Journal of Finance forthcoming

A Multifactor Perspective on Volatility-Managed Portfolios*

Victor DeMiguel   Alberto Martín-Utrera   Raman Uppal

May 1, 2024

Abstract

Moreira and Muir (2017) question the existence of a strong risk-return tradeoff by showing that investors can improve performance by reducing exposure to risk factors when their volatility is high. However, Cederburg, O'Doherty, Wang, and Yan (2020) show these strategies fail out-of-sample, and Barroso and Detzel (2021) show they do not survive transaction costs. We propose a conditional multifactor portfolio that outperforms its unconditional counterpart even out-of-sample and net-of-costs. Moreover, we show that factor risk prices generally decrease with market volatility. Our results demonstrate that the breakdown of the risk-return tradeoff is more puzzling than previously thought.

Keywords: Factor timing, transaction costs, estimation error, factor risk-return tradeoff, factor price of risk.
JEL Classification: G01, G11.

*DeMiguel, London Business School, e-mail: avmiguel@london.edu; Martín-Utrera, Iowa State University, e-mail: amutrera@iastate.edu; Uppal, EDHEC Business School, EDHEC Scientific Beta Research Chair, and CEPR, e-mail: raman.uppal@edhec.edu. We are grateful to Wayne Chang for sharing the code to construct the Fama-French factors and to Kewei Hou, Chen Xue, and Lu Zhang for help in constructing the ROE and IA factors. We would also like to acknowledge helpful comments from Pedro Barroso, Oliver Boguth, Svetlana Bryzgalova, Scott Cederburg, Joao Cocco, Andrew Detzel, Roberto Gómez-Cram, Björn Hagströmer, Hugues Langlois, Nathan Lassance, Alan Moreira, Michael O'Doherty, Anna Pavlova, Christoph Reschenhofer, Iñaki Rodriguez Longarela, Cisil Sarisoy, Nikolaos Tessaromatis, Greg Vilkov, Ran Xing, Paolo Zaffaroni, Abalfazl Zareei, Josef Zechner, Stavros Zenios, and seminar participants at the 3rd Frontiers of Factor Investing Conference, 5th International Workshop on Financial Markets and Nonlinear Dynamics, 6th Annual Global Quantitative and Macro Investment Conference, 14th Annual Imperial College London Hedge Fund Conference, 14th Financial Risks International Forum, American Finance Association Annual Meeting, European Finance Association Annual Meeting, Fidelity Investments, Financial Intermediation Research Society (FIRS) Conference, FMA Annual Meeting, Iowa State University, Lancaster University Management School, London Business School, New Jersey Institute of Technology, Northern Finance Association Annual Meeting, SKEMA Business School, Stockholm Business School, University of Bath, University of Bristol, University of Cyprus, University of Edinburgh, University of Gothenburg, University of Hong Kong, and University of Missouri.

1 Introduction

A fundamental premise in finance is that there is a strong risk-return tradeoff. Moreira and Muir (2017) challenge this by showing that investors can increase Sharpe ratios by reducing exposure to risk factors when their volatility is high. The intuition underlying their findings is that, in the absence of a strong risk-return tradeoff for factor returns, factor exposure can be scaled back during times of high volatility without a proportional reduction in returns. Their work is a challenge to structural models of time-varying expected returns, which typically predict that the market risk-return tradeoff strengthens during periods of high volatility. However, Cederburg et al. (2020) show that the performance gains from volatility management are not achievable out of sample because of estimation error, and Barroso and Detzel (2021) show that transaction costs erode these gains.

While the above papers focus on volatility-managed *individual-factor* portfolios, we provide a *multifactor* perspective by proposing a novel conditional mean-variance multifactor portfolio whose weights on each factor decrease with market volatility. We show that this strategy outperforms the unconditional multifactor portfolio even out of sample and net of costs. Our findings show that estimation error and transaction costs do not explain the gains from volatility management, and hence, the breakdown of the risk-return tradeoff is more puzzling than previously thought.

The key distinguishing feature of our approach to volatility management is that we focus on multifactor portfolios. While it is informative to study the effect of volatility management for each individual factor, the stochastic discount factor that prices all assets is determined by the conditional mean-variance *multifactor* portfolio. Thus, to evaluate the asset-pricing implications of volatility management, one needs to test whether the volatility-managed multifactor portfolio outperforms its unconditional counterpart. Moreover, as explained by Chernov, Lochstoer, and Lundeby (2022), this test provides information about the joint dynamics of factor returns. In particular, if our conditional multifactor portfolio, which reduces its weights on the factors when market volatility increases, outperforms its unconditional counterpart, then we must have the counterintuitive result that the relation between the conditional mean vector and covariance matrix of factor returns weakens with market volatility.[1]

Our approach to volatility management differs in three other ways from that in the existing literature. First, our conditional multifactor portfolios allow the relative weights on the different factors to *vary* with market volatility. In contrast, Moreira and Muir (2017, section I.E) consider a conditional *fixed-weight multifactor portfolio* whose relative weight on each factor does not vary with volatility and Barroso and Detzel (2021, ftn. 12) consider a portfolio that assigns an equal relative weight to each factor. Second, we evaluate conditional multifactor portfolios that are optimized accounting for transaction costs. Third, we account for the reduction in transaction costs associated with the netting of trades across the different factors combined in the multifactor portfolio, an effect termed *trading diversification* by DeMiguel, Martin-Utrera, Nogales, and Uppal (2020).[2]

Our conditional mean-variance multifactor portfolio achieves an out-of-sample and net-of-costs Sharpe ratio 13% higher than that of the unconditional mean-variance multifactor portfolio, with the difference being statistically significant at the 1% level.[3] We identify three main drivers of the favorable performance of our conditional multifactor portfolio. The first is trading diversification. In particular, although both the unconditional and conditional multifactor portfolios benefit from the netting of trades across multiple factors, the benefits are larger for the conditional portfolio because the transaction costs of the volatility-managed factors are much larger than those of the unmanaged factors. For instance, ignoring trading diversification, we find that while the net-of-costs mean return of all nine unmanaged factors is positive, that of four of the nine managed factors is negative. However, accounting for trading diversification, the net mean return of all nine managed factors becomes positive.

[1]To see this, note that the conditional mean-variance multifactor portfolio of a single-period investor with constant relative risk-aversion γ is $w_t = \Sigma_t^{-1}\mu_t/\gamma$, where μ_t and Σ_t are the conditional mean vector and covariance matrix of factor excess returns. Thus, the conditional mean vector and covariance matrix satisfy the relation $\mu_t = \gamma\Sigma_t w_t$. Therefore, if the conditional multifactor portfolio weights w_t decrease with market volatility, then the relation between the conditional mean and covariance matrix weakens with market volatility.

[2]Other papers that have also documented that combining characteristics reduces transaction costs include Barroso and Santa-Clara (2015a), Frazzini, Israel, and Moskowitz (2015), and Novy-Marx and Velikov (2016).

[3]Barroso and Detzel (2021) show that the volatility-managed *market* portfolio outperforms the market factor during high-sentiment periods, but it underperforms the market during low-sentiment periods. Section IA.8 of the Internet Appendix shows, however, that the out-of-sample performance of the conditional multifactor portfolio is significantly better than that of the unconditional portfolio during *both* high- and low-sentiment periods. We conclude that sentiment does not explain the out-of-sample and net-of-costs performance of our proposed multifactor strategy.

The second driver of the performance of our conditional multifactor portfolio is that it is optimized accounting for transaction costs, which significantly improves its performance relative to the unconditional multifactor portfolio. Again, even though the performance of both the conditional and unconditional portfolios improves when they are optimized accounting for transaction costs, the benefits are larger for the conditional portfolios because the transaction costs of trading the managed factors are larger.

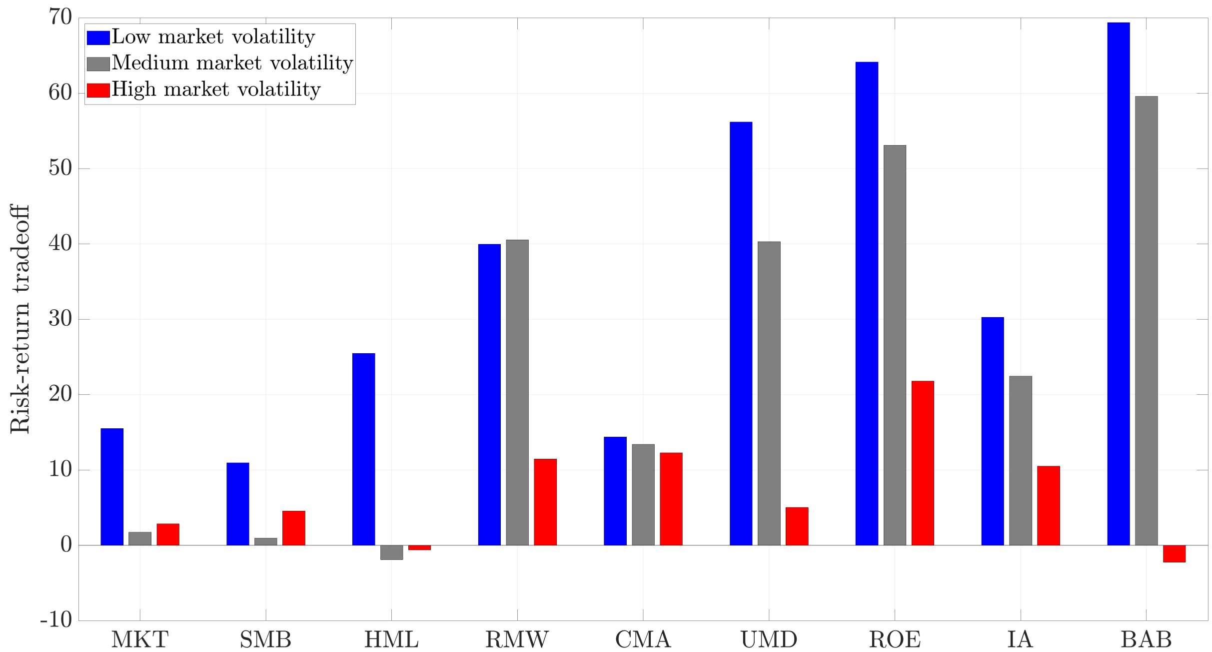
The third driver of the performance of our conditional portfolios is that they allow the *relative* weight on each factor to vary with market volatility. Indeed, our conditional multifactor portfolio optimally times some of the factors aggressively while assigning an almost-constant weight to others. As a result, the *average* exposure of our conditional portfolio to the various factors can differ substantially from that of the unconditional and fixed-weight portfolios. For instance, our conditional portfolio has a larger average exposure to the value, momentum, and betting-against-beta factors compared with the unconditional and fixed-weight portfolios but a smaller average exposure to the investment factors.

To explain the economic mechanism underlying the performance of the conditional multifactor portfolios, we illustrate in Figure 1 how the risk-return tradeoff for the nine factors varies with market volatility. In the figure, we use the monthly time series of realized market volatility to sort the months in our sample into volatility terciles and report the risk-return tradeoff for each factor averaged across the months in each tercile. Our key finding is that for all nine individual factors the risk-return tradeoff *weakens* with market volatility. This explains why our conditional multifactor portfolio, which reduces exposure to the risk factors when realized market volatility is high, outperforms its unconditional counterpart. Moreover, the weakening of the risk-return tradeoff is substantial for some of the factors (UMD, ROE, and BAB) but less striking for others (SMB and CMA). This motivates our choice to consider a conditional multifactor portfolio that allows the relative weights on the different factors to vary with market volatility.

To understand the asset-pricing implications of our findings, we also estimate a conditional stochastic discount factor whose price of risk for each of the nine factors is an affine function of inverse realized market volatility. Consistent with the results in Figure 1, we find that the price of risk for individual factors generally *decreases* with realized market volatility. This is a counterintuitive result because one expects the price of risk for systematic factors

Figure 1: Factor risk-return tradeoff and market volatility

This barplot illustrates how the risk-return tradeoff for the nine factors in our dataset varies with realized market volatility. We first use the monthly time series of realized market volatility to sort the months in our sample into terciles. For each factor, we then estimate the risk-return tradeoff for month t as the realized factor return for month $t+1$ divided by the monthly realized factor variance estimated as the sample variance of daily returns for month t. Finally, we report the risk-return tradeoff averaged across the months in each tercile. Blue bars correspond to the tercile containing low-market-volatility months, gray bars to medium-market-volatility months, and red bars to high-market-volatility months. The sample spans January 1977 to December 2020.



to remain constant or increase with market volatility. We also observe that the reduction in the price of risk with market volatility is more significant for some factors than others. Thus, our analysis shows that conditioning on volatility helps to construct a stochastic discount factor that better spans the investment opportunity set, but the importance of volatility management varies across factors.

Our work is related to the literature on factor timing. Early contributions include Fleming, Kirby, and Ostdiek (2001, 2003), who evaluate the gains from volatility timing across multiple asset classes, and Marquering and Verbeek (2004), who study volatility and return timing of a market index. More recently, Ehsani and Linnainmaa (2022) and Gupta and Kelly (2019) study the performance of factor-momentum strategies. Gómez-Cram (2021) shows that the market can be timed using a business-cycle predictor derived from macroeconomic data. There are also papers that, like ours, study the timing of *combinations* of factors. For instance, Miller, Li, Zhou, and Giamouridis (2015) develop a dynamic portfolio approach using classification-tree analysis. Bass, Gladstone, and Ang (2017), Hodges, Hogan, Peterson, and Ang (2017), Amenc, Esakia, Goltz, and Luyten (2019), and Bender, Sun, and Thomas (2018) study multifactor portfolios conditional on macroeconomic state variables. Blin, Ielpo, Lee, and Teiletche (2018) study alternative risk premia conditional on macroeconomic regimes identified using Nowcasting. De Franco, Guidolin, and Monnier (2017) consider a multivariate Markov regime-switching model for the three Fama-French factors. Haddad, Kozak, and Santosh (2020) time the market and the first five principal components of a large set of equity factors using the value spread of the principal components as the timing variable. In contrast to these papers, our focus is on multifactor portfolios whose relative weights change with market volatility.

Our work is also related to the literature on the relation between market risk and return. While some papers find a positive relation between market risk and return (French, Schwert, and Stambaugh, 1987; Campbell and Hentschel, 1992), others find a negative relation (Breen, Glosten, and Jagannathan, 1989; Nelson, 1991; Glosten, Jagannathan, and Runkle, 1993). In addition, Lochstoer and Muir (2022) show that slow-moving beliefs about stock-market volatility could explain a weak, or even negative, market risk-return tradeoff. We contribute to this literature by showing that the risk-return tradeoff for the nine factors we consider weakens with realized market volatility.

The rest of the paper is organized as follows. Section 2 describes our data and methodology for constructing conditional multifactor portfolios. Section 3 reports the performance gains of our conditional multifactor portfolios. Section 4 investigates the sources of these gains. Section 5 studies the broader economic implications of our work by estimating a conditional stochastic discount factor whose price of risk for each factor varies with market volatility. Section 6 concludes. Appendix A provides a description of the construction of the nine factors we consider and the Internet Appendix contains a large number of robustness tests and additional results.

2 Data and methodology

In this section, we first describe the data used for our empirical analysis and then explain how we construct conditional multifactor portfolios and account for transaction costs.

2.1 Data

We compile data for the same nine factors considered by Moreira and Muir (2017) and Barroso and Detzel (2021).[4] To do this, we first download from the authors' websites excess returns for the market (MKT), small-minus-big (SMB), high-minus-low (HML), robust-minus-weak (RMW), and conservative-minus-aggressive (CMA), and momentum (UMD) factors of Fama and French (2018), the profitability (ROE) and investment (IA) factors of Hou, Xue, and Zhang (2015), and the betting-against-beta (BAB) factor of Frazzini and Pedersen (2014). Every factor (other than MKT and BAB) is the return of a long-short portfolio of stocks with one dollar in the long leg and one dollar in the short leg. The MKT and BAB factors are also zero-cost portfolios because their investment in the long leg is equal to that in the short leg once we account for their negative position in the risk-free asset.

We also construct these nine value-weighted factor portfolios independently in order to calculate the transaction costs required to trade the stocks comprising the factor portfolios.[5] To do this, we combine data from CRSP and Compustat for every stock traded on the NYSE, AMEX, and NASDAQ exchanges from January 1967 to December 2020.[6] We then drop stocks for firms with a negative book-to-market ratio.

For the out-of-sample analysis, we use an expanding-window approach, with the first estimation window consisting of 120 months starting from January 1967. Thus, the out-of-sample results are for January 1977 to December 2020. To ensure a fair comparison with the out-of-sample results, the in-sample results are evaluated for the same period, January 1977 to December 2020.

[4] In the main body of the manuscript, we consider the same set of factors as Moreira and Muir (2017) and Barroso and Detzel (2021) so that we can compare our results to theirs. However, Section IA.4 in the Internet Appendix shows that our findings are robust to considering a larger set of 66 factors that includes the nine factors considered by Moreira and Muir (2017) plus 57 factors from Green, Hand, and Zhang (2017).

[5] We find that the correlation of each of our factor returns with that of the original factor is above 90%.

[6] Moreira and Muir (2017) use data from 1926 to 2015 for MKT, SMB, HML, and UMD, from 1963 to 2015 for RMW and CMA, and from 1967 to 2015 for ROE and IA. Our multifactor analysis exploits all nine factors, so in order to ensure that we have data for all the factors over the entire sample period, our sample spans 1967 to 2020. Section IA.7 of the Internet Appendix shows that our main findings are robust to evaluating the performance of the conditional *multifactor* portfolios over the first and second halves of our sample.

2.2 Conditional mean-variance multifactor portfolios

We start by defining *individual* volatility-managed factors, as in Moreira and Muir (2017). Specifically, the return of the kth volatility-managed factor is

$$r^{\sigma}_{k,t+1} = \frac{c}{\sigma^2_{k,t}} r_{k,t+1}, \tag{1}$$

where $r_{k,t+1}$ is the kth unmanaged factor return for month $t+1$, $\sigma^2_{k,t}$ is the realized variance of the kth factor for month t estimated as the sample variance of daily factor returns, and c is a scaling parameter that ensures the volatility of the managed factor coincides with that of the unmanaged factor. The volatility-managed individual-factor *portfolio* is then the mean-variance combination of the unmanaged factor with its managed counterpart.[7]

Although the bulk of their analysis focuses on individual factors, Moreira and Muir (2017) also consider timing the unconditional mean-variance multifactor portfolio. In particular, they construct the optimal combination of the unconditional mean-variance multifactor portfolio and its managed counterpart, obtained by scaling the unconditional portfolio by the inverse of its past-month return variance. The resulting portfolio assigns the same relative weight to each factor as the unconditional multifactor portfolio, and thus, we refer to it as the "conditional *fixed-weight* multifactor portfolio."

In contrast to timing individual factors or a conditional fixed-weight multifactor portfolio, we consider a conditional mean-variance multifactor portfolio that allows the relative weights of the different factors to vary with market volatility. For simplicity, we employ market volatility to time all nine factors, and Section IA.10 of the Internet Appendix shows that this is a conservative choice because the performance is even stronger when we time each fac-

[7] Liu, Tang, and Zhou (2019) show that the out-of-sample Sharpe ratio of the managed factor is smaller when they estimate the scaling parameter c using a rolling window of past data. However, the scaling parameter c does not affect the out-of-sample performance of the volatility-managed individual-factor *portfolio* because it is obtained by selecting the weight on the unmanaged and managed factors that maximizes the mean-variance utility in the estimation window. In particular, for the kth factor we compute the values of a_k and b_k that maximize the mean-variance utility of the returns of the portfolio of the kth unmanaged and managed factors $(a_k r_{k,t+1} + b_k r^{\sigma}_{k,t+1})$ over each estimation window. Using Equation (1), the mean-variance portfolio return can be rewritten as $a_k r_{k,t+1} + (b_k c) r_{k,t+1}/\sigma^2_{k,t}$. From this expression, it may appear that the mean-variance portfolio depends on c. However, using the change of variables $\hat{b}_k = b_k c$, we can rewrite the return of the mean-variance portfolio as $a_k r_{k,t+1} + \hat{b}_k r_{k,t+1}/\sigma^2_{k,t}$, which does not depend on c. Thus, the values of a_k and $\hat{b}_k$ that maximize the mean-variance utility of the volatility-managed individual-factor portfolio are independent of c, and hence, the volatility-managed individual-factor portfolio is also independent of c.

tor using its own volatility or the average volatility of factors other than the market.[8] Note also that we use market volatility instead of market variance as our conditioning variable because Moreira and Muir (2017, section II.B) and Barroso and Detzel (2021, section 3.3) point out that using volatility can help reduce the transaction costs of volatility-managed factor portfolios.[9]

A conditional multifactor portfolio at time t can be expressed as

$$w_t(\theta_t) = \sum_{k=1}^{K} x_{k,t}\theta_{k,t}, \tag{2}$$

where K is the number of factors, $x_{k,t} \in \mathbb{R}^{N_t}$ is the stock portfolio associated with the kth factor at time t, in which N_t is the number of stocks at time t, $\theta_{k,t}$ is the portfolio weight on the kth factor at time t, and $\theta_t = (\theta_{1,t}, \theta_{2,t}, \ldots, \theta_{K,t})$ is the factor-weight vector at time t. We parameterize each factor weight, $\theta_{k,t}$, as an affine function of the inverse of market volatility,

$$\theta_{k,t} = a_k + b_k \frac{1}{\sigma_t}, \tag{3}$$

where σ_t is the realized market volatility estimated as the sample volatility of the daily market returns in month t. Note that a positive b_k implies that the portfolio reduces exposure to the kth factor when realized market volatility is high. Also, this parameterization allows for the weight of each factor to vary differently with market volatility because, in general, $b_i \neq b_j$ for $i \neq j$.

Let $r_{t+1} \in \mathbb{R}^{N_t}$ be the vector of stock returns for month $t+1$ and $r_{k,t+1} \equiv x_{k,t}^{\top}(r_{t+1} - r_{f,t+1}e_t) \in \mathbb{R}$ the kth factor return for month $t+1$, where $r_{f,t+1}$ is the return of the risk-free asset at time $t+1$ and e_t is the N_t-dimensional vector of ones. Then the return of a conditional multifactor portfolio can be written as

$$r_{p,t+1}(\theta_t) = \sum_{k=1}^{K} r_{k,t+1}\theta_{k,t} = \sum_{k=1}^{K} r_{k,t+1}\left(a_k + b_k \frac{1}{\sigma_t}\right), \tag{4}$$

where the second equality follows from substituting (3) into (2).

[8] In Section IA.15 of the Internet Appendix, we also consider seven alternative conditioning variables and find that exploiting another conditioning variable, in addition to market volatility, does not help to improve performance significantly.

[9] Moreover, Cejnek and Mair (2021) show that using volatility also reduces leverage.

For convenience, we also define the "extended" factor portfolio-weight matrix $X_{\text{ext},t}$, factor-return vector $r_{\text{ext},t+1}$, and factor-weight vector η as

$$X_{\text{ext},t} \equiv \begin{bmatrix} x_{1t}^\top \\ x_{2t}^\top \\ \vdots \\ x_{Kt}^\top \\ x_{1t}^\top \times \frac{1}{\sigma_t} \\ x_{2t}^\top \times \frac{1}{\sigma_t} \\ \vdots \\ x_{Kt}^\top \times \frac{1}{\sigma_t} \end{bmatrix}^\top, \quad r_{\text{ext},t+1} \equiv \begin{bmatrix} r_{1,t+1} \\ r_{2,t+1} \\ \vdots \\ r_{K,t+1} \\ r_{1,t+1} \times \frac{1}{\sigma_t} \\ r_{2,t+1} \times \frac{1}{\sigma_t} \\ \vdots \\ r_{K,t+1} \times \frac{1}{\sigma_t} \end{bmatrix}, \quad \text{and} \quad \eta \equiv \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_K \\ b_1 \\ b_2 \\ \vdots \\ b_K \end{bmatrix}, \tag{5}$$

respectively. Then, the conditional *mean-variance* multifactor portfolio is given by the extended factor-weight vector, η, that optimizes the net-of-transaction-costs mean-variance utility of an investor with risk-aversion parameter γ:

$$\max_{\eta \geq 0} \quad \widehat{\mu}_{\text{ext}}^\top \eta - \widehat{\text{TC}}(\eta) - \frac{\gamma}{2} \eta^\top \widehat{\Sigma}_{\text{ext}} \eta, \tag{6}$$

in which $\widehat{\mu}_{\text{ext}}$ and $\widehat{\Sigma}_{\text{ext}}$ are the sample mean and covariance matrix of the extended factor-return vector, $\widehat{\mu}_{\text{ext}}^\top \eta$ and $\eta^\top \widehat{\Sigma}_{\text{ext}} \eta$ are the sample mean and variance of the conditional multifactor portfolio return, and $\widehat{\text{TC}}(\eta)$ is its sample transaction cost.[10] Note that because all of our factors are zero-cost portfolios, we do not need to impose a constraint that the weights of the conditional multifactor portfolio add up to one.

To alleviate the impact of estimation error, we discipline the conditional multifactor portfolios by assigning a nonnegative weight to each unmanaged factor $a_k \geq 0$ and a higher weight to each factor when volatility is low $b_k \geq 0$; that is, we impose the constraint that $\eta \geq 0$. These nonnegativity constraints are also economically meaningful in the sense of Campbell and Thompson (2008) because one would expect the optimal portfolio to load positively on the unmanaged factors and reduce exposure when volatility is high. However, in Section IA.12 of the Internet Appendix, we show that our findings are robust to both relaxing

[10]Following Moreira and Muir (2017), Barroso and Detzel (2021), and Cederburg et al. (2020), in the main body of the manuscript we study volatility management in the context of a short-term mean-variance investor with a one-month horizon, but Section IA.9 in the Internet Appendix shows that our main findings are robust to evaluating performance over investment horizons up to 18 months. Note also that Moreira and Muir (2019) solve the optimal portfolio for a long-term investor with Epstein-Zin utility and find that the intertemporal hedging demands are small, and thus, their findings about the volatility-managed market portfolio based on mean-variance utility are robust to considering more general utility functions.

and dropping entirely the nonnegativity constraints on the factor weights of the multifactor portfolios. Moreover, in Section IA.13 of the Internet Appendix, we show that our findings are robust also to constraining the leverage of the conditional multifactor portfolio to be at most 20% higher than that of the unconditional multifactor portfolio and to dropping low-institutional ownership stocks from the sample.

2.3 Modeling transaction costs

We now explain how we compute the sample transaction cost of a conditional multifactor portfolio. First, note that the vector of stock trades required to rebalance the conditional multifactor portfolio at time $t+1$ is

$$\Delta w_{t+1}(\eta) = w_{t+1}(\eta) - w_t(\eta)^+, \tag{7}$$

where

$$w_{t+1}(\eta) = X_{\text{ext},t+1}\eta \quad \text{and} \tag{8}$$

$$w_t(\eta)^+ = w_t(\eta) \circ (e_t + r_{t+1}) \tag{9}$$

are the conditional multifactor portfolio at time $t+1$ and the conditional multifactor portfolio before rebalancing at time $t+1$, respectively, in which $x \circ y$ is the Hadamard or componentwise product of vectors x and y.

Given an estimation window with T historical observations of stock returns and factor portfolios, the average transaction cost incurred for rebalancing the conditional multifactor portfolio can be estimated as

$$\widehat{\text{TC}}(\eta) = \frac{1}{T-1} \sum_{t=1}^{T-1} \|\Lambda_t \Delta w_{t+1}(\eta)\|_1, \tag{10}$$

where $\|a\|_1 = \sum_{i=1}^{N} |a_i|$ denotes the 1-norm of the N-dimensional vector a and the transaction-cost matrix at time t, Λ_t, is the diagonal matrix whose ith diagonal element contains the transaction-cost parameter $\kappa_{i,t}$ of stock i at time t. Note that the transaction-cost term in Equation (10) accounts for the netting of the rebalancing trades across multiple factors. That is, the transaction cost is computed by first netting the rebalancing trades across the K factor portfolios and then charging the transaction cost at the individual-stock level.

To isolate the benefit of trading diversification, we also compute the transaction costs ignoring the netting of trades across factors. In this case, in contrast to (10), we estimate the transaction costs of the conditional multifactor portfolio by charging for the transaction cost *before* aggregating the rebalancing trades across the K factors:

$$\widehat{\mathrm{TC}}(\eta) = \frac{1}{T-1}\sum_{t=1}^{T-1}\sum_{k=1}^{K}\|\Lambda_t\left(x_{k,t+1}\theta_{k,t+1} - x_{k,t}^{+}\theta_{k,t}\right)\|_1, \tag{11}$$

where $x_{k,t}^{+} = x_{k,t} \circ (e_t + r_{t+1})$.

To compute the stock-level transaction costs parameter $\kappa_{i,t}$, we use the two-day corrected method proposed by Abdi and Ranaldo (2017) to estimate the monthly bid-ask spread of the ith stock as:

$$\widehat{s}_{i,t} = \frac{1}{D}\sum_{d=1}^{D}\widehat{s}_{i,d}, \quad \widehat{s}_{i,d} = \sqrt{\max\{4(\mathrm{cls}_{i,d} - \mathrm{mid}_{i,d})(\mathrm{cls}_{i,d} - \mathrm{mid}_{i,d+1}), 0\}}, \tag{12}$$

where D is the number of days in month t, $\widehat{s}_{i,d}$ is the two-day bid-ask spread estimate, $\mathrm{cls}_{i,d}$ is the closing log-price on day d, and $\mathrm{mid}_{i,d}$ is the mid-range log-price on day d; that is, the mean of daily high and low log-prices.[11] Finally, because the effective trading cost is half the bid-ask spread, the transaction-cost parameter for the ith stock is $\kappa_{i,t} = \widehat{s}_{i,t}/2$.[12]

Figure 2 depicts the time series of transaction costs for January 1977 to December 2020 at the individual-stock level (Panel A) and the average transaction costs at the factor level (Panel B). Panel A shows that the transaction costs of individual stocks are highly time varying, with the variation being stronger for less liquid stocks. The transaction costs of individual stocks are particularly large during NBER recessions, which are shaded in gray.

Panel B of Figure 2 shows that the factor with the highest average transaction costs is momentum (UMD) and the factor with the lowest average cost is the market (MKT).[13]

[11] Following Novy-Marx and Velikov (2016) and Barroso and Detzel (2021), we replace missing observations of transaction-cost parameters in month t with the transaction cost for the stock that is the closest match in terms of market capitalization and idiosyncratic volatility, or the closest match in terms of only one of these two characteristics if the other is missing, or the cross-sectional mean transaction-cost parameter if both characteristics are missing. We estimate idiosyncratic volatility as the standard deviation of residuals from a CAPM regression over the three months of daily data ending in month t.

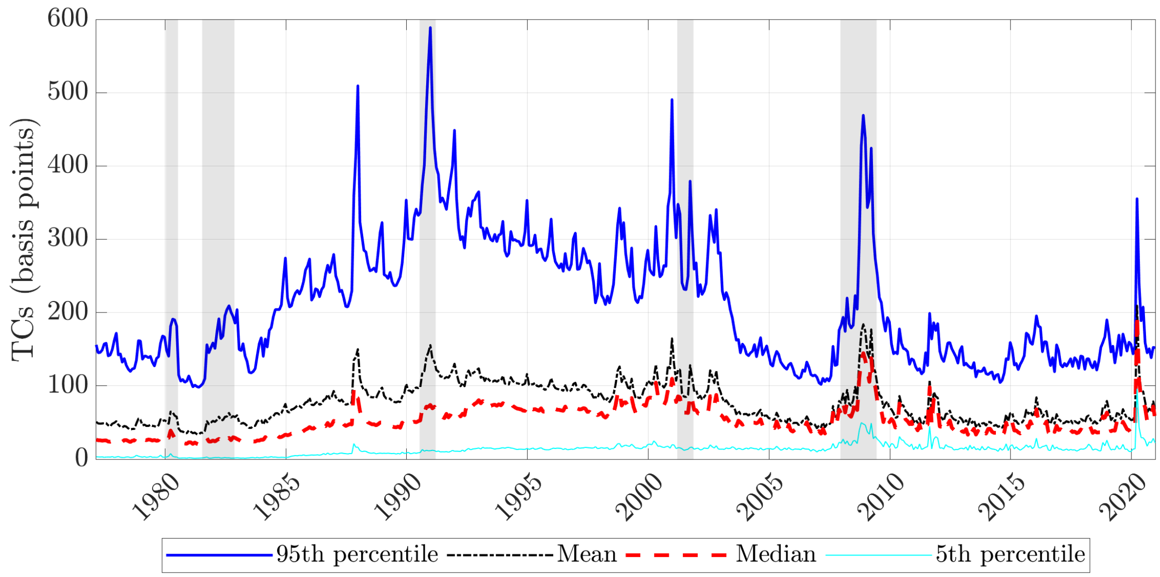
[12] Sections IA.16 and IA.17 of the Internet Appendix, respectively, show that our findings are robust to considering proportional transaction costs estimated using the low-frequency average bid-ask spread of Chen and Velikov (2023) and to considering quadratic price-impact costs estimated using the results of Novy-Marx and Velikov (2016).

[13] Section IA.18 of the Internet Appendix shows that the transaction costs that we estimate for trading the market factor are very small and do not drive any of the results in our manuscript.

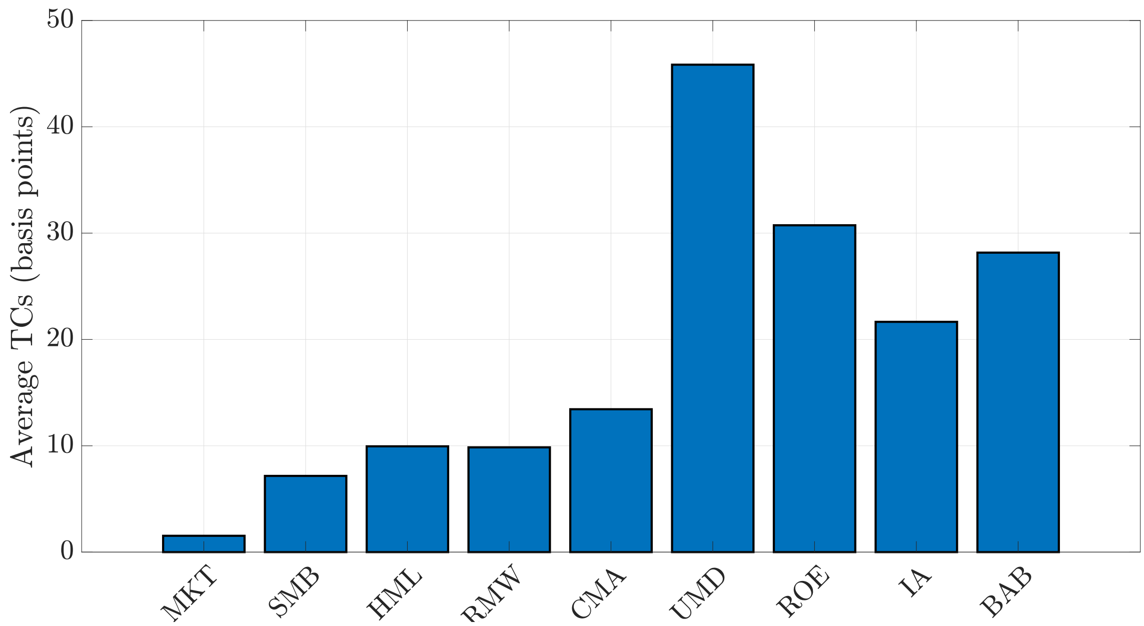
Figure 2: Transaction costs at the stock and factor levels

This figure depicts several descriptive statistics of the transaction costs estimated using the method of Abdi and Ranaldo (2017) described in Equation (12). Panel A depicts how the 95th percentile, mean, median, and 5th percentile of individual-stock transaction-cost parameters vary over time for the out-of-sample period from January 1977 to December 2020, with NBER recessions shaded in gray. Panel B depicts the average monthly transaction cost of trading the nine factors in our dataset.

Panel A: Individual-stock level



Panel B: Factor level



Panel B also shows that our transaction-cost estimates are very similar to those in Barroso and Detzel (2021). For instance, our estimate of the transaction cost to rebalance the unmanaged SMB factor is around 7.5 basis points and Barroso and Detzel (2021, Figure 2, Panel D) estimate 7 basis points; for HML, we estimate 10 basis points and they 8; for RMW, we estimate 10 basis points and they 10; for CMA, we estimate 13 basis points and they 14; for UMD, we estimate 46 basis points and they 50; for ROE, we estimate 31 basis points and they 29; for IA, we estimate 22 basis points and they 21; and for BAB, we estimate 28 basis points and they 33.

3 Performance gains from volatility management

In this section, we study the economic gains from volatility management. Section 3.1 evaluates the performance of the volatility-managed *individual*-factor portfolios, which are the focus of the existing literature. Section 3.2 evaluates the out-of-sample and net-of-costs performance of our proposed conditional *multifactor* portfolio.

3.1 Volatility-managed individual-factor portfolios

To set the stage for analyzing our multifactor portfolios, we first evaluate the in-sample performance of the volatility-managed *individual-factor* portfolios, which are the focus of Moreira and Muir (2017). We then assess the performance of these strategies net of transaction costs and out of sample, which allows us to confirm the findings of Barroso and Detzel (2021) and Cederburg et al. (2020), respectively.[14]

For each of the nine factors we consider, Table 1 reports the annualized Sharpe ratio of the unmanaged factor, $\mathrm{SR}(r_k)$, the volatility-managed individual-factor portfolio, $\mathrm{SR}(r_k, r_k^\sigma)$, which is the mean-variance combination of the unmanaged factor with its managed counterpart given in (1), and the p-value for the difference between these two Sharpe ratios.[15]

[14]To facilitate comparison with the existing literature, the individual-factor portfolios considered in this section are obtained by using inverse factor variance to time each of the factors. However, Section IA.3 of the Internet Appendix shows that the results are similar if, instead of using inverse factor variance to time the individual factors, we use inverse market volatility, which is the conditioning variable for our multifactor portfolios.

[15]We use bootstrap to construct one-sided p-values for the difference in Sharpe ratios. Specifically, we first generate 10,000 bootstrap samples of the returns of the volatility-managed individual-factor portfolio and the unmanaged factor using the stationary block-bootstrap method of Politis and Romano (1994) with an average block size of five. Second, we construct the empirical distribution of the difference between the

We consider an investor with risk-aversion parameter $\gamma = 5$. Panel A reports performance in-sample and ignoring transaction costs, Panel B in-sample and net of costs but ignoring trading diversification, Panel C out-of-sample and ignoring costs, Panel D out-of-sample and net of costs but ignoring trading diversification, and Panel E out-of-sample and net of costs with trading diversification. Our sample spans January 1967 to December 2020 and, similar to the base-case analysis in Cederburg et al. (2020), we evaluate out-of-sample performance using an expanding window with the first estimation window containing the first 120 months of data.[16] Thus, the out-of-sample results are for January 1977 to December 2020. The out-of-sample return of each volatility-managed individual-factor portfolio is evaluated for the month following the last month of each estimation window. To ensure a fair comparison with the out-of-sample results, the in-sample results are computed for the same period, January 1977 to December 2020.[17]

Panel A of Table 1 confirms the main finding by Moreira and Muir (2017): in-sample and ignoring transaction costs, the Sharpe ratio of the volatility-managed individual-factor portfolio, $\mathrm{SR}(r_k, r_k^\sigma)$, is greater than that of the unmanaged factor, $\mathrm{SR}(r_k)$, for all nine factors, with the difference being statistically significant at the 10% level for five of the factors (RMW, UMD, ROE, IA, and BAB).[18]

Panel B reports the performance in-sample and net of transaction costs but, as in Barroso and Detzel (2021), ignoring the trading-diversification benefits from combining the

---

Sharpe ratios of the returns of the volatility-managed individual-factor portfolio and the unmanaged factor, $\mathrm{SR}(r_k, r_k^\sigma) - \mathrm{SR}(r_k)$, across the 10,000 bootstrap samples. Third, we compute the p-value as the frequency with which this difference is smaller than zero across the bootstrap samples. Section IA.24 of the Internet Appendix shows that the inference is robust to using three other approaches to compute p-values: (i) the approach of Jobson and Korkie (1981), (ii) the approach of Ledoit and Wolf (2008), and (iii) an alternative bootstrap approach to compare in-sample Sharpe ratios that accounts for how the conditional portfolios are constructed.

[16] Cederburg et al. (2020) report that their results are not sensitive to the length of the estimation window: "We therefore consider specifications with 20-year ($K = 240$) and 30-year ($K = 360$) initial estimation periods. These designs produce roughly the same number of positive Sharpe ratio and CER differences that the base case does." They also report that their results are not sensitive to the value chosen for the risk aversion parameter: "Using a lower ($\gamma = 2$) or higher ($\gamma = 10$) risk aversion parameter leads to almost identical results to the base case with $\gamma = 5$."

[17] As mentioned in Footnote 6, we consider a sample spanning the period 1977 to 2020, for which there are data to construct all nine factors we consider, so that we can evaluate the performance of the conditional multifactor portfolios that exploit all nine factors. However, it is possible to evaluate the performance of each of the volatility-managed *individual-factor* portfolios for longer samples, and Section IA.1 of the Internet Appendix shows that our main findings are robust to considering such longer samples.

[18] In unreported results, we also observe that the alphas of the nine volatility-managed individual-factor portfolios with respect to their unmanaged counterparts are positive, and they are statistically significant for the same five factors.

Table 1: Performance of volatility-managed individual-factor portfolios

For each of the nine factors we consider, this table reports the annualized Sharpe ratios of the unmanaged factor, $\mathrm{SR}(r_k)$, the volatility-managed individual-factor portfolio, $\mathrm{SR}(r_k, r_k^\sigma)$, which is the mean-variance combination of the unmanaged factor with its managed counterpart given in (1), and the p-value for the difference between these two Sharpe ratios. We consider an investor with risk-aversion parameter $\gamma = 5$. Panel A reports performance in-sample and ignoring transaction costs, Panel B in-sample and net of costs but ignoring trading diversification, Panel C out-of-sample but ignoring costs, Panel D out-of-sample and net of costs but ignoring trading diversification, and Panel E out-of-sample and net of costs considering trading diversification. To facilitate comparison, both the in-sample and out-of-sample performance are evaluated for January 1977 to December 2020.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
Panel A: In-sample without transaction costs									
$\mathrm{SR}(r_k)$	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
$\mathrm{SR}(r_k, r_k^\sigma)$	0.585	0.246	0.215	0.739	0.419	1.088	1.153	0.621	1.397
p-value$(\mathrm{SR}(r_k, r_k^\sigma) - \mathrm{SR}(r_k))$	0.242	0.366	0.337	0.033	0.311	0.000	0.001	0.094	0.000
Panel B: In-sample and net of transaction costs but without trading diversification									
$\mathrm{SR}(r_k)$	0.519	0.125	0.053	0.356	0.159	0.114	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.521	0.125	0.053	0.356	0.159	0.251	0.331	0.107	0.703
p-value$(\mathrm{SR}(r_k, r_k^\sigma) - \mathrm{SR}(r_k))$	0.464	0.500	0.500	0.500	0.500	0.249	0.407	0.500	0.161
Panel C: Out-of-sample but ignoring transaction costs									
$\mathrm{SR}(r_k)$	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
$\mathrm{SR}(r_k, r_k^\sigma)$	0.408	0.068	0.194	0.527	0.355	1.035	1.094	0.605	1.321
p-value$(\mathrm{SR}(r_k, r_k^\sigma) - \mathrm{SR}(r_k))$	0.899	0.929	0.390	0.467	0.897	0.000	0.000	0.062	0.000
Panel D: Out-of-sample and net of transaction costs but ignoring trading diversification									
$\mathrm{SR}(r_k)$	0.519	0.125	0.053	0.356	0.159	0.114	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.324	−0.295	−0.041	−0.453	−0.047	0.194	0.269	−0.127	0.690
p-value$(\mathrm{SR}(r_k, r_k^\sigma) - \mathrm{SR}(r_k))$	0.976	1.000	0.879	1.000	1.000	0.342	0.672	1.000	0.281
Panel E: Out-of-sample and net of transaction costs with trading diversification									
$\mathrm{SR}(r_k)$	0.519	0.125	0.053	0.356	0.159	0.114	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.433	0.035	0.089	0.226	0.153	0.209	0.324	0.193	0.746
p-value$(\mathrm{SR}(r_k, r_k^\sigma) - \mathrm{SR}(r_k))$	0.917	0.858	0.243	0.965	0.547	0.097	0.405	0.029	0.064

unmanaged and managed factors. Comparing Panels A and B, we observe that transaction costs greatly diminish the performance of the volatility-managed individual-factor portfolios. In fact, the transaction cost of the managed factor is so large for five of the nine factors—SMB, HML, RMW, CMA, and IA—that when considering the optimal combination of the unmanaged and the volatility-managed factors, the investor assigns a zero weight to the managed factor, which explains why the Sharpe ratio of the individual-factor portfolio is the same as that of the unmanaged factor.[19] For the other four factors, the improvement

[19]Note that the p-values are not well defined in Panel B of Table 1 for the SMB, HML, RMW, CMA, and IA factors because the difference in Sharpe ratios is zero for every bootstrap sample, and thus, the entire

in Sharpe ratio from volatility management is not statistically significant. Thus, we conclude that even in sample, the gains from volatility management are completely eroded by transaction costs, confirming the result in Barroso and Detzel (2021).

Panel C shows that the *out-of-sample* Sharpe ratio of the volatility-managed individual-factor portfolios in the absence of transaction costs is lower than the in-sample Sharpe ratio in Panel A for all nine factors. We also observe that the optimal combination of the unmanaged and volatility-managed factors delivers an out-of-sample Sharpe ratio, $\mathrm{SR}(r_k, r_k^\sigma)$, that can be smaller than that of even the unmanaged factor, $\mathrm{SR}(r_k)$; this is the case for the MKT, SMB, and CMA factors.[20] The out-of-sample gains from volatility management are statistically significant at the 10% level for only four of the nine factors (UMD, ROE, IA, BAB). Overall, our results show that, consistent with Cederburg et al. (2020), estimation error diminishes the gains from volatility management.

Panel D shows that transaction costs erode the out-of-sample performance further. In particular, once we account for *both* estimation error and transaction costs ignoring trading diversification, the Sharpe ratio for five of the nine volatility-managed individual-factor portfolios becomes negative. Moreover, the Sharpe ratio of the optimal combination of the unmanaged and volatility-managed factor is lower than that of the corresponding unmanaged factor for all factors except UMD and BAB, with neither being statistically significant, which drives home the point that estimation error and transaction costs erode entirely the gains from volatility-managing individual factors.

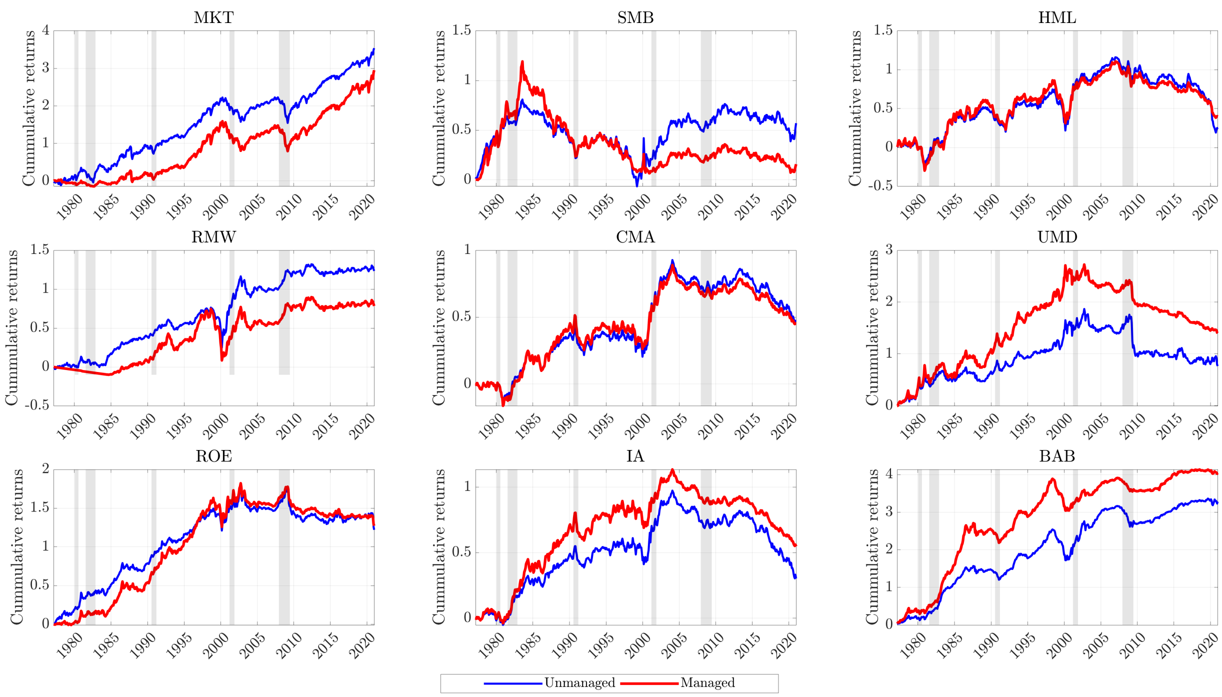
Comparing the Sharpe ratios of the volatility-managed individual-factor portfolios, $\mathrm{SR}(r_k, r_k^\sigma)$, in Panels D and E, we find that accounting for the netting of trades across the unmanaged and managed factors improves the performance of all nine portfolios. Moreover, with trading diversification, volatility management improves the performance for five of the

empirical distribution for the difference in Sharpe ratios is concentrated at zero. For these cases, we set the p-value equal to 0.5, which is the level one would expect for a one-sided test when the Sharpe ratios of two portfolios are statistically indistinguishable.

[20]The first row of each panel in Table 1 reports the performance of each of the *unmanaged individual factors*, that is, assuming the investor assigns a constant weight to the factor. Note that the Sharpe ratio of the unmanaged factor does not depend on the value of the constant weight because both the average and standard deviation of the factor returns are linear in the weight. Thus, we simply report the Sharpe ratio of the *unscaled* unmanaged factor return, which does not require any estimation, and thus, the in-sample and out-of-sample Sharpe ratios of the unmanaged factor reported in the first row of Panels A and C coincide. Moreover, there are no trading-diversification benefits from trading an unmanaged factor in isolation, and thus, its Sharpe ratio with costs is the same whether one accounts for trading diversification or not. Consequently, the Sharpe ratios reported in the first row of Panels B, D, and E also coincide.

Figure 3: Cumulative returns of individual-factor portfolios

The nine graphs in this figure depict the out-of-sample cumulative returns net of transaction costs with trading diversification of each unmanaged factor (blue line) and its associated volatility-managed individual factor portfolio (red line) over the out-of-sample period from January 1977 to December 2020. The cumulative returns are reported in dollars, and the volatility-managed individual factor portfolio is scaled to have the same volatility as the unmanaged factor.



nine factors, with the improvement being statistically significant at the 10% level for three factors (UMD, IA, BAB).[21] Thus, trading diversification partially alleviates the concerns raised by Barroso and Detzel (2021) and Cederburg et al. (2020), but it does not fully resurrect the gains from volatility managing individual factors.[22]

To illustrate these results, Figure 3 depicts the out-of-sample cumulative returns net of transaction costs with trading diversification of each unmanaged factor (blue line) and its associated volatility-managed individual-factor portfolio (red line) over the out-of-sample period from January 1977 to December 2020. The cumulative returns are reported in dollars

[21] This finding is consistent with Barroso and Santa-Clara (2015b), Cederburg and O'Doherty (2016), and Barroso, Detzel, and Maio (2021) who show that timing the volatility of momentum and betting-against-beta produces substantial gains.

[22] Section IA.2 of the Internet Appendix shows that it is possible to improve the out-of-sample and net-of-costs performance of the volatility-managed individual-factor portfolios by assigning equal weights to the unmanaged and managed factors and estimating realized volatility using a six-month window of daily returns instead of a one-month window. After implementing these strategies to alleviate the impact of estimation error and transaction costs, the number of volatility-managed individual-factor portfolios that significantly outperform their unconditional counterpart at the 10% level goes up from three to five.

and the volatility-managed individual factor portfolio is scaled to have the same volatility as the unmanaged factor.[23] These plots show again that, with trading diversification, volatility management improves the performance for five of the nine factors, although (as shown in Table 1) the difference is statistically significant for only three factors.

We conclude from the evidence presented above that, consistent with the findings of Barroso and Detzel (2021) and Cederburg et al. (2020), a volatility-managed portfolio based on an *individual* factor typically fails to significantly outperform its unmanaged counterpart when performance is measured out of sample and net of transaction costs.

3.2 Conditional mean-variance multifactor portfolio

In the previous section, we evaluated the performance of the volatility-managed individual-factor portfolios, which have been the focus of the existing literature. In this section, we provide a multifactor perspective by evaluating the benefits of volatility management for an investor who has access to multiple factors. To do this, we compare the out-of-sample and net-of-costs performance of two portfolios: the conditional mean-variance multifactor portfolio (CMV) obtained by solving problem (6) and the unconditional mean-variance multifactor portfolio (UMV) obtained by solving problem (6) under the additional constraint that $b_k = 0$ for $k = 1, 2, \ldots, K$; that is, under the constraint that its weights on the K factors do not vary with market volatility.

For each multifactor portfolio, Table 2 reports the out-of-sample annualized mean, standard deviation, Sharpe ratio of returns net of transaction costs, and p-value for the difference between the Sharpe ratios of the conditional and unconditional portfolios.[24] For completeness, the table also reports in percentage the annualized alpha of the time-series regression of the conditional portfolio out-of-sample returns net of transaction costs on those of the unconditional portfolio, the Newey-West t-statistic for the alpha, and the out-of-sample transaction costs accounting for trading diversification of the unconditional and conditional

[23]Note that the unmanaged factor and the volatility-managed individual factor portfolio are both self-financing portfolios, and thus, they earn payoffs rather than returns, but for simplicity we refer to the payoffs as returns. Then, we calculate the cumulative return of each self-financing portfolio by adding the dollar payoffs over the entire out-of-sample period.

[24]Consistent with the conditional multifactor portfolio problem (6), we compute the annualized net mean return as the difference between the out-of-sample gross mean return and the transaction cost, $E[r_{p,t+1}] - \text{TC}$, the standard deviation as $\text{stdev}(r_{p,t+1})$, and the Sharpe ratio as the ratio of these two quantities. We use the procedure described in Footnote 15 to construct one-sided p-values for the difference in Sharpe ratios.

Table 2: Performance of conditional mean-variance multifactor portfolio

This table reports the out-of-sample and net-of-costs performance of two multifactor portfolios: the conditional mean-variance multifactor portfolio (CMV) obtained by solving problem (6) and the unconditional mean-variance multifactor portfolio (UMV) obtained by solving problem (6) under the additional constraint that $b_k = 0$ for $k = 1, 2, \ldots, K$; that is, under the constraint that its weights on the K factors are constant over time. For each multifactor portfolio, the table reports the out-of-sample annualized mean, standard deviation, Sharpe ratio of returns net of transaction costs accounting for trading diversification, and p-value for the difference between the Sharpe ratios of the conditional and unconditional portfolios. The table also reports the annualized alpha of the time-series regression of the conditional portfolio out-of-sample returns net of transaction costs on those of the unconditional portfolio, alpha Newey-West t-statistic, and out-of-sample transaction costs of the unconditional and conditional portfolios. The portfolios are constructed using all nine factors in our dataset. We report out-of-sample performance from January 1977 to December 2020.

	UMV	CMV
Mean	0.430	0.477
Standard deviation	0.458	0.449
Sharpe ratio	0.940	1.062
p-value($\mathrm{SR_{CMV}} - \mathrm{SR_{UMV}}$)		0.006
α		0.066
$t(\alpha)$		3.637
TC	0.163	0.213

portfolios.[25] The portfolios are constructed exploiting all nine factors in our dataset. We use an expanding-window approach and the out-of-sample period spans January 1977 to December 2020.

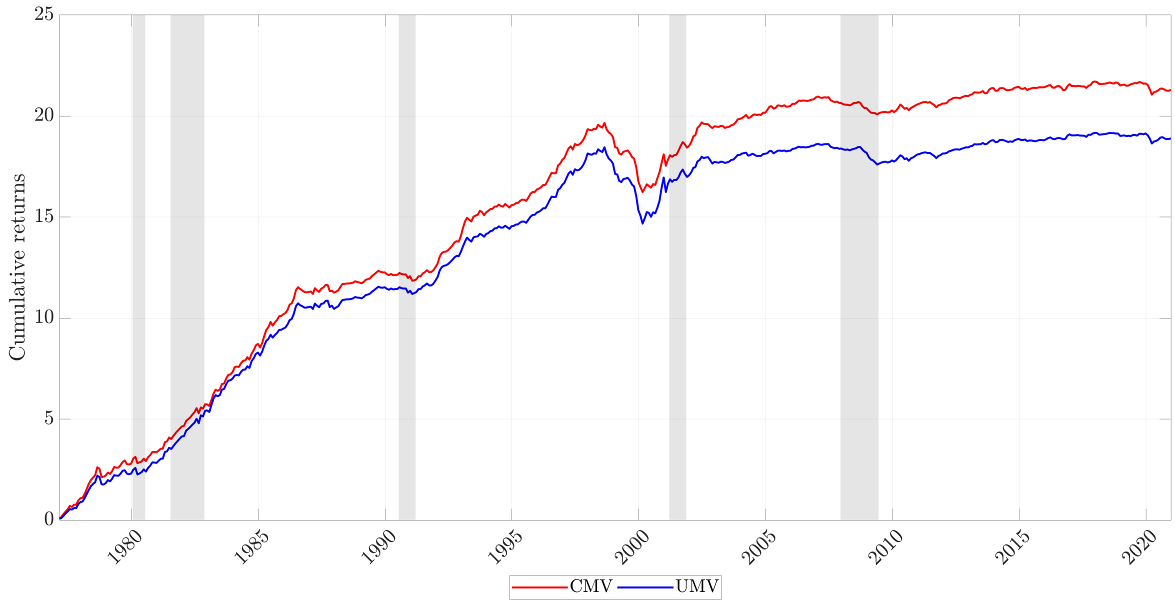
Table 2 shows that the conditional multifactor portfolio delivers an out-of-sample Sharpe ratio of net returns that is significantly larger than that of the unconditional portfolio. In particular, the conditional portfolio achieves a Sharpe ratio of 1.062, which is 13% higher than that of the unconditional portfolio, with the difference being statistically significant at the 1% level. The conditional portfolio also has a significantly positive annualized alpha with a t-statistic above three.[26] The table shows that the conditional portfolio has slightly lower volatility than its unconditional counterpart, and although the conditional portfolio incurs

[25] For the Newey-West alpha t-statistic, we use a one-month lag throughout the manuscript. We have computed all Newey-West alpha t-statistics using alternative lags of five and 10 months and the inference (that is, whether they are larger than two in absolute value) does not change for any of the t-statistics.

[26] Note that the magnitude of the alpha of the conditional multifactor portfolios is not comparable to that of standard asset-pricing factors. This is because while standard asset-pricing factors assign a weight of one dollar to each of their long and short legs, the multifactor portfolio assigns a weight to its long and short legs that varies over time and is not generally equal to one dollar on average. Moreover, multiplying the conditional multifactor portfolio by a scalar larger than one will proportionally inflate its alpha. However, the alpha t-statistic of the conditional multifactor portfolio is invariant to scaling, and thus, can be compared to the alpha t-statistics of standard asset-pricing factors.

Figure 4: Cumulative returns of multifactor portfolios

This figure depicts the out-of-sample cumulative returns net of transaction costs of the unconditional (UMV) and conditional (CMV) mean-variance multifactor portfolios over the out-of-sample period from January 1977 to December 2020. The returns are reported in dollars and the conditional multifactor portfolio is standardized to have the same volatility as its unconditional counterpart.



larger transaction costs, its gross mean return more than compensates for the additional trading costs associated with factor timing.[27]

Figure 4 plots the cumulative out-of-sample net returns of the unconditional (UMV) and conditional (CMV) multifactor portfolios. The returns are reported in dollars and the conditional multifactor portfolio is standardized to have the same volatility as its unconditional counterpart. Figure 4 shows that the conditional multifactor portfolio outperforms the unconditional portfolio steadily over the entire sample.[28]

[27]Section IA.19 of the Internet Appendix shows that the conditional multifactor portfolio is less risky than its unconditional counterpart also in terms of alternative risk measures such as value-at-risk, maximum drawdown, skewness, and kurtosis.

[28]Section IA.5 of the Internet Appendix shows that our findings are not driven by the performance of a particular factor because the conditional multifactor portfolio outperforms its unconditional counterpart even after excluding each of the nine factors one at a time or replacing the BAB factor with a more conventional *value-weighted* BAB factor. In addition, Section IA.6 of the Internet Appendix shows that the out-of-sample and net-of-costs performance of the conditional multifactor portfolio is significantly better than that of its unconditional counterpart during high-volatility and crises periods. Finally, Section IA.11 of the Internet Appendix shows that the performance of the conditional multifactor portfolio can be further improved by estimating realized market volatility using daily market returns over three-, six-, or 12-month

Finally, comparing the performance of the conditional *multifactor* portfolio in Table 2 with the performance of the volatility-managed *individual*-factor portfolios in Panel E of Table 1, we see that, not surprisingly, the conditional multifactor portfolio also outperforms substantially the volatility-managed individual-factor portfolios out of sample and net of transaction costs.

4 Understanding the conditional multifactor portfolio

The results in the previous section demonstrate that the conditional multifactor portfolio significantly outperforms its unconditional counterpart out of sample and net of transaction costs. In this section, we undertake various experiments to understand the sources of the favorable performance of the conditional multifactor portfolio.

4.1 Disentangling the source of the gains

Table 3 reports the performance of three multifactor portfolios optimized either ignoring or accounting for transaction costs. The three multifactor portfolios are the: (i) unconditional multifactor portfolio (UMV), (ii) conditional fixed-weight multifactor portfolio (CFW) of Moreira and Muir (2017),[29] and (iii) conditional multifactor portfolio (CMV). Columns (1) to (3) report the performance of the three multifactor portfolios optimized ignoring transaction costs and Columns (4) to (6) accounting for transaction costs. Each of the four panels in the table reports the performance of the portfolios *evaluated* in a different way: Panel A in-sample without transaction costs, Panel B out-of-sample but without transaction costs, Panel C out-of-sample with transaction costs but ignoring trading diversification, and Panel D out-of-sample with transaction costs and trading diversification.

We first discuss the performance of the three multifactor portfolios optimized *ignoring transaction costs*, which is reported in Columns (1) to (3) of Table 3. Comparing Columns (1) and (2) in Panel A, we confirm the finding in Moreira and Muir (2017) that, in-sample and ignoring transaction costs, timing the unconditional multifactor portfolio leads to a

estimation windows. However, to facilitate comparison with the existing literature, we focus our analysis on the conditional multifactor portfolios obtained using one-month realized volatility.

[29]Specifically, the CFW portfolio is the optimal combination of the unconditional mean-variance multifactor portfolio and its managed counterpart, obtained by scaling the unconditional portfolio by the inverse of its past-month return variance.

Table 3: Understanding the performance of the multifactor portfolios

This table reports the performance of three multifactor portfolios optimized either ignoring or accounting for transaction costs. The three multifactor portfolios are the: (i) unconditional multifactor portfolio (UMV), (ii) conditional fixed-weight multifactor portfolio (CFW) of Moreira and Muir (2017), and (iii) our conditional multifactor portfolio (CMV). Columns (1) to (3) report the performance of the three multifactor portfolios optimized ignoring transaction costs and Columns (4) to (6) accounting for transaction costs. Each of the four panels reports the performance of the portfolios *evaluated* in a different way: Panel A in-sample without transaction costs, Panel B out-of-sample without transaction costs, Panel C out-of-sample with transaction costs without trading diversification, and Panel D out-of-sample with transaction costs with trading diversification. The sample period and quantities reported for each portfolio are the same as in Table 2.

	Optimized ignoring TC			Optimized accounting for TC		
	UMV (1)	CFW (2)	CMV (3)	UMV (4)	CFW (5)	CMV (6)
Panel A: In-sample without transaction costs						
Mean	0.415	0.602	0.680	0.301	0.314	0.432
Standard deviation	0.288	0.347	0.369	0.218	0.222	0.250
Sharpe ratio	1.441	1.735	1.844	1.378	1.415	1.726
p-value($SR_{CMV} - SR_{UMV}$)		0.000	0.000		0.005	0.000
α		0.187	0.213		0.009	0.107
$t(\alpha)$		5.738	6.684		3.073	7.638
TC	0.000	0.000	0.000	0.000	0.000	0.000
Panel B: Out-of-sample without transaction costs						
Mean	0.753	0.783	0.925	0.593	0.626	0.690
Standard deviation	0.580	0.520	0.569	0.458	0.445	0.449
Sharpe ratio	1.299	1.506	1.625	1.295	1.407	1.537
p-value($SR_{CMV} - SR_{UMV}$)		0.000	0.000		0.002	0.000
α		0.140	0.239		0.059	0.126
$t(\alpha)$		5.012	5.797		3.808	6.414
TC	0.000	0.000	0.000	0.000	0.000	0.000
Panel C: Out-of-sample with transaction costs without trading diversification						
Mean	0.412	0.313	0.349	0.347	0.343	0.332
Standard deviation	0.580	0.520	0.569	0.458	0.445	0.449
Sharpe ratio	0.710	0.601	0.613	0.758	0.772	0.739
p-value($SR_{CMV} - SR_{UMV}$)		0.986	0.930		0.329	0.721
α		-0.035	-0.027		0.012	0.001
$t(\alpha)$		-1.610	-0.727		0.967	0.031
TC	0.341	0.470	0.576	0.246	0.283	0.358
Panel D: Out-of-sample with transaction costs and trading diversification						
Mean	0.517	0.511	0.575	0.430	0.457	0.477
Standard deviation	0.580	0.520	0.569	0.458	0.445	0.449
Sharpe ratio	0.891	0.984	1.010	0.940	1.026	1.062
p-value($SR_{CMV} - SR_{UMV}$)		0.017	0.072		0.002	0.006
α		0.072	0.103		0.046	0.066
$t(\alpha)$		3.094	2.759		3.407	3.637
TC	0.236	0.272	0.350	0.163	0.169	0.213

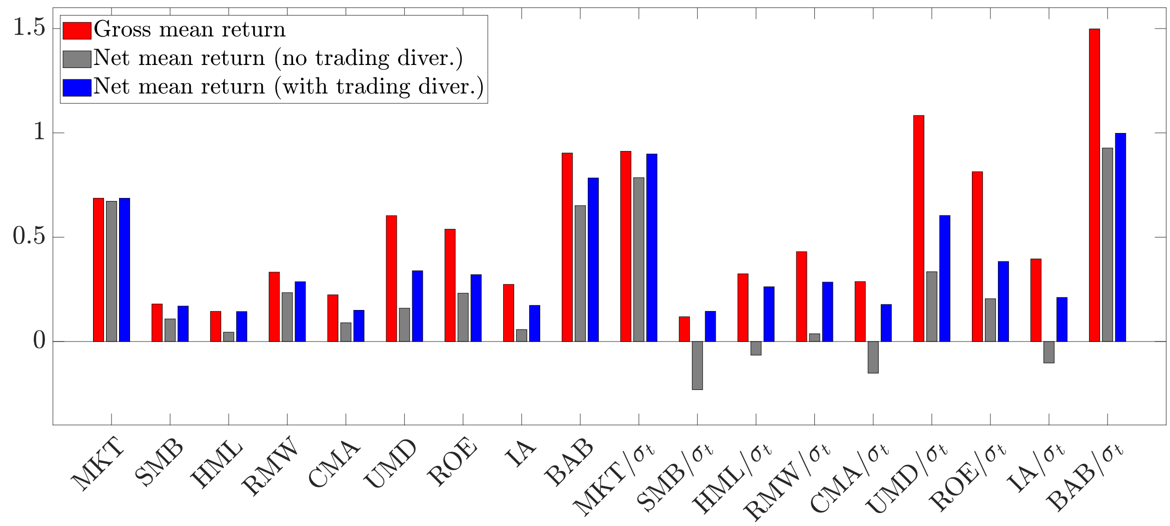
statistically significant increase in the Sharpe ratio from 1.441 to 1.735. Column (3) shows that allowing the relative weight of each factor to vary with market volatility, one can obtain a slightly higher Sharpe ratio of 1.844. Panel B shows that the gains from volatility managing multiple factors are significant even out of sample, if one ignores transaction costs, although they are smaller than those in sample. This is in contrast to the result in Table 1 that the volatility-managed *individual-factor* portfolios typically fail to outperform significantly the unmanaged factor out of sample. This suggests that combining multiple factors can help to alleviate the impact of the estimation error associated with some of the noisier factors. However, Columns (1) to (3) of Panel C show that accounting for transaction costs while ignoring trading diversification eliminates the out-of-sample gains from volatility-managing multiple factors. Finally, Panel D shows that if one accounts for trading diversification by netting out the rebalancing trades across the multiple factors, then both conditional multifactor portfolios outperform their unconditional counterpart even out of sample and net of transaction costs. Thus, the main finding from Columns (1) to (3) is that it is crucial to net out trades when accounting for transaction costs of volatility-managed multifactor portfolios.

We now discuss the performance of the three multifactor portfolios optimized accounting for transaction costs. Comparing Columns (2) and (5) for Panel D, we observe that optimizing the conditional fixed-weight portfolio for transaction costs increases its out-of-sample and net-of-costs Sharpe ratio by around 4%. More importantly, a key takeaway from comparing Columns (4) and (5) of Table 3 is that the conditional fixed-weight portfolio significantly outperforms its unconditional counterpart even in the presence of transaction costs and estimation error, once we optimize the portfolio for transaction costs and account for trading diversification. Moreover, Column (6) of Panel D shows that allowing the relative weights on the different factors to vary with market volatility leads to a further moderate increase in the out-of-sample and net-of-cost Sharpe ratio. The conditional multifactor portfolios optimized accounting for transaction costs in Columns (5) and (6) also incur lower transaction costs than the corresponding strategies in Columns (2) and (3), where the factor weights are not optimized for transaction costs.

Summarizing, Table 3 shows that the favorable performance of the conditional multifactor portfolios has three drivers: (i) taking trading diversification into account when eval-

Figure 5: Gross and net-of-costs mean factor returns

This barplot depicts the monthly average factor returns (in percentage) of the nine unmanaged and volatility-managed factors. The figure compares three quantities for each factor: (i) its mean gross return, (ii) its mean return net of transaction costs ignoring trading diversification, and (iii) its mean return net of transaction costs accounting for trading diversification. For the case in which we account for trading diversification, we use the factor weights that solve problem (6) in sample. The sample spans January 1977 to December 2020.



uating performance, (ii) accounting for transaction costs and trading diversification when optimizing portfolio weights, and (iii) allowing the relative weights on different factors to vary with market volatility. In the rest of this section, we examine these drivers more closely.

4.2 Trading diversification of multifactor portfolios

To investigate the source of the trading-diversification benefits that are one of the critical drivers of the favorable performance of the conditional multifactor portfolios, Figure 5 compares three quantities for each factor: (i) its mean gross return, (ii) its mean return net of transaction costs ignoring trading diversification, and (iii) its mean return net of transaction costs accounting for trading diversification.[30] For the case in which we account for trading diversification, we use the factor weights that solve problem (6) in sample.

We highlight four findings from Figure 5. First, comparing the mean gross return of each factor (red bar) with its mean net return when ignoring trading diversification (gray bar), we observe that transaction costs substantially reduce mean returns. For instance,

[30] We use Equations (12) and (14) of DeMiguel et al. (2020) to compute the transaction cost of each factor accounting for and ignoring trading diversification, respectively. We then report the difference between the gross mean return of each factor and its transaction cost without or with trading diversification.

Table 4: Sources of trading-diversification benefits

This table reports the out-of-sample and net-of-costs performance of the unconditional (UMV) and conditional (CMV) multifactor portfolios. We evaluate the performance of the conditional multifactor portfolio for three different cases: (1) taking trading diversification fully into account (that is, netting trades across all unmanaged and managed factors), (2) taking trading diversification into account only partially (netting trades only across the unmanaged and managed versions of each individual factor, but not across different factors), and (3) ignoring trading diversification altogether. The sample period and quantities reported for each portfolio are the same as in Table 2.

	Case (1) with full trading div. within & across factors		Case (2) with trading div. only within factors	Case (3) without any trading div.
	UMV	CMV	CMV	CMV
Mean	0.430	0.477	0.351	0.332
Standard deviation	0.458	0.449	0.449	0.449
Sharpe ratio	0.940	1.062	0.783	0.739
p-value($\mathrm{SR_{CMV}} - \mathrm{SR_{UMV}}$)		0.006	1.000	1.000
α		0.066	−0.060	−0.079
$t(\alpha)$		3.637	−3.228	−4.223
TC	0.163	0.213	0.339	0.358

when ignoring trading diversification, the mean net returns of three of the unmanaged factors (HML, UMD, and IA) are less than half their mean gross returns.

Second, transaction costs are even more critical for the profitability of the managed factors, with four of them (SMB/σ_t, HML/σ_t, CMA/σ_t, and IA/σ_t) having *negative* mean net returns when we ignore trading diversification.

Third, trading diversification helps to explain why the conditional multifactor portfolio outperforms the unconditional multifactor portfolio even in the presence of transaction costs. In particular, although both multifactor portfolios benefit from the netting of trades across factors, the benefits are relatively larger for the conditional portfolios because they exploit managed factors, which are more expensive to trade.[31]

Fourth, most of the benefits from trading diversification arise from the netting of trades across different factors rather than across just the managed and unmanaged versions of each individual factor. To demonstrate this, Table 4 reports the out-of-sample performance of the conditional multifactor portfolio evaluated for three different cases: (1) taking trading diversification fully into account (that is, netting trades across all unmanaged and managed

[31] Note that the managed SMB factor achieves a mean net return accounting for trading diversification that is larger than its mean gross return. This is because the rebalancing trades of the conditional mean-variance multifactor portfolio are negatively correlated with those of the managed SMB factor. Thus one can effectively exploit the managed SMB factor at a negative transaction cost.

factors), (2) taking trading diversification into account only partially (netting trades only across the unmanaged and managed versions of each individual factor, but not across different factors), and (3) ignoring trading diversification altogether.

Case (3) in Table 4 shows that the out-of-sample Sharpe ratio of the conditional multifactor portfolio when we ignore trading diversification is 0.739, which is smaller than even that of its unconditional counterpart when we ignore trading diversification, 0.758, as shown earlier in Table 3 (Column (4) of Panel C). Case (2) in Table 4 shows that allowing for trading diversification across just the unmanaged and managed versions of each individual factor increases the Sharpe ratio of the conditional multifactor portfolio only marginally from 0.739 to 0.783. However, Case (1) shows that allowing for trading diversification across all unmanaged and managed factors substantially increases the Sharpe ratio of the conditional multifactor portfolio from 0.783 to 1.062, making it significantly higher than that of the unconditional portfolio, 0.940.[32]

In summary, most of the trading-diversification benefits enjoyed by the conditional multifactor portfolio arise from the netting of trades across different factors. Thus, the favorable performance of the conditional *multifactor* portfolio compared to the volatility-managed *individual-factor* portfolios is explained partly by the benefits of trading diversification across *multiple* factors.[33]

4.3 Time variation of multifactor portfolio weights

We now study how the conditional multifactor portfolio benefits from the ability to time the various factors *differentially*, which is ruled out for the conditional fixed-weight portfolios. Figure 6 plots the in-sample weights from January 1977 to December 2020 of the unconditional multifactor portfolio (UMV, blue line) and the conditional multifactor portfolio (CMV, solid red line) that account for transaction costs and trading diversification.[34]

[32] One can make the same inference by comparing the alpha t-statistics or transaction costs of these three portfolios instead of their Sharpe ratios.

[33] Another reason that the multifactor portfolio outperforms the individual-factor portfolios is that it takes advantage of the risk-diversification benefits from combining multiple factors. Section IA.20 of the Internet Appendix shows that the market and size factors are moderately negatively correlated to the other seven factors, and thus multifactor portfolios benefit substantially from risk diversification across factors.

[34] We consider in-sample weights in this section so that the weights of the unconditional mean-variance portfolio are constant over time, which allows us to interpret the time variation of the conditional multifactor portfolio weights. However, we show in Section IA.21 of the Internet Appendix that our insights are robust to considering the out-of-sample weights of the conditional and unconditional multifactor portfolios.

The figure also depicts the *average* weights of the conditional multifactor portfolio (E[CMV], dashed red line).

Figure 6 shows that the unconditional multifactor portfolio assigns a strictly positive weight to every factor except value (HML), to which it assigns a zero weight. This is not surprising given that Panel B of Table 1 shows that the Sharpe ratio of net returns of the HML factor is only 5.3%, the smallest across the nine factors. The figure also shows that the conditional multifactor portfolio assigns an almost-constant weight to three factors (MKT, SMB, CMA), while timing aggressively the other six (HML, RMW, UMD, ROE, IA, BAB). For instance, the weights of the conditional multifactor portfolio on these six factors drop dramatically during the Great Recession and after the Early 2000's Recession, but they increase during periods of low market volatility, such as 1992 to 1997. Thus, our conditional portfolio takes advantage of the opportunity to time factors differentially.

Because our conditional multifactor portfolio times the factors differentially, it optimally assigns an *average* weight to each factor that differs substantially also from that of the unconditional portfolios, as shown in Figure 6. For example, the conditional portfolio assigns a much higher average weight to the value (HML), momentum (UMD), and betting-against-beta (BAB) factors than the unconditional portfolio. Interestingly, allowing the relative weight of each factor to vary with market volatility "resurrects" the value (HML) factor, to which the conditional multifactor portfolio assigns a substantial average weight of about 0.35. On the other hand, the conditional portfolio assigns a substantially lower average weight to the investment factors (CMA and IA) than the unconditional portfolio.[35]

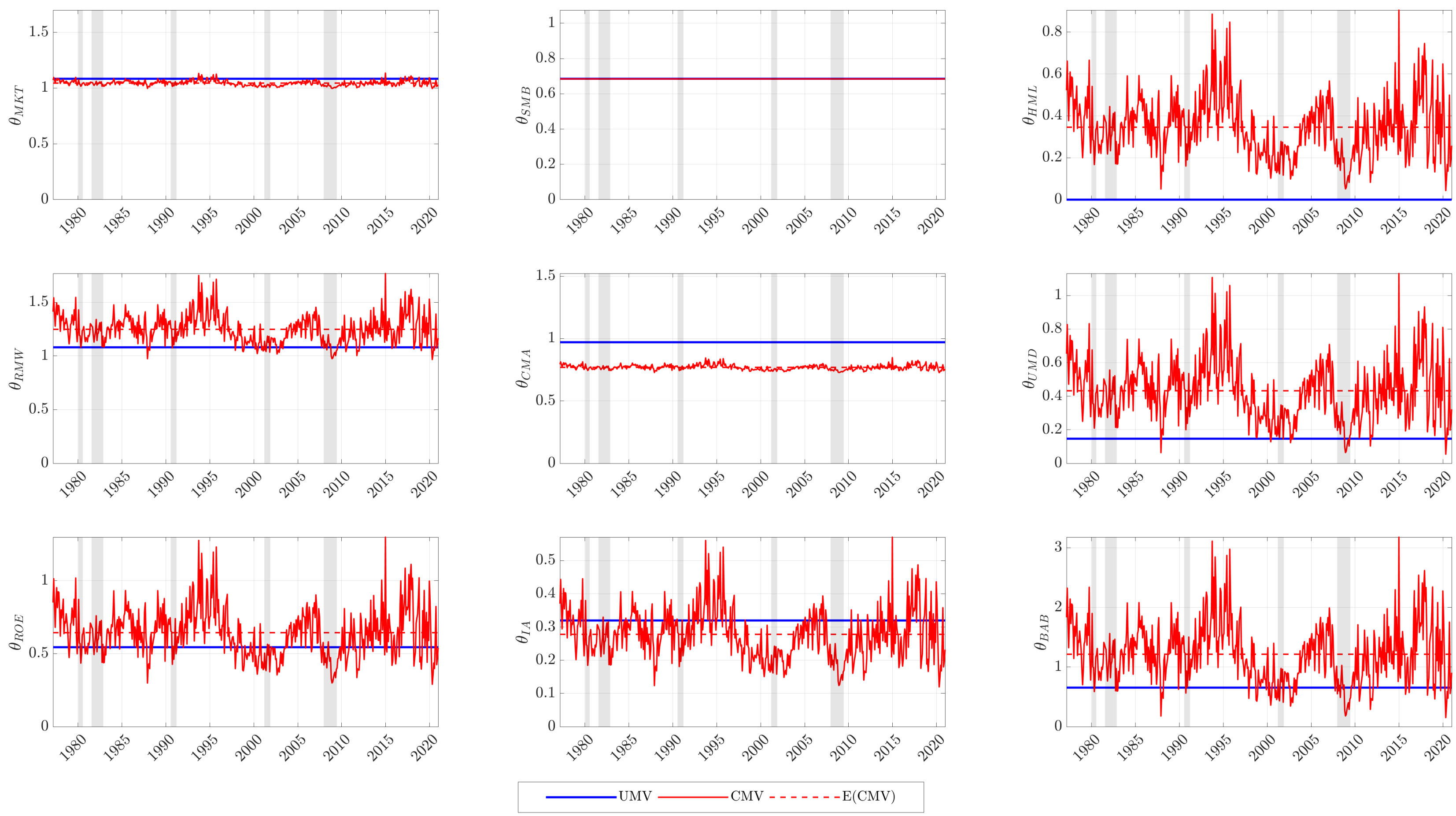
5 Economic mechanism and implications

In this section, we first characterize the economic mechanism driving the performance of the conditional multifactor portfolio by studying how the risk-return tradeoff for individual factors varies with market volatility. We then study the broader economic implications of

[35]In contrast to our conditional multifactor portfolio, the conditional *fixed-weight* portfolio is obtained by timing the unconditional multifactor portfolio in its entirety, and thus, its relative weight on each factor coincides with that of the unconditional portfolio. Consequently, the conditional fixed-weight portfolio has zero weight on HML, just like the unconditional portfolio. In unreported results, we find that for the other factors, the *average* weight assigned by the conditional fixed-weight portfolio is also similar to that assigned by the unconditional portfolio.

Figure 6: Weights of unconditional and conditional multifactor portfolios

This figure depicts the in-sample weights of the unconditional multifactor portfolio (UMV, blue line) and the conditional multifactor portfolio (CMV, solid red line) from January 1977 to December 2020. The figure also depicts the average weights of the conditional multifactor portfolio (E[CMV], dashed red line). Each of the nine graphs depicts the weights for a particular factor.



our work by estimating a conditional stochastic discount factor whose price of risk for each factor varies with market volatility.

5.1 Factor risk-return tradeoff and market volatility

Moreira and Muir (2017, figure 1) show that the risk-return tradeoff for the market weakens with market volatility and this explains the outperformance of the volatility-managed *market* portfolio. In particular, they find that "there is little relation between lagged volatility and average returns, but there is a strong relation between lagged volatility and current volatility. This means that the mean-variance tradeoff weakens in periods of high volatility." Thus, a mean-variance investor should decrease exposure to the market when realized market volatility is high.

We extend the analysis in Moreira and Muir (2017) to the other factors in our dataset, besides the market. Our key finding is that for all nine individual factors the risk-return tradeoff *weakens* with realized market volatility. This explains why our conditional multi-factor portfolio, which reduces exposure to the risk factors when realized market volatility is high, outperforms its unconditional counterpart.

Figure 1 in the introduction depicts how the risk-return tradeoff for the nine factors varies with realized market volatility. In the figure, we first use the monthly time series of realized market volatility to sort the months in our sample into terciles. For each factor, we then estimate the risk-return tradeoff for month t as the realized factor return for month $t + 1$ divided by the monthly realized factor variance estimated as the sample variance of daily returns for month t. Finally, we report the risk-return tradeoff averaged across the months in each tercile.[36]

Figure 1 shows that the risk-return tradeoff for all nine factors *weakens* with realized market volatility. To see this, note that the risk-return tradeoff for the low-market-volatility tercile (blue bars) is higher than that for the high-market-volatility tercile (red bars) for every factor. Moreover, the weakening of the risk-return tradeoff is substantial for some of the factors (UMD, ROE, and BAB) but less striking for others (MKT, SMB, and CMA).[37]

[36]Section IA.22 of the Internet Appendix shows that the findings from Figure 1 are robust to estimating the factor risk-return tradeoff for each volatility tercile, instead of estimating it for each month and then computing the average risk-return tradeoff across the months in that volatility tercile.

[37]Note that the mean-variance tradeoff for some of the factors (UMD, ROE, and BAB) is above 50 for the low-market-volatility tercile, which may seem unreasonably large. These large tradeoffs stem from a

This explains why the conditional multifactor portfolio assigns an almost-constant weight to the MKT, SMB, and CMA factors but a time-varying weight to the rest of the factors, as shown in Figure 6.

5.2 Conditional stochastic discount factor

To understand the broader economic implications of our work, we also estimate a conditional stochastic discount factor whose price of risk for each factor can vary with inverse market volatility. To do this, we extend the unconditional approach of Barroso and Maio (2021) to study how the prices of risk for the nine factors vary conditional on market volatility.

To set the stage for our empirical analysis, we assume that, conditional on realized market volatility σ_t, there is a stochastic discount factor (SDF) that prices tradable assets; that is, $0 = E_t(M^{\sigma}_{t+1} r^e_{t+1})$, where M^{σ}_{t+1} is the conditional SDF at time $t+1$ and r^e_{t+1} is the vector of excess asset returns at time $t+1$. Furthermore, we assume that the price of risk for the kth factor is an affine function of inverse realized market volatility at time t so that:

$$M^{\sigma}_{t+1} = 1 - \sum_{k=1}^{K} (\alpha_k + \beta_k \tilde{\sigma}_t^{-1}) \tilde{r}_{k,t+1}, \tag{13}$$

where $\tilde{\sigma}_t^{-1}$ is the demeaned inverse realized market volatility at time t, $\tilde{\sigma}_t^{-1} = 1/\sigma_t - E(1/\sigma_t)$, and $\tilde{r}_{k,t+1}$ is the conditionally demeaned kth factor return at time $t+1$, $\tilde{r}_{k,t+1} = r_{k,t+1} - E_t(r_{k,t+1})$. The SDF prices every traded factor return, and thus:[38]

$$E_t(r_{i,t+1}) = \sum_{k=1}^{K} (\alpha_k + \beta_k \tilde{\sigma}_t^{-1}) \mathrm{cov}_t(r_{i,t+1}, r_{k,t+1}), \quad \text{for } i = 1, 2, \ldots, K, \tag{14}$$

where $\mathrm{cov}_t(r_{i,t+1}, r_{k,t+1})$ is the conditional covariance between the ith and kth factor returns.

One could estimate the coefficients α_k and β_k by running a pooled conditional regression for the K equations in (14), but this would require estimating a large number of $K(K+1)/2$ realized factor variances and covariances each month. We consider two approaches to address the challenge of estimating a large covariance matrix. First, one could

combination of high mean returns and low variance. However, we have also computed the corresponding Sharpe ratios and we find that they are below one for every factor across all three terciles. For instance, for the low-market-volatility tercile, the Sharpe ratio for the BAB factor is 0.83 and for the ROE factor is 0.59.

[38] To see this, note that $0 = E_t(M^{\sigma}_{t+1} r_{i,t+1}) = E_t(M^{\sigma}_{t+1}) E_t(r_{i,t+1}) + \mathrm{cov}_t(M^{\sigma}_{t+1}, r_{i,t+1})$. Because $E_t(M^{\sigma}_{t+1}) = 1$, we have that $E_t(r_{i,t+1}) = -\mathrm{cov}_t(M^{\sigma}_{t+1}, r_{i,t+1}) = \sum_{k=1}^{K} (\alpha_k + \beta_k \tilde{\sigma}_t^{-1}) \mathrm{cov}_t(r_{i,t+1}, r_{k,t+1})$.

ignore the correlations between the different factors, which would lead to a more parsimonious (robust) approach. Second, one could estimate the pooled conditional regression in (14) but use a shrinkage estimator of the covariance matrix of factor returns. We pursue the first approach below and show in Section IA.23 of the Internet Appendix that our findings are similar when using the second approach. Ignoring the correlations between different factors, (14) simplifies to

$$E_t(r_{k,t+1}) = (\alpha_k + \beta_k \tilde{\sigma}_t^{-1})\text{var}_t(r_{k,t+1}), \quad \text{for } k = 1, 2, \ldots, K, \tag{15}$$

which requires estimating only K realized factor-return variances each month. Moreover, Equation (15) implies that, when factor returns are uncorrelated, the conditional risk-return tradeoff for the kth factor, given by $E_t(r_{k,t+1})/\text{var}_t(r_{k,t+1})$, is equal to the price of risk for the kth factor, $\alpha_k + \beta_k \tilde{\sigma}_t^{-1}$, that is:

$$\frac{E_t(r_{k,t+1})}{\text{var}_t(r_{k,t+1})} = \alpha_k + \beta_k \tilde{\sigma}_t^{-1}, \quad \text{for } k = 1, 2, \ldots, K. \tag{16}$$

To test how the price of risk for the kth factor varies with market volatility, we estimate the following time-series regression:

$$\frac{r_{k,t+1}}{\sigma_{k,t}^2} = \alpha_k + \beta_k \tilde{\sigma}_t^{-1} + \epsilon_{k,t+1}, \tag{17}$$

where $\sigma_{k,t}^2$ is the monthly realized variance of the kth factor estimated as the sample variance of daily returns over month t, and $\epsilon_{k,t+1}$ is the residual at time $t+1$. Note that one can estimate the unconditional price of risk for the kth factor as $E(r_{k,t+1}/\sigma_{k,t}^2) = E(\alpha_k + \beta_k \tilde{\sigma}_t^{-1} + \epsilon_{k,t+1}) = \alpha_k$ because $E(\tilde{\sigma}_t^{-1}) = 0$ and $E(\epsilon_{k,t+1}) = 0$. Thus, to test whether the *unconditional* price of risk for the kth factor is positive, we can use the t-statistic for the estimated coefficient α_k. More importantly, to test whether the *conditional* price of risk for the kth factor *weakens* with market volatility, we can use the t-statistic for the estimated coefficient β_k.

Table 5 reports the results for the time-series regressions in (17) for the nine factors. Our first observation is that the estimated coefficient α_k is positive for all nine factors. This indicates that, as one would expect, the unconditional price of risk for the nine factors is positive. Moreover, the unconditional price of risk is significant for MKT, RMW, CMA, UMD, ROE, IA, and BAB.

More importantly, our second observation from Table 5 is that, consistent with the results in Figure 1, the estimated coefficient β_k is positive for every individual factor, which

Table 5: Factor risk prices and market volatility

This table reports the coefficients α_k and β_k for the time-series regression defined in Equation (17) for the nine factors in our dataset. The numbers in square brackets are Newey-West t-statistics. The sample spans January 1977 to December 2020.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
α_k	6.668	5.479	7.573	30.467	13.341	33.565	46.118	20.979	41.824
	[3.790]	[1.494]	[1.258]	[4.499]	[2.240]	[6.676]	[7.101]	[3.820]	[8.678]
β_k	0.512	0.155	1.002	1.155	0.329	1.561	1.361	0.770	2.311
	[2.358]	[0.420]	[2.270]	[2.236]	[0.749]	[3.840]	[2.621]	[1.982]	[7.204]

indicates that the conditional price of risk for all nine factors decreases with realized market volatility. This is a counterintuitive result because one expects that the price of risk of systematic risk factors should *not* decrease with market volatility. We also observe that the reduction in the price of risk is significant for some of the factors (MKT, HML, RMW, UMD, ROE, and BAB), but not for others (SMB, CMA, and IA).[39] Thus, although conditioning on volatility helps to construct a stochastic discount factor that better spans the investment opportunity set, the importance of conditioning on volatility to achieve this goal varies across factors.[40]

6 Conclusion

We develop a new strategy that exploits market volatility to time investment in popular asset-pricing factors. Instead of timing an individual equity factor conditional on its variance or timing a *fixed* combination of factors conditional on the variance of that combination, we consider a conditional *multifactor* portfolio whose relative weight on each factor can vary with market volatility. We show that the conditional multifactor portfolio outperforms its unconditional counterpart even out of sample and net of transaction costs. To study the economic mechanism driving the performance of the conditional multifactor portfolio,

[39] The statistical significance of β_k for the UMD and BAB factors is consistent with the findings by Barroso and Santa-Clara (2015b), Cederburg and O'Doherty (2016), and Barroso et al. (2021) that timing the volatility of the UMD and BAB factors produces substantial gains.

[40] Note that $\tilde{\sigma}_t^{-1}$ can be negative, and thus, the expected risk-return tradeoff predicted by the conditional regression in (17), $E_t(r_{k,t+1})/\sigma^2_{k,t} = \alpha_k + \beta_k\tilde{\sigma}_t^{-1}$, could be negative. However, we find empirically that the expected risk-return tradeoff predicted by the regression is positive for every month for SMB, RMW, CMA, ROE and IA, for more than 95% of the months for UMD and BAB, for more than 85% of the months for MKT, and for around 70% of the months for HML. This is reassuring because, for the factors we consider, the expected risk-return tradeoff should be positive.

we estimate the factor risk-return tradeoff and prices of risk and find that they generally decrease with market volatility. This is counterintuitive because one would expect the price of risk of systematic factors to remain constant or increase with market volatility. Thus, the breakdown of the most fundamental premise in finance, that between risk and return, is more puzzling than previously thought.

A Appendix: Factor definition

We consider the same nine factors as Moreira and Muir (2017). This includes the six factors—MKT, SMB, HML, RMW, CMA, and UMD—as constructed in Fama and French (2018), the Hou et al. (2015) profitability and investment factors (ROE and IA), and the Frazzini and Pedersen (2014) BAB factor. The SMB and HML Fama-French factors are constructed using six value-weighted portfolios formed as the intersection of stocks sorted independently on two size buckets (big and small) and three book-to-market buckets (value, neutral, and growth). The RMW, CMA and UMD factors are constructed using six value-weighted portfolios formed as the intersection of stocks sorted independently on two size buckets (big and small) and three buckets using operating profitability, asset growth, and prior returns from month -12 to -2, respectively. Similarly, the ROE and IA factors of Hou et al. (2015) are constructed using 18 value-weighted portfolios formed as the intersection of stocks sorted independently on two size buckets (big and small), three profitability (return on equity) buckets, and three investment (asset growth) buckets. The Fama-French and Hou et al. (2015) factors use NYSE breakpoints to define the value-weighted portfolios for the construction of the factors. Below, we summarize how each factor is constructed.

1. Market (MKT): Excess return on the value-weighted portfolio of all NYSE, AMEX, and NASDAQ firms with a CRSP share code of 10 or 11.

2. Size (SMB): Average return on the three value-weighted small portfolios minus the average return on the three value-weighted big portfolios.

3. Value (HML): Average return on the two high-book-to-market value-weighted portfolios minus the average return on the two low-book-to-market value-weighted portfolios.

4. Robust-minus-weak (RMW): Average return on the two robust (i.e., high) operating-profitability value-weighted portfolios minus the average return on the two weak (i.e., low) operating-profitability value-weighted portfolios.

5. Conservative minus aggressive (CMA): Average return on the two conservative-investment (i.e., low asset growth) value-weighted portfolios minus the average return on the two aggressive-investment (i.e., high asset growth) value-weighted portfolios.

6. Momentum (UMD): Average return on the two high-prior-return value-weighted portfolios minus the average return on the two low-prior-return value-weighted portfolios.

7. Profitability (ROE): Average return on the six high-return-on-equity value-weighted portfolios minus average return on six low-return-on-equity value-weighted portfolios.

8. Investment (IA): Average return on the six low-asset-growth value-weighted portfolios minus the average return on the six high-asset-growth value-weighted portfolios.

9. Betting against beta (BAB): Excess return of market neutral portfolio that buys a rank-weighted portfolio of low-beta stocks and shorts a rank-weighted portfolio of high-beta stocks.[41]

[41] Section IA.5 of the Internet Appendix shows that our findings are robust to considering a more conventional *value-weighted* BAB factor.

References

Abdi, Farshid, and Angelo Ranaldo, 2017, A simple estimation of bid-ask spreads from daily close, high, and low prices, *Review of Financial Studies* 30, 4437–4480.

Aït-Sahalia, Yacine, Felix Matthys, Emilio Osambela, and Ronnie Sircar, 2021, When uncertainty and volatility are disconnected: Implications for asset pricing and portfolio performance, NBER Working Paper.

Amenc, Noël, Mikheil Esakia, Felix Goltz, and Ben Luyten, 2019, Macroeconomic risks in equity factor investing, *Journal of Portfolio Management* 45, 39–60.

Baker, Malcolm, and Jeffrey Wurgler, 2006, Investor sentiment and the cross-section of stock returns, *Journal of Finance* 61, 1645–1680.

Baker, Scott R., Nicholas Bloom, and Steven J. Davis, 2016, Measuring economic policy uncertainty, *Quarterly Journal of Economics* 131, 1593–1636.

Barroso, Pedro, and Andrew L. Detzel, 2021, Do limits to arbitrage explain the benefits of volatility-managed portfolios?, *Journal of Financial Economics* 140, 744–767.

Barroso, Pedro, Andrew L. Detzel, and Paulo F. Maio, 2021, The volatility puzzle of the low-risk anomaly, SSRN Working Paper.

Barroso, Pedro, and Paulo F. Maio, 2021, The risk-return tradeoff among equity factors, SSRN Working Paper.

Barroso, Pedro, and Pedro Santa-Clara, 2015a, Beyond the carry trade: Optimal currency portfolios, *Journal of Financial and Quantitative Analysis* 50, 1037–1056.

Barroso, Pedro, and Pedro Santa-Clara, 2015b, Momentum has its moments, *Journal of Financial Economics* 116, 111–120.

Bass, Robert, Scott Gladstone, and Andrew Ang, 2017, Total portfolio factor, not just asset, allocation, *Journal of Portfolio Management* 43, 38–53.

Bender, Jennifer, Jerry Sun, and Ric Thomas, 2018, Asset allocation vs. factor allocation—Can we build a unified method?, *Journal of Portfolio Management* 45, 9–22.

Blin, Olivier, Florian Ielpo, Joan Lee, and Jerome Teiletche, 2018, Factor timing revisited: Alternative risk premia allocation based on Nowcasting and valuation signals, SSRN Working Paper.

Breen, William, Lawrence R. Glosten, and Ravi Jagannathan, 1989, Economic significance of predictable variations in stock index returns, *Journal of Finance* 44, 1177–1189.

Campbell, John Y., and Ludger Hentschel, 1992, No news is good news: An asymmetric model of changing volatility in stock returns, *Journal of Financial Economics* 31, 281–318.

Campbell, John Y., and Samuel B. Thompson, 2008, Predicting excess stock returns out of sample: Can anything beat the historical average?, *Review of Financial Studies* 21, 1509–1531.

Cederburg, Scott, and Michael S. O'Doherty, 2016, Does it pay to bet against beta? On the conditional performance of the beta anomaly, *Journal of Finance* 71, 737–774.

Cederburg, Scott, Michael S. O'Doherty, Feifei Wang, and Xuemin (Sterling) Yan, 2020, On the performance of volatility-managed portfolios, *Journal of Financial Economics* 138, 95–117.

Cejnek, Georg, and Florian Mair, 2021, Understanding volatility-managed portfolios, SSRN Working Paper.

Chen, Andrew Y., and Mihail Velikov, 2023, Zeroing in on the expected returns of anomalies, *Journal of Financial and Quantitative Analysis* 58, 968–1004.

Chernov, Mikhail, Lars A. Lochstoer, and Stig R. H. Lundeby, 2022, Conditional dynamics and the multihorizon risk-return trade-off, *Review of Financial Studies* 35, 1310–1347.

De Franco, Carmine, Massimo Guidolin, and Bruno Monnier, 2017, The robustness of the volatility factor: Linear versus nonlinear factor model, *Journal of Index Investing* 8, 75–88.

DeMiguel, Victor, Lorenzo Garlappi, and Raman Uppal, 2009, Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy?, *Review of Financial Studies* 22, 1915–1953.

DeMiguel, Victor, Alberto Martin-Utrera, Francisco J. Nogales, and Raman Uppal, 2020, A transaction-cost perspective on the multitude of firm characteristics, *Review of Financial Studies* 33, 2180–2222.

DeMiguel, Victor, Francisco J. Nogales, and Raman Uppal, 2014, Stock return serial dependence and out-of-sample portfolio performance, *Review of Financial Studies* 27, 1031–1073.

Ehsani, Sina, and Juhani T. Linnainmaa, 2022, Factor momentum and the momentum factor, *The Journal of Finance* 77, 1877–1919.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

Fama, Eugene F., and Kenneth R. French, 2018, Choosing factors, *Journal of Financial Economics* 128, 234–252.

Fleming, Jeff, Chris Kirby, and Barbara Ostdiek, 2001, The economic value of volatility timing, *Journal of Finance* 56, 329–352.

Fleming, Jeff, Chris Kirby, and Barbara Ostdiek, 2003, The economic value of volatility timing using "realized" volatility, *Journal of Financial Economics* 67, 473–509.

Frazzini, Andrea, Ronen Israel, and Tobias J. Moskowitz, 2015, Trading costs of asset pricing anomalies, Fama-Miller Working Paper.

Frazzini, Andrea, and Lasse H. Pedersen, 2014, Betting against beta, *Journal of Financial Economics* 111, 1–25.

French, Kenneth R., G. William Schwert, and Robert F. Stambaugh, 1987, Expected stock returns and volatility, *Journal of Financial Economics* 19, 3–29.

Glosten, Lawrence R., Ravi Jagannathan, and David E. Runkle, 1993, On the relation between the expected value and the volatility of the nominal excess return on stocks, *Journal of Finance* 48, 1779–1801.

Green, Jeremiah, John R. M. Hand, and Frank Zhang, 2017, The characteristics that provide independent information about average US monthly stock returns, *Review of Financial Studies* 30, 4389–4436.

Gupta, Tarun, and Bryan Kelly, 2019, Factor momentum everywhere, *Journal of Portfolio Management* 45, 13–36.

Gârleanu, Nicolae, and Lasse H. Pedersen, 2013, Dynamic trading with predictable returns and transaction costs, *The Journal of Finance* 68, 2309–2340.

Gómez-Cram, Roberto, 2021, Late to recessions: Stocks and the business cycle, *Journal of Finance* 77, 923–966.

Haddad, Valentin, Serhiy Kozak, and Shrihari Santosh, 2020, Factor timing, *Review of Financial Studies* 33, 1980–2018.

Harvey, Campbell R., and Yan Liu, 2021, Lucky factors, *Journal of Financial Economics* 141, 413–435.

Herskovic, Bernard, Alan Moreira, and Tyler Muir, 2020, Hedging risk factors, UCLA Working Paper.

Hodges, Philip, Ked Hogan, Justin R. Peterson, and Andrew Ang, 2017, Factor timing with cross-sectional and time-series predictors, *Journal of Portfolio Management* 44, 30–43.

Hou, Kewei, Chen Xue, and Lu Zhang, 2015, Digesting anomalies: An investment approach, *Review of Financial Studies* 28, 650–705.

Jobson, John D., and Bob M. Korkie, 1981, Performance hypothesis testing with the Sharpe and Treynor measures, *Journal of Finance* 889–908.

Ledoit, Olivier, and Michael Wolf, 2004, A well-conditioned estimator for large-dimensional covariance matrices, *Journal of Multivariate Analysis* 88, 365–411.

Ledoit, Olivier, and Michael Wolf, 2008, Robust performance hypothesis testing with the Sharpe ratio, *Journal of Empirical Finance* 15, 850–859.

Ledoit, Olivier, and Michael Wolf, 2020, Analytical nonlinear shrinkage of large-dimensional covariance matrices, *Annals of Statistics* 48, 3043–3065.

Liu, Fang, Xiaoxiao Tang, and Guofu Zhou, 2019, Volatility-managed portfolio: Does it really work?, *Journal of Portfolio Management* 46, 38–51.

Lo, Andrew W., and A. Craig MacKinlay, 1990, When are contrarian profits due to stock market overreaction?, *Review of Financial Studies* 3, 175–205.

Lochstoer, Lars, and Tyler Muir, 2022, Volatility expectations and returns, *Journal of Finance* 77, 1055–1096.

Marquering, Wessel, and Marno Verbeek, 2004, The economic value of predicting stock index returns and volatility, *Journal of Financial and Quantitative Analysis* 39, 407–429.

Miller, Keith L., Hong Li, Tiffany G. Zhou, and Daniel Giamouridis, 2015, A risk-oriented model for factor timing decisions, *Journal of Portfolio Management* 41, 46–58.

Moreira, Alan, and Tyler Muir, 2017, Volatility-managed portfolios, *Journal of Finance* 72, 1611–1644.

Moreira, Alan, and Tyler Muir, 2019, Should long-term investors time volatility?, *Journal of Financial Economics* 131, 507–527.

Nelson, Daniel B., 1991, Conditional heteroskedasticity in asset returns: A new approach, *Econometrica* 59, 347–370.

Novy-Marx, Robert, and Mihail Velikov, 2016, A taxonomy of anomalies and their trading costs, *Review of Financial Studies* 29, 104–147.

Novy-Marx, Robert, and Mihail Velikov, 2022, Betting against betting against beta, *Journal of Financial Economics* 143, 80–106.

Olivares-Nadal, Alba V., and Victor DeMiguel, 2018, A robust perspective on transaction costs in portfolio optimization, *Operations Research* 66, 733–739.

Politis, Dimitris N., and Joseph P. Romano, 1994, The stationary bootstrap, *Journal of the American Statistical Association* 89, 1303–1313.

Yu, Jianfeng, and Yu Yuan, 2011, Investor sentiment and the mean-variance relation, *Journal of Financial Economics* 100, 367–381.

Internet Appendix to

A Multifactor Perspective on Volatility-Managed Portfolios

This Internet Appendix reports the following robustness checks and additional results:

1. evaluating performance of individual-factor portfolios using a longer sample,
2. evaluating performance of individual-factor portfolios using cost-mitigation strategies,
3. evaluating performance of individual-factor portfolios using market volatility instead of factor variance as a conditioning variable,
4. considering a larger set of 66 equity risk factors,
5. evaluating performance after excluding one factor at a time, or using a more conventional BAB factor,
6. evaluating performance during periods of high market volatility,
7. evaluating performance in the first and second halves of our sample,
8. evaluating performance during high- and low-sentiment periods,
9. evaluating performance using different investment horizons,
10. evaluating performance exploiting other volatility measures instead of market volatility,
11. evaluating performance of the conditional multifactor portfolio using transaction-cost-mitigation strategies,
12. relaxing the nonnegativity constraints on the factor weights,
13. constraining the leverage of the conditional multifactor portfolio,
14. using a less parsimonious conditional multifactor portfolio,
15. exploiting other conditioning variables in addition to inverse market volatility,
16. evaluating performance using alternative proportional-transaction costs,
17. evaluating performance using quadratic price-impact costs,
18. evaluating performance after setting the cost for trading the market to zero,
19. evaluating performance using alternative measures of risk,
20. reporting the risk-diversification benefits that arise because the returns on the nine factors are less than perfectly correlated,
21. considering the out-of-sample factor weights to explain performance,
22. considering an alternative estimator of conditional factor risk-return tradeoff, and

23. considering an alternative estimator of factor risk prices,

24. considering alternative methods for computing p-values for Sharpe-ratio differences.

IA.1 Individual-factor portfolios: Longer sample

In the main body of the manuscript, we consider a sample spanning the period 1967 to 2020, for which there are data to construct all nine factors we consider. This allows us to evaluate the performance of the conditional multifactor portfolios that exploit all nine factors. However, in this section we evaluate the performance of each of the volatility-managed *individual-factor* portfolios for longer samples.

Table IA.1 reports the performance of each volatility-managed individual-factor portfolio for longer samples. In particular, for the MKT, SMB, and HML factors the sample is from October 1926 to December 2020, for RMW and CMA from August 1963 to December 2020, for UMD from February 1927 to December 2020, for ROE and IA from January 1967 to December 2020, and for BAB from January 1931 to December 2020. Note that the sample for ROE and IA that we use in Table IA.1 coincides with that in the main body of the manuscript because we do not have access to the data required to construct the ROE and IA factors for a longer sample. To facilitate comparison, both the in-sample and out-of-sample performance are reported for the same period, which excludes the first 100 months in the sample that are used as the first estimation window in the out-of-sample evaluation.

Comparing Table IA.1 with Table 1 in the manuscript, we observe that, although there are some differences in the performance of specific individual-factor portfolios across the two samples, the key takeaways from each panel are similar across the two tables. Comparing Panel A in the two tables, we observe that in-sample and without transaction costs the same five individual-factor portfolios significantly outperform their corresponding unmanaged factors at the 10% confidence level for both samples (RMW, UMD, ROE, IA, and BAB). Comparing Panel B in the two tables, we observe that in-sample and net of transaction costs without trading diversification, none of the individual-factor portfolios is significantly better than their corresponding unmanaged factor for either sample. Comparing Panel C in the two tables, we observe that out-of-sample and without transaction costs the same four individual-factor portfolios significantly outperform their corresponding unmanaged factors at the 10% level (UMD, ROE, IA, and BAB). Comparing Panel D in the two tables, we observe that out-of-sample and net of transaction costs without trading diversification, none

Table IA.1: Performance of individual-factor portfolios: Longer sample

For each of the nine factors we consider, this table reports the annualized Sharpe ratios of the unmanaged factor, $\mathrm{SR}(r_k)$, and the volatility-managed individual-factor portfolio, $\mathrm{SR}(r_k, r_k^\sigma)$, which is the mean-variance combination of the unmanaged factor with its managed counterpart, and the p-value for the difference in Sharpe ratios. We consider an investor with risk-aversion parameter $\gamma = 5$. Panel A reports performance in-sample and ignoring transaction costs, Panel B in-sample and net of costs but ignoring trading diversification, Panel C out-of-sample and ignoring costs, Panel D out-of-sample and net of costs but ignoring trading diversification, and Panel E out-of-sample and net of costs considering trading diversification. We use a longer sample for each factor compared to the main body of the manuscript. In particular, for the MKT, SMB, and HML factors the sample is from October 1926 to December 2020, for RMW and CMA from August 1963 to December 2020, for UMD from February 1927 to December 2020, for ROE and IA from January 1967 to December 2020, and for BAB from January 1931 to December 2020. To facilitate comparison, both the in-sample and out-of-sample performance are reported for the same period, which excludes the first 100 months in the sample that are used as the first estimation window in the out-of-sample evaluation.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
Panel A: In-sample without transaction costs									
$\mathrm{SR}(r_k)$	0.509	0.216	0.367	0.411	0.512	0.533	0.722	0.508	0.831
$\mathrm{SR}(r_k, r_k^\sigma)$	0.545	0.226	0.400	0.623	0.538	0.974	1.153	0.621	1.128
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.211	0.400	0.256	0.048	0.238	0.000	0.001	0.099	0.000
Panel B: In-sample net of transaction costs but without trading diversification									
$\mathrm{SR}(r_k)$	0.499	0.140	0.262	0.262	0.275	0.184	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.500	0.140	0.262	0.262	0.275	0.232	0.331	0.107	0.623
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.470	0.500	0.500	0.500	0.500	0.296	0.400	0.500	0.326
Panel C: Out-of-sample without transaction costs									
$\mathrm{SR}(r_k)$	0.509	0.216	0.367	0.411	0.512	0.533	0.722	0.508	0.831
$\mathrm{SR}(r_k, r_k^\sigma)$	0.440	0.149	0.289	0.517	0.407	0.832	1.094	0.605	1.115
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.790	0.809	0.824	0.278	0.996	0.014	0.000	0.062	0.001
Panel D: Out-of-sample net of transaction costs but without trading diversification									
$\mathrm{SR}(r_k)$	0.499	0.140	0.262	0.262	0.275	0.184	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.333	−0.078	−0.095	−0.482	−0.017	0.116	0.269	−0.127	0.535
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.975	0.992	1.000	0.999	1.000	0.741	0.672	1.000	0.796
Panel E: Out-of-sample net of transaction costs with trading diversification									
$\mathrm{SR}(r_k)$	0.499	0.140	0.262	0.262	0.275	0.184	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.366	0.068	0.196	0.180	0.214	0.103	0.324	0.193	0.596
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.966	0.902	0.922	0.835	0.939	0.793	0.403	0.026	0.596

of the individual-factor portfolios is significantly better than its corresponding unmanaged factor for either sample. More importantly, comparing Panel E in the two tables, we observe that out-of-sample and with transaction costs and trading diversification, three out of the nine individual-factor portfolios significantly outperform their unmanaged counterparts at the 10% level for the shorter sample, and only one for the longer sample (IA).[42]

[42]Note that Moreira and Muir (2017) show that the volatility-managed market portfolio outperforms the unmanaged market factor in the sample from 1926 to 2015 even in the presence of transaction costs.

Overall, we observe that the takeaways from Table 1 of the manuscript are robust to considering longer samples.

IA.2 Individual-factor portfolios: Cost mitigation

In the main body of the manuscript, we evaluate the performance of the volatility-managed individual-factor portfolios constructed as in Moreira and Muir (2017), Cederburg et al. (2020), and Barroso et al. (2021) so that we can replicate their findings and compare our results to theirs. In this section, we evaluate the performance of the volatility-managed individual-factor portfolios after implementing several techniques to alleviate the impact of estimation error and transaction costs. First, we rely on the naive approach that assigns an equal weight to the unmanaged and volatility-managed factor. This guarantees that the exposure to the factor is always positive and alleviates the impact of estimation error (DeMiguel, Garlappi, and Uppal, 2009). Second, we estimate factor volatility using six months of daily returns, rather than a single month. Third, to increase statistical power, we perform this test using for each individual factor the longer sample considered in Section IA.1.

Table IA.2 reports the annualized Sharpe ratios of the unmanaged factors and the volatility-managed individual-factor portfolios, now obtained as the equally-weighted combination of the unmanaged factor with its managed counterpart, and the p-value for the difference in Sharpe ratios (in parenthesis). To alleviate the impact of transaction costs and estimation error, we consider volatility-managed factors scaled by the factor standard deviation, instead of the factor variance, and we estimate the standard deviation for the return of the factor using not only the prior month's daily returns (second row), but also the prior six month's daily returns (third row).

Table IA.2 shows that for the case where factor-return volatility is estimated using a six-month window, the volatility-managed individual-factor portfolios outperform significantly at the 10% level the unmanaged factors net of costs for five out of the nine factors we consider, compared to three in Table 1 of the manuscript. Thus, it is possible to improve

In contrast, Panel B of Table IA.1 shows that the volatility-managed market portfolio does not survive transaction costs. The explanation for this difference is that we drop the first 100 months in our sample so that we can compare the in-sample and out-of-sample performance of the different portfolios, and the volatility-managed market portfolio performs particularly well during the Great Depression that falls in these 100 months. In unreported analysis, we evaluate the in-sample and net-of-costs performance of the volatility-managed market portfolio from January 1926 to December 2020 and, consistent with the results in Moreira and Muir (2017), we find that it significantly outperforms the unmanaged market factor.

Table IA.2: Individual-factor portfolios: Cost-mitigating strategies

For each of the nine factors we consider, this table reports the annualized Sharpe ratios of the unmanaged factor, $\mathrm{SR}(r_k)$, and the volatility-managed individual-factor portfolios obtained as the equally weighted combination of the managed and unmanaged factors for the cases in which the managed factor is obtained using realized factor volatility estimated using one month of daily factor returns, $\mathrm{SR}(r_k, r_k^{\sigma})^{ew}$, and using six months of daily factor returns, $\mathrm{SR}(r_k, r_k^{\sigma_6})^{ew}$. To alleviate the impact of transaction costs and estimation error, we consider volatility-managed factors scaled by the factor standard deviation, instead of the factor variance. All results are net of costs considering trading diversification. We use a longer sample for each factor compared to the main body of the manuscript. In particular, for the MKT, SMB, and HML factors the sample is from October 1926 to December 2020, for RMW and CMA from August 1963 to December 2020, for UMD from February 1927 to December 2020, for ROE and IA from January 1967 to December 2020, and for BAB from January 1931 to December 2020.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
$\mathrm{SR}(r_k)$	0.430	0.136	0.212	0.251	0.227	0.145	0.272	0.224	0.512
$\mathrm{SR}(r_k, r_k^{\sigma})^{ew}$	0.474	0.041	0.157	0.170	0.066	0.266	0.331	0.136	0.608
	(0.079)	(0.999)	(0.961)	(0.976)	(1.000)	(0.001)	(0.075)	(0.997)	(0.004)
$\mathrm{SR}(r_k, r_k^{\sigma_6})^{ew}$	0.483	0.109	0.252	0.268	0.219	0.325	0.392	0.236	0.619
	(0.042)	(0.816)	(0.084)	(0.326)	(0.604)	(0.000)	(0.000)	(0.302)	(0.001)

the performance of the volatility-managed individual-factor portfolios using techniques to alleviate the impact of transaction costs and estimation error, but even after implementing these techniques four out of nine individual-factor portfolios fail to significantly outperform their associated unmanaged factors.

IA.3 Individual-factor portfolios: Market volatility

In the main body of the manuscript, we report the performance of the volatility-managed individual-factor portfolios for which the managed factor is scaled using inverse factor variance because this allows us to confirm the results in Moreira and Muir (2017), Barroso and Detzel (2021), and Cederburg et al. (2020). However, it is also useful to evaluate the performance of the volatility-managed individual-factor portfolios scaled using inverse market volatility, which we use as the conditioning variable for the multifactor portfolios.

Table IA.3 reports the performance of the volatility-managed individual-factor portfolios obtained when we scale each unmanaged factor using inverse market volatility instead of inverse factor variance (as in Table 1 in the manuscript). Comparing Table IA.3 to Table 1, we find that our key findings are similar when we use inverse market volatility instead of inverse factor variance to scale the unmanaged factors.

Table IA.3: Individual-factor portfolios: Inverse market volatility

This table reports the performance of the volatility-managed individual-factor portfolios obtained when we scale each unmanaged factor using inverse market volatility, instead of inverse factor variance (as in Table 1 of the manuscript). For each of the nine factors we consider, the table reports the annualized Sharpe ratios of the unmanaged factor, $\mathrm{SR}(r_k)$, and the volatility-managed individual-factor portfolio, $\mathrm{SR}(r_k, r_k^\sigma)$, which is the mean-variance combination of the unmanaged factor with its managed counterpart, and the p-value for the difference in Sharpe ratios. We consider an investor with risk-aversion parameter $\gamma = 5$. Panel A reports performance in-sample and ignoring transaction costs, Panel B in-sample and net of costs but ignoring trading diversification, Panel C out-of-sample and ignoring costs, Panel D out-of-sample and net of costs but ignoring trading diversification, and Panel E out-of-sample and net of costs considering trading diversification. To facilitate comparison, both the in-sample and out-of-sample performance are reported for January 1977 to December 2020.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
Panel A: In-sample without transaction costs									
$\mathrm{SR}(r_k)$	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
$\mathrm{SR}(r_k, r_k^\sigma)$	0.581	0.270	0.375	0.611	0.413	0.944	1.046	0.576	1.511
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.254	0.275	0.100	0.135	0.318	0.001	0.013	0.155	0.000
Panel B: In-sample net of transaction costs but without trading diversification									
$\mathrm{SR}(r_k)$	0.519	0.125	0.053	0.356	0.159	0.114	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.524	0.125	0.053	0.356	0.159	0.233	0.311	0.107	0.798
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.441	0.500	0.500	0.500	0.500	0.084	0.500	0.500	0.004
Panel C: Out-of-sample without transaction costs									
$\mathrm{SR}(r_k)$	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
$\mathrm{SR}(r_k, r_k^\sigma)$	0.413	0.136	0.286	0.434	0.354	0.881	0.928	0.568	1.444
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.862	0.899	0.132	0.744	0.920	0.002	0.022	0.112	0.000
Panel D: Out-of-sample net of transaction costs but without trading diversification									
$\mathrm{SR}(r_k)$	0.519	0.125	0.053	0.356	0.159	0.114	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.333	−0.121	−0.173	−0.496	−0.070	−0.170	0.058	−0.001	0.453
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.960	1.000	0.979	1.000	1.000	0.985	0.997	0.969	0.908
Panel E: Out-of-sample net of transaction costs with trading diversification									
$\mathrm{SR}(r_k)$	0.519	0.125	0.053	0.356	0.159	0.114	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.435	0.050	0.096	0.229	0.156	0.206	0.317	0.196	0.788
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.908	0.835	0.193	0.961	0.505	0.061	0.467	0.022	0.020

For instance, comparing the results for Panel A in the two tables, we find that in-sample and without transaction costs there are four individual-factor portfolios that significantly outperform their corresponding unmanaged factor (HML, UMD, ROE, and BAB) at the 10% level when we use inverse market volatility to time the individual factors compared to five factors when we use inverse factor variance (RMW, UMD, ROE, IA, and BAB).

Panel B shows that in-sample and net of transaction costs without trading diversification there are only two individual-factor portfolios that significantly outperform their corresponding unmanaged factor (UMD and BAB) at the 10% level when we use inverse market volatility to time the factors compared to none when using inverse factor variance.

Panel C shows that out-of-sample and without transaction costs there are three individual-factor portfolios that significantly outperform their corresponding unmanaged factor (UMD, ROE, and BAB) at the 10% level when we use inverse market volatility compared to four when we use inverse factor variance (UMD, ROE, IA, and BAB).

Panel D in both tables shows that none of the individual-factor portfolios outperform their associated unmanaged factors out of sample and net of costs but ignoring trading diversification. More importantly, Panel E in both tables shows that out of sample and net of costs with trading diversification, the same three individual-factor portfolios (UMD, IA, BAB) significantly outperform their corresponding unmanaged factors at the 10% level.

Overall, Table IA.3 confirms the main takeaways from Table 1 in the manuscript—estimation error and transaction costs erode the performance of the volatility-managed individual-factor portfolios and, although trading diversification partially alleviates the concerns raised by Barroso and Detzel (2021) and Cederburg et al. (2020), it does not fully resurrect the gains from volatility managing individual factors.

IA.4 Performance for a larger set of factors

In the main body of the manuscript, we evaluate the performance of the conditional and unconditional multifactor portfolios using the nine factors considered by Moreira and Muir (2017) and Barroso and Detzel (2021). We now study the robustness of our findings to considering 57 other factors in addition to the nine factors in the main body of the manuscript.

To construct the 57 additional factors, we compute the returns on value-weighted long-short portfolios obtained from single sorts on 57 of the firm characteristics in Green et al. (2017) for the same period as in the main body of the manuscript—January 1967 to December 2020. We start with a database that contains every firm traded on the NYSE, AMEX, and NASDAQ exchanges. As in DeMiguel et al. (2020), we then drop firms with negative book-to-market or with a market capitalization below the 20th cross-sectional percentile. We also drop characteristics with more than five percent of missing observations for more than five percent of firms with CRSP returns available for the entire sample. We then rank stocks at the beginning of every month based on a particular firm characteristic and build a long value-weighted portfolio of stocks with a value of the characteristic above the 70th percentile

Table IA.4: List of characteristics considered

This table lists the acronyms and firm characteristics used to construct the 57 value-weighted factors used in Section IA.4 of the Internet Appendix. The table reports the acronyms following Green et al. (2017). See the Appendix in Green et al. (2017) for a detailed description of how each firm characteristic is constructed.

#	Characteristic and definition	Acronym	Author(s)	Date and Journal
1	Number of years since first Compustat coverage	age	Jiang, Lee, & Zhang	2005, RAS
2	Asset growth: Annual percent change in total assets	agr	Cooper, Gulen & Schill	2008, JF
3	Bid-ask spread: Monthly average of daily bid-ask spread divided by average of daily spread	baspread	Amihud & Mendelson	1989, JF
4	Beta: Estimated market beta from weekly returns and equal weighted market returns for 3 years ending month $t-1$ with at least 52 weeks of returns	beta	Fama & MacBeth	1973, JPE
5	Book to market: Book value of equity divided by end of fiscal-year market capitalization	bm	Rosenberg, Reid & Lanstein	1985, JPM
6	Industry adjusted book to market: Industry adjusted book-to-market ratio	bm_ia	Asness, Porter & Stevens	2000, WP
7	Earnings before depreciation and extraordinary items (ib+dp) divided by avg. total liabilities (lt)	cashdebt	Ou & Penman	1989, JAE
8	Cash productivity: Fiscal year-end market capitalization plus long term debt minus total assets divided by cash and equivalents	cashpr	Chandrashekar & Rao	2009 WP
9	Industry adjusted change in asset turnover: 2-digit SIC fiscal-year mean adjusted change in sales divided by average total assets	chatoia	Soliman	2008, TAR
10	Change in shares outstanding: Annual percent change in shares outstanding	chcsho	Pontiff & Woodgate	2008, JF
11	Industry adjusted change in employees: Industry-adjusted change in number of employees	chempia	Asness, Porter & Stevens	1994, WP
12	Change in inventory (inv) scaled by average total assets (at)	chinv	Thomas & Zhang	2002, RAS
13	Change in 6-month momentum: Cumulative returns from months $t-6$ to $t-1$ minus months $t-12$ to $t-7$	chmom	Gettleman & Marks	2006 WP
14	Industry adjusted change in profit margin: 2-digit SIC fiscal-year mean adjusted change in income before extraordinary items divided by sales	chpmia	Soliman	2008, TAR
15	An indicator equal to 1 if company has convertible debt obligations	convind	Valta	2016, JFQA
16	Current assets / current liabilities	currat	Ou & Penman	1989, JAE
17	Depreciation divided by PP&E	depr	Holthausen & Larcker	1992, JAE
18	An indicator variable equal to 1 if company pays dividends but did not in prior year	divi	Michaely, Thaler, & Womack	1995, JF
19	An indicator variable equal to 1 if company does not pay dividend but did in prior year	divo	Michaely, Thaler, & Womack	1995, JF
20	Dividends-to-price: Total dividends divided by market capitalization at fiscal year-end	dy	Litzenberger & Ramaswamy	1982, JF
21	Change in common shareholder equity: Annual percent change in book value of equity	egr	Richardson, Sloan, Soliman & Tuna	2005, JAE
22	Earnings to price: Annual income before extraordinary items divided by end of fiscal year market cap	ep	Basu	1977, JF
23	Gross profitability: Revenues minus cost of goods sold divided by lagged total assets	gma	Novy-Marx	2013 JFE
24	Industry sales concentration: Sum of squared percent of sales in industry for each company	herf	Hou & Robinson	2006, JF
25	Employee growth rate: Percent change in number of employees	hire	Bazdresch, Belo & Lin	2014 JPE
26	Idiosyncratic return volatility: Standard deviation of residuals of weekly returns on weekly equal weighted market returns for 3 years prior to month-end	idiovol	Ali, Hwang & Trombley	2003, JFE
27	Average of daily (absolute return / dollar volume)	ill	Amihud	2002, JFM
28	Industry momentum: Equal weighted average industry 12-month returns	indmom	Moskowitz & Grinblatt	1999, JF
29	Annual change in gross property, plant, and equipment (ppegt) + annual change in inventories (invt) all scaled by lagged total assets (at)	invest	Chen & Zhangt	2010, JF

Table IA.4 continued: List of characteristics considered

#	Characteristic and definition	Acronym	Author(s)	Date and Journal
30	An indicator variable equal to 1 if first year available on CRSP monthly stock file	IPO	Loughran & Ritter	1995, JF
31	Leverage: Total liabilities divided by fiscal year-end market capitalization	lev	Bhandari	1988, JF
32	12-month momentum: 11-month cumulative returns ending one month before month-end	mom12m	Jegadeesh	1990, JF
33	1-month momentum: 1-month cumulative return	mom1m	Jegadeesh	1990, JF
34	Market capitalization: Natural log of market capitalization at end of month $t-1$	mve	Banz	1981, JFE
35	Industry-adjusted firm size: 2-digit SIC industry-adjusted fiscal year-end market capitalization	mve_ia	Asness, Porter & Stevens	2000, WP
36	Revenue minus cost of goods sold - SG&A expense - interest expense divided by lagged common shareholders' equity	operprof	Fama & French	2015, JFE
37	Δ% CAPEX - industry Δ% AR: 2-digit SIC fiscal-year mean adjusted percent change in capital expenditures	pchcapx_ia	Abarbanell & Bushee	1998, TAR
38	Percent change in currat.	pchcurrat	Ou & Penman	1989, JAE
39	Percent change in depr	pchdepr	Holthausen & Larcker	1992, JAE
40	Δ% gross margin - Δ% sales: Percent change in gross margin minus percent change in sales	pchgm_pchsale	Abarbanell & Bushee	1998, TAR
41	Δ% sales - Δ% AR: Annual percent change in sales minus annual percent change in receivables	pchsale_pchrect	Abarbanell & Bushee	1998, TAR
42	Price delay: The proportion of variation in weekly returns for 36 months ending in month t explained by 4 lags of weekly market returns incremental to contemporaneous market return	pricedelay	How & Moskowitz	2005, RFS
43	Financial-statements score: Sum of 9 indicator variables to form fundamental health score	ps	Piotroski	2000, JAR
44	An indicator variable equal to 1 if R&D expense as a percentage of total assets has an increase greater than 5%.	rd	Eberhart, Maxwell, & Siddique	2004, JF
45	Return volatility: Standard deviation of daily returns from month $t-1$	retvol	Ang, Hodrick, Xing & Zhanf	2006, JF
46	Annual earnings before interest and taxes (ebit) minus nonoperating income (nopi) divided by non-cash enterprise value (ceq+lt-che)	roic	Brown & Rowe	2007, WP
47	Sales to cash: Annual sales divided by cash and cash equivalents	salecash	Ou & Penman	1989, JAE
48	Sales to receivables: Annual sales divided by accounts receivable	salerec	Ou & Penman	1989, JAE
49	An indicator equal to 1 if company has secured debt obligations	securedind	Valta	2016, JFQA
50	Annual percent change in sales (sale)	sgr	Lakonishok, Shleifer, & Vishny	1994, JF
51	An indicator variable equal to 1 if a company's primary industry classification is in smoke or tobacco, beer or alcohol, or gaming	sin	Hong & Kacperczyk	2009, JFE
52	Annual revenue (sale) divided by fiscal year-end market capitalization	sp	Barbee, Mukherji, & Raines	1996, FAJ
53	Volatility of dollar trading volume: Monthly standard deviation of daily dollar trading volume	std_dolvol	Chordia, Subrahmanyan & Anshuman	2001, JFE
54	Volatility of share turnover: Monthly standard deviation of daily share turnover	std_turn	Chordia, Subrahmanyan & Anshuman	2001, JFE
55	Cash holdings + 0.715 × receivables +0.547 × inventory + 0.535 × PPE/ totl assets	tang	Almeida & Campello	2007, RFS
56	Share turnover: Average monthly trading volume for most recent 3 months scaled by number of shares outstanding in current month	turn	Datar, Naik & Radcliffe	1998, JFM
57	Zero trading days: Turnover weighted number of zero trading days for most recent month	zerotrade	Liu	2006, JFE

Table IA.5: Performance for a large set of factors

This table reports the out-of-sample and net-of-costs performance of the conditional and unconditional multifactor portfolios constructed using the nine factors we consider in the main body of the manuscript, plus a large set of 57 characteristic portfolio returns. The conditional mean-variance multifactor portfolio (CMV) is obtained by solving problem (6) and the unconditional mean-variance multifactor portfolio (UMV) obtained by solving problem (6) under the additional constraint that $b_k = 0$ for $k = 1, 2, \ldots, K$; that is, under the constraint that its weights on the K factors are constant over time. To alleviate the impact of the estimation error associated with exploiting a large set of factors, we construct both the unconditional and conditional multifactor portfolios using the shrinkage estimator of Ledoit and Wolf (2004) to estimate the covariance matrix of factor returns. The sample period and quantities reported for each portfolio are the same as in Table 2.

	UMV	CMV
Mean	0.510	0.618
Standard deviation	0.444	0.494
Sharpe ratio	1.150	1.251
p-value(SR_{CMV}-SR_{UMV})		0.036
α		0.091
$t(\alpha)$		2.976
TC	0.188	0.288

and a short value-weighted portfolio of stocks with a value of characteristic below the 30th percentile. Table IA.4 lists the acronyms and firm characteristics used to construct the 57 value-weighted factors we use. The table reports the acronyms following Green et al. (2017). We evaluate out-of-sample performance using an expanding window with the first estimation window containing the first 120 months of data; hence, the out-of-sample results are for January 1977 to December 2020.

Table IA.5 reports the performance of the unconditional and conditional multifactor portfolios. To alleviate the impact of the estimation error associated with exploiting a large set of factors, we construct both the unconditional and conditional multifactor portfolios using the shrinkage estimator of Ledoit and Wolf (2004) to estimate the covariance matrix of factor returns. The table confirms that our findings are robust to considering a larger set of factors: the conditional multifactor portfolio attains an out-of-sample Sharpe ratio of returns net of transaction costs of 1.251, which is around 9% larger than that of the unconditional multifactor portfolio, 1.150, with the difference being significant with a p-value of 3.6%. In addition, relative to the unconditional multifactor portfolio, the conditional multifactor portfolio delivers a significantly positive alpha with a t-stat of 2.976.

Table IA.6: Performance after excluding one of the nine factors

This table reports the performance of the multifactor portfolios constructed after excluding one of the nine factors. The first row indicates the factor that we exclude from the multifactor portfolios. For instance, the second and third columns report the out-of-sample performance of the unconditional and conditional multifactor portfolios obtained after excluding the market (MKT) factor. The conditional multifactor portfolio (CMV) is obtained by solving problem (6) and the unconditional multifactor portfolio (UMV) by solving problem (6) under the additional constraint that $b_k = 0$ for $k = 1, 2, \ldots, K$; that is, under the constraint that its weights on the K factors are constant over time. The sample period and quantities reported for each portfolio are the same as in Table 2.

	Without MKT		Without SMB		Without HML		Without RMW		Without CMA		Without UMD		Without ROE		Without IA		Without BAB	
	UMV	CMV	UMV	CMV	UMV	CMV	UMV	CMV	UMV	CMV	UMV	CMV	UMV	CMV	UMV	CMV	UMV	CMV
Mean	0.369	0.419	0.372	0.421	0.369	0.408	0.363	0.402	0.426	0.474	0.420	0.455	0.360	0.404	0.423	0.467	0.381	0.401
Standard deviation	0.448	0.437	0.423	0.414	0.401	0.395	0.401	0.393	0.457	0.448	0.454	0.443	0.403	0.399	0.452	0.442	0.423	0.417
Sharpe ratio	0.824	0.958	0.878	1.018	0.918	1.032	0.905	1.022	0.932	1.059	0.925	1.027	0.893	1.013	0.935	1.057	0.902	0.962
p-value(SR_{CMV}-SR_{UMV})		0.001		0.005		0.006		0.004		0.004		0.010		0.006		0.006		0.090
α		0.070		0.069		0.054		0.055		0.068		0.053		0.060		0.066		0.032
$t(\alpha)$		3.841		3.832		3.470		3.523		3.746		3.423		3.498		3.602		2.074
TC	0.156	0.209	0.140	0.191	0.155	0.202	0.164	0.211	0.164	0.215	0.165	0.210	0.128	0.182	0.154	0.206	0.157	0.188

Table IA.7: Conditional multifactor portfolio using conventional BAB factor

This table reports the out-of-sample and net-of-costs performance of the unconditional multifactor portfolio (UMV) and the conditional multifactor portfolio (CMV). The portfolios are constructed exploiting all nine factors in our dataset but instead of using the BAB factor of Frazzini and Pedersen (2014), we use a more conventional value-weighted BAB factor where the long leg is formed with the stocks whose estimated beta falls in the bottom cross-sectional tercile and the short leg is formed with stocks whose estimated beta falls in the top cross-sectional tercile. The sample period and quantities reported for each portfolio are the same as in Table 2.

	UMV	CMV
Mean	0.435	0.461
Standard deviation	0.460	0.453
Sharpe ratio	0.946	1.018
p-value($SR_{CMV} - SR_{UMV}$)		0.066
α		0.042
$t(\alpha)$		2.332
TC	0.149	0.181

IA.5 Performance after excluding a particular factor

In this section, we study whether a particular factor drives the favorable performance of the conditional multifactor portfolio. This is an important robustness check because Barroso and Detzel (2021) find that the volatility-managed *market* portfolio outperforms its unmanaged counterpart even net of transaction costs. Therefore, a legitimate concern is whether the good performance of the conditional multifactor portfolio is driven entirely by the performance of the market factor. Similarly, Novy-Marx and Velikov (2022) show that the performance of the betting-against-beta (BAB) factor is driven by its large weight in small (illiquid) stocks. Although our analysis in the main body of the manuscript takes this into account by evaluating performance net of transaction costs estimated using the bid-ask spreads of Abdi and Ranaldo (2017), which are larger for small stocks compared to large stocks, it is useful to study whether the performance of the conditional multifactor portfolio is robust to excluding the BAB factor or considering a more conventional version of the BAB factor.

To address these concerns, Table IA.6 reports the out-of-sample performance of the conditional and unconditional multifactor portfolios obtained after excluding one factor at a time. We see that in all nine cases, the conditional multifactor portfolio delivers an out-of-sample Sharpe ratio net of transaction costs that is larger than that of the corresponding unconditional multifactor portfolio, and in all cases, the difference is statistically significant at the 1% level, except when we exclude the BAB factor, for which the difference is significant at the 10% level.

In addition, we undertake an additional experiment. Novy-Marx and Velikov (2022) identify three non-standard procedures that the BAB factor of Frazzini and Pedersen (2014) employs. First, it is a rank-weighted portfolio instead of the standard value-weighted one. Second, the BAB factor invests less in the short leg than in the long leg and uses leverage to achieve market neutrality. Third, they use a novel method for the estimation of market betas. We evaluate the performance of the conditional multifactor portfolio that exploits a more conventional BAB factor that does not rely on the three non-standard procedures employed in the construction of the Frazzini and Pedersen (2014) BAB factor. We use a BAB factor that is a value-weighted long-short portfolio with market betas of individual stocks estimated using the last year of daily returns. The long leg of this BAB factor contains the stocks whose estimated beta falls in the bottom cross-sectional decile, and the short leg contains the stocks whose estimated beta falls in the top cross-sectional decile. Table IA.7 reports the results and shows that the conditional multifactor portfolio outperforms the unconditional multifactor portfolio, with the p-value for the difference in Sharpe ratios being 6.6%.

IA.6 Performance in high-volatility periods

One appealing feature of volatility timing is that it reduces exposure to risk factors at times of high volatility. To study whether this leads to improved performance not only over the entire sample but also during periods of high volatility and market crises, in this section we report the performance of the unconditional and conditional portfolios during such periods.

Table IA.8 reports the out-of-sample annualized mean return, standard deviation, and Sharpe ratio of the unconditional multifactor portfolio (UMV), the conditional fixed-weight multifactor portfolio (CFW), and the conditional multifactor portfolio (CMV) during high-market-volatility and crises periods. We consider periods with market volatility above the 80th, 85th, and 90th percentiles, the Early 2000's Recession, the Great Recession of 2007–09, and the COVID-crisis periods.[43]

Table IA.8 shows that the performance of the conditional multifactor portfolio is robust during periods of high market volatility. In particular, the out-of-sample Sharpe ratio net of transaction costs of the conditional multifactor portfolio is higher than that of the unconditional portfolio by about 7%, 7%, and 5% for subperiods where volatility is greater

[43] We define the Early 2000's Recession as the period from February to November 2001, the Great Recession from December 2007 to June 2009, and the COVID crisis from January to December 2020.

Table IA.8: Performance during high-volatility periods

This table reports the out-of-sample and net-of-costs annualized average return, standard deviation, and Sharpe ratio of the unconditional multifactor portfolio (UMV), the conditional fixed-weight multifactor portfolio (CFW), and the conditional multifactor portfolio (CMV) during high market volatility and crises periods. We consider periods with market volatility above the 80th, 85th, and 90th percentiles, as well as the 2000-Recession, the Great-Recession, and the COVID-Crisis periods.

	UMV	CFW	CMV
Panel A: Mean			
σ_t >80th percentile	0.235	0.227	0.232
σ_t >85th percentile	0.114	0.095	0.111
σ_t >90th percentile	0.108	0.084	0.099
2000 crisis	0.896	0.620	1.057
Great Recession	−0.462	−0.345	−0.315
COVID crisis	−0.242	−0.206	−0.314
Panel B: Standard deviation			
σ_t >80th percentile	0.525	0.479	0.483
σ_t >85th percentile	0.543	0.485	0.493
σ_t >90th percentile	0.558	0.493	0.487
2000 crisis	0.688	0.478	0.566
Great Recession	0.256	0.193	0.221
COVID crisis	0.364	0.249	0.400
Panel C: Sharpe ratio			
σ_t >80th percentile	0.448	0.474	0.480
σ_t >85th percentile	0.211	0.195	0.225
σ_t >90th percentile	0.193	0.170	0.202
2000 crisis	1.303	1.297	1.869
Great Recession	−1.803	−1.791	−1.424
COVID crisis	−0.665	−0.826	−0.786

than the 80th, 85th, and 90th percentile, respectively, and higher than that of the conditional fixed-weight portfolio by about 1%, 14%, and 17%, respectively. The conditional multifactor portfolio also outperforms the unconditional and conditional fixed-weight portfolios for the Early 2000's Recession and the Great Recession. For instance, during the Early 2000s Recession, the out-of-sample Sharpe ratio net of transaction costs of the conditional multifactor portfolio is around 43% higher than those of the unconditional and conditional fixed-weight portfolios. During the Great Recession, all three multifactor portfolios attain a negative out-of-sample Sharpe ratio of returns net of costs, but the conditional multifactor portfolio has a substantially larger mean return than the unconditional and conditional fixed-weight portfolios, with a standard deviation of returns that is between those of the unconditional and conditional fixed-weight portfolios. An exception to the overall favorable performance of the conditional multifactor portfolio is the COVID-crisis period, during which it underperforms the unconditional and conditional fixed-weight portfolios.

IA.7 Subsample analysis

Table 2 in the main body of the manuscript shows that the conditional multifactor portfolio outperforms its unconditional counterpart in terms of out-of-sample and net-of-costs performance for January 1977 to December 2020. We now study the robustness of this finding by considering two subsamples with an equal number of observations. Table IA.9 reports the out-of-sample and net-of-costs performance of the conditional and unconditional multifactor portfolios for the first and second halves of our overall sample. The table shows that both multifactor portfolios perform much better in terms of out-of-sample and net-of-costs Sharpe ratio in the first half of the sample. More importantly, our findings are robust to considering these subsamples. In particular, the conditional multifactor portfolio outperforms its unconditional counterpart in terms of out-of-sample and net-of-costs Sharpe ratio for both subsamples, with the p-value being significant for the second subsample.

IA.8 Does sentiment explain performance?

Barroso and Detzel (2021) document that the volatility-managed *market* portfolio outperforms the market only during high-sentiment periods, and it underperforms the market during low-sentiment periods. This suggests that, consistent with the finding in Yu and Yuan (2011), the performance of the volatility-managed market portfolio is driven by sentiment traders who undermine the strong risk-return tradeoff during high-sentiment periods. However, in this section we show that sentiment does not explain the out-of-sample performance of the conditional multifactor portfolio.

Table IA.10 compares the Sharpe ratios of the unconditional and conditional versions of the market portfolio and the multifactor portfolio for the entire sample, high-sentiment periods, and low-sentiment periods. Panels A and B report the in-sample and out-of-sample Sharpe ratios of returns net of transaction costs accounting for trading diversification. We consider the Baker and Wurgler (2006) sentiment index orthogonalized to economic conditions and, like Barroso and Detzel (2021), we define high-sentiment (low-sentiment) years as those for which the sentiment index at the end of the prior year is above (below) its median value for the entire sample. Our sample spans the period from January 1967 to December

Table IA.9: Performance of conditional portfolio: Subsample analysis

This table reports the subsample analysis for the out-of-sample and net-of-costs performance of the conditional and unconditional multifactor portfolios. Panel A reports the performance for the first half of our out-of-sample period (January 1977 to January 1999), and Panel B for the second (February 1999 to December 2020). We evaluate out-of-sample performance using an expanding window that starts from January 1967. The out-of-sample return of each multifactor portfolio is assessed for the month following the last month of each estimation window. The conditional multifactor portfolio (CMV) is obtained by solving problem (6) and the unconditional multifactor portfolio (UMV) by solving problem (6) under the additional constraint that $b_k = 0$ for $k = 1, 2, \ldots, K$; that is, under the constraint that its weights on the K factors are constant over time. The portfolios are constructed exploiting all nine factors in our dataset. The quantities reported for each portfolio are the same as in Table 2.

	UMV	CMV
Panel A: January 1977 to January 1999		
Mean	0.779	0.825
Standard deviation	0.446	0.464
Sharpe ratio	1.747	1.779
p-value(SR_{CMV}-SR_{UMV})		0.276
α		0.030
$t(\alpha)$		1.569
TC	0.210	0.252
Panel B: February 1999 to December 2020		
Mean	0.080	0.127
Standard deviation	0.441	0.405
Sharpe ratio	0.182	0.314
p-value(SR_{CMV}-SR_{UMV})		0.021
α		0.056
$t(\alpha)$		2.113
TC	0.114	0.175

2018, for which the sentiment index of Baker and Wurgler (2006) is available. We evaluate out-of-sample performance using an expanding window with the first estimation window containing the first 120 months of data. Thus, the out-of-sample results are for January 1977 to December 2018. To ensure a fair comparison with the out-of-sample results, the in-sample results are reported for the same period.

We start by confirming the finding by Barroso and Detzel (2021) that sentiment explains the in-sample performance of the volatility-managed *market* portfolio. Panel A of Table IA.10 shows that the in-sample performance of the volatility-managed market portfolio is better than that of the unmanaged market factor only during high-sentiment periods, with a p-value that is significant at the 10% confidence level. In contrast, in sample the conditional multifactor portfolio outperforms its unconditional counterpart during both low- and high-sentiment periods, with the difference being statistically significant for high-sentiment periods.

Table IA.10: Performance for high- and low-sentiment periods

This table compares the Sharpe ratios of the unconditional and conditional versions of the market portfolio and the multifactor portfolio for the entire sample, high-sentiment periods, and low-sentiment periods. Panels A and B report the in-sample and out-of-sample Sharpe ratios of returns net of transaction costs accounting for trading diversification. We consider the Baker and Wurgler (2006) sentiment index orthogonalized to economic conditions and, like Barroso and Detzel (2021), define high-sentiment (low-sentiment) years as those for which the sentiment index at the end of the prior year is above (below) its median value for the entire sample. Our sample spans the period from January 1967 to December 2018, for which the sentiment index of Baker and Wurgler (2006) is available. We evaluate out-of-sample performance using an expanding window with the first estimation window containing the first 120 months of data. Thus, the out-of-sample results are for January 1977 to December 2018. To ensure a fair comparison with the out-of-sample results, the in-sample results are reported for the same period.

	Entire sample			High sentiment			Low sentiment		
	Ucond.	Cond.	p-val.	Uncond.	Cond.	p-val.	Unman.	Mana.	p-val.
Panel A: In sample									
Market	0.519	0.532	0.299	0.178	0.217	0.094	0.954	0.940	0.654
Multifactor	1.155	1.372	0.000	1.281	1.637	0.000	1.121	1.172	0.251
Panel B: Out of sample									
Market	0.519	0.433	0.921	0.178	0.070	0.913	0.954	0.922	0.602
Multifactor	0.940	1.062	0.006	1.370	1.538	0.011	0.379	0.495	0.056

We then study whether sentiment explains the performance of the conditional multifactor portfolio *out of sample*, which is the focus of our work. Panel B of Table IA.10 shows that out of sample the conditional multifactor portfolio outperforms its unconditional counterpart during both high- and low-sentiment periods, with the difference being significant at the 10% level. Therefore, we conclude that sentiment does not explain the out-of-sample and net-of-costs performance of the conditional multifactor portfolio.

IA.9 Different investment horizons

In the main body of the manuscript, we follow Moreira and Muir (2017), Barroso and Detzel (2021), and Cederburg et al. (2020) and study volatility management in the context of a short-term mean-variance investor. We now check the robustness of our main findings to considering mean-variance investors with longer investment horizons.

To do this, we start from the monthly out-of-sample returns net of transaction costs of the conditional and unconditional multifactor portfolios used to produce Table 2 in the main body of the manuscript, and we cumulate these returns over nonoverlapping periods of 3, 6, 12, and 18 months. We then compare the Sharpe ratio of the cumulated returns of

Table IA.11: Performance for different investment horizons

This table reports the out-of-sample and net-of-costs annualized Sharpe ratio for different investment horizons of the conditional multifactor portfolio (CMV) and the unconditional multifactor portfolio (UMV). We start from the *monthly* out-of-sample returns net of transaction costs of the conditional and unconditional multifactor portfolios used to produce Table 2 in the main body of the manuscript, and we cumulate these returns over nonoverlapping periods of 3, 6, 12, and 18 months. We then report the Sharpe ratio of the cumulated returns of the conditional and unconditional portfolios for the different investment horizons. The table also reports the p-value for the difference in Sharpe ratios. The sample period is the same as in Table 2.

	Sharpe ratio		
Horizon	UMV	CMV	p-val
1 month	0.940	1.062	0.006
3 months	0.845	0.919	0.019
6 months	0.694	0.758	0.021
12 months	0.609	0.670	0.031
18 months	0.571	0.607	0.047

the conditional and unconditional portfolios for the different investment horizons. Note that the number of observations decreases sharply with the horizon; thus, we use bootstrap to compute p-values for the differences in Sharpe ratios.[44]

Table IA.11 shows that our findings are robust to considering different investment horizons: the conditional multifactor portfolio significantly outperforms its unconditional counterpart for investment horizons up to 18 months.[45]

IA.10 Exploiting other volatility measures

We now study the robustness of the performance of the conditional multifactor portfolios to using other measures of volatility to time the factors instead of realized market volatility

[44]Specifically, we consider 10,000 bootstrap samples. For each bootstrap sample, we use the circular block-bootstrap of Politis and Romano (1994) with an average block size of five months to generate a sample of monthly out-of-sample returns net of costs for the conditional and unconditional portfolios with the same number of observations as our original out-of-sample series. We then cumulate the returns of the conditional and unconditional portfolios at different horizons and compute their Sharpe ratios for the bootstrap sample. Finally, we calculate the p-value as the proportion of bootstrap samples for which the Sharpe ratio of the conditional multifactor portfolio is higher than that of its unconditional counterpart.

[45]Another observation from Table IA.11 is that the out-of-sample Sharpe ratio of both conditional and unconditional portfolios declines with the investment horizon. The explanation for this is that the out-of-sample returns of the conditional and unconditional portfolios are positively autocorrelated, consistent with existing empirical evidence about positive autocorrelation in portfolio returns (Lo and MacKinlay, 1990). While this positive autocorrelation has no effect on estimates of *annualized* average returns when cumulating the returns over longer investment horizons, it increases the estimates of *annualized* standard deviation of returns over longer horizons, which explains why the annualized Sharpe ratios of portfolio returns decline with the investment horizon.

Table IA.12: Conditional multifactor portfolios: Other volatility measures

This table reports the out-of-sample and net-of-costs performance of the unconditional multifactor portfolio (UMV) and the conditional multifactor portfolios obtained using three different measures of volatility to time the factors: market volatility (CMV_1), each factor's own volatility (CMV_{own}), and the average volatility of factors other than the market (CMV_{avg}), all estimated using one month of daily returns. The portfolios are constructed exploiting all nine factors in our dataset. The sample period is the same as in Table 2 of the manuscript. For each portfolio, we report the same quantities as in Table 2 of the manuscript and p-values for the differences between the Sharpe ratios of CMV_{own} and CMV_{avg} with respect to CMV.

	UMV	CMV	CMV_{own}	CMV_{avg}
Mean	0.430	0.477	0.481	0.502
Standard deviation	0.458	0.449	0.435	0.438
Sharpe ratio	0.940	1.062	1.106	1.144
p-value($\text{SR}_{\text{CMV}_k} - \text{SR}_{\text{UMV}}$)		0.005	0.001	0.007
p-value($\text{SR}_{\text{CMV}_k} - \text{SR}_{\text{CMV}}$)			0.098	0.057
α		0.066	0.087	0.119
$t(\alpha)$		3.637	3.988	3.926
TC	0.163	0.213	0.211	0.210

estimated using one month of daily returns. In particular, we consider using each factor's own volatility and the average volatility of factors other than the market.

Table IA.12 reports the out-of-sample and net-of-costs performance of the unconditional multifactor portfolio (UMV) and the conditional multifactor portfolios obtained using three different measures of volatility to time the factors: market volatility (CMV), each factor's own volatility (CMV_{own}), and the average volatility of factors other than the market (CMV_{avg}), all estimated using one month of daily returns. The table shows that all three conditional multifactor portfolios significantly outperform the unconditional multifactor portfolio, demonstrating that our findings are robust to considering conditional multifactor portfolios that exploit measures of volatility different from market volatility. The table also shows that, although the conditional multifactor portfolios that exploit each factor's own volatility (CMV_{own}) and the average volatility of factors other than the market (CMV_{avg}) achieve higher Sharpe ratios than the conditional multifactor portfolio that exploits market volatility (CMV), the differences in Sharpe ratios (relative to CMV) are not significant at the 5% level.

IA.11 Transaction-cost mitigation strategies

In the main body of the manuscript, we alleviate the impact of transaction costs on the performance of the multifactor portfolios in two ways. First, we exploit the benefits of trading

Table IA.13: Conditional portfolio: 3-, 6-, and 12-month realized volatility

This table reports the out-of-sample and net-of-costs performance of the unconditional multifactor portfolio (UMV) and the conditional multifactor portfolios constructed using a one-month (CMV_1), three-month (CMV_3), six-month (CMV_6), and 12-month (CMV_{12}) window of daily returns to compute sample market volatility. The portfolios are constructed exploiting all nine factors in our dataset. The sample period is the same as in Table 2 of the manuscript. For each portfolio, we report the same quantities as in Table 2 of the manuscript and the p-values for the differences between the Sharpe ratios of CMV_m with respect to CMV_1, for $m = 3, 6$ and 12 months.

	UMV	CMV_1	CMV_3	CMV_6	CMV_{12}
Mean	0.430	0.477	0.546	0.561	0.523
Standard deviation	0.458	0.449	0.463	0.468	0.448
Sharpe ratio	0.940	1.062	1.181	1.199	1.167
p-value($\text{SR}_{\text{CMV}_m} - \text{SR}_{\text{UMV}}$)		0.006	0.002	0.000	0.001
p-value($\text{SR}_{\text{CMV}_m} - \text{SR}_{\text{CMV}_1}$)			0.010	0.002	0.015
α		0.066	0.136	0.147	0.121
$t(\alpha)$		3.637	4.349	4.895	4.446
TC	0.163	0.213	0.203	0.193	0.175

diversification that arise when combining multiple characteristics. Second, we explicitly optimize the multifactor portfolios accounting for the impact of transaction costs. The results in Table 2 show that the conditional multifactor portfolio delivers a significantly higher out-of-sample and net-of-costs Sharpe ratio than the unconditional multifactor portfolio. This shows that our transaction-cost optimized portfolios survive the impact of proportional transaction costs. In this section, we consider two additional strategies to mitigate the impact of transaction costs on the performance of the conditional multifactor portfolios: (i) using longer windows to estimate realized market volatility and (ii) dropping small-cap stocks.

The first strategy consists of constructing conditional multifactor portfolios that exploit market volatility estimated using a three-month, six-month, and 12-month window of daily returns instead of the one-month window used in the main body of the manuscript. This alternative procedure gives a smoother estimate of market volatility and helps reduce the turnover and transaction costs of our conditional multifactor strategy. Table IA.13 reports the out-of-sample and net-of-costs performance of the unconditional multifactor portfolio (UMV) and the conditional multifactor portfolios constructed using a one-month (CMV_1), three-month (CMV_3), six-month (CMV_6), and 12-month (CMV_{12}) window of daily returns to compute sample market volatility. The table shows that all three conditional multifactor portfolios based on estimation windows longer than one month significantly outperform the conditional multifactor portfolio that uses a one-month window to estimate market volatility, and performance peaks for an estimation window of around six months. Although using a

Table IA.14: Conditional multifactor portfolio: Dropping small-cap stocks

This table reports the out-of-sample and net-of-costs performance of the unconditional multifactor portfolio (UMV) and the conditional multifactor portfolio (CMV) after dropping small-cap stocks. In particular, every month we drop stocks whose market capitalization falls below the kth cross-sectional percentile. We consider three different thresholds with $k = 10$, 20, and 30. The portfolios are constructed exploiting all nine factors in our dataset. The sample period and quantities reported for each portfolio are the same as in Table 2.

	Drop 10%		Drop 20%		Drop 30%	
	UMV	CMV	UMV	CMV	UMV	CMV
Mean	0.331	0.369	0.319	0.352	0.316	0.343
Standard deviation	0.356	0.353	0.346	0.345	0.341	0.342
Sharpe ratio	0.929	1.046	0.924	1.019	0.926	1.004
p-value($\mathrm{SR_{CMV}} - \mathrm{SR_{UMV}}$)		0.010		0.036		0.063
α		0.052		0.043		0.037
$t(\alpha)$		3.179		2.679		2.359
TC	0.107	0.152	0.099	0.140	0.094	0.131

longer window to estimate realized market volatility substantially improves the performance of the conditional multifactor portfolios, we focus the discussion in the main body of the manuscript on the one-month estimate of realized volatility in order to facilitate comparison with the existing literature.

The second strategy to reduce transaction costs is to drop small-cap stocks. In particular, we compute the conditional multifactor portfolio after dropping stocks with market capitalization below the 10th, 20th, and 30th percentile. Table IA.14 shows that dropping small-cap stocks helps to reduce the transaction costs of the conditional multifactor portfolio from 0.213 in Table 2 for the case when all small-cap stocks are included to 0.152, 0.140, and 0.131 for the cases where we drop the 10%, 20%, and 30% smallest stocks, respectively. However, the mean net return also decreases and, as a result, the Sharpe ratio of the conditional multifactor portfolio decreases slightly as we drop small-cap stocks. Nonetheless, the conditional multifactor portfolio continues to significantly outperform the unconditional multifactor portfolio at the 10% level even after we drop the 30% smallest stocks.

IA.12 Relaxing nonnegativity constraints

In the main body of the manuscript, we discipline the conditional multifactor portfolios by assigning a nonnegative weight to each unmanaged factor $a_k \geq 0$ and a higher weight to each factor when volatility is low $b_k \geq 0$. As explained in Section 2.2, these are also economically meaningful constraints because one would expect the optimal portfolio to load positively on

Table IA.15: Performance after relaxing nonnegativity constraints

This table reports the out-of-sample and net-of-costs performance of the multifactor portfolios constructed after relaxing the nonnegativity constraints on the factor weights. Panel A reports the performance of the conditional multifactor portfolio computed using the weaker nonnegativity constraints $a_k + b_k/\sigma_t \geq 0$ and the unconditional multifactor portfolio using the constraint $a_k \geq 0$ and $b_k = 0$ for $k = 1, 2, \ldots, K$. Panel B reports the results after dropping the nonnegativity constraints entirely but using the shrinkage estimator of Ledoit and Wolf (2004) to estimate the covariance matrix of the extended factor-return vector, Σ_{ext} defined in (6), to construct both the unconditional and conditional multifactor portfolios. The portfolios are constructed exploiting all nine factors described in Section 2.1. The sample period and quantities reported for each portfolio are the same as in Table 2.

	UMV	CMV
Panel A: Weaker nonnegativity constraints		
Mean	0.430	0.465
Standard deviation	0.458	0.450
Sharpe ratio	0.940	1.034
p-value(SR_{CMV}-SR_{UMV})		0.058
α		0.060
$t(\alpha)$		2.588
TC	0.163	0.239
Panel B: Without nonnegativity constraints		
Mean	0.387	0.442
Standard deviation	0.425	0.427
Sharpe ratio	0.909	1.035
p-value(SR_{CMV}-SR_{UMV})		0.021
α		0.071
$t(\alpha)$		3.224
TC	0.153	0.227

the unmanaged factors and reduce exposure when volatility is high. In this section, we study the robustness of our findings to relaxing the nonnegativity constraints.

First, we consider imposing a constraint that the out-of-sample weight on the kth factor of the conditional multifactor portfolio, $\theta_{k,t}$, is nonnegative; that is, $a_k + b_k/\sigma_t \geq 0$ for $k = 1, 2, \ldots, K$. Note that this is a weaker constraint than imposing that both a_k and b_k are nonnegative. Panel A of Table IA.15 shows that the conditional multifactor portfolio computed with the weaker nonnegativity constraints significantly outperforms its unconditional counterpart, achieving an out-of-sample and net-of-costs Sharpe ratio of 1.034, which is around 10% higher than that of the unconditional multifactor portfolio (0.940) with a p-value of 5.8%.

Second, we drop entirely the nonnegativity constraints on the factor weights of the conditional multifactor portfolio. To alleviate the impact of the estimation error associated with exploiting a large set of 18 factors (nine unmanaged and nine managed), we construct both the unconditional and conditional multifactor portfolios using the shrinkage estimator of Ledoit and Wolf (2004) to estimate the covariance matrix of extended factor-return vector, $\widehat{\Sigma}_{\text{ext}}$, defined in (6). Panel B of Table IA.15 shows that the unconstrained conditional multifactor portfolio significantly outperforms its unconditional counterpart, achieving an out-of-sample and net-of-costs Sharpe ratio of 1.035, which is around 12% higher than that of the unconstrained unconditional multifactor portfolio (0.909) with a p-value of 2.1%.

IA.13 Leverage constraints

In the main body of the manuscript, our conditional multifactor portfolio is optimized to invest in nine equity factors that, with the exception of the market factor, require shorting stocks. One concern is that to be profitable, our conditional multifactor portfolio may require a much larger degree of leverage than the unconditional multifactor portfolio.

To address this concern, we first compute the average short position of the conditional and unconditional multifactor portfolios in our out-of-sample period and find that the average short position of the conditional portfolio (\$1.83) is only moderately larger than that of its unconditional counterpart (\$1.47).

Second, we evaluate the out-of-sample performance of the conditional multifactor portfolio after dropping every quarter from our sample stocks with institutional ownership below the kth cross-sectional percentile. Like Barroso and Detzel (2021), we define institutional ownership as the percentage of total shares owned by institutional investors, which we obtain from the Thomson Financial 13(F) Institutional Holdings database. Table IA.16 reports the results for the cases with $k = 10$, 20, and 30. For all three cases, we observe that the conditional multifactor portfolio delivers an out-of-sample and net-of-costs Sharpe ratio that is larger than that of its unconditional counterpart, with the difference being statistically significant at the 5% level. This confirms that the favorable performance of the conditional multifactor portfolio is not explained by its shorting of stocks with low institutional ownership that are difficult to short.

Finally, we rescale the conditional multifactor portfolio every month in our out-of-sample period so that the dollar investment in short positions in stocks is at most 20% higher than that of the unconditional multifactor portfolio.[46] Table IA.17 shows that after imposing

[46] Moreira and Muir (2017) consider a 50% limit on the extra leverage that the volatility-managed individual-factor portfolios can have over that of the unmanaged factors and find that their results are robust to this leverage constraint. In unreported results, we confirm that our results are virtually unchanged

Table IA.16: Performance after dropping low-institutional-ownership stocks

This table reports the out-of-sample and net-of-costs performance of the unconditional multifactor portfolio (UMV) and the conditional multifactor portfolio (CMV) after dropping low-institutional-ownership stocks. In particular, every quarter we drop those stocks whose institutional ownership falls below the kth cross-sectional percentile. We consider three different thresholds with $k = 10$, 20, and 30. The portfolios are constructed exploiting all nine factors in our dataset. The sample period and quantities reported for each portfolio are the same as in Table 2.

	Drop 10%		Drop 20%		Drop 30%	
	UMV	CMV	UMV	CMV	UMV	CMV
Mean	0.310	0.343	0.303	0.336	0.286	0.318
Standard deviation	0.363	0.359	0.359	0.356	0.349	0.349
Sharpe ratio	0.854	0.955	0.846	0.942	0.819	0.912
p-value(SR_{CMV}-SR_{UMV})		0.017		0.020		0.024
α		0.045		0.043		0.042
$t(\alpha)$		2.924		2.849		2.787
TC	0.114	0.158	0.110	0.152	0.104	0.145

Table IA.17: Performance with leverage constraints

This table reports the out-of-sample and net-of-costs performance of the conditional multifactor portfolio (CMV), subject to the constraint that its leverage is at most 20% higher than that of the unconditional multifactor portfolio (UMV). The portfolios are constructed exploiting all nine factors in our dataset. The sample period and quantities reported for each portfolio are the same as in Table 2.

	UMV	CMV
Mean	0.430	0.479
Standard deviation	0.458	0.445
Sharpe ratio	0.940	1.076
p-value(SR_{CMV}-SR_{UMV})		0.001
α		0.072
$t(\alpha)$		4.271
TC	0.163	0.197

such a leverage constraint, the conditional multifactor portfolio delivers an out-of-sample and net-of-costs Sharpe ratio that is around 14.5% higher than that of the unconditional portfolio, with the difference being statistically significant at the 1% level.

when we consider a 50% constraint on the leverage that the conditional multifactor portfolio can have over that of the unconditional multifactor portfolio. However, in this section, we present the results for the more conservative case with a 20% leverage constraint.

Table IA.18: Performance of alternative conditional multifactor portfolio

This table reports the performance of the unconditional multifactor portfolio (UMV), our conditional multifactor portfolio that exploits inverse market volatility (CMV), and the alternative conditional multifactor portfolio (ALT) obtained by solving problem (IA2) and scaled to have the same standard deviation as the unconditional multifactor portfolio. The portfolios are constructed exploiting all nine factors in our dataset. The sample period and quantities reported for each portfolio are the same as in Table 2.

	UMV	CMV	ALT
Mean	0.430	0.477	0.435
Standard deviation	0.458	0.449	0.458
Sharpe ratio	0.940	1.062	0.950
p-value(SR_{CMV}-SR_{UMV})		0.006	0.483
α		0.066	0.145
$t(\alpha)$		3.637	2.169
TC	0.163	0.213	0.142

IA.14 Alternative conditional multifactor portfolio

Our conditional multifactor portfolio relies on the assumption that the conditional weight on the kth factor is an affine function of inverse market volatility:

$$\theta_{k,t} = a_k + b_k \frac{1}{\sigma_t}. \tag{IA1}$$

One may wonder whether a less parsimonious conditional multifactor portfolio would perform better. To address this concern, in this section, we evaluate the performance of an alternative conditional multifactor portfolio obtained by using the estimator of the conditional covariance matrix considered by Chernov et al. (2022): $\widehat{\Sigma}_t = (1-\lambda)V_t + \lambda\widehat{\Sigma}_{t-1}$, where V_t is the nonlinear shrinkage covariance matrix of Ledoit and Wolf (2020) using the daily returns in month t and $\lambda = 0.94$, which is the value used by the RiskMetrics model. We then compute the alternative conditional multifactor portfolio by solving the following problem:

$$\max_{\theta_t \geq 0} \quad \theta_t^\top \widehat{\mu} - \frac{\gamma}{2}\theta_t^\top \widehat{\Sigma}_t \theta_t - \widehat{\mathrm{TC}}(\theta_t), \tag{IA2}$$

where $\widehat{\mu}$ is the vector of means estimated from an expanding window of factor returns.[47]

Table IA.18 reports the performance of the alternative conditional multifactor portfolio. We observe that our proposed conditional multifactor portfolio outperforms the alternative portfolio in terms of out-of-sample Sharpe ratio.[48]

[47]It is challenging to estimate conditional mean returns, and thus, we use an unconditional estimate of means for simplicity.

[48]Note that although the out-of-sample alpha of the alternative conditional multifactor portfolio (ALT) is larger than that of the conditional multifactor portfolio (CMV), the t-statistic for the alpha of the ALT portfolio is smaller.

IA.15 Exploiting additional conditioning variables

Several papers have used other conditioning variables to time equity risk factors (Haddad et al., 2020; Gómez-Cram, 2021). We now study whether the performance of the conditional multifactor portfolio can be improved further by using another conditioning variable in addition to inverse market volatility. We consider seven additional conditioning variables.

First, we exploit the value spread (Haddad et al., 2020), which we compute for each factor as the difference between the (lagged) aggregate book-to-market ratio of the stocks in the long leg minus that of the stocks in the short leg.[49] We then define the weight assigned to the kth factor in our portfolio as

$$\theta_{k,t} = a_k + b_k \frac{1}{\sigma_t} + c_k \text{ValSpread}_{k,t},$$

where σ_t is the realized market volatility in month t and $\text{ValSpread}_{k,t}$ is the kth factor's value spread in month t.

We also consider six macroeconomic variables, which include the four business-cycle variables considered by Herskovic, Moreira, and Muir (2020): Moody's Baa-Aaa spread, the slope of the term structure, initial claims, and industrial production. In addition, we consider a sentiment dummy variable based on the Baker and Wurgler (2006) index. Finally, Aït-Sahalia, Matthys, Osambela, and Sircar (2021) find that the equity risk premium is earned mostly during periods of high uncertainty when investors have greater difficulty forecasting future cash flows. Therefore, our final macroeconomic variable is, as in Aït-Sahalia et al. (2021), the Economic Policy Uncertainty (EPU) index of Baker, Bloom, and Davis (2016). For each of the six macroeconomic conditioning variables, we define the weight on the kth factor as

$$\theta_{k,t} = a_k + b_k \frac{1}{\sigma_t} + c_k \text{Macro}_{\ell t}, \tag{IA3}$$

where $\text{Macro}_{\ell t}$ is the value of the macroeconomic variable ℓ at time t. Parameters a_k, b_k, and c_k are estimated for the nine factors to maximize net-of-costs mean-variance utility, subject to nonnegativity constraints on these parameters.

Table IA.19 reports the Sharpe ratio of the conditional multifactor portfolios that exploit another conditioning variable in addition to inverse market volatility. The table

[49] Like Fama and French (1992), the book-to-market ratio for June of year t is the book equity for the fiscal year ending in $t-1$ divided by market capitalization for December of year $t-1$.

Table IA.19: Performance exploiting market volatility and another variable

This table reports the performance of the conditional multifactor portfolio that exploits only inverse market volatility (CMV) and the performance of the conditional multifactor portfolios that exploit inverse market volatility *plus* another variable. We consider seven conditioning variables in addition to market volatility: 1) value spread (ValSpread) of each factor, 2) Moody's Baa-Aaa spread, 3) slope of the term structure, 4) initial claims, 5) industrial production, 6) sentiment dummy variable based on the Baker and Wurgler (2006) index, and 7) the Economic Policy Uncertainty (EPU) index of Baker et al. (2016). For each multifactor portfolio, we report the p-value for the difference between the Sharpe ratios of the conditional multifactor portfolio exploiting inverse market volatility and one macroeconomic variable (SR_{CMV+}) and the conditional multifactor portfolio exploiting only inverse market volatility (SR_{CMV}). The portfolios are constructed exploiting all nine factors in our dataset. The sample period and quantities reported for each portfolio are the same as in Table 2.

		Conditioning on market volatility plus						
	CMV	ValueSpread	BaaAaa	Slope	Claims	Production	Sentiment	EPU
Mean	0.477	0.536	0.519	0.559	0.475	0.476	0.583	0.510
Standard deviation	0.449	0.493	0.481	0.498	0.455	0.451	0.557	0.465
Sharpe ratio	1.062	1.086	1.080	1.121	1.043	1.056	1.046	1.097
p-value($SR_{CMV+} - SR_{CMV}$)		0.224	0.320	0.164	0.971	0.693	0.560	0.243
α		0.031	0.018	0.062	−0.007	−0.001	0.028	0.026
$t(\alpha)$		1.416	1.127	1.943	−1.627	−0.134	0.798	1.312
TC	0.213	0.218	0.227	0.226	0.223	0.222	0.225	0.223

shows that none of the seven additional conditioning variables helps to significantly improve the out-of-sample and net-of-cost Sharpe ratio of the conditional portfolio that exploits only inverse market volatility.[50]

IA.16 Alternative proportional transaction costs

In the main body of the manuscript, we use the approach of Abdi and Ranaldo (2017) to estimate the proportional transaction-cost parameter of individual stocks. We now evaluate the performance of the volatility-managed individual-factor portfolios and the conditional multifactor portfolios using the low-frequency average bid-ask spread measure of Chen and Velikov (2023) to estimate transaction costs. In particular, we estimate the transaction-cost parameter of each stock as half of the effective spread of Chen and Velikov (2023) instead of half the effective spread of Abdi and Ranaldo (2017), which we use in the main body of the manuscript. The results are reported in Tables IA.20 and IA.21, which are the counterparts of Tables 1 and 2 in the manuscript.

Comparing Table IA.20 to Table 1 in the manuscript, we observe that the performance of the volatility-managed individual-factor portfolios is similar when considering the

[50]In unreported results, we confirm that the results presented in this section are robust to considering also interactions between inverse market volatility and the macroeconomic variables.

Table IA.20: Individual-factor portfolios: Alternative proportional costs

For each of the nine factors we consider, this table reports the annualized Sharpe ratios of the unmanaged factor, $\mathrm{SR}(r_k)$, and the volatility-managed individual-factor portfolio, $\mathrm{SR}(r_k, r_k^\sigma)$, which is the mean-variance combination of the unmanaged factor with its managed counterpart, and the p-value for the difference in Sharpe ratios. We estimate the transaction-cost parameter of each stock as half the effective spread of Chen and Velikov (2023) and consider an investor with risk-aversion parameter $\gamma = 5$. Panel A reports performance in-sample and ignoring transaction costs, Panel B in-sample and net of costs but ignoring trading diversification, Panel C out-of-sample and ignoring costs, Panel D out-of-sample and net of costs but ignoring trading diversification, and Panel E out-of-sample and net of costs considering trading diversification. To facilitate comparison, both the in-sample and out-of-sample performance are reported for January 1977 to December 2020.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
Panel A: In-sample without transaction costs									
$\mathrm{SR}(r_k)$	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
$\mathrm{SR}(r_k, r_k^\sigma)$	0.585	0.246	0.215	0.739	0.419	1.088	1.153	0.621	1.397
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.238	0.366	0.339	0.036	0.306	0.000	0.001	0.099	0.000
Panel B: In-sample net of transaction costs but without trading diversification									
$\mathrm{SR}(r_k)$	0.521	0.135	0.065	0.372	0.186	0.154	0.365	0.164	0.561
$\mathrm{SR}(r_k, r_k^\sigma)$	0.529	0.135	0.065	0.372	0.186	0.260	0.406	0.164	0.580
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.415	0.500	0.500	0.500	0.500	0.281	0.326	0.500	0.361
Panel C: Out-of-sample without transaction costs									
$\mathrm{SR}(r_k)$	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
$\mathrm{SR}(r_k, r_k^\sigma)$	0.408	0.068	0.194	0.527	0.355	1.035	1.094	0.605	1.321
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.896	0.928	0.388	0.462	0.898	0.000	0.000	0.062	0.000
Panel D: Out-of-sample net of transaction costs but without trading diversification									
$\mathrm{SR}(r_k)$	0.521	0.135	0.065	0.372	0.186	0.154	0.365	0.164	0.561
$\mathrm{SR}(r_k, r_k^\sigma)$	0.339	−0.293	−0.030	−0.431	−0.014	0.215	0.362	−0.033	0.537
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.972	1.000	0.881	0.999	1.000	0.384	0.546	0.998	0.595
Panel E: Out-of-sample net of transaction costs with trading diversification									
$\mathrm{SR}(r_k)$	0.521	0.135	0.065	0.372	0.186	0.154	0.365	0.164	0.561
$\mathrm{SR}(r_k, r_k^\sigma)$	0.430	0.039	0.091	0.238	0.167	0.209	0.365	0.226	0.653
p-value($\mathrm{SR}(r_k, r_k^\sigma)$-$\mathrm{SR}(r_k)$)	0.927	0.881	0.302	0.967	0.726	0.167	0.496	0.053	0.148

proportional transaction cost estimates of Chen and Velikov (2023) instead of those of Abdi and Ranaldo (2017). Panels A and C of the two tables are identical because they ignore transaction costs. Panel B of the two tables show that in-sample and net of transaction costs but without trading diversification, none of the individual-factor portfolios significantly outperforms its associated unmanaged factor; Panel D of the two tables show that out-of-sample and net of transaction costs but without trading diversification, none of the individual-factor portfolios significantly outperforms its associated unmanaged factor. More importantly, Panel E of the two tables show that out-of-sample and net of transaction costs with trading diversification, three individual-factor portfolios (UMD, IA, BAB) significantly

Table IA.21: Conditional multifactor portfolio: Alternative proportional costs

This table reports the out-of-sample and net-of-costs performance of the unconditional multifactor portfolio (UMV) and the conditional multifactor portfolio (CMV). We estimate the transaction-cost parameter of each stock as half the effective spread of Chen and Velikov (2023) and consider an investor with risk-aversion parameter $\gamma = 5$. The portfolios are constructed exploiting all nine factors in our dataset. The sample period and quantities reported for each portfolio are the same as in Table 2 of the manuscript.

	UMV	CMV
Mean	0.428	0.467
Standard deviation	0.449	0.441
Sharpe ratio	0.952	1.059
p-value(SR_{CMV}-SR_{UMV})		0.008
α		0.057
$t(\alpha)$		3.359
TC	0.151	0.194

outperform their unmanaged factor when we estimate costs using Abdi and Ranaldo (2017) and only one factor (IA) when using Chen and Velikov (2023). Thus, our main finding is robust to considering the alternative estimate of proportional transaction costs in Chen and Velikov (2023)—transaction costs and estimation error erode the gains from volatility managing individual factors and, although trading diversification partially alleviates the concerns of Barroso and Detzel (2021) and Cederburg et al. (2020), it does not fully resurrect the gains from volatility managing *individual* factors.

Comparing Table IA.21 with Table 2 in the manuscript, we find that the performance of the conditional *multifactor* portfolio is also robust to considering this alternative estimator of the proportional transaction-cost parameter. In particular, the conditional multifactor portfolio continues to outperform its unconditional counterpart when we estimate transaction costs using the effective spreads of Chen and Velikov (2023), with the difference being statistically significant at the 5% level.

IA.17 Quadratic price-impact costs

We now evaluate the performance of the conditional multifactor and individual-factor portfolios in the presence of price-impact costs and find that our results are robust to this.

We consider quadratic price-impact costs, that is, we assume that the amount of trading has a linear impact on prices. Given a sample with T historical observations of stock returns and factor portfolios, the price-impact cost incurred for rebalancing the conditional multifactor portfolio can be estimated as

$$\widehat{\mathrm{PIC}}(\eta) = \frac{1}{T-1}\sum_{t=1}^{T-1} \Delta w_{t+1}(\eta)^\top D_t \Delta w_{t+1}(\eta), \tag{IA4}$$

where $\Delta w_{t+1}(\eta) \in \mathbb{R}^{N_t}$, defined in Equation (7) of the manuscript, is the vector of stock trades required to rebalance the conditional multifactor portfolio at time $t+1$, which is a function of the vector of factor weights $\eta \in \mathbb{R}^{2K}$ defined in Equation (5) of the manuscript, and $D_t \in \mathbb{R}^{N_t \times N_t}$ is a diagonal matrix whose nth diagonal element d_{nt} is the nth stock's Kyle lambda.

Our estimate of the Kyle lambdas relies on the results of Novy-Marx and Velikov (2016), who estimate the Kyle lambda of the nth stock at time t, d_{nt}, for the case with linear price impact and find that the R-square of a cross-sectional regression of $\log(d_{nt})$ on market capitalization is 70% and the slope is not statistically distinguishable from minus one. This suggests that a good approximation for the d_{nt} of individual stocks is to assume that they are inversely proportional to the market capitalization of each firm. Moreover, Novy-Marx and Velikov (2016) find that the average price elasticity of supply, defined as the product of the price-impact parameter of the nth stock and its market capitalization at time t, $d_{nt} \times me_{nt}$, is about 6.5. Based on this evidence, we estimate the stock-level price-impact parameter as $d_{nt} = 6.5/me_{nt}$. Finally, to make price-impact costs comparable throughout our sample period, we scale the price-impact parameter, d_{nt}, by multiplying it with the ratio of the aggregate market capitalization in month t to that in December 2020.

The conditional multifactor portfolio problem for an investor who accounts for price-impact costs can be written as:

$$\max_{\eta \geq 0} \quad \widehat{\mu}_{\text{ext}}^\top \eta - \widehat{\mathrm{PIC}}(\eta) - \frac{\gamma_a}{2}\eta^\top \widehat{\Sigma}_{\text{ext}}\eta, \tag{IA5}$$

in which $\widehat{\mu}_{\text{ext}}^\top \eta$ and $\eta^\top \widehat{\Sigma}_{\text{ext}}\eta$ are the sample mean and variance of the conditional multifactor portfolio return, $\widehat{\mu}_{\text{ext}}$ and $\widehat{\Sigma}_{\text{ext}}$ are the sample mean and covariance matrix of the extended factor-return vector defined in Equation (5) of the manuscript, $\widehat{\mathrm{PIC}}(\eta)$ is the sample price-impact cost defined in Equation (IA4) above, and γ_a is the investor's absolute risk aversion parameter.

As pointed out by Gârleanu and Pedersen (2013), the absolute risk-aversion parameter γ_a can be interpreted as the investor's relative risk-aversion parameter γ over the investor's endowment B, $\gamma_a = \gamma/B$. We consider investors with four levels of absolute risk aversion:

Table IA.22: Conditional multifactor portfolio: Price-impact costs

This table reports the out-of-sample and net-of-costs performance of the unconditional multifactor portfolio (UMV) and the conditional multifactor portfolio (CMV) in the presence of quadratic price-impact costs. The portfolios are constructed exploiting all nine factors in our dataset and, to alleviate the effect of price-impact costs, we estimate market volatility over a six-month window of daily returns instead of the one-month window we consider in the main body of the manuscript. We consider investors with different absolute risk-aversion parameters of $\gamma_a = 5\times10^{-6}$, 5×10^{-9}, 10^{-9}, 5×10^{-10}. For each multifactor portfolio, the table reports the annualized Sharpe ratio of returns net of price-impact costs, p-value for the difference between the Sharpe ratios of the conditional and unconditional portfolios, and the average total investment in billions of dollars on each leg of the long-short portfolio. The sample period is the same as in Table 2 of the manuscript.

	$\gamma_a = 5 \times 10^{-6}$		$\gamma_a = 5 \times 10^{-9}$		$\gamma_a = 10^{-9}$		$\gamma_a = 5 \times 10^{-10}$	
	UMV	CMV	UMV	CMV	UMV	CMV	UMV	CMV
Sharpe ratio	1.297	1.519	0.928	1.169	0.636	0.774	0.548	0.624
p-value(SR_{CMV}-SR_{UMV})		0.002		0.001		0.020		0.089
Total investment	0.008	0.008	4.205	4.373	10.598	10.511	14.309	14.063

$\gamma_a = 5 \times 10^{-6}$, 5×10^{-9}, 10^{-9}, and 5×10^{-10}, which is equivalent to considering investors with a relative risk aversion of $\gamma = 5$ and an endowment of $B =$ \$1 million, \$1 billion, \$5 billion, and \$10 billion, respectively.[51] To alleviate the effect of price-impact costs, we time each factor using market volatility estimated over a six-month window of past daily returns instead of the one-month window we use in the main body of the manuscript. This helps to smooth out changes in market volatility and reduce the turnover and price-impact costs required to implement the conditional multifactor portfolio.

Table IA.22 reports the out-of-sample and net-of-costs performance of the unconditional multifactor portfolio (UMV) and the conditional multifactor portfolio (CMV) in the presence of quadratic price-impact costs. For each multifactor portfolio, the table reports the annualized Sharpe ratio of returns net of price-impact costs, p-value for the difference between the Sharpe ratios of the conditional and unconditional portfolios, and the total investment in billions of dollars on each leg of the long-short portfolio.[52] We observe that the conditional multifactor portfolio outperforms the unconditional multifactor portfolio, with

[51] As explained in the second paragraph in Section 2.1 of the manuscript, all nine factors we consider are returns of zero-cost portfolios. Therefore, the conditional multifactor portfolio problem defined in Equation (6) of the manuscript does not contain a budget constraint, and thus, the conditional multifactor portfolio depends on the endowment of the investor only via her absolute risk-aversion parameter.

[52] We compute the total investment in the long leg by adding all stock positive positions of the conditional multifactor portfolio. Note that the MKT and BAB factors short the risk-free asset, and thus, the sum of the stock positive positions is larger than the absolute value of the sum of the stock negative positions for the conditional multifactor portfolio. However, the total investment in the short leg of the conditional multifactor portfolio coincides with that in the long leg if we account for the negative positions in the risk-free asset.

Table IA.23: Individual-factor portfolios: Price-impact costs

For each of the nine factors we consider and in the presence of price-impact costs, this table reports the annualized Sharpe ratios of the unmanaged factor, $\mathrm{SR}(r_k)$, and the volatility-managed individual-factor portfolio, $\mathrm{SR}(r_k, r_k^\sigma)$, which is the mean-variance combination of the unmanaged factor with its managed counterpart, the p-value for the difference in Sharpe ratio, and the average total investment in billions of dollars on each leg of the long-short portfolio. We consider an investor with an absolute risk-aversion parameters of $\gamma_a = 10^{-9}$. The sample period is the same as in Table 2 of the manuscript.

	MKT		SMB		HML		RMW		CMA		UMD		ROE		IA		BAB	
	UMV	CMV	UMV	CMV	UMV	CMV	UMV	CMV	UMV	CMV	UMV	CMV	UMV	CMV	UMV	CMV	UMV	CMV
Sharpe ratio	0.467	0.466	0.120	0.111	0.074	0.077	0.273	0.281	0.145	0.142	0.245	0.327	0.373	0.374	0.274	0.274	−0.048	−0.057
p-value($\mathrm{SR}_{\mathrm{CMV}}$-$\mathrm{SR}_{\mathrm{UMV}}$)		0.506		0.912		0.428		0.310		0.796		0.028		0.310		0.437		0.674
Total investment	1.788	1.873	2.169	2.175	3.365	3.413	2.134	2.180	2.628	2.621	1.778	1.801	0.743	0.742	0.874	0.874	3.443	3.578

the difference being statistically significant at the 5% level for all absolute risk-aversion parameters except for $\gamma_a = 5 \times 10^{-10}$, for which the difference is statistically significant at the 10% level. Moreover, Table IA.22 shows that it is possible to invest more than ten billion dollars on each leg of the conditional multifactor portfolio and outperform the unconditional multifactor portfolio. For instance, the investor with absolute risk-aversion parameter $\gamma_a = 10^{-9}$ invests around \$10.5 billion on each leg of the conditional multifactor portfolio. Overall, these results suggest that the gains from volatility timing in our multifactor framework survive price-impact costs.

We also report the performance of the *individual-factor* portfolios. In particular, Table IA.23 reports the performance of the volatility-managed individual-factor portfolios for each of the nine factors for the case with $\gamma_a = 10^{-9}$. We find that only one of the nine individual-factor portfolios (UMD) significantly outperforms its corresponding unmanaged factor. Thus, our finding that estimation error and transaction costs erode the gains from volatility managing individual factors continues to hold when we consider price-impact costs instead of proportional transaction costs, as in Table 1 of the manuscript.

IA.18 Transaction costs to trade the market

In this section, we show that the transaction costs that we compute for trading the market factor are very small and do not drive any of the results in our manuscript. Indeed, setting the transaction cost of trading the market to be zero has a negligible effect on our results.

Figure 5 of the manuscript shows that the transaction costs we compute for rebalancing the unmanaged and managed market factors are very small. In particular, the figure reports the gross mean returns (red bars), the net-of-transaction-cost mean returns ignoring trading diversification (gray bars), and the net-of-transaction-cost mean returns accounting for trading diversification (blue bars) for the unmanaged and managed versions of the nine factors we consider. The first set of bars in this figure is for the unmanaged market (MKT) factor and the tenth set of bars is for the managed market (MKT/σ_t) factor. Comparing the red bars to the gray bars, we observe that the difference between the gross and net-of-transaction-cost return is negligible for the unmanaged market factor and small for the managed market factor even when we ignore trading diversification. Comparing the red bars to the blue bars, we see that the transaction cost of rebalancing the managed market factor is also negligible once one accounts for the benefits of trading diversification across all factors.

Table IA.24: Conditional multifactor portfolio: Zero transaction cost for market

This table reports the out-of-sample and net-of-costs performance of two multifactor portfolios: the conditional mean-variance multifactor portfolio (CMV) and the unconditional mean-variance multifactor portfolio (UMV) after setting the transaction costs for the market portfolio equal to zero. The sample period and quantities reported for each portfolio are the same as in Table 2 of the manuscript.

	UMV	CMV
Mean	0.430	0.476
Standard deviation	0.458	0.449
Sharpe ratio	0.940	1.060
p-value(SR_{CMV}-SR_{UMV})		0.006
α		0.065
$t(\alpha)$		3.603
TC	0.163	0.213

It is also possible to see that the transaction costs for the market factor do not drive any of our results. For instance, Panel A of Table 1 in the manuscript shows that in-sample and in the absence of transaction costs, the volatility-managed market portfolio does improve the Sharpe ratio of the unmanaged factor: the annualized Sharpe ratio for the unmanaged market factor is 0.530 and for the mean-variance combination of the unmanaged and managed factors is 0.585. However, Panel C, which *excludes* transaction costs, shows that these gains are eroded by estimation error: the out-of-sample annualized Sharpe ratio for the unmanaged market factor is 0.530 but for the mean-variance combination of the unmanaged and managed factors drops to 0.408. Thus, estimation error has a substantial detrimental effect on the performance of the volatility-managed market portfolio. Panel E shows that when we account for transaction costs and trading diversification across the unmanaged and managed factors, the out-of-sample and net-of-costs Sharpe ratio of the volatility-managed market portfolio is 0.433, which is even higher than that in Panel C, which ignores transaction costs, 0.408. Thus, transaction costs actually help to improve the performance of the volatility-managed market portfolio. The reason for this is that a transaction-cost term in a portfolio optimization problem can work as a regularization term that alleviates the impact of estimation error (Olivares-Nadal and DeMiguel, 2018). Thus, our results indicate that the poor performance of the volatility-managed market portfolio is not driven by transaction costs and instead it can be traced back entirely to the impact of estimation error.

Similarly, the transaction costs associated with trading the market do not drive the performance of the conditional *multifactor* portfolio. In particular, we evaluate the perfor-

Table IA.25: Performance using alternative risk measures

This table reports the out-of-sample and net-of-costs performance of the unconditional multifactor portfolio (UMV) and conditional multifactor portfolio (CMV). For each multifactor portfolio, Panel A reproduces the base-case results in Table 2. Panel B reports the value-at-risk with a 95% confidence level, VaR(95), maximum drawdown, skewness, and kurtosis of the unconditional and conditional multifactor portfolios. The portfolios are constructed exploiting all nine factors in our dataset. The sample period is the same as in Table 2.

	UMV	CMV
Panel A: Base-case results		
Mean	0.430	0.477
Standard deviation	0.458	0.449
Sharpe ratio	0.940	1.062
p-value(SR_{CMV}-SR_{UMV})		0.006
Panel B: Alternative risk measures		
VaR(95)	−0.156	−0.145
Max. drawdown	−0.994	−0.988
Skewness	−0.565	−0.480
Kurtosis	8.710	6.630

mance of the conditional multifactor portfolio after setting the transaction costs for trading both the unmanaged and managed markets factors to be zero. These results are in Table IA.24. Comparing Table IA.24 with Table 2 in the manuscript, which reports the performance of the conditional multifactor portfolio when we account for the costs associated with trading the market, we observe that the Sharpe ratio for the unconditional multifactor portfolio (UMV) is 0.940 in both cases and for the conditional multifactor portfolio (CMV) barely changes from 1.062 to 1.060 (with this change being small enough to be explained by the tolerance of the optimization solver). Thus, our finding that the conditional multifactor portfolio significantly outperforms its unconditional counterpart is not driven by our estimates of the cost of trading the market.

IA.19 Alternative risk measures

In Section 3.2, we show that the conditional multifactor portfolio delivers an out-of-sample Sharpe ratio net of transaction costs larger than that of the unconditional multifactor portfolio. Panel A of Table IA.25 reproduces the measures reported in Table 2 from which we see that the conditional portfolio achieves both a higher out-of-sample mean return and lower standard deviation. Panel B of Table IA.25 shows that the conditional multifactor portfolio is less risky than its unconditional counterpart also in terms of alternative measures such as

Table IA.26: Correlation of factor returns

This table reports the correlation matrix for the nine unmanaged-factor returns from January 1977 to December 2020. The numbers in bold font are for negative correlations.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
MKT	1.00	0.26	**−0.22**	**−0.26**	**−0.37**	**−0.14**	**−0.25**	**−0.34**	**−0.15**
SMB		1.00	**−0.20**	**−0.46**	**−0.14**	0.03	**−0.38**	**−0.21**	**−0.07**
HML			1.00	0.20	0.67	**−0.24**	**−0.04**	0.66	0.36
RMW				1.00	0.11	0.11	0.68	0.19	0.36
CMA					1.00	**−0.02**	**−0.01**	0.91	0.33
UMD						1.00	0.52	**−0.02**	0.22
ROE							1.00	0.08	0.33
IA								1.00	0.34
BAB									1.00

value-at-risk, maximum drawdown, skewness, and kurtosis. Intuitively, the explanation for this finding is that the timing strategy scales back exposure to the risk factors when market volatility is high, which leads to a reduction in risk in terms of the aforementioned measures.

IA.20 Risk diversification of multifactor portfolios

Compared to the volatility-managed *individual-factor* portfolios, the unconditional and conditional multifactor portfolios reduce risk by diversifying across multiple factors. To identify the source of this diversification benefit, Table IA.26 reports the correlation matrix for the returns of the nine unmanaged factors.[53] The correlation matrix reveals that risk diversification is an important driver of the favorable performance of the conditional *multifactor* portfolios, compared to the volatility-managed *individual-factor* portfolios. In particular, the market (MKT) and size (SMB) factors are generally negatively correlated with the other factors, and thus combining MKT and SMB with the other factors helps to reduce the overall risk of the unconditional and conditional multifactor portfolios.

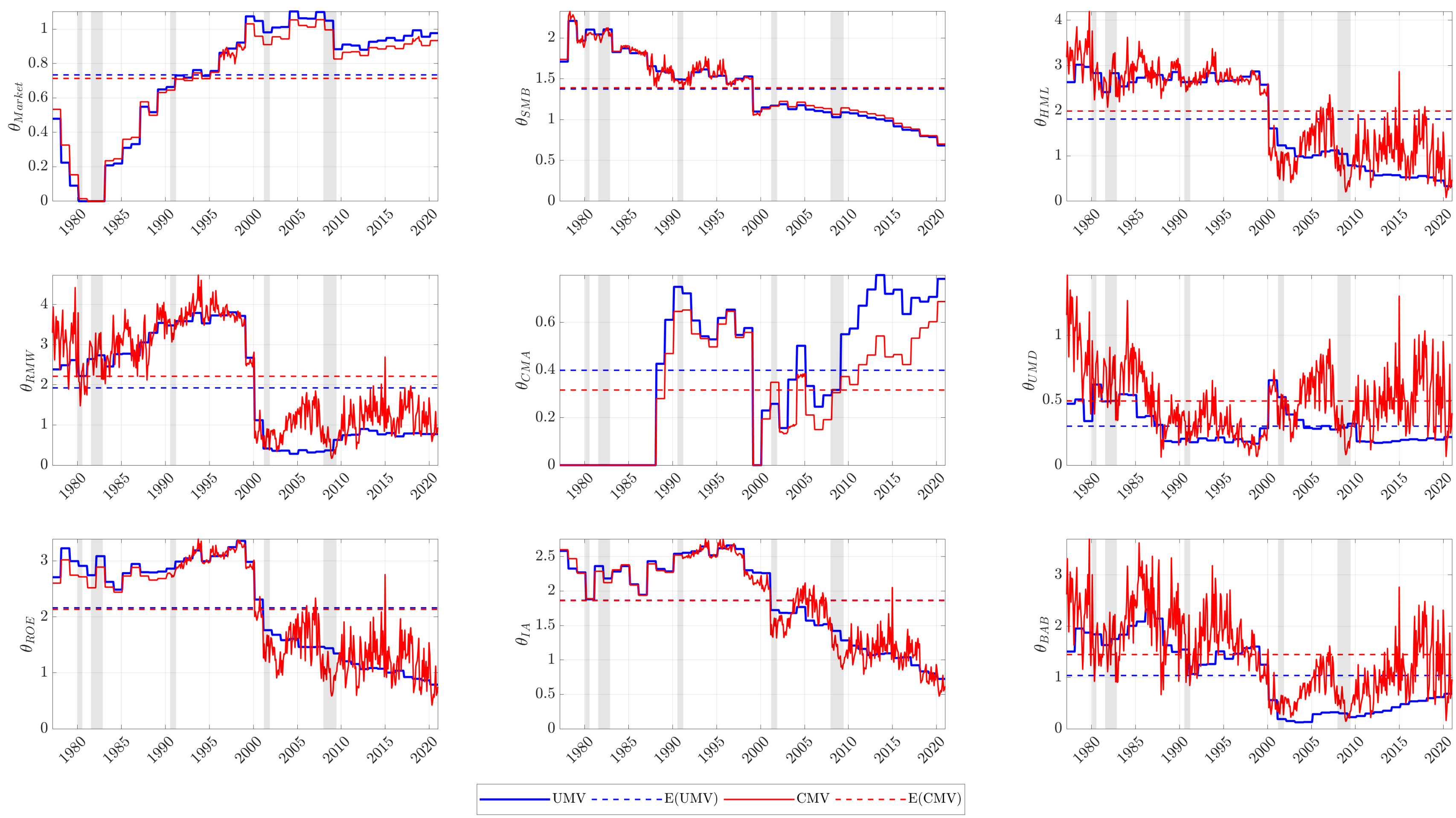
IA.21 Out-of-sample multifactor portfolio weights

Section 4.3 shows that the conditional multifactor portfolio assigns an almost constant weight to the MKT, SMB, and CMA factors while it times the rest of the factors aggressively. The analysis in Section 4.3 is in sample, that is, we estimate the optimal conditional multifactor

[53] In unreported results, we find that the managed factors are highly correlated with their unmanaged counterparts (correlation in excess of 86% for every factor) and that the correlations between the managed factors are similar to those between their unmanaged counterparts.

Figure IA.1: Out-of-sample weights of unconditional and conditional multifactor portfolios

This figure depicts the out-of-sample weights from January 1977 to December 2020 of the unconditional multifactor portfolio (UMV, blue line) and the conditional multifactor portfolio (CMV, solid red line). The figure also depicts the average weights of the unconditional multifactor portfolio (E[UMV], dashed blue line) and the conditional multifactor portfolio (E[CMV], dashed red line). Each of the nine graphs depicts the weights for a particular factor.



portfolio using the entire sample from January 1977 to December 2020. We now confirm that this insight is true also for the out-of-sample factor weights computed using an expanding window. Figure IA.1 shows that out of sample the conditional multifactor portfolio also assigns a weight to MKT, SMB, and CMA that is almost constant within each calendar year; that is, the conditional multifactor portfolio does not time these factors every month using realized market volatility.[54] However, the conditional portfolio times the other six factors aggressively every month throughout most of the sample. Thus, we conclude that, even out of sample, the conditional multifactor portfolio times factors differentially.

Note also that Table 1 in the main body of the manuscript shows that the volatility-managed individual-factor portfolios for the MKT, SMB, and CMA factors perform poorly out of sample. In contrast, Figure IA.1 shows that when we combine the three aforementioned factors with the other six factors, the conditional multifactor portfolio chooses not to time the MKT, SMB, and CMA factors. This suggests that combining factors may help to alleviate the impact of the estimation error associated with some of the noisier factors and obtain more robust volatility-managed portfolios.

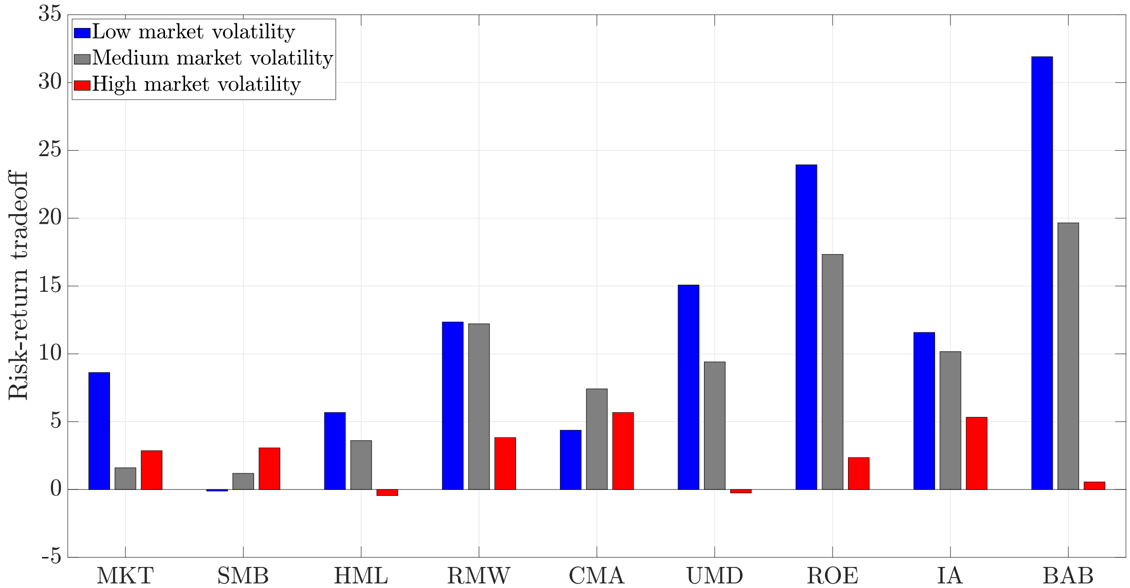
IA.22 Alternative estimator of risk-return tradeoff

In this section, we produce an alternative estimator of the conditional factor risk-return tradeoff given in Figure 1 of the manuscript. In the new figure, we average factor returns over each tercile. In particular, we first use the monthly time series of realized market volatility to sort the months in our sample into terciles. Then, for each factor, we estimate the risk-return tradeoff as the average of factor returns divided by the variance of factor returns over the corresponding volatility tercile, instead of computing the risk-return tradeoff for each month and then averaging across each tercile, as in Figure 1 of the main body of the manuscript. The results are depicted in Figure IA.2, with blue bars corresponding to the tercile containing low-market-volatility months, gray bars to medium-market-volatility months, and red bars to high-market-volatility months. Consistent with Figure 1 in the manuscript, Figure IA.2 shows that the risk-return tradeoff deteriorates substantially with market volatility for seven factors (MKT, HML, RMW, UMD, ROE, IA, BAB). For SMB and CMA, Figure IA.2 shows that the risk-return tradeoff improves slightly with market volatility, which is also broadly

[54] The reason the weight for these three factors does change at the beginning of each calendar year is that we re-optimize the conditional multifactor portfolio each year using an expanding window.

Figure IA.2: Alternative estimator of conditional factor risk-return tradeoff

This barplot illustrates how the risk-return tradeoff for the nine factors in our dataset varies with realized market volatility. We first use the monthly time series of realized market volatility to sort the months in our sample into terciles. Then, for each factor, we estimate the risk-return tradeoff for each volatility tercile as the average of factor returns for months following each month in the volatility tercile divided by the sample variance of factor returns for the months in the volatility tercile. Blue bars correspond to the tercile containing low-market-volatility months, gray bars to medium-market-volatility months, and red bars to high-market-volatility months. The sample spans January 1977 to December 2020.



consistent with Figure 1 in the manuscript, which shows only a modest deterioration of the risk-return tradeoff for SMB and CMA with market volatility.

IA.23 Alternative parametric estimators of risk prices

In the main body of the manuscript, we estimated the coefficients α_k and β_k in (14) under the assumption that factor returns are uncorrelated. We now show that our findings are robust to estimating the coefficients α_k and β_k accounting for factor-return correlations. To alleviate the estimation error associated with estimating a large number of $K(K+1)/2$ realized factor variances and covariances each month, we use the shrinkage approach of Ledoit and Wolf (2004) to estimate the realized covariance matrix of factor returns.

We run the following pooled conditional regression:

$$r_{i,t+1} = \sum_{k=1}^{K} (\alpha_k + \beta_k \tilde{\sigma}_t^{-1}) \sigma_{ik,t}^{\mathrm{LW}} + \epsilon_{i,t+1}, \quad \text{for } k = 1, 2, \ldots, K, \tag{IA6}$$

Table IA.27: First alternative estimator of factor risk-return tradeoff

This table reports the coefficients α_k and β_k for the pooled conditional regression defined in Equation (IA6) for the nine factors in our dataset. The numbers in square brackets are Newey-West t-statistics. The sample spans January 1977 to December 2020.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
α_k	10.764	3.426	7.934	8.670	6.827	2.487	7.525	−0.200	11.906
	[4.720]	[1.389]	[2.549]	[2.117]	[1.230]	[1.109]	[1.565]	[−0.036]	[3.413]
β_k	0.395	0.076	0.600	0.154	0.449	0.195	0.847	−0.462	0.394
	[3.653]	[0.603]	[3.382]	[0.676]	[1.492]	[1.637]	[2.447]	[−1.471]	[2.422]

where $\sigma_{ik,t}^{\text{LW}}$ is the element on the ith row and kth column of the shrinkage estimator of the realized covariance matrix of factor returns estimated from daily returns over month t, and $\epsilon_{i,t+1}$ is the residual for the ith regression equation at time $t+1$.

Table IA.27 reports the results for the pooled conditional regression in (IA6). The table shows that, consistent with our findings in Table 5 of the main body of the manuscript, the estimated unconditional price of risk α_k is generally positive across the nine factors in our dataset, with the exception of IA. Moreover, the unconditional price of risk α_k is significantly positive for MKT, HML, RMW, and BAB. Furthermore, consistent with the results in Table 5, the estimated coefficient β_k is again generally positive across all factors, with the exception of IA, and it is significantly positive for MKT, HML, ROE, and BAB. Thus, the conditional price of risk decreases with market volatility for all factors except IA.

IA.24 Alternative methods for computing p-values

In the main body of the manuscript, we compute p-values for the difference between the Sharpe ratios of two portfolios using a bootstrap test based on the stationary bootstrap of Politis and Romano (1994), as explained in Footnote 15 of the manuscript. We now study the robustness of our inference to considering other methods for computing p-values. In Section IA.24.1, we show that the inference obtained using our approach is very similar to that obtained using the approaches of Jobson and Korkie (1981) and Ledoit and Wolf (2008). In Section IA.24.2, we propose an alternative bootstrap test to compute p-values for the difference between the Sharpe ratios of the *in-sample* returns of two portfolios, and show that the inference using this alternative test is similar to that using the bootstrap test in the main body of the manuscript.

IA.24.1 The Jobson-Korkie and Ledoit-Wolf tests

Our approach for computing p-values for the difference in Sharpe ratios is consistent with the approach of Jobson and Korkie (1981), which is used in the closely related papers by Barroso and Detzel (2021) and Cederburg et al. (2020).[55] A limitation of the Jobson and Korkie (1981) test is that it relies on the assumption that the returns of the two portfolios being compared are independently and identically distributed as a bivariate Normal distribution. To address this limitation, Ledoit and Wolf (2008) propose a circular block bootstrap method that accounts for heavy tails and serial dependence. In the main body of the manuscript, we use a variant of the Ledoit-Wolf test based on the stationary bootstrap of Politis and Romano (1994), which is also used in DeMiguel, Nogales, and Uppal (2014). In this section, we compute p-values using the approaches of Jobson and Korkie (1981) and Ledoit and Wolf (2008) for all the tables in the main body of the manuscript and we find that the inference using our approach is very similar to that from the other two approaches.[56]

Table IA.28 replicates Table 1 in the main body of the manuscript, but reports also the p-values obtained using the approaches of Jobson and Korkie (1981) and Ledoit and Wolf (2008). The table shows that the inference from the three alternative p-values is very similar. Indeed, out of the 45 comparisons carried out in Table IA.28, the inference changes for just three cases. For the IA factor in Panel A, the p-value in the main body of the manuscript and that of Ledoit and Wolf are significant at the 10% level, but the Jobson-Korkie p-value is 11%. For UMD in Panel E, the p-value in the manuscript and the Jobson-Korkie p-value are significant at the 10% level, but the Ledoit-Wolf p-value is significant at the 5% level. Finally, for BAB in Panel E, the p-value in the main body of the manuscript and the Ledoit-Wolf p-value are significant at the 10% level, whereas the Jobson-Korkie p-value is significant at the 5% level.

Table IA.29 replicates Table 2 in the main body of the manuscript, but reports also the p-values obtained using the approaches of Jobson and Korkie (1981) and Ledoit and Wolf (2008). The table shows that the inference obtained from the three alternative p-values is identical, the difference in out-of-sample Sharpe ratios between the conditional and

[55] Specifically, Barroso and Detzel (2021, table II) use the p-values of Jobson and Korkie (1981) to compare the in-sample Sharpe ratio of the unmanaged and managed factors, and Cederburg et al. (2020, table 5) use the approach of Jobson and Korkie (1981) to compare the out-of-sample Sharpe ratio of the unmanaged factor and the volatility-managed individual-factor portfolio, which they term *combination strategy*.

[56] For the Ledoit-Wolf test and our test based on the stationary bootstrap of Politis and Romano (1994), we use a block size of five months. In unreported results, we find that the inference is robust to considering block sizes between one and 10 months.

Table IA.28: Individual-factor portf.: Jobson-Korkie and Ledoit-Wolf p-vals

This table replicates Table 1 in the main body of the manuscript, but reports also the p-values obtained using the approaches of Jobson and Korkie (1981) and Ledoit and Wolf (2008). For each of the nine factors we consider, this table reports the annualized Sharpe ratios of the unmanaged factor, $\mathrm{SR}(r_k)$, the volatility-managed individual-factor portfolio, $\mathrm{SR}(r_k, r_k^\sigma)$, which is the mean-variance combination of the unmanaged factor with its managed counterpart given in (1), and the p-value for the difference between these two Sharpe ratios obtained using the approach in the main body of the manuscript as well as the approaches of Jobson and Korkie (1981) and Ledoit and Wolf (2008). We consider an investor with risk-aversion parameter $\gamma = 5$. Panel A reports performance in-sample and ignoring transaction costs, Panel B in-sample and net of costs but ignoring trading diversification, Panel C out-of-sample but ignoring costs, Panel D out-of-sample and net of costs but ignoring trading diversification, and Panel E out-of-sample and net of costs considering trading diversification. To facilitate comparison, both the in-sample and out-of-sample performance are evaluated for January 1977 to December 2020.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
Panel A: In-sample without transaction costs									
$\mathrm{SR}(r_k)$	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
$\mathrm{SR}(r_k, r_k^\sigma)$	0.585	0.246	0.215	0.739	0.419	1.088	1.153	0.621	1.397
p-val (manuscript)	0.242	0.366	0.337	0.033	0.311	0.000	0.001	0.094	0.000
p-val (Jobson and Korkie)	0.204	0.325	0.322	0.028	0.334	0.000	0.001	0.110	0.000
p-val (Ledoit and Wolf)	0.257	0.363	0.378	0.020	0.347	0.001	0.002	0.080	0.001
Panel B: In-sample net of transaction costs but without trading diversification									
$\mathrm{SR}(r_k)$	0.519	0.125	0.053	0.356	0.159	0.114	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.521	0.125	0.053	0.356	0.159	0.251	0.331	0.107	0.703
p-val (manuscript)	0.464	0.500	0.500	0.500	0.500	0.249	0.407	0.500	0.161
p-val (Jobson and Korkie)	0.433	0.500	0.500	0.500	0.500	0.183	0.351	0.500	0.114
p-val (Ledoit and Wolf)	0.459	0.500	0.500	0.500	0.500	0.150	0.242	0.500	0.124
Panel C: Out-of-sample without transaction costs									
$\mathrm{SR}(r_k)$	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
$\mathrm{SR}(r_k, r_k^\sigma)$	0.408	0.068	0.194	0.527	0.355	1.035	1.094	0.605	1.321
p-val (manuscript)	0.899	0.929	0.390	0.467	0.897	0.000	0.000	0.062	0.000
p-val (Jobson and Korkie)	0.905	0.967	0.361	0.460	0.883	0.000	0.000	0.073	0.000
p-val (Ledoit and Wolf)	0.867	0.964	0.407	0.428	0.892	0.001	0.001	0.062	0.003
Panel D: Out-of-sample net of transaction costs but without trading diversification									
$\mathrm{SR}(r_k)$	0.519	0.125	0.053	0.356	0.159	0.114	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.324	−0.295	−0.041	−0.453	−0.047	0.194	0.269	−0.127	0.690
p-val (manuscript)	0.976	1.000	0.879	1.000	1.000	0.342	0.672	1.000	0.281
p-val (Jobson and Korkie)	0.981	1.000	0.914	1.000	1.000	0.281	0.660	1.000	0.241
p-val (Ledoit and Wolf)	0.960	1.000	0.863	0.981	1.000	0.213	0.411	1.000	0.239
Panel E: Out-of-sample net of transaction costs with trading diversification									
$\mathrm{SR}(r_k)$	0.519	0.125	0.053	0.356	0.159	0.114	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.433	0.035	0.089	0.226	0.153	0.209	0.324	0.193	0.746
p-val (manuscript)	0.917	0.858	0.243	0.965	0.547	0.097	0.405	0.029	0.064
p-val (Jobson and Korkie)	0.903	0.863	0.184	0.971	0.568	0.059	0.356	0.019	0.027
p-val (Ledoit and Wolf)	0.876	0.897	0.212	0.957	0.571	0.048	0.176	0.044	0.086

Table IA.29: Multifactor portfolios: Jobson-Korkie and Ledoit-Wolf p-vals

This table replicates Table 2 in the main body of the manuscript, but reports also the p-values obtained using the approaches of Jobson and Korkie (1981) and Ledoit and Wolf (2008). The table reports the out-of-sample and net-of-costs performance of two multifactor portfolios: the conditional mean-variance multifactor portfolio (CMV) obtained by solving problem (6) and the unconditional mean-variance multifactor portfolio (UMV) obtained by solving problem (6) under the additional constraint that $b_k = 0$ for $k = 1, 2, \ldots, K$; that is, under the constraint that its weights on the K factors are constant over time. For each multifactor portfolio, the table reports the out-of-sample annualized mean, standard deviation, Sharpe ratio of returns net of transaction costs accounting for trading diversification, and p-value for the difference between the Sharpe ratios of the conditional and unconditional portfolios obtained using the approach in the main body of the manuscript as well as the approaches of Jobson and Korkie (1981) and Ledoit and Wolf (2008). The table also reports the annualized alpha of the time-series regression of the conditional portfolio out-of-sample returns net of transaction costs on those of the unconditional portfolio, alpha Newey-West t-statistic, and out-of-sample transaction costs of the unconditional and conditional portfolios. The portfolios are constructed using all nine factors in our dataset. We report out-of-sample performance from January 1977 to December 2020.

	UMV	CMV
Mean	0.430	0.477
Standard deviation	0.458	0.449
Sharpe ratio	0.940	1.062
p-val (manuscript)		0.006
p-val (Jobson and Korkie)		0.001
p-val (Ledoit and Wolf)		0.002
α		0.066
$t(\alpha)$		3.637
TC	0.163	0.213

unconditional multifactor portfolios is significant at the 1% confidence level for all three methods.

Table IA.30 replicates Table 3 in the main body of the manuscript, but reports also the p-values obtained using the approaches of Jobson and Korkie (1981) and Ledoit and Wolf (2008). The table shows that the inference obtained from the three alternative p-values is very similar. For instance, all p-values for differences in Sharpe ratios obtained using the three alternative methods are significant at the 1% level for Panels A and B. For Panel C, none of the p-values are significant at the 10% level. For Panel D, all p-values are significant at the 10% level, although there are minor differences in the level of significance. In particular, Column (3) in Panel D shows that the p-value in the main body of the manuscript for the difference in Sharpe ratio between the CMV and UMV portfolios that ignore transaction costs is significant at the 10% level, but the p-values for the Jobson-Korkie and Ledoit-Wolf methods are significant at the 5% level. Importantly, the p-values

Table IA.30: Multifactor portfolios: Jobson-Korkie and Ledoit-Wolf p-vals

This table replicates Table 3 in the main body of the manuscript, but reports also the p-values obtained using the approaches of Jobson and Korkie (1981) and Ledoit and Wolf (2008). The table reports the performance of three multifactor portfolios optimized either ignoring or accounting for transaction costs. The three multifactor portfolios are the: (i) unconditional multifactor portfolio (UMV), (ii) conditional fixed-weight multifactor portfolio (CFW) of Moreira and Muir (2017), and (iii) our conditional multifactor portfolio (CMV). Columns (1) to (3) report the performance of the three multifactor portfolios optimized ignoring transaction costs and Columns (4) to (6) accounting for transaction costs. Each of the four panels reports the performance of the portfolios *evaluated* in a different way: Panel A in-sample without transaction costs, Panel B out-of-sample without transaction costs, Panel C out-of-sample with transaction costs without trading diversification, and Panel D out-of-sample with transaction costs with trading diversification. The sample period and quantities reported for each portfolio are the same as in Table 2.

	Optimized ignoring TC			Optimized accounting for TC		
	UMV (1)	CFW (2)	CMV (3)	UMV (4)	CFW (5)	CMV (6)
Panel A: In-sample without transaction costs						
Mean	0.415	0.602	0.680	0.301	0.314	0.432
Standard deviation	0.288	0.347	0.369	0.218	0.222	0.250
Sharpe ratio	1.441	1.735	1.844	1.378	1.415	1.726
p-val (manuscript)		0.000	0.000		0.004	0.000
p-val (Jobson and Korkie)		0.001	0.000		0.004	0.000
p-val (Ledoit and Wolf)		0.001	0.001		0.007	0.001
α		0.187	0.213		0.009	0.107
$t(\alpha)$		5.738	6.684		3.073	7.638
TC	0.000	0.000	0.000	0.000	0.000	0.000
Panel B: Out-of-sample without transaction costs						
Mean	0.753	0.783	0.925	0.593	0.626	0.690
Standard deviation	0.580	0.520	0.569	0.458	0.445	0.449
Sharpe ratio	1.299	1.506	1.625	1.295	1.407	1.537
p-val (manuscript)		0.000	0.000		0.002	0.000
p-val (Jobson and Korkie)		0.000	0.000		0.000	0.000
p-val (Ledoit and Wolf)		0.001	0.001		0.002	0.001
α		0.140	0.239		0.059	0.125
$t(\alpha)$		5.012	5.797		3.808	6.414
TC	0.000	0.000	0.000	0.000	0.000	0.000
Panel C: Out-of-sample with transaction costs without trading diversification						
Mean	0.412	0.313	0.349	0.347	0.343	0.332
Standard deviation	0.580	0.520	0.569	0.458	0.445	0.449
Sharpe ratio	0.710	0.601	0.613	0.758	0.772	0.739
p-val (manuscript)		0.986	0.933		0.337	0.724
p-val (Jobson and Korkie)		0.989	0.955		0.247	0.718
p-val (Ledoit and Wolf)		0.940	0.907		0.258	0.694
α		-0.035	-0.027		0.012	0.001
$t(\alpha)$		-1.610	-0.727		0.967	0.031
TC	0.341	0.470	0.576	0.246	0.283	0.358
Panel D: Out-of-sample with transaction costs with trading diversification						
Mean	0.517	0.511	0.575	0.430	0.457	0.477
Standard deviation	0.580	0.520	0.569	0.458	0.445	0.449
Sharpe ratio	0.891	0.984	1.010	0.940	1.026	1.062
p-val (manuscript)		0.019	0.074		0.003	0.005
p-val (Jobson and Korkie)		0.008	0.025		0.000	0.001
p-val (Ledoit and Wolf)		0.004	0.034		0.004	0.003
α		0.072	0.103		0.046	0.066
$t(\alpha)$		3.094	2.759		3.407	3.637
TC	0.236	0.272	0.350	0.163	0.169	0.213

in Column (6), which compares the performance of the CMV and UMV portfolios optimized accounting for transaction costs, are significant at the 1% level for all three methods.

IA.24.2 Alternative bootstrap test for in-sample results

All three approaches considered in the previous section to compute p-values for Sharpe-ratio differences rely on the assumption that the returns of the two portfolios being compared follow a stationary bivariate distribution, and thus, they do not explicitly account for how the portfolios are constructed. For *out-of-sample* returns, this is not a concern because we recompute the portfolio weights for each out-of-sample month using a different estimation window. Thus, the out-of-sample returns of the portfolios being compared already account for how the portfolios are constructed in each estimation window.

However, one may wish to account for how the portfolios are constructed when computing p-values also for the *in-sample* returns. Although this is not standard practice in the literature (see for instance, Barroso and Detzel (2021) and Cederburg et al. (2020)), we use an approach based on Harvey and Liu (2021, sec. 2) and DeMiguel et al. (2020, sec. IA.6) to adjust the sample returns of the unmanaged and managed factors so that (in the adjusted sample) they satisfy the null hypothesis that the Sharpe ratio of the conditional portfolio is equal to that of the unconditional portfolio. In particular, as in Harvey and Liu (2021), we change *only* the sample mean of the factor returns while leaving their sample covariance matrix unchanged, so that the adjusted sample data preserves the time-series properties of the original sample. Then, we use the stationary block-bootstrap method of Politis and Romano (1994) to generate bootstrap samples of the unmanaged and managed factor returns under the null. We then compute the conditional and unconditional portfolios for each bootstrap sample, estimate the empirical distribution of the difference between the bootstrap sample Sharpe ratios of the conditional and unconditional portfolios under the null, and produce a one-sided p-value for the in-sample difference in Sharpe ratios.

In the remainder of this section, we first describe this alternative bootstrap method to produce p-values for the in-sample Sharpe ratios that accounts for how the portfolios are constructed, and then compare the p-values obtained with this methodology with those in the main body of the manuscript. For brevity, we focus our description on the multi-factor portfolios, but we use the same formulae for the volatility-managed *individual*-factor portfolios by setting the number of unmanaged factors $K = 1$.

For the case without transaction costs, Equation (6) implies that the conditional multifactor portfolio can be written as

$$\begin{bmatrix} \widehat{\theta} \\ \widehat{\theta}^{\sigma} \end{bmatrix} = \frac{1}{\gamma}\widehat{\Sigma}_{\text{ext}}^{-1}\widehat{\mu}_{\text{ext}},$$

where $\widehat{\theta} \in \mathbb{R}^K$ and $\widehat{\theta}^{\sigma} \in \mathbb{R}^K$ are the sample conditional multifactor portfolio weights on the unmanaged and managed factors, respectively, γ is the investor's relative risk aversion, and $\widehat{\mu}_{\text{ext}}$ and $\widehat{\Sigma}_{\text{ext}}$ are the sample mean vector and covariance matrix of the extended factor-return vector, $r_{\text{ext},t+1}$, which is defined in Equation (6) of the main body of the manuscript as the vector that concatenates the returns of the unmanaged and managed factors.

We adjust the extended factor-return data so that the null hypothesis holds in the adjusted sample. To do this, as in Harvey and Liu (2021), we change *only* the sample mean of the extended factor-return vector while leaving its sample covariance matrix unchanged, so that the adjusted sample data preserves the time-series properties of the original sample. Specifically, we add to the time-series of extended factor returns the time-invariant vector $\widehat{\mu}_{\text{ext}}^{\text{adj}} - \widehat{\mu}_{\text{ext}}$, where

$$\widehat{\mu}_{\text{ext}}^{\text{adj}} = \gamma\widehat{\Sigma}_{\text{ext}} \begin{bmatrix} \widehat{\theta} \\ 0 \end{bmatrix}. \tag{IA7}$$

Thus, the sample mean of the adjusted extended factor return is $\widehat{\mu}_{\text{ext}}^{\text{adj}}$ and (IA7) implies that, in the absence of transaction costs, the conditional multifactor portfolio $[\theta, \theta^{\sigma}]$ that solves problem (6) for the adjusted sample assigns a weight of $\widehat{\theta}$ to the unmanaged factors and zero to the managed factors. Thus, the null hypothesis holds in the adjusted sample.

For the case with transaction costs, we apply a similar approach iteratively. At each iteration, we first compute the sample conditional multifactor portfolio $[\widehat{\theta}, \widehat{\theta}^{\sigma}]$ by solving problem (6) accounting for transaction costs for the current adjusted sample. We then add to the current adjusted time-series of extended factor returns the time-invariant vector $\widehat{\mu}_{\text{ext}}^{\text{adjCost}} - \widehat{\mu}_{\text{ext}}$, where

$$\widehat{\mu}_{\text{ext}}^{\text{adjCost}} = \gamma\widehat{\Sigma}_{\text{ext}} \begin{bmatrix} \widehat{\theta} \\ 0 \end{bmatrix} + \partial\widehat{\text{TC}}(\widehat{\theta}, 0), \tag{IA8}$$

in which $\widehat{\text{TC}}(\widehat{\theta}, 0)$ is the transaction-cost term defined in Equation (10) of the main body of the manuscript evaluated at the conditional portfolio defined by the unmanaged weight vector $\widehat{\theta}$ and the managed weight vector $\theta^{\sigma} = 0$ and $\partial\widehat{\text{TC}}(.)$ is the subdifferential of the transaction-cost function computed using Equation (12) of DeMiguel et al. (2020). Because the subdifferential of the transaction-cost term is a set-valued function and the optimality conditions given in Proposition 2 of DeMiguel et al. (2020) are satisfied only at a particular element of the subdifferential, we iterate this process until the null hypothesis is satisfied. We find that the procedure converges to an adjusted sample that satisfies the null hypothesis in just a few iterations.

Once we have an adjusted sample for which the null holds, we use bootstrap to construct one-sided p-values for the in-sample difference in Sharpe ratios. Specifically, we first generate 10,000 bootstrap samples from the adjusted sample of extended factor-return vectors using the stationary block-bootstrap method of Politis and Romano (1994) with an average block size of five. Second, we construct the empirical distribution of the difference between the Sharpe ratios of the returns of the conditional multifactor portfolio and the unconditional multifactor portfolio, $\mathrm{SR}_{\mathrm{CMV}} - \mathrm{SR}_{\mathrm{UMV}}$, across the 10,000 bootstrap samples. Third, we compute the p-value as the frequency with which this difference is larger than the difference of Sharpe ratios in the original sample. The one-sided test rejects the null hypothesis that the two Sharpe ratios are equal when the p-value is smaller than the confidence threshold.

We generate p-values for all the *in-sample* Sharpe-ratio comparisons in the manuscript and Internet Appendix and we find that, although the p-values generated using this alternative bootstrap method are different from those in the manuscript, the inferences from the alternative bootstrap p-values are similar. For instance, Table IA.31 replicates the in-sample results (Panels A and B) of Table 1 in the main body of the manuscript, but reports the alternative bootstrap p-values. Comparing Panel A in the two tables, we observe that the inference at the 10% confidence level is identical except for the RMW and IA factors, for which the p-values are significant at the 10% level (3.3% and 9.4%, respectively) in Table 1, but insignificant (13.7% and 28.2%, respectively) in Table IA.31. In addition, we observe that the p-values for ROE is significant only at the 10% level in Table IA.31, but significant at the 1% level in Table 1. Comparing Panel B in the two tables, we observe that the inference is identical at the 10% confidence level for all factors except BAB, for which the p-value is significant at the 10% level when using the alternative bootstrap procedure, but is not significant when using the bootstrap procedure in the main body of the manuscript.

Table IA.32 replicates the in-sample results (Panel A) of Table 3 in the main body of the manuscript, but reports the alternative bootstrap p-values. The inference from the

Table IA.31: Individual-factor portfolios: Alternative bootstrap p-vals

This table replicates Panels A and B of Table 1 in the main body of the manuscript, but reports the alternative bootstrap p-values described in Section IA.24 of the Internet Appendix. For each of the nine factors we consider, the table reports the annualized Sharpe ratios of the unmanaged factor, $\mathrm{SR}(r_k)$, the volatility-managed individual-factor portfolio, $\mathrm{SR}(r_k, r_k^\sigma)$, which is the mean-variance combination of the unmanaged factor with its managed counterpart given in (1), and the p-value for the difference between these two Sharpe ratios. We consider an investor with risk-aversion parameter $\gamma = 5$. Panel A reports performance in-sample and ignoring transaction costs and Panel B in-sample and net of costs but ignoring trading diversification. The in-sample performance is evaluated for January 1977 to December 2020.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
Panel A: In-sample without transaction costs									
$\mathrm{SR}(r_k)$	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
$\mathrm{SR}(r_k, r_k^\sigma)$	0.585	0.246	0.215	0.739	0.419	1.088	1.153	0.621	1.397
p-val(alternative)	0.323	0.744	0.675	0.137	0.490	0.010	0.081	0.282	0.049
Panel B: In-sample net of transaction costs but without trading diversification									
$\mathrm{SR}(r_k)$	0.519	0.125	0.053	0.356	0.159	0.114	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.521	0.125	0.053	0.356	0.159	0.251	0.331	0.107	0.703
p-val(alternative)	0.362	0.500	0.500	0.500	0.500	0.147	0.258	0.500	0.053

Table IA.32: Multifactor portfolios: Alternative bootstrap

This table replicates Panel A of Table 3 in the main body of the manuscript, but reports the alternative bootstrap p-values described in Section IA.24 of the Internet Appendix. The table reports the performance of three multifactor portfolios optimized either ignoring or accounting for transaction costs. The three multifactor portfolios are the: (i) unconditional multifactor portfolio (UMV), (ii) conditional fixed-weight multifactor portfolio (CFW) of Moreira and Muir (2017), and (iii) our conditional multifactor portfolio (CMV). Columns (1) to (3) report the performance of the three multifactor portfolios optimized ignoring transaction costs and Columns (4) to (6) accounting for transaction costs.Panel A reports the performance of the portfolios *evaluated* in-sample without transaction costs, Panel B out-of-sample without transaction costs. The sample period and quantities reported for each portfolio are the same as in Table 2.

	Optimized ignoring TC			Optimized accounting for TC		
	UMV	CFW	CMV	UMV	CFW	CMV
Panel A: In-sample without transaction costs						
Mean	0.415	0.602	0.680	0.301	0.314	0.432
Standard deviation	0.288	0.347	0.369	0.218	0.222	0.250
Sharpe ratio	1.441	1.735	1.844	1.378	1.415	1.726
p-value($\mathrm{SR_{CMV}}$-$\mathrm{SR_{UMV}}$)		0.000	0.000		0.000	0.000
α		0.187	0.213		0.009	0.107
$t(\alpha)$		5.738	6.684		3.073	7.638
TC	0.000	0.000	0.000	0.000	0.000	0.000

alternative bootstrap p-values is identical to that from the bootstrap p-values in the main body of the manuscript, with all p-values being significant at the 1% level.

In addition to comparing the inference in the tables presented in the main body of the manuscript for the two bootstrap methods, we also compare the inference in the tables

Table IA.33: Individual-factor portf.: longer sample and alternative bootstrap

This table replicates Panels A and B of Table IA.1, but reports the alternative bootstrap p-values described in Section IA.24 of the Internet Appendix. For each of the nine factors we consider, the table reports the annualized Sharpe ratios of the unmanaged factor, $\mathrm{SR}(r_k)$, the volatility-managed individual-factor portfolio, $\mathrm{SR}(r_k, r_k^\sigma)$, which is the mean-variance combination of the unmanaged factor with its managed counterpart given in (1), and the p-value for the difference between these two Sharpe ratios. We consider an investor with risk-aversion parameter $\gamma = 5$. Panel A reports performance in-sample and ignoring transaction costs and Panel B in-sample and net of costs but ignoring trading diversification. The in-sample performance is evaluated for January 1977 to December 2020.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
Panel A: In-sample without transaction costs									
$\mathrm{SR}(r_k)$	0.509	0.216	0.367	0.411	0.512	0.533	0.722	0.508	0.831
$\mathrm{SR}(r_k, r_k^\sigma)$	0.545	0.226	0.400	0.623	0.538	0.974	1.153	0.621	1.128
p-val(alternative)	0.204	0.609	0.344	0.255	0.351	0.024	0.081	0.282	0.014
Panel B: In-sample net of transaction costs but without trading diversification									
$\mathrm{SR}(r_k)$	0.499	0.140	0.262	0.262	0.275	0.184	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.500	0.140	0.262	0.262	0.275	0.232	0.331	0.107	0.623
p-val(alternative)	0.376	0.500	0.500	0.500	0.500	0.168	0.258	0.500	0.159

presented in the Internet Appendix. Table IA.33 replicates Panels A and B of Table IA.1, but reports the alternative bootstrap p-values. The inference from the two tables is again very similar. For Panel B, none of the factors are significant at the 10% level using either approach for computing p-values. Comparing Panel A in the two tables, we observe that the inference at the 10% confidence level is identical except for the RMW and IA factors. For RMW, the p-value is significant at the 5% level in Table IA.1, but insignificant (25.5%) in Table IA.33. For IA, the p-value is significant at the 10% level in Table IA.1, but insignificant (28.2%) in Table IA.33.

Table IA.34 replicates Panels A and B of Table IA.3, but reports the alternative bootstrap p-values. The inference from the two tables is similar. Comparing Panel A in the two tables, we observe that the inference at the 10% confidence level is identical. In addition, we observe that the p-value for HML is significant only at the 10% level in Table IA.3, but significant at the 5% level in Table IA.34. Comparing Panel B in the two tables, we observe that the inference is identical at the 10% confidence levels, except for UMD, for which the p-value is significant at the 10% level when using the bootstrap procedure in the main body of the manuscript, but is not significant when using the alternative bootstrap procedure.

Table IA.35 replicates Panels A and B of Table IA.20, but reports the alternative bootstrap p-values. The inference from the two tables is very similar. For Panel B, none

Table IA.34: Individual-factor portf.: inverse market volatility, alt. bootstrap

This table replicates Panels A and B of Table IA.3, but reports the alternative bootstrap p-values described in Section IA.24 of the Internet Appendix. For each of the nine factors we consider, the table reports the annualized Sharpe ratios of the unmanaged factor, $\mathrm{SR}(r_k)$, the volatility-managed individual-factor portfolio, $\mathrm{SR}(r_k, r_k^\sigma)$, which is the mean-variance combination of the unmanaged factor with its managed counterpart given in (1), and the p-value for the difference between these two Sharpe ratios. We consider an investor with risk-aversion parameter $\gamma = 5$. Panel A reports performance in-sample and ignoring transaction costs and Panel B in-sample and net of costs but ignoring trading diversification. The in-sample performance is evaluated for January 1977 to December 2020.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
Panel A: In-sample without transaction costs									
$\mathrm{SR}(r_k)$	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
$\mathrm{SR}(r_k, r_k^\sigma)$	0.581	0.270	0.375	0.611	0.413	0.944	1.046	0.576	1.511
p-val(alternative)	0.651	0.148	0.029	0.291	0.653	0.000	0.000	0.615	0.001
Panel B: In-sample net of transaction costs but without trading diversification									
$\mathrm{SR}(r_k)$	0.519	0.125	0.053	0.356	0.159	0.114	0.311	0.107	0.606
$\mathrm{SR}(r_k, r_k^\sigma)$	0.524	0.125	0.053	0.356	0.159	0.233	0.311	0.107	0.798
p-val(alternative)	0.307	0.500	0.500	0.500	0.500	0.172	0.500	0.500	0.061

Table IA.35: Individual-factor portf.: alternative costs and bootstrap

This table replicates Panels A and B of Table IA.20, but reports the alternative bootstrap p-values described in Section IA.24 of the Internet Appendix. For each of the nine factors we consider, the table reports the annualized Sharpe ratios of the unmanaged factor, $\mathrm{SR}(r_k)$, the volatility-managed individual-factor portfolio, $\mathrm{SR}(r_k, r_k^\sigma)$, which is the mean-variance combination of the unmanaged factor with its managed counterpart given in (1), and the p-value for the difference between these two Sharpe ratios. We consider an investor with risk-aversion parameter $\gamma = 5$. Panel A reports performance in-sample and ignoring transaction costs and Panel B in-sample and net of costs but ignoring trading diversification. The in-sample performance is evaluated for January 1977 to December 2020.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
Panel A: In-sample without transaction costs									
$\mathrm{SR}(r_k)$	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
$\mathrm{SR}(r_k, r_k^\sigma)$	0.585	0.246	0.215	0.739	0.419	1.088	1.153	0.621	1.397
p-val(alternative)	0.323	0.744	0.675	0.137	0.490	0.010	0.081	0.282	0.049
Panel B: In-sample net of transaction costs but without trading diversification									
$\mathrm{SR}(r_k)$	0.521	0.135	0.065	0.372	0.186	0.154	0.365	0.164	0.561
$\mathrm{SR}(r_k, r_k^\sigma)$	0.529	0.135	0.065	0.372	0.186	0.260	0.406	0.164	0.580
p-val(alternative)	0.292	0.500	0.500	0.500	0.500	0.147	0.185	0.500	0.218

of the factors are significant at the 10% level using either approach for computing p-values. Comparing Panel A in the two tables, we observe that the inference at the 10% confidence level is identical except for the RMW and IA factors, for which the p-values are significant at the 10% level in Table IA.20, but insignificant in Table IA.35.

Overall, the inference from using the alternative bootstrap method is similar to that from using the standard bootstrap method. Moreover, in the small number of cases in which the inference differs, there is no systematic bias of one method with respect to the other. For instance, focusing on the p-values for the case with transaction costs, which is our main focus, the inference at the 10% level is identical for 43 out of the 45 p-values reported in Panel B of Tables IA.31–IA.35. For the two cases in which the inference at the 10% level changes, the p-value for the alternative bootstrap method is more significant for BAB in Table IA.31 but less significant for UMD in Table IA.34.