SOME DISCRETE DISTRIBUTIONS

Name	Genesis	Notation	Probability function	$E(X)$	$\mathrm{Var}(X)$	Applications	Comments
Uniform (discrete)	Set of k equally likely outcomes (usually, not necessarily, the integers)	$U(1,...,k)$ (not standard)	$p(x)=1/k$ $x=1,...,k$	$\frac{k+1}{2}$	$\frac{k^2-1}{12}$	Dice	
Bernoulli trial	Expt. with two outcomes: 'success' w.p. θ and 'failure' w.p. $1-\theta$ $X\equiv$ no. successes	$Ber(\theta)$	$p(x)=\theta^x(1-\theta)^{1-x}$ $x=0,1$ $\theta\epsilon[0,1]$	θ	$\theta(1-\theta)$	Coins, constituent of more complex distributions	
Binomial	$X\equiv$ no. successes in n ind. $Ber(\theta)$ trials	$Bi(n,\theta)$	$p(x)={}^n C_x\theta^x(1-\theta)^{n-x}$ $x=0,1,2,...,n$ $\theta\epsilon[0,1]$	$n\theta$	$n\theta(1-\theta)$	Sampling with replacement	$Bi(1,\theta)\equiv Ber(\theta)$
Geometric	$X\equiv$ no. failures until 1st success in sequence of ind. $Ber(\theta)$ trials	$\mathrm{Ge}(\theta)$	$p(x)=\theta(1-\theta)^x$ $x=0,1,2,...$ $\theta\epsilon[0,1]$	$\frac{1-\theta}{\theta}$	$\frac{1-\theta}{\theta^2}$	Waiting times (for single events)	For alternative formulation see next line.
Geometric (alternative definition)	$X\equiv$ total no. trials until 1st success in sequence of ind. $Ber(\theta)$ trials	$\mathrm{Ge}(\theta)$	$p(x)=\theta(1-\theta)^{x-1}$ $x=1,2,...$ $\theta\epsilon[0,1]$	$\frac{1}{\theta}$	$\frac{1-\theta}{\theta^2}$	Waiting times (for single events)	Includes the first success itself, so in a given situation gives a value one higher than the previous definition
Negative binomial (or Pascal)	$X\equiv$ no. failures to mth success in sequence of ind. $Ber(\theta)$ trials. Generalization of Geometric	$Neg\ Bi\ (m,\theta)$ (not standard)	$p(x)={}^{m+x-1} C_x\theta^m(1-\theta)^x$ $x=0,1,2,...$ $\theta\epsilon[0,1]$	$\frac{m(1-\theta)}{\theta}$	$\frac{m(1-\theta)}{\theta^2}$	Waiting times (for compound events)	Neg $Bi(1,\theta)\equiv Ge(\theta)$ Remains valid for any $k>0$ (not necessarily integer). Alternative formulation as for Geometric.
Hypergeometric	$X\equiv$ no. of defectives in sample of size n taken without replacement from population of size N of which N^* are defective	Hypergeom (N,N^*,n) (not standard, esp. order of arguments)	$p(x)=\frac{{}^{N^*}C_x\ {}^{N-N^*}C_{n-x}}{{}^N C_n}$ $x=0,...,n$ (strictly $x=\max(0,n+N^*-N),...,$ $...,\min(n,N^*)$)	$\frac{nN^*}{N}$	$\frac{N-n}{N-1}n\frac{N^*}{N}(1-\frac{N^*}{N})$	Sampling without replacement	Sampling with replacement leads to the $Bi(n,\frac{N^*}{N})$ - a suitable approx if $\frac{n}{N}<0.1$
Poisson	Arises empirically or via Poisson Process for counting events. For Po. Proc. rate ν the no. of events in time $t\sim Po(\nu t)$. Also as an approx. to the Binomial	$Po(\lambda)$	$p(x)=\frac{e^{-\lambda}\lambda^x}{x!}$ $x=0,1,2,...$ $\lambda>0$	λ	λ	Counting events occurring 'at random' in space or time	$Bi(n,\theta)\equiv Po(n\theta)$ if n large, θ small

SOME CONTINUOUS DISTRIBUTIONS

Name	**Notation**	**p.d.f.**	$E(X)$	$\mathrm{Var}(X)$	**Applications**	**Comments**
Uniform (continuous) (or Rectangular)	$Un(\alpha, \beta)$	$f(x) = \frac{1}{\beta - \alpha}$ $x \in [\alpha, \beta]$ $\alpha < \beta$	$\frac{\alpha + \beta}{2}$	$\frac{(\beta - \alpha)^2}{12}$	Rounding errors $Un(-\frac{1}{2}, \frac{1}{2})$. Simulating other distributions from $Un(0, 1)$.	
Exponential	$Ex(\lambda)$	$f(x) = \lambda e^{-\lambda x}$ $x > 0$ $\lambda > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	Inter-event times for Poisson Process. Models lifetimes of non-ageing items.	Alternative parameterization in terms of $1/\lambda$ $Ga(1, \lambda) \equiv Ex(\lambda)$
Gamma	$Ga(\alpha, \beta)$	$f(x) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}$ $x \geq 0$ $\alpha, \beta > 0$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$	Times between k events for Poisson Process. Lifetimes of ageing items.	Alternative parameterization in terms of $1/\beta$ $Ga(1, \lambda) \equiv Ex(\lambda)$, $Ga(\nu/2, 1/2) \equiv X_\nu^2$,
Beta	$Be(\alpha, \beta)$	$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)}$ $x \in [0, 1]$ $\alpha, \beta > 0$	$\frac{\alpha}{\alpha + \beta}$	$\frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$	Useful model for variables with finite range. Also as a Bayesian conjugate prior.	$Be(1, 1) \equiv Un(0, 1)$ $Be(\alpha, \beta)$ is reflection about $\frac{1}{2}$ of $Be(\beta, \alpha)$. Can transform $Be(\alpha, \beta)$ on $[0, 1]$ to any finite range $[a, b]$ by $Y = (b - a)X + a$
Normal	$N(\mu, \sigma^2)$	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - \mu}{\sigma}\right)^2\right]$ $x \in (-\infty, \infty)$	μ	σ^2	Empirically and theoretically (via CLT etc.) a good model in many situations. Often easy to handle mathematically.	$X \sim N(\mu, \sigma^2) \Longrightarrow aX + b \sim N(a\mu + b, a^2\sigma^2)$ $\Longrightarrow Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$ So $P[X \in (u, v)] = P[Z \in \left(\frac{u-\mu}{\sigma}, \frac{v-\mu}{\sigma}\right)]$ $N(0, 1)$ special case has p.d.f. denoted ϕ, c.d.f. Φ (tabulated). Note $\Phi(-z) = 1 - \Phi(z)$.
Chi-square	χ_ν^2	$f(x) = 2^{-\nu/2}\Gamma(\nu/2)^{-1}x^{\nu/2-1}e^{-x/2}$ $x > 0$ $\nu > 0$	ν	2ν	Sum of squares of ν standard normals	$X_\nu^2 \equiv Ga(\nu/2, 1/2)$ If $X_1, X_2, \ldots, X_n \sim N(0, 1)$ independent, then $\sum_{i=1}^n X_i^2 \sim X_n^2$
Student t	t_ν	$f(x) =$ $\nu^{-1/2}B\left(\frac{1}{2}, \frac{\nu}{2}\right)^{-1}\left(1 + x^2/\nu\right)^{-(\nu+1)/2}$ $x \in (-\infty, \infty)$ $\nu > 0$	0 (if $\nu > 1$)	$\frac{\nu}{\nu - 2}$ (if $\nu > 2$)	Useful alternative to Normal for variables with heavy tails.	If $X \sim N(0, 1)$ and $Y \sim \chi_\nu^2$ independent then $\frac{X}{\sqrt{Y/\nu}} \sim t_\nu$. $t_1 \equiv$ Cauchy. $t_\nu^2 \equiv F_{1,\nu}$.
F	$F_{\nu,\delta}$	$f(x) = \frac{\nu^{\nu/2}\delta^{\delta/2}x^{\nu/2-1}}{B(\nu/2, \delta/2)(\nu x + \delta)^{(\nu+\delta)/2}}$ $x > 0$ $\nu, \delta > 0$	$\frac{\delta}{\delta - 2}$ (if $\delta > 2$)	$\frac{2\delta^2(\nu+\delta-2)}{\nu(\delta-2)^2(\delta-4)}$ (if $\delta > 4$)	Scaled ratio of chi-squares. Used in tests to compare variances	If $X \sim \chi_\nu^2$ and $Y \sim \chi_\delta^2$ independent then $\frac{X/\nu}{Y/\delta} \sim F_{\nu,\delta}$. If $T \sim t_\nu$ then $T^2 \sim F_{1,\nu}$. If $Z \sim Be(\alpha, \beta)$ then $\frac{\beta Z}{\alpha(1-Z)} \sim F_{2\alpha,2\beta}$.