CS 511
Formal Methods for High-Assurance Software Engineering
Homework Assignment 05

Out: 4 October 2024
Due: Thursday, 10 October 2024, by 11:59 pm

Repeated below are administrative issues already mentioned in the handout of Assignment 01:

- You need to open a Gradescope account, after which you need to add yourself to the CS511 roster for this semester. The entry code for CS511, Fall 2023, is `WWX2NW`.

  If you want to read more on adding yourself to the CS511 roster, go to Adding a Course .

- You also need to create a *GitHub repository* where you store your solutions for *coding exercises with* Lean_4.

  To create a GitHub repository, you need to open a GitHub account. Instructions for how to do this are at the following webpages: Set Up a GitHub Account and Create a GitHub Repository.

- Typically, each weekly assignment consists of two parts:

  1. One part includes ***hand exercises***, *i.e.* ***pencil-and-paper exercises***, and
  2. One part includes ***coding exercises*** in Lean_4.

  And each of the two parts will consist of:

  - **2 easy exercises**, and
  - **1 demanding exercise**, which we will call a **problem**,

  for a total of **4 easy exercises** and **2 problems** in each weekly assignment.

- Typeset your solutions with Latex to produce a single '`.pdf`' file containing:

  1. All your solutions for the ***hand exercises***, and
  2. Links to your ***coding exercises***, which are stored in your GitHub repository. (You should insert the links as active, *i.e.* clickable, *hyperlinks* in your '`.pdf`' file.)

  It is the '`.pdf`' file produced with Latex that you will submit in Gradescope.

  You do not need to use any particular format in naming your '`.pdf`' file, because Gradescope will keep track of who is submitting it. Nonetheless, it is nice to use suggestive names in case of a mishap and we need to recover your file. So, here is a possible naming:

  ```
  <your last name>_<your first name>.hw01.pdf
  ```

  For example, for myself, I would call my file '`kfoury_assaf.hw01.pdf`'.

1 By Hand

Exercise 1 [LCS, page 160]: Exercise 2.3.1, do parts (a) and (b) only.

Use $\approx$, instead of $=$, for the formal symbol whose interpretation is equality. In LaTeX, you can typeset $\approx$ with `\approx`. □

Exercise 2 [LCS, page 161]: Exercise 2.3.9, do parts (a) and (d) only. □

PROBLEM 1 Let ψ_1, ψ_2, and ψ_3 be the three axioms of group theory, which are written as first-order wff's on page 11 of **Lecture Slides 20**. Let φ be the wff in the middle of the same page 11 of **Lecture Slides 20**. The wff φ expresses the uniqueness of inverses in groups. Your task is to produce a formal proof, as a natural deduction, of the following judgment:

$$\psi_1, \psi_2, \psi_3 \quad \vdash \quad \varphi$$

Hint: Do Exercises 1 and 2 above before this problem. Also use $\approx$ for the formal symbol whose interpretation is equality, leaving $=$ for equality at the meta-level. □

2 With Lean_4

Exercise 3 From Macbeth's book:

1. Exercise 3.3.4 ,
2. Exercise 3.3.5 ,
3. Exercise 3.3.12.2 . □

Exercise 4 From Macbeth's book:

1. Exercise 3.3.12.3 ,
2. Exercise 3.3.12.6 ,
3. Exercise 4.3.5.2 . □

PROBLEM 2 Prove in Lean 4 the judgment for which you produced a formal proof as a natural deduction in Problem 1 above. We will post an appropriate Lean 4 template for this homework on Piazza, by the end of Friday, October 4 (today). □