SOME DISCRETE DISTRIBUTIONS

Name	**Parameters**	**Genesis / Usage**	**Notation**	$p(x) = \mathbb{P}[X = x]$ **(and non-zero range)**	$\mathbb{E}[X]$	$\text{Var}(X)$	**Comments**
Uniform (discrete)	$k \in \mathbb{N}$	Set of k equally likely outcomes	$Unif(1, \ldots, k)$ (not standard)	$p(x) = 1/k$ $x = 1, ..., k$	$\frac{k+1}{2}$	$\frac{k^2-1}{12}$	Fair dice roll ($k = 6$)
Bernoulli trial	$\theta \in [0, 1]$	Experiment with two outcomes (typically, success = 1, fail = 0)	$Bernoulli(\theta)$	$p(x) = \theta^x(1 - \theta)^{1-x}$ $x = 0, 1$	θ	$\theta(1 - \theta)$	Coin toss
Binomial	$n \in \mathbb{N}$, $\theta \in [0, 1]$	Number of successes in n i.i.d. Bernoulli trials	$Bi(n, \theta)$ $B(n, p)$	$p(x) = \binom{n}{x}\theta^x(1 - \theta)^{n-x}$ $x = 0, 1, 2, ..., n$	$n\theta$	$n\theta(1 - \theta)$	Sampling with replacement $Bi(1, \theta) \equiv Bernoulli(\theta)$
Geometric	$\theta \in [0, 1)$	Number of failed i.i.d. Bernoulli trials before first success	$Geom(\theta)$ $Geo(\theta)$	$p(x) = \theta^x(1 - \theta)$ $x = 0, 1, 2, \ldots$	$\frac{\theta}{1-\theta}$	$\frac{\theta^2}{(1-\theta)^2}$	Alternative formulations might swap θ and $1-\theta$, or use $X' = X + 1$ to include the successful trial
Negative Binomial	$k \in \mathbb{N}$, $\theta \in (0, 1]$	Number of i.i.d. Bernoulli trials until k^{th} success	$NegBin(k, \theta)$ (not standard)	$p(x) = \binom{x-1}{k-1}\theta^k(1-\theta)^{x-k}$ $x = k, k + 1, k + 2, \ldots$	$\frac{k}{\theta}$	$\frac{k(1-\theta)}{\theta^2}$	Several alternative formulations exist.
Hypergeometric	$N \in \mathbb{N}$ $k \in \{0, \ldots, N\}$ $n \in \{0, \ldots, n\}$	Number of special objects in a random sample of n objects, from a population of N objects with k special objects	$HypGeom(N, k, n)$ (not standard)	$p(x) = \binom{k}{x}\binom{N-k}{n-x}/\binom{N}{n}$ $x = 0, ..., n$	$\frac{nk}{N}$	$n\frac{N-n}{N-1}\frac{k}{N}(1 - \frac{k}{N})$	
Poisson	$\lambda \in (0, \infty)$	Counting events occurring 'at random' within space or time	$Poi(\lambda)$	$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $x = 0, 1, 2, ...$	λ	λ	

SOME CONTINUOUS DISTRIBUTIONS

Name	**Parameters**	**Genesis / Usage**	**Notation**	$f(x) =$ **p.d.f. (and non-zero range)**	$\mathbb{E}[X]$	$\mathrm{Var}(X)$	**Comments**
Uniform (continuous)	$\alpha, \beta \in \mathbb{R}$ with $\alpha < \beta$	The uniform distribution for a continuous interval	$Unif(\alpha, \beta)$ $U(a, b)$	$f(x) = \frac{1}{\beta-\alpha}$ $x \in (\alpha, \beta)$	$\frac{\alpha+\beta}{2}$	$\frac{(\beta-\alpha)^2}{12}$	Also written as $U[\alpha, \beta]$ and similarly for open and half-open intervals.
Normal	$\mu \in \mathbb{R}$, $\sigma \in (0, \infty)$	Empirically and theoretically (via CLT, etc.) a good model in many situations.	$N(\mu, \sigma^2)$	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ $x \in \mathbb{R}$	μ	σ^2	$N(0, 1) \equiv$ standard normal. $X \sim N(\mu, \sigma^2) \Rightarrow$ $aX + b \sim N(a\mu + b, a^2\sigma^2)$ Hence $Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$
Exponential	$\lambda \in (0, \infty)$	Inter-arrival times of random events	$Exp(\lambda)$	$f(x) = \lambda e^{-\lambda x}$ $x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	Alternative parametrization: $\theta = \frac{1}{\lambda}$
Gamma	$\alpha, \beta \in (0, \infty)$	Lifetimes of ageing items, multi-inter-arrival times	$Ga(\alpha, \beta)$ $\Gamma(\alpha, \beta)$	$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ $x > 0$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$	Alternative parametrization: $\theta = 1/\beta$, $Ga(1, \lambda) \equiv Exp(\lambda)$, $Ga(n/2, 1/2) \equiv \chi^2_n$
Log-normal	$\mu \in \mathbb{R}, \sigma \in (0, \infty)$	Quantities related to exponential growth	$logN(\mu, \sigma^2)$ (not standard)	$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\log x-\mu)^2}{\sqrt{2}\sigma}\right)$ $x > 0$	$e^{\mu+\frac{1}{2}\sigma^2}$	$(e^{\sigma^2} - 1)e^{2\mu+\sigma^2}$	If $X \sim logN(\mu, \sigma^2)$ then $\log X \sim N(\mu, \sigma^2)$
Chi-squared	$n \in \mathbb{N}$	Squared (normally distributed) errors, statistical tests	χ^2_n	$f(x) = \frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2}$ $x > 0$	n	$2n$	$X^2_n \equiv Ga(n/2, 1/2)$ $X_i \sim N(0, 1)$ i.i.d. $\Rightarrow \sum_{i=1}^n X_i^2 \sim \chi^2_n$
Beta	$\alpha, \beta \in (0, \infty)$	Quantities constrained to be within intervals	$Be(\alpha, \beta)$ $Beta(\alpha, \beta)$	$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$ $x \in [0, 1]$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$	$Be(1, 1) \equiv Unif(0, 1)$
Cauchy	$a, b \in \mathbb{R}$	Heavy tailed, pathological examples	$Cauchy(a, b)$	$f(x) = \frac{1}{\pi b} \frac{b^2}{(x-a)^2+b^2}$ $x \in \mathbb{R}$	undefined	undefined	$Cauchy(0, 1)$ is often called 'the' Cauchy distribution
Pareto	$\alpha, \beta \in (0, \infty)$	Heavy tailed quantities	$Pareto(\alpha, \beta)$	$f(x) = \frac{\alpha\beta^\alpha}{x^{\alpha+1}}$ $x > \beta$	$\frac{\alpha\beta}{\alpha+1}$ if $\alpha > 1$	$\frac{\alpha^2\beta}{(\alpha-1)^2(\alpha-2)}$ if $\alpha > 2$	If $X \sim Pareto(\alpha, \beta)$ then $\log \frac{X}{\beta} \sim Exp(\alpha)$
Weibull	$\lambda, k \in (0, \infty)$	Lifetimes, extreme values, particle sizes	$Weibull(\lambda, k)$	$f(x) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}$ $x > 0$	$\lambda\Gamma(1 + 1/k)$	$\lambda^2 [\Gamma(1 + 2/k)$ $+\Gamma(1 + 1/k)^2]$	If $X \sim Weibull(\lambda, k)$ then $(X/\lambda)^k \sim Exp(1)$
Student t	$n \in \mathbb{N}$	Statistical tests	t_n	$f(x) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}(1 + \frac{x^2}{n})^{-\frac{n+1}{2}}$ $x \in \mathbb{R}$	0 if $n > 1$	$\frac{n}{n-2}$ if $n > 2$	$t_1 \equiv Cauchy(0, 1)$ Can take $n \in (0, \infty)$
F	$\nu, \delta \in (0, \infty)$	Statistical tests	$F_{\nu,\delta}$	$f(x) = \frac{\nu^{\nu/2}\delta^{\delta/2}x^{\nu/2-1}}{B(\nu/2,\delta/2)(\nu x+\delta)^{(\nu+\delta)/2}}$ $x > 0$	$\frac{\delta}{\delta-2}$ if $\delta > 2$	$\frac{2\delta^2(\nu+\delta-2)}{\nu(\delta-2)^2(\delta-4)}$ if $\delta > 4$	If $X \sim \chi^2_\nu$ and $Y \sim \chi^2_\delta$ are independent then $\frac{X/\nu}{Y/\delta} \sim F_{\nu,\delta}$. If $T \sim t_\nu$ then $T^2 \sim F_{1,\nu}$. If $Z \sim Be(\alpha, \beta)$ then $\frac{\beta Z}{\alpha(1-Z)} \sim F_{2\alpha,2\beta}$.