MASx52: Assignment 3

1. Consider the binomial model with $r = \frac{1}{11}$, $d = 0.9$, $u = 1.2$, $s = 100$ and time steps $t = 0, 1, 2$.

   (a) Draw a recombining tree of the stock price process, for time $t = 0, 1, 2$.

   (b) Find the value, at time $t = 0$, of a European call option that gives its holder the option to purchase one unit of stock at time $t = 2$ for a strike price $K = 90$. Write down the hedging strategy that replicates the value of this contract, at all nodes of your tree.

   You may annotate your tree from (a) to answer (b).

2. Let $S_n = \sum_{i=1}^{n} X_i$, be the simple symmetric random walk, in which $(X_i)_{i\in\mathbb{N}}$ is a sequence of i.i.d. random variables with common distribution $\mathbb{P}[X_i = 1] = \mathbb{P}[X_i = -1] = \frac{1}{2}$.

   Let $\mathcal{F}_n = \sigma(S_1, \ldots, S_n)$, and note that $(\mathcal{F}_n)_{n\in\mathbb{N}}$ is a filtration. Which of the following stochastic processes are previsible? Which are adapted? (Justification is not required.)

   (a) $A_n = \mathbb{1}\{S_n \geq 0\}$

   (b) $B_n = \mathbb{E}[S_n \,|\, \mathcal{F}_{n-1}]$

   (c) $C_n = \min\{S_1, S_2, \ldots, S_{n-1}\}$

   (d) $D_n = \max\{S_1, S_2, \ldots, S_{n+1}\}$

3. Let Z be a random variable taking values in $[1, \infty)$ and for $n \in \mathbb{N}$ define

$$X_n = \begin{cases} Z & \text{if } Z \in [n, n+1) \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Show that, if $Z \in L^1$, then $\mathbb{E}[X_n] \to 0$ as $n \to \infty$.

   (b) Let Z be the continuous random variable with probability density function

$$f(x) = \begin{cases} x^{-2} & \text{if } x \geq 1, \\ 0 & \text{otherwise.} \end{cases}$$

   Show that Z is not in L^1, but that $\mathbb{E}[X_n] \to 0$.

   (c) Comment on what (a) and (b) tell us about the dominated converge theorem.