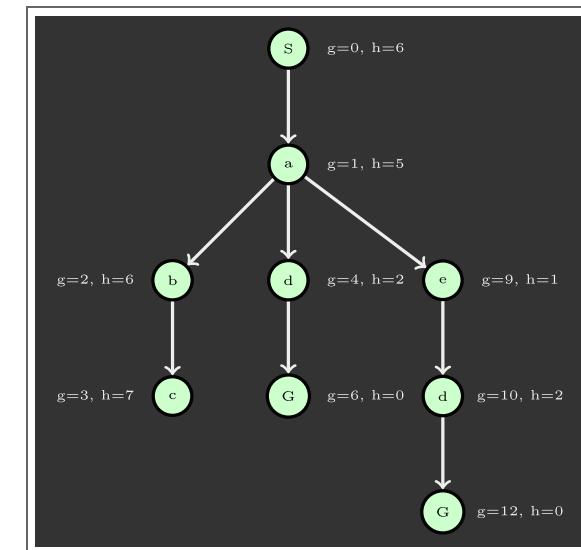
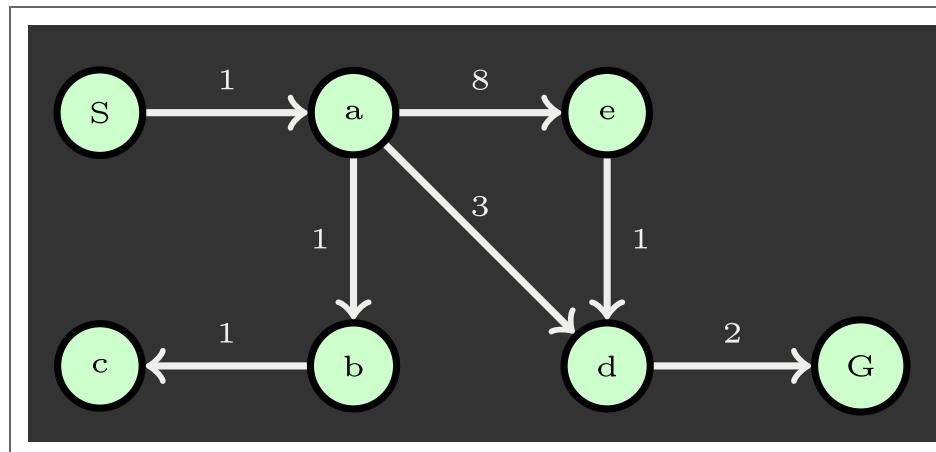
KEY IDEA: COMBINE UCS AND GREEDY

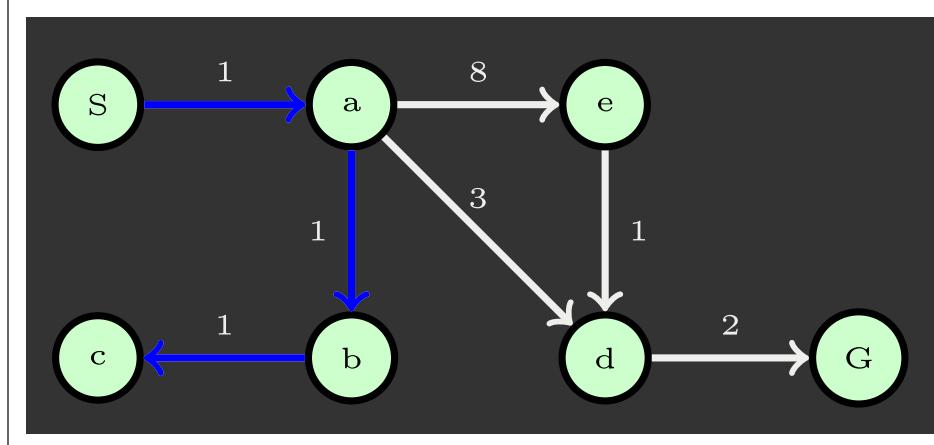
# COMBINING UCS AND GREEDY

**Uniform-cost** orders by path cost, or backward cost  $g(n)$ .

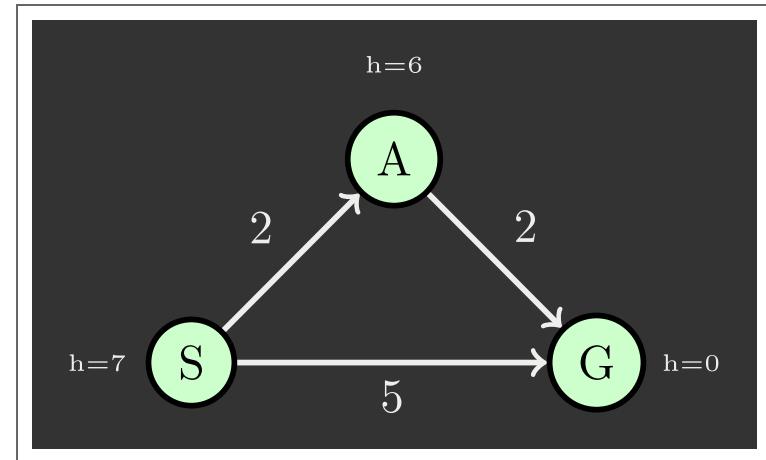
**Greedy** orders by goal proximity, or forward cost  $h(n)$ .

**$A^*$**  Search orders by the sum:  $f(n) = g(n) + h(n)$



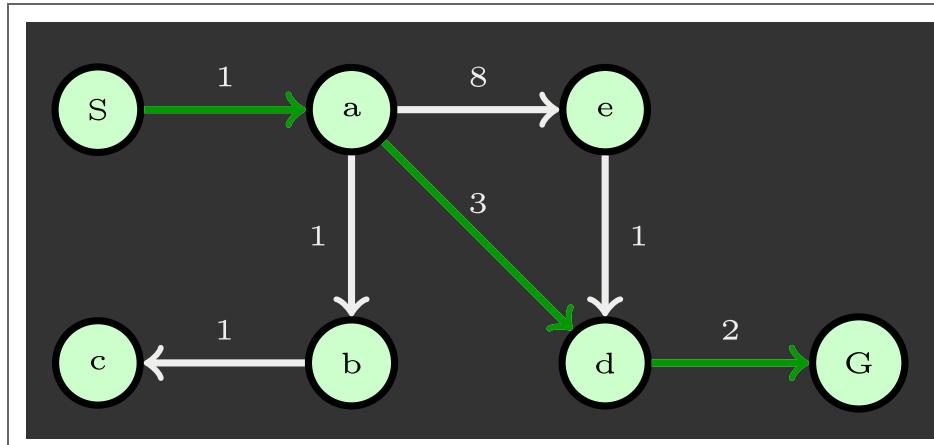


# IS A\* OPTIMAL?



What went wrong?

$\text{Actual goal cost} \leq \text{estimated goal cost}$



We need estimates to be less than actual costs

## ADMISSIBILITY

---

Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe.

Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

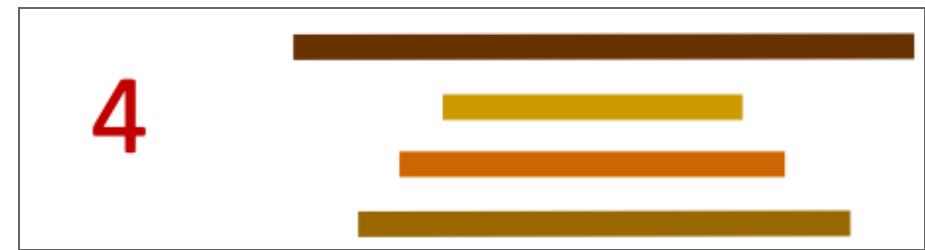
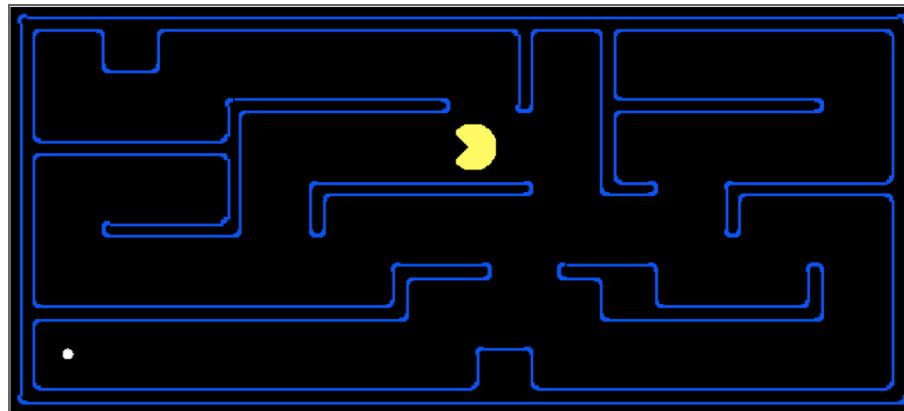
## ADMISSIBLE HEURISTICS

A heuristic  $h$  is admissible (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where  $h^*(n)$  is the true cost to a nearest goal

Example:



In practice, coming up with an admissible heuristic is most of what is involved in using  $A^*$ .

# OPTIMALITY OF $A^*$ TREE SEARCH

---

Assume:

- › A is an optimal goal node
- › B is a suboptimal goal node
- ›  $h$  is admissible

Claim:

- › A will exit the fringe before B

# OPTIMALITY OF $A^*$ TREE SEARCH

Proof:

Imagine  $B$  is on the fringe

$$f(n) = g(n) + h(n)$$

Some ancestor  $n$  of  $A$  is on the fringe too

$$f(n) \leq g(A) \text{ admissibility of } h$$

Claim:  $n$  will be expanded before  $B$

$$g(A) = f(A) \quad h = 0 \text{ at goal}$$

- $f(n)$  is less or equal to  $f(A)$

$$g(A) \leq g(B) \quad B \text{ is suboptimal}$$

- $f(A)$  is less than  $f(B)$

$$f(A) \leq f(B) \quad h = 0 \text{ at goal}$$

- $n$  expands before  $B$

$$f(n) \leq f(A) < f(B)$$

All ancestors of  $A$  expand before  $B$

$A$  expands before  $B$

$A^*$  search is optimal

## UCS VS $A^*$ CONTOURS

---

Uniform-cost expands equally in all "directions"

$A^*$  expands mainly toward the goal, but does hedge its bets to ensure optimality

# $A^*$ DEMO

---

# *A\** APPLICATIONS

---

Video games

Pathing / routing problems

Resource planning problems

Robot motion planning

Language analysis

Machine translation

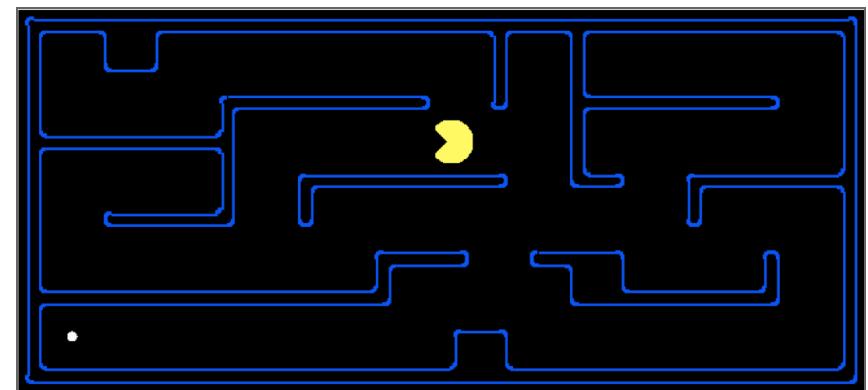
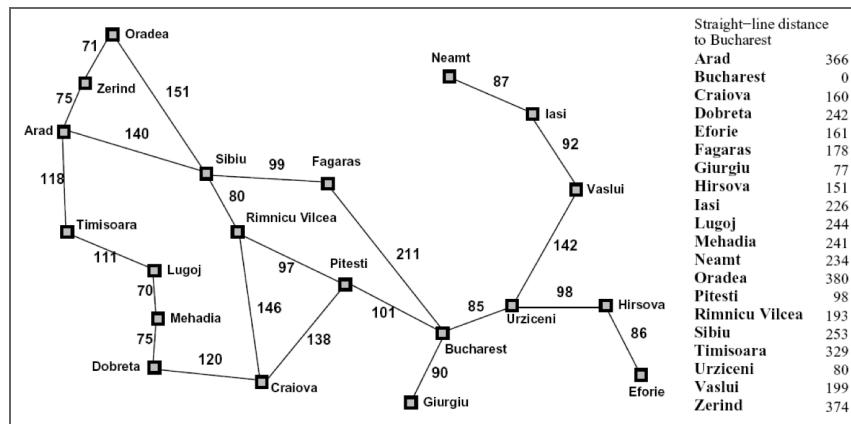
Speech recognition

...

# CREATING ADMISSIBLE HEURISTICS

Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

Often, admissible heuristics are solutions to relaxed problems, where new actions are available



Inadmissible heuristics can often be useful too.

## *A\** SUMMARY

---

*A\** uses both backward costs and (estimates of) forward costs

*A\** is optimal with admissible heuristics

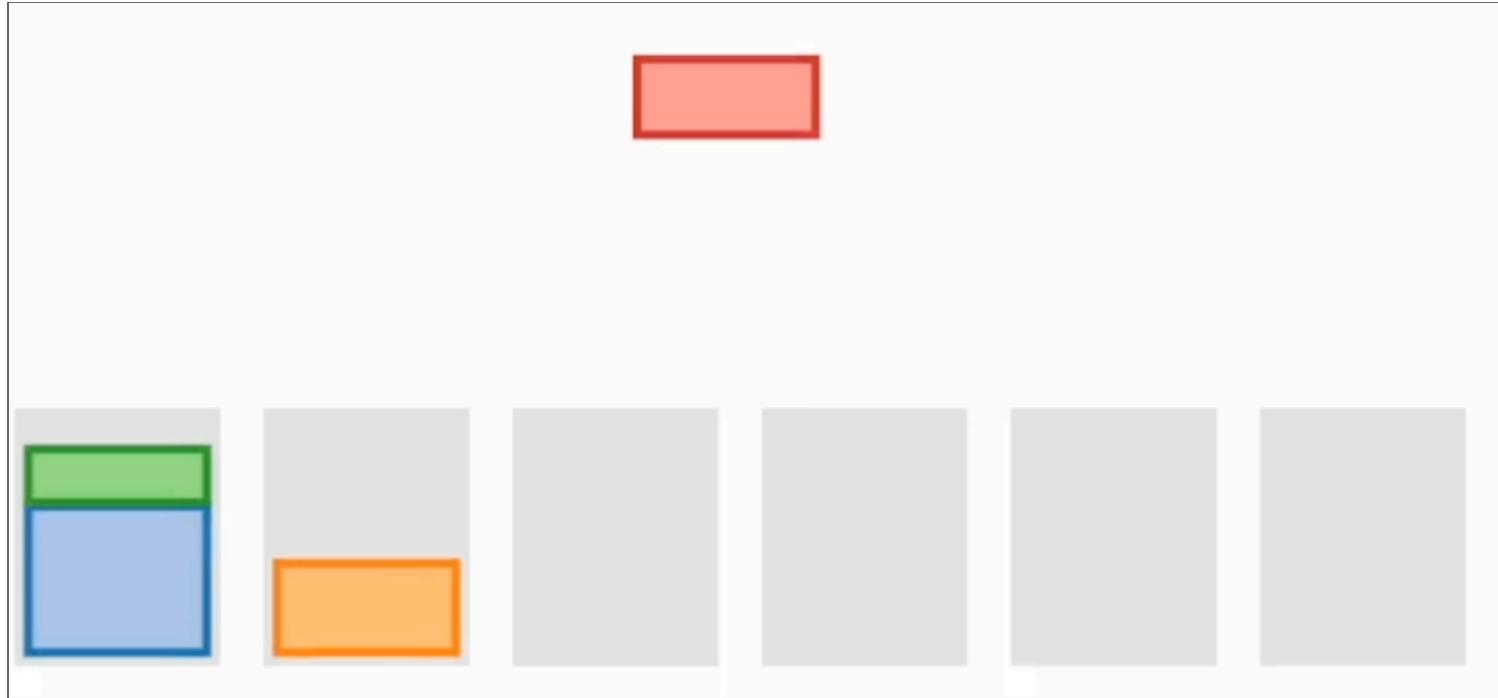
Heuristic design is key: often use relaxed problems

# DESIGNING HEURISTIC AS A SEARCH PROBLEM

The screenshot shows a journal article page on the Nature website. The article is titled "Mathematical discoveries from program search with large language models". It is authored by Bernardino Romera-Paredes, Mohammadamin Barekatian, Alexander Novikov, Matej Balog, M. Pawan Kumar, Emilien Dupont, Francisco J. R. Ruiz, Jordan S. Ellenberg, Pengming Wang, Omar Fawzi, Pushmeet Kohli, and Alhussein Fawzi. The article was published in Nature 625, 468–475 (2024). The abstract discusses the use of large language models (LLMs) for scientific discovery, specifically FunSearch, which pairs LLMs with a systematic evaluator to solve complex problems like extremal combinatorics. The page includes sections for Abstract, Main, FunSearch, Extremal combinatorics, Bin packing, Discussion, Methods, Data availability, Code availability, References, Acknowledgements, and Author information.

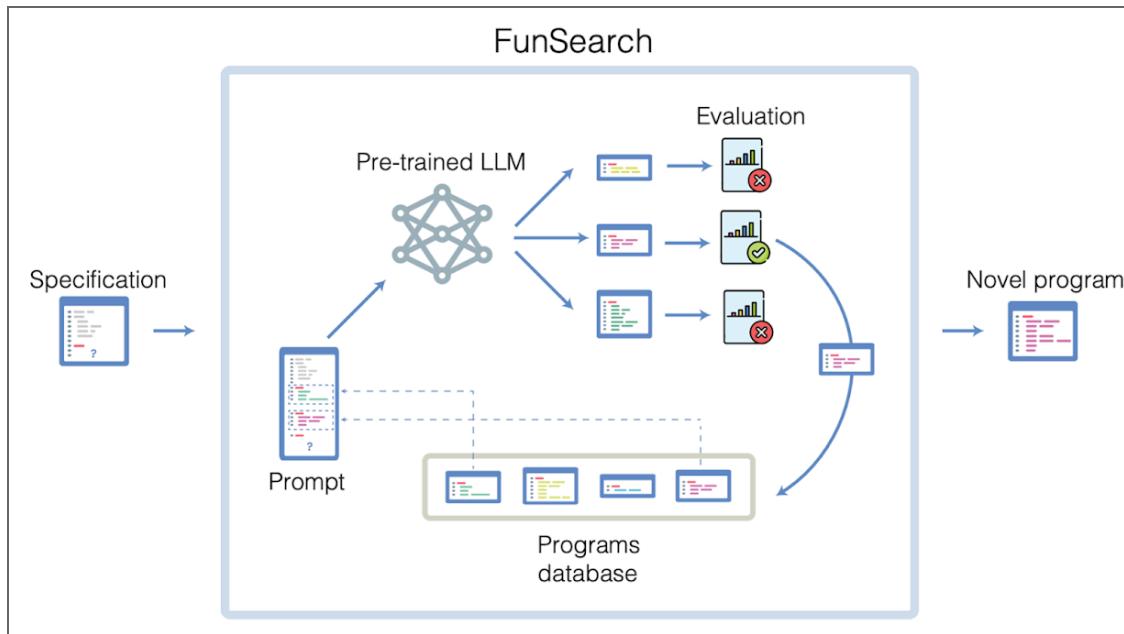
# DESIGNING HEURISTIC AS A SEARCH PROBLEM

## ONLINE BIN PACKING



# DESIGNING HEURISTIC AS A SEARCH PROBLEM

## ONLINE BIN PACKING



# DESIGNING HEURISTIC AS A SEARCH PROBLEM

## ONLINE BIN PACKING

```
def best_fit_heuristic(item: float, bins: np.ndarray) -> np.ndarray:  
    """Returns priority with which we want to add item to each bin.  
  
    Args:  
        item: Size of item to be added to the bin.  
        bins: Array of capacities for each bin.  
  
    Return:  
        Array of same size as bins with priority score of each bin.  
    """  
  
    return np.argmin(bins - item)
```

```
def FunSearch_heuristic(item: float, bins: np.ndarray) -> np.ndarray:  
    """Heuristic discovered for the Weibull datasets."""  
    max_bin_cap = max(bins)  
    score = (bins - max_bin_cap)**2 / item + bins**2 / (item**2)  
    score += bins**2 / item**3  
    score[bins > item] = -score[bins > item]  
    score[1:] -= score[:-1]  
    return np.argmax(score)
```

# DESIGNING HEURISTIC AS A SEARCH PROBLEM

---

## ONLINE BIN PACKING

# Q & A



XKCD







## Speaker notes

