

## MASx52: Assignment 1

1. Recall the one-period market, and its parameters  $r, u, d, p_u, p_d$  and  $s$ . We assume that  $d < 1 + r < u$ .
  - (a) At time  $t = 0$  our portfolio contains 2 unit of cash and 3 units of stock. What is the value of our portfolio at time  $t = 0$ ? If we hold this portfolio until time  $t = 1$ , what is its new value?
  - (b) A rival investor holds a portfolio containing 3 units of cash and 2 unit of stock. Under what condition (on the parameters) can we be *certain* that our own portfolio will have a strictly greater value at time  $t = 1$ ?

2. Let  $\Omega = \{HH, HT, TH, TT\}$ , representing two coin tosses each of which may show either  $H$  (head) or  $T$  (tail). Let  $X : \Omega \rightarrow \mathbb{R}$  be the toss in which the first head occurred, or zero if no heads occurred:

$$X = \begin{cases} 0 & \text{if } \omega = TT \\ 1 & \text{if } \omega = HT \text{ or } \omega = HH \\ 2 & \text{if } \omega = TH. \end{cases}$$

Let  $Y$  be the total number of heads that occurred in both tosses.

- (a) Write down the sets  $X^{-1}(0)$ ,  $X^{-1}(1)$  and  $X^{-1}(2)$ .
  - (b) State the definition of  $\sigma(X)$ , and list its elements.
  - (c) Is  $Y$  measurable with respect to  $\sigma(X)$ ? Why, or why not?
3. Let  $\Omega = \{1, 2, 3, 4, 5\}$ , representing one roll of a five sided dice. Let  $\mathcal{F}$  be the set of all subsets of  $\Omega$ . Describe, in words, the information contained within the following sub- $\sigma$ -fields of  $\mathcal{F}$ .
  - (a)  $\mathcal{G}_1 = \{\emptyset, \Omega, \{1\}, \{2, 3, 4, 5\}\}$
  - (b)  $\mathcal{G}_2 = \{\emptyset, \Omega, \{1, 3, 5\}, \{2, 4\}\}$
  - (c)  $\mathcal{G}_3 = \{\emptyset, \Omega, \{1\}, \{2, 3, 4\}, \{5\}, \{1, 2, 3, 4\}, \{2, 3, 4, 5\}, \{1, 5\}\}$
4. Let  $X$  be a random variable.
  - (a) Show that  $Y = \cos X$  is also a random variable.
  - (b) For which  $p \in [1, \infty)$  do we have  $Y \in L^p$ ?