SOME DISCRETE DISTRIBUTIONS

Name	**Parameters**	**Genesis / Usage**	**Notation**	$p(x) = \mathbb{P}[X = x]$ **(and non-zero range)**	$\mathbb{E}[X]$	$\mathrm{Var}(X)$	**Comments**
Uniform (discrete)	$k \in \mathbb{N}$	Set of k equally likely outcomes	$Unif(1, \ldots, k)$ (not standard)	$p(x) = 1/k$ $x = 1, ..., k$	$\frac{k+1}{2}$	$\frac{k^2-1}{12}$	Fair dice roll ($k = 6$)
Bernoulli trial	$\theta \in [0, 1]$	Experiment with two outcomes (typically, success = 1, fail = 0)	$Bernoulli(\theta)$	$p(x) = \theta^x(1-\theta)^{1-x}$ $x = 0, 1$	θ	$\theta(1-\theta)$	Coin toss
Binomial	$n \in \mathbb{N}$, $\theta \in [0, 1]$	Number of successes in n i.i.d. Bernoulli trials	$Bi(n, \theta)$	$p(x) = \binom{n}{x}\theta^x(1-\theta)^{n-x}$ $x = 0, 1, 2, ..., n$	$n\theta$	$n\theta(1-\theta)$	Sampling with replacement $Bi(1, \theta) \equiv Bernoulli(\theta)$
Geometric	$\theta \in (0, 1]$	Number of failed i.i.d. Bernoulli trials before first success	$Geom(\theta)$	$p(x) = (1-\theta)^x\theta$ $x = 0, 1, 2, \ldots$	$\frac{1-\theta}{\theta}$	$\frac{1-\theta}{\theta^2}$	Alternative formulations might swap p and $1-p$, or use $X' = X + 1$ to include the successful trial
Negative Binomial (or Pascal)	$k \in \mathbb{N}$, $\theta \in (0, 1]$	Number of i.i.d. Bernoulli trials until k^{th} success	$NegBin(k, \theta)$ (not standard)	$p(x) = \binom{x-1}{k-1}\theta^k(1-\theta)^{x-k}$ $x = k, k+1, k+2, \ldots$	$\frac{k}{\theta}$	$\frac{k(1-\theta)}{\theta^2}$	Several alternative formulations exist.
Hypergeometric	$N \in \mathbb{N}$, $k \in \{0, \ldots, N\}$, $n \in \{0, \ldots, n\}$	Number of special objects in a random sample of n objects, from a population of N objects with k special objects	$HypGeom(N, k, n)$ (not standard)	$p(x) = \binom{n}{x}\binom{N-n}{k-x}/\binom{N}{n}$ $x = 0, ..., n$	$\frac{nk}{N}$	$n\frac{N-n}{N-1}\frac{k}{N}(1-\frac{k}{N})$	
Poisson	$\lambda \in (0, \infty)$	Counting events occurring 'at random' within space or time	$Poi(\lambda)$	$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $x = 0, 1, 2, ...$	λ	λ	

SOME CONTINUOUS DISTRIBUTIONS

Name	**Parameters**	**Genesis / Usage**	**Notation**	$f(x) =$ **p.d.f.** **(and non-zero range)**	$\mathbb{E}[X]$	$\mathrm{Var}(X)$	**Comments**
Uniform (continuous)	$\alpha, \beta \in \mathbb{R}$ with $\alpha < \beta$	The uniform distribution for a continuous interval	$Unif(\alpha, \beta)$	$f(x) = \frac{1}{\beta-\alpha}$ $x \in (\alpha, \beta)$	$\frac{\alpha+\beta}{2}$	$\frac{(\beta-\alpha)^2}{12}$	Also written as $U[\alpha, \beta]$ and similarly for half-open intervals.
Normal	$\mu \in \mathbb{R}$, $\sigma \in (0, \infty)$	Empirically and theoretically (via CLT, etc.) a good model in many situations.	$N(\mu, \sigma^2)$	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ $x \in \mathbb{R}$	μ	σ^2	$N(0,1) \equiv$ standard normal. $X \sim N(\mu, \sigma^2) \Rightarrow$ $aX + b \sim N(a\mu + b, a^2\sigma^2)$ Hence $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$
Exponential	$\lambda \in (0, \infty)$	Inter-arrival times of random events	$Exp(\lambda)$	$f(x) = \lambda e^{-\lambda x}$ $x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	Alternative parametrization: $\theta = \frac{1}{\lambda}$
Gamma	$\alpha, \beta \in (0, \infty)$	Lifetimes of ageing items, multi-inter-arrival times	$Ga(\alpha, \beta)$	$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ $x > 0$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$	Alternative parametrization: $\theta = 1/\beta$, $Ga(1, \lambda) \equiv Exp(\lambda)$, $Ga(n/2, 1/2) \equiv \chi^2_n$
Chi-squared	$n \in \mathbb{N}$	Squared (normally distributed) errors, statistical tests	χ^2_n	$f(x) = \frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2}$ $x > 0$	n	$2n$	$X^2_n \equiv Ga(n/2, 1/2)$ $X_i \sim N(0,1)$ i.i.d. $\Rightarrow \sum_{i=1}^n X_i^2 \sim \chi^2_n$
Beta	$\alpha, \beta \in (0, \infty)$	Quantities constrained to be within intervals	$Be(\alpha, \beta)$	$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$ $x \in [0, 1]$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$	$Be(1,1) \equiv Unif(0,1)$
Cauchy	$a, b \in \mathbb{R}$	Heavy tailed	$Cauchy(a, b)$	$f(x) = \frac{1}{\pi b} \frac{b^2}{(x-a)^2+b^2}$ $x \in \mathbb{R}$	undefined	undefined	$Cauchy(0,1)$ is often called 'the' Cauchy distribution
Student t	$n \in \mathbb{N}$	Statistical tests	t_n	$f(x) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})} (1 + \frac{x^2}{n})^{-\frac{n+1}{2}}$ $x \in \mathbb{R}$	0 if $n > 1$	$\frac{n}{n-2}$ if $n > 2$	$t_1 \equiv Cauchy(0,1)$ Can take $n \in (0, \infty)$