

MASx52: Assignment 1

1. Recall the one-period market, and its parameters r, u, d, p_u, p_d and s . We assume that $d < 1 + r < u$.
 - (a) At time $t = 0$ our portfolio contains 2 unit of cash and 3 units of stock. What is the value of our portfolio at time $t = 0$? If we hold this portfolio until time $t = 1$, what is its new value?
 - (b) A rival investor holds a portfolio containing 3 units of cash and 2 unit of stock. Under what condition (on the parameters) can we be *certain* that our own portfolio will have a strictly greater value at time $t = 1$?

2. Let $\Omega = \{HH, HT, TH, TT\}$, representing two coin tosses each of which may show either H (head) or T (tail). Let $X : \Omega \rightarrow \mathbb{R}$ be the toss in which the first head occurred, or zero if no heads occurred:

$$X = \begin{cases} 0 & \text{if } \omega = TT \\ 1 & \text{if } \omega = HT \text{ or } \omega = HH \\ 2 & \text{if } \omega = TH. \end{cases}$$

Let Y be the total number of heads that occurred in both tosses.

- (a) Write down the sets $X^{-1}(0)$, $X^{-1}(1)$ and $X^{-1}(2)$.
 - (b) List the elements of $\sigma(X)$.
 - (c) Is Y measurable with respect to $\sigma(X)$? Justify your answer.
3. Let $\Omega = \{1, 2, 3, 4, 5\}$, representing one roll of a five sided dice. In each case, match the σ -field to the information it contains.
 - (a) $\{\emptyset, \Omega, \{1\}, \{2, 3, 4, 5\}\}$
 - (b) $\sigma(\{1, 2, 3\}, \{3, 4, 5\})$
 - (c) $\{\emptyset, \Omega, \{1\}, \{2, 3, 4\}, \{5\}, \{1, 2, 3, 4\}, \{2, 3, 4, 5\}, \{1, 5\}\}$
 - (i) If the roll was less than or equal to 3.
 - (ii) If the roll was the minimum possible value, or the maximum possible value, or neither.
 - (iii) If the roll was equal to 3, or strictly less three, or strictly greater than 3.
 - (iv) If the roll was a 1 or not.
4. Let X be a random variable.
 - (a) Show that $Y = \cos X$ is also a random variable.
 - (b) For which $p \in [1, \infty)$ do we have $Y \in L^p$?