

## Adversarial Search - II

CS5491: Artificial Intelligence  
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Content Credits: Prof. Wei's CS4486 Course  
and Prof. Boddeti's AI Course

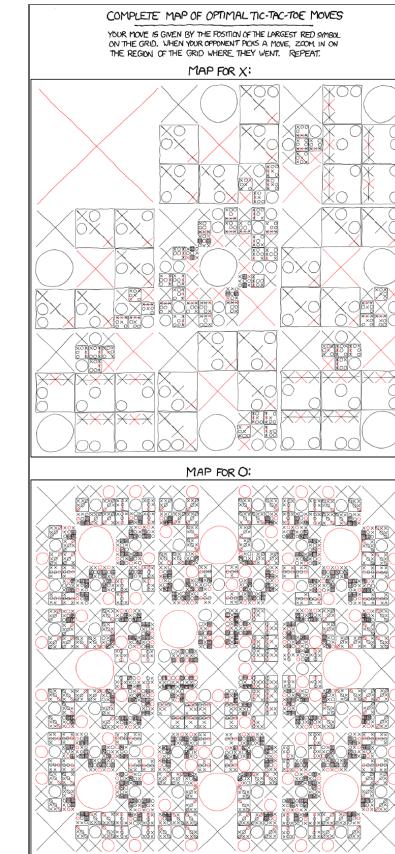
# TODAY

Expectimax Games

Utilities

Reading

- Today's Lecture: RN Chapters 16.1-16.3

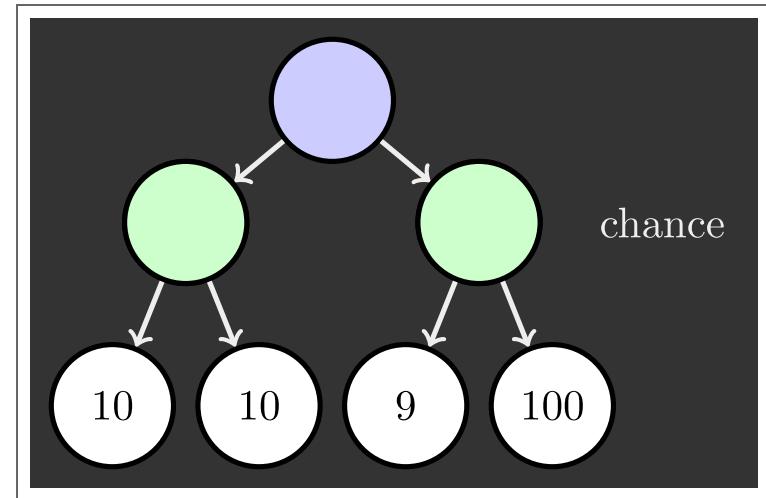
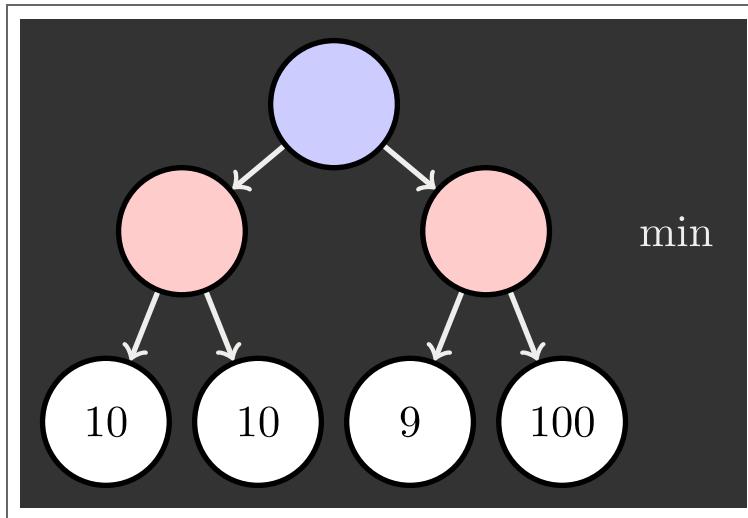


**XKCD**

# UNCERTAIN OUTCOMES

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## WORST CASE VS AVERAGE CASE



Idea: Uncertain outcomes controlled by chance, not an adversary.

## STOCHASTIC GAMES

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Why would we not know what the result of an action will be?

- › Explicit randomness: rolling dice
- › Unpredictable opponents: the ghosts respond randomly
- › Actions can fail: when moving a robot, wheels might slip

Values should now reflect average-case (expectimax) outcomes, not worst-case (minimax) outcomes

# EXPECTIMAX SEARCH

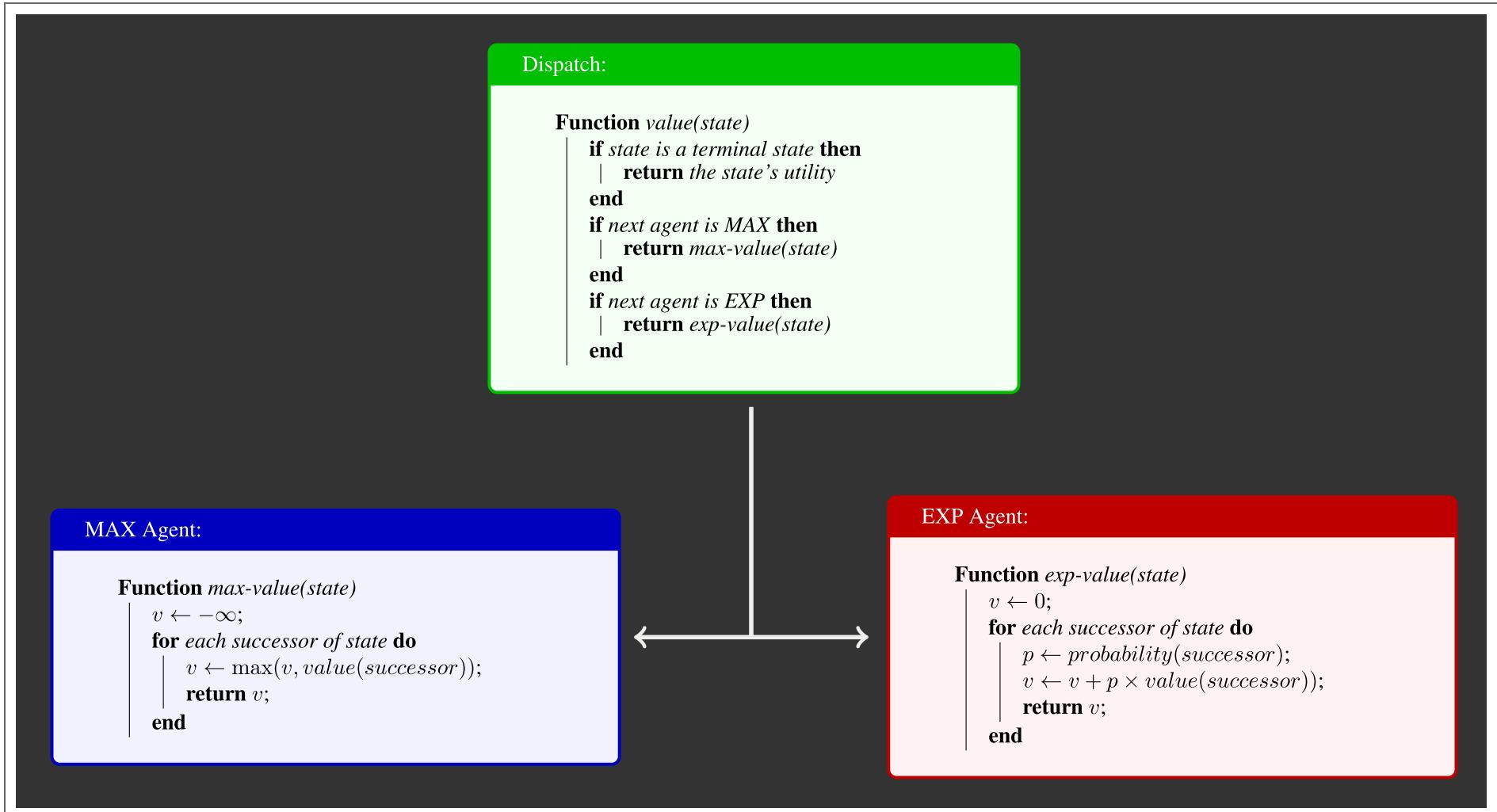
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Expectimax search: compute the average score under optimal play

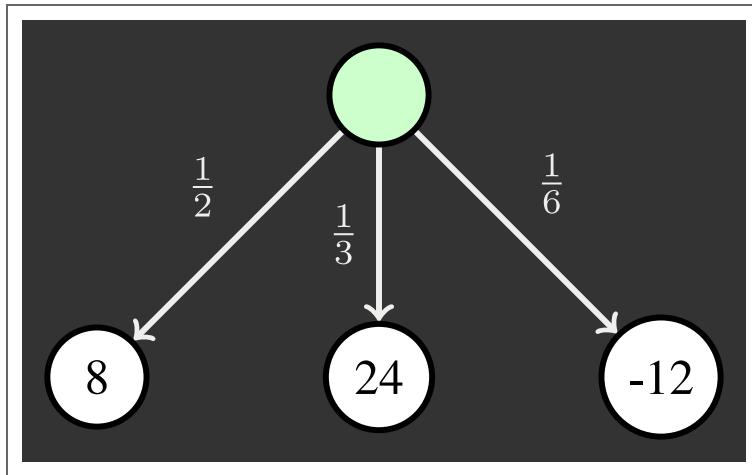
- › Max nodes as in minimax search
- › Chance nodes are like min nodes but the outcome is uncertain
- › Calculate their **expected utilities**
- › i.e., take weighted average (expectation) of children

Later, we will learn how to formalize the underlying uncertain-result problems as  
**Markov Decision Processes**

# EXPECTIMAX IMPLEMENTATION



# EXPECTIMAX IMPLEMENTATION

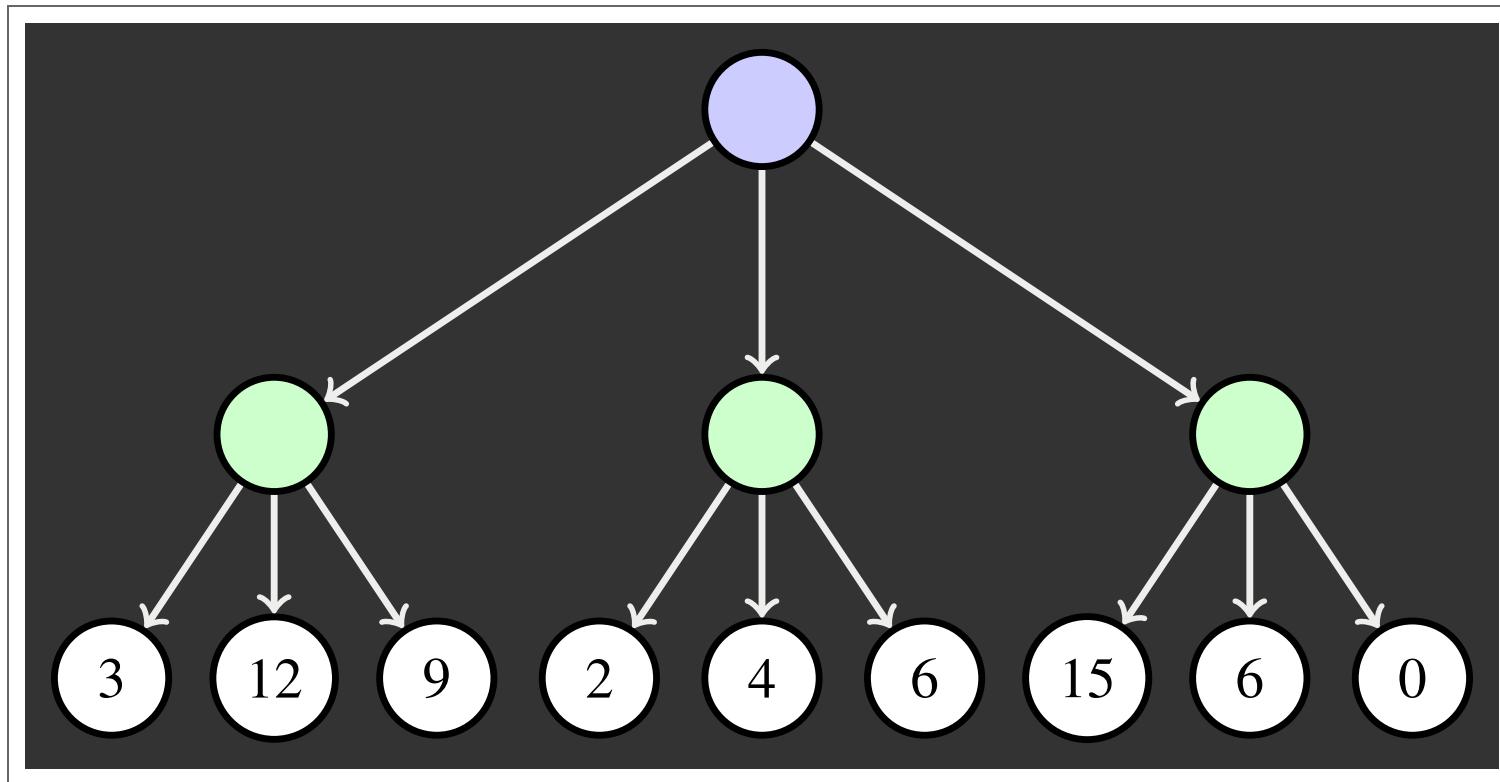


EXP Agent:

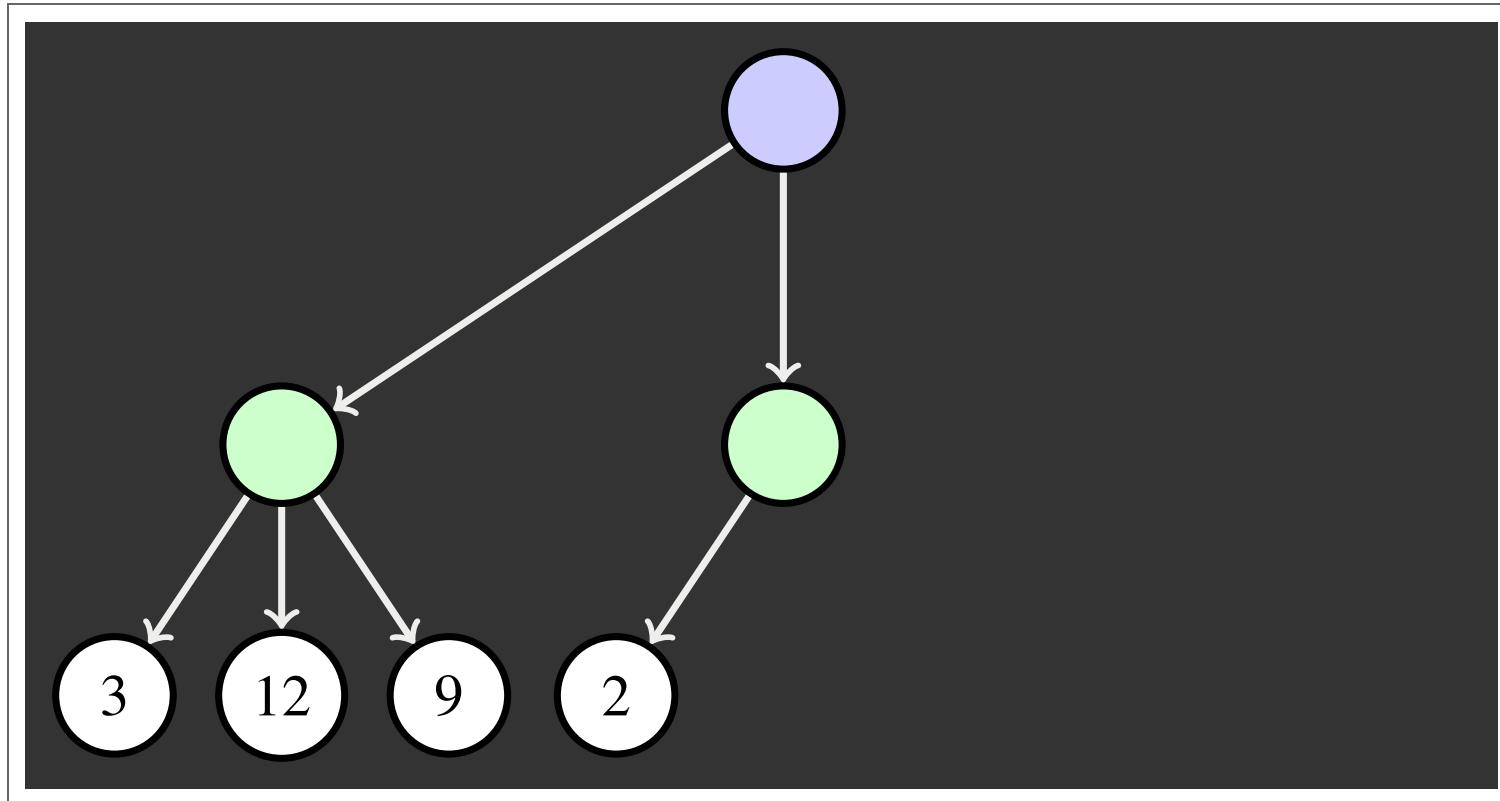
```
Function exp-value(state)
    v ← 0;
    for each successor of state do
        p ← probability(successor);
        v ← v + p × value(successor));
    return v;
end
```

$$v = \frac{1}{2} * 8 + \frac{1}{3} * 24 - \frac{1}{6} * 12 = 10 \quad (1)$$

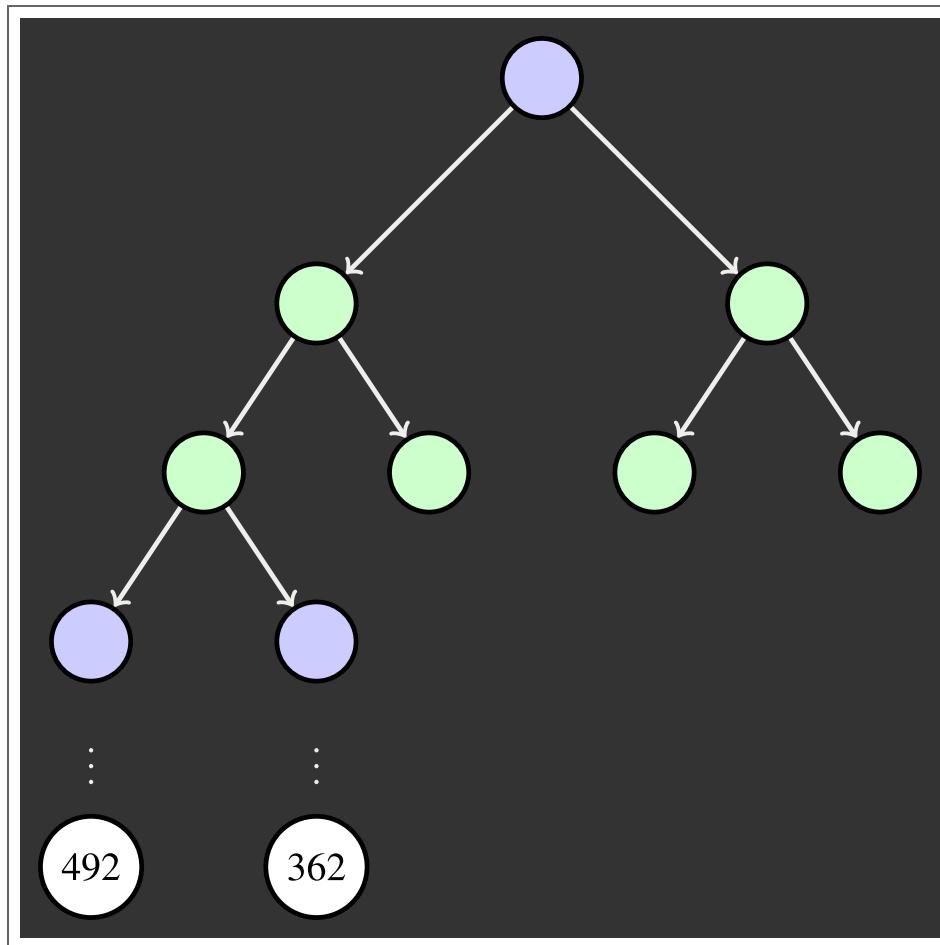
# EXPECTIMAX EXAMPLE

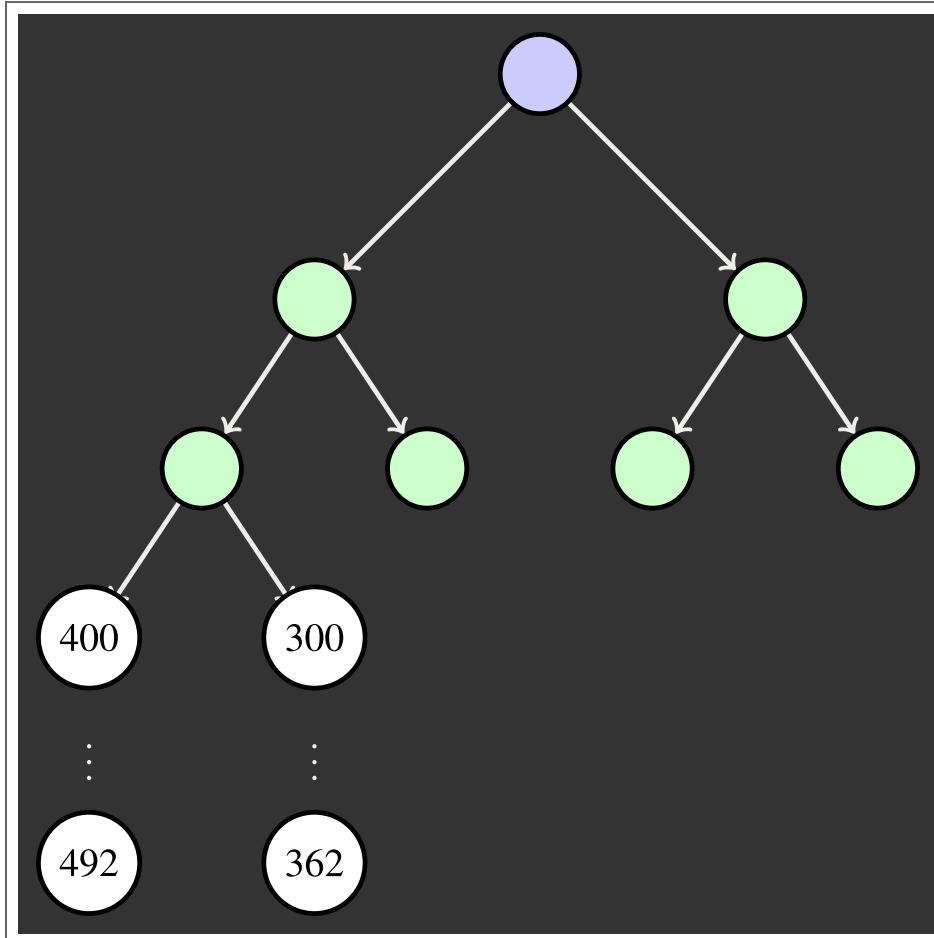


# EXPECTIMAX PRUNING



# DEPTH-LIMITED EXPECTIMAX





# PROBABILITIES

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## PRIMER: PROBABILITIES

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A **random variable** represents an event whose outcome is unknown.

A **probability distribution** is an assignment of weights to outcomes

Example: Traffic on freeway

- › Random variable:  $T =$  whether there is traffic
- › Outcomes:  $T \in \{none, mild, heavy\}$
- › Distribution:  $P(T = none) = 0.25, P(T = mild) = 0.50, P(T = heavy) = 0.25$

## PRIMER: PROBABILITIES...

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Some laws of probability (more later):

- Probabilities are always non-negative
- Probabilities over all possible outcomes sum to one

As we get more evidence, probabilities may change:

- $P(T = \text{heavy}) = 0.25, P(T = \text{heavy} \mid \text{Hour} = 8\text{am}) = 0.60$
- We will talk about methods for reasoning and updating probabilities later

## PRIMER: EXPECTATIONS

The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes

Example: How long to get to the airport?

	<i>None</i>	<i>mild</i>	<i>heavy</i>
Time:	20	30	60
Probability:	0.25	0.50	0.25

	<i>None</i>	<i>mild</i>	<i>heavy</i>	
Time:	20	30	60	
	×	+	×	=
Probability:	0.25	0.50	0.25	35

## WHAT PROBABILITIES TO USE?

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In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state

- › Model could be a simple uniform distribution (roll a die)
- › Model could be sophisticated and require a great deal of computation
- › We have a chance node for any outcome out of our control: opponent or environment
- › The model might say that adversarial actions are likely.

For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes

## QUIZ: INFORMED PROBABILITIES

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Let us say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise.

Question: What tree search should you use?

Answer: Expectimax

- › To figure out EACH chance node's probabilities, you have to run a simulation of your opponent
- › This kind of thing gets very slow very quickly
- › Even worse if you have to simulate your opponent simulating you
- › Except for minimax, which has the nice property that it all collapses into one game tree

# MODELING ASSUMPTIONS

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# THE DANGERS OF OPTIMISM AND PESSIMISM

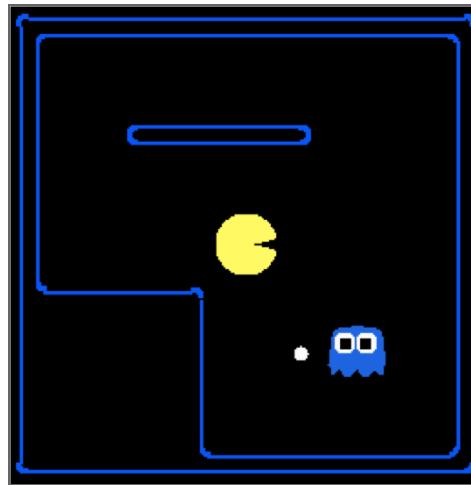
## Dangerous Optimism

- Assuming chance when the world is adversarial

## Dangerous Pessimism

- Assuming the worst case when it is not likely

# ASSUMPTIONS VS REALITY

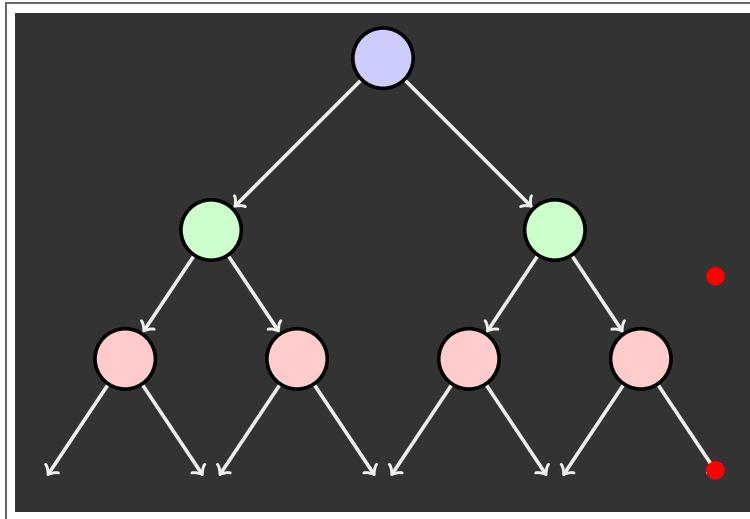


Pacman used depth 4 search with an eval function that avoids trouble

Ghost used depth 2 search with an eval function that seeks Pacman

	Adversarial Ghost	Random Ghost
Minimax Pacman	Won 5/5	Won 5/5
	Avg. Score: 483	Avg. Score: 493
Expectimax Pacman	Won 1/5	Won 5/5
	Avg. Score: -303	Avg. Score: 503

# MIXED AGENT GAMES



E.g. Backgammon

Expectiminimax

- Environment is an extra “random agent” player that moves after each min/max agent
- Each node computes the appropriate combination of its children

## MIXED AGENT GAMES: BACKGAMMON

Dice rolls increase b: 21 possible rolls with 2 dice

- ›  $\approx 20$  legal moves
- › Depth 2 =  $20 \times (21 \times 20)^3 = 1.2 \times 10^9$



As depth increases, probability of reaching a given search node shrinks

- › So usefulness of search is diminished
- › So limiting depth is less damaging
- › But pruning is trickier

Historic AI: TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play

1st AI world champion in any game !!

# UTILITIES

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# MAXIMUM EXPECTED UTILITY

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Why should we average utilities? Why not minimax?

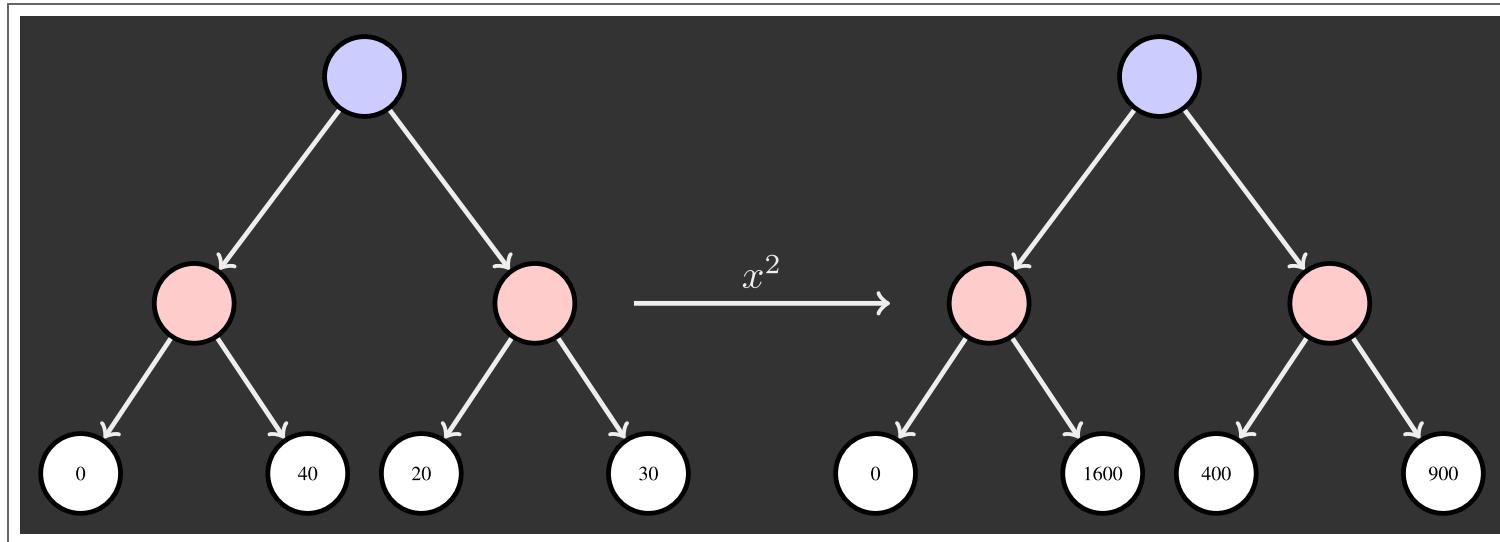
Principle of maximum expected utility:

- › A rational agent should choose the action that **maximizes its expected utility, given its knowledge**

Questions:

- › Where do utilities come from?
- › How do we know such utilities even exist?
- › How do we know that averaging even makes sense?
- › What if our behavior (preferences) cannot be described by utilities?

## WHAT UTILITIES TO USE?



For worst-case minimax reasoning, terminal function scale does not matter

- We just want better states to have higher evaluations (get the ordering right)
- We call this **insensitivity to monotonic transformations**.

For average-case expectimax reasoning, we need magnitudes to be meaningful

# UTILITIES

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Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences

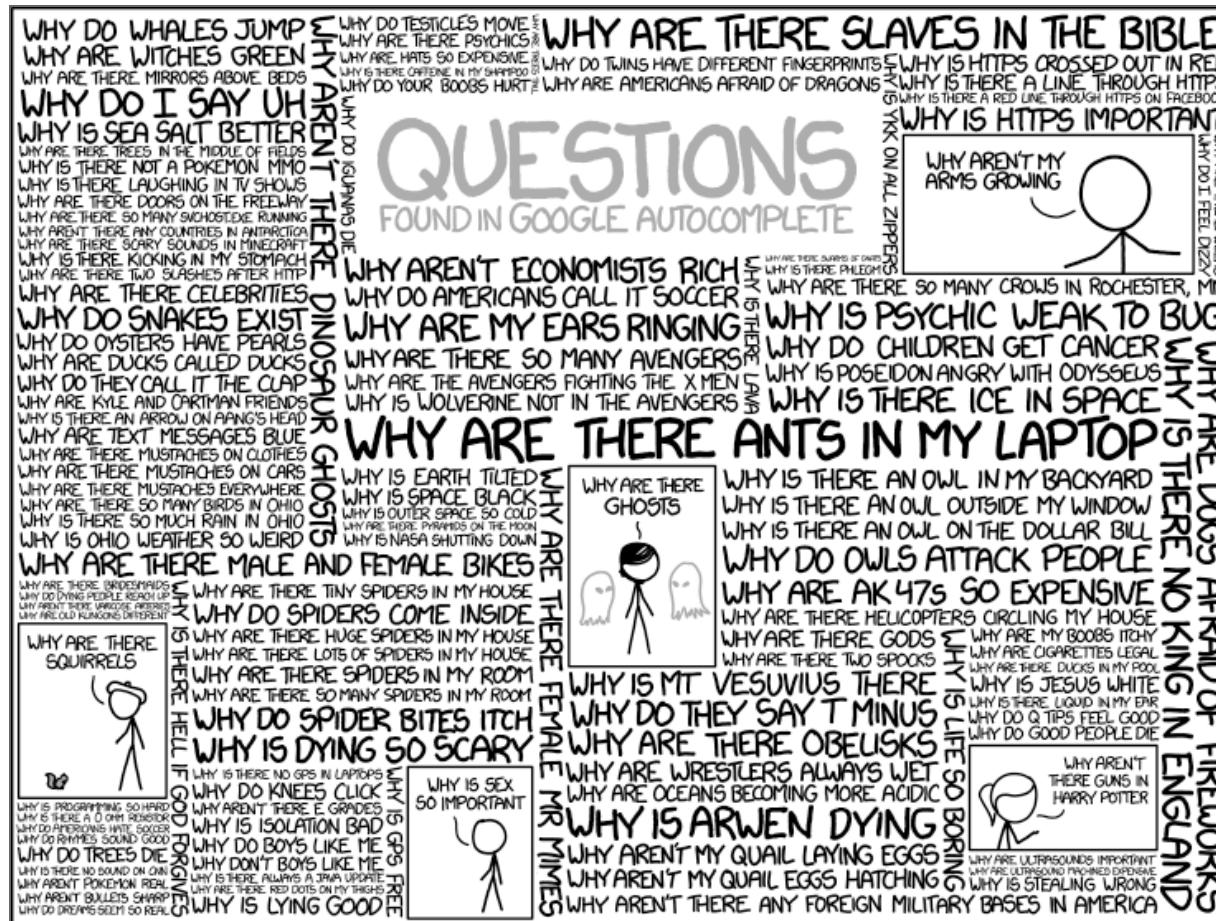
Where do utilities come from?

- In a game, may be simple (+1 / -1)
- Utilities summarize the agent's goals
- Theorem: any "rational" preferences can be summarized as a utility function

We hard-wire utilities and let behaviors emerge

- Why don't we let agents pick utilities?
- Why don't we prescribe behaviors?

# Q & A









## Speaker notes

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Q & A

