Prof. Wei

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TODAY

Markov Decision Processes (MDPs)

NON-DETERMINSTIC SEARCH

EXAMPLE: GRID WORLD

A maze like problem

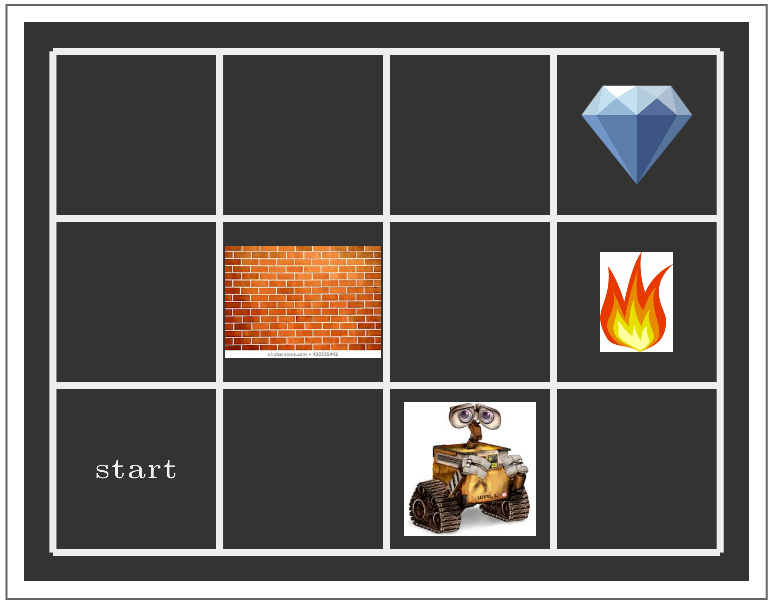
- The agent lives in a grid
- Walls block the agent's path

Noisy movement: actions do not always go as planned

- 80% of the time, the action North takes the agent North (if there is no wall there)
- 10% of the time, North takes the agent West; 10% East
- If there is a wall in the direction the agent would have been taken, the agent stays put

The agent receives rewards each time step

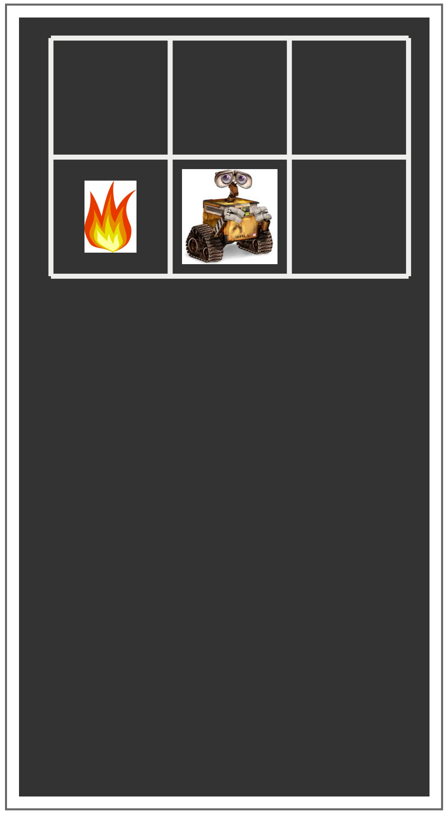
- Small "living" reward each step (can be negative)
- Big rewards come at the end (good or bad)



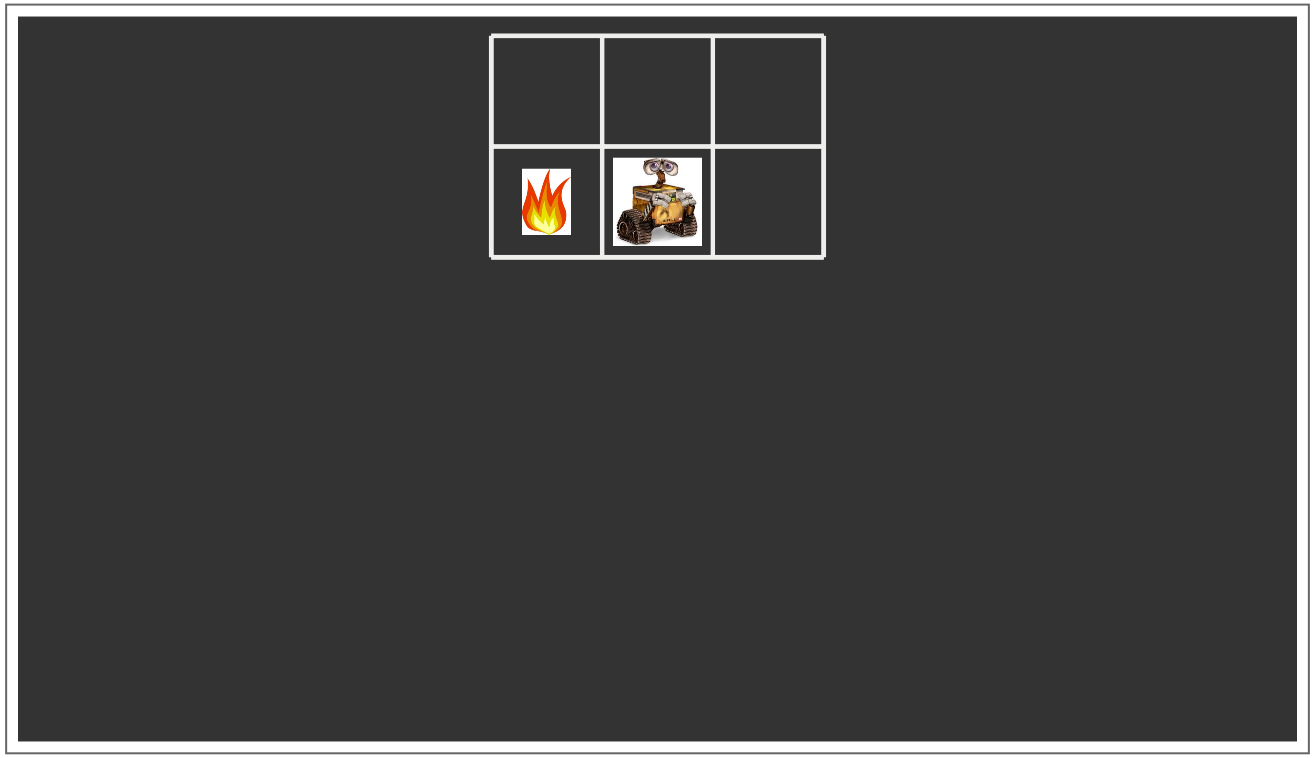
Goal: maximize sum of rewards

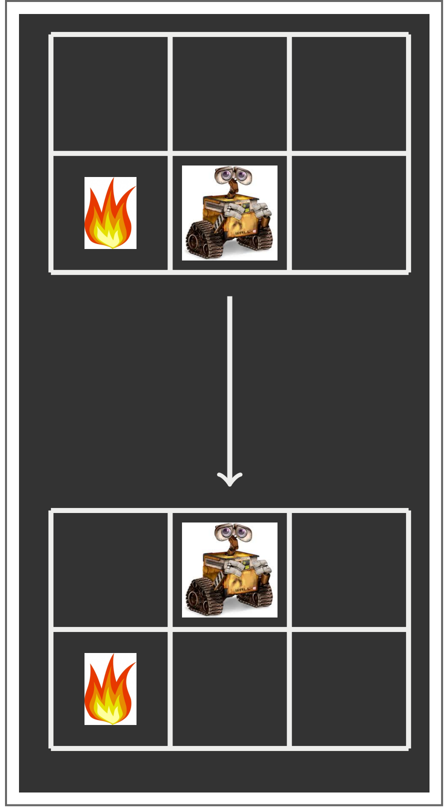
GRID WORLD ACTIONS

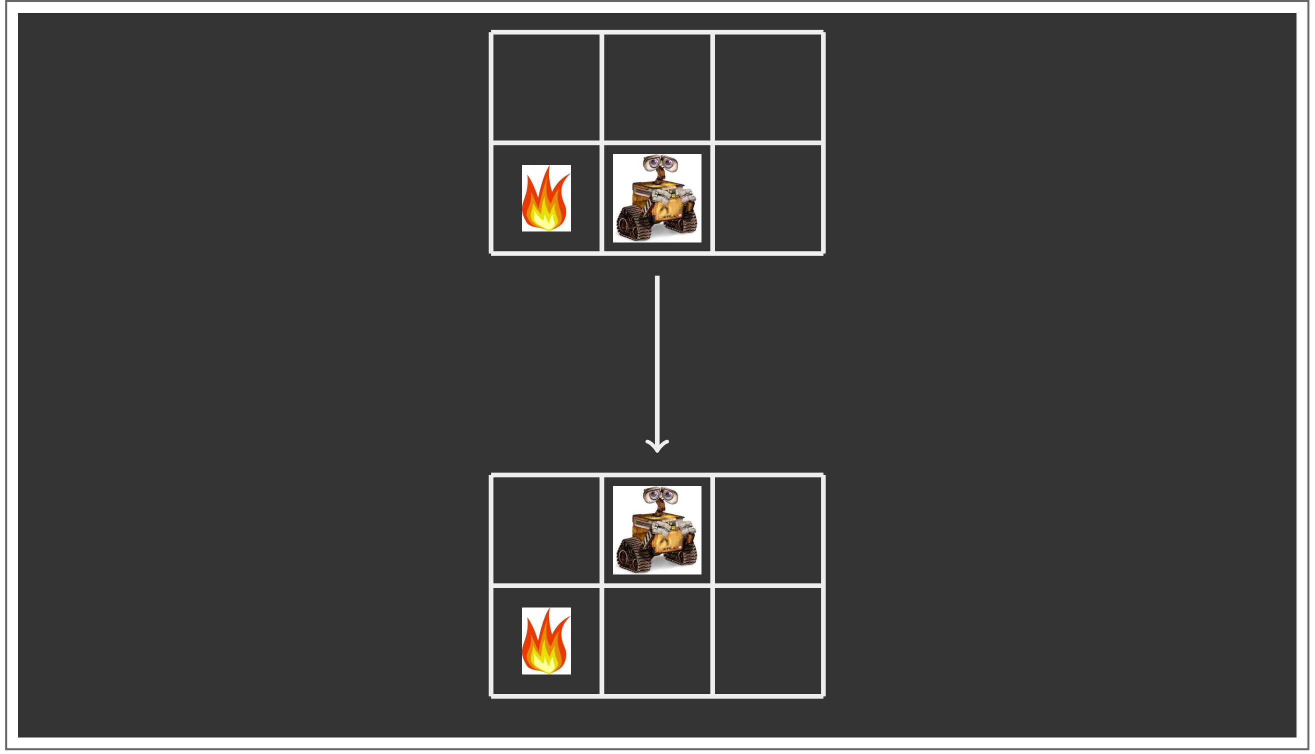
Deterministic Grid World

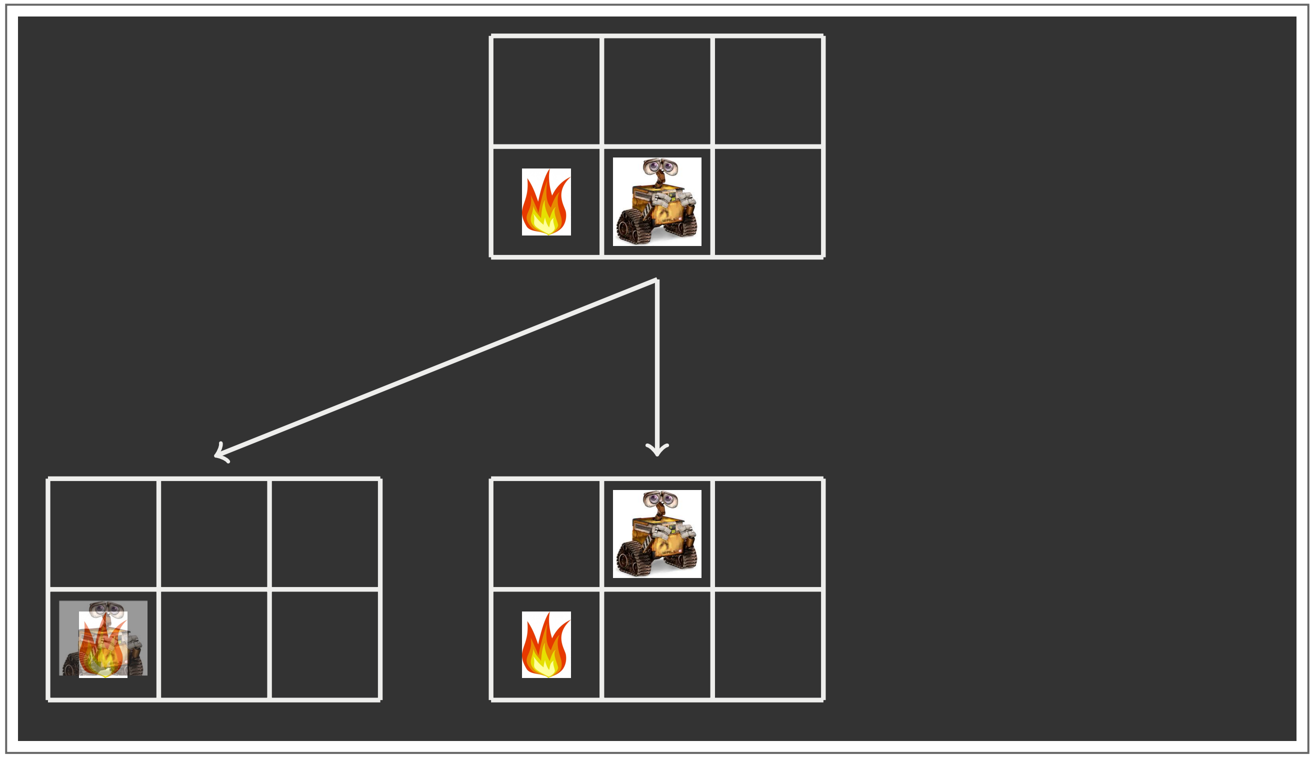


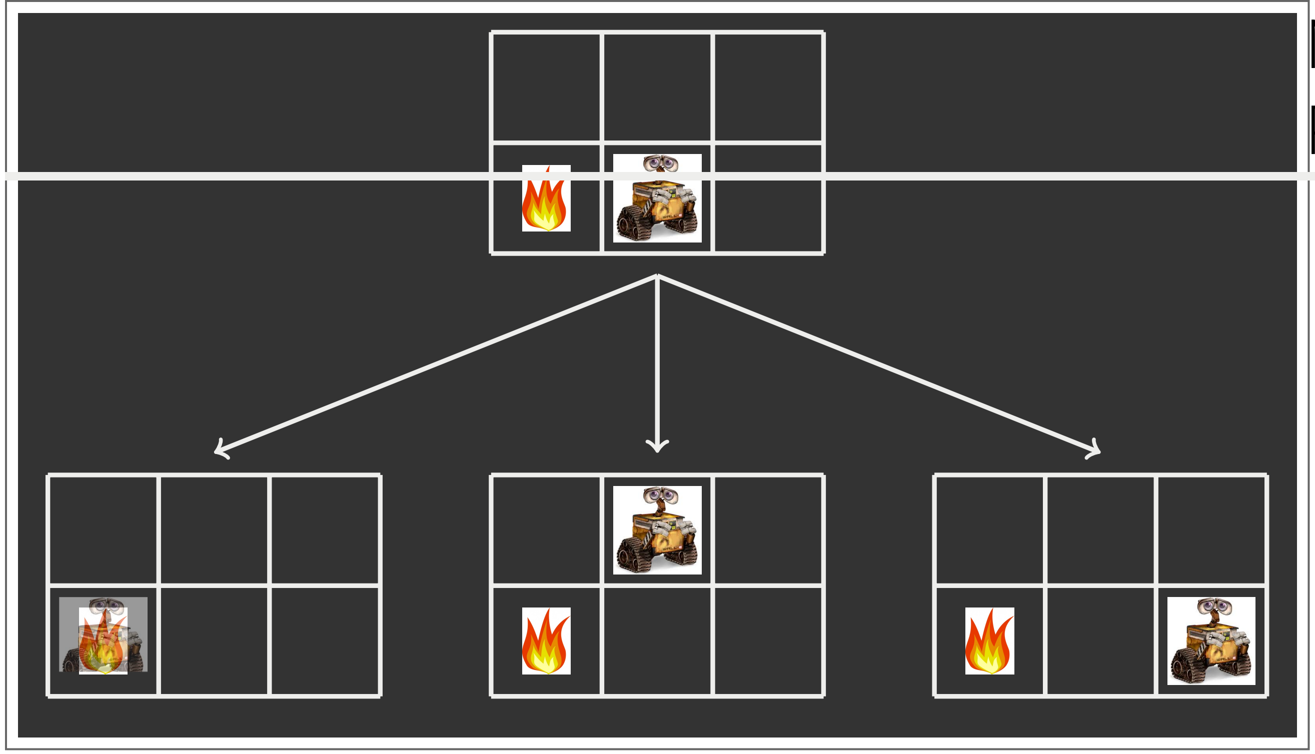
Stochastic Grid World







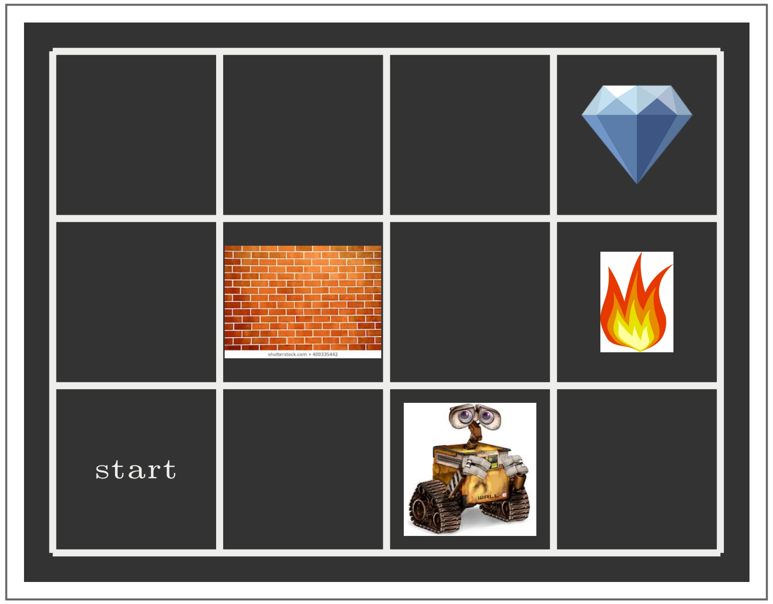




MARKOV DECISION PROCESSES

An MDP is defined by:

- A set of states $s \in S$
- A set of actions $a \in A$
- A transition function $T(s, a, s')$
- Probability that a from s leads to s', i.e., $P(s'|s, a)$
- Also called the model or the dynamics
- A reward function $R(s, a, s')$



- Sometimes just $R(s)$ or $R(s')$
- A start state, s_0
- Maybe a terminal state

WHAT IS MARKOV ABOUT MDPS

MDPs are non-deterministic search problems

- One way to solve them is with expectimax search
- We will have a new tool soon

"Markov" generally means that given the present state, the future and the past are independent

For Markov decision processes, "Markov" means action outcomes depend only on the current state

$$\begin{aligned}&p(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \ldots, S_0 = s_0)\\&= p(S_{t+1} = s' | S_t = s_t, A_t = a_t)\end{aligned}$$

This is just like search, where the successor function could only depend on the current state (not the history)

POLICIES

In deterministic single-agent search problems, we wanted an optimal plan, or sequence of actions, from start to a goal

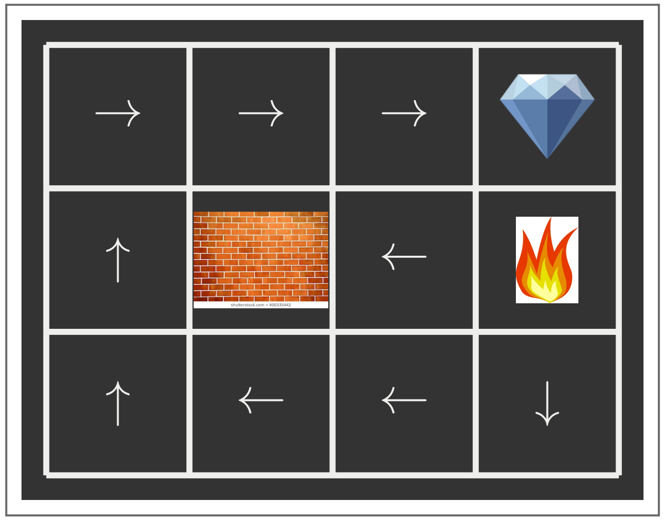
For MDPs, we want an optimal policy $\pi^*: S \rightarrow A$

- A policy π gives an action for each state
- An optimal policy is one that maximizes expected utility if followed
- An explicit policy defines a reflex agent

Expectimax did not compute entire policies

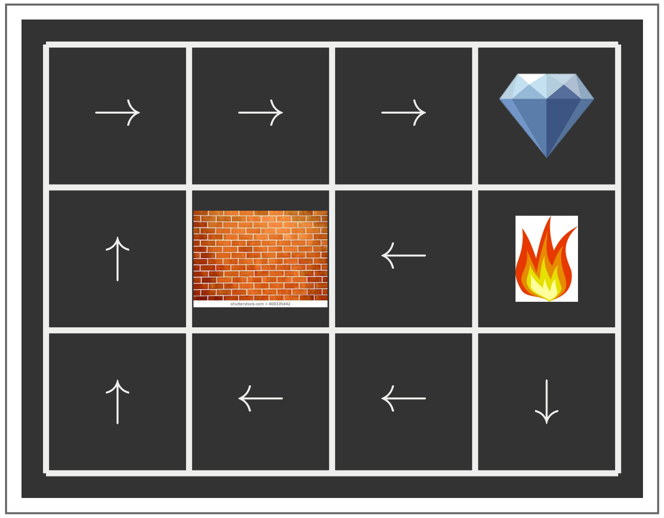
- It computed the action for a single state only

Andrey Markov (1856-1922)

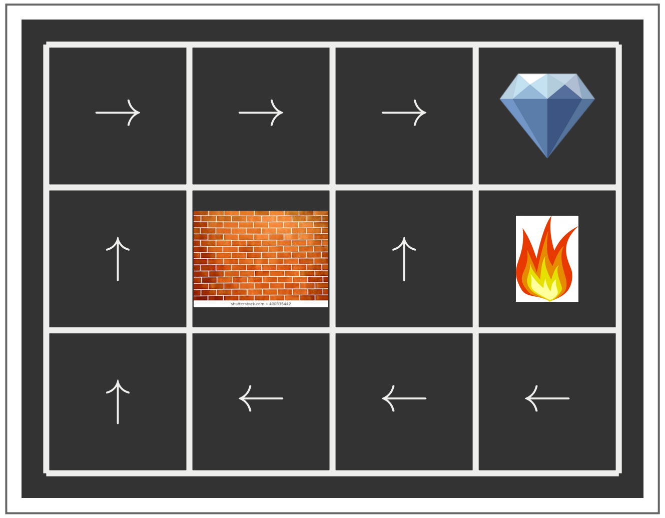


OPTIMAL POLICIES

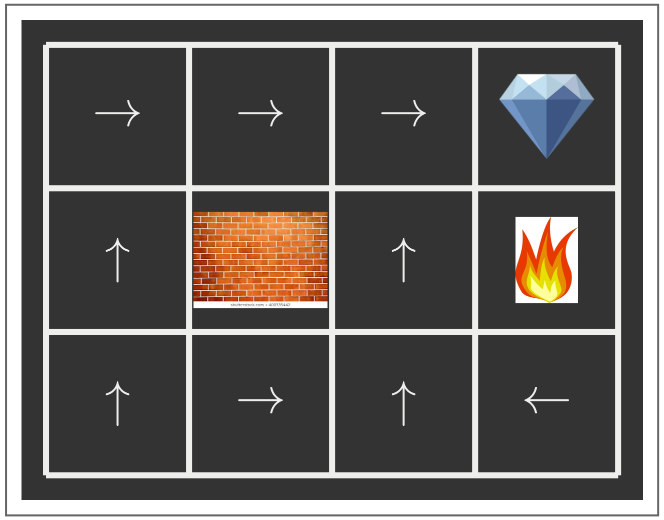
Optimal policy when $\boldsymbol{R(s, a, s') = -0.01}$ for all non-terminals $\boldsymbol{s}$.



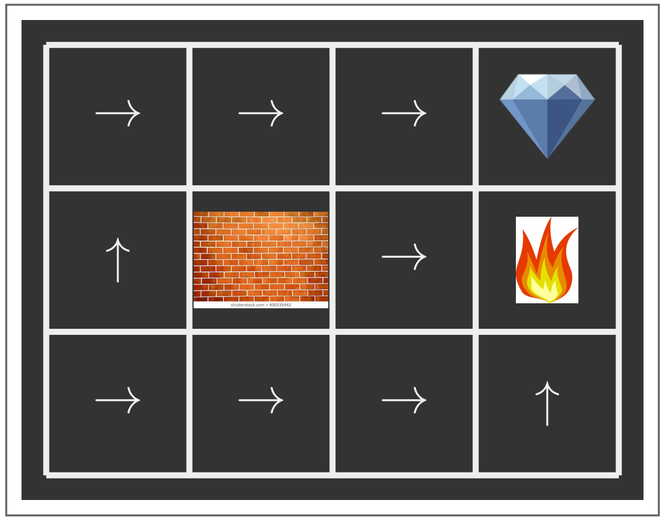
$R(s) = -0.01$



$R(s) = -0.03$



$R(s) = -0.4$



$R(s) = -2.0$

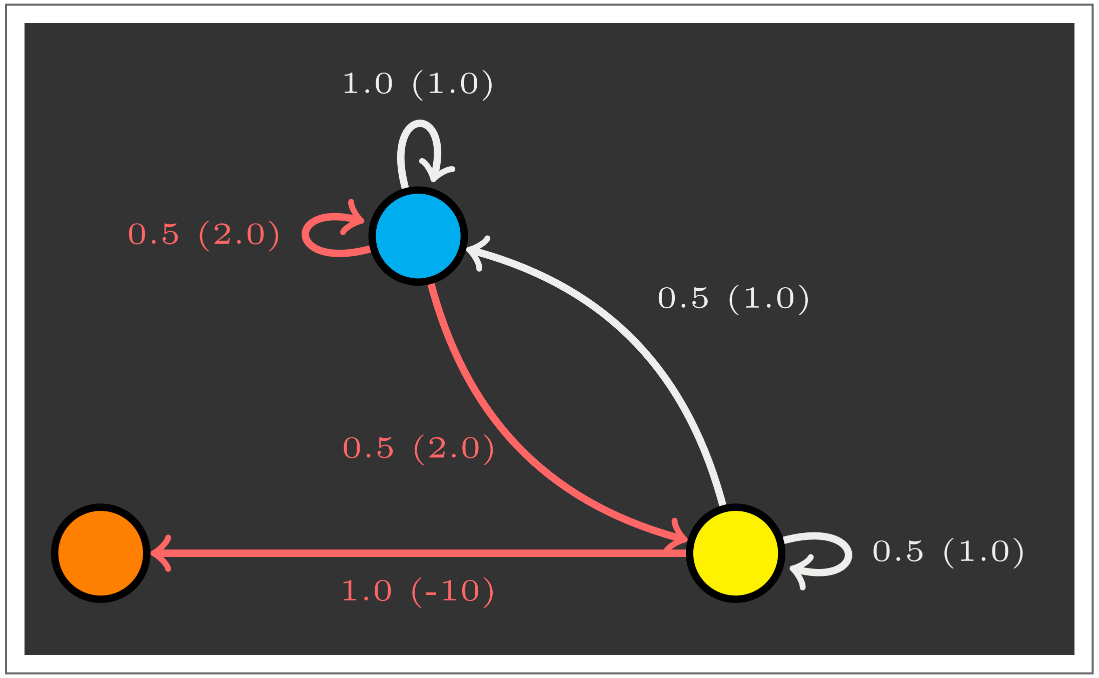
EXAMPLE: CAR RACING

A robot car wants to travel far, quickly

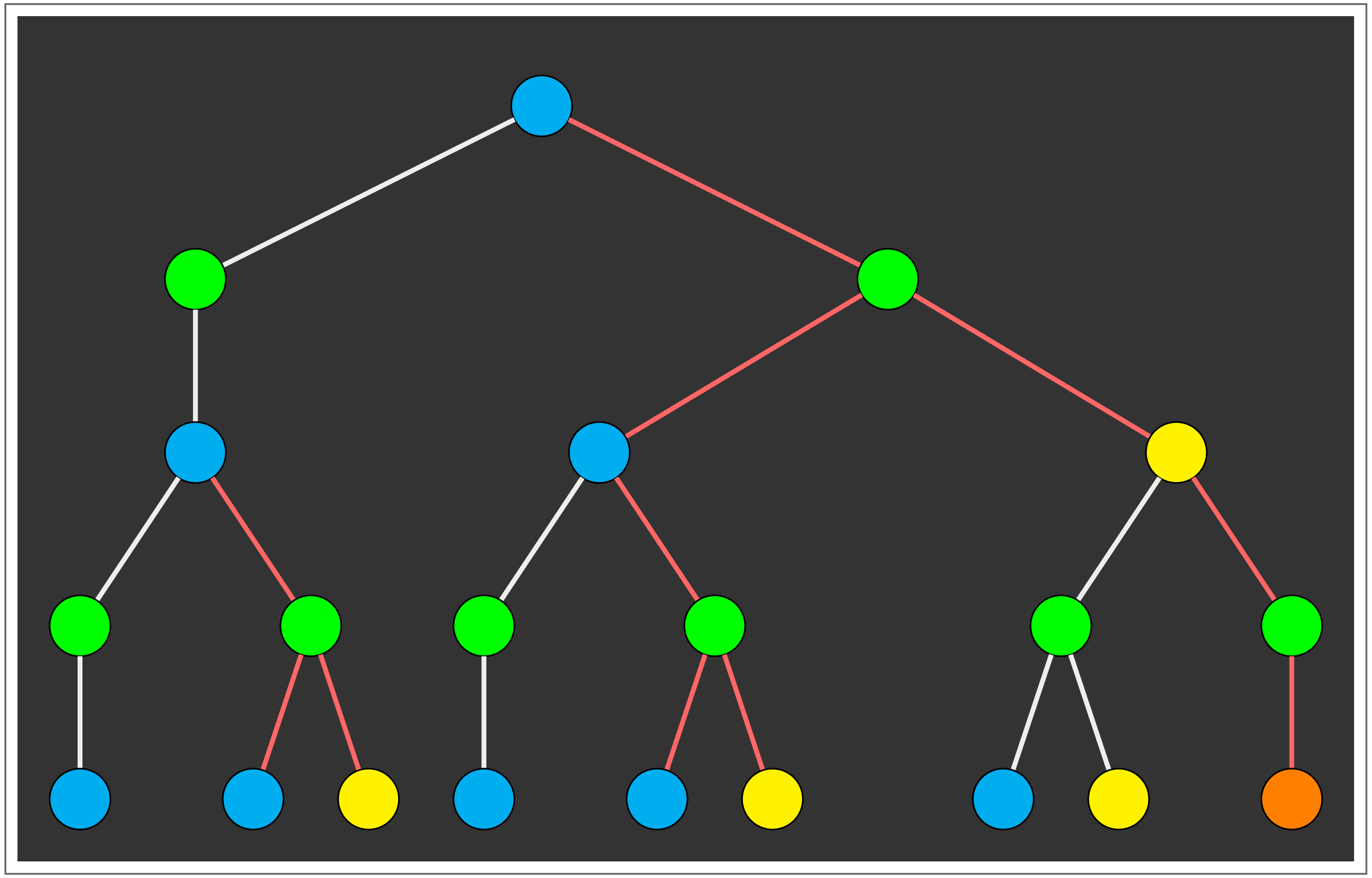
Three states: Cool, Warm, Overheated

Two actions: Slow, Fast

Going faster gets double reward

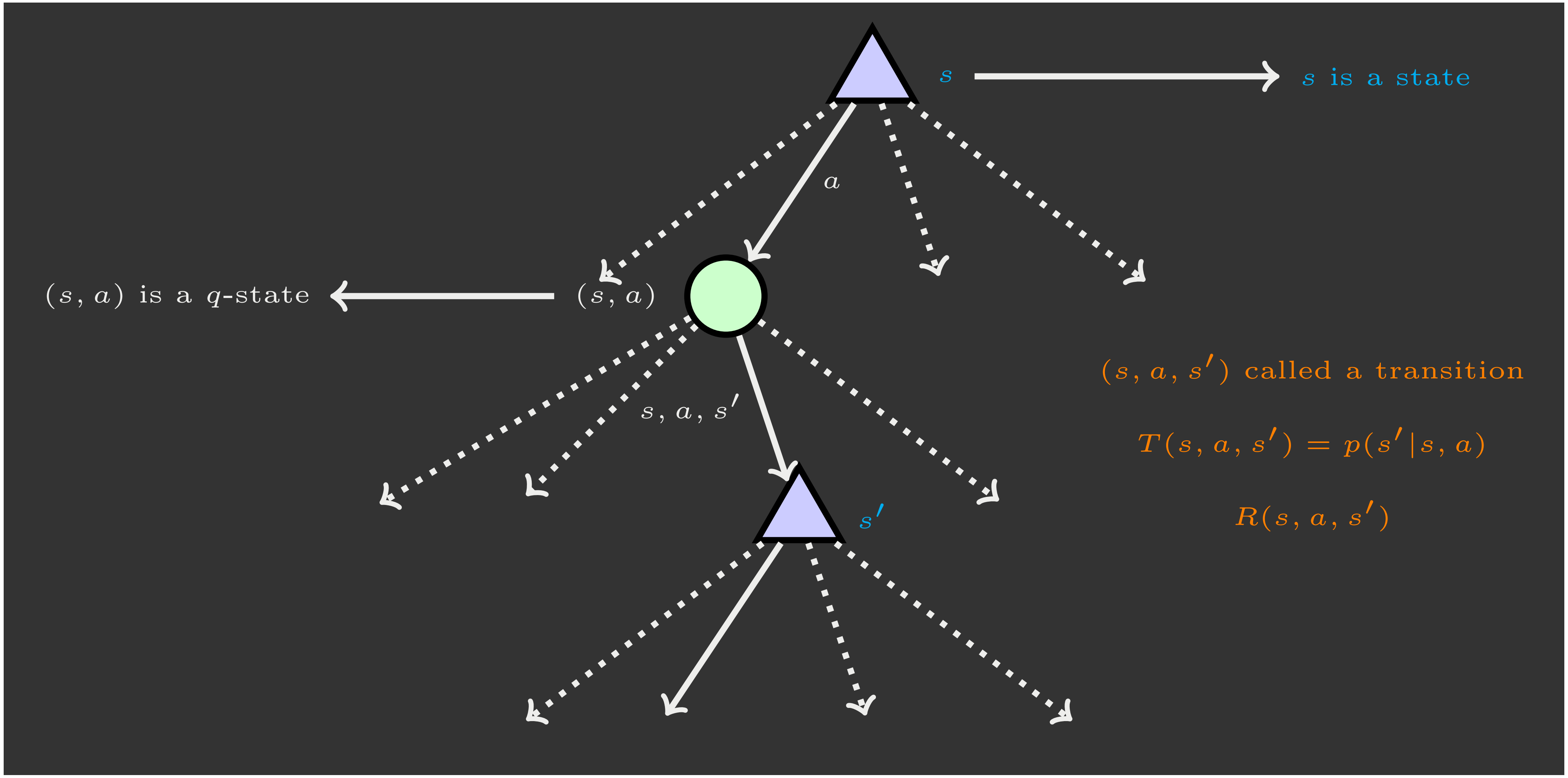


RACING SEARCH TREE



MDP SEARCH TREE

Each MDP state projects an expectimax-like search tree


s
s is a state
a
(s, a) is a q-state
(s, a)
s, a, s'
s'
(s, a, s') called a transition
$T(s, a, s') = p(s'|s, a)$
$R(s, a, s')$

UTILITY OF SEQUENCES

What preferences should an agent have over reward sequences?

More or less? [1,2,2] or [2,3,4]

Now or later? [0,0,1] or [1,0,0]

DISCOUNTING

It is reasonable to maximize the sum of rewards

It is also reasonable to prefer rewards now to rewards later

One solution: values of rewards decay exponentially

Worth Now

Worth Next Step

Worth in Two Steps

DISCOUNTING

How to discount?

- Each time we descend a level, we multiply in the discount once

Why discount?

- Sooner rewards probably do have higher utility than later rewards
- Also helps our algorithms converge

Example: discount of 0.5

- $\boldsymbol{U([1,2,3]) = 1*1 + 0.5*2 + 0.25*3}$
- $\boldsymbol{U([1,2,3]) < U([3,2,1])}$

STATIONARY PREFERENCES

Assumption: if we assume stationary preferences:

$$[a_1, a_2, \ldots] \succ [b_1, b_2, \ldots]$$
$$\Updownarrow$$
$$[r, a_1, a_2, \ldots] \succ [r, b_1, b_2, \ldots]$$

Then: there are only two ways to define utilities

- Additive utility:

$$U([r_0, r_1, r_2, \ldots]) = r_0 + r_1 + r_2 + \ldots$$

- Discounted utility:

$$U([r_0, r_1, r_2, \ldots]) = r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots$$

INFINITE UTILITIES

Problem: What if the game lasts forever? Do we get infinite rewards?

Solutions:

- Finite horizon: (similar to depth-limited search)
- Terminate episodes after a fixed T steps (e.g. life)
- Gives non-stationary policies (π depends on time left)
- Discounting: use $0 < \gamma < 1$

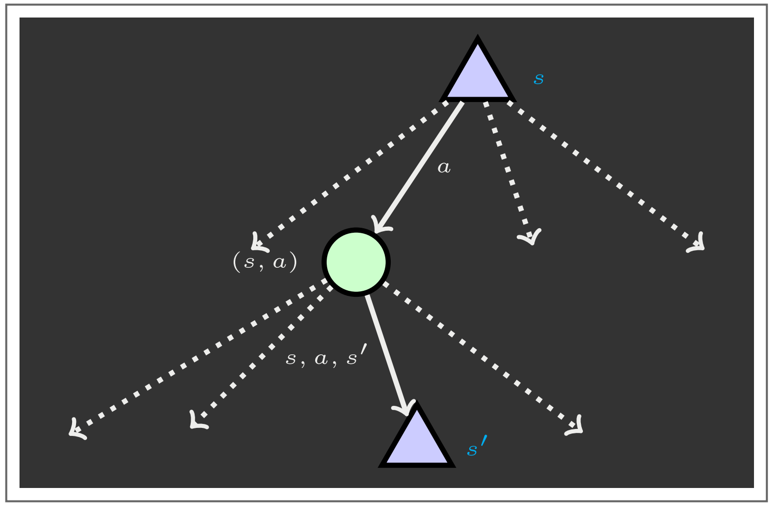
$$U([r_0, \ldots, r_\infty]) = \sum_{t=0}^{\infty} \gamma^t r_t \leq \frac{R_{max}}{1-\gamma}$$

- Smaller γ means smaller "horizon" – shorter term focus
- Absorbing state: guarantee that for every policy, a terminal state will eventually be reached (like "overheated" for racing)

RECAP: DEFINING MDPS

Markov decision processes:

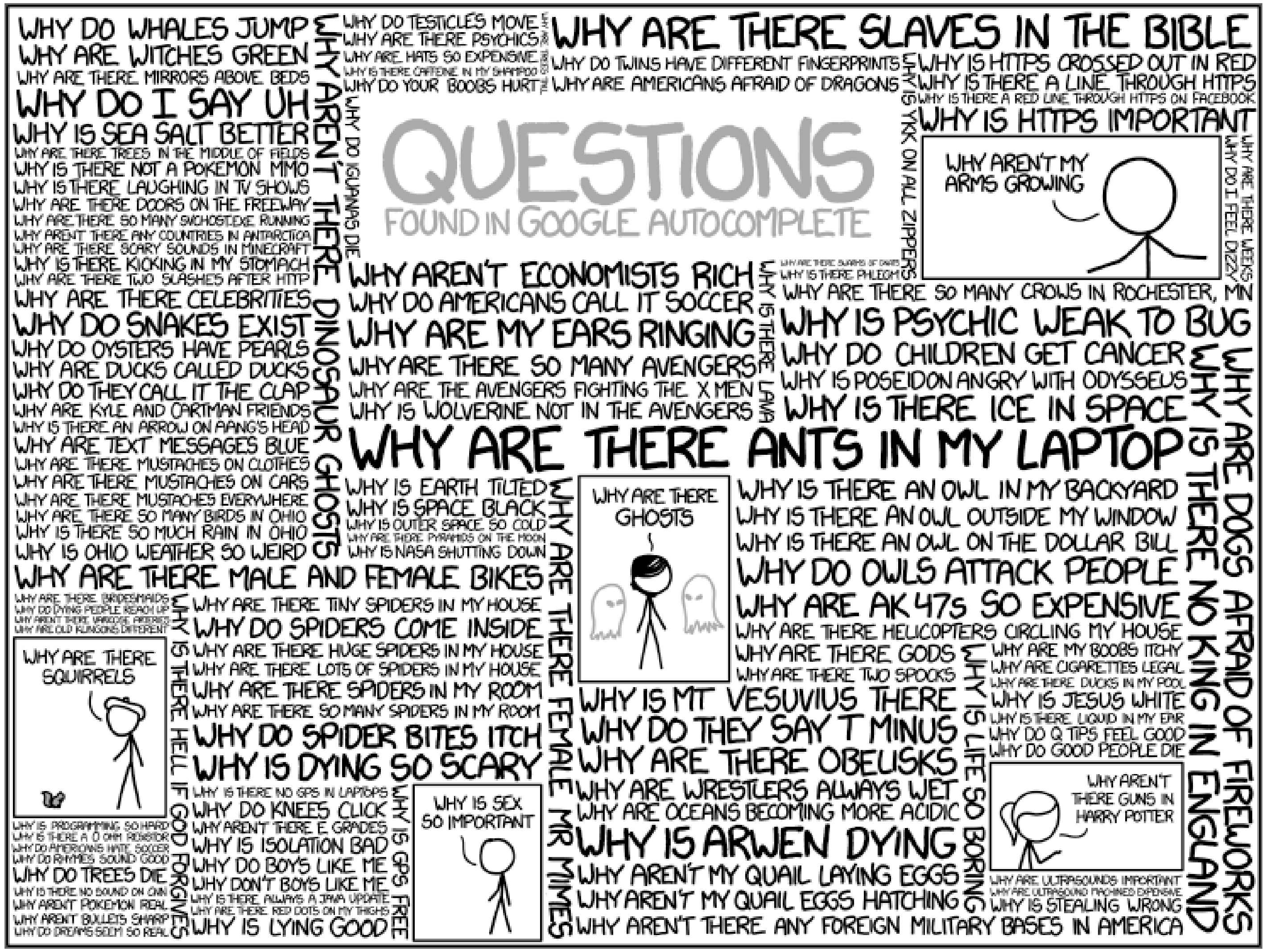
- Set of states S
- Start state s_0
- Set of actions A
- Transitions $P(s'|s,a)$ (or $T(s,a,s')$)
- Rewards R(s,a,s') (and discount γ)



MDP quantities so far:

- Policy = Choice of action for each state
- Utility = sum of (discounted) rewards

Q & A


QUESTIONS
FOUND IN GOOGLE AUTOCOMPLETE
WHY DO WHALES JUMP
WHY ARE WITCHES GREEN
WHY ARE THERE MIRRORS ABOVE BEDS
WHY DO I SAY UH
WHY IS SEA SALT BETTER
WHY ARE THERE TREES IN THE MIDDLE OF FIELDS
WHY IS THERE NOT A POKEMON MMO
WHY IS THERE LAUGHING IN TV SHOWS
WHY ARE THERE DOORS ON THE FREEWAY
WHY ARE THERE SO MANY SVCHOST.EXE RUNNING
WHY AREN'T THERE ANY COUNTRIES IN ANTARCTICA
WHY ARE THERE SCARY SOUNDS IN MINECRAFT
WHY IS THERE KICKING IN MY STOMACH
WHY ARE THERE TWO SLASHES AFTER HTTP
WHY ARE THERE CELEBRITIES
WHY DO SNAKES EXIST
WHY DO OYSTERS HAVE PEARLS
WHY ARE DUCKS CALLED DUCKS
WHY DO THEY CALL IT THE CLAP
WHY ARE KYLE AND CARTMAN FRIENDS
WHY IS THERE AN ARROW ON AANG'S HEAD
WHY ARE TEXT MESSAGES BLUE
WHY ARE THERE MUSTACHES ON CLOTHES
WHY ARE THERE MUSTACHES ON CARS
WHY ARE THERE MUSTACHES EVERYWHERE
WHY ARE THERE SO MANY BIRDS IN OHIO
WHY IS THERE SO MUCH RAIN IN OHIO
WHY IS OHIO WEATHER SO WEIRD
WHY ARE THERE MALE AND FEMALE BIKES
WHY AREN'T THERE DINOSAUR GHOSTS
WHY DO IGUANAS DIE
WHY DO TESTICLES MOVE
WHY ARE THERE PSYCHICS
WHY ARE HATS SO EXPENSIVE
WHY IS THERE CAFFEINE IN MY SHAMPOO
WHY DO YOUR BOOBS HURT
WHY ARE THERE SLAVES IN THE BIBLE
WHY DO TWINS HAVE DIFFERENT FINGERPRINTS
WHY ARE AMERICANS AFRAID OF DRAGONS
WHY IS YKK ON ALL ZIPPERS
WHY IS HTTPS CROSSED OUT IN RED
WHY IS THERE A LINE THROUGH HTTPS
WHY IS THERE A RED LINE THROUGH HTTPS ON FACEBOOK
WHY IS HTTPS IMPORTANT
WHY AREN'T MY ARMS GROWING
WHY ARE THERE WEEKS
WHY DO I FEEL DIZZY
WHY AREN'T ECONOMISTS RICH
WHY DO AMERICANS CALL IT SOCCER
WHY ARE MY EARS RINGING
WHY ARE THERE SO MANY AVENGERS
WHY ARE THE AVENGERS FIGHTING THE X MEN
WHY IS WOLVERINE NOT IN THE AVENGERS
WHY IS THERE LAVA
WHY ARE THERE SWARMS OF GNATS
WHY IS THERE PHLEGM
WHY ARE THERE SO MANY CROWS IN ROCHESTER, MN
WHY IS PSYCHIC WEAK TO BUG
WHY DO CHILDREN GET CANCER
WHY IS POSEIDON ANGRY WITH ODYSSEUS
WHY IS THERE ICE IN SPACE
WHY ARE THERE ANTS IN MY LAPTOP
WHY IS EARTH TILTED
WHY IS SPACE BLACK
WHY IS OUTER SPACE SO COLD
WHY ARE THERE PYRAMIDS ON THE MOON
WHY IS NASA SHUTTING DOWN
WHY ARE THERE FEMALE MR MIMES
WHY ARE THERE GHOSTS
WHY IS THERE AN OWL IN MY BACKYARD
WHY IS THERE AN OWL OUTSIDE MY WINDOW
WHY IS THERE AN OWL ON THE DOLLAR BILL
WHY DO OWLS ATTACK PEOPLE
WHY ARE AK 47s SO EXPENSIVE
WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE
WHY ARE THERE GODS
WHY ARE THERE TWO SPOCKS
WHY IS THERE NO KING IN ENGLAND
WHY ARE DOGS AFRAID OF FIREWORKS
WHY ARE THERE BRIDESMAIDS
WHY DO DYING PEOPLE REACH UP
WHY AREN'T THERE VARICOSE ARTERIES
WHY ARE OLD KLINGONS DIFFERENT
WHY ARE THERE SQUIRRELS
WHY IS THERE HELL IF GOD FORGIVES
WHY ARE THERE TINY SPIDERS IN MY HOUSE
WHY DO SPIDERS COME INSIDE
WHY ARE THERE HUGE SPIDERS IN MY HOUSE
WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE
WHY ARE THERE SPIDERS IN MY ROOM
WHY ARE THERE SO MANY SPIDERS IN MY ROOM
WHY DO SPIDER BITES ITCH
WHY IS DYING SO SCARY
WHY IS PROGRAMMING SO HARD
WHY IS THERE A 0 OHM RESISTOR
WHY DO AMERICANS HATE SOCCER
WHY DO RHYMES SOUND GOOD
WHY DO TREES DIE
WHY IS THERE NO SOUND ON CNN
WHY AREN'T POKEMON REAL
WHY AREN'T BULLETS SHARP
WHY DO DREAMS SEEM SO REAL
WHY IS THERE NO GPS IN LAPTOPS
WHY DO KNEES CLICK
WHY AREN'T THERE E GRADES
WHY IS ISOLATION BAD
WHY DO BOYS LIKE ME
WHY DON'T BOYS LIKE ME
WHY IS THERE ALWAYS A JAVA UPDATE
WHY ARE THERE RED DOTS ON MY THIGHS
WHY IS LYING GOOD
WHY IS GPS FREE
WHY IS SEX SO IMPORTANT
WHY IS MT VESUVIUS THERE
WHY DO THEY SAY T MINUS
WHY ARE THERE OBELISKS
WHY ARE WRESTLERS ALWAYS WET
WHY ARE OCEANS BECOMING MORE ACIDIC
WHY IS ARWEN DYING
WHY AREN'T MY QUAIL LAYING EGGS
WHY AREN'T MY QUAIL EGGS HATCHING
WHY AREN'T THERE ANY FOREIGN MILITARY BASES IN AMERICA
WHY IS LIFE SO BORING
WHY ARE MY BOOBS ITCHY
WHY ARE CIGARETTES LEGAL
WHY ARE THERE DUCKS IN MY POOL
WHY IS JESUS WHITE
WHY IS THERE LIQUID IN MY EAR
WHY DO Q TIPS FEEL GOOD
WHY DO GOOD PEOPLE DIE
WHY AREN'T THERE GUNS IN HARRY POTTER
WHY ARE ULTRASOUNDS IMPORTANT
WHY ARE ULTRASOUND MACHINES EXPENSIVE
WHY IS STEALING WRONG

XKCD

Speaker notes