

CS 511, Fall 2024, Lecture Slides 01

Syntax of Propositional Logic

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Syntax of the wff's of Propositional Logic

- ▶ Reading: [LCS, Section 1.3], [EML.Appendix, pp 1-2]

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- ▶ The wff's of propositional logic are obtained by applying the construction rules below, and only these, **finitely many times**.

One **basis step**:

0. every propositional atom (*i.e.*, propositional variable) p is a WFF

Four **induction steps**:

1. if φ is a wff, then so is $(\neg\varphi)$
2. if φ and ψ are wff's, then so is $(\varphi \wedge \psi)$
3. if φ and ψ are wff's, then so is $(\varphi \vee \psi)$
4. if φ and ψ are wff's, then so is $(\varphi \rightarrow \psi)$

Remember this inductive definition: We use it later in syntax-directed proofs.

Syntax of the wff's of Propositional Logic

- More succinctly, in **BNF (Backus Naur Form)**:

$$\varphi ::= p \mid (\neg \varphi) \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid (\varphi \rightarrow \varphi)$$

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$$\varphi ::= p \mid (\neg\varphi) \mid (\varphi \wedge \psi) \mid (\varphi \vee \psi) \mid (\varphi \rightarrow \psi)$$

- ▶ Or, more abstractly by omitting parentheses, in **Extended BNF**:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \rightarrow \psi$$

Parentheses are used only to set an order of precedence among logical connectives $\{\neg, \wedge, \vee, \rightarrow\}$.

Parse Trees of wff's

- ▶ A fully-parenthesized wff:

$$\begin{aligned} & \left(\left(\neg \left(\left(\neg P \right) \vee \left(Q \wedge \left(\neg P \right) \right) \right) \right) \right. \\ & \quad \left. \rightarrow \left(\neg \left(\left(\neg P \right) \rightarrow \left(Q \vee \left(\neg R \right) \right) \right) \right) \right) \end{aligned}$$

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- ▶ Same wff minimally parenthesized:

$$\begin{aligned} & \neg \left(\neg P \vee \left(Q \wedge \neg P \right) \right) \\ & \rightarrow \neg \left(\neg P \rightarrow \left(Q \vee \neg R \right) \right) \end{aligned}$$

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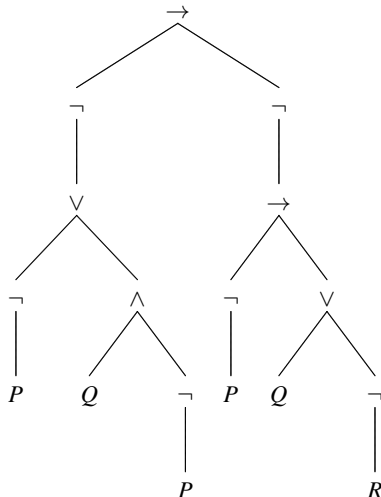
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$$\begin{aligned} & \neg \left(\neg P \vee \left(Q \wedge \neg P \right) \right) \\ & \rightarrow \neg \left(\neg P \rightarrow \left(Q \vee \neg R \right) \right) \end{aligned}$$

No parentheses in the parse tree



Parse trees are very nice,
but more difficult to store.

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