HW 6 Solutions

Problem 1

Part a

Check the Grashof Criterion:

$$\begin{aligned}1.5 + 2.75 &< 2 + 2.5\\ 4.25 &< 4.5\end{aligned}$$

This is a Grashof linkage. Since $r_3 = r_s$, then it is classified as a double-rocker (rocker-rocker.)

Part b

$$\begin{aligned}
\Delta\theta_2 =& |\theta_2' - \theta_2''|\\
\theta_2' =& \arccos\left(\frac{r_1^2 + r_2^2 - (r_3 + r_4)^2}{2r_1r_2}\right)\\
=& \arccos\left(\frac{2^2 + 2.5^2 - (1.5 + 2.75)^2}{2 \times 2 \times 2.5}\right)\\
=& 141.4^\circ\\
\theta_2'' =& \arccos\left(\frac{r_1^2 + r_2^2 - (r_3 - r_4)^2}{2r_1r_2}\right)\\
=& \arccos\left(\frac{2^2 + 2.5^2 - (1.5 - 2.75)^2}{2 \times 2 \times 2.5}\right)\\
=& 29.7^\circ\\
\Delta\theta_2 =& |141.4 - 29.7| = 111.7^\circ
\end{aligned}$$

Part c

$$
\begin{aligned}
\Delta\theta_4 =& |\theta_4' - \theta_4''| \\
\theta_4' =& \arccos\left(\frac{r_1^2 + r_4^2 - (r_3 + r_2)^2}{2r_1r_4}\right) \\
=& \arccos\left(\frac{2^2 + 2.75^2 - (1.5 + 2.5)^2}{2 \times 2 \times 2.75}\right) \\
=& 113.8^\circ \\
\theta_4'' =& \arccos\left(\frac{r_1^2 + r_4^2 - (r_3 - r_2)^2}{2r_1r_4}\right) \\
=& \arccos\left(\frac{2^2 + 2.75^2 - (1.5 - 2.5)^2}{2 \times 2 \times 2.75}\right) \\
=& 16.2^\circ \\
\Delta\theta_2 =& |113.8 - 16.2| = 97.6^\circ
\end{aligned}
$$

Problem 2

$$\theta_4 - \theta_3 = 105^\circ - 15^\circ = 90^\circ$$

Problem 3

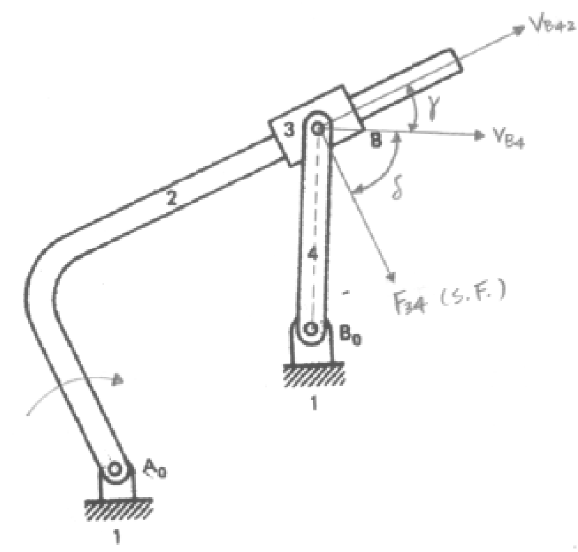

Image 1: Mechanism a, link 2 input

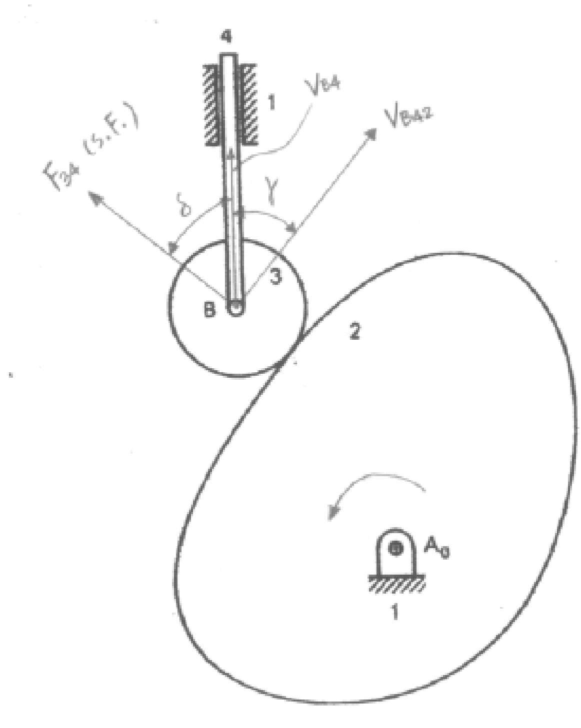

Image 2: Mechanism b, link 2 input

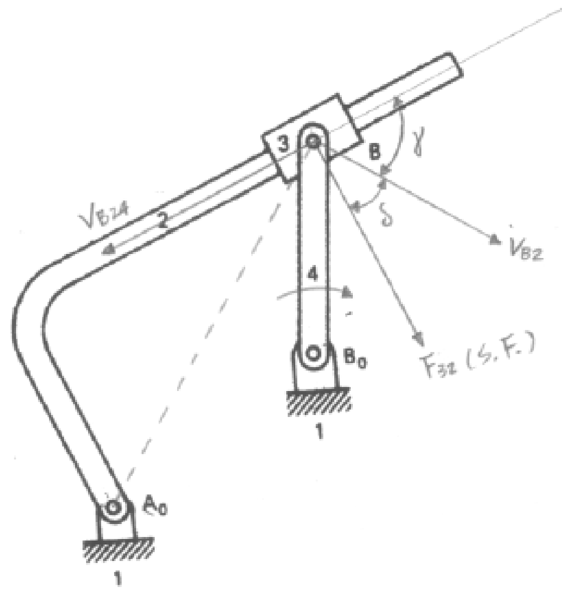

Image 3: Mechanism a, link 4 input

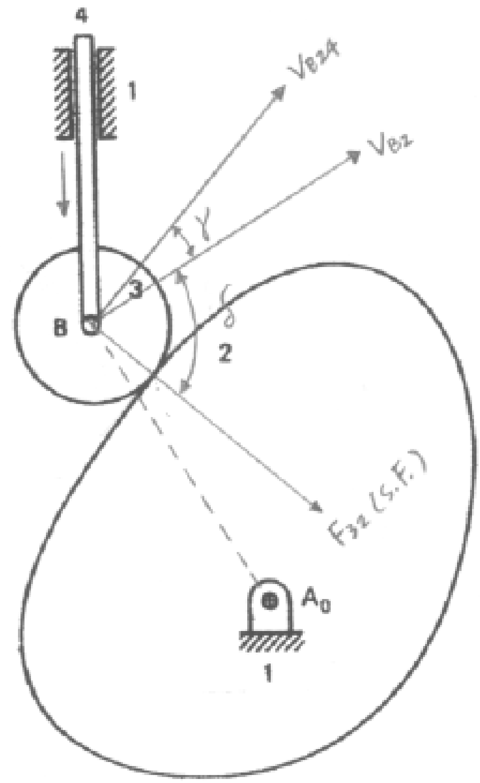

Image 4: Mechanism b, link 4 input

Problem 4

Part a

There are 2 loops in this mechanism. Derive the loop closure equations where $r_4' = \overline{B_0C}$:

$$\begin{aligned} r_1 + r_4 &= r_2 + r_3 \\ r_1 + r_4' + r_5 &= r_6 \end{aligned}$$

Put vectors with unknowns on the left hand side:

$$\begin{aligned} r_3 - r_4 &= r_1 - r_2 \\ r_6 - r_5 &= r_1 + r_4' \end{aligned}$$

Then set $Z_1 = r_1 - r_2$ and $Z_2 = r_1 + r_4'$ and rewrite the equations in polar form. Recall that $\theta_4' = \theta_4$:

$$\begin{aligned} r_3 e^{i\theta_3} - r_4 e^{i\theta_4} =& Z_1 = X_1 + iY_1 \quad (1) \\ r_6 e^{i\theta_6} - r_5 e^{i\theta_5} =& Z_2 = X_2 + iY_2 \quad (2) \end{aligned}$$

Split equations (1) and (2) into their real and imaginary components.

$$\begin{aligned} r_3 cos(\theta_3) - r_4 cos(\theta_4) =& X_1 \quad (3) \\ r_3 sin(\theta_3) - r_4 sin(\theta_4) =& Y_1 \quad (4) \\ r_6 cos(\theta_6) - r_5 cos(\theta_5) =& X_2 \quad (5) \\ r_6 sin(\theta_6) - r_5 sin(\theta_5) =& Y_2 \quad (6) \end{aligned}$$

Calculate X_1 and Y_1 first.

$$\begin{aligned} X_1 =& r_1 cos(\theta_1) - r_2 cos(\theta_2) \\ =& 6cos(-30°) - cos(45°) \\ =& 4.49 \\ Y_1 =& r_1 sin(\theta_1) - r_2 sin(\theta_2) \\ =& 6sin(-30°) - sin(45°) \\ =& -3.71 \end{aligned}$$

Plug in variables and solve equations (3) and (4) first.

$$\begin{aligned} 3cos(\theta_3) - 5cos(\theta_4) =& 4.49 \\ 3sin(\theta_3) - 5sin(\theta_4) =& -3.71 \\ \Rightarrow \theta_3 =& 0.342 = 19.6° \quad (7) \\ \Rightarrow \theta_4 =& 1.910 = 109.4° \end{aligned}$$

Now you can calculate X_2 and Y_2:

$$\begin{aligned} X_2 =& r_1 cos(\theta_1) + r_4' cos(\theta_4) \\ =& 6cos(-30°) + 4cos(109.4°) \\ =& 3.87 \\ Y_2 =& r_1 sin(\theta_1) + r_4' sin(\theta_4) \\ =& 6sin(-30°) + 4sin(109.4°) \\ =& 0.77 \end{aligned}$$

Now solve equations (5) and (6):

$$\begin{aligned} r_6 cos(0) - 4cos(\theta_5) =& 3.87 \\ r_6 sin(0) - 4sin(\theta_5) =& 0.77 \\ \Rightarrow r_6 =& 7.8 \\ \Rightarrow \theta_5 =& -0.194 = -11.1^\circ \end{aligned}$$

To solve for the location of the coupler point P, set up the vector equation:

$$P = r_2 + r_p$$

Now rewrite this in polar form knowing that $\theta_p = \beta + \theta_3$:

$$P = r_2 e^{i\theta_2} + r_p e^{i(\beta+\theta_3)}$$

Then, decompose this equation into real and imaginary components:

$$\begin{aligned} P_x =& r_2 cos(\theta_2) + r_p cos(\beta + \theta_3) \\ P_y =& r_2 sin(\theta_2) + r_p sin(\beta + \theta_3) \end{aligned}$$

Now solve by plugging in known values and using θ_3 from the solution of equation (7):

$$\begin{aligned} P_x =& cos(45^\circ) + 2.5cos(20^\circ + 19.6^\circ) \\ =& 2.63 \\ P_y =& sin(45^\circ) + 2.5sin(20^\circ + 19.6^\circ) \\ =& 2.30 \end{aligned}$$

Part b

```
At 45 degrees,
branches = 2
Circuit 1:
theta3 = 19.634638
theta4 = 109.432520
theta5 = -11.129897
r6 = 7.790135
P = complex(2.632426,2.301831)

Circuit 2:
theta3 = -98.735229
theta4 = 171.466889
theta5 = 36.985943
r6 = 4.435564
P = complex(1.195465,-1.744731)

Circuit 3:
theta3 = 19.634638
```

```
theta4 = 109.432520
theta5 = -168.870103
r6 = -0.059402
P = complex(2.632426,2.301831)

Circuit 4:
theta3 = -98.735229
theta4 = 171.466889
theta5 = 143.014057
r6 = -1.954701
P = complex(1.195465,-1.744731)
```

Part c

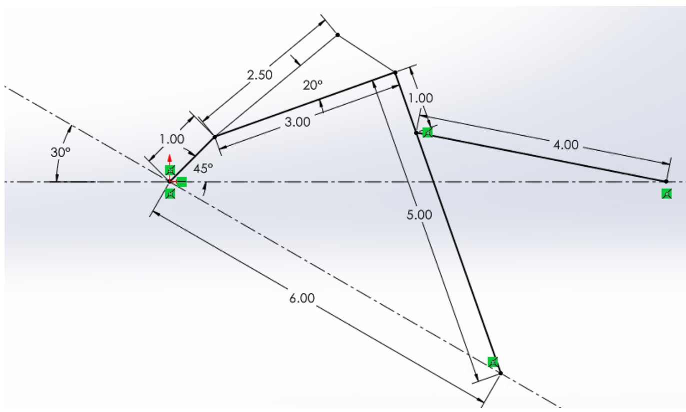

Image 5: Drawing to scale

Part d

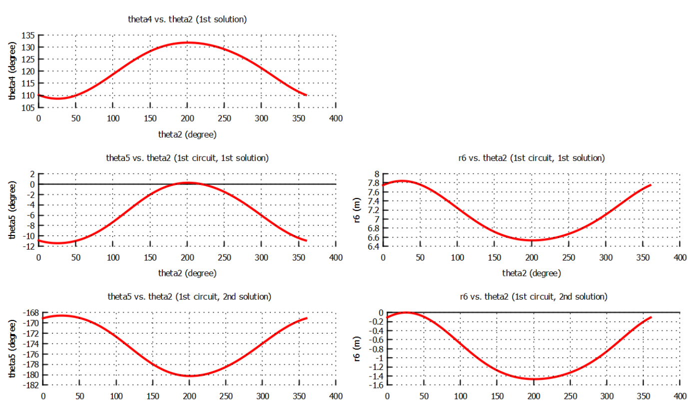

Image 6: First set of solutions

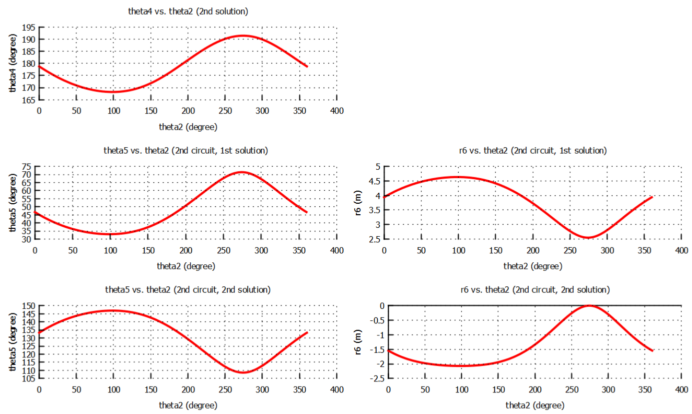

Image 7: Second set of solutions

Code

```
/* h5p5.ch */
#include <math.h>
#include <complex.h>
#include <array.h>
#include <chplot.h>

#define NUMPOINTS 361
#define DEG2RAD(a) ((M_PI/180)*a)
#define RAD2DEG(a) ((180/M_PI)*a)

/* Consult Figure P3.50 in the Mechanism Design textbook. */

int main() {
    int n1, n2;
    double r[1:7], theta1, rp, beta, theta6;

    int i;
    double theta2[NUMPOINTS],
           theta3_1[NUMPOINTS], theta3_2[NUMPOINTS],
           x1, x2, x3, x4;
    array double theta4_1[NUMPOINTS], theta4_2[NUMPOINTS],
          theta5_1_1[NUMPOINTS], theta5_1_2[NUMPOINTS],
          theta5_2_1[NUMPOINTS], theta5_2_2[NUMPOINTS];
    double complex z,
                   P_1[NUMPOINTS], P_2[NUMPOINTS],
                   B_1[NUMPOINTS], B_2[NUMPOINTS];

    array double r6_1_1[NUMPOINTS], r6_1_2[NUMPOINTS],
          r6_2_1[NUMPOINTS], r6_2_2[NUMPOINTS];
    class CPlot mainplot1;
    class CPlot mainplot2;
    class CPlot* subplot;
    int branches[NUMPOINTS];

    //-------------------------Part A
    ---------------------------------
```

```
    //Determine theta4, point B on the output link, and coupler point P
    //-----------------------------------------------------------------

    // Define the fourbar linkage
    n1 = 2;
    n2 = 4;
    r[1] = 6;
    r[2] = 1;
    r[3] = 3;
    r[4] = 5;
    r[5] = 4;
    r[7] = r[4] - 1; /* Length of B0 - C */
    theta1 = DEG2RAD(-30);
    rp = 2.5;
    beta = DEG2RAD(20);
    theta6 = 0;

    // Analyze the fourbar with theta2 rotating from 0-360
    for(i=0; i<=360; i++) {
        theta2[i] = DEG2RAD(i);

        // Determine theta3 and theta4
        z = polar(r[1], theta1) - polar(r[2], theta2[i]);
        branches[i] = complexsolve(n1, n2, r[3], -r[4], z, x1, x2, x3, x4);

        // First set of solutions
        theta3_1[i] = x1;
        theta4_1[i] = x2;
        P_1[i] = polar(r[2], theta2[i]) + polar(rp, theta3_1[i]+beta);
        B_1[i] = polar(r[2], theta2[i]) + polar(r[3], theta3_1[i]);

        // Second set of solutions
        theta3_2[i] = x3;
        theta4_2[i] = x4;
        P_2[i] = polar(r[2], theta2[i]) + polar(rp, theta3_2[i]+beta);
        B_2[i] = polar(r[2], theta2[i]) + polar(r[3], theta3_2[i]);

        // Find solutions for r5 and r6 : Circuit 1
        complexsolve(1, 4, theta6, -r[5],
            polar(r[1], theta1) + polar(r[7], theta4_1[i]),
            x1, x2, x3, x4 );
        r6_1_1[i] = x1;
        theta5_1_1[i] = x2;
        r6_1_2[i] = x3;
        theta5_1_2[i] = x4;

        // Find solutions for r5 and r6 : Circuit 2
        complexsolve(1, 4, theta6, -r[5],
            polar(r[1], theta1) + polar(r[7], theta4_2[i]),
            x1, x2, x3, x4 );
        r6_2_1[i] = x1;
```

```
        theta5_2_1[i] = x2;
        r6_2_2[i] = x3;
        theta5_2_2[i] = x4;
    }
    // Smooth theta4_1, theta4_2
    unwrap(theta4_1, theta4_1);
    unwrap(theta4_2, theta4_2);
    unwrap(theta5_1_2, theta5_1_2);

    // Convert required angles from radian to degree
    for(i=0; i<=360; i++) {
        theta2[i] = RAD2DEG(theta2[i]);
        theta3_1[i] = RAD2DEG(theta3_1[i]);
        theta3_2[i] = RAD2DEG(theta3_2[i]);
        theta4_1[i] = RAD2DEG(theta4_1[i]);
        theta4_2[i] = RAD2DEG(theta4_2[i]);
        theta5_1_1[i] = RAD2DEG(theta5_1_1[i]);
        theta5_1_2[i] = RAD2DEG(theta5_1_2[i]);
        theta5_2_1[i] = RAD2DEG(theta5_2_1[i]);
        theta5_2_2[i] = RAD2DEG(theta5_2_2[i]);
    }

    printf("At 45 degrees, \n");
    printf("branches = %d\n", branches[45]);
    printf("Circuit 1:\n");
    printf("theta3 = %lf\n", theta3_1[45]);
    printf("theta4 = %lf\n", theta4_1[45]);
    printf("theta5 = %lf\n", theta5_1_1[45]);
    printf("r6 = %lf\n", r6_1_1[45]);
    printf("P = %lf\n", P_1[45]);
    printf("\nCircuit 2:\n");
    printf("theta3 = %lf\n", theta3_2[45]);
    printf("theta4 = %lf\n", theta4_2[45]);
    printf("theta5 = %lf\n", theta5_2_1[45]);
    printf("r6 = %lf\n", r6_2_1[45]);
    printf("P = %lf\n", P_2[45]);
    printf("\nCircuit 3:\n");
    printf("theta3 = %lf\n", theta3_1[45]);
    printf("theta4 = %lf\n", theta4_1[45]);
    printf("theta5 = %lf\n", theta5_1_2[45]);
    printf("r6 = %lf\n", r6_1_2[45]);
    printf("P = %lf\n", P_1[45]);
    printf("\nCircuit 4:\n");
    printf("theta3 = %lf\n", theta3_2[45]);
    printf("theta4 = %lf\n", theta4_2[45]);
    printf("theta5 = %lf\n", theta5_2_2[45]);
    printf("r6 = %lf\n", r6_2_2[45]);
    printf("P = %lf\n", P_2[45]);

    /* Display Plots */
    mainplot1.subplot(3,2);
    subplot=mainplot1.getSubplot(0,0);
    subplot->data2D(theta2, theta4_1);
    subplot->title("theta4 vs. theta2 (1st solution)");
    subplot->label(PLOT_AXIS_X,"theta2 (degree)");
    subplot->label(PLOT_AXIS_Y,"theta4 (degree)");

```

```
    subplot=mainplot1.getSubplot(1,0);
    subplot->data2D(theta2, theta5_1_1);
    subplot->title("theta5 vs. theta2 (1st circuit, 1st solution)")
    ;
    subplot->label(PLOT_AXIS_X,"theta2 (degree)");
    subplot->label(PLOT_AXIS_Y,"theta5 (degree)");

    subplot=mainplot1.getSubplot(1,1);
    subplot->data2D(theta2, r6_1_1);
    subplot->title("r6 vs. theta2 (1st circuit, 1st solution)");
    subplot->label(PLOT_AXIS_X,"theta2 (degree)");
    subplot->label(PLOT_AXIS_Y,"r6 (m)");

    subplot=mainplot1.getSubplot(2,0);
    subplot->data2D(theta2, theta5_1_2);
    subplot->title("theta5 vs. theta2 (1st circuit, 2nd solution)")
    ;
    subplot->label(PLOT_AXIS_X,"theta2 (degree)");
    subplot->label(PLOT_AXIS_Y,"theta5 (degree)");

    subplot=mainplot1.getSubplot(2,1);
    subplot->data2D(theta2, r6_1_2);
    subplot->title("r6 vs. theta2 (1st circuit, 2nd solution)");
    subplot->label(PLOT_AXIS_X,"theta2 (degree)");
    subplot->label(PLOT_AXIS_Y,"r6 (m)");

    mainplot1.plotting();

    /* Display Plots */
    mainplot2.subplot(3,2);
    subplot=mainplot2.getSubplot(0,0);
    subplot->data2D(theta2, theta4_2);
    subplot->title("theta4 vs. theta2 (2nd solution)");
    subplot->label(PLOT_AXIS_X,"theta2 (degree)");
    subplot->label(PLOT_AXIS_Y,"theta4 (degree)");

    subplot=mainplot2.getSubplot(1,0);
    subplot->data2D(theta2, theta5_2_1);
    subplot->title("theta5 vs. theta2 (2nd circuit, 1st solution)")
    ;
    subplot->label(PLOT_AXIS_X,"theta2 (degree)");
    subplot->label(PLOT_AXIS_Y,"theta5 (degree)");

    subplot=mainplot2.getSubplot(1,1);
    subplot->data2D(theta2, r6_2_1);
    subplot->title("r6 vs. theta2 (2nd circuit, 1st solution)");
    subplot->label(PLOT_AXIS_X,"theta2 (degree)");
    subplot->label(PLOT_AXIS_Y,"r6 (m)");

    subplot=mainplot2.getSubplot(2,0);
    subplot->data2D(theta2, theta5_2_2);
    subplot->title("theta5 vs. theta2 (2nd circuit, 2nd solution)")
    ;
    subplot->label(PLOT_AXIS_X,"theta2 (degree)");
    subplot->label(PLOT_AXIS_Y,"theta5 (degree)");

    subplot=mainplot2.getSubplot(2,1);
```

```
    subplot->data2D(theta2, r6_2_2);
    subplot->title("r6 vs. theta2 (2nd circuit, 2nd solution)");
    subplot->label(PLOT_AXIS_X,"theta2 (degree)");
    subplot->label(PLOT_AXIS_Y,"r6 (m)");

    mainplot2.plotting();
    return 0;
}
```