

1. Simulate a 5-neuron MAXNET with lateral connection weight of -0.15 , and external input vector of $(0.1, 0.3, 0.5, 0.7, 0.9)$. Do the same for the kWTA network with $k = 1$ and 2 .

```
import numpy as np

def maxnet(input_vector, lateral_weight=-0.15, iterations=10):
    n = len(input_vector)
    output = np.copy(input_vector)

    for _ in range(iterations):
        net_input = np.dot(lateral_weight * np.ones((n, n)) + np.eye(n), output)
        output = np.maximum(0, net_input)

    return output

input_vector = np.array([0.1, 0.3, 0.5, 0.7, 0.9])
maxnet_output = maxnet(input_vector)
print("MAXNET Output:", maxnet_output)
```

MAXNET Output: [0. 0. 0. 0. 0.08679375]

```
def kwta(input_vector, k, lateral_weight=-0.15):
    n = len(input_vector)
    output = np.copy(input_vector)

    for _ in range(10): # Fixed number of iterations
        net_input = np.dot(lateral_weight * np.ones((n, n)) + np.eye(n), output)

        top_k_indices = np.argsort(net_input)[-k:]
        output = np.zeros_like(net_input)
        output[top_k_indices] = net_input[top_k_indices]

    return output
```

```
# k-WTA outputs for k=1 and k=2
kwta_output_k1 = kwta(input_vector, k=1)
kwta_output_k2 = kwta(input_vector, k=2)
```

```
print("kWTA Output (k=1):", kwta_output_k1)
print("kWTA Output (k=2):", kwta_output_k2)
```

kWTA Output (k=1): [0. 0. 0. 0. 0.1215989]
kWTA Output (k=2): [0. 0. 0. -0.01459294 0.09940771]

2. Simulate a 9-neuron discrete Hopfield network as an associative memory of 3-by-3 digital images as shown

- in Fig. 1. First determine the connection weights using the outer product rule. Then retrieve the two stored patterns by using two keys (probes) with variations of one pixel.

```
import numpy as np

P1 = np.array([+1, +1, +1, -1, +1, -1, -1, +1, -1])
P2 = np.array([+1, +1, +1, +1, -1, -1, +1, +1, +1])

W = np.outer(P1, P1) + np.outer(P2, P2) - 2 * np.eye(9)

def sign(x):
    return np.where(x >= 0, 1, -1)

def retrieve_pattern(W, noisy_input, max_iter=10):
    s = noisy_input.copy()
    for _ in range(max_iter):
        s_new = sign(np.dot(W, s))
        if np.array_equal(s, s_new):
            break
        s = s_new
    return s

noisy_input1 = np.array([+1, +1, +1, -1, +1, -1, +1, +1, -1])
retrieved1 = retrieve_pattern(W, noisy_input1)

noisy_input2 = np.array([+1, +1, +1, +1, -1, -1, +1, +1, -1])
retrieved2 = retrieve_pattern(W, noisy_input2)

print("Retrieved Pattern 1:", retrieved1)
print("Retrieved Pattern 2:", retrieved2)
```

→ Retrieved Pattern 1: [1 1 1 -1 1 -1 -1 1 -1]
Retrieved Pattern 2: [1 1 1 1 -1 -1 1 1 1]