MPS350/4111: Assignment 2

★ 1. (a) A random quantity θ is known to take real values, believed to be moderately close to zero. Arrange the following four candidate prior distributions for θ in order, from the most informative (first) to the least informative (last).

$$\mathrm{N}(0,5), \qquad \mathrm{N}(0,1), \qquad \mathrm{Cauchy}(0,1), \qquad \mathrm{Uniform}\left([-\tfrac{1}{4},\tfrac{1}{4}]\right)$$

(b) Which, if any, of the distributions in part (a) would usually be viewed as weakly informative?

★★ 2. (a) The following statements and formulae are written in Bayesian shorthand. Write a version in precise mathematical notation, for continuous random variables.

i. $f(x,y) = f(x|y)f(y)$.

ii. If $(u,t) \sim \mathrm{NGamma}(m,p,a,b)$ then $u|t \sim \mathrm{N}(m,\frac{1}{pt})$.

(b) On the reference sheet you will find a conjugate pair involving both the Poisson and Gamma distributions. State this relationship using Bayesian shorthand.

★★ 3. (a) i. Show that the reference prior for the exponential distribution is given by

$$f(\lambda) \propto \begin{cases} \frac{1}{\lambda} & \text{for } \lambda > 0 \\ 0 & \text{otherwise.} \end{cases}$$

ii. Find the reference prior for the $\mathrm{N}(0,v)$ distribution.

iii. In each of i and ii, is the result a proper or improper prior density function?

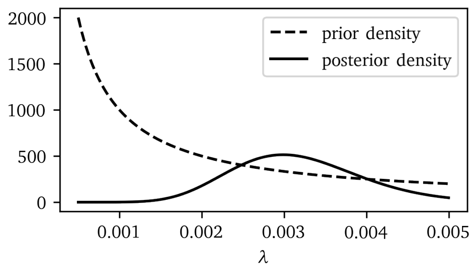
(b) Electronic devices are, in general, most likely to fail soon after they are manufactured or after their warranty period has expired. In between these times, their lifetime is often modelled using the Exponential distribution. A manufacturer of industrial high-temperature thermometers tracks a sample of twenty devices. The duration before the devices fail are measured and recorded in days as

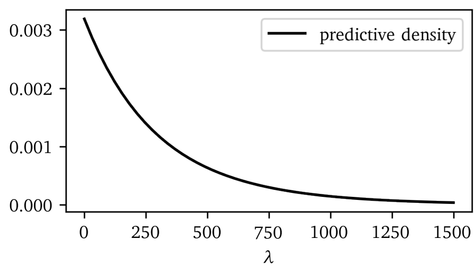
$$(3,5,9,11,23,58,89,113,124,190,221,262,279,280,281,396,565,587,647,1062).$$

It is decided to to exclude devices that failed during their first 10 days from the analysis. With these removed, the resulting data $y_i = x_i - 10$ satisfies $\sum y_i = 5018$.

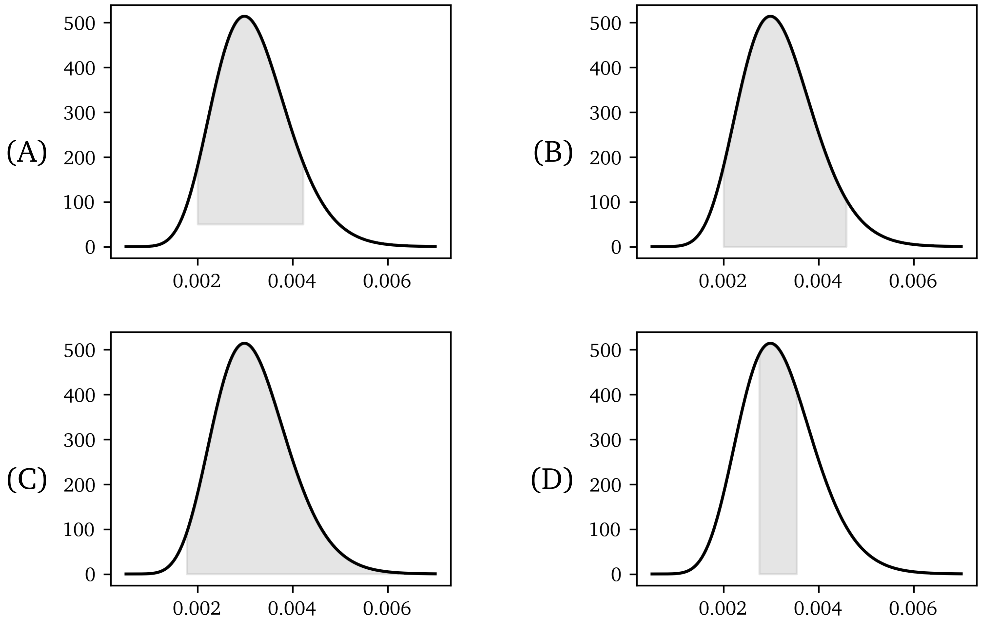
i. Using the reference prior from part (a) and the data y_i, find the posterior density of λ. Is this a proper or improper density function?

ii. A colleague produces the following plots of some of the density functions associated to the Bayesian update in part i. One of these plots contains an error. What is the error and what should be done to correct it?





iii. It is required to give a credible interval for λ. Which one of the following plots best represents the results in this way? For each of the other plots, give a reason why you did not choose it.



iv. A draft report of the analysis includes the sentence:

> *The lifetime of a thermometer was modelled by* $\mathrm{Exp}(\lambda)$*, based on a reference prior and the available data.*

Is this a fair description of the analysis? If not, suggest a correction.