

MASx52: Assignment 4

1. Let B_t be a standard Brownian motion.
 - (a) Write down the distribution of B_t , and write down $\mathbb{E}[B_t]$ and $\mathbb{E}[B_t^2]$.
 - (b) Let $0 \leq u \leq t$. Show that $\mathbb{E}[(B_t - B_u)^2 \mid \mathcal{F}_u] = t - u$.
2. Write down the following stochastic differential equations in integral form, over the time interval $[0, t]$.
 - (a) $dX_t = t dt + 2B_t dB_t$.
 - (b) $dY_t = 3Y_t dt$.

Write down a differential equation satisfied by Y_t , and find its solution with the initial condition $Y_0 = 1$. Is X_t differentiable?

3. Use Ito's formula to calculate the stochastic differential of dZ_t where
 - (a) $Z_t = tB_t$
 - (b) $Z_t = B_t^2 - t$
 - (c) $Z_t = 1 + t^2 X_t$ where $dX_t = \mu dt + \sigma B_t dB_t$ and μ, σ are deterministic constants.
 - (d) $Z_t = e^{-2t} S_t$ where $dS_t = 2S_t dt + 5S_t dB_t$.

Explain briefly why Z_t is a martingale in cases (b) and (d).

4. Let S_t be a geometric Brownian motion, with drift $\mu \in \mathbb{R}$, volatility $\sigma > 0$, and (deterministic) initial condition S_0 .
 - (a) Find $\mathbb{E}[S_t]$ and deduce that S_t is not a Brownian motion when $\mu \neq 0$.
 - (b) Is S_t a Brownian motion when $\mu = 0$?