Prof. Wei

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TODAY

Value Iteration

Policy Iteration

- Policy Evaluation
- Policy Extraction

EXAMPLE: GRID WORLD

A maze like problem

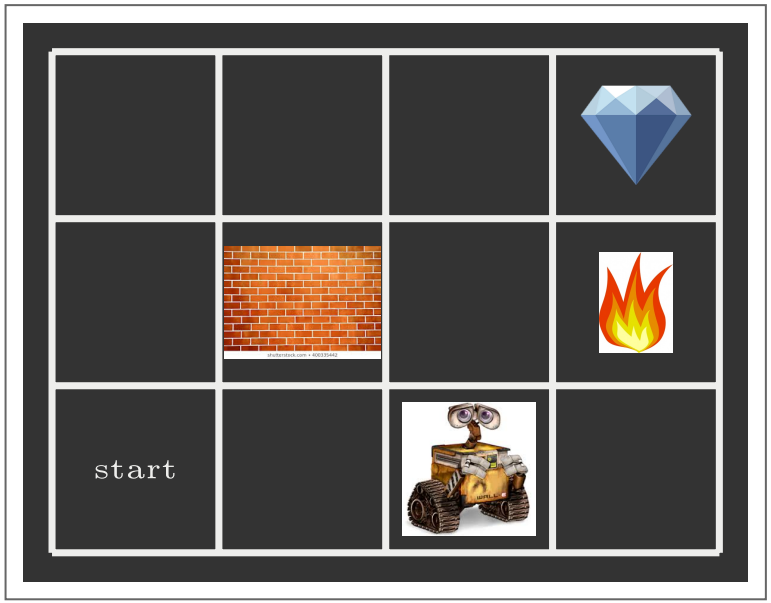
- The agent lives in a grid
- Walls block the agent's path

Noisy movement: actions do not always go as planned

- 80% of the time, the action North takes the agent North (if there is no wall there)
- 10% of the time, North takes the agent West; 10% East
- If there is a wall in the direction the agent would have been taken, the agent stays put

The agent receives rewards each time step

- Small "living" reward each step (can be negative)
- Big rewards come at the end (good or bad)

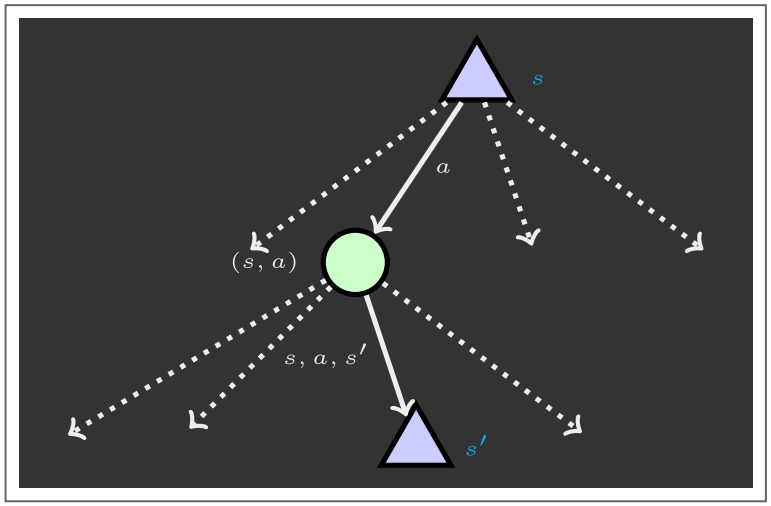


Goal: maximize sum of rewards

RECAP: DEFINING MDPS

Markov decision processes:

- Set of states S
- Start state s_0
- Set of actions A
- Transitions $P(s'|s,a)$ (or $T(s,a,s')$)
- Rewards R(s,a,s') (and discount γ)



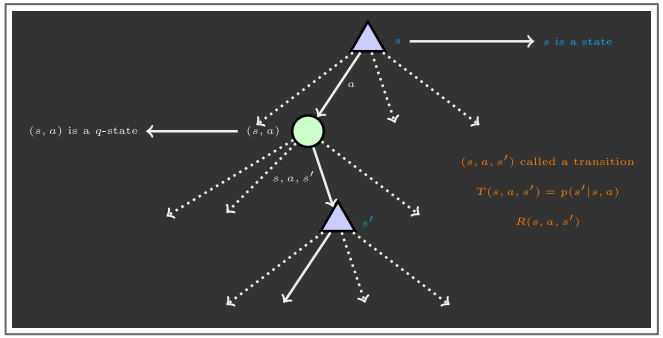
MDP quantities so far:

- Policy = Choice of action for each state
- Utility = sum of (discounted) rewards

SOLVING MDPS

OPTIMAL QUANTITIES

The value (utility) of a state s: $V^*(s)$ = expected utility starting in s and acting optimally



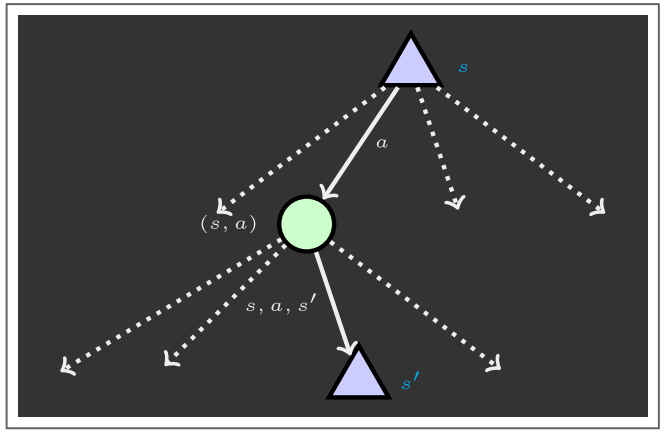
The value (utility) of a q-state (s, a): $Q^*(s, a)$ = expected utility starting out having taken action a from state s and (thereafter) acting optimally

The optimal policy: $\pi^*(s)$ = optimal action from state s

VALUES OF STATES

Fundamental operation: compute the (expectimax) value of a state

- Expected utility under optimal action
- Average sum of (discounted) rewards
- This is just what expectimax computed



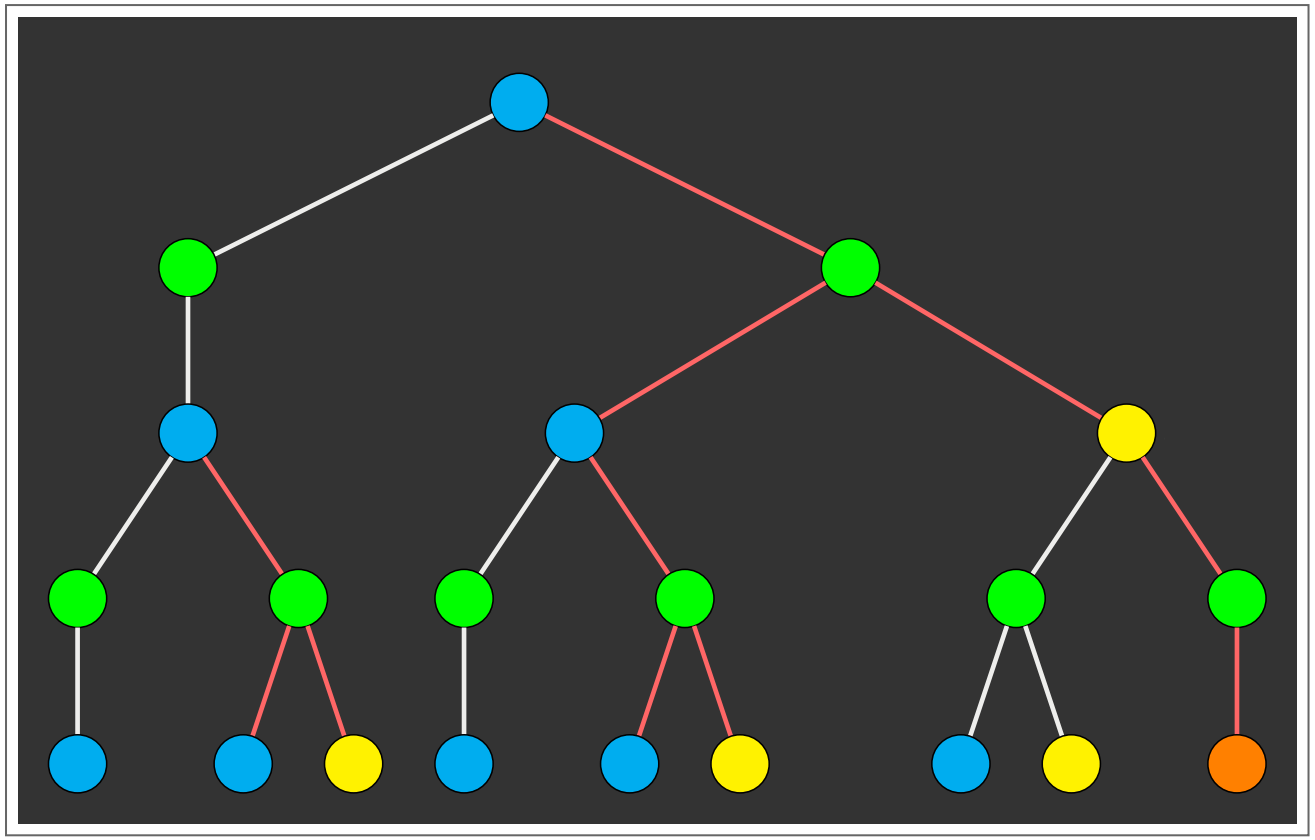
Recursive definition of value:

$$V^*(s) = \max_a Q^*(s,a) \tag{1}$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

$$V^*(s) = \max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')] \tag{2}$$

RACING SEARCH TREE



RACING SEARCH TREE

We are doing way too much work with expectimax.

Problem 1: States are repeated

- Idea: Only compute needed quantities once

Problem 2: Tree goes on forever

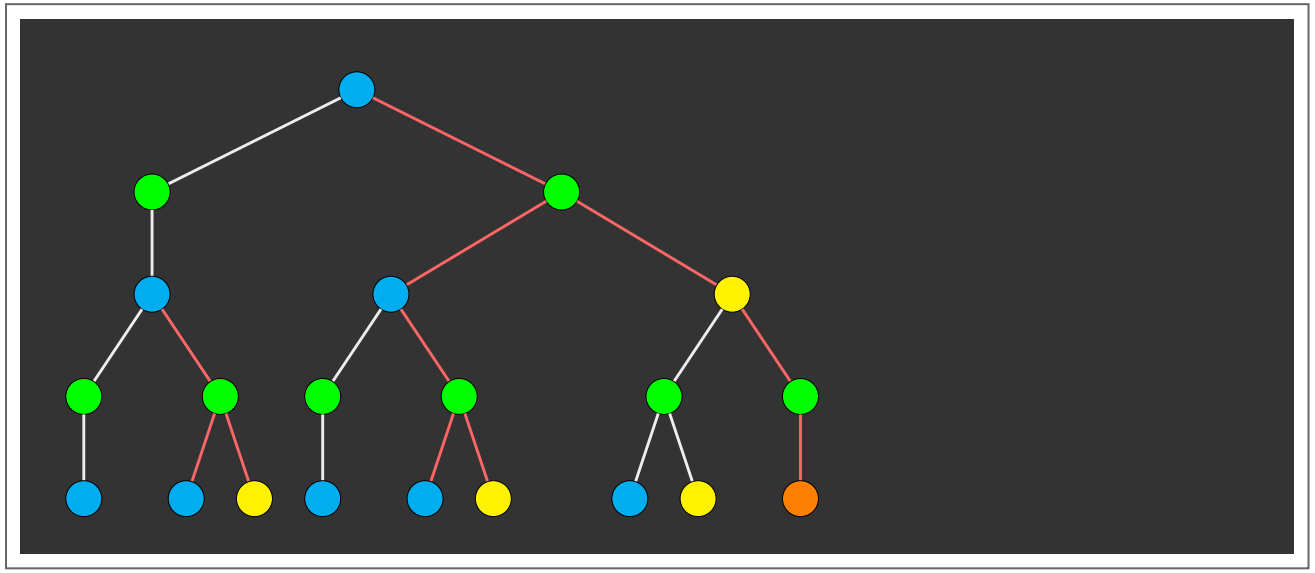
- Idea: Do a depth-limited computation, but with increasing depths until change is small
- Note: deep parts of the tree eventually do not matter if $\gamma < 1$

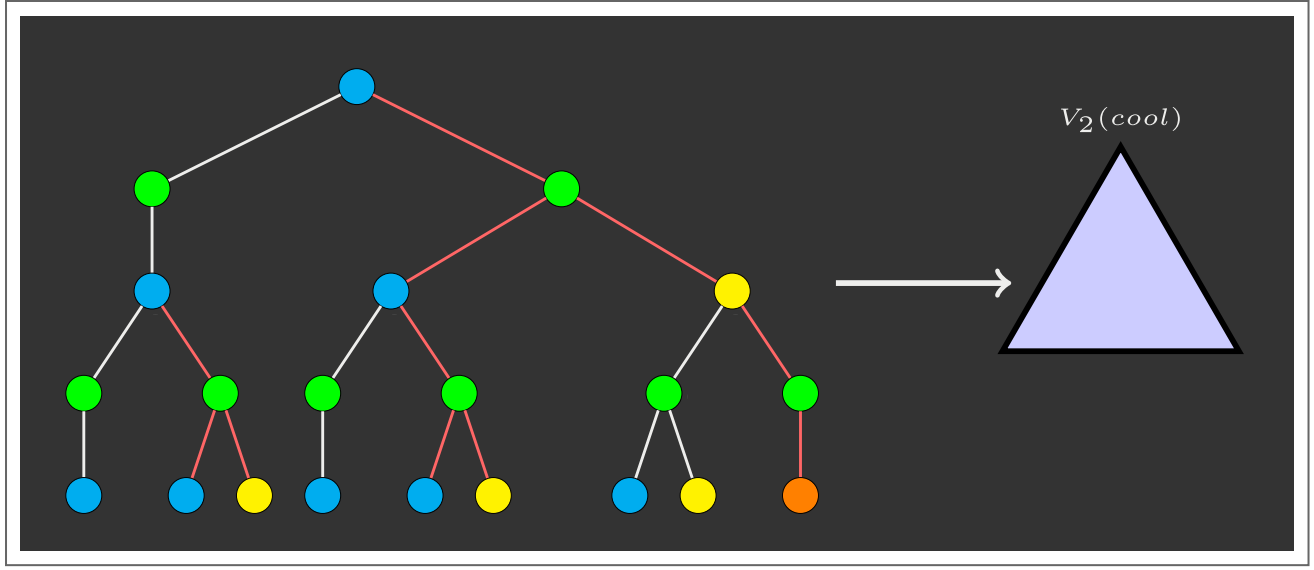
TIME-LIMITED VALUES

Key idea: time-limited values

Define $V_k(s)$ to be the optimal value of s if the game ends in k more time steps

- Equivalently, it is what a depth-k expectimax would give from s




$V_2(cool)$

VALUE ITERATION

VALUE ITERATION

Start with $V_0(s) = 0$: no time steps left means an expected reward sum of zero

Given vector of $V_k(s)$ values, do one ply of expectimax from each state:

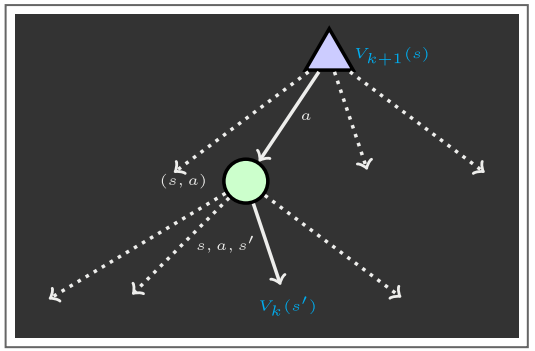
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V_k(s')]$$

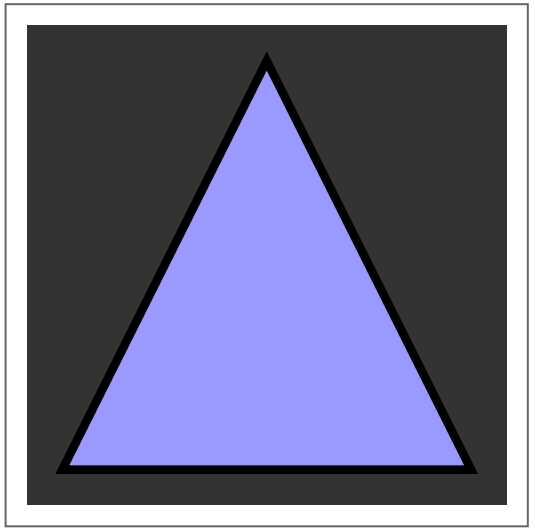
Repeat until convergence

Complexity of each iteration: $\mathcal{O}(S^2 A)$

Theorem: will converge to unique optimal values

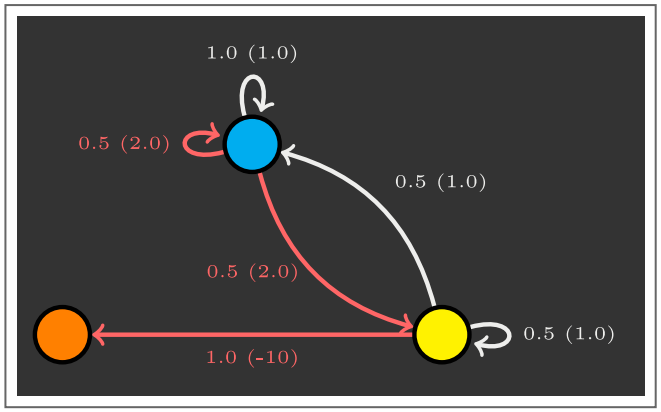
- Basic idea: approximations get refined towards optimal values
- Policy may converge long before values do





EXAMPLE VALUE ITERATION

	Cool	Warm	Overheated
V_2	3.5	2.5	0
V_1	2	1	0
V_0	0	0	0



$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s')[R(s,a,s') + V_k(s')]$$

Assume no discount

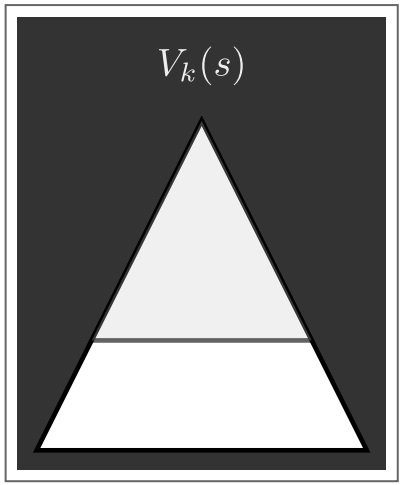
CONVERGENCE

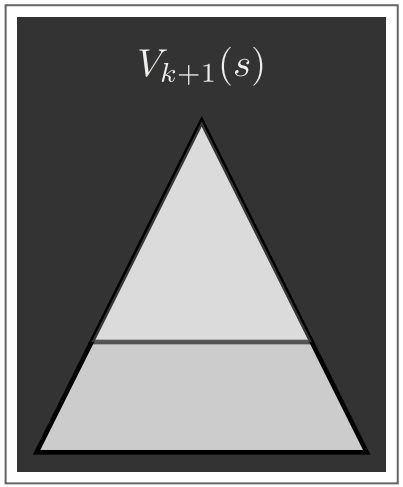
How do we know the V_k vectors are going to converge?

Case 1: If the tree has maximum depth M, then V_M holds the actual untruncated values

Case 2: If the discount is less than 1

- Sketch: For any state V_k and V_{k+1} can be viewed as depth $k+1$ expectimax results in nearly identical search trees
- The difference is that on the bottom layer, V_{k+1} has actual rewards while V_k has zeros
- That last layer is at best all R_{MAX}
- It is at worst R_{MIN}
- But everything is discounted by γ^k that far out
- So V_k and V_{k+1} are at most $\gamma^k \max |R|$ different

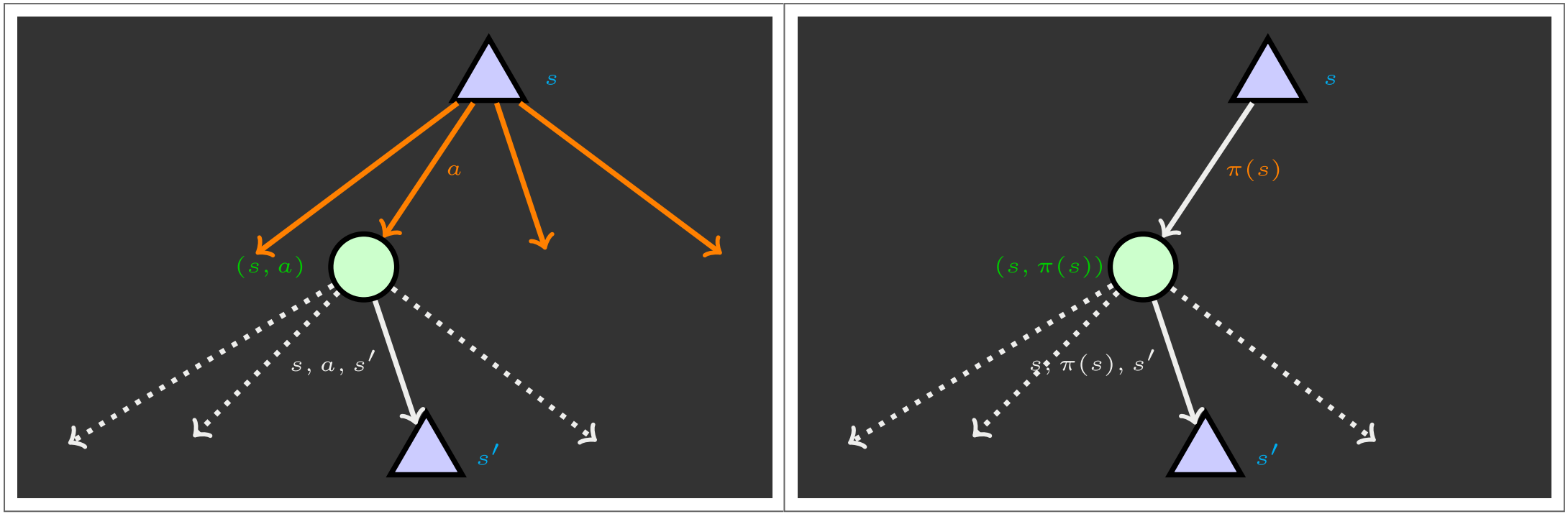




- So as k increases, the values converge

POLICY EVALUATION

FIXED POLICIES



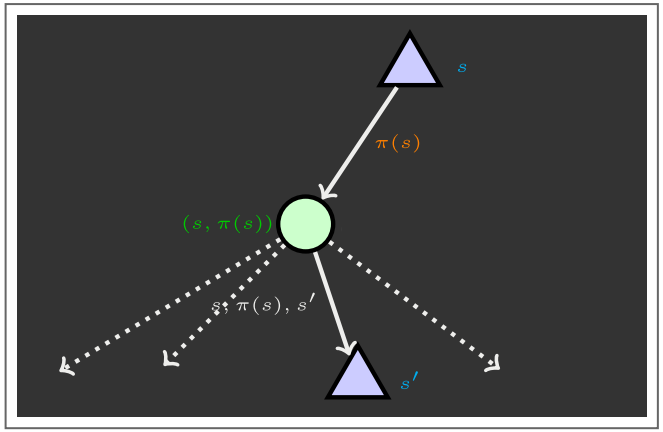
Expectimax trees max over all actions to compute the optimal values

If we fixed some policy $\pi(s)$, then the tree would be simpler – only one action per state

. . . though the tree's value would depend on which policy we fixed

UTILITIES FOR A FIXED POLICY

Another basic operation: compute the utility of a state s under a fixed (generally non-optimal) policy



Define the utility of a state s, under a fixed policy π:

$V^\pi(s)$ = expected total discounted rewards starting in s and following π

Recursive relation (one-step look-ahead / Bellman equation):

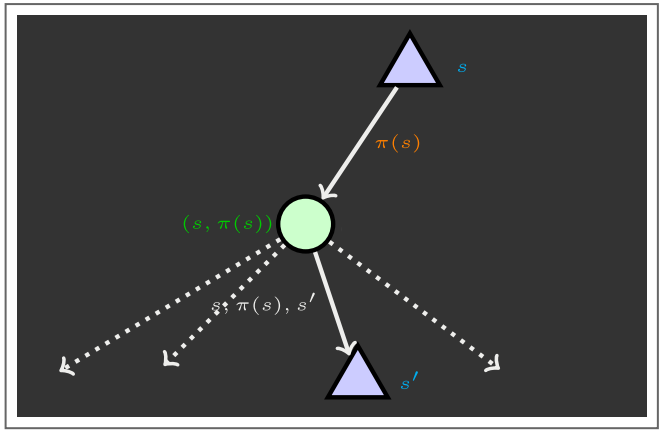
$$V^\pi(s) = \sum_{s'} T(s, a, s')[R(s, a, s') + \gamma V^\pi(s')]$$

POLICY EVALUATION

How do we calculate the V's for a fixed policy π?

Idea 1: Turn recursive Bellman equations into updates (like value iteration)

$$V_0^\pi(s) = 0$$
$$V_{k+1}^\pi(s) = \sum_{s'} T(s, a, s')[R(s, a, s') + \gamma V_k^\pi(s')]$$



Efficiency: $\mathcal{O}(S^2)$ per iteration

Idea 2: Without the maxes, the Bellman equations are just a linear system

- Solve with Matlab (or your favorite linear system solver)

POLICY EXTRACTION

COMPUTING VALUES FROM ACTIONS

Imagine we have the optimal values $V^*(s)$

How should we act?

- It is not obvious

We need to do a mini-expectimax (one step)

$$\pi^*(s) = \operatorname*{argmax}_a \sum_{s'} T(s, a, s')[R(s, a, s') + \gamma V^*(s')]$$

This is called policy extraction, since it gets the policy implied by the values.

COMPUTING ACTIONS FROM Q-VALUES

Imagine we have the optimal q-values: $\boldsymbol{Q^*(s,a)}$

How should we act?

- Completely trivial to decide !!

$$\pi^*(s) = \underset{a}{\operatorname{argmax}} Q^*(s,a)$$

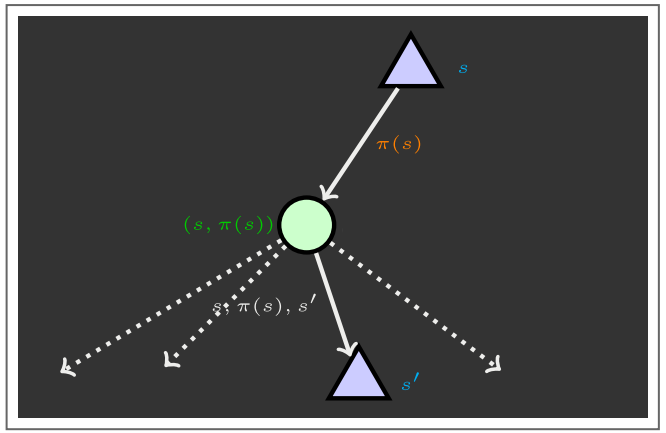
Important lesson: actions are easier to select from q-values than values !!

POLICY ITERATION

PROBLEMS WITH VALUE ITERATION

Value iteration repeats the Bellman updates:

$$V_{k+1}^*(s) = \max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V_k^*(s')]$$



Problem 1: It is slow – $\mathcal{O}(S^2A)$ per iteration

Problem 2: The "max" at each state rarely changes

Problem 3: The policy often converges long before the values

POLICY ITERATION

Alternative approach for optimal values:

- Step 1: Policy evaluation: calculate utilities for some fixed policy (not optimal utilities) until convergence
- Step 2: Policy improvement: update policy using one-step look-ahead with resulting converged (but not optimal) utilities as future values
- Repeat steps until policy converges

This is policy iteration

- It is still optimal !!
- Can converge (much) faster under some conditions

POLICY ITERATION

Evaluation: For fixed current policy π, find values with policy evaluation:

- Iterate until values converge:

$$V_{k+1}^{\pi_i}(s) = \sum_{s'} T(s, \pi_i(s), s')[R(s, \pi_i(s), s') + \gamma V_k^{\pi_i}(s')]$$

Improvement: For fixed values, get a better policy using policy extraction

- One-step look-ahead:

$$\pi_{i+1}(s) = \operatorname*{argmax}_a \sum_{s'} T(s, a, s')[R(s, a, s') + \gamma V^{\pi_i}(s')]$$

COMPARISON

Both value iteration and policy iteration compute the same thing (all optimal values)

In value iteration:

- Every iteration updates both the values and (implicitly) the policy
- We do not track the policy, but taking the max over actions implicitly recomputes it.

In policy iteration:

- We do several passes that update utilities with fixed policy (each pass is fast because we consider only one action, not all of them)
- After the policy is evaluated, a new policy is chosen (slow like a value iteration pass)
- The new policy will be better (or we are done)

Both are dynamic programs for solving MDPs

SUMMARY: MDP ALGORITHMS

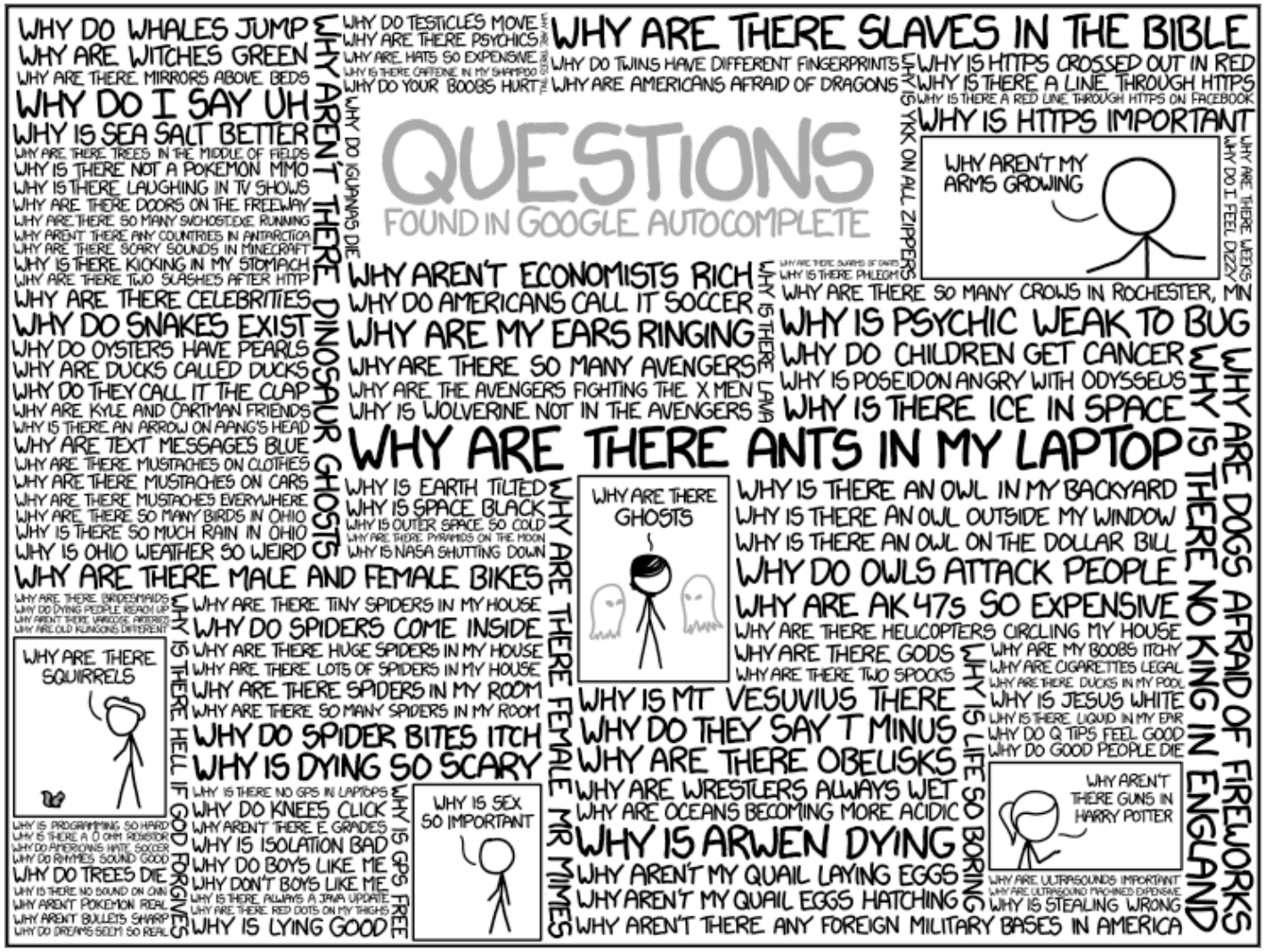
So you want to . . .

- Compute optimal values: use value iteration or policy iteration
- Compute values for a particular policy: use policy evaluation
- Turn your values into a policy: use policy extraction (one-step lookahead)

These all look the same . . .

- They basically are – they are all variations of Bellman updates
- They all use one-step lookahead expectimax fragments
- They differ only in whether we plug in a fixed policy or max over actions

Q & A


QUESTIONS
FOUND IN GOOGLE AUTOCOMPLETE
WHY DO WHALES JUMP
WHY ARE WITCHES GREEN
WHY ARE THERE MIRRORS ABOVE BEDS
WHY DO I SAY UH
WHY IS SEA SALT BETTER
WHY ARE THERE TREES IN THE MIDDLE OF FIELDS
WHY IS THERE NOT A POKEMON MMO
WHY IS THERE LAUGHING IN TV SHOWS
WHY ARE THERE DOORS ON THE FREEWAY
WHY ARE THERE SO MANY SKYSCRAPERS RUNNING
WHY AREN'T THERE ANY COUNTRIES IN ANTARCTICA
WHY ARE THERE SCARY SOUNDS IN MINECRAFT
WHY IS THERE KICKING IN MY STOMACH
WHY ARE THERE TWO SLASHES AFTER HTTP
WHY ARE THERE CELEBRITIES
WHY DO SNAKES EXIST
WHY DO OYSTERS HAVE PEARLS
WHY ARE DUCKS CALLED DUCKS
WHY DO THEY CALL IT THE CLAP
WHY ARE KYLE AND CARTMAN FRIENDS
WHY IS THERE AN ARROW ON AANG'S HEAD
WHY ARE TEXT MESSAGES BLUE
WHY ARE THERE MUSTACHES ON CLOTHES
WHY ARE THERE MUSTACHES ON CARS
WHY ARE THERE MUSTACHES EVERYWHERE
WHY ARE THERE SO MANY BIRDS IN OHIO
WHY IS THERE SO MUCH RAIN IN OHIO
WHY IS OHIO WEATHER SO WEIRD
WHY ARE THERE MALE AND FEMALE BIKES
WHY ARE THERE BRIDESMAIDS
WHY DO DYING PEOPLE REACH UP
WHY AREN'T THERE VARICOSE AVERIES
WHY ARE OLD KLINGONS DIFFERENT
WHY ARE THERE SQUIRRELS
WHY IS PROGRAMMING SO HARD
WHY IS THERE A 0 OHM RESISTOR
WHY DO AMERICANS HATE SOCCER
WHY DO RHYMES SOUND GOOD
WHY DO TREES DIE
WHY IS THERE NO SOUND ON CNN
WHY AREN'T POKEMON REAL
WHY AREN'T BULLETS SHARP
WHY DO DREAMS SEEM SO REAL
WHY AREN'T THERE DINOSAUR GHOSTS
WHY DO TESTICLES MOVE
WHY ARE THERE PSYCHICS
WHY ARE HATS SO EXPENSIVE
WHY IS THERE CAFFEINE IN MY SHAMPOO
WHY DO YOUR BOOBS HURT
WHY DO IGUANAS DIE
WHY ARE THERE SLAVES IN THE BIBLE
WHY DO TWINS HAVE DIFFERENT FINGERPRINTS
WHY ARE AMERICANS AFRAID OF DRAGONS
WHY IS HTTPS CROSSED OUT IN RED
WHY IS THERE A LINE THROUGH HTTPS
WHY IS THERE A RED LINE THROUGH HTTPS ON FACEBOOK
WHY IS HTTPS IMPORTANT
WHY AREN'T MY ARMS GROWING
WHY IS YKK ON ALL ZIPPERS
WHY ARE THERE WEEKS
WHY DO I FEEL DIZZY
WHY AREN'T ECONOMISTS RICH
WHY DO AMERICANS CALL IT SOCCER
WHY ARE MY EARS RINGING
WHY ARE THERE SO MANY AVENGERS
WHY ARE THE AVENGERS FIGHTING THE X MEN
WHY IS WOLVERINE NOT IN THE AVENGERS
WHY ARE THERE SWARMS OF DAVES
WHY IS THERE PHLEGM
WHY ARE THERE SO MANY CROWS IN ROCHESTER, MN
WHY IS PSYCHIC WEAK TO BUG
WHY DO CHILDREN GET CANCER
WHY IS POSEIDON ANGRY WITH ODYSSEUS
WHY IS THERE ICE IN SPACE
WHY IS THERE LAVA
WHY ARE THERE ANTS IN MY LAPTOP
WHY IS EARTH TILTED
WHY IS SPACE BLACK
WHY IS OUTER SPACE SO COLD
WHY ARE THERE PYRAMIDS ON THE MOON
WHY IS NASA SHUTTING DOWN
WHY ARE THERE GHOSTS
WHY ARE THERE TINY SPIDERS IN MY HOUSE
WHY DO SPIDERS COME INSIDE
WHY ARE THERE HUGE SPIDERS IN MY HOUSE
WHY ARE THERE LOTS OF SPIDERS IN MY HOUSE
WHY ARE THERE SPIDERS IN MY ROOM
WHY ARE THERE SO MANY SPIDERS IN MY ROOM
WHY DO SPIDER BITES ITCH
WHY IS DYING SO SCARY
WHY IS THERE NO GPS IN LAPTOPS
WHY DO KNEES CLICK
WHY AREN'T THERE E GRADES
WHY IS ISOLATION BAD
WHY DO BOYS LIKE ME
WHY DON'T BOYS LIKE ME
WHY IS THERE ALWAYS A JAVA UPDATE
WHY ARE THERE RED DOTS ON MY THIGHS
WHY IS LYING GOOD
WHY IS SEX SO IMPORTANT
WHY ARE THERE FEMALE MR MIMES
WHY IS THERE AN OWL IN MY BACKYARD
WHY IS THERE AN OWL OUTSIDE MY WINDOW
WHY IS THERE AN OWL ON THE DOLLAR BILL
WHY DO OWLS ATTACK PEOPLE
WHY ARE AK 47s SO EXPENSIVE
WHY ARE THERE HELICOPTERS CIRCLING MY HOUSE
WHY ARE THERE GODS
WHY ARE THERE TWO SPOOKS
WHY IS MT VESUVIUS THERE
WHY DO THEY SAY T MINUS
WHY ARE THERE OBELISKS
WHY ARE WRESTLERS ALWAYS WET
WHY ARE OCEANS BECOMING MORE ACIDIC
WHY IS ARWEN DYING
WHY AREN'T MY QUAIL LAYING EGGS
WHY AREN'T MY QUAIL EGGS HATCHING
WHY AREN'T THERE ANY FOREIGN MILITARY BASES IN AMERICA
WHY IS LIFE SO BORING
WHY ARE MY BOOBS ITCHY
WHY ARE CIGARETTES LEGAL
WHY ARE THERE DUCKS IN MY POOL
WHY IS JESUS WHITE
WHY IS THERE LIQUID IN MY EAR
WHY DO Q TIPS FEEL GOOD
WHY DO GOOD PEOPLE DIE
WHY AREN'T THERE GUNS IN HARRY POTTER
WHY ARE ULTRASOUNDS IMPORTANT
WHY ARE ULTRASOUND MACHINES EXPENSIVE
WHY IS STEALING WRONG
WHY ARE DOGS AFRAID OF FIREWORKS
WHY IS THERE NO KING IN ENGLAND
WHY IS GPS FREE
WHY GOD FORGIVES

XKCD

Speaker notes